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Endogenous Entry and Growth of Firms with Heterogeneous Firms

Markus Thomas Münter¹

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Abstract

We analyze the impact of persistent cost heterogeneity of potential firms on free entry equilibria under a Cournot setting. We show that firm heterogeneity always reduces the number of firms in equilibrium, and that vertical and horizontal market growth have different effects on industry population and firm dynamics. Vertical market growth always promotes entry and growth of new firms, but heterogeneity weakens this effect. For horizontal market growth, we identify two growth regimes: incumbents dominate in large markets with significant heterogeneity, while new entrants grow faster in smaller markets with low heterogeneity. In contrast to existing research, we show that higher degrees of heterogeneity among potential firms reduces entry rates in both cases.

Keywords Endogenous market structure · Heterogeneity · Number of firms · Entry rates · Growth of firms · Cournot-nash-equilibria · L1 · L11 · L22

JEL Classifications L1 · L11 · L22

1 Market Size, Number of Firms and Entry Rates

Does the number of firms increase when a market grows? Is the entry rate into an industry identical to the growth rate of a market? Or do only incumbents grow when markets grow—without leaving much space for new entrants? Does it make a difference if markets grow vertically or horizontally? A large body of studies has examined the impact of changes in market size on market structure, especially with regard to the number and size distribution of firms, as well as growth patterns of firms (among others Bresnahan & Reiss, 1991; Campbell & Hopenhayn, 2005; Neumann et al., 2001; Sutton, 1997).

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From both a strategy and a competition policy perspective, it is important to understand whether increasing demand predominately gives rise to the entry of new firms or whether incumbents will mainly grow, even without barriers to entry. Theoretical approaches often assume potential firms to be uniform—all firms have access to the same technology and know-how, and hence have identical marginal costs—at least in the long run. Entry rates under Cournot competition with uniform firms are well-understood: If markets grow horizontally, the elasticity of entry lies between 0.5 and 0.66; but if markets grow vertically, the elasticity of entry is equal to 1 (see Neumann et al., 2001, and Sect. 3 below).

However, cost heterogeneity among firms within the same industry is a well-documented empirical fact (Belitski et al., 2023; Bernard et al., 2012; Geroski et al., 2003; Hottman et al., 2016; Jensen & McGuckin, 1997; Syverson, 2014).

This paper analyzes the effects of persistent heterogeneity of firms' marginal costs on endogenous market structure—on the number of firms, growth rates, and entry rates—under a simple Cournot setting for a homogeneous product market without barriers to entry. We will demonstrate that—contrary to intuition—if markets expand, sometimes incumbents with a cost advantage grow faster than do new firms, and sometimes vice versa. These different growth regimes depend on the interplay of the degree of heterogeneity of firms and the type of market growth (horizontal versus vertical). We aim to provide a way to understand the comparative statics of endogenous market structure.

1.1 Organization of this Paper

In Sect. 2, we develop a model of Cournot competition with free entry, where potential firms are heterogeneous due to differences in their marginal costs. We identify Cournot-Nash equilibria that are determined by the size of the market, following Bresnahan and Reiss (1991), Sutton (1997) and Neumann et al. (2001). In Sect. 3, we consider two types of market growth: changes in vertical and in horizontal market size. When the market size changes, all firms adjust to their equilibrium values through individual growth, entry, or exit.

We find that (i) firm heterogeneity reduces the number of firms in equilibrium. We show that (ii) vertical market growth always promotes entry and growth of new firms, whereas (iii) horizontal market growth gives rise to two different growth regimes: incumbents dominate in large markets with significant heterogeneity, while new entrants grow faster in smaller markets with low heterogeneity. Finally, we show that (iv) higher degrees of heterogeneity reduce entry rates for both vertical and horizontal market growth.

Similar approaches have been proposed, e.g. by Melitz (2003) as well as by Melitz and Ottaviano (2008). These are based on extensions of monopolistic competition along the lines of Dixit and Stiglitz (1977) in the context of international trade including product differentiation and uncertainty about firm productivity. A discussion and comparison of their results is provided in the concluding remarks.

2 Cost Heterogeneity Among Firms in a Cournot Model

In this model, we assume that firms are persistently cost heterogeneous: Firms are not able to catch up with the most efficient firm in an industry. This may be due to organizational inertia, path dependencies, and/or incomplete learning, or simply limited access to state-of-the-art know-how. Hence, marginal costs are constant for each firm, but they differ permanently across firms.

2.1 Number of Firms with Discrete Marginal Costs

In this section, we show that differences in marginal costs reduce the number of firms in equilibrium. We consider a homogeneous product market with a linear demand function $p(Q) = a - bQ$ with parameters $a, b > 0$, $a > c_i$ and total production of all firms i given by $Q = \sum q_i$. Assume that there is a certain ranking of all potential firms $i = 1, 2, \dots, z, z+1, \dots, \infty$ with respect to their marginal costs $c_i = c_1, c_2, \dots, c_z, c_{z+1}, \dots, c_\infty$ or efficiencies, where $c_1 < c_2 < \dots < c_\infty$ applies (Bergstrom & Varian, 1985, and Münter, 2017). With a cost function $C_i = c_i q_i + F$ that includes firm-specific marginal costs c_i and industry-specific fixed costs F that are identical to all firms, the profit function of each firm i is $\pi_i(q_i, Q - q_i) = p(Q)q_i - C_i$.

Under a Cournot setting, each firm i maximizes its profits by choosing its individual production q_i as

$$q_i = \frac{a - c_i}{b} - Q. \quad (1)$$

Total quantity and price are

$$Q = n \frac{a - \bar{c}}{b} - nQ = \frac{n}{n+1} \frac{a - \bar{c}}{b} \quad (2)$$

and

$$p(Q) = a - bQ = \frac{a + n\bar{c}}{n+1}, \quad (3)$$

with $\bar{c} = \frac{1}{n} \sum_{i=1}^n c_i$ as average marginal costs across all firms.

