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**OPTIMAL DYNAMIC PRICING  
UNDER NETWORK EXTERNALITIES**

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# Optimal Dynamic Pricing under Network Externalities\*

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## Abstract

This paper studies the relationship between optimal dynamic pricing for network goods and the coordination of consumers' adoption decisions. We show that based on risk dominance criterion, consumers face the risk of coordination failure, and introductory pricing is optimal if the risk is higher in period 1 without network. We find that under threshold coordination, the impact of price on the network size varies according to consumer beliefs. In pessimistic (optimistic) threshold coordination, the network size expands (shrinks) as the price increases. Lowering (Raising) the price in period 2 implies a smaller network size, so introductory (skim) pricing is optimal.

**Keywords:** Network Externalities, Dynamic Pricing, Coordination Game

## 1 Introduction

A good has network externalities when a consumer derives a higher utility from its adoption when more consumers adopt the good. Goods with network externalities abound in the real world ranging from smartphones to online games. One important feature of a good with network externalities is the multiplicity of equilibria in the adoption game played by potential consumers of any such good. More specifically, when a consumer believes that other consumers adopt, he has high valuation of the good and hence will also adopt the good for a given price. On the other hand, if a consumer believes that other consumers do not adopt, he has low valuation of the good and will not adopt the good either. Turning to the supply side of the market, one fundamental and unsettled question concerns the optimal pricing strategy of a firm selling such a good over multiple rounds. Should it start with a low price to build a network base first and reap the benefit of a large network later on through a higher price, or should it rather price high first and sell to consumers

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with higher willingness to pay before raising the value of the good for consumers with lower willingness to pay? The first type of pricing strategy is known as introductory pricing, whereas the second type is known as skim pricing.<sup>1</sup> Both are discussed in the literature as viable pricing strategies for the dynamic sale of a network good. For example, Bensaid and Lesne (1996), Gabszewicz and Garcia (2008) and Hattori and Zenny (2018) show that the optimal strategy is introductory pricing whereas Mason (2000) and Laussel et al. (2015) identify skim pricing as optimal.

In this paper, we formulate a stylized model of dynamic sale of network goods and study if and when either of these two pricing strategies can be optimal in the monopoly and duopoly settings. Our key observation is that there is a close relationship between how consumers coordinate in their adoption decisions and which of introductory and skim pricing is optimal.

Our monopoly model is described as follows. A mass of consumers with private types adopt a network good over two periods. Each consumer demands at most one unit of the good and decides strategically if and when to adopt the good. Their private type represents the marginal impact of the network size on their utility. In period 1, the monopolist sets the price and consumers respond by making a purchase or waiting until period 2. In period 2, the monopolist sets the price based on the publicly observable period 1 history and the consumers who chose to wait in period 1 make purchase decisions. As mentioned above, how consumers coordinate in their adoption decisions is a critical determinant of the monopolist's optimal pricing strategy. We consider the following two types of coordination behavior:

1. Payoff and risk dominance with binary consumer types.
2. Threshold coordination with continuous consumer types.

Under payoff dominance, the consumers adopt the good if and only if the payoff from coordinated adoption is higher than that from no adoption. Under risk dominance, the consumers adopt the good if and only if the payoff from the coordinated adoption is higher than that from no adoption by a certain margin. The payoff margin is interpreted as the premium associated with the risk of coordination failures. Payoff dominance is the limiting case of risk dominance as the payoff premium approaches zero. If the consumers require a higher payoff premium, the monopolist must price lower in period 1 than in period 2 since there is no existing network base in period 1, and a possible coordination failure is more serious than in period 2 where the network base is already formed. This implies that introductory pricing is optimal when the consumers require a higher coordination payoff premium. For the same reason, introductory pricing is optimal when there are few high-type consumers even if the premium is not so high: Although the high-type consumers having a high willingness to pay for the good are supposed to adopt in period 1, the coordination payoff

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<sup>1</sup>See Besanko and Winston (1990), Du and Chen (2017), and Su (2007).

generated by the high-type alone is not high in period 1, inducing the monopolist to lower its period 1 price. On the other hand, when the coordination payoff premium is not high and there are sufficiently many high type consumers, the monopolist finds it optimal to use skim pricing to focus on extracting surplus from the high type consumers.

In our monopoly model of threshold coordination, consumer types are continuous and represent the marginal impact of the network size on their utility as before. For any price offered by the monopolist, the set of adopting consumers adjusts so that the marginal consumer type is just indifferent between adoption and no adoption. In this setting, we find that there are two types of threshold coordination referred to as pessimistic and optimistic threshold coordination. Pessimistic threshold coordination is one in which consumers expect that only relatively high types of consumers adopt, leading to a higher marginal consumer type; conversely, in optimistic threshold coordination, consumers expect that even relatively low types of consumers adopt, leading to a lower marginal consumer type. If the threshold coordination is pessimistic in both periods, introductory pricing is optimal. On the other hand, if the threshold coordination is optimistic in both periods, skim pricing is optimal. The key observation here is the impact of price on the network size. In the pessimistic case, the marginal consumer type decreases as the price increases so as to justify the higher price with the larger network. If the monopolist lowers the price in period 2, it will imply a smaller demand in period 2. From the period 1 perspective, this implies a smaller expansion in the network size and discourages early adoption. Hence, skim pricing is never feasible. Conversely, in the optimistic case, the marginal consumer type increases as the price increases as adopter becomes less attractive for lower consumer types. Hence, a price increase leads to a smaller network. If the monopolist raises the price in period 2, the network expansion in period 2 is smaller, and the value of the good in period 1 is lower. This implies that introductory pricing is not optimal.

In our duopoly model, we adopt the threshold coordination framework described above and assume that two firms  $A$  and  $B$  sell incompatible network goods  $A$  and  $B$ , respectively. Consumer types now represent the relative preference for  $B$  over  $A$  with respect to the marginal impact of the network size on their utility. In the duopoly model, we again demonstrate that both introductory pricing and skim pricing can be sustained as pricing strategies in a symmetric equilibrium where both firms set the same prices. The logic, however, is very different from the monopoly case. Consumers who adopt in period 2 benefit from the final network size, regardless of the period 1 network. Therefore, the range of equilibrium prices in period 2 is independent of the outcome in period 1. In symmetric equilibrium, for example, the adoption in period 2 is such that each firm splits the market in half. This implies that although the period 1 price changes the network size in period 1, it will have no impact in the adoption pattern in period 2. As a result, a range of period 1 prices is consistent with equilibrium. Some of these prices correspond to introductory pricing, whereas others

correspond to skim pricing.

The paper is organized as follows. After the discussion of the related literature in Section 2, Section 3 describes the monopoly model in which consumers perceive risk in adoption. In Section 4, we proceed to the model with threshold coordination, with Subsections 4.1 and 4.2 focusing the monopoly model and the duopoly model, respectively. Section 5 concludes with a discussion.

## 2 Related Literature

This paper contributes to two strands of literature as summarized in Table 1.

paper	Model	monopoly /duopoly	network effect	strategic consumer	consumer type	pricing	adoption criterion
Keser et al. (2012)	static	D	NS	N/A	homogeneous	N/A	network size
Ruffle et al. (2010)	static	D	NS	N/A	homogeneous	N/A	network size
Griva and Vettas (2011)	static	D	NS	N/A	continuous	N/A	threshold
Caillaud and Jullien (2003)	static	D	NS	N/A	homogeneous	DC	iterative deletion
Bensaid and Lesne (1996)	dynamic	M	S	Yes	continuous	IP	preexisting base
Makhdoumi et al. (2017)	dynamic	M	S	Yes	continuous	IP	preexisting base
Gabszewicz and Garcia (2008)	dynamic	M	S	No	continuous	IP	preexisting base
Xie and Sirbu (1995)	dynamic	D	S	No	continuous	IP	preexisting base
Mason (2000)	dynamic	M	S	Yes	continuous	SP	preexisting base
Fudenberg and Tirole (2000)	dynamic	M	NS	No	discrete	IP	payoff dominance
Hattori and Zenryo (2018)	dynamic	M	NS	Yes	homogeneous	IP	payoff dominance
Laussel et al. (2015)	dynamic	M	NS	Yes	continuous	SP	threshold
Cabral (2011)	dynamic	D	NS	No	continuous	IP	probabilistic
Belleflamme and Toulemonde (2009)	dynamic	M	NS	No	homogeneous	DC	iterative deletion
This paper	dynamic	M & D	NS	Yes	discrete & continuous	IP & SP	payoff and risk dominance & threshold coordination

Table 1: M=monopoly, D=duopoly, S=stationary, NS=non-stationary, IP=introductory pricing, SP=skim pricing, DC=divide-and-conquer. The network effect is stationary if consumers do not consider network expansion after their own adoption. Consumers are strategic if they may defer adoption until the next period. Network size: Consumers adopt a technology with a smaller network size requirement. Threshold: Consumers adopt if and only if their type is above some threshold. Iterative deletion: All consumers adopt the good of incumbent by repeating pessimistic expectations. Preexisting base: Consumer choice is based on the preexisting network size. Payoff dominance: Consumers choose the equilibrium that yields the highest payoff. Probabilistic: Consumers make probabilistic choices about which goods to adopt based on future network size and price expectations.

