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Working Paper Competing Stochastic Thresholds: The Green Transition as a Race Between "The Good" and "The Ugly"

CESifo Working Paper, No. 11418

Provided in Cooperation with: Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Lorentzen, Linnea; Strøm, Steinar; Vislie, Jon (2024) : Competing Stochastic Thresholds: The Green Transition as a Race Between "The Good" and "The Ugly", CESifo Working Paper, No. 11418, CESifo GmbH, Munich

This Version is available at: https://hdl.handle.net/10419/307348

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Impressum:

CESifo Working Papers ISSN 2364-1428 (electronic version) Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute Poschingerstr. 5, 81679 Munich, Germany Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de Editor: Clemens Fuest https://www.cesifo.org/en/wp An electronic version of the paper may be downloaded • from the SSRN website: www.SSRN.com

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Competing Stochastic Thresholds: The Green Transition as a Race Between "The Good" and "The Ugly"

Abstract

We derive policy rules for a highly aggregated fossil-based world economy with two competing stochastic thresholds or tipping points. Current production generates emissions that add to a stock of GHGs that affect the probability distribution of hitting a climate threshold with severe consequences (the "ugly" scenario). The fossil-intensive output is used for current consumption and as investment in knowledge production, with the stock of knowledge affecting the probability distribution for hitting a "good" threshold or having a technological breakthrough (the "good" scenario). The new technology will provide a clean emission-free substitute to fossil energy. Given that no threshold has been hit, the decision rules are being continuously revised due to the induced changes in the derived probability distributions. To avoid the ugly scenario, while pushing for the good one, we find that the conditional expected marginal benefit or willingnessto-pay for knowledge will increase over time, with a non-decreasing rate of R&D investment and non-increasing rate of consumption. Implementation of this strategy requires a global organization with coercive power, equipped with instruments to tax the negative stock externality and to eventually subsidize the provision of a public good; the stock of knowledge. The optimal carbon tax is derived and shown to depend on the hazard rate for a climate change, modified by the "odds ratio" for a technological breakthrough.

JEL-Codes: C020, H230, H410, Q540, Q550.

Keywords: competing stochastic thresholds, climate change, technological innovation, optimal carbon tax, global implementation.

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September 25, 2024

Comments from Snorre Kverndokk are appreciated. Lorentzen gratefully acknowledges financial support from CREE Oslo Centre for Research on Environmentally friendly Energy. All errors are our own.

1. Introduction

One of the most pressing issues of our time is the danger of a climate change caused by the atmospheric stock of Green House Gases (GHGs) due to the use of fossil fuels. There seems to be consensus among scientists that we should take actions to stabilize the stocks of GHGs to avoid a too high increase in temperature with serious consequences for future generations, but some disagreement between those that call for immediate actions, e.g., Stern (2007,2008). Disagreement stems from different views about the global impact of temperature increases, different estimates of damage costs and costs of reducing emissions, as well as different views about ethics, risk and uncertainty.¹ Policies towards combating a severe climate change seems therefore highly required. "The Green Transition", embodying a large array of national and international measures for reducing current emissions of GHGs, stabilizing the stocks of GHGs and prevent the temperature to increase too much, is high on the global political agenda. Another measure is related to policies designed towards allocating R&D resources to find renewable, clean (emission-free) energy substitutes for fossil fuels. Combining these conflicting issues is the novelty of the paper, where the focus is to consider this green transition as a "competition" between stochastic thresholds.

Many studies rely on damage and cost structures that are smooth and continuous functions of, say, the stock of GHGs or the increase in temperature. However, as pointed out by Nordhaus in his Nobel Lecture; see Nordhaus (2019; p. 1999): "Scientists are particularly concerned about "tipping points" in the earth's systems. These involve processes in which sudden or irreversible changes occur as systems cross thresholds. Many of these systems operate at such a large scale that they are effectively unmanageable by humans with existing technologies. Important global

¹ The suggested social rates of discount to evaluate future costs and benefits from environmental policy actions, and hence the proposed path of the tax-adjusted carbon price will differ substantially between the fringes. See Nordhaus (2007) and Weitzman (2007) for a critical discussion of the Stern Review.

tipping points include the rapid melting of large ice sheets (such as Greenland and West Antarctic) and large-scale changes in ocean circulation such as the Gulf Stream. These tipping points are particularly dangerous because they are not easily reversed once they are triggered." Crossing a tipping point or a threshold might not only be dangerous but might be catastrophic with devastating, irreversible impact on the living conditions for future generations of people around the world. In a worst casescenario, the whole civilization might go under. On the other hand, fighting the consequences or the realization of a catastrophic outcome seems therefore highly warranted. One measure in that direction is the effort to develop alternative technologies or energy substitutes to fossil fuels that cut emissions of GHGs. However, we don't know when such a technology will be available or what will be required of accumulated R&D investments to achieve a successful invention.

