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# Modeling inflation expectations in forward-looking interest rate and money growth rules $\stackrel{\text{\tiny{$\widehat{}}}}{\sim}$



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# ABSTRACT

We propose a novel approach that directly embeds rational expectations (RE) into a lowdimensional structural vector autoregression (SVAR) without the need for any mapping to a dynamic stochastic general equilibrium (DSGE) model. Beginning from a fully specified "consensus" structural model, we establish an instrumental variable procedure internal to the SVAR to obtain RE-consistent structural responses to identified monetary policy shocks. Our *RE-SVAR* framework facilitates a comparison across two alternative monetary policy indicators that accommodate long horizons in the formation of inflation expectations in the policy rule. We construct clouds of responses to innovations in the federal funds rate. This suggests that indicator often requires being augmented with more information in standard VAR settings. A money growth rule characterization—with Divisia M4 as a policy indicator—exhibits comparatively larger regions of sensible responses within a low-dimensional textbook model of the economy.

#### 1. Introduction

The Great Financial Crisis (GFC) of 2007 seemed to upend one of the most visible and well-established features of the New Keynesian general equilibrium literature; namely, that the effective federal funds rate stands alone as the single best indicator of the stance of monetary policy.<sup>1</sup> The decade following 2008 opened the door for a reconsideration of money growth rules. Belongia and Ireland (2022) advance a theoretical model that compares a money growth rule to a *Taylor (1993)-type* rule for the federal funds rate

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<sup>&</sup>lt;sup>1</sup> See the definitive work of Bernanke and Blinder (1992), Gertler and Gilchrist (1994), Eichenbaum and Evans (1995), Bernanke and Mihov (1998), and Christiano et al. (1999).

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and demonstrate conclusively that properly specified money growth rules provide useful guidance for monetary policy—particularly during periods when extremely low levels of short-term interest rates can raise concerns about the effectiveness of monetary policy.

But even prior to the GFC, there was extensive evidence that interest rate rules in structural vector autoregression (SVAR) specifications often give rise to what is commonly known as the *price puzzle*. The price puzzle, first coined by Eichenbaum (1992), pertains to the aberrant result in SVAR models when the price level responds in the same direction to innovations in the short-term rate—while the sensible dynamic is that the price level should respond in the opposite direction. This anomaly called into question the validity of SVARs to properly capture the "consensus" macroeconomic model comprising a *Phillips curve* (*PC*), an investment-savings (*IS*) equation, and a monetary policy (*MP*) rule—when the latter is modeled from innovations in the federal funds rate, as motivated by Taylor (1993).

Given this seemingly deficient signal from the short-term interest rate within this low-dimensional system—that is the *textbook* model of the macroeconomy—a large literature set about augmenting the information set spanned by monetary SVARs. This often involves adding more variables to the model.<sup>2</sup> Importantly, while adding variables that better span the Federal Reserve's information set provides empirical expediency, there is no explicit theoretical role for many of these suggested variables in the consensus macroeconomic model. For example, while there may be comparatively wider agreement about the law of motion for inflation within a *PC* structure, no similarly accepted structural equation that governs the Federal Reserve's Greenbook forecasts (for instance) is available in the textbook model of the macroeconomy.<sup>3</sup>

Returning to the post-GFC period, the Federal Reserve engaged in a number of unconventional policies, since 2008, that included the creation of new liquidity facilities and large-scale asset purchases, which swelled bank reserves.<sup>4</sup> The effective federal funds rate could not be indicative of changes in the stance of monetary policy during the protracted seven years it remained at its effective lower bound (ELB). Even after the Federal Reserve's liftoff from ELB in December 2015, the new economic environment limited the monetary policy signal that could be derived from movements in the federal funds rate.<sup>5</sup>

In light of these developments, there is renewed interest in the dynamic effects of monetary policy shocks that might be gleaned from augmenting the information set beyond innovations in the federal funds rate. A popular alternative has been to dramatically increase the dimension of the system by adding a broad number of factors in what Bernanke et al. (2005) called factor-augmented VARs (FAVARs). Boivin et al. (2010) consider a benchmark recursive VAR, as well as a time-varying FAVAR, that allows for changes in regimes, and find the incidence of price puzzles is sensitive to changes in the VAR specification. Given the wide agreement among monetary economists that the Federal Reserve has become more forward-looking in modern times, FAVARs present a trade-off. As high-dimensional models, they incorporate a larger information set that could better approximate the expectations of the Federal Reserve. On the other hand, they constitute a wide departure from the low dimension that characterizes the textbook model.

Moreover, as Batini and Haldane state: "It has long been recognized that economic policy in general, and monetary policy in particular, needs a forward-looking dimension." (Batini and Haldane, 1999, p. 157). In this historical context, SVAR models of monetary policy should address (i) the important role of expectations in a modern period that encompasses different monetary regimes, and (ii) whether the low-dimensional textbook model remains viable in characterizing monetary policy in a more complex post-GFC world. On the first point, the revolutionary work of Lucas (1972) has had lasting implications on macroeconomic modeling. Whereas dynamic stochastic general equilibrium (DSGE) models are essentially founded on rational expectations (RE), SVAR models' connection with RE has typically been more ephemeral. Regarding the second point, an important question to address is whether SVARs can preserve the relevance of a Taylor Rule when accompanied by a *PC*—or aggregate supply (*AS*)—equation, as well as an *IS* equation in a modern sample.

To answer these questions, we propose a novel SVAR approach that directly embeds expectations into a low-dimensional specification of a consensus macroeconomic model, which does not rely on the delayed-reaction assumption from the typical recursive schemes. Our *rational expectations-augmented structural vector autoregression* (RE-SVAR) methodology facilitates the construction of theoretically-consistent clouds of structural impulse response functions (IRFs). We compare an interest rate rule operationalized through movements in the federal funds rate with a money growth rule substantiated by an index-theoretic consistent measure of the money supply, known as Divisia.

We find that—when allowing for long horizons in the formation of inflation expectations—our results show much larger regions of puzzling behavior stemming from our assumed interest rate rules than from our assumed money growth rules. We conclude that the information content from a Divisia monetary aggregate—which encapsulates price and quantity signals from a much larger set of various money markets than what is reflected in the more segmented federal funds market—is richer and more capable of capturing a wider array of monetary shocks in a low-dimensional model. This is consistent with Chen and Valcarcel (2021) who reach a similar conclusion with a vastly different methodological approach.<sup>6</sup>

<sup>&</sup>lt;sup>2</sup> For example: commodity prices, (as in Christiano et al. (1999), among many other since); federal funds futures data, (see Kuttner (2001), Cochrane and Piazzesi (2002), Gürkaynak et al. (2005), Brissimis and Magginas (2006), Gertler and Karadi (2015), among others); or Federal Reserve's Greenbook forecasts (see Stock and Watson (2001), Romer and Romer (2004), Barth and Ramey (2002), and Coibion (2012), among others.).

<sup>&</sup>lt;sup>3</sup> Hanson (2004) and Giordani (2004) leveled a similar indictment for the inclusion of commodity prices in SVARs as "Ad Hoc."

<sup>&</sup>lt;sup>4</sup> A relatively large literature engaged in explaining the effects of these policies on (*i*) transmission mechanisms to interest rates [Krishnamurthy and Vissing-Jorgensen (2011), Gagnon et al. (2011), and D'Amico et al. (2012)]; (*ii*) transmission onto asset markets [Carpenter et al. (2015)]; (*iii*) forward guidance onto expected rates [Bundick and Smith (2020)]; and (*iv*) the transmission onto the balances of various money markets [Chen and Valcarcel (2021)].

<sup>&</sup>lt;sup>5</sup> Smith and Valcarcel (2023) show the new ample reserve environment, along with the gradual manner in which the Federal Reserve undertook its normalization procedures between 2015 and 2019, attenuated the signal associated with Federal Open Market Committee (FOMC) announcements in this period.

<sup>&</sup>lt;sup>6</sup> Chen and Valcarcel (2021) specify a SVAR with a recursive setting and time-varying parameters. Importantly, they do not consider a consensus low-dimensional model. Instead, they augment their specification with a large number of monetary factors.

The rest of the paper is organized as follows. Section 2 establishes a general methodology for directly embedding expectations in a low-dimensional SVAR without the need to assume a delayed reaction to policy shocks or a specific mapping to a reduced-form VAR based on exclusion restrictions. Section 3 provides a theoretical illustration that motivates a comparison between interest rate rules and money growth rules. Section 4 describes the application of our method for our two candidate monetary policy indicators: the federal funds rate and the broad Divisia M4 monetary aggregate. Section 5 presents results from our two specifications and reports the percentages of puzzling responses obtained from the interest rate specification as well as the model with money growth as the indicator. Section 6 delves down into an analysis of expectations formation by allowing for various horizons of inflation expectations in the respective policy rules from our two competing rules. Section 7 outlines an extension to a four-variable RE-SVAR, while emphasizing that our procedure is not "modular" so it does not admit variables that can be simply incorporated absent a theoretical construct. Section 8 concludes.

#### 2. Embedding expectations directly into a low-dimensional structural VAR

The assumption that monetary policy shocks affect output and prices with a lag can be formulated in a SVAR setting by ordering the policy indicator variable after economic activity variables so that a Cholesky decomposition recovers the SVAR forecast error of the policy equation as the structural shock. However, the incidence of price puzzles is particularly common in recursive SVARs identified with a delayed reaction of the economy to monetary policy shocks. Recursive SVARs are largely constructed as backward-looking econometric mechanisms; thus—under certain settings—they may fail to properly capture the role of expectations. As mentioned above, these problems have been broached by enlarging the information set spanned by the SVAR.<sup>7</sup> Whereas the DSGE modeling framework was built from the ground up with the RE paradigm, recursive SVARs are less well-equipped to directly accommodate RE.