Differences in marginal costs enter Eqs. (2) and (3) regardless of their distribution only as averages of marginal costs. Hence, the outcome of the competitive process represented in this Nash equilibrium is independent of the exact distribution of firms' characteristics. It only depends on the average (respectively the sum) of marginal costs. This result remains valid for any empirical or theoretical distribution of firms' marginal costs: There is no necessity for a smooth or well-defined distribution function.

Under free entry, all firms will enter the market when they can produce a positive quantity q_i with non-negative profits π_i given their individual marginal costs c_i . Suppose that there is currently a positive number z of firms in the market that achieve

non-negative profits. Given the discrete positions of the firms, it follows that there is a firm $i = z + 1$, for which profits, as expressed below,

$$\pi_i = \left(\frac{a + n\bar{c}}{n+1} - c_i \right) \left(\frac{a - c_i}{b} - \frac{n}{n+1} \frac{a - \bar{c}}{b} \right) - F = \frac{1}{b} \left(\frac{a - c_i + n(\bar{c} - c_i)}{n+1} \right)^2 - F \quad (4)$$

eventually become negative. Hence, using Eq. (2) and setting profits (4) to equal zero for firm z , the number of firms n is endogenously determined as

$$n_{\bar{c} < c_z} = \frac{a - c_z - \sqrt{bF}}{\sqrt{bF} + (c_z - \bar{c})} = \frac{a - \bar{c} - \sqrt{bF} - (c_z - \bar{c})}{\sqrt{bF} + (c_z - \bar{c})}. \quad (5)$$

c_z equals the marginal costs of firm z , which—for a given size of the market—is the cut-off firm with the highest marginal costs within the industry (see Mnter, 2017 and Melitz, 2003).

The number of firms in equilibrium is lower with heterogeneous marginal costs as compared to a situation with uniform firms, each exactly having average costs \bar{c} ,

$$n_{\bar{c} = c_z} = \frac{a - \bar{c} - \sqrt{bF}}{\sqrt{bF}} > \frac{a - \bar{c} - \sqrt{bF} - (c_z - \bar{c})}{\sqrt{bF} + (c_z - \bar{c})} = n_{\bar{c} < c_z}, \quad (6)$$

since for all types of distributions, the average value \bar{c} is lower than the maximum value c_z , and hence $(c_z - \bar{c}) > 0$. Therefore, heterogeneity of marginal costs at the firm-level reduces the number of firms in equilibrium. The larger the cost asymmetries, the smaller the free-entry equilibrium number of firms in an industry.

From (6), it also becomes clear that the number of firms always has an upper bound, as long as $(c_z - \bar{c}) > 0$: Differences in marginal costs are sufficient to limit the number of firms in equilibrium, even if fixed costs F are zero. From this point of view, fixed costs and differences in marginal costs have a similar impact: Both limit the number of firms in a Cournot-Nash equilibrium.

2.2 Number of Firms with Continuous Marginal Costs

In this section, we enhance the model above to analyze the impact of various degrees of heterogeneity. We assume a continuum of potential firms that can be measured by a continuous variable $n > 0$. For simplicity and convenience, we will not deal with problems that involve a non-integer number of firms. Let there be a continuous function that assigns marginal costs c_i to any firm's index i (for a similar approach see Martin, 1993), hence,

$$c_i = \chi + i^\varphi. \quad (7)$$

The exponent φ reflects various degrees of heterogeneity of firms' marginal costs, while χ denotes the basic level of marginal costs. One might think of a ladder of efficiencies, with each potential firm having different marginal costs, where φ controls

the spacing between the rungs of the ladder. For $\varphi = 0$, all firms have marginal costs equal to $\chi + 1$. For $\varphi > 0$, all firms differ in marginal costs: For larger values of φ , the spacing between the firms' marginal costs increases.

If marginal costs of firms are described by Eq. (7), then based on

$$\int_{i-1}^i c_i = \int_{i-1}^i (\chi + i^\varphi), \quad (8)$$

average marginal costs \bar{c}_i of a firm i are

$$\begin{aligned} \bar{c}_i &= \frac{1}{i-1} \int_{i-1}^i (\chi + i^\varphi) = i\chi + \frac{1}{1+\varphi} i^{1+\varphi} - (i-1)\chi - \frac{1}{1+\varphi} (i-1)^{1+\varphi} \\ &= \chi + \frac{1}{1+\varphi} (i^{1+\varphi} - (i-1)^{1+\varphi}). \end{aligned} \quad (9)$$

To determine industry equilibria, we first identify the number of firms and average marginal cost in the industry. As in the section above, the number of firms n_z in an industry is determined by the highest-cost firm z , which exists in equilibrium with non-negative profits. The sum of marginal costs within the industry then equals $n_z \bar{c}$. For continuous marginal costs, $n_z \bar{c}$ equals

$$\int_0^z c_i = \int_0^z (\chi + i^\varphi), \quad (10)$$

with a primitive function

$$\Gamma(n_z) = n_z \chi + \frac{1}{1+\varphi} n_z^{1+\varphi} + \mu - \frac{1}{1+\varphi} n_0^{1+\varphi} - \mu = n_z \chi + \frac{1}{1+\varphi} n_z^{1+\varphi}, \quad (11)$$

where μ is a constant. It follows that average marginal costs across all firms in the industry are

$$\bar{c} = \frac{1}{n_z} \Gamma(n_z) = \chi + \frac{1}{1+\varphi} n_z^\varphi \quad (12)$$