First, it contributes to the literature on the adoption game of network goods under different assumptions on consumer coordination. The literature on network effects includes Keser et al. (2012) and Ruffle et al. (2010), who analyze which of the two equilibria, one in which all consumers adopt technology  $A$  and the other in which all adopt  $B$ , prevails. They consider the minimum network size as a criterion for the adoption of either technology and find that the adoption of technology with a lower value prevails. Griva and Vettas (2011) considers a continuum of consumer types and derives multiple equilibria in a static setting.<sup>2</sup> Compared with these papers, our model considers coordination based on the payoff criterion in a dynamic setting. Second, the present paper contributes to the literature on the optimal pricing strategy of a firm selling network goods. Bensaid and Lesne (1996), Gabszewicz and Garcia (2008), and Xie and Sirbu (1995) demonstrate the optimality of introductory pricing, while Mason (2000) finds skim pricing to be optimal. These papers focus on the “stationary network effect,” meaning that consumers do not experience the externality effect of a network formed after their own adoption.<sup>3</sup> In contrast, our paper assumes a “non-stationary network effect,” implying that consumers benefit from the network formed both before and after their own adoption. Laussel et al. (2015) finds that skim pricing is optimal in a model where a firm sells a network good along with a complementary good that enhances the value of the network good. Fudenberg and Tirole (2000) studies the role of introductory pricing as a way to discourage potential entry, while Hattori and Zenryo (2018) considers introductory pricing as a means to convince consumers with the expectations that the current network size will continue.<sup>4</sup> In Cabral (2011), firms compete on prices to attract new consumers, and a firm with a larger network sets a higher price. Although formally a static pricing strategy, a divide-and-conquer strategy in a two-sided market is similar to introductory pricing, as studied in a monopoly model by Belleflamme and Toulemonde (2009) and duopoly models by Caillaud and Jullien (2001, 2003).

Our paper extends the adoption game with a coordination problem to a dynamic setting. In contrast to previous studies such as Bensaid and Lesne (1996), Gabszewicz and Garcia (2008), Makhdoumi et al. (2017), Mason (2000) and Xie and Sirbu (1995), our model assumes that the good is durable and yields utility over two periods once adopted. Additionally, unlike Fudenberg and Tirole (2000) and Cabral (2011), our model assumes strategic consumers who choose the timing of adoption. Under these assumption, we show the impact of consumer coordination as a key driver on dynamic pricing strategies.

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<sup>2</sup>See also Economides (1996), Grilo et al. (2001), and Tolotti and Yezpe (2020).

<sup>3</sup>See Long (2015).

<sup>4</sup>In Hattori and Zenryo (2018), the consumer is referred as naive consumer. See Cabral (2011, 2019).

### 3 Payoff versus Risk Dominance

We begin with the discussion of different types of consumer coordination in a monopoly model. A monopolist firm sells a network good to a mass of consumers over two periods. Each consumer has type  $\theta$  which represents a marginal impact of the network size on his utility. Let  $\Theta = \{\theta_\ell, \theta_h\}$  be the set of consumer types and assume that  $\theta_h > \theta_\ell > 0$ . Let  $\lambda \in (0, 1)$  be the proportion of type  $\theta_h$  consumers. Consumers demand at most one unit of the good and make strategic adoption decisions. Specifically, they decide if and when to adopt. Given a price  $p_t$  quoted by the monopolist in period  $t = 1, 2$ , the utility of a type  $\theta$  consumer adopting the good in period 1 is given by

$$(\theta + v)(n_1 + \delta n_2) - p_1,$$

where  $n_t$  is the size of adoption in period  $t$ , also referred to as the network size,  $\delta \in (0, 1)$  is a discount factor, and  $v > 0$  is a constant. The first term represents the utility from consuming the network good: Consumers benefit not only from the current network but also from the network formed after their adoption. The future network benefit is discounted by  $\delta$ . Similarly, the utility of a type  $\theta$  consumer adopting the good in period 2 is given by

$$\delta \{(\theta + v)(n_1 + n_2) - p_2\}.$$

The utility is zero if a consumer does not adopt in either period. The monopolist has zero marginal cost of production, and its profit function is given by

$$\pi(p) = p_1 n_1 + \delta p_2 n_2.$$

The game proceeds as follows: In period 1, the monopolist sets price  $p_1$ , and consumers decide whether to adopt the good or not. Once a consumer adopts the good, he does not make any further decision. In period 2, the network size for period 1 becomes public, and the monopolist sets price  $p_2$ . Consumers who did not adopt in period 1 then decide whether to adopt or not.

The monopolist determines the price  $p_t$  in period  $t$  from the set  $\mathbb{R}_+$ , whereas each consumer chooses his action  $d_t$  in period  $t$  from the set  $D = \{A, \emptyset\}$ , where  $A$  and  $\emptyset$  represent adoption and no adoption, respectively. We can interpret  $d_1 = \emptyset$  as the decision to wait until period 2. A period 1 history  $h = (p_1, n_1)$  consists of the price in period 1 as well as the network size in period 1. The set of period 1 history is denoted by  $H = \mathbb{R}_+ \times [0, 1]$ . The strategy of the monopolist consists of its price  $p_1$  in period 1 as well as its price  $p_2$  as a function of the period 1 history  $h \in H$ . On the other hand, each consumer's strategy is a mapping  $s_1 : \Theta \times \mathbb{R}_+ \rightarrow D$  that determines his period 1 choice as a function of his type and period 1 price



and a mapping  $s_2 : \Theta \times \mathbb{R}_+ \times H \rightarrow D$  that determines his period 2 choice as a function of his type and period 2 price as well as the period 1 history. Since consumers demand at most one unit of the good,  $s_2 = \emptyset$  if  $s_1 = A$ .

We will define payoff dominant equilibrium (PDE) and risk dominant equilibrium (RDE) as follows. Given a price  $p$ , each type of consumer faces the problem shown in Tables 2 and 3, where the numbers represent the consumer's payoff corresponding to his choice. For example, the top left cell of Table 2 shows the payoff of a type  $\theta_h$  consumer when he and all other type  $\theta_h$  consumers adopt. Let  $\alpha$  be a proportion of other consumers of the same type who adopt. Then, payoff dominant equilibrium (PDE) and risk dominant equilibrium (RDE) are defined as follows:

	$\theta_h$ adopts	Neither type adopts
Adopt	$(\theta_h + v)\lambda - p$	$-p$
Don't	0	0

Table 2: Type  $\theta_h$

	Both $\theta_h$ and $\theta_\ell$ adopts	$\theta_h$ adopts
Adopt	$\theta_\ell + v - p$	$(\theta_\ell + v)\lambda - p$
Don't	0	0

Table 3: Type  $\theta_\ell$

**Definition 1.** The strategy profile where every type  $\theta_h$  consumer adopts and no type  $\theta_\ell$  consumer adopts is payoff dominant equilibrium (PDE) if

$$p \leq (\theta_h + v)\lambda$$

and is  $\alpha$ -risk dominant equilibrium (RDE) if

$$(\theta_h + v)\alpha\lambda - p \geq 0 \Leftrightarrow \alpha \geq \frac{p}{(\theta_h + v)\lambda}.$$

The strategy profile where both types of consumer adopt is PDE if

$$p \leq \theta_\ell + v$$

and is RDE if

$$(\theta_\ell + v)\{\lambda + \alpha(1 - \lambda)\} - p \geq 0 \Leftrightarrow \alpha \geq \frac{p - (\theta_\ell + v)\lambda}{(\theta_\ell + v)(1 - \lambda)}.$$

Note that the above definition of RDE is based on the comparison of the two coordination payoffs and, hence, is different from the standard definition based on both the coordination and miscoordination payoffs.<sup>5</sup> When network externalities are involved in the payoff, the probability can be regarded as the network size. Adoption involves the risk of a coordination failure. If a player believes that the network size is small, the

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<sup>5</sup>See Harsanyi and Selten (1988) and Van Huyck et al. (1990)

risk will be high. The RDE expresses the fact that an inefficient “no-adoption” equilibrium will be chosen if the risk is perceived to be high. Conversely, extremely low risk,  $\alpha \rightarrow 1$ , implies payoff dominance.

### 3.1 Coordination based on payoff dominance

We derive a separating equilibrium in which type  $\theta_h$  consumers adopt in period 1 and type  $\theta_\ell$  consumers adopt in period 2 based on the payoff dominance criterion. Consider a period 1 history  $h \in H$  along which mass  $\mu \leq \lambda$  of type  $\theta_h$  consumers adopt in period 1. A type  $\theta_h$  consumer believes that even a type  $\theta_\ell$  consumer may adopt in period 2. If so, he should adopt in period 2. Hence, each consumer believes that type  $\theta_h$  consumers will adopt in period 2. For a given price  $p_2$  in period 2, the utility of a type  $\theta_\ell$  consumers is  $\theta_\ell + v - p_2$  if he adopts when all consumers adopt and 0 if he does not adopt when only type  $\theta_h$  consumers adopt. Thus, a type  $\theta_\ell$  consumer adopts in period 2 if  $p_2 \leq \theta_\ell + v$  and the optimal price in period 2 is  $p_2^* = \theta_\ell + v$ .<sup>6</sup>

In period 1, for a given price  $p_1$  in period 1, the utility of a type  $\theta_h$  consumer is  $(\theta_h + v)\{\lambda + \delta(1 - \lambda)\} - p_1$  if he adopts when only other type  $\theta_h$  consumers adopt. On the other hand, if he waits until period 2, then all remaining consumers will adopt in period 2, and his utility is  $\delta(\theta_h + v - p_2^*)$ . Thus, a type  $\theta_h$  consumer adopts in period 1 if  $p_1 \leq (\theta_h + v)\{\lambda + (1 - \delta)\} + \delta p_2^*(h_0)$  and the optimal price in period 1 is  $p_1^* = (\theta_h + v)\{\lambda + (1 - \delta)\} + \delta p_2^*(h_0)$ .