Partly in an effort to address the backward-lookingness of VARs, a growing literature has engaged in looking for conditions under which the state-space framework that underwrites many DSGEs can be mapped into a VAR—or a VAR with moving average components (VARMA)—representation (see Fernández-Villaverde et al. (2007), Ravenna (2007), Morris (2016), Morris (2017), and Martínez-García (2020) as a non-exhaustive list.) However, this mapping approach never scales from a VAR with expectations into a DSGE. Instead, these methodologies largely begin from a higher-dimensional DSGE to a lower-dimensional VAR *representation* that would not accommodate RE unless: a moving average process were also appended, lags were truncated, or some algebraic mechanism for dimension reductionality were advanced.<sup>8</sup>

#### 2.1. Trade-offs in our approach

A prototypical *n*-variable SVAR begins with a reduced-form estimation of the data from which residuals are extracted. Subsequently, the researcher appeals to economic theory, some market mechanism(s), or empirical regularities about the data for help in pinning down a mapping matrix connecting the statistical residuals to disturbances that can be interpreted with some semblance of economic meaning. Dynamics that are imposed by construction are often de-emphasized in favor of results that are driven by the combination of the researcher's identifying restrictions and the data.<sup>9</sup> The validity of a reduced-form VAR is rarely called into question for assessing the suitability of a model of economic inference. Rather, the scrutiny typically rests on the restriction strategy itself. Thus, if the restriction scheme is doubted, one may call into question the mapping of the innovations to the structural shocks. Conversely, the efficacy of our approach must rest solely on the suitability of the theoretical construct itself. For example, if one did not believe in a consensus *AS-IS-MP* model, or if one eschewed the particular characterization of the Taylor Rule we employ, then the approach would be a non-starter.

We propose an identification strategy founded on RE that is *directly* derived—rather than loosely mapped—from a theoretical construct. Our VAR begins with structural shock identification out of the gate. Of course, we are not the first to discipline a VAR with a fully written theoretical construct. Important forerunners include Keating (1992) for monetary policy and Blanchard and Perotti (2002) for fiscal policy, as specified in Kilian and Lütkepohl (2017), and citations therein. What distinguishes our method is that we incorporate forward-looking expectation terms in our theoretical construct—whereas the older literature considered more static structural relationships. We model forward-looking behavior directly within the VAR, rendering our approach to be expectational and structural from the outset.

Quantifying forward-lookingness is a difficult proposition. Our approach allows us to broach this issue, in an imperfect way, by iterating over possible horizons for inflation  $(h_{\pi})$  and output  $(h_{y})$  in our assumed interest rate and money growth rules.<sup>10</sup> An additional advantage of our approach for identifying RE-consistent structural shocks is that our RE-SVAR method does not require adding dynamics from unobservables to the information set, as is common practice in DSGEs. Thus, the low dimensionality of the model can be maintained so as to provide a more direct correspondence with the textbook theoretical macroeconomic model.

 $<sup>^{7}</sup>$  Christiano et al. (1999) expand their block-recursive SVAR with information on commodity prices, which—for samples extending through the mid 1990s seemed to put to bed this price-puzzling behavior. This conclusion of a *price puzzle fix* with added information on commodity prices proved to be premature. Ramey (2016) documents a large set of empirical investigations that find this puzzling behavior both in historical and modern samples. Keating et al. (2019) augment their block-recursive SVAR with information from monetary aggregates with strong evidence of a price puzzle resolution in samples that predate or extend beyond GFC.

<sup>&</sup>lt;sup>8</sup> As the literature cited above shows, not every DSGE has a viable VAR representation. In fact, the more tractable representation is VARMA, with VAR representations representing a minute portion of the classes of DSGE models available in the literature.

<sup>&</sup>lt;sup>9</sup> Responsible researchers will not want to conclude their assumptions; see Uhlig (2005).

<sup>&</sup>lt;sup>10</sup> Batini and Haldane argued that rules that "allow flexibility over both the forecast horizon and the feedback parameter—both of which affect output stabilization" were a close analog to "Svensson's flexible inflation-forecast-targeting rule" (Batini and Haldane, 1999, p. 160).

The small dimension of our model may be advantageous but it also presents the disadvantage that our approach is not "modular." While additional variables may be easily appended with facility to the prototypical *n*-variable SVAR, our methodology requires a fully specified structural equation for each additional variable incorporated into the system. Specifically, if we wanted to add a fourth variable to our three-equation consensus model, we would need an explicit equation for it. As examples mentioned earlier, commodity prices<sup>11</sup> or Federal Reserve's Greenbook forecasts<sup>12</sup> may provide statistical convenience in SVARs, yet no wide acceptance on a theoretical construct for those variables currently exists. We investigate in greater detail the possibility of expanding the dimension of our system in Section 7 below.

#### 2.2. A non-recursive RE-augmented SVAR framework

This section outlines a three-equation-system consistent with the consensus model of the economy. Our identification strategy pins down shocks to the monetary policy indicator with a method that does not rely on a Cholesky decomposition premised on the paradigm of a delayed reaction to policy shocks. Instead, our proposed technique orders the policy function first in the system. We leverage a simple application of the RE methodology to find a relationship between the reduced-form innovations in the policy equation and the associated structural shock. Once we have isolated the monetary policy shock, we can obtain dynamic effects for output, inflation, and short-term nominal rates or growth rates in money balances.<sup>13</sup>

Our proposed methodology is substantially different in numerous ways from the standard VAR approach. First, our model does not impose a delayed reaction through an exclusion restriction built on a Cholesky ordering. Second, we propose a novel instrumental variable methodology for modeling RE directly in a structural VAR setting. Third, we circumvent the more standard methods of estimating a reduced-form VAR first, and then advocating a mapping to the structural shock of interest. In a way congruent with the persuasive descriptions in Arias et al. (2019), the approach in our paper (as in many other VAR applications) must deal with the joint problem of VAR modeling: *statistical uncertainty* and *model uncertainty*. In essence, our construct takes model uncertainty off the table. We begin with the assumption that the structural model we generate responses from is appropriate. Contingent on this theoretical structure, we generate impulse responses by imposing values on some of the structural parameters in what becomes a pseudo-calibration exercise.

Each response we report will be a separate realization of a structural VAR. This allows us to produce a "cloud" of structural responses, each of which is unique for a given calibration of the structural parameters.<sup>14</sup> Consider the following structural VAR:

$$A_0 x_t = \sum_{i=1}^{\nu} A_i x_{t-i} + \varepsilon_t \tag{1}$$

where *p* is the number of lags and where  $E(\epsilon_t \epsilon'_t)$  is a diagonal covariance matrix of the structural shocks. The structural model consists of three equations:

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+h_\pi} + \phi_y \mathbb{E}_t y_{t+h_y} + A^{MP}(L) x_{t-1} + \varepsilon_t^{MP}$$
<sup>(2)</sup>

$$y_{t} = \mathbb{E}_{t} y_{t+1} - \alpha_{1} (i_{t} - \mathbb{E}_{t} \pi_{t+1}) + A^{IS}(L) x_{t-1} + \varepsilon_{t}^{IS}$$
(3)

$$\pi_{t} = \alpha_{2} \mathbb{E}_{t} \pi_{t+1} + \alpha_{3} y_{t} + A^{AS}(L) x_{t-1} + \varepsilon_{t}^{AS}$$
(4)

where  $h_{\pi}$  and  $h_{y}$  refer to the number of forward-looking horizons in the policy reaction function to inflation and output, respectively;  $\phi_{\pi}$ ,  $\phi_{y}$ ,  $\alpha_{1}$ ,  $\alpha_{2}$ , and  $\alpha_{3}$  are structural parameters in the policy rule, *IS* and *AS* equations; and  $A^{MP}$ ,  $A^{IS}$ , and  $A^{AS}$  are the autoregressive matrices containing the remaining structural parameters.

Our specification of monetary policy may be described as an interest feedback rule or a money growth rule. Therefore, in our application we will alternate between equation (2) and  $\mu_t = \tilde{\phi}_{\pi} \mathbb{E}_t \pi_{t+h_{\pi}} + \tilde{\phi}_y \mathbb{E}_t y_{t+h_y} + \tilde{A}^{MP}(L) x_{t-1} - \tilde{\epsilon}_t^{MP}$  where  $\mu_t$  denotes the growth rate of Divisia M4. Our use of tildes on these parameters is to highlight these are analogous to, but separate from, those in (2).

Let the reduced-form VAR in companion form be given by  $X_t = \Psi X_{t-1} + De_t$ , where  $X_t = [x'_t, x'_{t-1}, ..., x'_{t-p-1}]'$  is  $np \times 1$ ,  $D = (I_n, 0_n, ..., 0_n)'$  is  $np \times n$ , and  $\Psi$  is the  $np \times np$  companion-form autoregressive matrix containing the  $n \times n$  autoregressive matrices  $(\psi_1, \psi_2, ..., \psi_n)$  along with  $n \times n$  identity  $(I_n)$  and zero  $(0_n)$  matrices. Define a selection vector  $S_v$  such that

$$S_{\nu}X_{t} = v_{t} \tag{5}$$

where  $v_t$  is some component of  $X_t$  (such as  $i_t$ ,  $\mu_t$ ,  $y_t$ , or  $\pi_t$ , in our model above). We can forecast, or rationally expect, the movement of a given variable using the VAR:

$$\mathbb{E}_t v_{t+j} = \mathbb{E}_t S_v X_{t+j} = S_v \Psi^j X_t \tag{6}$$

<sup>&</sup>lt;sup>11</sup> See, e.g. Christiano et al. (1999).

<sup>&</sup>lt;sup>12</sup> See, e.g. Stock and Watson (2001).

 $<sup>^{13}</sup>$  Our methodology describes a way to identify monetary policy (*MP*) shocks. Some of the information from recovering those shocks can be used to subsequently identify *IS* shocks and *AS* shock. We show this in Section 7.

<sup>&</sup>lt;sup>14</sup> This voids the need for the construction of confidence bounds. While ours is a frequentist approach, Inoue and Kilian (2022) argue against constructing confidence bounds around median responses in Bayesian VARs.