Thus we introduce another threshold in addition to the climate threshold. With two stochastic thresholds, one related to the "ugly" scenario – a climate change – and another one, related to the "good" scenario – having a technological breakthrough providing us with a clean emission-free substitute to dirty fossil fuels – we have a race or competition between "what will happen first?". Should a catastrophic outcome happen first, the global economy enters a new regime; called "the bad continuation regime", as an absorbing state. If, on the other hand, a technological breakthrough should happen first, then we will enter "the good continuation regime", with no further risk of a climate change. The question is how a global planner should take these conflicting issues into account and what policy rules should be adopted.

Related literature. Our paper combines two strands in the literature, one from the typical environmental and resource literature and one from the R&D literature. During the last five decades a flow of papers dealing with such threshold uncertainty, mostly based on the assumption of only *one* stochastic threshold, have been published. In the first category, a common feature is that the current use of

fossil fuels generates emissions that add to a stock of GHGs that affects the probability distribution for hitting a climate threshold with social consequences, and to derive policy measures (a carbon tax) so as to reduce the damages; see e.g. the papers by Cropper (1976), Reed and Heras (1992), Clarke and Reed (1994), Aronsson et al. (1998), Gjerde et al. (1999), Nævdal and Oppenheimer (2007), Tsur and Zemel (2008, 2009, 2016), Lemoine and Traeger (2014, 2016), Lontzek et al. (2015), Engström and Gars (2016), and van der Ploeg and de Zeeuw (2018).² Most papers have only one stochastic threshold for the climate change, but a catastrophic outcome might be triggered by different factors; say either if a stock variable should exceed some critical level or if the increase in global temperature should become too high (as in Nævdal and Oppenheimer (op.cit.)). Another approach, with several thresholds is the domino approach taken by Lemoine and Traeger (2016), where the probability of occurrence for hitting one threshold is affected by the probability of hitting another threshold, causing a "domino effect.

The second category of related papers is found in the R&D literature. Because there is uncertainty about the social consequences of a climate change and, also, about what measures should be taken to avoid it or how to mitigate its effects. One such measure is the effort in developing alternative technologies or energy substitutes to fossil fuel that cut future emissions of GHGs. We don't know how much accumulated effort is required to have a drastic invention. This relationship is modelled as another stochastic threshold, along the lines found in the seminal papers by Lucas (1971), Kamien and Schwartz (1971, 1978), Davison (1978) and Dasgupta (1982). The accumulated level of knowledge will affect the probability distribution for a technological breakthrough.³

² See also the special issues of Environmental and Resource Economics (2016), and Journal of Economic Behavior and Organization (2016) for further references.

³ We admit that assuming the occurrence of a sudden transition to a new technology – and not a gradual one – is not so realistic, but it helps to focus primarily on the interplay between the two competing thresholds and the corresponding dynamic trade-offs that are derived from the global planner's optimal program. Dixit (2023) discusses whether transition from investing and using

The main findings. On combining these competing thresholds, with probability distributions being affected by economic activities (fossil-based production, consumption, and R&D investment) we derive some policy rules and the associated design of instruments as long as no continuation regime has been reached.

R&D investment should increase over time along with a declining consumption path. However, depending on the induced probability distribution, the fossil-based output itself might increase so as to provide sufficient resources for knowledge production without increasing the likelihood for hitting the bad threshold. In the opposite case, the use of fossil fuels and hence output, should decline over time. To avoid the occurrence of the "ugly" regime, consumption must be reduced to release resources for investment in knowledge production. Current generations must bear the cost of avoiding the "ugly" regime, while at the same time promote the likelihood of entering the "good" regime. The policy rules, based on the hazards rates for either regime must be continuously updated, implying that the negative externality of CO2 emissions is internalized by imposing a carbon tax or a "precautionary tax on fossil fuel", also called a "Pigouvian hazard tax". Such a tax alone will not be sufficient for implementing an optimal outcome. The reason why is that we need to invest in activities that can lead to a technological breakthrough. The "ugly" outcome is a common bad, while investing in knowledge to get the "good" outcome is an investment in a global public good. To implement optimal investment in knowledge, it might be necessary to subsidize investment in R&D according to the conditional expected marginal valuation of knowledge.

