

Lindbeck, Assar; Snower, Dennis J.

Working Paper

The firm as a pool of factor complementarities

CESifo Working Paper, No. 1046

Provided in Cooperation with:

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Lindbeck, Assar; Snower, Dennis J. (2003) : The firm as a pool of factor complementarities, CESifo Working Paper, No. 1046, Center for Economic Studies and Ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/3085>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

THE FIRM AS A POOL OF FACTOR COMPLEMENTARITIES

ASSAR LINDBECK
DENNIS J. SNOWER

CESIFO WORKING PAPER NO. 1046
CATEGORY 9: INDUSTRIAL ORGANISATION
SEPTEMBER 2003

An electronic version of the paper may be downloaded

- *from the SSRN website:* www.SSRN.com
- *from the CESifo website:* www.CESifo.de

THE FIRM AS A POOL OF FACTOR COMPLEMENTARITIES

Abstract

This paper presents a new approach to the theory of the firm by identifying factor complementarities as central to the determination of the firm's boundaries. The factor complementarities may take a variety of forms: technological and informational complementarities, as well as economies of scale and scope. We examine the tradeoff between the gains from these complementarities and transactions costs. In so doing, we must abandon the standard dichotomy between the determinants of plant size and firm size. The influence of factor complementarities on firm size is examined in partial and general equilibrium frameworks.

JEL Code: D2, D4, D8.

Assar Lindbeck
Institute for International Economic Studies
Stockholm University
SE-106 91 Stockholm
Sweden
Assar.Lindbeck@iies.su.se

Dennis J. Snower
Department of Economics
Birkbeck College
University of London
7 Gresse Street
London W1P 1PA
United Kingdom

The modern literature on the boundaries of the firm focuses primarily on three major costs of organizing firms: communication and coordination costs (e.g. Coase (1937) and Williamson (1975)); principal agent problems (e.g. Alchian and Demsetz (1972), Holmstrom (1982)); and hold-up problems (e.g. Klein, Crawford and Alchian (1978), Grout (1984)). Broadly speaking, these costs may be viewed as different types of transactions costs. They have also given rise to the notion that the firm may be identified as a system of property rights (e.g. Grossman and Hart (1986)), an incentive system (e.g. Holmstrom and Milgrom (1994)), and a communication network (e.g. Bolton and Dewatripont (1994)), and so on.

Another literature deals with plant size (e.g. Viner (1932), Robinson (1958), Baumol, Panzer and Willig (1982)). This literature emphasizes technological considerations, such as fixed costs and economies of scale and scope. In the literature on the size of firms, it is generally taken for granted that the factors affecting plant size (in particular, economies of scale and scope) are not relevant to firm size. Our paper calls this conventional wisdom into question.

Our analysis instead identifies the firm as a “pool of factor complementarities,” and we examine how these complementarities interact with transactions costs.¹ When these interactions are taken into account, we must abandon the dichotomy between the determinants of plant size and of firm size. Both sets of influences have a role to play in the determination of the firm’s boundaries. But since the role of factor complementarities has not received much attention in the recent literature on the firm’s boundaries, we will focus on this aspect here, while transactions costs will be modeled quite schematically.

The paper is organized as follows. Section 1 deals with preliminaries, outlining various types of factor complementarities and their implications for the boundaries of the firm. Section 2 runs through some simple, partial equilibrium models to show how these complementarities influence the firm’s boundaries. Section 3 presents a general equilibrium model in which the boundaries of different firms affect one another and are determined simultaneously, so as to yield a model of market structure (viz., the degree of imperfect competition). Section 4 concludes.

1 Preliminaries

Factor complementarities (and substitutabilities) come in various guises. First, the *technological complementarities* (and substitutabilities) may be identified in terms of the cross-partial derivatives in

¹Our work extends the analysis of Lindbeck and Snower (1996, 2000). Whereas the latter focuses on intra-personal complementarities, we are concerned with inter-factor (particularly inter-personal) complementarities.

a production function. For the production function $Q = f(\mathbf{F})$, where $\mathbf{F} = (F_1, \dots, F_n)$ is a vector of factors, the factors F_i and F_j are technological complements when $\frac{\partial^2 Q}{\partial F_i \partial F_j} > 0$ and technological substitutes when $\frac{\partial^2 Q}{\partial F_i \partial F_j} < 0$.

Second, there are *informational complementarities* among different types of labor. Specifically, suppose that through learning-by-doing, each type of worker gains information that is useful to other types of workers. In practice, the natural domain for such informational complementarities is the firm (rather than its plants), because firms generally encourage the exchange of knowledge among their employees, but often strongly discourage them from sharing it with employees in other firms². Thus the natural domain of informational complementarities is the firm, rather than its plants. Let H be the firm's knowledge capital, which is a public good within the firm but not beyond it. Let $H = H(\mathbf{L})$ be the firm's production function for knowledge capital,³ where $\mathbf{L} = (L_1, \dots, L_m)$ is a vector of labor types. The firm's production function (different from the one above) may be expressed as $Q = f[g_1(H)L_1, \dots, g_m(H)L_m]$, where $g_i(H)$ describes how the firm's knowledge capital enhances the productivity of type- i labor, so that $g_i(H)L_i$ is type- i labor in efficiency units. Then, in the absence of technological complementarities ($f_{L_1 L_2} = 0$), the informational complementarities between labor of types i and j ($i \neq j$) are⁴ $\frac{\partial^2 Q}{\partial L_i \partial L_j} = (H_{L_i L_j} + H_{L_i} H_{L_j}) (f_{L_i} g'_i + f_{L_j} g'_j) > 0$.