In the separating equilibrium, type  $\theta_h$  consumers adopt in period 1 and type  $\theta_\ell$  consumers adopt in period 2, and the prices in each period are given by

$$p_1^{\text{PDE}} = (1 - \delta)(\theta_h + v)\lambda + \delta(\theta_\ell + v) \quad \text{and} \quad p_2^{\text{PDE}} = \theta_\ell + v.$$

We obtain the following proposition regarding the pricing strategy in the separating equilibrium.

**Proposition 1.** When consumers coordinate based on the payoff dominance criterion, the monopolist uses introductory pricing if  $(\theta_h + v)\lambda < \theta_\ell + v$  and skim pricing if  $(\theta_h + v)\lambda > \theta_\ell + v$  in the separating equilibrium in which  $\theta_h$  adopts in period 1 and  $\theta_\ell$  adopts in period 2.

The equilibrium price in period 1 is the average discounted sum of the benefit each type of consumer derives from the network when he adopts, and the equilibrium price in period 2 is the benefit a type  $\theta_\ell$  consumer derives from the network. Since waiting is more costly to type  $\theta_h$  than type  $\theta_\ell$ , type  $\theta_h$  consumers will adopt the good earlier than type  $\theta_\ell$  consumers. If the proportion of type  $\theta_h$  consumers is small, then the network benefit he receives upon adoption in period 1 is reduced, so their incentive to wait becomes

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<sup>6</sup>See the Appendix for the formal description of the equilibrium.

stronger. Therefore, the monopolist must price lower in period 1. Conversely, if the proportion of  $\theta_h$  is large, the monopolist sets a higher price in period 1 and extract the surplus from type  $\theta_h$  consumers.

### 3.2 Coordination based on risk dominance

In this section, we derive a separating equilibrium when consumers coordinate based on the risk dominance criterion and compare it with the result in the previous section on payoff dominance. Consider a period 1 history  $h \in H$  along which mass  $\mu \leq \lambda$  of type  $\theta_h$  consumers adopt the good in period 1. As with Subsection 3.1, a type  $\theta_h$  consumer in period 2, if any, will adopt as long as type  $\theta_\ell$  adopts. On the other hand, type  $\theta_\ell$  consumers are at the risk of a coordination failure. Let  $\beta$  be the proportion of type  $\theta_\ell$  consumers who adopt. A type  $\theta_\ell$  consumer adopts in period 2 if

$$\delta [(\theta_\ell + v)\{\lambda + \beta(1 - \lambda)\} - p_2] \geq 0 \Leftrightarrow p_2 \leq (\theta_\ell + v)\lambda + (\theta_\ell + v)\beta(1 - \lambda).$$

Thus, the optimal price in period 2 is  $p_2^* = (\theta_\ell + v)\lambda + (\theta_\ell + v)\beta(1 - \lambda)$ . We can interpret  $\beta$  as the risk, where a lower  $\beta$  indicates a higher risk of a coordination failure. The right-hand side of the inequality on the right represents the network benefit enjoyed by type  $\theta_\ell$  when he expects type  $\theta_h$  adopts but only fraction  $\beta$  of type  $\theta_\ell$  adopts. If there is no such risk, i.e.  $\beta = 1$ , the network benefit is  $\theta_\ell + v$ . Thus, the difference  $(\theta_\ell + v)(1 - \lambda)(1 - \beta)$  can be understood as the premium associated with the risk of a coordination failure. If the payoff from the coordinated adoption is large enough to offset the risk, the type  $\theta_\ell$  will adopt. In other words, when adoption involves risk, consumers require a high coordination payoff premium.

Type  $\theta_\ell$  consumers will not adopt in period 1 because the period 1 price is set so that type  $\theta_h$  consumers would be indifferent between adoption and no adoption. Expecting the period 2 outcome above, if a type  $\theta_h$  consumer waits until period 2, his payoff is  $\delta(\theta_h + v - p_2^*)$ . Let  $\alpha$  be the proportion of type  $\theta_h$  consumers who adopt in period 1. A type  $\theta_h$  consumer adopts in period 1 if

$$\begin{aligned} (\theta_h + v)\{\alpha\lambda + \delta(1 - \alpha\lambda)\} - p_1 &\geq \delta\{(\theta_h + v) - p_2^*\} \\ \Leftrightarrow p_1 &\leq (1 - \delta)(\theta_h + v)\alpha\lambda + \delta\{(\theta_\ell + v)\lambda + (\theta_\ell + v)\beta(1 - \lambda)\}. \end{aligned}$$

Hence, the price in the separating equilibrium in each period is given by

$$\begin{aligned} p_1^{\text{RDE}} &= (1 - \delta)(\theta_h + v)\alpha\lambda + \delta\{(\theta_\ell + v)\lambda + (\theta_\ell + v)\beta(1 - \lambda)\} \quad \text{and} \\ p_2^{\text{RDE}} &= (\theta_\ell + v)\lambda + (\theta_\ell + v)\beta(1 - \lambda). \end{aligned}$$

Without risk of a coordination failure, the network benefit for type  $\theta_h$  consumer is  $(\theta_h + v)\{\lambda + \delta(1 - \lambda)\}$ . Hence, the premium associated with the risk is  $(1 - \delta)(\theta_h + v)(1 - \alpha)\lambda$ . As in period 2, consumers require a coordination payoff premium in order to adopt. The extent of the premium, however, varies depends on the discount factor. When the discount factor is small, the benefit of the network formed in period 2 is discounted, which means that the loss due to coordination failure is higher. As a result, a higher premium is required in such cases.

**Proposition 2.** When consumers coordinate based on the risk dominance criterion, the monopolist uses introductory pricing if  $\alpha(\theta_h + v)\lambda < (\theta_\ell + v)\lambda + \beta(\theta_\ell + v)(1 - \lambda)$  and skim pricing if  $\alpha(\theta_h + v)\lambda > (\theta_\ell + v)\lambda + \beta(\theta_\ell + v)(1 - \lambda)$ .

The equilibrium price in period 1 is the average discounted sum of the benefit each type of consumer will receive from the expected network upon adoption. On the other hand, the equilibrium price in period 2 is the benefit a type  $\theta_\ell$  consumer derives from the expected network. The second period has a network base formed in period 1, but no network base exists in period 1. Therefore, adopting in period 1 is more risky, so if consumers are more pessimistic, which means that  $\alpha$  and  $\beta$  are low, introductory pricing prevails to mitigate the risk. On the other hand, if consumers are more optimistic and perceive less risk, both introductory and skim pricing can be optimal depending on the proportion of type  $\theta_h$  consumers. If the proportion of type  $\theta_h$  consumers is small, even if the risk is low, the benefit derived from the period 1 network is also small. Introductory pricing is optimal in such a case. Conversely, if the proportion of type  $\theta_h$  consumers is large, the benefits derived from the period 1 network is large and type  $\theta_h$  has high willingness to pay. Therefore, the monopolist extracts surplus from them through skim pricing.

In the extreme case, when consumers perceive no risk of a coordination failure,  $\alpha$  and  $\beta$  equal 1, and  $p^{\text{RDE}}$  equals  $p^{\text{PDE}}$ . This implies that risk dominance is equivalent to payoff dominance in the limit.

## 4 Threshold coordination

We now suppose that the consumer type  $\theta$  is uniformly distributed over  $\Theta = [0, 1]$ . This allows the same model to be used to analyze the monopoly and duopoly cases.

### 4.1 Monopoly

As mentioned above, adoption incentive depends critically on the expected size of adoption by others. Price changes have a significant impact on these expectations. In the monopoly model, we consider an equilibrium in which consumers' coordination behavior in response to a price change is punitive in the sense that the

firm loses its customer base after any deviation from the equilibrium price. Specifically, for an equilibrium price profile  $p^* = (p_1^*, p_2^*)$  and prices  $p'_1 \neq p_1^*$  and  $p'_2 \neq p_2^*$ , the consumer's strategy is given by

$$s_1^*(\theta, p'_1) = s_2^*(\theta, p'_2, h) = \emptyset \quad \forall \theta \in \Theta, \forall h \in H.$$

Observing a price deviation, consumers with punitive response coordinate to wait until period 2 in period 1 and not to adopt in period 2. This coordination behavior presents the worst-case scenario after a deviation for the monopolist, and minimizes his incentive to deviate from the equilibrium price. Consequently, it supports the widest range of potential equilibrium prices.

Suppose  $\theta > \theta'$  and compare the consumer's strategy when he has type  $\theta$  and when he has type  $\theta'$ . If  $\theta'$  adopts in a given period in equilibrium, then  $\theta$  will also adopt in that period. Since  $\theta$  has a higher valuation for the good than  $\theta'$ ,  $\theta$  receives higher utility from adoption than  $\theta'$  in both periods. If  $\theta'$  adopts in period 2,  $\theta$  also adopts if he has not already done so. Waiting is more costly to type  $\theta$  than type  $\theta'$ . Therefore, if  $\theta'$  adopts in period 1,  $\theta$  also adopts in period 1 without waiting until period 2. This implies that the set of consumer types is divided into three intervals in equilibrium: From right to left, the first interval is the set of consumers who adopt in period 1, the second is the set of consumers who adopt in period 2, and the third is the set of consumers who do not adopt in either period. Note that the third interval is empty if the market is fully covered. We consider a threshold coordination equilibrium (TCE) defined as follows:

**Definition 2.** An SPE is a threshold coordination equilibrium (TCE) if there exist  $\theta_1^*$  and  $\theta_2^*$  such that  $0 \leq \theta_2^* < \theta_1^* < 1$  for the equilibrium price profile  $(p_1^*, p_2^*)$  and type  $\theta \in [\theta_1^*, 1]$  adopts in period 1,  $\theta \in [\theta_2^*, \theta_1^*)$  adopts in period 2, and  $\theta \in [0, \theta_2^*)$  does not adopt.