Putting these results into Eqs. (4), (2), and (3) from above, we see that the free entry-equilibrium number of firms, total quantity of all firms, and price are, respectively:

$$n_z = \frac{a - (\chi + n_z^\varphi) - \sqrt{bF}}{\sqrt{bF} - \left(\chi + \frac{1}{1+\varphi} n_z^\varphi - \chi - n_z^\varphi\right)} = \frac{a - (\chi + n_z^\varphi) - \sqrt{bF}}{\sqrt{bF} + \frac{\varphi}{1+\varphi} n_z^\varphi}, \quad (13)$$

$$Q = \frac{n_z}{n_z + 1} \frac{a - \left(\chi + \frac{1}{1+\varphi} n_z^\varphi\right)}{b}, \quad (14)$$

and

$$p(Q) = \frac{a + n_z \left(\chi + \frac{1}{1+\varphi} n_z^\varphi \right)}{n_z + 1} = \frac{a + n_z \chi + \frac{1}{1+\varphi} n_z^{1+\varphi}}{n_z + 1}. \quad (15)$$

For uniform firms with $\varphi = 0$, the well-known textbook results ensue. With regard to the number of firms n_z in equilibrium, we do not require positive fixed costs F to limit the number of firms: As long as $\varphi > 0$, the number of firms n_z is always smaller than infinity (compare Eq. (6)). However, to keep the model flexible in the remainder of this approach, we assume $F > 0$, in particular to allow $\varphi = 0$.

Due to the structure of (13), (14), and (15), one has to solve only Eqs. (13) and (14). Taking the expression

$$\frac{n_z}{n_z + 1} = \frac{a - \sqrt{bF} - (\chi + n_z^\varphi)}{a - \left(\chi + \frac{1}{1+\varphi} n_z^\varphi \right)} \quad (16)$$

from (13) into (14), one gets

$$Q = \frac{a - \sqrt{bF} - (\chi + n_z^\varphi)}{b}. \quad (17)$$

For $\varphi = 0$ —uniform firms—we obtain the well-known solutions

$$n_z = \frac{a - (\chi + 1) - \sqrt{bF}}{\sqrt{bF}} \quad (18)$$

and

$$Q = \frac{a - (\chi + 1) - \sqrt{bF}}{b}. \quad (19)$$

For $\varphi = 1$ —a linear increase in marginal costs across firms—it follows that

$$n_z = -1 - \sqrt{bF} \pm \sqrt{2(a - \chi) + bF + 1} \quad (20)$$

and

$$Q = \frac{(a - \chi) - \sqrt{2(a - \chi) + bF + 1} + 1}{b}. \quad (21)$$

The sign of the last term of (20) must be positive to ensure a positive number of firms. When comparing (18) and (20), analogous to the discrete number case in the section above, the number of firms in equilibrium is lower due to heterogeneity, since

$$n_{z(\varphi=0)} = \frac{a - (\chi + 1) - \sqrt{bF}}{\sqrt{bF}} > -1 - \sqrt{bF} + \sqrt{2(a - \chi) + bF + 1} = n_{z(\varphi=1)} \quad (22)$$

Comparing (19) and (21) shows that total quantity is lower when firms are heterogeneous,

$$Q_{(\varphi=0)} = \frac{a - (\chi + 1) - \sqrt{bF}}{b} > \frac{(a - \chi) - \sqrt{2(a - \chi) + bF + 1} + 1}{b} = Q_{(\varphi=1)}, \quad (23)$$

for $a > \chi$, as assumed.

The typical effects of changes in the cost structure remain true: Higher fixed costs F and/or higher levels of marginal costs χ lead to a smaller number of firms n and a smaller total quantity Q . However, while it is easy to calculate equilibrium values and comparative statics for the case of uniform firms, this proves to be impossible for heterogeneous firms with arbitrary values $\varphi > 0$.

3 Computation of Industry Equilibria

In this section, we discuss industry equilibria under free entry. Computation of numerical solutions is necessary since the simultaneous equation system (13) and (17) determining n and Q cannot be solved analytically for most parameter settings. In Sect. 3.1, we examine the effects of varying degrees of heterogeneity among firms controlled by φ on market structure, with a view to identifying lower and upper bounds of firm size. In Sect. 3.2, we analyze how changes in market size due to the dynamics of a and b affect the number of firms in order to differentiate the share of market growth imputable to new firms from the share explained by the growth of incumbents. In Sect. 3.3, we develop a measure for entry elasticity in growing markets, and show that entry rates are significantly lower under heterogeneity.

3.1 Industry Equilibrium Solutions

In this section, we derive equilibrium solutions for: the number of firms n ; total quantity Q ; price p ; and average firm size q_{avg} —as well as upper and lower bounds on firm size. Equilibria are computed with the use of the initial parameter choices $a = 100, b = 1, F = 50, \chi = 5$ to ensure that the number of firms is larger than 1, including for substantial variations of all parameters. We assume that fixed costs F are positive, so as to keep all results in an economically meaningful range—but more important, to allow $\varphi = 0$ and to inspect differences between industries that are populated by uniform ($\varphi = 0$) or heterogeneous firms ($\varphi > 0$). The model is quite robust against changes of parameters. Only large values of φ reduce the number of firms below 1.