Third, in contrast to the inter-factor complementarities above, there are *intra-factor complementarities* (or substitutabilities). Specifically, consider a vector of factors $\mathbf{F} = (F_1, \dots, F_I)$ producing the output $Q = f(\mathbf{F})$, and now consider a proportional increase in all the factors $\Delta \mathbf{F} = \mu \mathbf{F}$ producing the additional output ΔQ , where $\mu (> 1)$ is a constant. If the two sets of factors, \mathbf{F} and $\Delta \mathbf{F}$, are complementary, then $\Delta Q > \mu Q$ (increasing returns to scale). If the two sets are substitutable, then $\Delta Q < \mu Q$ (diminishing returns to scale); and if the two sets are independent, then $\Delta Q = \mu Q$ (constant returns to scale). In this way, returns to scale may be identified as the outcome of intra-factor complementarities or substitutabilities.

Fourth, there are *complementarities* (or substitutabilities) among the same set of factors *in the production of additional products*. In particular, suppose that the vector of factors \mathbf{F} is used to produce a vector of goods $\mathbf{Q} = (Q_1, \dots, Q_n)$ via the production function $\Phi(\mathbf{Q}) = \Psi(\mathbf{F})$. Let the cost function $C(\mathbf{Q})$ be the solution to the problem of minimizing the factor cost $\mathbf{p}\mathbf{F}$ (where \mathbf{p} is a vector of factor

²They often even have various sanctions - legal and economic - to prevent sensitive information about the firm from reaching their competitors.

³Under learning by doing, knowledge is created as an automatic by-product of working in the firm. The production function for knowledge capital shows how the stock of knowledge available to the firm depends on the labor services of all labor types.

⁴Observe that these complementarities operate solely through the exchange of knowledge, and thus are distinct from the technological complementarities that operate through the cross-partial of the production function.

prices) subject to the above production function (for given \mathbf{Q}). Similarly, let $C_i(Q_i)$ be the minimum factor cost of producing just output Q_i (an element of the output vector \mathbf{Q}). If the use of factors in the production of goods Q_i and Q_j ($j \neq i$) is complementary, then there are increasing returns to scope, so that $C(\mathbf{Q}) < \sum_{i=1}^n C_i(Q_i)$, where n is the number of goods under consideration. Alternatively, if $C(\mathbf{Q}) > \sum_{i=1}^M C_i(Q_i)$, then there are diminishing returns to scope; and if $C(\mathbf{Q}) = \sum_{i=1}^M C_i(Q_i)$, there are constant returns to scope. In this way, returns to scope may be viewed in terms of factor complementarities or substitutabilities in the production of different goods.⁵

We will show how the factor complementarities above interact with transactions costs in setting the boundaries of the firm. We define the firm's boundaries as an array (Q_1, \dots, Q_n) , specifying the amounts of all outputs that the firm produces. The number of elements in this vector describes the firm's scope and the magnitude of all the elements describes its size. To highlight how the various factor complementarities above can affect the boundaries of the firm (rather than merely influencing plant size), we will focus on complementarities and transactions costs that are firm-wide (rather than merely plant-wide).⁶

It is commonly alleged that although technological phenomena - such as inter-factor complementarities, economies of scale and scope - are relevant to plant size, they are irrelevant to firm size, for two reasons. First, technological economies allegedly set no lower limit to firm size, since the underlying factors may be controlled by more than one firm. Second, technological diseconomies allegedly set no upper limit to firm size, since the firm is always at liberty to split into independent subsidiaries and thus avoid such diseconomies.⁷

One reason for calling this conventional wisdom into question is that it is generally inefficient for more than one firm to control a common set of complementary factors. If there were multiple domains of authority to decide how such factors are to be used, the different firms would need to be engaged in an ongoing process of bargaining. Conducting these bargains would be costly and possibly vulnerable to hold-up, without countervailing benefits. For this reason, it is efficient for a single firm to have exclusive right over a given set of factors.

Moreover, firms generally cannot avoid diseconomies through the creation of subsidiaries as an alternative to market transactions among independent firms, because there is an important difference

⁵Returns to scope may of course also arise if factor prices change with factor use in such a way as to drive a wedge between $C(\mathbf{Q})$ and $\sum_{i=1}^M C_i(Q_i)$.

⁶For example, technological complementarities between different factors may span several plants, as when several plants make use of a common firm facility, e.g. a storage facility, advertising, or recruitment. For analogous reasons, economies of scale and scope may cover several plants as well. Informational complementarities may also extend across plants, such as when workers in different plants share a common data base or participate in common teams. The transactions costs in our analysis will also be firm-wide.

⁷See, for example, Tirole (1989, p. 20-21).

between a subsidiary and an independent firm. If the subsidiary goes bankrupt, the parent company is financially liable; whereas bankruptcy of another firm has no direct financial implications for the company in question. Thus it is legally impossible for a firm to split itself up into totally independent units. It follows that the managers of a firm have a natural responsibility, and hence interest, in the running of their subsidiaries. For these reasons, managers are unable to avoid intervening in the activities of subsidiaries, and thus firms cannot escape the standard diseconomies of scale and scope - such as those caused by bureaucratic waste, internal politiking, the scarcity of entrepreneurial talent and the associated cost of giving entrepreneurs additional responsibilities, and so on⁸ - by having subsidiaries with complete autonomy.