By definition, positive measures of consumer types adopt the good in both periods in TCE.

The period 1 history  $h$  reveals the distribution of consumers remaining in period 2. Let  $\theta_1(h)$  be the lowest type who adopted in period 1 for a period 1 history  $h = (p_1, n_1)$ ,  $n_1 = 1 - \theta_1(h)$ . Thus, consumers in the interval  $[0, \theta_1(h))$  remain in period 2. Given a period 2 price  $p_2$ , there exists a type  $\theta_2$  such that he is indifferent between adoption in period 2 and no adoption:

$$\delta \{(\theta_2 + v)(1 - \theta_2) - p_2\} = 0 \Leftrightarrow \theta_2 = \frac{1 - v}{2} \pm \sqrt{\frac{(1 + v)^2}{4} - p_2}.$$

The two values with different signs in the second term are two threshold values.

Expecting the equilibrium price  $p_2^*$  and threshold value  $\theta_2^*$  in period 2, a type  $\theta_1$  consumer is indifferent

between adopting in period 1 and waiting until period 2 for a period 1 price  $p_1$ :

$$\begin{aligned} & (\theta_1 + v) \{1 - \theta_1 + \delta(\theta_1 - \theta_2^*)\} - p_1 = \delta \{(\theta_1 + v)(1 - \theta_2^*) - p_2^*\} \\ \Leftrightarrow \quad & \theta_1 = \frac{1-v}{2} \pm \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2^*}{1-\delta}}. \end{aligned}$$

Similar to the threshold value in period 2, there can be two threshold values in period 1 with different signs of the second term.

For a given price profile  $p = (p_1, p_2)$ , if the sign of the square root term of each threshold value is positive, then the period 1 network size is smaller than in the negative case, and the final network size is smaller than in the negative case. We therefore refer to the case where the sign is positive as *pessimistic* and the case where it is negative as *optimistic*.

**Proposition 3.** When  $v < 1$ , the optimal pricing strategy of the monopolist is introductory pricing in case (i), and skim pricing in case (ii). In case (iii), both introductory and skim pricing can be optimal depending on parameter. The prices and threshold values in the threshold coordination equilibrium (TCE) are as follows:

(i) Pessimistic in both periods

$$\begin{aligned} p_1^* & \in \left\{ p_1 : \delta p_2^* < p_1 < p_2^* \right\}, \quad p_2^* \in \left\{ p_2 : (\theta_1^* + v)(1 - \theta_1^*) < p_2 \leq \frac{(1+v)^2}{4} \right\}, \\ \theta_1^* & = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad \text{and} \quad \theta_2^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - p_2^*}. \end{aligned}$$

(ii) Optimistic in both periods

$$\begin{aligned} p_1^* & \in \left\{ p_1 : p_2^* < p_1 \leq \delta p_2^* + (1-\delta) \frac{(1+v)^2}{4} \right\}, \quad p_2^* \in \left\{ p_2 : v \leq p_2 \leq \frac{(1+v)^2}{4} \right\}, \\ \theta_1^* & = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad \text{and} \quad \theta_2^* = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - p_2^*}. \end{aligned}$$

(iii) Pessimistic in period 1 and optimistic in period 2

$$\begin{aligned} p_1^* & \in \left\{ p_1 : \delta p_2^* < p_1 \leq \delta p_2^* + (1-\delta) \frac{(1+v)^2}{4} \right\}, \quad p_2^* \in \left\{ p_2 : v \leq p_2 \leq \frac{(1+v)^2}{4} \right\}, \\ \theta_1^* & = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad \text{and} \quad \theta_2^* = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - p_2^*}. \end{aligned}$$

When  $v > 1$ , the optimal pricing strategy of the monopolist is introductory pricing, and the prices and

threshold values in TCE are as follows:

$$p_1^* \in \left\{ p_1 : \delta p_2^* < p_1 < p_2^* \right\}, \quad p_2^* \in \left\{ p_2 : (\theta_1^* + v)(1 - \theta_1^*) < p_2 \leq v \right\},$$

$$\theta_1^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad \text{and} \quad \theta_2^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - p_2^*}.$$

In TCE, threshold values satisfy  $0 \leq \theta_2^* < \theta_1^* < 1$ . If the network effect is small ( $v < 1$ ), so both optimistic and pessimistic threshold values are valid, resulting in three possible TCE: (i) pessimistic in both periods, (ii) optimistic in both periods and (iii) pessimistic in period 1 and optimistic in period 2. If thresholds are pessimistic in both periods, the marginal consumer type is high, and a price increase leads to a higher demand. If the monopolist offers a lower price in period 2, the network does not expand much in period 2, reducing the incentive to adopt in period 1. Therefore, skim pricing is not optimal, and introductory pricing is optimal. Conversely, if thresholds are optimistic in both periods, a higher price results in a lower demand. In this case, a higher price in period 2 reduces the incentive to adopt in period 1 due to smaller expansion in the network size in period 2. Therefore, introductory pricing is not optimal, and skim pricing is optimal. If the threshold values are pessimistic in period 1 and optimistic in period 2, only relatively few consumer types adopt in period 1, and relatively more consumer types adopt in period 2. This means that  $\theta_1^* > \theta_2^*$  tends to hold more easily, making both introductory and skim pricing consistent with TCE. If  $v$  is large ( $v > 1$ ), consumers can enjoy a large enough network effect to adopt. Consequently, the optimistic threshold value is not consistent with TCE. Since only the pessimistic threshold value is valid, introductory pricing is optimal just as in the case of pessimistic thresholds when  $v < 1$ .

## 4.2 Duopoly

We now turn to a duopoly model. Two firms,  $A$  and  $B$ , sell incompatible network goods,  $A$  and  $B$ , respectively, to a mass of consumers. Firm  $A$  is located at 0, and firm  $B$  is located at 1. Each consumer demands at most one unit of either good. The consumer's type  $\theta$  represents a relative preference for good  $B$  over  $A$ . A consumer with a higher  $\theta$  derives higher utility from  $B$  and lower utility from  $A$ . Consumers can choose which good to adopt and when to adopt it. Given prices  $p_t$  and  $q_t$  quoted by firms  $A$  and  $B$ , respectively, in period  $t$ , the utilities of a type  $\theta$  consumer adopting  $A$  and  $B$  in period 1 are respectively given by

$$(1 - \theta + v)(n_1^A + \delta n_2^A) - p_1 \quad \text{and} \quad (\theta + v)(n_1^B + \delta n_2^B) - q_1,$$

where  $n_t^f$  is the network size of firm  $f$  in period  $t$ . The utility function is identical to the one used in the monopoly model, and a consumer adopting one good does not benefit from the network of the other good.

Similarly, the utilities of a type  $\theta$  consumer adopting  $A$  and  $B$  in period 2 are respectively given by

$$\delta \{ (1 - \theta + v)(n_1^A + n_2^A) - p_2 \} \quad \text{and} \quad \delta \{ (\theta + v)(n_1^B + n_2^B) - q_2 \}.$$

The utility is zero if the consumer does not adopt either good. Each firm has zero marginal cost of production, and the profit functions of firms  $A$  and  $B$  are respectively given by

$$\pi_A((p_1, q_1), (p_2, q_2), n^A) = p_1 n_1^A + \delta p_2 n_2^A \quad \text{and} \quad \pi_B((p_1, q_1), (p_2, q_2), n^B) = q_1 n_1^B + \delta q_2 n_2^B.$$

Firms  $A$  and  $B$  simultaneously choose prices  $p_t$  and  $q_t$ , respectively, in period  $t$  from the set  $\mathbb{R}_+$ , whereas each consumer chooses his action  $d_t$  in period  $t$  from the set  $D = \{A, B, \emptyset\}$ . A period 1 history  $h = ((p_1, q_1), n_1)$  consists of the price profile in period 1 as well as a pair  $n_1 = (n_1^A, n_1^B) \in [0, 1]^2$  of network size of each firm in period 1. Denote the set of period 1 histories by  $H = \mathbb{R}_+^2 \times [0, 1]^2$ . The strategies of firm  $A(B)$  consist of the price  $p_1(q_1)$  in period 1 as well as the price  $p_2(q_2)$  as a function of the period 1 history  $h \in H$ . On the other hand, the strategy of each consumer is a mapping  $s_1 : \Theta \times \mathbb{R}_+^2 \rightarrow D$  that determines his period 1 choice as a function of his type  $\theta$  and period 1 price profile  $(p_1, q_1)$  and a mapping  $s_2 : \Theta \times \mathbb{R}_+^2 \times H \rightarrow D$  that determines his period 2 choice as a function of his type  $\theta$ , period 2 price profile  $(p_2, q_2)$ , and the period 1 history  $h \in H$ . Since consumers demand at most one unit of either good,  $s_2 = \emptyset$  if  $s_1 \neq \emptyset$ .