The paper is organized as follows: In Section 2 the main ingredients of a model will be outlined. In Section 3 the global planning problem is formulated, whereas in Section 4 and 5 we discuss the optimal strategies conditional on no threshold being

polluting technologies to modern costly non-polluting ones should be gradual or sudden. In his model the (initial high) cost of clean technologies is reduced over time due to learning by doing, and once transition becomes worthwhile it should be sudden and complete.

hit. In Section 6 we summarize our main findings. The details of the model are found in appendices.

2. The Structure of the Model

We formulate a rather stylized model for a global economy run by a planner ("the climate protocol"). The model captures the two competing stochastic thresholds to analyze an uncertain race between technological development and an environmental catastrophe or climate change. We assume a very simple aggregated production structure, with one single output, produced by using energy as the sole input, which is given by the sum of fossil fuels (the dirty or "brown" input), x, and a "green" or clean perfect substitute, y. The clean input is available only after a technological breakthrough or "the invention". The macro production function is an ordinary neoclassical one, f(x + y), with net output f(x + y) - ax - by, being used for current consumption, c, and as input («R&D-investment») in knowledge production, given by the flow variable m. The coefficients (a,b) are unit costs of providing the two categories of energy, in terms of output, with a > b, by assumption. The production function has standard properties; twice continuously differentiable, strictly increasing and strictly concave, with f(0) = 0, $f'(0) = \infty$, and f' approaches a number below *b* as the input use becomes sufficiently high. In the macro production function, we have implicitly assumed that other inputs are used in fixed proportions to energy. This is a crude simplification but enables us to concentrate on the main issue dealing with competing stochastic thresholds.

To avoid some technical issues related to complex dynamic trade-offs, we will assume that fossil (dirty) fuels are in abundance, say by having access to huge reserves of shale oil or coal. Then we can ignore the pro blem related to exhaustibility of resources, as analyzed by Kamien and Schwartz (1978), and Davison (1978).

Fossil fuel can be produced at a known unit cost a, whereas the substitute, when available, can be produced at a known cost b < a. The assumption of the substitute being cheaper will imply that one will stop using fossil fuels once the substitute has become available, given that no catastrophe yet having occurred. Our justification for this rather drastic assumption is that we then can highlight directly on the competing interaction between the two thresholds. Otherwise, we have to discuss the transition from the brown to a green technology in detail, as done by Dixit (op.cit.).

At each instant of time *t*, we have

(1)
$$f(x(t) + y(t)) - ax(t) - by(t) = c(t) + m(t)$$

with $x(t) \ge 0, m(t) \ge 0, y(t) \ge 0, c(t) \ge 0$. Note that y = 0, prior to the date of invention, and m = 0 once a successful R&D project has been realized or if the climate threshold has been realized first. A climate change cannot be reversed. Thus, if the climate threshold is hit first, the economy will enter an absorbing state.

The use of fossil fuel generates emissions of GHGs, which accumulate without decay, to a stock at t, z(t) as given by

(2)
$$\dot{z}(t) \coloneqq \frac{dz(t)}{dt} = g(x(t))$$
 with $z(t) = \int_{0}^{t} g(x(s))ds$

with z(0) = 0 (by assumption) and the function g(x) being increasing and convex. This relationship is a reduced form of a complex physical relationship between emissions and the concentration of atmospheric GHGs. To simplify even more, we let g be linear and we rule out or do not incorporate explicitly any abatement or natural decay. There is by assumption no damage or cost caused by *current* flow emissions, only by *stock pollution* itself. However, the level of z that triggers a catastrophe, through a substantial rise in temperature, is stochastic. We translate this complex relationship into a known probability distribution for the «location» of a tipping point. To have a tractable analytical framework within which we might discuss some interaction between multiple stochastic thresholds, we need some simplifying assumptions. Let the stochastic climate tipping state be given by a variable Z, and let $F(z) = \Pr(Z \le z)$, with the distribution function F being increasing and differentiable, and with F(0) = 0 and $\lim_{z\to\infty} F(z) = 1$. Because z is increasing over time, we can derive a probability distribution for the critical point in time space T_1 when the threshold is hit, as

$$(3-i) \quad F(z(t)) = \Pr(Z \leq z(t)) = \Pr(z^{-1}(Z) \leq t) := \Pr(T_1 \leq t) := \Omega(t)$$

From an analytical perspective the stochastic nature of the climate threshold is similar to the extraction of a non-renewable resource of unknown size, as analyzed very elegantly by Loury (1978).