With this in mind, we now proceed to show how factor complementarities, in conjunction with transactions costs, affect the boundaries of the firm.

2 Factor Complementarities and the Boundaries of a Firm

This section presents a partial-equilibrium analysis of the firm, examining the firm in isolation from other firms. (General equilibrium is covered in the next section.) We consider the different types of factor complementarities in turn.

2.1 Economies of Scale

In the presence of economies of scale for firms (such as economies of marketing and product design, or those related to customer good will for a firm), there is a tradeoff: while increasing the size of the firm enables it to exploit these economies, it also generates additional transactions costs. To focus on firm-wide economies, we consider a firm that produces a homogeneous good (Q) through x identical plants, each of which may be viewed as a bundle of factors (\mathbf{F}).⁹ The firm's economies of scale across plants be represented by the following production function

$$q = Ax^{1+\alpha} \tag{1}$$

where A and α are positive constants, measuring the magnitude of the scale economies.

Let each plant have a fixed operating cost of κ per period of analysis. The firm's transactions costs

⁸Further limits to the boundaries of the firm are given by sources of firms' finance. To achieve portfolio diversification, lenders commonly prefer lending to a number of independent firms rather than to a single firm with an equivalent number of subsidiaries.

⁹Then the size of the firm can be measured by the number of its plants.

are specified in a simple, stylized way, to capture the usual picture of transactions costs increasing with the size of the firm.¹⁰ In particular, we assume that the firm's transactions costs (viz., the sum of the internal and external transactions costs) are given by

$$z = Bx^{1+\beta} \quad (2)$$

where B and β are positive constants, so that the transactions costs rise at an increasing rate with the number of plants. This general representation is convenient and appropriate for our purposes because our analysis is concerned only with transactions costs insofar as they are affected by the number of plants (or scale of factor use). We may interpret our transactions cost function as showing how a rise in the number of plants increases search costs for factor inputs and customers, communication costs among employees, or the cost of hold-up and principal-agent problems within the firm.

In this context, we can analyze the degree of horizontal integration. The firm maximizes its profit $\pi = Ax^{1+\alpha} - \kappa x - Bx^{1+\beta}$ with respect to the number of plants x . The first-order condition for the optimal number of plants (or scale of factor use) is

$$\frac{\partial \pi}{\partial x} = A(1 + \alpha)x^\alpha - \kappa - B(1 + \beta)x^\beta = 0 \quad (3)$$

This condition implies that the magnitude of the scale economies has a positive effect on the size of the firm:¹¹

$$\frac{\partial x}{\partial A}, \frac{\partial x}{\partial \alpha} > 0. \quad (4)$$

It is straightforward to extend this analysis to cover a firm's degree of vertical integration. Specifically, suppose that the final output is produced by means of a chain of intermediate goods. For

¹⁰These are the sum of the internal transactions costs (arising within the firm) and external transactions costs (arising from the firm's market transactions with other firms). Although in practice this sum is not always monotonically increasing in the size of the firm, the firm in our model has an incentive to expand until it reaches the range in which further increases in firm size lead to increases in the sum of the transactions costs.

¹¹By the implicit function theorem, $\frac{\partial(\frac{\partial \pi}{\partial x})}{\partial A} = -\frac{\frac{\partial^2 \pi}{\partial x^2} / \frac{\partial \pi}{\partial x}}{\frac{\partial^2 \pi}{\partial x^2} / \frac{\partial \pi}{\partial x}}$; $\frac{\partial \pi}{\partial x} / \frac{\partial \pi}{\partial x} < 0$ by the second-order condition, and $\frac{\partial^2 \pi}{\partial x^2} / \frac{\partial \pi}{\partial x} > 0$.

simplicity, consider the following vertical production chain:

$$\begin{aligned}
q(1) &= Ax^{1+\alpha(0)} \\
q(2) &= q(1)^{1+\alpha(1)} \\
q(3) &= q(2)^{1+\alpha(2)} \\
&\dots \\
q(S) &= q(S-1)^{1+\alpha(S-1)}
\end{aligned}$$

In the first stage of production, the intermediate good $q(1)$ is produced by means of the factor bundles x (which were interpreted as plants in the previous model). In the next stage, the intermediate good $q(2)$ is produced by means of the intermediate good $q(1)$; and so on. At each production stage, the economies of scale are given by the parameter $\alpha(i)$, $i = 0, \dots, S$, where S is the number of productive stages integrated (vertically) in the firm. Thus the firm's production function may be expressed as

$$q(S) = x^{\gamma(S)} \quad (5)$$

where $\gamma(S) = \prod_{i=1}^{S-1} (1 + \alpha(i))$.

Furthermore, let the firm's transactions costs¹² be given by

$$z = B(0)x^{1+\beta(0)} + \sum_{i=2}^S B(1)q(i)^{1+\beta(i)} \quad (6)$$

The firm's profit now is

$$\begin{aligned}
\pi &= q(S) - \kappa x - z \\
&= x^{\gamma(S)} - \kappa x - B(0)x^{1+\beta(0)} + \sum_{i=2}^S B(1)x^{\prod_{j=1}^{i-1} (1+\alpha(j))(1+\beta(i))}
\end{aligned}$$

to be maximized with respect to x . In this context, it is easy to show that a sufficiently large increase in returns to scale $\alpha(j)$ leads to an increase in the profit-maximizing number of production stages S^* . Thus the firm's degree of vertical integration is affected positively by its economies of scale across production stages.