As in the monopoly model, we consider punitive coordination following a price deviation: Let  $p^* = (p_1^*, p_2^*)$  and  $q^* = (q_1^*, q_2^*)$  be equilibrium price profiles of firms  $A$  and  $B$ .

$$s_1(\theta, (p_1, q_1)) = \begin{cases} A & \text{if } p_1 = p_1^* \text{ and } q_1 \neq q_1^*, \\ B & \text{if } p_1 \neq p_1^* \text{ and } q_1 = q_1^*, \text{ and} \\ \emptyset & \text{if } p_1 \neq p_1^* \text{ and } q_1 \neq q_1^*, \end{cases}$$

$$s_2(\theta, (p_2, q_2), h) = \begin{cases} A & \text{if } p_2 = p_2^* \text{ and } q_2 \neq q_2^*, \\ B & \text{if } p_2 \neq p_2^* \text{ and } q_2 = q_2^*, \\ \emptyset & \text{if } p_2 \neq p_2^* \text{ and } q_2 \neq q_2^*, \end{cases} \quad \forall \theta \in \Theta, \forall h \in H.$$

With punitive response, a price deviation by either one firm causes consumers to coordinate on the other firm.

As in the monopoly model described in Subsection 4.1, consider the consumer's strategy when  $\theta < \theta'$ . In equilibrium,  $\theta'$  never adopts  $A$  before  $\theta$  and  $\theta$  never adopts  $B$  before  $\theta'$ . These imply that the set of consumer types is divided into four intervals in equilibrium when the market is fully covered: From left to



right, the first interval is the set of consumers who adopt  $A$  in period 1, the second is the set of consumers who adopt  $A$  in period 2, the third is the set of consumers who adopt  $B$  in period 2, and the fourth is the set of consumers who adopt  $B$  in period 1. We define a threshold coordination equilibrium in the duopoly model as follows:

**Definition 3.** An SPE is a threshold coordination equilibrium (TCE) if there exist  $x^*, y^*$  and  $z^*$  such that  $0 < x^* < z^* < y^* < 1$  for the equilibrium price profile and type  $\theta \in [0, x^*]$  adopts  $A$  in period 1,  $\theta \in (x^*, z^*]$  adopts  $A$  in period 2,  $\theta \in (z^*, y^*)$  adopts  $B$  in period 2, and  $\theta \in [y^*, 1]$  adopts  $B$  in period 1.

The strategy in period 2 depends on period 1 history. For any period 1 history  $h \in H$ , let  $x(h)$  be the highest type who adopts  $A$  in period 1 and  $y(h)$  be the lowest type who adopts  $B$  in period 1, respectively, consumers in the interval  $(x(h), y(h))$  remain in period 2. Given a price profile  $(p_2(h), q_2(h))$  in period 2, there exists a type  $z(h)$  who is indifferent between adoption in period 2 and no adoption:

$$\begin{aligned} \delta\{(1 - z(h) + v)z(h) - p_2(h)\} &= \delta\{(z(h) + v)(1 - z(h)) - q_2(h)\} \\ \Leftrightarrow z(h) &= \frac{1}{2} + \frac{p_2(h) - q_2(h)}{2v} \quad \forall h \in H. \end{aligned}$$

If  $(p_2^*, q_2^*)$  is the equilibrium price profile in period 2, then the threshold value  $z^* \in (x^*, y^*)$  and the utility of type  $z^*$  must be non-negative. Thus the equilibrium price profile in period 2 is given as follows:

$$(p_2^*, q_2^*) \in \left\{ (p_2, q_2) : v(2x^* - 1) < p_2 - q_2 < v(2y^* - 1) \text{ and } \frac{(p_2 - q_2)^2}{v^2} + 2(p_2 + q_2) \leq 1 + 2v \right\}.$$

In Figure 1, we present the equilibrium price profile in period 2. For  $z^* \in (x^*, y^*)$ , it must satisfy  $v(2x^* - 1) < p_2^* - q_2^* < v(2y^* - 1)$ . Furthermore, for type  $z^*$ 's utility to be non-negative, it must satisfy  $\frac{(p_2^* - q_2^*)^2}{v^2} + 2(p_2^* + q_2^*) \leq 1 + 2v$ . Therefore, the region (a) corresponds to the equilibrium price profile in period 2. In the duopoly model, unlike the monopoly model, the presence of the other firm reduces the potential market size of each firm. For example, when firm  $A$  builds a network of size  $x^*$  in period 1, the maximum network firm  $B$  can build is  $1 - x^*$ . This implies that as firm  $A$  builds a large network in period 1, the willingness to pay for  $B$  decreases, making it impossible for firm  $B$  to set a high price in period 2.

Expecting the equilibrium price profile in period 2 shown above, for a period 1 price profile  $(p_1, q_1)$ , a type  $x$  is indifferent between adopting  $A$  in periods 1 and 2, and type  $y$  is indifferent between adopting  $B$  in periods 1 and 2:

$$x = \frac{1+v}{2} - \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2^*}{1-\delta}} \quad \text{and} \quad y = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{q_1 - \delta q_2^*}{1-\delta}}.$$

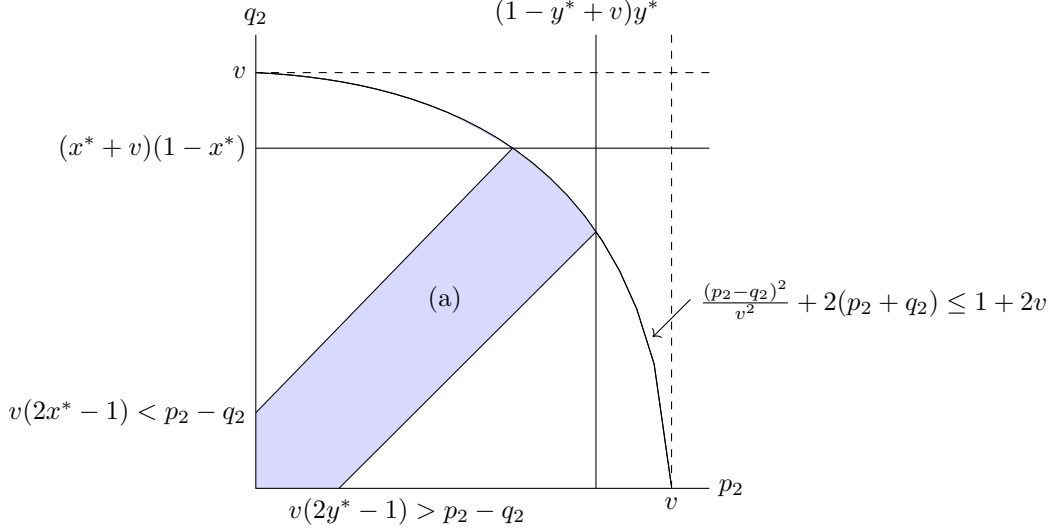


Figure 1: By  $z^* \in (x^*, y^*)$ , the equilibrium price profile in period 2 must be  $v(2x^* - 1) < p_2 - q_2$  and  $v(2y^* - 1) > p_2 - q_2$ . Furthermore, since the utility of type  $z^*$  is non-negative,  $\frac{(p_2 - q_2)^2}{v^2} + 2(p_2 + q_2) \leq 1 + 2v$ .

Since  $0 < x^* < z^* < y^* < 1$  must be satisfied in equilibrium for the threshold value profile  $(x^*, y^*, z^*)$ , the equilibrium price profile in period 1 is given as follows:

$$(p_1^*, q_1^*) \in \left\{ (p_1, q_1) : \begin{array}{l} \delta p_2^* < p_1 < \delta p_2^* + (1 - \delta) \left\{ \frac{-d_2^{*2} + 2v^2 d_2^* + (1 + 2v)v^2}{4v^2} \right\} \text{ and} \\ \delta q_2^* < q_1 < \delta q_2^* + (1 - \delta) \left\{ \frac{-d_2^{*2} - 2v^2 d_2^* + (1 + 2v)v^2}{4v^2} \right\} \end{array} \right\},$$

where  $d_2^* \equiv p_2^* - q_2^*$ .

**Proposition 4.** In the duopoly model, firm  $A$  uses introductory pricing if  $(1 - x^* + v)x^* < p_2^*$  and skim pricing if  $(1 - x^* + v)x^* > p_2^*$ , and firm  $B$  uses introductory pricing if  $(y^* + v)(1 - y^*) < q_2^*$  and skim pricing if  $(y^* + v)(1 - y^*) > q_2^*$ . The price profiles in each period and threshold values in the threshold coordination equilibrium (TCE) are as follows:

$$(p_1^*, q_1^*) \in \left\{ (p_1, q_1) : \begin{array}{l} \delta p_2^* < p_1 < \delta p_2^* + (1 - \delta) \left\{ \frac{-d_2^{*2} + 2v^2 d_2^* + (1 + 2v)v^2}{4v^2} \right\} \text{ and} \\ \delta q_2^* < q_1 < \delta q_2^* + (1 - \delta) \left\{ \frac{-d_2^{*2} - 2v^2 d_2^* + (1 + 2v)v^2}{4v^2} \right\} \end{array} \right\},$$

$$(p_2^*, q_2^*) \in \left\{ (p_2, q_2) : v(2x^* - 1) < p_2 - q_2 < v(2y^* - 1) \text{ and } \frac{(p_2 - q_2)^2}{v^2} + 2(p_2 + q_2) \leq 1 + 2v \right\},$$

$$x^* = \frac{1 + v}{2} - \sqrt{\frac{(1 + v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1 - \delta}}, \quad y^* = \frac{1 - v}{2} + \sqrt{\frac{(1 + v)^2}{4} - \frac{q_1^* - \delta q_2^*}{1 - \delta}}, \text{ and } z^* = \frac{1}{2} + \frac{d_2^*}{2v},$$

where  $d_2^* = p_2^* - q_2^*$ .