Equation (6) follows from the fact that, via recursive substitution, it can be shown that  $X_{i+j} = \Psi^j X_i + \Psi^{j-1} e_{i-1} + \Psi^{j-2} e_{i+2} + \dots + e_{i+j}$ . Given equation (6), along with the companion form of the reduced-form VAR, and the assumption that  $\mathbb{E}_t e_{i+i} = 0$  for i > 0, the following equation holds:

$$\mathbb{E}_{t-1}v_{t+i} = \mathbb{E}_{t-1}S_v X_{t+i} = S_v \Psi^j \mathbb{E}_{t-1}X_t = S_v \Psi^j \Psi X_{t-1}$$

it is, then, straightforward to show the expectation, or forecast, revision is given by:

$$\mathbb{E}_{t}v_{t+j} - \mathbb{E}_{t-1}v_{t+j} = S_{v}\Psi^{j}X_{t} - S_{v}\Psi^{j}\Psi X_{t-1} = S_{v}\Psi^{j}(X_{t} - \Psi X_{t-1}) = S_{v}\Psi^{j}De_{t}$$
<sup>(7)</sup>

We use this general result to identify the structural monetary policy shock from the VAR model defined above. Taking a stand on the coefficients in equation (2), we can derive the monetary policy shocks by expressing them as a linear combination of the reduced form residuals without estimating any structural parameters. The policy feedback rule is given by:

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+h_\pi} + \phi_y \mathbb{E}_t y_{t+h_y} + A^{MP}(L) x_{t-1} + \varepsilon_t^{MP}$$
(8)

or

$$\mu_t = \tilde{\phi}_{\pi} \mathbb{E}_t \pi_{t+h_{\pi}} + \tilde{\phi}_y \mathbb{E}_t y_{t+h_y} + \tilde{A}^{MP}(L) x_{t-1} - \tilde{\varepsilon}_t^{MP} \tag{9}$$

Rewriting equation (8) in expectational difference form and subtracting the expectation of the policy rule at time t - 1 from (8) yields:

$$i_t - \mathbb{E}_{t-1}i_t = \phi_\pi \left( \mathbb{E}_t \pi_{t+h_\pi} - \mathbb{E}_{t-1}\pi_{t+h_\pi} \right) + \phi_y \left( \mathbb{E}_t y_{t+h_y} - \mathbb{E}_{t-1}y_{t+h_y} \right) + \varepsilon_t^{MP}$$
(10)

We can then use equation (7) to find expressions in the expectational difference in the above equation. Solving for the structural shock in the interest rate feedback rule obtains the following:

$$\varepsilon_t^{MP} = e_t^i - (\phi_\pi S_\pi \Psi^{h_\pi} D e_t)' - (\phi_y S_y \Psi^{h_y} D e_t)' \tag{11}$$

Analogously, we can repeat the analysis—starting from (9) instead of (8)—and obtain the following structural shock for the money growth feedback rule:

$$\tilde{\epsilon}_{t}^{MP} = e_{t}^{\mu} - (\tilde{\phi}_{\pi} S_{\pi} \Psi^{h_{\pi}} D e_{t})' - (\tilde{\phi}_{y} S_{y} \Psi^{h_{y}} D e_{t})'$$
(12)

In the following sections, we focus our attention exclusively on identifying monetary policy shocks within our RE framework. However, this mechanism can be leveraged in a sequential fashion for a full recovery of all the structural shocks in the system. In Section 7, we expand our analysis to show how to recover structural *IS* and *AS* shocks from our RE-SVAR.

#### 3. Interest rate and money growth rules

The landmark paper by Christiano et al. (1999) undertook an exhaustive investigation of the most widely used methods for identifying monetary policy shocks at the time. They compared various specifications of (block-)recursive SVARs. Imposing a delayed reaction of economic activity to monetary shocks, they found that innovations in the federal funds rate—when augmented with commodity prices to better account for the forward-looking nature of the information available to the central bank—elicited responses generally free from the price puzzle.

In a more recent paper, Keating et al. (2019) revisit the delayed reaction assumption and find that shocks consistent with a money growth rule exert a similar qualitative influence to federal funds rate shocks for the pre-GFC period. Importantly, federal funds rate specifications exhibit price puzzles (a price increase follows a monetary contraction) for samples that extend through the ELB period—whereas the money growth rules continue to work well beyond the GFC period.

The delayed reaction can also be accommodated in a New Keynesian general equilibrium setting. Belongia and Ireland (2022) show results from such a model that allows for protracted delayed reactions operationalized as forward guidance.<sup>15</sup> Keating et al. (2019) also motivate their identification with a New Keynesian model, which can be solved with (rational) agents' expectations. Importantly, they subsequently turn to a block-recursive SVAR for most of their empirical conclusions, which is not equipped to directly incorporate expectations.

## 3.1. A consensus new Keynesian model

Keating et al. (2019) describe the following sticky price model, which contains some forward-lookingness in the inflation and output variables.

$$y_{t} = \frac{1}{1+\bar{c}} \mathbb{E}_{t} y_{t+1} + \frac{\bar{c}}{1+\bar{c}} y_{t-1} - \frac{1-\bar{c}}{1+\bar{c}} \left( i_{t} - \mathbb{E}_{t} \pi_{t+1} \right)$$
(13)

<sup>&</sup>lt;sup>15</sup> See Kulish et al. (2017).

Z. Chen and V.J. Valcarcel

Journal of Economic Dynamics and Control 170 (2025) 104999

$$m_{t} = \frac{1 + \chi(1 - \bar{c})}{(1 + \chi)(1 - \bar{c})} y_{t} - \frac{\bar{c}}{(1 + \chi)(1 - \bar{c})} y_{t-1} - \eta i_{t}$$

$$\pi_{t} = \pi_{t-1} + \kappa \left(\frac{1}{1 + \bar{c}} y_{t} - \frac{\bar{c}}{1 + \bar{c}} y_{t-1}\right) + \beta \mathbb{E}_{t} (\pi_{t+1} - \pi_{t})$$
(14)
(15)

In the above equations,  $y_t$ ,  $\pi_t$ , and  $m_t \equiv log\left(\frac{M_t}{P_t}\right)$  denote the log of output, the inflation rate, and the log of real money balances, respectively, and  $i_t$  is the nominal interest rate on one-period bonds. Equation (13) is the household's Euler equation where  $0 \le \bar{c} \le 1$  modulates the degree of external habit that relates the habit-adjusted rate of output growth to the real return on a one-period bond. Equation (14) is the household's money demand equation in which  $\eta > 0$  is the interest semi-elasticity pinned down for a given value of  $\chi > 0$ , which quantifies the elasticity of the amount of time the household must allocate to shopping with respect to the velocity of money. Equation (15) outlines firms' decision to occasionally re-optimize their own price. The parameter  $0 < \beta < 1$  is the household's discount factor. The parameter  $\kappa = (1 - \alpha)(1 - \beta\alpha)/\alpha$  is the slope of this *PC*, in which  $1 - \alpha$  is the probability that any given firm is able to re-optimize its price in the current period. Those firms unable to revise their prices will index their current prices to last period's inflation rate.<sup>16</sup>

One deviation of Keating et al. (2019) from the standard models found in Woodford (2003) and Gali (2008) is that—instead of using an unweighted monetary aggregator—a broad measure of nominal money balances is specified as a *constant elasticity of substitution* (CES) aggregate of non-interest-bearing ( $N_t$ ) and interest-bearing ( $D_t$ ) assets, first advanced by Belongia and Ireland (2014), as follows:

$$M_t = \left[ v^{\frac{1}{\omega}} N_t^{\frac{\omega-1}{\omega}} + (1-v)^{\frac{1}{\omega}} D_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}},\tag{16}$$

where  $0 \le v \le 1$  calibrates the weight placed on each asset in the CES aggregate, and  $\omega \ge 0$  is the elasticity of substitution between each asset.

Any unweighted aggregate that simply sums the nominal value of non-interest-bearing- and interest-bearing assets together would fail to capture substitution effects among these assets.<sup>17</sup> Keating et al. (2019) show that using unweighted monetary aggregates (such as the Federal Reserve's current measure of M2) results in an endogenous, time-varying gap between the unweighted aggregate and  $M_t$  even if v and  $\omega$  remained constant. Conversely, an expenditure-weighted Divisia monetary aggregate tracks  $M_t$  perfectly up to second order (Diewert, 1976). This proves that in a log-linear model as the one advanced above, the Divisia monetary aggregate will equal  $M_t$ .<sup>18</sup>

The empirical reasons for rejecting simple-sum monetary aggregates in favor of weighted aggregates are powerful. However, in a theoretical setting, it is tractable to accommodate a monetary aggregate, whether it is weighted or unweighted. While the micro-foundations of monetary aggregation are more clearly spelled out with this specification, it can be shown that combining equations (14) and (16) above with the proper log linearization yields a money demand equation of a form that is similar to those found in textbook New Keynesian models, which generally consider an unweighted aggregate for  $M_i$ . We eschew using unweighted measures of M1 or M2, in favor of a weighted (Divisia) measure on the empirical grounds advanced in Barnett (1980), Belongia and Ireland (2014), Belongia and Ireland (2018), Keating et al. (2019), and Chen and Valcarcel (2021), among others.

The model can be closed with a specification of monetary policy, which Keating et al. (2019) describe by an interest rate feedback  $rule^{19}$ 

$$i_t = \rho i_{t-1} + (1 - \rho)(\phi_\pi \pi_t + \phi_y(y_t - y_{t-1})) + \varepsilon_t^{mp},$$
(17)

where  $\varepsilon_t^{mp}$  is an i.i.d. monetary policy shock. Similarly, monetary policy can be described by a nominal money growth rule

$$\mu_t = \bar{\rho}\mu_{t-1} + (1 - \bar{\rho})(\bar{\phi}_{\pi}\pi_t + \bar{\phi}_y(y_t - y_{t-1})) - \bar{\varepsilon}_t^{mp}, \tag{18}$$

where  $\mu_t = m_t - m_{t-1} + \pi_t$ .