¹²These transactions costs are specified along the same lines as in (2).

2.2 Economies of Scope

To analyze economies of scope (the degree of horizontal integration), let the potential goods that the firm could produce lie on a unit circle, where the distance between two points on this circle is inversely related to their economies of scope. Moreover, the larger the number of goods the firm produces, the greater are its internal transactions costs. Thus the firm faces a tradeoff between economies of scope and transactions costs. The firm's problem is to find the profit-maximizing length of its product segment on the circle of potential products.

For simplicity, let the revenue from good i be $R(Q_i)$, where $i = 1, \dots, N$, $R_{Q_i}(Q_i) > 0$, and $R_{Q_i Q_i}(Q_i) < 0$. Let the production cost be $vNQ_i - b\xi(N)$, where v and b are positive constants and $b\xi(N)$ specifies the economies of scope, with $\xi_N, \xi_{NN} > 0$, so that there are positive economies of scope. The constant b measures the magnitude of these economies of scope. Let the firm's transactions costs associated with the production of each good be $z^p(Q_i)$ and its transactions costs associated with the coordination of the production of different goods be $z^c(N)$. Note that, for simplicity, the revenues and costs are symmetric across products. Thus the profit-maximizing amounts of different outputs will be equal: $Q_i^* = Q^*$.

Then the firm's profit is $\pi = NR(Q) - vNQ + b\xi(N) - Nz^p(Q) - z^c(N)$. The first-order condition for each output is

$$R_{Q_i}(Q_i) - v - z_{Q_i}^p(Q_i) = 0 \quad (7)$$

which determines the profit-maximizing level of each output. The first-order condition with respect to the number of products is

$$R(Q^*) - vQ^* + b\xi_N(N) - z^p(Q) - z_N^c(N) = 0 \quad (8)$$

From this condition, it is evident that the greater are the economies of scope (b), the greater the number of goods (N) that the firm produces (i.e. the greater the degree of horizontal integration), and hence the larger the firm:

$$\frac{\partial N}{\partial b} > 0 \quad (9)$$

Note that this result is the outcome of the interaction between economies of scope and transactions costs. In the absence of internal transactions costs, the number of goods per firm and the amount of each good produced would be indeterminate. It is only on account of the internal transactions costs that economies of scope directly affect the boundaries of the firm.

2.3 Inter-Factor Complementarities

Since the role of technological inter-factor complementarities in determining the firm's boundaries may be analyzed along similar lines to the role of economies of scale (above), we focus on informational complementarities here. As employees gain information about their customers, their suppliers, and so on, this new information is added to the firm's stock of knowledge capital. Through this knowledge capital, employees within a firm become complements, even if they do not work within the same plant. The information gained by one employee is communicated and becomes useful in enhancing the productivity of other employees. We present a simple model in which the firm faces a tradeoff between these complementarities and the transactions costs considered above. This model is not meant to be comprehensive or general; it just provides an illustration of how inter-factor complementarities can affect the boundaries of the firm.

Consider a firm that comprises N plants.¹³ Plant i ($i = 1, \dots, N$) employs L_i workers. Workers accumulate knowledge through learning-by-doing; however, unlike the conventional learning-by doing models, the resulting knowledge is useful not just to the employee who acquired it, but to other employees as well.¹⁴ Let the production of knowledge capital be given by $H = (aL)^\alpha$, where $L = \sum_{i=1}^N L_i$, and a and α are positive constants, $0 < \alpha < 1$. Moreover, let the output of plant i be $Q = (aL)^\alpha L_i$. In short, the average productivity of the employees in each plant depends on the knowledge gained by all employees in the firm, which in turn depends on the total employment of the firm.¹⁵ The constant a measures the degree of informational complementarity among the employees.

Let the transactions costs associated with the employees in plant i be $\frac{1}{1+\eta_i} L_i^{1+\eta_i}$, $\eta_i > 0$ (a constant), and let those transactions costs arising from the coordination of the various plants be $\frac{1}{1+\eta_x} N^{1+\eta_x}$, $\eta_x > 0$ (a constant). Let wages be determined through bargaining in which workers capture a fraction μ of the available rent. Then the firm's profit may be expressed as $\pi = (1 - \mu) \sum_{i=1}^N g(aL) L_i - \sum_{i=1}^N \frac{1}{1+\eta_i} L_i^{1+\eta_i} - \frac{1}{1+\eta_x} N^{1+\eta_x}$. The first-order conditions are $(\partial\pi/\partial L_i) = 0$ and $(\partial\pi/\partial N) = 0$, which can be shown to imply:

$$\begin{aligned} (1 - \mu) a^\alpha \frac{1 + \alpha}{\alpha} N^{\eta_i} &= L_i^{\eta_i - \alpha} \\ \frac{1 - \mu}{\alpha} a^\alpha L &= N^{\eta_x + 1} \end{aligned}$$

¹³It makes no substantive difference whether these plants produce the same product or differentiated products.

¹⁴The mechanisms are analogous to those covered in some endogenous growth models.