From the threshold values  $x^*$  and  $y^*$ , we know  $p_1^* = (1 - \delta)(1 - x^* + v)x^* + \delta p_2^*$  and  $q_1^* = (1 - \delta)(y^* + v)(1 - y^*) + \delta q_2^*$ . As in the monopoly model, the equilibrium price of each firm in period 1 is the average discounted sum of the benefit the lowest type who adopts in period 1 derives from the period 1 network size and the equilibrium price in period 2. However, unlike in the monopoly model, both introductory and skim pricing can be optimal. Figure 2 shows the range of price profiles in symmetric equilibrium ( $p_1^* = q_1^*$  and  $p_2^* = q_2^*$ ). The threshold values must be such that  $0 < x^* < z^* < y^* < 1$ , and the equilibrium price

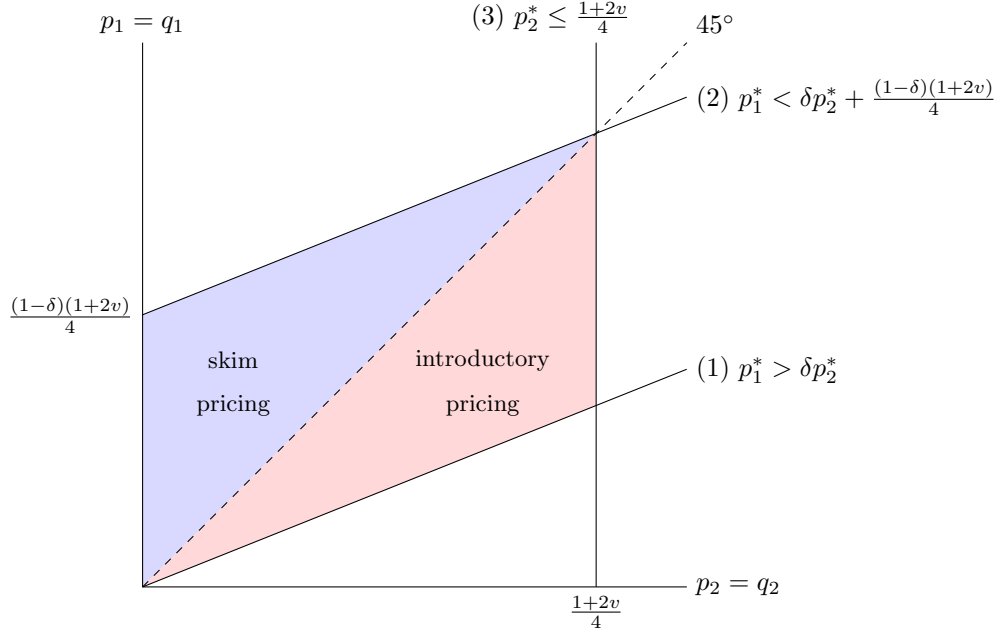


Figure 2: Symmetric equilibrium

profiles lie between (1) and (2) and to the left of (3), which ensures that type  $z^*$ 's utility is non-negative. The region above the 45-degree line corresponds to skim pricing and the region below the line corresponds to introductory pricing. This is understood as follows: As seen, the range of symmetric equilibrium prices in period 2 is independent of the set of remaining consumers in period 2. Consumers who adopt in period 2 benefit from the final network size only, regardless of the period 1 network size. Therefore, the period 2 price must be set so that the consumer's utility, who is indifferent between adopting  $A$  in period 2 and  $B$ , is non-negative. In the symmetric equilibrium, the market eventually adjusts so that each firm splits the market in half. The utility of type  $z^* = \frac{1}{2}$  is, therefore,  $\frac{1}{2}(\frac{1}{2} + v)$  and does not depend on the outcome of period 1. Suppose then that we consider moving  $x^* = 1 - y^*$  while fixing  $p_2^* = q_2^*$ . Since the period 1 equilibrium price is increasing in  $x^* = 1 - y^*$ , more adoptions in period 1 (higher  $x^* = 1 - y^*$ ) imply a higher equilibrium price  $p_1^* = q_1^*$  in period 1. Since  $p_1^* = q_1^*$  is related to  $p_2^* = q_2^*$  through a discount factor  $\delta$ , the range of the period 1 equilibrium prices covers both regions above and below the period 2 equilibrium prices. In other

words, since the period 2 equilibrium price is insulated from the period 1 history, the period 1 outcome in equilibrium simply reflects the relationship between the size of adoption and the price level in that period for each good separately. In the model of threshold coordination, the positive relationship between these two is not unique, and hence the period 1 price can be higher or lower than the period 2 price.

## 5 Conclusion

There are multiple equilibria in the adoption game of network goods depending on how consumers coordinate their decisions. Our analysis considers coordination based on payoff and risk dominance for binary types as well as pessimistic and optimistic thresholds with continuous types and shows how consumer coordination determines the optimal dynamic pricing strategy.

Under risk dominance, consumers are aware of the risk of a coordination failure. Consumers adopt when there is a coordination payoff premium associated with the risk. Since the network base already exists in period 2 but not in period 1, adoption is riskier in period 1 than in period 2. Therefore, if consumers perceive the risk to be high, the monopolist uses introductory pricing to give consumers a payoff premium for coordination. Payoff dominance is consistent when risk is sufficiently low.

Under threshold coordination, the impact of price changes on demand is influenced by consumers' expectations regarding network size. In the monopoly model, if consumers have pessimistic expectations, a higher price increases demand. A higher price in period 2 implies network expansion in that period, so introductory pricing is optimal. Conversely, if consumers have optimistic expectations, a higher price results in decreased demand. If the monopolist sets a higher price in period 2, the network size will not expand much, reducing the incentive to adopt in period 1, so introductory pricing is not optimal, but rather skim pricing is optimal. On the other hand, in the duopoly model, both introductory pricing and skim pricing can be optimal. In symmetric equilibrium, each firm shares half the market. Since only the final network size is important for consumers who adopt in period 2, the range of equilibrium period 2 price is independent of the outcome in period 1. More adoption in period 1 implies a higher equilibrium price in period 1, and the equilibrium price in period 1 exceeds a given equilibrium price in period 2, i.e., skim pricing. On the other hand, less adoption in period 1 implies a lower equilibrium price in period 1, and the equilibrium price in period 1 is lower than that in period 2, i.e., introductory pricing.

Our analysis assumes that all consumers are homogeneous in terms of their expectations about the network size. However, information asymmetry can cause heterogeneity in expectations, as discussed in Cabral (2019). Coordination behaviors can be different with such heterogeneity and firms' optimal pricing needs adjustment. This topic would be worth exploring in future research.

## Appendix A. Payoff dominance and proof of Proposition 1

In period 2, we assume that each consumer believes that type  $\theta_h$  consumers adopt. Consider a period 1 history  $h \in H$  along which mass  $\mu \leq \lambda$  of type  $\theta_h$  consumers adopt. The utility of a type  $\theta_\ell$  consumer is  $\delta(\theta_\ell + v - p_2)$  if he adopts when all consumers adopt and 0 if he does not adopt when only type  $\theta_h$  consumers adopt. Hence, the strategy of type  $\theta_\ell$  is given by

$$s_2(\theta_\ell, p_2, h) = \begin{cases} A & \text{if } p_2 \leq \theta_\ell + v, \\ \emptyset & \text{if } p_2 > \theta_\ell + v, \end{cases} \quad \forall h \in H.$$

Thus, the optimal price in period 2 is  $p_2^{\text{PD}} = \theta_\ell + v$ . In period 1, we assume that each consumer believes that type  $\theta_\ell$  consumers do not adopt. The utility of type  $\theta_h$  consumer is  $(\theta_h + v)\{\lambda + \delta(1 - \lambda)\} - p_1$  if he adopts when only type  $\theta_h$  consumers adopt and  $\delta(\theta_h + v - p_2^{\text{PD}})$  if he does not adopt when nobody adopts. Hence, the strategy of type  $\theta_h$  is given by

$$s_1(\theta_h, p_1) = \begin{cases} A & \text{if } p_1 \leq (1 - \delta)(\theta_h + v)\lambda + \delta p_2^{\text{PD}}, \\ \emptyset & \text{if } p_1 > (1 - \delta)(\theta_h + v)\lambda + \delta p_2^{\text{PD}}. \end{cases}$$

Thus, the optimal price in period 1 is  $p_1^{\text{PD}} = (1 - \delta)(\theta_h + v)\lambda + \delta(\theta_\ell + v)$ . Taking the difference between the price in period 1 and 2, we can obtain

$$p_1^{\text{PD}} - p_2^{\text{PD}} = (1 - \delta)\{(\theta_h + v)\lambda - (\theta_\ell + v)\}.$$

Thus,  $p_1^{\text{PD}} < p_2^{\text{PD}}$  if  $(\theta_h + v)\lambda < \theta_\ell + v$  and  $p_1^{\text{PD}} > p_2^{\text{PD}}$  if  $(\theta_h + v)\lambda > \theta_\ell + v$ .