3.1.1 Market Structure and Firm Characteristics

In Fig. 1, we show results that are related to the degree of heterogeneity $\varphi \in [0; 1.2]$. The top row of Fig. 1 shows that as heterogeneity increases, total quantity and the

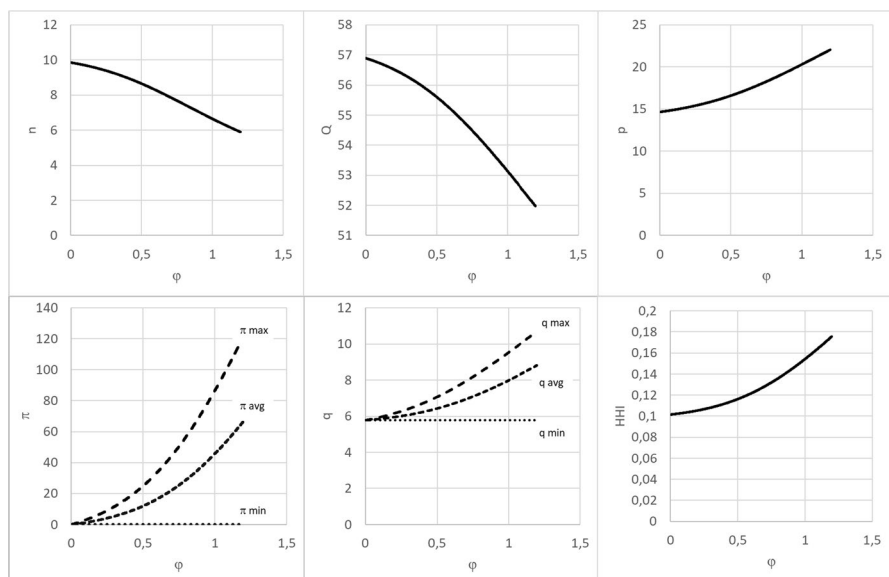


Fig. 1 Market structure with heterogeneous firms, with number of firms n , total quantity Q , price p , profits $\pi_{\min}, \pi_{\text{avg}}, \pi_{\max}$, firm size $q_{\min}, q_{\text{avg}}, q_{\max}$, and the Hirschmann-Herfindahl Index HHI (increasing degree of heterogeneity $\phi \in [0; 1.2]$, $a = 100, b = 1, F = 50, \chi = 5$)

number of firms in equilibrium decrease. For each size a and b of the market and given technology F and χ , the active number of firms decreases along an S-shaped curve as the heterogeneity of potential firms increases. Total quantity declines as well, accompanied by an increase in price.

Dividing total output Q by the number of firms shows that the average firm size q_{avg} in equilibrium increases. This effect is because large firms grow disproportionately with increasing values of ϕ , while minimum firm size q_{\min} in equilibrium is independent of ϕ , as is shown in the bottom row of Fig. 1. Taking these two effects together, the Hirschmann-Herfindahl Index HHI increases more than proportionately for larger values of ϕ : The number of firms decreases and, at the same time, the average firm size increases.

A key finding is that heterogeneity favors incumbents over new entrants and favors large firms over small firms: The intensity of competition decreases, prices are higher, and individual firms' profits increase (see Fig. 1, bottom left). Hence, persistent heterogeneity of potential firms has effects that are similar to barriers to entry.

Comparing these findings with results of Melitz and Ottaviano (2008), which are based on monopolistic competition, provides some additional insights: We are able not only to show effects on average profitability and firm size, but also explicitly to give lower and upper bounds on profits, marginal costs, and firm size in equilibrium, as is shown in Fig. 1. Moreover, Melitz (2003, p. 1700) shows that total production is only dependent on average productivity and the number of active firms;

whereas in the model that is presented here, the exact distribution and asymmetry of marginal costs (and not just the average) determines the total quantity produced. In contrast to the results of Melitz and Ottaviano (2008), we can now show that the variance of marginal costs increases with market size.

3.1.2 Vertical Market Size

To gain a deeper understanding of the connection between the number of firms and firm size we now look at the impact of vertical and horizontal market size. Levels of a indicate the willingness to pay, and changes of a can be considered vertical market growth. Levels of b indicate the slope of the demand curve and changes of b can be considered horizontal market growth (see Neumann et al., 2001, for a similar approach). Figure 2 shows equilibria for n and Q as a function of different levels of a .

Inspecting Fig. 2, we see that higher levels of a lead to a larger number of firms n and a larger total quantity Q for all levels of heterogeneity φ . Moreover, for any given increase in heterogeneity φ , the higher are the values of a , the larger is the absolute decrease in the number of firms. The same is true for Q , but the absolute change is significantly smaller.

If we now combine the change in the number of firms dn given a change in total quantity dQ and define a sensitivity dn/dQ (Fig. 2, top row right), two observations can be made: First, the values of dn/dQ converge for higher heterogeneity; and second, the course of the sensitivity dn/dQ depends on the level of a and appears to be strongly nonlinear.

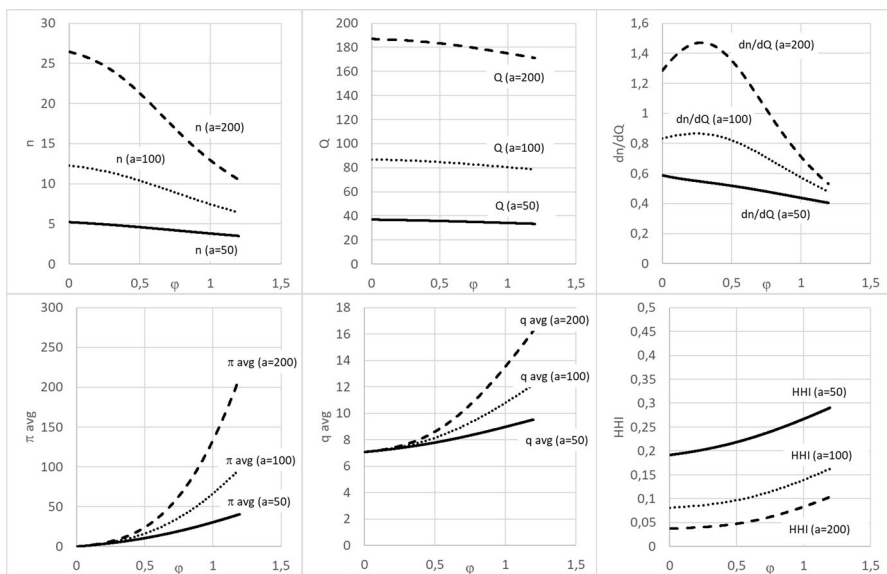


Fig. 2 Impact of vertical market size on the number of firms n , total quantity Q , sensitivity dn/dQ , average profits π_{avg} , average firm size q_{avg} , and the Hirschmann-Herfindahl Index HHI (increasing degree of heterogeneity $\varphi \in [0; 1.2]$, $b = 1$, $F = 50$, $\chi = 5$, different levels of a)

settings, for levels of a larger than 100, dn/dQ is a one-humped curve: dn/dQ is increasing for low levels of heterogeneity φ and decreasing for high levels of φ . We will inspect this relationship and possible explanations in more detail in Sect. 3.2.