¹⁵For simplicity, our model is static. For this purpose, we make the implicit assumption that knowledge depreciates 100 percent in moving from one period of analysis to the next. In general, of course, knowledge depreciates more slowly and thus a worker's productivity comes to depend on the stock of knowledge accumulated through all the work done in the firm over the present and past. It is straightforward to extend our model accordingly and generate analogous qualitative results in the steady state.

respectively. Solving these equations simultaneously, we obtain the profit-maximizing number of plants and the profit-maximizing employment level (over all the firm's plants):

$$N^* = \left(a \frac{1-\mu}{\alpha} \right)^{\frac{\eta_l - \alpha + 1}{\eta_x \eta_l + \alpha (\eta_x - 1)}} (1 + \alpha)^{\frac{1}{\eta_x \eta_l + \alpha (\eta_x - 1)}} \quad (10)$$

$$L^* = \frac{\alpha}{1-\mu} a^{-\alpha \frac{\eta_x (\eta_l + \alpha) - (1 - \eta_l)}{\eta_x \eta_l + \alpha (\eta_x - 1)}} \quad (11)$$

Since $\eta_l - \alpha + 1 > 0$, $\eta_x - 1$, and $\eta_x (\eta_l + \alpha) - (1 - \eta_l) > 0$, we find that an increase in the complementarity among workers (a rise in a) leads the firm to increase the number of plants and employees per plant:

$$\frac{\partial N}{\partial a} > 0, \quad \frac{\partial L}{\partial a} > 0 \quad (12)$$

3 General Equilibrium

Thus far we have considered the boundaries of an individual firm independently from the boundaries of other firms. We now extend our analysis by putting the behavior of firms into a general equilibrium context, enabling us to investigate the determination of market structure. In a simple analytical framework, we derive simultaneously the number of firms and the size of each firm, and show that these two variables are naturally interdependent.

Our analysis points to a broad vision of firms as institutions designed to exploit factor complementarities. We will show that the greater are these complementarities, relative to the firms' internal transactions costs, the greater will be the size of firms in the general equilibrium and the smaller will be the equilibrium number of firms.¹⁶

To express this vision, let us think of factors positioned in a production space in accordance with their complementarities: the shorter the distance between two factors in this space, the greater the complementarity between them. Our analysis will indicate that firms position themselves in this production space so as to maximize the profit opportunities from the factor complementarities. In this way, factor complementarities are shown to influence both the boundaries of each firm and the number of firms.

How such a general equilibrium system is modeled depends on the types of factor complementarity under consideration. For brevity, we will consider only the first type of complementarity above, namely, that which gives rise to returns to scale. In particular, suppose that homogeneous bundles of

¹⁶The partial equilibrium analysis above of course does not deal with the equilibrium number of firms at all.

factors are distributed uniformly around a unit circle, where the circumference of the circle (unity) represents the aggregate factor supply. Different firms occupy different segments of the factor circle.

Figure 1, for example, illustrates an economy containing three firms. The segment occupied by a single firm (indexed by $f = 1, 2, 3$), consists of two parts, a factor bundle $x(f)$ that is used in production (the production segment) and a factor bundle $z(f)$ that covers the firm's internal transactions (the transactions segment). The sum of the two factor bundles comprises the total factor use of the firm and thereby provides a measure of the size of the firm.

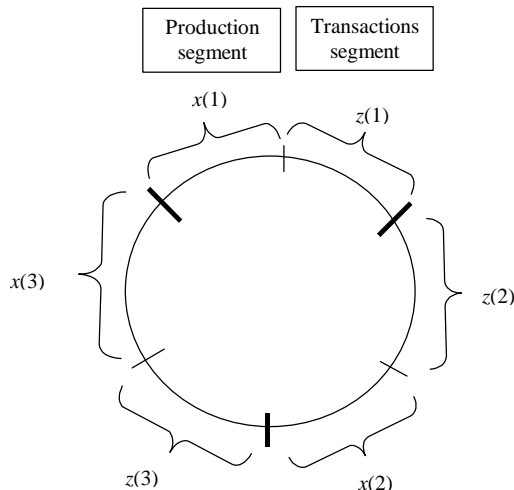


Fig. 1: Equilibrium Boundaries of Firms

As in the previous section, the firm faces a tradeoff between returns to scale and internal transactions costs. The returns to scale of firm f are given by the production function

$$q(f) = Ax(f)^{1+\alpha} \quad (13)$$

where the output $q(f)$ is assumed to be a nondurable consumption good, and A and α are positive constants. For simplicity, let us now interpret $x(f)$ as firm f 's employment level (rather than as a factor bundle, as above). The firm's internal transactions costs (measured as real factor costs) are given by

$$z(f) = Bx(f)^{1+\beta} \quad (14)$$

where B and β are positive constants. Note that all firms are assumed to face symmetric production and transactions technologies, and thus symmetric revenues and costs. The total length of the segment that firm f occupies on the unit circle is $x(f) + z(f)$. The number of firms in the economy is M^F , so

that $f = 1, \dots, M^F$.

Suppose that the economy contains a fixed number M^H of identical households. For simplicity, let household h ($h = 1, \dots, M^H$) have the following utility function:

$$U(h) = q(h)^\sigma - e[x(h) + z(h)] \quad (15)$$

where $q(h)$ is the household's consumption, σ (a positive constant) is the elasticity of utility with respect to consumption, $x(h)$ is the hours of work supplied by household h , and e is a positive constant.