We adopt a rather simple structure of knowledge production similar to what was proposed in the literature mentioned above. Current investment in R&D accumulates, without loss in memory, into a stock of "knowledge" – a public good – denoted M(t) at t, according to the "knowledge production function" h(m). We assume, for simplicity, that the h – function is twice differentiable, strictly increasing and strictly concave, with h(0) = 0 and h'(0) sufficiently large and with accumulated knowledge at some point in time t given by

(4)
$$M(t) = \int_{0}^{t} h(m(s))ds$$
; with $M(0) = 0$ and $\dot{M}(t) \coloneqq \frac{dM(t)}{dt} = h(m(t))$

Once knowledge has reached a critical but unknown level, and no environmental threshold has been hit by then, the clean substitute will become available. This substitute is emission-free with no damaging waste or pollution from its use. Hence, the new technology will free us from any catastrophe in the future.

Let then the level or state of knowledge that will trigger a successful invention be given by a stochastic variable *K*, with a probability distribution $G(M) = \Pr(K \le M)$; *G* increasing and differentiable, with G(0) = 0 and $\lim_{M\to\infty} G(M) = 1$. Again, we can transform this distribution into a probability distribution in time space, here for the critical point in time, denoted T_2 , as given by, cf. (3) above

$$(3-ii) \quad \Psi(t) = \Pr(T_2 \le t) := G(M(t))$$

The objective of our global planner or "protocol" is to maximize the expected present discounted value (PDV) of future utility flows, $\int_{0}^{\infty} e^{-rt}U(c(t))dt$, subject to all relevant constraints, with r being a positive "pure" utility discount rate, constant over time, by assumption. We assume that current welfare depends only on the flow of current consumption, with U(c) being twice differentiable, increasing and strictly concave, with $U(0) = 0, U'(0) = \infty$, and U'(c) small as c increases beyond limit. (Implicit in our formulation is a constant population, normalized to unity.)

To solve this planning problem, we must specify what the world will look like after having crossed either threshold, from the point in time when the economy enters a new regime. We call such a regime "a continuation regime". There are two exclusive continuation regimes, triggered by the realization τ of a stochastic variable $T = \min[T_1, T_2]$.

Given that a successful invention happens first, the "good" continuation regime starts at $T_2 = \tau$, with a corresponding value function, given by the present discounted value of all future utility flows due to the availability of clean energy, defined as

(5)
$$W \coloneqq Max_y \int_{\tau}^{\infty} e^{-r(t-\tau)} U(f(y(t)) - by(t)) dt = \frac{U(c(b))}{r}$$

where c(b) = f(y(b)) - by(b) with $y(b) = \arg \max_{y} [f(y) - by]$, obeying f'(y) = b. The continuation output for this regime is a constant flow per unit of time, y(b), and with a corresponding continuation payoff W.

On the other hand, should T_1 occur first, we have a climate change with a dramatic fall in welfare, caused by destruction of global production capacity, people and land. The continuation payoff for this outcome is simply assumed to be "small" and given by $\frac{\alpha}{r} \coloneqq J < W$, where $\alpha \ge 0$ is an exogeneous (expected) constant utility flow per unit of time for this regime.

The two stochastic variables (Z, K) have known (subjective) distributions, independent by assumption. We can then derive the probability distribution for the critical date $T = \min[T_1, T_2]$, which is:

(6)

$$Pr(T \le t) = 1 - Pr(T \ge t) = 1 - Pr(Min(T_1, T_2) \ge t) = 1 - Pr(T_1 \ge t \cap T_2 \ge t)$$

$$= 1 - (1 - F(z(t)) \cdot (1 - G(M(t))) := P(t)$$

with a corresponding unconditional density function given by:

(7)
$$P'(t) = G'(M(t))h(m(t))[1 - F(z(t))] + F'(z(t))x(t)[1 - G(M(t))]$$

Because a regime shift is triggered either by a climate change or by a technological breakthrough, we must specify the probability distribution for each outcome to provide a precise representation of the corresponding *expected* continuation payoffs. (Details are found in Appendix A.)