Higher values of a give rise to increasing profit levels, larger average firm size, but also greater intensity of competition (as measured by the Hirschmann-Herfindahl Index HHI). The greater is the heterogeneity across firms, the greater is the dispersion of profits and firm size.

3.1.3 Horizontal market size

Similar results are obtained when considering different levels of horizontal market size measured by b , as is shown in Fig. 3.

Lower values of b lead to a larger number of firms and higher total quantity in equilibrium. Once again, we see relatively larger effects in the number of firms compared to an only moderate decline in total quantity for a higher degree of heterogeneity φ . In addition, the sensitivity dn/dQ is again a one-humped curve that peaks at low levels of heterogeneity.

With growing market size (lower levels of b), average profits and average firm size increase. Horizontal concentration increases as markets become smaller, while the values of the Hirschmann-Herfindahl Index HHI converge when the degree of heterogeneity is high—which reflects a sharp decline in the number of firms that clearly outweighs the effect of the increasing variance of firm sizes.

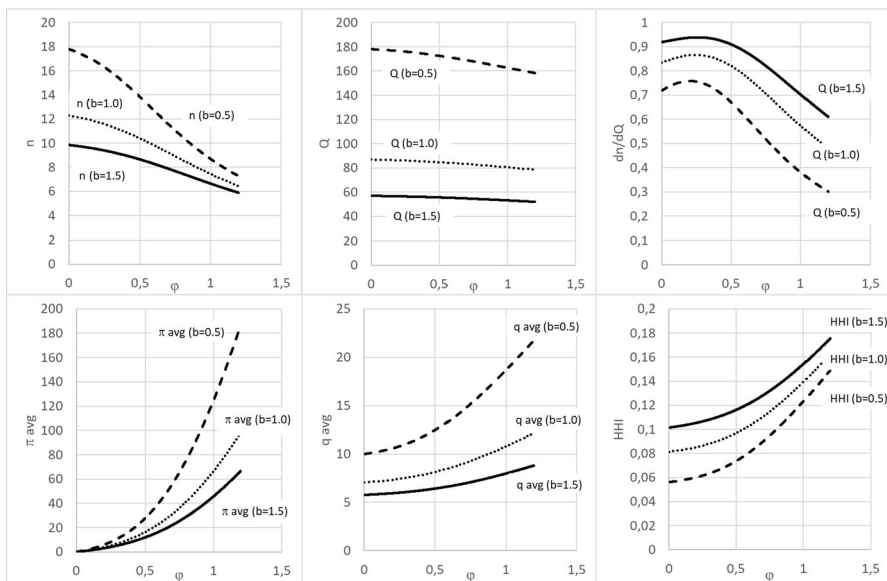


Fig. 3 Impact of horizontal market size on the number of firms n , total quantity Q , sensitivity dn/dQ , average profits π , average firm size q_{avg} , and the Hirschmann-Herfindahl Index HHI (increasing degree of heterogeneity $\varphi \in [0; 1.2]$, $a = 100$, $F = 50$, $\chi = 5$, different levels of b)

3.2 Growth Patterns of Incumbents and New Firms

In this section, we analyze how market growth is divided between incumbents and new firms: What share of total market growth— $\Delta Q/Q$ —is captured by growing incumbents compared to the share that new firms take by entering and reaching their equilibrium size. As before, we consider two different types of market growth: We first analyze a vertical increase in the market size; we then consider a horizontal increase in market size.

3.2.1 Vertical Market Growth

First, we inspect vertical market growth: an increase of a . We define $\frac{\Delta Q}{Q} = \frac{\Delta Q_{inc}}{Q} + \frac{\Delta Q_{ent}}{Q} = S_{inc} + S_{ent} = 100\%$ as the sum of the shares of market growth accounted for by incumbents S_{inc} and new entrants S_{ent} , respectively. Based on our simulations, Fig. 4 shows patterns of firm growth that are related to the degree of heterogeneity of firms. The three panels depict changes in vertical market size a , with the degree of firm heterogeneity φ increasing as we move from left to right.

A first finding is that in vertically growing markets new entrants always capture most of the market growth ($\frac{\Delta Q}{Q}$): $S_{ent} > S_{inc}$. If firms' marginal costs are identical, all market growth is taken by new entrants. Hence, incumbent firms' size remains the same, and new entrants reach an equilibrium size that is identical to that of an incumbent firm (left panel). However, if firms differ in their marginal costs, incumbents can also grow; consequently, market growth is not imputable to new firms alone. This effect is stronger when the degree of firm heterogeneity is greater (middle and right panels).