Let X be the aggregate amount of factors devoted to production and Z be the aggregate amount of factors devoted to internal transactions. In equilibrium, the aggregate factor supplies (by the households) is equal to the aggregate factor demands (by the firms), in both production and internal transactions activities:

$$X = \sum_{h=1}^{M^H} x(h) = \sum_{f=1}^{M^F} x(f) \quad (16)$$

$$Z = \sum_{h=1}^{M^H} z(h) = \sum_{f=1}^{M^F} z(f) \quad (17)$$

By symmetry,

$$M^H x(h) = M^F x(f) \quad (18)$$

$$M^H z(h) = M^F z(f) \quad (19)$$

In the general equilibrium, firms position themselves around the factor circle so as to exploit the available gains from trade. For simplicity, we assume that externalities, imperfect competition and distributional issues are absent. (In particular, imperfect competition is absent in our analysis despite economies of scale in production, because at the margin these economies are dominated by diseconomies in transactions activities.) Consequently the general equilibrium coincides with the social optimum. This implies that each firm expands until the marginal utility from producing more output is exactly equal to the marginal disutility from using factors to cover the costs of its internal transactions.

The general equilibrium number of firms (M^{F*}) and the general equilibrium size of each firm (measured by $x^*(f) + z^*(f)$) is such that there are no further profit opportunities to be exploited.

Profit opportunities arise when it is possible to change the number of firms (and thus, for given factor supplies, change the size of each firm) so as to make the households better off. To find the equilibrium values M^{F*} , X^* , and Z^* , we maximize the social welfare function:

$$\sum_{h=1}^{M^H} (q(h)^\sigma - e[x(h) + z(h)]) \quad (20)$$

subject to the production function (13), the transactions function (14), and the factor constraints (16) - (19).

The social welfare function (20) may be rewritten as $\left(\frac{Q}{M^H}\right)^\sigma - e$, where Q is aggregate output.¹⁷ The firm f 's production function (13) may be expressed as an aggregate production function: $Q = M^F A \left(\frac{1}{M^F} X\right)^{1+\alpha}$. Similarly, using (18) and (19), firm f 's transaction function (14) may be expressed as an aggregate transactions function: $(1 - X) = M^F B \left(\frac{X}{M^F}\right)^{1+\beta}$.

Thus, the market equilibrium may be derived as the solution to the following problem:

$$\underset{X, M^F}{\text{Maximize}} \left(M^F A \left(\frac{1}{M^F} X \right)^{1+\alpha} \right)^\sigma \quad (21)$$

subject to

$$(1 - X) = M^F B \left(\frac{X}{M^F} \right)^{1+\beta}$$

The solution is

$$M^{F*} = \frac{1}{\theta (1 + B\theta^\beta)} \quad (22)$$

$$X^* = \frac{1}{1 + B\theta^\beta} \quad (23)$$

where

$$\theta = \left(\frac{\alpha}{\beta(\beta(1+\alpha) - \alpha(1+\beta))} \right)^{\frac{1}{\beta}} \quad (24)$$

From these equations it is clear that

$$\frac{\partial M^{F*}}{\partial \alpha}, \frac{\partial X^*}{\partial \alpha} < 0, \text{ and } \frac{\partial Z^*}{\partial \alpha} > 0$$

¹⁷Recall that $\sum_{h=1}^H (x(h) + z(h)) = 1$.

Furthermore, in equilibrium, $x(f) = \frac{X^*}{M^{F^*}} = \theta$, and since $\frac{\partial \theta}{\partial \alpha} > 0$, we infer that

$$\frac{\partial x^*(f)}{\partial \alpha}, \frac{\partial z^*(f)}{\partial \alpha} > 0$$

In words, the greater are the economies of scale in production (the greater is α):

- the smaller will be the equilibrium number of firms (F^*),
- the greater will be the size of each firm ($x^*(f) + z^*(f)$),
- the smaller will be the aggregate amount of factors devoted to production (X^*),¹⁸ and
- the larger will be the aggregate amount of factors devoted to internal transactions (Z^*).

4 Conclusion

This paper has provided an account of the firm as a pool of factor complementarities. Through a sequence of models we have shown how factor complementarities, together with the standard transactions costs, can determine the boundaries of the firm. Identifying factor complementarities as a unifying concept, our analysis is an attempt to integrate recent theories of the firm (that emphasize communication and coordination costs, principal-agent problems, and hold-up) with the literature on economies of scale and scope for individual production plants.

¹⁸Intuitively, the firm takes advantage of greater scale economies by economizing on its use of factors in production, while utilizing more factors for internal transactions.