## 3.2. Comparing the effects of monetary policy shocks under interest rate rules and money growth rules

Over the years, a large literature on interest rate rules has offered a range of evidence in support of a high degree of persistence in estimated interest rate reaction functions. Clarida et al. (2000) find evidence of inertia in monetary policy over the 1960-1979 sample as well as the 1979-1996 sample. Similarly, using Federal Reserve's Greenbook forecast data, Orphanides (2001) finds substantial

<sup>&</sup>lt;sup>16</sup> Christiano et al. (2005) show the assumption that prices are indexed to lagged inflation is important for generating empirically plausible dynamics to a monetary policy shock. They omit any non-monetary shocks and focus their attention exclusively on the response to monetary policy shocks. However, the consensus on backward price indexation has been criticized for its lack of rigorous micro-foundations by Cogley and Sbordone (2008), and for its inconsistency with documented empirical evidence on the frequency of price adjustments [Bils and Klenow (2004) and Chari et al. (2009)].

<sup>&</sup>lt;sup>17</sup> Only if the assets were perfect substitutes (i.e.  $\omega$  goes to infinity) would an unweighted aggregate be equal to  $M_i$ .

<sup>&</sup>lt;sup>18</sup> The empirical results in Keating et al. (2019) allow for two conclusions regarding simple-sum monetary aggregates. First, the response of an unweighted aggregate to a monetary policy shock is likely to be biased upwards. The severity of this bias will vary widely when the substitutability among some of the underlying assets shifts over time. Second, using unweighted aggregates as the policy indicator could easily result in substantial liquidity puzzles.

<sup>&</sup>lt;sup>19</sup> This rule includes output growth as opposed to the output gap for better alignment with the information set available to the Federal Reserve to be spanned by our RE-SVAR model later in the paper.

#### Z. Chen and V.J. Valcarcel

persistence in interest rate changes over the 1987-1994 period. Coibion and Gorodnichenko (2012) also present evidence that target interest rate changes are persistent because of an explicit desire to gradually adjust rates over the 1987-2006 sample.

As this literature documents, interest rate inertia has been an important feature of the U.S. economy since the 1960s. Keating et al. (2019) show that interest rate persistence facilitates the approximation of interest rate rules with money growth rules in a sample that ends at the GFC. However, it is the coefficients to inflation and output growth that matter from the perspective of stabilization. To illustrate this point, combining the money growth rule in equation (18) with the money demand function in equation (14) (and assuming that  $\bar{c} = 0$ ) yields the following implied interest rate rule (for  $i_r$ ) under a money growth instrument:

$$i_{t} = i_{t-1} + (1/\eta)[(1 - \bar{\phi}_{\pi})\pi_{t} + (1 - \bar{\phi}_{y})(y_{t} - y_{t-1}) + \bar{\varepsilon}_{t}^{mp}].$$
<sup>(19)</sup>

So long as  $\bar{\phi}_{\pi} < 1$ , and  $\bar{\phi}_{y} < 1$ , the central bank actively stabilizes the economy where non-negative values of  $\bar{\phi}_{\pi}$  and  $\bar{\phi}_{y}$  that are less than one simply moderate a counter-cyclical monetary policy response. On the other hand, absent persistence in interest rates ( $\rho = 0$ ) a Taylor-type interest rate can be obtained as follows:

$$i_{t} = \phi_{\pi} \pi_{t} + \phi_{y} (y_{t} - y_{t-1}) + \varepsilon_{t}^{mp}$$
<sup>(20)</sup>

This theoretical formulation is premised on the basis that the interest rate is non-zero—an assumption that does not bind in the December 2008–December 2015 period. However, a money growth rule is not bound to a zero constraint. Keating et al. (2019) exploit this fact by comparing shocks from a money growth rule with a Taylor-type rule when the interest rate is replaced with a shadow federal funds rate, which is not bound at zero between 2008 and 2015. These authors draw from earlier papers that impose a delayed reaction of economic activity variables to policy shocks (see Eichenbaum (1992), Gordon and Leeper (1994) and Christiano et al. (1999), among others).

These VAR models that rely on a delayed reaction to policy shocks are inherently "backward-looking." And the theoretical construct of Section 3.1 is similarly backward-looking when it comes to the interest or money growth rule characterizations. Our RE-SVAR methodology described in Section 2.2 does not rely on a Cholesky ordering and accommodates forward-looking behavior in the policy reaction function. In the next section we apply our methodology to compare responses to the two different monetary policy indicators we consider.

# 4. An application for two candidate monetary policy indicators

Our VAR specification consists of three variables stacked in  $x_t = [i_t, y_t, \pi_t]'$ , where  $i_t$  is the monthly average of the federal funds rate, which is replaced by the Wu and Xia (2016) shadow effective federal funds rate for the ELB period,  $y_t$  is the natural log difference of industrial production, and  $\pi_t$  is the inflation rate constructed from the monthly annualized consumer price index (CPI).

We then replace the short-term interest rate  $(i_t)$  with the growth rate of a Divisia aggregate  $(DM_t)$ . Divisia monetary indexes are provided by the Center for Financial Stability (CFS) in the U.S. We focus on Divisia M4 (DM4) as the broadest standard aggregate CFS produces.<sup>20</sup> While CFS also provides narrower Divisia aggregates, broader monetary aggregates are less likely to arbitrarily discard substitution information across monetary assets.<sup>21</sup> Keating et al. (2019) posit a similar argument for broader Divisia aggregates as more informative indicators of U.S. monetary policy.<sup>22</sup>

We report results from 377 observations of a monthly sample encompassing October 1988—February 2020.<sup>23</sup> While our main model investigates the modern (1988-2020) sample, we also considered two other samples. A historical sample spanning January 1967—February 2020, and a post-ELB period spanning December 2008—February 2020. We also explored specifications, across these three samples, that replaced headline CPI with the personal consumption expenditure (PCE) index. We outline results of this exploration in Section 5 below.<sup>24</sup>

Equation (11) shows a way to construct a series of structural monetary policy shocks ( $\varepsilon_t^{MP}$ ). Rather than estimating the structural parameters by *ordinary least squares* (OLS), we opt for the following approach. Given a time series construction of  $\varepsilon_t^{MP}$ , as described in the previous section—along with given values for  $\phi_{\pi}$ ,  $\phi_y$ ,  $h_{\pi}$ , and  $h_y$ —we can compute a unique realization of the responses of variables in  $x_t$  to shocks in  $\varepsilon_t^{MP}$ . Throughout the analysis, we consider an expansionary policy shock—first with a standard deviation reduction in the Wu and Xia (2016) shadow rate, and later with a standard deviation increase in the growth rate of *DM*4.

<sup>&</sup>lt;sup>20</sup> CFS also produces and disseminates credit-card-augmented Divisia indexes.

<sup>&</sup>lt;sup>21</sup> Kelly et al. (2011) find stronger liquidity effects using broader Divisia aggregates.

 $<sup>^{22}</sup>$  The importance of the proper capture of substitution effects with broad Divisia aggregates—like the ones CFS disseminates—is also advanced by Chen and Valcarcel (2021), Belongia and Ireland (2022), and Chen and Valcarcel (forthcoming). Nevertheless, we find our results are robust to replacing *DM*4 with the narrower Divisia M2 (*DM*2).

 $<sup>^{23}</sup>$  In a previous version of this paper, we investigated a sample that extended through April 2022 spanning the COVID shock and the post-pandemic inflationary period that followed. Schorfheide and Song (2021) suggest models that include economic activity data post-2020 may severely distort estimates. Though scientific caution suggests we stop our sample before the onset of this monumental disruption, our conclusions from the longer sample remained unchanged to those learned from our results here. IRFs for the longer sample that extends beyond 2020 are available upon request.

<sup>&</sup>lt;sup>24</sup> We also conducted a quarterly estimation with the PCE inflation rate and the log of real GDP as a measure of output. Giordani (2004) suggests that replacing the standard real GDP measure with a GDP gap measure in quarterly VARs may help ameliorate the incidence of price puzzles. Thus, we also estimated a specification with the headline Bureau of Economic Analysis (BEA) measure of real GDP relative to the Congressional Budget Office (CBO) measure of real potential output. While the quantifications of the puzzling response regions change, by and large, our conclusions from the quarterly model remain robust to our monthly specifications. Those results are available upon request.

#### Z. Chen and V.J. Valcarcel

We proceed as follows. We produce a response for each variable of interest to an  $\varepsilon_t^{MP}$  shock by imposing a value for  $\phi_{\pi}$ ,  $\phi_{y}$ ,  $h_{\pi}$ , and  $h_y$ . We record the response and repeat the analysis. We iterate over a grid search of values of these parameters. We let the  $\phi_{\pi}$  coefficient cycle between values of zero and four in 1/15 increments. We pose a similar treatment on the other coefficient in the interest rate rule, with a different response for each  $\phi_y = 0,0.067,0.133,0.2,\ldots,3.8,3.867,4$ . This grid search is motivated by robust evidence in the empirical literature of structural change in the Federal Reserve's systematic response to economic fluctuations. Coibion et al. (2012) conduct a similar search over a range of values for  $\phi_{\pi}$  but in the context of welfare gains instead of puzzle detection.

As we mentioned above, quantifying the degree of forward-lookingness in the central bank's policy rule is a difficult proposition at best. We get at this issue in an imperfect way by also letting  $h_{\pi}$  take on values between zero and 12 months, as well as iterating over  $h_{\nu}$  to take on values between zero and five months ahead.