3. The global ex ante planning problem

We assume, perhaps a bit naïvely, the presence of a global benevolent planner ("the climate protocol" or "An Ideal Paris-like Agreement"), representing all living and unborn people, with the ambitious objective of designing an optimal strategy for all human generations. The relevant constraints are given by (1), (2) and (4), along with the probability beliefs for the location of the thresholds or tipping points, as summarized in (6) and (7) above, and (a-i) to (a-v) in Appendix A.

A feasible outcome is now a point in time t > 0 when we enter one of the regimes. As seen from ex ante, the payoff for this specific outcome is,

$$\int_{0}^{t} e^{-r\tau} U(c(\tau)) d\tau + e^{-rt} \Phi(t), \text{ where } \Phi(t) \text{ is the expected continuation payoff from t and}$$

onwards as seen from ex ante, defined as:

(8)
$$\Phi(t) \coloneqq \Gamma(z(t), M(t), x(t), m(t)) \cdot J + \Lambda(z(t), M(t), x(t), m(t)) \cdot W = \frac{\eta(z)xJ + \gamma(M)h(m)W}{\eta(z)x + \gamma(M)h(m)}$$

This expected continuation payoff, which is continuously revised over time, is a weighted average of the welfare levels, J and W, with the hazard rates in time space as weights. Note that from using (8) along with the hazard rate for $Min[T_1, T_2]$ from Appendix A, we have

(9)
$$\frac{P'(t)}{1-P(t)}\Phi(t) = \left[\eta(z(t))x(t)J + \gamma(M(t))h(m(t))W\right]$$

Because the *date* of a regime shift is stochastic, with a density function (7), we can express the overall ex ante planning problem as:

$$Max_{(x,m)}\int_{0}^{\infty}P'(t)\Biggl[\int_{0}^{t}e^{-r au}U(c(au))d au+e^{-rt}\Phi(t)\Biggr]dt$$

s.t.

(1)'
$$c(t) = f(x(t)) - ax(t) - m(t)$$

$$(2)'$$
 $\dot{z}(t) = x(t); z(0) = 0;$; no conditions on $\lim_{t \to \infty} z(t)$

(4)'
$$\dot{M}(t) = h(m(t)); M(0) = 0;$$
 no conditions on $\lim_{t \to \infty} M(t)$

On integrating by parts, the objective function can be expressed as

$$\int_{0}^{\infty} e^{-rt} \Big[(1 - P(t))U(c(t)) + P'(t)\Phi(t) \Big] dt = \int_{0}^{\infty} e^{-rt} (1 - P(t)) \bigg[U(c(t)) + \frac{P'(t)}{1 - P(t)} \Phi(t) \bigg] dt$$

The term within brackets in the last integral can be interpreted as a conditional expected welfare flow at t, conditional on the non-occurrence of a tipping point prior to t. This is for short written as

(10)
$$\lambda(t) \coloneqq U(c(t)) + \frac{P'(t)}{1 - P(t)} \Phi(t) = U(c(t)) + \eta(z(t))x(t)J + \gamma(M(t))h(m(t))W$$
.)

The details of the optimal ex ante program are found in Appendix B.

From these optimality conditions, we can formulate a set of decisions rules or a strategy, showing what should be done at any point in time, conditional on no threshold being hit. We assume both control variables, (x,m) being strictly positive outside the continuation regimes. In that case we have:

$$(11-i) \quad U'(c(t)) \Big[f'(x(t)) - a \Big] = \frac{p(t)}{1 - P(t)} - \frac{\frac{d}{dx} \Big[\Pr(A_1) J \Big]}{1 - P(t)} \coloneqq \pi(t) - \eta(z(t)) \cdot J$$

$$(11-ii) \quad U'(c(t)) = \frac{q(t)}{1-P(t)}h'(m(t)) + \frac{\frac{d}{dm}\left[\Pr(A_2)W\right]}{1-P(t)} \coloneqq \left[Q(t) + \gamma(M(t))W\right] \cdot h'(m(t))$$

Here (p,q) are the current (unconditional) costate variables, related to the state equations (2)' and (4)', with p as the shadow *cost* of the stock of GHGs (or (-p)being the shadow value of "environmental capital"), whereas q is the shadow value of knowledge, both in units of utility. Later we will impose a set of "truncated" or conditional shadow prices, $\pi(t) \coloneqq \frac{p(t)}{1-P(t)}$ and $Q(t) \coloneqq \frac{q(t)}{1-P(t)}$, conditional on no regime shift has occurred prior to t.