References

- [1] Alchian, Alchian, and Harold Demsetz (1972), "Production, Information Costs, and Economic Organization," *American Economic Review*, Dec, 62(5), 777-95.
- [2] Baumol, W., J. Panzer and R. Willig (1982), *Contestable Markets and the Theory of Industry Structure*, New York: Harcourt Brace Jovanovich.
- [3] Bolton, Patrick, and Mathias Dewatripont (1994), "The Firm as a Communication Network," *Quarterly Journal of Economics*, 109(4), 809-839.
- [4] Coase, Ronald (1937), "The Nature of the Firm," *Economica*, Nov., 4(4), 386-405.
- [5] Grossman, Sanford and Oliver Hart (1986), "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, August, 94(4), 691-719.
- [6] Grout, Paul (1984), "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach," *Econometrica*, 52, 449-60.
- [7] Holmstrom, Bengt (1982), "Moral Hazard in Teams," *Bell Journal of Economics*, Autumn, 13(2), 324-40.
- [8] Klein, Benjamin, Robert Crawford and Armen Alchian (1978), "Vertical Integration, Appropriate Rents, and the Competitive Contracting Process," *Journal of Law and Economics*, 21, 297-326.
- [9] Lindbeck, Assar, and Dennis J. Snower (1996), "Reorganization of Firms and Labor Market Inequality", *American Economic Review*, 86(2), 315-321.
- [10] Lindbeck, Assar, and Dennis J. Snower, (2000), "Multi-task Learning and the Reorganization of Work", *Journal of Labor Economics*, 18 (3), 353-376.
- [11] Robinson, E. (1958), *The Structure of Competitive Industry*, revised edition, University of Chicago Press.
- [12] Tirole, Jean (1989), *The Theory of Industrial Organization*, MIT Press.
- [13] Viner, Jacob (1932), "Cost Curves and Supply Curves," *Zeitschrift fur Nationalokonomie*, 3, 23-46.

- [14] Williamson, Oliver (1975), *Markets and Hierarchies: Analysis and Antitrust Implications*, New York: Free Press.

CESifo Working Paper Series

(for full list see www.cesifo.de)

- 983 Syed M. Ahsan and Panagiotis Tsigaris, Choice of Tax Base Revisited: Cash Flow vs. Prepayment Approaches to Consumption Taxation, July 2003
- 984 Campbell Leith and Jim Malley, A Sectoral Analysis of Price-Setting Behavior in US Manufacturing Industries, July 2003
- 985 Hyun Park and Apostolis Philippopoulos, Choosing Club Membership under Tax Competition and Free Riding, July 2003
- 986 Federico Etro, Globalization and Political Geography, July 2003
- 987 Dan Ariely, Axel Ockenfels and Alvin E. Roth, An Experimental Analysis of Ending Rules in Internet Auctions, July 2003
- 988 Paola Conconi and Carlo Perroni, Self-Enforcing International Agreements and Domestic Policy Credibility, July 2003
- 989 Charles B. Blankart and Christian Kirchner, The Deadlock of the EU Budget: An Economic Analysis of Ways In and Ways Out, July 2003
- 990 M. Hasham Pesaran and Allan Timmermann, Small Sample Properties of Forecasts from Autoregressive Models under Structural Breaks, July 2003
- 991 Hyun Park, Apostolis Philippopoulos and Vangelis Vassilatos, On the Optimal Size of Public Sector under Rent-Seeking competition from State Cooffers, July 2003
- 992 Axel Ockenfels and Alvin E. Roth, Late and Multiple Bidding in Second Price Internet Auctions: Theory and Evidence Concerning Different Rules for Ending an Auction, July 2003
- 993 Pierre Salmon, The Assignment of Powers in an Open-ended European Union, July 2003
- 994 Louis N. Christofides and Chen Peng, Contract Duration and Indexation in a Period of Real and Nominal Uncertainty, July 2003
- 995 M. Hashem Pesaran, Til Schuermann, Björn-Jakob Treutler, and Scott M. Weiner, Macroeconomic Dynamics and Credit Risk: A Global Perspective, July 2003
- 996 Massimo Bordignon and Sandro Brusco, On Enhanced Cooperation, July 2003
- 997 David F. Bradford, Addressing the Transfer-Pricing Problem in an Origin-Basis X Tax, July 2003

- 998 Daniel Gros, Who Needs Foreign Banks?, July 2003
- 999 Wolfram Merzlyn and Heinrich W. Ursprung, Voter Support for Privatizing Education: Evidence on Self-Interest and Ideology, July 2003
- 1000 Jo Thori Lind, Fractionalization and the Size of Government, July 2003
- 1001 Daniel Friedman and Donald Wittman, Litigation with Symmetric Bargaining and Two-Sided Incomplete Information, July 2003
- 1002 Matthew Clarke and Sardar M. N. Islam, Health Adjusted GDP (HAGDP) Measures of the Relationship Between Economic Growth, Health Outcomes and Social Welfare, July 2003
- 1003 Volker Grossmann, Contest for Attention in a Quality-Ladder Model of Endogenous Growth, August 2003
- 1004 Marcel Gérard and Joan Martens Weiner, Cross-Border Loss Offset and Formulary Apportionment: How do they affect multijurisdictional firm investment spending and interjurisdictional tax competition ?, August 2003
- 1005 Burkhard Heer, Nonsuperneutrality of Money in the Sidrauski Model with Heterogeneous Agents, August 2003
- 1006 V. Anton Muscatelli, Piergiiovanna Natale, and Patrizio Tirelli, A Simple and Flexible Alternative to the Stability and Growth Pact Deficit Ceilings. Is it at hand?, August 2003
- 1007 Reto Foellmi and Josef Zweimüller, Inequality and Economic Growth: European Versus U.S. Experiences, August 2003
- 1008 James S. Costain and Michael Reiter, Business Cycles, Unemployment Insurance, and the Calibration of Matching Models, August 2003
- 1009 Marco Runkel, Optimal Contest Design when the Designer's Payoff Depends on Competitive Balance, August 2003
- 1010 Donald O. Parsons, Torben Tranaes and Helene Bie Lilleør, Voluntary Public Unemployment Insurance, August 2003
- 1011 Rüdiger Pethig and Andreas Wagener, Profit Tax Competition and Formula Apportionment, August 2003
- 1012 Johan Willner, Privatisation and Public Ownership in Finland, August 2003
- 1013 Seppo Kari and Jouko Ylä-Liedenpohja, Taxation and Valuation of International Real Investments, August 2003
- 1014 James Heckman, Rosa Matzkin and Lars Nesheim, Simulation and Estimation of Hedonic Models, August 2003