We produce IRFs to  $\varepsilon_t^{MP}$  shocks from a total of **241,865 different structural VAR specifications** (comprising a combination of 61 possible values of  $\phi_x$ , with another 61 possible values for  $\phi_y$ , and 13 potential horizons  $h_{\pi} = 0, ..., 12$  for the inflation expectations and five potential horizons for output expectations.<sup>25</sup> We, then, replace the Wu and Xia (2016) shadow effective federal funds rate (ordered first) in the VAR with *DM*4 and generate another 241,865 responses.<sup>26</sup> All these 241,865 permutations of parameters yield clouds of structural IRFs with a few combinations rendering outsize shapes of responses, which dominate the scaling, but in all cases constitute a minute proportion. Dominated by these infrequent inordinately large responses, there are often thousands upon thousands of responses (including the median responses of each set) that simply show to be nearly superimposed on each other. We tally up the incidence of puzzling responses with the following arbitrary heuristic:

Following an expansionary shock to the indicator variable, we count as a puzzle any output or inflation response with a negative value at any time within the first year following the shock.

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We report these tallies as follows. Any response of industrial production that is negative at any, or all, horizons between impact and the 12-month post-shock, we designate an output puzzle. Similarly, we keep a separate count for the incidence of inflation puzzles. There may often be realizations that show both of these puzzling responses simultaneously. Therefore, we also report the sets of *surviving* responses that show neither puzzle. We call these the "*no (joint) puzzles responses*," which exclude the incidence of either or both puzzles according to our criterion described above.

#### 5. Searching for sensible responses: two monetary policy indicators

Fig. 1 corresponds to the Wu and Xia (2016) shadow rate specification and it contains six charts organized in two columns. The top charts in both columns show the responses of the monetary policy indicator to a one-standard-deviation expansionary shock to itself.<sup>27</sup> Beginning with the left column, each line in the next chart down (the middle row left column) shows a puzzling response— containing negative values at any time during the first year—of industrial production to an expansionary monetary policy shock. This chart includes a count of these puzzling responses expressed as a percent of the 241,865 total number of specifications. The bottom chart on the left column repeats the analysis for CPI inflation reporting the corresponding percentage of inflation puzzles—the share of responses that contain a negative value at any time during the first year post shock.

We now describe the right column. Each blue line in the middle chart shows the industrial production response for a given monetary policy shock specification that did not incur a puzzle in either variable. The label on the y-axis counts the number of these blue lines in this cloud, which corresponds to the responses that *"survive"* any puzzling behavior as we define it above. The bottom chart repeats the analysis for CPI inflation.

Fig. 1 shows that the Wu and Xia (2016) shadow rate gives rise to a large set of puzzling responses for our sample, with 98.68% of output puzzles and 99.13% of inflation puzzles. Only 2,109 responses out of a possible 241,865 survive our criterion of non-negative responses of industrial production and CPI inflation within the first year.

Fig. 2 shows the analogous responses for the DM4 specification as the monetary policy indicator. These results provide stark contrast to those of the Wu and Xia (2016) shadow rate in Fig. 1. There is a far smaller incidence of puzzles for this second set of VARs. The number of surviving responses to the same criterion here increases dramatically. Whereas the interest rate specification yielded 2,109 sensible (no-joint-puzzle) responses, the Divisia specification yields 231,825 surviving responses, out of the total 241,865 possible combinations—indicating a 95.85% of sensible responses. The top chart on the left column of Fig. 2 shows the cloud of 241,865 responses of DM4 to its own expansionary shock. At 4.02%, the middle chart in the left column shows a minimal incidence of industrial production puzzles. Similarly, the bottom panel of Fig. 2 shows a small percent of CPI inflation puzzles—4.13% of the total 241,865.<sup>28</sup>

<sup>&</sup>lt;sup>25</sup> Our focus on inflation expectations leads us to consider longer horizons for  $h_{\pi}$ . We impose a smaller range of  $h_{y}$  horizons only for computational tractability.

<sup>&</sup>lt;sup>26</sup> AIC selected eight lags for the shadow rate specification. For comparability we set the lag length to eight across all specifications.

<sup>&</sup>lt;sup>27</sup> Some of the abnormally large responses dominate the scaling, creating the incorrect impression that the median response is zero. Therefore, we do not report the median responses. We follow this practice along the lines argued by Inoue and Kilian (2022) who—from a Bayesian perspective—favor reporting clouds of responses to denote a credible set.

<sup>&</sup>lt;sup>28</sup> It is worth mentioning that extending the horizon of our *correct-sign-heuristic* beyond 12 months (say, for 24 months), would have no impact on the percent of sensible inflation responses—and it would minimally shrink the sensible set of industrial production responses—from the Divisia specification. Conversely, extending our heuristic to 24 months would eliminate the surviving 2,109 sensible responses to virtually zero from the interest rate specification.

Journal of Economic Dynamics and Control 170 (2025) 104999



Fig. 1. Responses to expansionary monetary policy shocks across 241,865 realizations: *Federal Funds Rate* specification of a pseudo-calibrated structural VAR. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Obtaining over 245K structural IRFs from our fully specified model in (2)–(4) is computationally expensive. Extending some facets of our exploration requires some trade-offs in others. First, we want to consider alternative measures to the headline CPI measure we study, such as the PCE index. More important, we want to see how sensitive our conclusions are to alternative samples. A larger search over variables and samples, necessitates that we conduct a coarser grid search in the values of the hyper-parameters.

In order to facilitate the identification of  $\varepsilon_t^{MP}$  shocks under a wider exploration of variables and samples, we impose a less granular grid search by reducing the 1/15 increments in our main model—described in Fig. 1 and Fig. 2—to steps of 1/4 within our cycles of parameters. We let the  $\phi_{\pi}$  and  $\phi_y$  coefficients in equation (11) for the interest rate—or  $\phi_{\pi}$  and  $\phi_y$  coefficients in equation (27) for money growth—cycle between values of zero and five in 1/4 increments, and we let  $h_{\pi} = 0, ..., 18$  and  $h_y = 0, ..., 2$  for horizons in the inflation and output expectations, respectively. We re-estimate various specifications and produce impulse responses to  $\varepsilon_t^{MP}$  shocks from a total of **25,137 different structural VAR specifications**, which is an order of magnitude lower than our main model.<sup>29</sup>

We re-estimate our model in a modern sample (October 1988—February 2020) within this less granular grid search. We then turn to a historical examination by extending our sample back to January 1967, when our Divisia measures first become available. Finally, we focus on a post-GFC sample spanning December 2008—February 2020. We then, repeat all our analysis by replacing our headline CPI measure with the PCE index. Moreover, we also replace our DM4 money measure with the narrower DM2 monetary aggregate. We obtain 25,137 IRFs from each sample and variable rotation and record the percentages of inflation and output puzzles from each model. Table 1 summarizes the puzzle percentages for each specification.<sup>30</sup>

<sup>&</sup>lt;sup>29</sup> The resulting smaller combination—of 21 possible values of  $\phi_x$ , with another 21 possible values for  $\phi_y$ , with 19 and three potential horizons for inflation and output, respectively—makes a wider search of specifications tractable.

<sup>&</sup>lt;sup>30</sup> To save space, we do not report the IRFs but we can make them available upon request.



Fig. 2. Responses to expansionary monetary policy shocks across 241,865 realizations: Divisia M4 specification of a pseudo-calibrated structural VAR.

Table 1 shows the conclusions we draw from the modern sample we consider (left column of the table) do not seem sensitive to the sharpness of the grid search. Both the 245K main specifications and the smaller 25K specifications show that Divisia money shocks seem to dominate shocks to the Wu and Xia (2016) shadow rate in their ability to elicit sensible responses between 1988 and 2020. This conclusion seems robust to alternative samples as well. Inspecting the first row in the top panel of Table 1 shows that for the specifications with CPI inflation, money growth rules, both to broader DM4 and narrower DM2, obtain low percentages of output puzzles—never exceeding 4%—across the three samples. Similarly, the first row in the bottom panel shows that money growth rules elicit low percentages of CPI-inflation puzzles in all three samples. The highest inflation puzzle for the Divisia growth measures tops at 4.1% in the historical (1967–2020) sample.

Conversely, the Wu and Xia (2016) shadow rate shows large regions of CPI-inflation puzzles. The first row in the bottom panel shows that—at 93%—the lowest incidence of price puzzles for the Wu and Xia (2016) shadow rate is found for the 2008–2020 sample spanning ELB. The top row in the top panel also reveals a high incidence of output puzzles for the Wu and Xia (2016) shadow rate with CPI inflation in the specifications—99.5%, 72.0%, and 98.8% for the modern sample, the post-2008 sample, and the historical sample, respectively. The sample with the best performance for the Wu and Xia (2016) shadow rate is the 2008–2020 period with a 93% inflation puzzle and a 72% output puzzle. Simple inspection of the second rows in the top and bottom panels of Table 1 reveals our conclusions are largely insensitive to the price level measure we use to construct inflation. Replacing the CPI with the PCE index in all our specifications does not materially affect price puzzle percentages found across all specifications. In addition, the percentages of output puzzles found are close between the PCE and CPI specifications in the modern and the post-2008 samples. Overall, money growth rules with the broader *DM*4 and the narrower *DM*2 vastly outperform the interest rate rules with Wu and Xia (2016) shadow rate.<sup>31</sup>

<sup>&</sup>lt;sup>31</sup> In the historical 1967–2020 sample for the PCE specification, the incidence of output puzzles is closer between the two rules: 53.3% for the Wu and Xia (2016) shadow rate and 56% and 47.9% for *DM*4 and *DM*2, respectively. However, the incidences of PCE puzzles for the money growth rule specifications are far lower

#### Table 1

Performance of 25K realizations a three-variable RE-SVAR mapped from (2)–(4) for three different policy indicators and two price indexes across three sample periods.