## Appendix B. Risk dominance and proof of Proposition 2

In period 2, we assume that each consumer believes that type  $\theta_h$  consumers adopt. Consider a period 1 history  $h \in H$  along which mass  $\mu \leq \lambda$  of type  $\theta_h$  consumers adopt. For a type  $\theta_\ell$  consumer, his utility when other type  $\theta_\ell$  consumers adopt and do not is as shown in Table 4. Let  $\beta$  be the proportion of type  $\theta_\ell$

	Both $\theta_h$ and $\theta_\ell$ adopt	$\theta_h$ adopts
Adopt	$\theta_\ell + v - p_2$	$(\theta_\ell + v)\lambda - p_2$
Don't	0	0

Table 4: Utility of a type  $\theta_\ell$  consumer

consumers who adopt. Given  $\beta$ , the utility of a type  $\theta_\ell$  consumer is  $\beta(\theta_\ell + v - p_2) + (1 - \beta)\{(\theta_\ell + v)\lambda - p_2\}$  if he adopts, and his utility is 0 if he does not adopt. Hence, the strategy of type  $\theta_\ell$  is given by

$$s_2(\theta_\ell, p_2, h) = \begin{cases} A & \text{if } p_2 \leq (\theta_\ell + v)\lambda + \beta(\theta_\ell + v)(1 - \lambda), \\ \emptyset & \text{if } p_2 > (\theta_\ell + v)\lambda + \beta(\theta_\ell + v)(1 - \lambda). \end{cases} \quad \forall h \in H.$$

Thus, the optimal price in period 2 is  $p_2^{\text{RD}} = (\theta_\ell + v)\lambda + \beta(\theta_\ell + v)(1 - \lambda)$ . In period 1, we assume that each consumer believes that type  $\theta_\ell$  consumers do not adopt. For a type  $\theta_h$  consumer, his utility when other type  $\theta_h$  consumers adopt and do not is as shown in Table 5. Let  $\alpha$  be the proportion of type  $\theta_h$  consumers

	$\theta_h$ adopts	Neither type adopts
Adopt	$(\theta_h + v)\{\lambda + \delta(1 - \lambda)\} - p_1$	$\delta(\theta_h + v) - p_1$
Don't	$\delta(\theta_h + v - p_2^{\text{RD}})$	$\delta(\theta_h + v - p_2^{\text{RD}})$

Table 5: Utility of a type  $\theta_h$  consumer

who adopt. Given  $\alpha$ , if a type  $\theta_h$  consumer adopts, his utility is given by

$$\alpha\{(\theta_h + v)\{\lambda + \delta(1 - \lambda)\} - p_1\} + (1 - \alpha)\{\delta(\theta_h + v) - p_1\},$$

and if he does not adopt, his utility is  $\delta(\theta_h + v - p_2^{\text{RD}})$ . Hence, the strategy of type  $\theta_h$  is given by

$$s_1(\theta_h, p_1) = \begin{cases} A & \text{if } p_1 \leq (1 - \delta)\alpha(\theta_h + v)\lambda + \delta p_2^{\text{RD}}, \\ \emptyset & \text{if } p_1 > (1 - \delta)\alpha(\theta_h + v)\lambda + \delta p_2^{\text{RD}}. \end{cases}$$

Thus, the optimal price in period 1 is  $p_1^{\text{RD}} = (1 - \delta)\alpha(\theta_h + v)\lambda + \delta p_2^{\text{RD}}$ . Taking the difference between the price in period 1 and 2, we can obtain

$$p_1^{\text{RD}} - p_2^{\text{RD}} = (1 - \delta)\left[\alpha(\theta_h + v)\lambda - \{(\theta_\ell + v)\lambda + \beta(\theta_\ell + v)(1 - \lambda)\}\right].$$

Thus,  $p_1^{\text{RD}} < p_2^{\text{RD}}$  if  $\alpha(\theta_h + v)\lambda < (\theta_\ell + v)\lambda + \beta(\theta_\ell + v)(1 - \lambda)$  and  $p_1^{\text{RD}} > p_2^{\text{RD}}$  if  $\alpha(\theta_h + v)\lambda > (\theta_\ell + v)\lambda + \beta(\theta_\ell + v)(1 - \lambda)$ .

## Appendix C. Proof of Proposition 3

Let  $\theta_1$  be a type such that adoption in period 1 is indifferent to adoption in period 2 and  $\theta_2$  be a type such that adoption in period 2 is indifferent to no adoption. Note that  $n_1 = 1 - \theta_1$  and  $n_2 = \theta_1 - \theta_2$ .

$$\begin{aligned}
& (\theta_1 + v)\{1 - \theta_1 + \delta(\theta_1 - \theta_2)\} - p_1 = \delta\{(\theta_1 + v)(1 - \theta_2) - p_2\} \\
\Leftrightarrow \quad & \theta_1 = \frac{1-v}{2} \pm \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2}{1-\delta}} \quad \text{and} \\
& \delta\{(\theta_2 + v)(1 - \theta_2) - p_2\} = 0 \\
\Leftrightarrow \quad & \theta_2 = \frac{1-v}{2} \pm \sqrt{\frac{(1+v)^2}{4} - p_2}.
\end{aligned}$$

For TCE, the prices must be such that these thresholds satisfy  $0 \leq \theta_2 < \theta_1 < 1$ . There exists two thresholds for each period according to the sign of the second term, and there are four possible combinations of thresholds  $(\theta_1, \theta_2)$  that can be the TCE. In the following, the sign  $+$  is referred to as pessimistic and the sign  $-$  as optimistic, and the combinations are denoted as (Pessimistic, Pessimistic), (Optimistic, Optimistic), (Pessimistic, Optimistic), and (Optimistic, Pessimistic). Before considering each case, note that each radicand is non-negative in the TCE:

$$\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta} \geq 0 \quad \Leftrightarrow \quad p_1^* - \delta p_2^* \leq (1-\delta) \frac{(1+v)^2}{4} \quad \text{and} \quad (1)$$

$$\frac{(1+v)^2}{4} - p_2^* \geq 0 \quad \Leftrightarrow \quad p_2^* \leq \frac{(1+v)^2}{4}. \quad (2)$$

From now on, we consider each case. First, we assume  $v < 1$ .

(i) (Pessimistic, Pessimistic)

Let  $\theta_1 = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2}{1-\delta}}$  and  $\theta_2 = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - p_2}$ . Since  $v < 1$  and the radicand is non-negative,  $\theta_2^* \geq 0$  always holds. For any period 1 history consisting of period 1 price  $p_1$  and threshold  $\theta_1$ , the threshold in period 2 must be  $\theta_2^* < \theta_1$  in TCE:

$$\theta_2^* < \theta_1 \quad \Leftrightarrow \quad p_2^* > (\theta_1 + v)(1 - \theta_1). \quad (3)$$

For the threshold in period 1,  $\theta_2^* < \theta_1^* < 1$  must be satisfied in TCE:

$$\theta_2^* < \theta_1^* < 1 \quad \Leftrightarrow \quad \delta p_2^* < p_1^* < p_2^*. \quad (4)$$

From (1), (2), (3) and (4), the TCE is

$$p_1^* \in \left\{ p_1 : \delta p_2^* < p_1 < p_2^* \right\}, \quad p_2^* \in \left\{ p_2 : p_1^* < p_2 \leq \frac{(1+v)^2}{4} \right\},$$

$$\theta_1^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad \text{and} \quad \theta_2^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - p_2^*}.$$

Thus,  $p_1^* < p_2^*$ .

(ii) (Optimistic, Optimistic)

Let  $\theta_1 = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2}{1-\delta}}$  and  $\theta_2 = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - p_2}$ . For any period 1 history consisting of period 1 price  $p_1$  and threshold  $\theta_1$ , the threshold in period 2 must be  $0 \leq \theta_2^* < \theta_1$  in TCE:

$$0 \leq \theta_2^* < \theta_1 \quad \Leftrightarrow \quad v \leq p_2^* < (\theta_1 + v)(1 - \theta_1). \quad (5)$$

For the threshold in period 1, since  $v < 1$  and the radicand is non-negative,  $\theta_1^* < 1$  always holds. The threshold in period 1 must be  $\theta_2^* < \theta_1^*$  in TCE:

$$\theta_2^* < \theta_1^* \quad \Leftrightarrow \quad p_2^* < p_1^*. \quad (6)$$

From (1), (2), (5) and (6), the TCE is as follows:

$$p_1^* \in \left\{ p_1 : p_2^* < p_1 \leq \delta p_2^* + (1-\delta) \frac{(1+v)^2}{4} \right\}, \quad p_2^* \in \left\{ p_2 : v \leq p_2 \leq \frac{(1+v)^2}{4} \right\},$$

$$\theta_1^* = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad \text{and} \quad \theta_2^* = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - p_2^*}.$$

Thus,  $p_1^* > p_2^*$ .

(iii) (Pessimistic, Optimistic)

Let  $\theta_1 = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2}{1-\delta}}$  and  $\theta_2 = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - p_2}$ . For any period 1 history consisting of period 1 price  $p_1$  and threshold  $\theta_1$ , the threshold in period 2 must be  $0 \leq \theta_2^* < \theta_1$  in TCE:

$$0 \leq \theta_2^* < \theta_1 \quad \Leftrightarrow \quad v \leq p_2^* < (\theta_1 + v)(1 - \theta_1). \quad (7)$$

For the threshold in period 1, since the radicand is non-negative,  $\theta_1^* > \theta_2^*$  always holds unless  $p_1 =$



$p_2 = \frac{(1+v)^2}{4}$ . The threshold in period 1 must be  $\theta_1^* < 1$  in TCE:

$$\theta_1^* < 1 \quad \Leftrightarrow \quad p_1^* > \delta p_2^*. \quad (8)$$

From (1), (2), (7) and (8), the TCE is as follows:

$$p_1^* \in \left\{ p_1 : \delta p_2^* < p_1 \leq \delta p_2^* + (1-\delta) \frac{(1+v)^2}{4} \right\}, \quad p_2^* \in \left\{ p_2 : v \leq p_2 \leq \frac{(1+v)^2}{4} \right\},$$

$$\theta_1^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad \text{and} \quad \theta_2^* = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - p_2^*}.$$

Thus, it can be  $p_1^* < p_2^*$  or  $p_1^* > p_2^*$ .