- 1015 Biswa N. Bhattacharyay, Towards a Macro-Prudential Leading Indicators Framework for Monitoring Financial Vulnerability, August 2003
- 1016 J. Stephen Ferris and Stanley L. Winer, Searching for Keynes: With Application to Canada, 1870-2000, August 2003
- 1017 Massimo Bordignon, Luca Colombo and Umberto Galmarini, Fiscal Federalism and Endogenous Lobbies' Formation, August 2003
- 1018 Annette Alstadsæter, The Dual Income Tax and Firms' Income Shifting through the Choice of Organizational Form and Real Capital Investments, August 2003
- 1019 Peter Fredriksson and Bertil Holmlund, Optimal Unemployment Insurance Design: Time Limits, Monitoring, or Workfare?, August 2003
- 1020 Kashif S. Mansori, Following in their Footsteps: Comparing Interest Parity Conditions in Central European Economies to the Euro Countries, August 2003
- 1021 Christoph Borgmann and Matthias Heidler, Demographics and Volatile Social Security Wealth: Political Risks of Benefit Rule Changes in Germany, August 2003
- 1022 Kjell Erik Lommerud, Bjørn Sandvik and Odd Rune Staume, Good Jobs, Bad Jobs and Redistribution, August 2003
- 1023 Patrick Karl O'Brien, The Governance of Globalization: The Political Economy of Anglo-American Hegemony, 1793-2003, September 2003
- 1024 Antonio Ciccone and Giovanni Peri, Skills' Substitutability and Technological Progress: U.S. States 1950-1990, September 2003
- 1025 Bjørn Sandvik, Optimal Taxation and Normalisations, September 2003
- 1026 Massimo Bordignon and Gilberto Turati, Bailing Out Expectations and Health Expenditure in Italy, September 2003
- 1027 José A. Herce, Namkee Ahn, Ricard Génova, and Joaquín Pereira, Bio-Demographic and Health Aspects of Ageing in the EU, September 2003
- 1028 John Komlos and Marieluise Baur, From the Tallest to (One of) the Fattest: The Enigmatic Fate of the American Population in the 20th Century, September 2003
- 1029 Stefan Napel and Mika Widgrén, Bargaining and Distribution of Power in the EU's Conciliation Committee, September 2003
- 1030 Kai Li and Dale J. Poirier, Relationship Between Maternal Behavior During Pregnancy, Birth Outcome, and Early Childhood Development: An Exploratory Study, September 2003
- 1031 Ivar Ekeland, James J. Heckman, and Lars Nesheim, Identification and Estimation of Hedonic Models, September 2003

- 1032 Kjetil Bjorvatn and Alexander W. Cappelen, Decentralization and the Fate of Minorities, September 2003
- 1033 Lars-Erik Borge and Jørn Rattsø, The Relationships Between Costs and User Charges: The Case of a Norwegian Utility Service, September 2003
- 1034 Maureen Were and Nancy N. Nafula, An Assessment of the Impact of HIV/AIDS on Economic Growth: The Case of Kenya, September 2003
- 1035 A. Lans Bovenberg, Tax Policy and Labor Market Performance, September 2003
- 1036 Peter Birch Sørensen, Neutral Taxation of Shareholder Income: A Norwegian Tax Reform Proposal, September 2003
- 1037 Roberta Dessi and Sheilagh Ogilvie, Social Capital and Collusion: The Case of Merchant Guilds, September 2003
- 1038 Alessandra Casarico and Carlo Devillanova, Capital-skill Complementarity and the Redistributive Effects of Social Security Reform, September 2003
- 1039 Assaf Razin and Efraim Sadka, Privatizing Social Security Under Balanced-Budget Constraints: A Political-Economy Approach, September 2003
- 1040 Michele Moretto, Paolo M. Panteghini, and Carlo Scarpa, Investment Size and Firm's Value under Profit Sharing Regulation, September 2003
- 1041 A. Lans Bovenberg and Peter Birch Sørensen, Improving the Equity-Efficiency Trade-off: Mandatory Savings Accounts for Social Insurance, September 2003
- 1042 Bas van Aarle, Harry Garretsen, and Florence Huart, Transatlantic Monetary and Fiscal Policy Interaction, September 2003
- 1043 Jerome L. Stein, Stochastic Optimal Control Modeling of Debt Crises, September 2003
- 1044 Thomas Stratmann, Tainted Money? Contribution Limits and the Effectiveness of Campaign Spending, September 2003
- 1045 Marianna Grimaldi and Paul De Grauwe, Bubbling and Crashing Exchange Rates, September 2003
- 1046 Assar Lindbeck and Dennis J. Snower, The Firm as a Pool of Factor Complementarities, September 2003