Percentages of	of <b>Output Puz</b>	zles Rep	orted by Ea	ach Specificati	on				
Price Index	Modern Sample (1988.10 - 2020.02) W&X FFR DM2 DM4			Post-2008 (ELB) Sample (2008.12 - 2020.02) W&X FFR DM2 DM4			Long Sample (1967.01 - 2020.02) W&X FFR DM2 DM4		
CPI	99.5%	2.6%	3.7%	72.0%	2.4%	2.4%	98.9%	2.9%	3.9%
PCE	99.6%	2.7%	23.7%	90.8%	6.5%	9.1%	53.3%	47.9%	56.0%
Percentages of	of Price Puzzl	es Report	ted by Eacl	h Specificatior	l				
	Modern Sample			Post-2008 (ELB) Sample			Long Sample		
	(1988.10 - 2020.02)			(2008.12 - 2020.02)			(1967.01 - 2020.02)		
Price Index	W&X FFR	DM2	DM4	W&X FFR	DM2	DM4	W&X FFR	DM2	DM4
CPI	99.4%	3.1%	3.8%	93.0%	1.9%	1.6%	98.8%	3.8%	4.1%
PCE	99.4%	3.2%	4.2%	96.1%	5.2%	5.1%	94.7%	5.5%	7.4%

Note: The top (bottom) panel reports the percentage of monetary policy specifications resulting in output (price) puzzles for each baseline RE-SVAR model. Each specification loads the policy indicator as the first variable, which rotates through: W&X FFR, DM2 and DM4 denoting the Wu and Xia (2016) shadow rate, the (log-differenced) Divisia M2 or Divisia M4 aggregate, respectively. The second variable is always the (log-differenced) industrial production across all specifications. The third variable rotates between the CPI and the PCE inflation rate. Each model accommodates **25,137 realizations** from the grid search specified above as:  $\phi_{\pi} \in \left\{0, \frac{1}{4}, \frac{2}{4}, \dots, 5\right\}$  and  $h_{\pi} \in \{0, \dots, 18\}$ , as well as

 $\phi_y \in \{0, \frac{1}{4}, \frac{2}{4}, \dots, 5\}$  and  $h_y \in \{0, \dots, 2\}$  for the coefficients in equation (11) or equation (27).

#### 6. Quantifying forward-looking inflation expectations with median responses at various horizons

In this section, we return to the main models (comprising 245K IRFs) we discussed at the top of Section 5. From there, we report median responses for various horizons of CPI inflation resulting from our two competing specifications. For example, if we are interested in the responses where  $h_{\pi}$  is fixed at six months, we can simply compute all the responses from a combination of 61 possible values of  $\phi_{\pi}$ , with another 61 possible values for  $\phi_y$ , and five potential horizons ( $h_y = 0, ..., 5$ ) for output expectations fixed at a single value of  $h_{\pi} = 6$ . This comprises a response cloud of 18,605 possible combinations. We can also collate responses for fixed values of the policy rule coefficients. Setting  $\phi_{\pi} = 1.5$  for horizons of inflation expectations between one and six would render a cloud of 1,830 possible combinations (61 possible values for  $\phi_y$  times six possible values of  $1 \le h_{\pi} \le 6$  times five potential  $h_y = 0, ..., 5$  horizons). Finally, one may be interested in a specific subset of the responses obtained in Figs. 1–2 without fixing any given parameter to a single value. For example, a region of responses that would fall under the following constraints:  $7 \le h_{\pi} \le 12$  and  $\phi_{\pi} \ge 1.5$  would yield a cloud of 69,540 possible responses (61 possible values for  $\phi_y$  times 38 possible values for  $\phi_{\pi} \ge 1.5$  times six possible values for  $\phi_{\pi} \le 12$  times five potential  $h_y = 0, ..., 5$  horizons).

Fig. 3 shows the IRFs of these various subsets of clouds to an expansionary shock in the Wu and Xia (2016) shadow rate. The top row shows the clouds of 18,605 responses—when  $h_{\pi}$  is fixed at six months—for industrial production and the CPI, respectively. Recall that nearly all of the total 241,865 responses (99.13%) elicited a joint puzzle in the full model. This percentage is shown here again, for convenience, in the vertical axes of the top charts. The middle charts in Fig. 3 report the median puzzling responses at various horizons of  $h_{\pi}$ . The green lines for  $h_{\pi} = 6$  are the median responses of those reported on the top charts. While these responses—both for CPI and industrial production—are overwhelmingly negative, overall the magnitudes of the negative responses at horizons one and three are very close across horizons. The bottom charts show the median responses across the possible 1,830 responses for  $\phi_{\pi} = 1.5$  and  $1 \le h_{\pi} \le 6$  (the solid blue line) along with the black solid line depicting the median responses across the possible 69,540 responses for  $\phi_{\pi} \ge 1.5$  and  $7 \le h_{\pi} \le 12.^{32}$  Given the large proportion of puzzling responses that emanate from the Wu and Xia (2016) shadow rate shock, the median puzzling responses do not much vary along expectations horizons ( $h_{\pi}$ ) for the Federal Reserve's reaction to inflation in the policy function ( $\phi_{\pi}$ ).

We now inspect the Divisia money specification. Fig. 4 provides a saliently different picture to that of Fig. 3. The top charts, again, show the clouds of 18,605 responses when  $h_{\pi} = 6$ . Recall that in the full model, this Divisia specification yielded a smaller proportion of joint puzzles (4.15% of the total 241,865 responses)—this information is again appended to the y-axes of these top charts. In stark contrast to the IRFs from Fig. 3, the median responses to an expansionary shock in DM4 exhibit the expected sign—up to two years for industrial production and across all horizons for inflation. The middle-right chart shows that the 18,605 CPI responses for the short and medium-term expectation horizons ( $h_{\pi} = 1, 3, 6$ ) have sensible dynamics and are close in magnitude. The magnitude of the

than those of the interest rate rule in this sample. Thus, when considering the *joint* puzzling responses, the offending set of responses the interest rate rule yields is nearly twice as large as those of the money growth rules for the PCE specification in the historical sample. Moreover, the shape of the IRFs (available on request) reveals that negative output responses to expansionary money growth shocks tend to be less severe—i.e. tend to be less negative—than the counterpart responses to the Wu and Xia (2016) shadow rate.

<sup>&</sup>lt;sup>32</sup> We select  $\phi_{\pi} = 1.5$ , a value consistent with the original specification of the landmark Taylor (1993) paper.



Fig. 3. Responses to expansionary monetary policy shocks at various horizons: Federal Funds Rate specification of a pseudo-calibrated structural VAR.

 $h_{\pi} = 12$  median response is nearly twice as large within the first few months following the shock. It subsequently returns to zero faster than the other responses. This price behavior looks reasonable. The middle-left chart also shows the median responses of industrial production which, by and large, also look sensible. They do turn negative roughly 24 months after the shock, but that magnitude is comparatively small at shorter horizons. Finally, the bottom charts of Fig. 4 show the median responses to an expansionary *DM*4 shock across the possible 1,830 responses for  $\phi_{\pi} = 1.5$  and  $1 \le h_{\pi} \le 6$  (the solid blue line) along with the median responses across the possible 69,540 responses for  $\phi_{\pi} \ge 1.5$  and  $7 \le h_{\pi} \le 12$  (the solid blue line). Both median responses here are diametrically opposite to the responses of these variables to the Wu and Xia (2016) shadow rate shock. The magnitudes of these median responses are markedly larger at shorter horizons for the CPI, whereas there is a delayed reaction of industrial production before it begins to grow in response to the expansionary monetary policy shock.

Figs. 3–4 show the clouds comprising the totality of responses for various expectation horizons. We now delve deeper by extracting the subsets of these two *horizon-specific clouds* that render non-puzzling responses. Fig. 5 is the analogous chart to (and a subset of) Fig. 3 now showing the small clouds where an expansionary Wu and Xia (2016) shadow rate shock yields sensible responses. The top charts show that, of the total 18,605 responses at  $h_{\pi} = 6$ , only 195 responses do not exhibit a joint puzzle. The middle charts in Fig. 5 show that the small subset of 392 responses from  $h_{\pi} = 1$  actually provides the largest set of surviving responses (out of the 18,605) for this specification. The incidence of joint puzzles increases with the length of the horizon in inflation expectations. At  $h_{\pi} = 12$ , a mere five responses out of the total 18,605 show a non-puzzling response. While the magnitudes of the sensible responses of industrial production and CPI to a reduction in the interest rate increase at longer horizons, statistical uncertainty demands caution when some of these median responses constitute a tiny portion of the total number of responses (0.027% of the responses with  $h_{\pi} = 12$ ). The



Fig. 4. Responses to expansionary monetary policy shocks at various horizons: Divisia M4 specification of a pseudo-calibrated structural VAR.

bottom charts show the median responses of the 20 surviving IRFs (out of the total 1,830) when  $\phi_{\pi} = 1.5$  and  $1 \le h_{\pi} \le 6$ . Out of the total 69,540 combinations for  $\phi_{\pi} \ge 1.5$  and  $7 \le h_{\pi} \le 12$  a mere 85 exhibit the expected sign (less than 0.005%).

Fig. 6 presents the counterpart charts for the *DM4* shock to those of Fig. 5 for the Wu and Xia (2016) shadow rate shock. The shapes of these non-puzzling responses here do not greatly vary over the total sets in Fig. 4. This is not surprising since only a small portion of these responses yielded puzzling behavior for the money growth indicator. The top chart of Fig. 6 shows the clouds of responses for  $h_{\pi} = 6$ . Out of the total 18,605 responses at the top of the figure, 17,973 responses are non-puzzling (contrasting the 195 non-puzzling responses from the interest rate specification). Whereas 4.15% of the total 241,865 number of responses in the main *DM4* model were puzzling, when fixing  $h_{\pi}$  at one, the puzzling number of responses decreases to less than 1% (17,973 out of 18,605). Another point of departure between the interest rate and the money specifications arises when inspecting the middle charts. The number of surviving responses decreases nearly monotonically as the horizon ( $h_{\pi}$ ) increases for the interest rate specifications—the number of surviving responses are much larger—and the incidence of puzzles is much smaller—for the *DM4* RE-SVAR than for the Wu and Xia (2016) shadow rate shock yields 303, 195, and five surviving responses. These constitute the following puzzle percentages: 11.6% and 97.9% at  $h_{\pi} = 1$ ; 4.8% and 98.4% at  $h_{\pi} = 3$ ; 3.4% and 98.9% at  $h_{\pi} = 6$ ; and 0.9% and 99.9% at  $h_{\pi} = 12$  for the Divisia M4 and the Wu and Xia (2016) shadow rate, respectively.