The economic intuition is twofold: For uniform firms, (i) vertical market growth will always trigger entry of new firms and lead to an equilibrium of firms with identical size. However, if firms differ in marginal costs, then (ii) incumbents with a cost advantage can capture some of the growth opportunities that are due to their inherent advantage of having lower marginal costs and thus limit the growth of new entrants. In addition, the location of the curves that are shown in Fig. 4 is affected by the level of fixed costs F : If there are differences in marginal costs, a higher level of

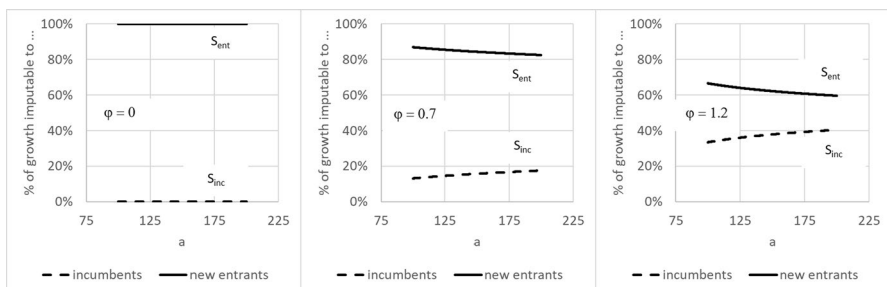


Fig. 4 Distribution of market growth between incumbents and new entrants (vertical increase of market size: $a = 100 \Rightarrow a = 200; b = 1; F = 50; \chi = 5$)

fixed costs shifts the curves closer toward the 50% growth split, while a lower level widens the spread between incumbents and new entrants.

3.2.2 Horizontal market growth

The dynamics appear to be quite different when we analyze horizontal market growth (Fig. 5). We begin our analysis with a range of b between 0.2 and 1.2. If firms are uniform (left panel, where $\varphi = 0$), at any market size new entrants account for little more than 50% of market growth. However, the growth rates of incumbents and new firms converge to 50% when the market is larger (when b is smaller), and hence the larger is the number of firms in a market (see also Sect. 3.3). The split of market growth ($\frac{\Delta Q}{Q}$) between incumbents and new entrants for $\varphi = 0$ would converge to $S_{ent} = S_{inc} = 50\%$ each, at any level of b if there were no fixed costs. However, without fixed costs, the number of firms would always be infinite, and a distinction between incumbents and new entrants would then be impossible.

With significant heterogeneity (middle panel, where $\varphi = 0.7$), gains in market share reverse: Incumbents now increasingly take the lion's share, and the growth that is due to new entrants is significantly smaller, so that $S_{ent} < S_{inc}$. These effects become stronger as heterogeneity increases: With $\varphi = 1.2$ (right panel), almost all market growth is taken by incumbents, which implies that the increase in the number of firms comes almost to a halt. The economic intuition is that the more-efficient incumbent firms can now expand due to their significantly lower marginal costs, thereby blocking entry and limiting the growth opportunities for new firms (which have higher marginal costs). However, this effect is not based on an increase of minimum efficient size, but is due merely to lower entry rates (see also Sect. 3.3 below).

To examine the impact of heterogeneity φ and fixed costs F in more detail, Fig. 6 shows the relationship of the distribution of market growth between incumbents and new entrants for different levels of fixed costs F and for larger ranges of b , in particular converging toward 0.

For uniform firms and $\varphi = 0$ (left panel) we find that higher fixed costs F spread the distribution of market growth between incumbents and new entrants for any level of horizontal market size b , while $S_{ent} > S_{inc}$ remains true. As markets become larger

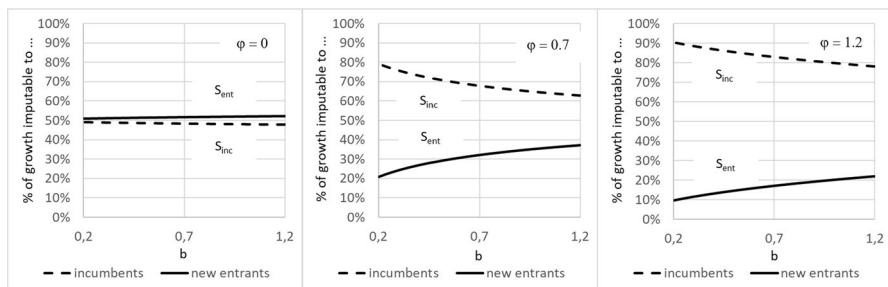


Fig. 5 Distribution of market growth between incumbents and new entrants (horizontal increase of market size = 1.2 $\Rightarrow b = 0.2, a = 100, F = 50, \chi = 5$)

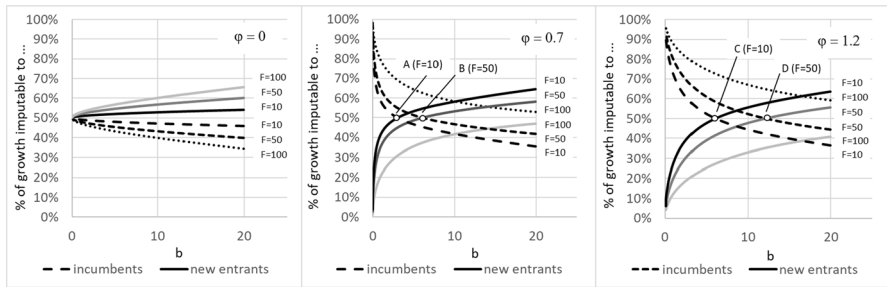


Fig. 6 Distribution of market growth between incumbents and new entrants for different levels of fixed costs (horizontal increase of market size = $20 \Rightarrow b \rightarrow 0$, $a = 100$, $F = 10; 50; 100$, $\chi = 5$)

and b converges to zero, this gap gets smaller and values converge to 50% for every level of fixed costs F .

When firms are heterogeneous (middle and right panels), for sufficiently large values of b there is an intersection—where $S_{ent} = S_{inc}$ —at which growth shares of incumbents (dashed and dotted lines) and growth shares of new entrants (straight lines) are identical. For example: To the left of point A (at a level of fixed costs $F = 10$) the share of the growth of incumbents S_{inc} is larger than that of new entrants S_{ent} , while to the right of point A we find the reverse.