(iv) (Optimistic, Pessimistic)

Let  $\theta_1 = \frac{1-v}{2} - \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2}{1-\delta}}$  and  $\theta_2 = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - p_2}$ . Since  $v < 1$  and the radicand is non-negative,  $\theta_2^* \geq 0$  always holds. For any period 1 history consisting of period 1 price  $p_1$  and threshold  $\theta_1$ , the threshold in period 2 must be  $\theta_2^* < \theta_1$  in TCE:

$$\theta_2^* < \theta_1 \quad \Leftrightarrow \quad p_2^* > (\theta_1 + v)(1 - \theta_1). \quad (9)$$

For the threshold in period 1, since  $v < 1$  and the radicand is non-negative,  $\theta_1^* < 1$  always holds.

Taking the difference between  $\theta_1$  and  $\theta_2$ ,

$$\theta_1 - \theta_2 = - \left( \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2}{1-\delta}} + \sqrt{\frac{(1+v)^2}{4} - p_2} \right).$$

Since the radicands are non-negative, no price profile satisfies  $\theta_1 > \theta_2$ .

Second, we assume  $v \geq 1$ . Then, since  $\frac{1-v}{2} \leq 0$ , the second terms of  $\theta_1$  and  $\theta_2$  must be positive. Let  $\theta_1 = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1 - \delta p_2}{1-\delta}}$  and  $\theta_2 = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - p_2}$ . For any period 1 history consisting of period 1 price  $p_1$  and threshold  $\theta_1$ , the threshold in period 2 must be  $0 \leq \theta_2^* < \theta_1$  in TCE:

$$0 \leq \theta_2^* < \theta_1 \quad \Leftrightarrow \quad v \geq p_2^* > (\theta_1 + v)(1 - \theta_1). \quad (10)$$

For the threshold in period 1,  $\theta_2^* < \theta_1^* < 1$  must be satisfied in TCE:

$$\theta_2^* < \theta_1^* < 1 \quad \Leftrightarrow \quad \delta p_2^* < p_1^* < p_2^*. \quad (11)$$

From (1), (2), (10) and (11), the TCE is as follows:

$$p_1^* \in \left\{ p_1 : \delta p_2^* < p_1 < p_2^* \right\}, \quad p_2^* \in \left\{ p_2 : (\theta_1^* + v)(1 - \theta_1^*) < p_2 \leq v \right\},$$

$$\theta_1^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad \text{and} \quad \theta_2^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - p_2^*}.$$

## Appendix D. Proof of Proposition 4

We show that ACE if and only if

$$(p_1^*, q_1^*) \in \left\{ (p_1, q_1) : \begin{array}{l} \delta p_2^* < p_1 < \delta p_2^* + (1-\delta) \left\{ \frac{-d_2^{*2} + 2v^2 d_2^* + (1+2v)v^2}{4v^2} \right\} \quad \text{and} \\ \delta q_2^* < q_1 < \delta q_2^* + (1-\delta) \left\{ \frac{-d_2^{*2} - 2v^2 d_2^* + (1+2v)v^2}{4v^2} \right\} \end{array} \right\},$$

$$(p_2^*, q_2^*) \in \left\{ (p_2, q_2) : \begin{array}{l} v(2x^* - 1) < p_2 - q_2 < v(2y^* - 1) \quad \text{and} \\ \frac{(p_2 - q_2)^2}{v^2} + 2(p_2 + q_2) \leq 1 + 2v \end{array} \right\}, \quad \text{and}$$

$$x^* = \frac{1+v}{2} - \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}, \quad y^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{q_1^* - \delta q_2^*}{1-\delta}}, \quad \text{and}$$

$$z^* = \frac{1}{2} + \frac{d_2^*}{2v}, \quad \text{where } d_2^* = p_2^* - q_2^*.$$

Since  $(\Rightarrow)$  is already shown in the main text, only  $(\Leftarrow)$  is shown. These sets of equilibrium prices are obtained by the following two conditions:  $0 < x^* < z^* < y^* < 1$  and the utility of type  $z^*$  is non-negative. If firm  $A$  deviates from the equilibrium price  $p_1^*$  in period 1 or  $p_2^*$  in period 2, it loses all of its demand for that period to firm  $B$ . In other words, if firm  $A$  deviates in period 1, then  $x = y = 0$ , this is a contradiction to  $0 < x^* < y^*$ , while if firm  $A$  deviates in period 2, then  $z = x^*$ , this is a contradiction to  $x^* < z^*$ . The same holds true for firm  $B$ .

Suppose that  $x^* = \frac{1+v}{2} - \sqrt{\frac{(1+v)^2}{4} - \frac{p_1^* - \delta p_2^*}{1-\delta}}$ ,  $y^* = \frac{1-v}{2} + \sqrt{\frac{(1+v)^2}{4} - \frac{q_1^* - \delta q_2^*}{1-\delta}}$ , and  $z^* = \frac{1}{2} + \frac{d_2^*}{2v}$ , where

$d_2^* = p_2^* - q_2^*$ . For any type  $\theta$  consumer, his utility is given by

$$\begin{cases} (1 - \theta + v)\{x^* + \delta(z^* - x^*)\} - p_1^* & \text{if adopt } A \text{ in period 1,} \\ \delta\{(1 - \theta + v)z^* - p_2^*\} & \text{if adopt } A \text{ in period 2,} \\ (\theta + v)\{(1 - y^*) + \delta(y^* - z^*)\} - q_1^* & \text{if adopt } B \text{ in period 1,} \\ \delta\{(\theta + v)(1 - z^*) - q_2^*\} & \text{if adopt } B \text{ in period 2,} \\ 0 & \text{if no adoption.} \end{cases}$$

The utility when adopting  $A$  is decreasing in  $\theta$ , while the utility when adopting  $B$  is increasing in  $\theta$ . Let  $\Delta_{12}^F(\theta)$  be a difference in utility of a type  $\theta$  between adopting  $F = A, B$  in periods 1 and 2.

$$\Delta_{12}^A(\theta) = (1 - \delta)(1 - \theta + v)x^* - p_1^* + \delta p_2^*,$$

$$\Delta_{12}^B(\theta) = (1 - \delta)(\theta + v)(1 - y^*) - q_1^* + \delta q_2^*,$$

implying that  $\Delta_{12}^A(\theta)$  is decreasing in  $\theta$  and  $\Delta_{12}^B(\theta)$  is increasing in  $\theta$ . When  $\theta = x^*$ ,  $\Delta_{12}^A(x^*) = 0$ , and when  $\theta = y^*$ ,  $\Delta_{12}^B(y^*) = 0$ . Thus, for type  $\theta \in [0, x^*]$ , adopting  $A$  in period 1 is better than adopting  $A$  in period 2, and for type  $\theta \in [y^*, 1]$ , adopting  $B$  in period 1 is better than adopting  $B$  in period 2. Let  $\Delta_{22}(\theta)$  be a difference in utility of a type  $\theta$  between adopting  $A$  in periods 2 and adopting  $B$  in period 2.

$$\Delta_{22}(\theta) = \delta\{-\theta - v + (1 + 2v)z^* - p_2 + q_2\},$$

implying that  $\Delta_{22}(\theta)$  is decreasing in  $\theta$ . When  $\theta = z^*$ ,  $\Delta_{22}(z^*) = 0$ . Thus, for type  $z^*$ , adopting  $A$  in period 2 is indifferent to adopting  $B$  in period 2, and its utility then is given by

$$\frac{1}{4} \left[ (1 + 2v) - \left\{ \frac{d_2^*}{v^2} + 2(p_2^* + q_2^*) \right\} \right].$$

Since  $1 + 2v \geq \frac{d_2^*}{v^2} + 2(p_2^* + q_2^*)$ , its utility is non-negative. For type  $\theta \in [0, z^*]$ , adopting  $A$  in period 2 is better than adopting  $B$  in period 2, and for type  $\theta \in (z^*, 1]$ , adopting  $B$  in period 2 is better than adopting  $A$  in period 2. Since  $0 < x^* < z^* < y^* < 1$ , for type  $\theta \in [0, x^*]$ , adopting  $A$  in period 1 is optimal; for type  $\theta \in (x^*, z^*]$ , adopting  $A$  in period 2 is optimal; for type  $\theta \in (z^*, y^*)$ , adopting  $B$  in period 2 is optimal; and for type  $\theta \in [y^*, 1]$ , adopting  $B$  in period 1 is optimal.

By the threshold values  $x^*$  and  $y^*$ , we can obtain

$$p_1^* = (1 - \delta)(1 - x^* + v)x^* + \delta p_2^* \quad \text{and} \quad q_1^* = (1 - \delta)(y^* + v)(1 - y^*) + \delta q_2^*.$$

Taking the difference between the price in period 1 and 2 for each firm,

$$p_1^* - p_2^* = (1 - \delta)\{(1 - x^* + v)x^* - p_2^*\}, \quad \text{and} \\ q_1^* - q_2^* = (1 - \delta)\{(y^* + v)(1 - y^*) - q_2^*\}.$$

Therefore, for firm  $A$ ,  $p_1^* < p_2^*$  if  $(1 - x^* + v)x^* < p_2^*$ , that is, introductory pricing is optimal,  $p_1^* > p_2^*$  if  $(1 - x^* + v)x^* > p_2^*$ , that is, skim pricing is optimal, whereas for firm  $B$ ,  $q_1^* < q_2^*$  if  $(y^* + v)(1 - y^*) < q_2^*$ , that is, introductory pricing is optimal,  $q_1^* > q_2^*$  if  $(y^* + v)(1 - y^*) > q_2^*$ , that is, skim pricing is optimal.

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