Fig. 5. Non-puzzling responses to expansionary monetary policy shocks at various horizons: Federal Funds Rate specification of a pseudo-calibrated structural VAR.

# 7. Expanding the dimension of the RE-SVAR

As mentioned in previous sections, augmenting the RE-SVAR is far from trivial, as it requires a fully specified structural equation for each added variable in the system. We substantiated our analysis thus far on a general consensus for a system of *AS-IS-MP* equations from measures of inflation, economic activity, and the interest rate, or money growth. However, there is a wide array of data that has been used in the past to better capture expectations, such as: commodity prices, federal funds futures, Federal Reserve forecasts, or surveys of inflation expectations. Importantly, no commonly agreed structural equations governing the behavior of these variables emerge from the theory.

This section illustrates that our method can be dimensionally augmented only if a theoretical stance is assumed from the outset. Given that greater attention to financial conditions has been placed in monetary models—particularly since GFC—we expand our system with the Gilchrist and Zakrajšek (2012) excess bond premium (EBP) measure.<sup>33</sup> This requires appending a fourth equation to our (2)–(4) system.

<sup>&</sup>lt;sup>33</sup> Belongia and Ireland (2018) make a persuasive argument that the EBP may provide good information to condition against financial stress before, during, and after the U.S. Great Recession.



Fig. 6. Non-puzzling responses to expansionary monetary policy shocks at various horizons: Divisia M4 specification of a pseudo-calibrated structural VAR.

# 7.1. A sequential extraction of structural shocks contingent on the identified monetary policy shock

Our new structural model now consists of four equations:

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$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+h_\pi} + \phi_y \mathbb{E}_t y_{t+h_y} + A^{MP}(L) x_{t-1} + \varepsilon_t^{MP}$$
(21)

$$y_{t} = \mathbb{E}_{t} y_{t+1} - \alpha_{1} (i_{t} - \mathbb{E}_{t} \pi_{t+1}) + A^{IS}(L) x_{t-1} + \varepsilon_{t}^{IS}$$
(22)

$$\pi_t = \alpha_2 \mathbb{E}_t \pi_{t+1} + \alpha_3 y_t + A^{AS}(L) x_{t-1} + \varepsilon_t^{AS}$$
(23)

$$b_{t} = \mathbb{E}_{t} b_{t+1} + \alpha_{4} \mathbb{E}_{t} i_{t+1} + \alpha_{5} \mathbb{E}_{t} \pi_{t+1} + \alpha_{6} \mathbb{E}_{t} y_{t+1} + A^{BR}(L) x_{t-1} + \varepsilon_{t}^{BR}$$
(24)

which now contains an added equation for bond risk shocks (BR).<sup>34</sup> Section 2 shows how to obtain the monetary policy shocks from money growth  $(\tilde{\epsilon}_t^{MP})$  or from the interest rate  $(\epsilon_t^{MP})$  in the (2)–(4) system. Since our four-variable system in (21)–(24) has the same first three equations as that of Section 2, we begin from the identified monetary policy shock to show how to obtain the remaining structural shocks.

<sup>&</sup>lt;sup>34</sup> Given the lack of a theoretical foundation for a law of motion that might describe the Gilchrist and Zakrajšek (2012) EBP, we specify a very unrestricted form of this equation where the EBP is allowed to respond to expected changes in all the variables in the system.

Beginning on the second equation of the system, we take  $\mathbb{E}_{t-1}$  of equation (22) and subtract it from itself, rendering the following expectational difference:

$$y_t - \mathbb{E}_{t-1}y_t = \mathbb{E}_t y_{t+1} - \mathbb{E}_{t-1}y_{t+1} - \alpha_1((i_t - \mathbb{E}_{t-1}i_t) - (\mathbb{E}_t \pi_{t+1} - \mathbb{E}_{t-1}\pi_{t+1})) + \varepsilon_t^{IS}$$
(25)

We can then use the general forecast revision equation (7) to find expressions for the expectational differences in the above equation:

$$e_t^{\gamma} = (S_{\gamma} \Psi D e_t)' - \alpha_1 (e_t^{\gamma} - (S_{\pi} \Psi D e_t)') + \varepsilon_t^{TS}$$
<sup>(26)</sup>

which can be rewritten as a linear equation with slope coefficient  $\alpha_1$  and can be solved for the structural shock  $\varepsilon_t^{IS}$  as follows:

$$\varepsilon_t^{IS} = e_t^y - (S_y \Psi D e_t)' + \alpha_1 (e_t^i - (S_\pi \Psi D e_t)')$$
(27)

Given that the reduced-form innovation  $(e_t^y)$  in this equation may be correlated with  $e_t$  in general, and  $e_t^i$  in particular, the estimate of  $\alpha_1$  could be biased. However, leveraging (11), we can use  $\varepsilon_t^{MP}$  as an instrument—which is potentially correlated with  $e_t^i$  but must be uncorrelated with  $\varepsilon_t^{IS}$ —to gain an unbiased estimate of  $\alpha_1$ . This can be done in two stages. First, we regress the second term in parentheses on the right side of equation (27) on our recovered structural shock  $\varepsilon_t^{MP}$  from equation (11), and obtain an OLS coefficient estimate  $(\hat{\rho}_1)$ . Second, we regress  $[e_t^y - (S_y \Psi D e_t)']$  in equation (27) on  $[\hat{\rho}_1 \times \varepsilon_t^{MP}]$  and we obtain  $\hat{\alpha}_1$ . Once this is done, a time series for  $\varepsilon_t^{IS}$  can be recovered by replacing  $\alpha_1$  with  $\hat{\alpha}_1$  in equation (27).

Repeating the analysis for the third equation, we have the following expectational difference for the AS equation:

$$\pi_{t} - \mathbb{E}_{t-1}\pi_{t} = \alpha_{2}(\mathbb{E}_{t}\pi_{t+1} - \mathbb{E}_{t-1}\pi_{t+1}) + \alpha_{3}(y_{t} - \mathbb{E}_{t-1}y_{t}) + \varepsilon_{t}^{AS}$$

where, once more, applying equation (7) to the above equation yields the following linear equation with slope coefficients  $\alpha_2$  and  $\alpha_3$ :

$$e_t^{\pi} = \alpha_2 (S_{\pi} \Psi D e_t)' + \alpha_3 e_t^{\gamma} + \epsilon_t^{AS}$$
<sup>(28)</sup>

Again, estimates of  $\alpha_2$  and  $\alpha_3$  from this equation may generally be biased. Therefore, we conduct a two-stage regression again. In the first stage, we obtain two OLS estimates:  $\hat{\rho}_2$ —by regressing  $[S_{\pi}\Psi De_t]$  on  $[\epsilon_t^{MP}, \epsilon_t^{IS}]'$ —and  $\hat{\rho}_3$  by regressing  $e_t^{\gamma}$  on these two structural shocks again. In the second stage, we use  $\hat{\rho}_2$  and  $\hat{\rho}_3$  to regress  $[e_t^{\pi}]$  on  $[\epsilon_t^{MP}, \epsilon_t^{IS}]'$  and obtain  $\hat{\alpha}_2$  and  $\hat{\alpha}_3$ , which can then be substituted into equation (28) to recover a time series of the structural shock  $(\epsilon_t^{AS})$ .

Finally, we can apply equation (7) to ultimately arrive at an equation for the structural shock to the EBP as follows:

$$\varepsilon_t^{BR} = \varepsilon_t^h - (S_b \Psi D \varepsilon_t)' - \alpha_4 (S_i \Psi D \varepsilon_t)' - \alpha_5 (S_x \Psi D \varepsilon_t)' - \alpha_6 (S_v \Psi D \varepsilon_t)'$$
<sup>(29)</sup>

Given the possible bias in the estimates of  $\alpha_4$ ,  $\alpha_5$ , and  $\alpha_6$  that may result from the endogeneity of  $e_t^h$  and  $e_t$ , once again we conduct two-stage least squares using the identified structural shocks to the previous equations as instruments. In the first stage, we estimate three regressions, one for each of the first three terms in brackets in equation (29), and derive three OLS coefficients. In the second stage, we regress the residuals  $e_t^h$  from the fourth equation on the combinations of the first-stage estimates and the structural shocks to obtain sample estimates for  $\hat{\alpha}_4$ ,  $\hat{\alpha}_5$ , and  $\hat{\alpha}_6$ . A time series for  $\varepsilon_t^{BR}$  can then be generated by replacing the coefficients  $\alpha_4$ ,  $\alpha_5$ , and  $\alpha_6$  with their corresponding sample estimates.