This implies that there is a critical value of b that separates growth regimes in larger markets (lower values of b) that favor incumbents—where $S_{ent} < S_{inc}$ prevails—from growth regimes in smaller markets (higher values of b) that favor new entrants, with $S_{ent} > S_{inc}$. The separation of these two regimes depends on the level of fixed costs F . Comparing points A and B (middle panel), we find that higher fixed costs F shift this intersection to the right, hence the critical value of b increases. Our simulations also show that for very large fixed costs F , S_{ent} and S_{inc} may not intersect at all. The same is true if we increase ϕ : For higher degrees of heterogeneity, the critical values of b increase (points C and D in the right panel). Hence, the range in which $S_{ent} > S_{inc}$ holds becomes smaller when ϕ is larger.

Summarizing these results, we find that vertical market growth favors new firms over incumbents, although this growth advantage weakens as the degree of heterogeneity across firms increases. When a market grows horizontally, the emerging patterns are ambiguous: When there is little heterogeneity across firms and fixed costs are small, new entrants capture most of the market growth; but increasing both fixed costs F and heterogeneity ϕ limit this growth regime, thereby reducing growth and entry rates of new firms.

Section 3.3 will examine this relationship in more detail: We identify a bound for entry rates in growing markets as a function of the degree of heterogeneity.

3.3 Dynamics of Entry

We now examine the effects of vertical and horizontal market growth on entry rates and show that the share taken by incumbents significantly depends on the degree of firm heterogeneity: The larger are the cost asymmetries, the smaller is the entry rate of new firms and the larger is the growth rate of incumbents, and vice versa.

We introduce a measure β to study the elasticity of the number of firms to an increase in total production—the net entry rate—and again consider two different types of market growth: a vertical increase in market size that is represented by an increase in a ; and a horizontal increase of the market that is captured by a decrease of b . In the absence of exit barriers, all entry results can be easily transformed into predictions of exit in shrinking markets.

3.3.1 Vertical Increase in Market Size

Using (18) and (19), it is easy to see that the coefficient β_a —which measures the elasticity of the number of firms to a vertical increase in market size (changes in a)—is equal to one in the uniform firm case, since

$$\beta_a = \frac{Q}{n} \frac{\partial n / \partial a}{\partial Q / \partial a} = \frac{\sqrt{bF}}{b} \frac{b}{\sqrt{bF}} = 1. \quad (24)$$

Thus, with free entry, uniform firms, and vertical growth of the market, β_a predicts that market growth will be completely captured by new entrants, and average firm size will not be affected (compare also Sect. 3.2 and Fig. 4). When firms are uniform ($\varphi = 0$), net entry is proportional to the increase of market size (total quantity Q), which means that the number of firms grows at the same rate as the size of the market, with average firm size remaining constant.

For heterogeneous firms ($\varphi > 0$), as explained above, it is impossible to determine this ratio analytically, so we have to rely on the numerical results of our simulations to estimate β_a .

As can be seen from Fig. 7 (left panel), the greater is the heterogeneity of firms, the lower is the net entry rate (β_a) and the greater is the average size of firms. However, depending on the degree of heterogeneity, there is now a lower bound for the net entry rate. This lower bound is determined by the degree of heterogeneity among (potential) firms and the extent of fixed costs relative to market size, and indicates how a vertical increase of the market size is split between net entry rates of new entrants and the growth of incumbents.

3.3.2 Horizontal Increase in Market Size

Calculating the elasticity β_b of the number of firms to the size of the market for the case of uniform firms (for (18) and (19)) and for changes in the slope b of the demand curve, we obtain

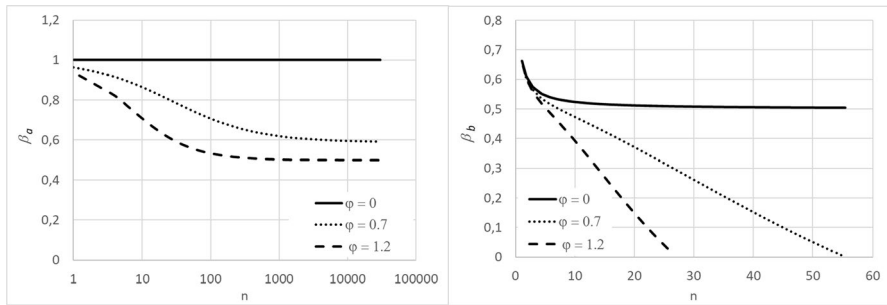


Fig. 7 Elasticity β_a of the number of firms given a vertical increase of the market (left panel, log scale) and elasticity β_b of the number of firms given a horizontal increase of the market (right panel), $F = 50$; $\chi = 5$

$$\beta_b = \frac{Q}{n} \frac{\partial n / \partial b}{\partial Q / \partial b} = \frac{\sqrt{bF}}{b} \frac{\frac{1}{2} b^2 (\chi + 1 - a) F}{b F \sqrt{bF} \left(-\frac{1}{2} \sqrt{bF} - a + \chi + 1 \right)} = \frac{a - (\chi + 1)}{2(a - (\chi + 1)) - \sqrt{bF}}. \quad (25)$$

The level of β_b increases as fixed costs F increase.

From (18), we get

$$\frac{n_z}{n_z + 1} = \frac{a - (\chi + 1) - \sqrt{bF}}{a - (\chi + 1)}. \quad (26)$$

Substituting this into (24), we obtain, after a few rearrangements.

$$\beta_b = \frac{n_z + 1}{2n_z + 1}, \quad (27)$$

with a spread of β_b between 0.66 ($n = 1$) and 0.5 ($n \rightarrow \infty$), depending on the number of firms. This result is identical to that of Neumann et al. (2001): Note that these bounds per se do not appear to be independent of the level of fixed costs F , as can be seen from Eq. (25) (see also Neumann et al., 2001, p. 830). However, given Eq. (27), the impact of fixed costs F is fully represented and accounted for in the number of firms n_z , and the range of β_b with respect to the number of firms is always between 0.66 ($n = 1$) and 0.5 ($n \rightarrow \infty$). If—as is argued in Neumann et al. (2001)—incurring higher fixed costs does reduce marginal costs, the lower and upper bounds of β_b might even be lower.