#### 7.2. Implications from a rational expectations solution to the RE-SVAR

Section 7.1 shows our sequential estimation procedure to identify all four structural shocks of the (21)-(24) system equationby-equation. However, our approach does not allow for the "modularity" of simply appending additional variables without a fully specified additional equation for each new variable added to the system. To further illustrate this point, additional insights may be obtained from a multivariate rational expectations solution to the system as a whole. A multivariate RE representation of (21)-(24)can be written as:

$$Cx_{t} = \sum_{k=1}^{h} \Phi_{k} \mathbb{E}_{t} x_{t+k} + \sum_{l=1}^{p} A_{l} x_{t-l} + \epsilon_{t}$$
(30)

where the first sum on the right hand side comprises the collection of forward-looking structural parameters with an arbitrary number of forward horizons  $h = \max \{1, h_{\pi}, h_{y}\}$ , and  $\Phi_{k} = 0$ , for all  $k \notin \{1, h_{\pi}, h_{y}\}$ . The second term on the equation shows the sum of the lagged structural parameters with an arbitrary number of p lags. We employ Binder and Pesaran's (1997) method to solve the RE model, by assuming that the solution has the following form:

$$x_t = \sum_{l=1}^{p} B_l x_{t-l} + \mathbf{A}^+ \epsilon_t \tag{31}$$

To find a correspondence between the parameters in (31) and the parameters in our theoretical (21)–(24) system, we first iterate (31) forward, assuming that future shocks are unobserved, i.e.  $\mathbb{E}_t \epsilon_{t+k} = 0$ , for all  $k \ge 1$ ,

$$\mathbb{E}_{t} x_{t+k} = \sum_{l=1}^{p} \bar{B}_{k+l} x_{t-l} + \bar{B}_{k} \mathbf{A}^{+} \epsilon_{t} \quad k = 1, 2, \dots$$
(32)

where

$$\bar{B}_{j} = \sum_{l=1}^{\min(j,p)} B_{l}\bar{B}_{j-l}, \quad j = 1, 2, \dots$$
(33)

and  $\bar{B}_0 = I$ . Replacing the expectation terms in (32) into (30) yields

$$x_{t} = \sum_{l=1}^{p} C^{-1} \left( A_{l} + \sum_{k=1}^{h} \Phi_{k} \bar{B}_{k+l} \right) x_{t-l} + C^{-1} \left( I + \sum_{k=1}^{h} \Phi_{k} \bar{B}_{k} A^{+} \right) \epsilon_{t}$$
(34)

combining coefficients with equation (31) reveals the following:

$$A^{+} = C^{-1} \left( I + \sum_{k=1}^{h} \Phi_{k} \bar{B}_{k} A^{+} \right),$$
  

$$B_{l} = C^{-1} \left( A_{l} + \sum_{k=1}^{h} \Phi_{k} \bar{B}_{k+l} \right), l = 1, \dots, p$$
(35)

Hence, matrix  $A^+$  maps the RE system in (21)–(24) to the SVAR representation in (31):

$$\mathbf{A}^{+-1} = C - \sum_{k=1}^{n} \Phi_k \bar{B}_k$$
(36)

and loading the matrices *C* and  $\{\Phi_1, \dots, \Phi_h\}$  in (36) with the parameters in the system (21)–(24) yields:

Importantly, while the values of the matrices  $B_j$  and  $\bar{B}_j$ , for all *j* could be obtained directly from estimation of a reduced-form VAR, estimates of (36) would likely be biased. The reason is that  $A^{+-1}$  contains the structural hyper-parameters for the remaining (22)–(24) equations in the system. Estimates of  $\hat{a}_i$  for i = 1, 2, ..., 6 require the two-stage IV procedure outlined in the previous section because unbiased estimates for these cannot be obtained from the reduced-form VAR alone.

This RE solution highlights the general lack of modularity in our approach. To expand the dimension of the RE-SVAR, a fully specified structural equation is required for each additional variable in order to implement the two-stage IV procedure and, thereby, extract each additional structural shock. The ensuing cross-equation restrictions implied by (36) are not enough to satisfy the rank condition in (31) for global identification from Theorem 1 in Rubio-Ramírez et al. (2010). It is straightforward to show that the system requires two additional restrictions for point identification of this four-variable RE-SVAR. Therefore, we must complement our identification strategy with the two additional assumptions, which we impose on the policy rule coefficients  $\phi_{\pi}$  and  $\phi_{y}$ , as we describe in the next section.

#### 7.3. Results from the expanded RE-SVAR

Our analysis in Section 7 illustrates that our RE-SVAR approach is not modular. Augmenting the dimension of our RE-SVAR requires a full formulation of the theoretical model. Given that the purpose of this section is to emphasize this point, we concentrate the rest of our analysis on a four-variable specification with the growth rate of DM4 as the indicator of monetary policy. Thus, we replace the first equation for the interest rate in the (21)–(24) system with DM4 growth. And we focus on the money growth rule in (9) to identify monetary policy shocks ( $\tilde{e}_{l}^{MP}$ ) from Divisia money for a new sample spanning July 1979—February 2020. The starting date is governed by the availability of the Gilchrist and Zakrajšek (2012) EBP measure.

We proceed in the same way we did for the three-variable consensus model. Instead of assigning a single value for parameters in equation (9):  $\{\phi_y, \phi_\pi, h_y, h_\pi\}$ , we opt for a set-identification approach with a grid of the following parameter values: for  $\phi_y \in \{0, \frac{1}{15}, \frac{2}{15}, \dots, 4\}$ ,  $h_y \in \{0, \dots, 4\}$ ,  $\phi_\pi \in \{0, \frac{1}{15}, \frac{2}{15}, \dots, 4\}$ , and for  $h_\pi \in \{0, \dots, 12\}$ . Similar to the three-variable model, this comprises identification of  $\varepsilon_i^{MP}$  shocks from a total of **241,865 different structural VAR specifications** in this augmented RE-SVAR.

Fig. 7 shows responses to an expansionary *DM*4 shock. Out of the total 241K structural responses estimated in this four-variable specification for the July 1979—February 2020 sample, 198,988 responses did not incur a puzzle—which constitutes an 81.5% survival rate. The survival rate of (*"no-joint-puzzle"*) responses drops, somewhat, from 95.9% in our consensus three-equation model to 81.5% in our augmented model. This may be explained by starting the sample in the augmented model in 1979—a time of tighter financial pressures relative to 1988. Nevertheless, *DM*4 continues to perform well as a monetary policy indicator in our augmented four-variable RE-SVAR.

While the emphasis of our application surrounds the identification of monetary policy shocks, Section 7.1 shows how to sequentially recover the remaining shocks of the system. Though we can identify *BR* shocks from our method, that is outside our emphasis



Fig. 7. Responses to expansionary (Divisia M4) monetary policy shocks across 241,865 realizations of the augmented RE-SVAR.

on the consensus model. Our (2)—(4) consensus model in Section 2.2 also includes *IS* and *AS* shocks. Similar to the monetary policy shock, our method allows us to construct clouds comprising 241,865 responses to  $\varepsilon_l^{IS}$  and  $\varepsilon_l^{AS}$  shocks. A detailed quantitative analysis of those shocks merits a deeper discussion, which is left for future work. Instead, we provide here a simple verification of the qualitative effects of those shocks on economic activity and inflation. Fig. 8 reports the median responses of industrial production and CPI inflation to the second ( $\varepsilon_l^{IS}$ ) and third ( $\varepsilon_l^{AS}$ ) shocks in the (21)–(24) system across the 245K realizations. The red lines denote the median responses of the variable that is being shocked. The left column (a) of Fig. 8 shows the responses of industrial production at the top, and CPI inflation at the bottom, to the second shock in the system. Both variables respond in the same direction, which is qualitatively consistent with the standard prediction of an *IS* shock.<sup>35</sup> Conversely, the right column (b) shows an exogenous increase in CPI inflation in conjunction with a reduction in industrial production, which is consistent with the textbook prediction of an adverse *AS* shock. Given that we are omitting the clouds of structural responses, the scale of these charts is not informative—but, in principle, the median responses are qualitatively consistent with the textbook prediction.

<sup>&</sup>lt;sup>35</sup> Column (a) shows the median response of industrial production to the *IS* shock remains positive for the first two years before turning negative roughly 24 months after the shock, when the CPI response bottoms out and begins to increase again.



Fig. 8. Median responses across 241,865 realizations of the augmented RE-SVAR to: (a): Shocks to industrial production and (b): Shocks to CPI inflation.

#### 8. Conclusion

We advance a new approach for directly embedding rational expectations into a small-dimension SVAR. Structural shocks can be recovered from a direct application of rational expectations that quantifies the effects across expectation horizons for two candidate monetary policy rules. This methodology comprises observable variables exclusively—it does not require transition equations with laws of motions for unobservables that may further constrain the parameter space.

A potential benefit of our low-dimensional approach is that it can be more directly built from a "consensus" theoretical model of the economy—which is itself low dimensional. This makes it tractable to model forward-lookingness in a way consistent with a textbook rational expectations mechanism.

If the theoretical model we construct our RE-SVAR from is not sensible, it renders the whole enterprise a non-starter. But a similar concern also applies to more standard SVARs. Given that no econometric technique currently delivers incontrovertible identification, an unconvincing restriction strategy to map reduced-form innovations to structural shocks can also be raised to call into question the validity of the analysis. Absent a suggested mapping, our approach requires credibility of the underlying theoretical model.

The framework is useful for drawing inference from a low-dimensional system. It is, however, not modular. While standard VARs admit an increase in the information set by simply appending additional variables or factors to the system, our mechanism requires a fully specified equation for each variable included in the structure. Our implementation is directed to an investigation of monetary policy rules but our approach could be leveraged for additional applications of monetary policy and other extended inquiries. For example, a financial RE-SVAR founded on a CAPM mechanism, or a fiscal extension of Blanchard and Perotti (2002) seems like potentially interesting avenues to pursue.

Our investigation is motivated by the unremitting evidence in the literature of the relatively poor performance of the short-term interest rate as a sensible indicator of monetary policy in low-dimensional recursive SVARs—which is amply documented in Ramey (2016) and references therein. Based on our assumed structure, we conduct a grid search over the parameters of the policy reaction functions to construct clouds of responses consistent with an interest rate and a money growth rule.

We find overwhelming evidence that a forward-looking money growth rule elicits large regions of sensible responses of economic activity and inflation. The fact that the federal funds rate seems to perform less well in our system suggests that indicator often benefits from being augmented with more information in traditional VAR settings. Given the backward-looking econometric machinery of most recursive SVARs, interest rate specifications often require appending more variables to better approximate the forward-looking information set available to the Federal Reserve. Importantly, while the facile addition of these variables may provide empirical utility in VAR estimation, there is no explicit theoretical role for many of them in the consensus model of the economy.

Overall, our findings suggest that even a low-dimensional model can perform well with Divisia money growth as an indicator of monetary policy. A search over the parameter space for an interest rate rule seems substantially more arduous—and likely requires more work—than a similar search over a money growth rule to elicit textbook-consistent economic responses. A preponderance of evidence suggests monetary aggregates may be more informative for policy description than previously thought.

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