Although it is impossible to determine analytically the value of β_b for heterogeneous firms, we can examine our numerical data from simulations to estimate its value. Figure 7 (right panel) shows β_b values for uniform and heterogeneous firms. Heterogeneity has a significant effect on the net entry rate of an industry. Unlike the case of uniform firms (with $\varphi = 0$), there is no stable lower bound for the net entry rate that accompanies an increase in market size. Instead, the larger is the number of firms in

that industry, the smaller is the net entry rate, which eventually converges to zero (as b approaches zero).

The main explanation is that incumbent firms have lower marginal costs than potential entrants. As markets grow, given heterogeneity among potential firms, the relative competitiveness of incumbents increases. Market growth then ensures that incumbents expand faster, thereby impeding the entry of potential entrants (see also Sect. 3.2). The resulting firm growth exceeds the average firm growth for uniform firms, as is shown in Fig. 7. Notice, however, that the exact shape of the β_b curve for heterogeneous firms depends to some extent on the ratio of fixed costs F to total quantity Q : Higher fixed costs shift the β_b curve farther away from the case of uniform firms, making it steeper.

Of course, one should be very careful when deriving predictions for empirical work from this model: All results depend heavily on the assumption of Cournot competition, the specific cost structure, and the firms' ability to grow to an equilibrium size. Taking together the cases of horizontal and vertical market growth, these equilibrium results nevertheless suggest that persistent heterogeneity among (potential) firms might very well explain why long-run net entry rates lag behind predictions that are derived for uniform firms.

4 Conclusion

We have developed a simple and flexible framework that allows for firm-specific cost heterogeneity under Cournot conditions, where free-entry equilibria of the number and size distribution of firms are determined by the size of the market. Our key findings are that:

- the number of firms in equilibrium is reduced by a higher degree of cost heterogeneity;
- vertical market growth favors the entry and growth of new firms;
- horizontal market growth involves two different growth regimes: incumbents dominate in large markets with significant cost heterogeneity, while new entrants grow faster in smaller markets with low cost heterogeneity; and
- a higher degree of firm heterogeneity reduces entry rates for both vertical and horizontal market growth.

Therefore, when viewed dynamically, it may well be that the number and rate of entry and exit decrease when the market size changes if firms are heterogeneous. Moreover, market growth might lead to only a small increase of the number of firms, while firm sizes increase disproportionately—this is in strong contrast to the results for uniform firms. However, the findings in this paper are in line with recent empirical research: e.g., Syverson (2004, 2014) and Campbell and Hopenhayn (2005). However, both studies indicate that a larger market size gives rise to higher average productivity and firm size, and the distribution of firm sizes is less dispersed—this in contrast to the model presented here.

Our model resembles some of the findings of Melitz and Ottaviano (2008), and—despite our model’s being much simpler in structure and examining fewer dimensions—our model provides some additional insights. Both approaches imply that changes in market size induce changes in the number of firms, as well in firm heterogeneity in equilibrium. While our setup focuses on endogenous size differences and entry rates based only on heterogeneity in marginal costs, Melitz (2003), as well as Melitz and Ottaviano (2008), consider endogenous product differentiation and mark-ups across firms in a multidimensional way. The major differences between Melitz and Ottaviano (2008) and the results that are presented here (apart from the differences due to dynamic versus static considerations, international trade, and product differentiation) are not merely that all of our firms generally grow larger as markets grow. As we have shown, this crucially depends on the degree of cost heterogeneity, and it might well be that the minimum efficient size of firms is independent of heterogeneity: The smallest firms remain of the same size (see Fig. 2).

Consequently, both models imply increasing distributions of firms’ sizes but differ in their lower bounds. Moreover, Melitz (2003) and Melitz and Ottaviano (2008) show that the least productive firms are forced out of the market as the market grows. In our model, this again depends on the degree of heterogeneity (see Sects. 3.1 and 3.2): Less efficient firms could be crowded out; but the relative share of growth that is imputable to incumbents versus new entrants shifts in favor of incumbents if markets grow horizontally. Thus, a major feature of our model, which departs from Melitz and Ottaviano (2008), is the distinction between horizontal and vertical market growth.

It is important to note that there are limitations to the greatly simplified model that is presented here: First, the model is based on the Cournot conjecture: on the assumption that all firms react perfectly to reach a new Cournot-Nash equilibrium. In reality, even if firms compete predominately in capacities, neither perfect reactions nor instantaneous adoptions can be expected. Accordingly, while the model can provide predictions for Cournot-Nash equilibria, it is insufficiently operational to inform directly the design of empirical research. The second limitation is that the model is static, although it is intended to analyze indirectly the dynamic processes of changing market size. Moreover, in real markets changes of vertical and horizontal market size might well occur simultaneously, with the consequence that simulations with randomized parameters will show more rugged landscapes than the graphs in Figs. 2 and 3. In addition, the degree of firm heterogeneity might change as a result of innovation or learning.

These limitations ought to guide future research: On the one hand, large-scale simulations with numerous exogenous parameters are necessary to test the robustness of the model. On the other hand, careful predictions for empirical testing of entry rates need to be developed along the lines that are set out by Berry and Reiss (2007) and Neumann et al. (2001) and, in any case, should include a larger set of strategic choices than just Cournot competition.

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Declarations

Conflict of interest The author has no competing interests to declare that are relevant to the content of this article.

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