At any point in time given that no threshold has been hit, (11-i) says that the utility valuation of the increased consumption from a marginal increase in fossil fuel should be balanced against the conditional net marginal cost, as given by the conditional shadow cost for the stock of GHGs, $\pi(t)$, minus the conditional expected immediate welfare from entering the bad continuation regime. The second condition (11-ii) shows that the optimal R&D investment trades off the direct utility cost of spending more resources on R&D (for a given output), and the expected utility valuation of the marginal productivity of investment in R&D: On intensifying R&D now, the probability of hitting the good threshold will immediately increase, along with making the future value of having more knowledge higher if no threshold is hit now.⁴

From Appendix B, the unconditional costate variables must obey a set of dynamic efficiency or no-arbitrage conditions. The shadow cost of the stock of GHGs has to obey

$$(11 - iii) \quad rp(t) = \dot{p}(t) + (1 - G(M(t)))F'(z(t)) \left[\lambda(t) - \frac{\eta'(z(t))}{\eta(z(t))}x(t)J\right]$$

with the following interpretation: On delaying the use of fossil fuel to avoid an increase in the stock of GHGs by one unit, the economy will suffer a time cost per unit of time, rp(t), because we sacrifice immediate welfare valued at p(t). This time cost is traded off against the current expected benefit from an improved environmental capital per unit of time, as given by a "capital gain" $\dot{p}(t)$ or reduced future shadow cost of GHGs, and a term showing expected welfare gain from lowering the likelihood for a bad outcome as the probability distribution for hitting a threshold is moved "outwards".⁵ We observe that the expected return from investing in environmental capital is smaller the more likely it is to hit the *good* threshold. This makes sense. The closer we are to get a technological breakthrough, the less

⁴ See Dasgupta (op.cit.) for a similar line of reasoning.

⁵ A lower stock of GHGs will reduce the unconditional probability for hitting a threshold $Pr(T \ge t) = 1 - P(t)$, so it will be possible to reap the expected welfare flow λ somewhat longer, adjusted for the rate of change in the hazard rate $\eta(z)$.

dangerous will it be to use fossil fuel today. It might then even be desirable to *speed up* production to have more resources available for R&D. We also note that for an increasing hazard rate; $\eta'(z) > 0$, the expected return is lower the higher is the continuation payoff should the bad threshold be hit. The lower is the welfare in the bad scenario, the higher is the expected return from investing in a clean atmosphere.

On the other hand, if there is no prospect at all for having a technological breakthrough, then the only way to avoid or delay the bad threshold is by lowering current use of fossil fuel or lowering consumption. Then we are back to the onethreshold case studied in the literature referred to above. To implement an optimal policy in that case is to impose a tax on using fossil fuel. The optimality-supporting price of fossil fuel at t conditional on no bad threshold has been hit, can then be found to be⁶

$$(11-i)' \quad f'(x(t)) = a + \frac{1}{U'(c(t))} \int_{t}^{\infty} e^{-r(\tau-t)} \frac{F'(z(\tau))x(\tau)}{1 - F(z(t))} \frac{U(c(\tau)) - rJ}{x(\tau)} d\tau$$

The last term on the RHS of (11-i)', is the *carbon tax* or what Tsur and Zemel (2008) called a "*Pigouvian hazard tax*", measured in units of output, per unit fossil fuel, showing the conditional expected marginal cost from using one more unit of fossil fuel at t. If the bad threshold were to be hit at some future point in time τ , a unit increase in the use of fossil fuel at t will reduce the time period until the bad threshold is hit by $\frac{1}{x(\tau)}$ time units, with a utility loss over the assigned period, discounted back to t, as given by $e^{-r(\tau-t)} \frac{U(c(\tau)) - rJ}{x(\tau)}$. Because the location of the bad threshold is stochastic, we must weigh this utility loss with the conditional density (in time space), and then integrate from t and onwards, for all possible outcomes,

⁶ See Kamien & Schwartz (1978), Loury (op.cit.) and Strøm and Vislie (2019) for a similar derivation.

conditional on not having hit the threshold by t. Later we show this tax rule will be affected by the prospect of having a technological breakthrough.

In an optimal program the unconditional shadow value of knowledge should obey

$$(11 - iv) \quad rq(t) = \dot{q}(t) + G'(M(t))(1 - F(z(t))\left[\frac{\gamma'(M(t))}{\gamma(M(t))}h(m(t))W - \lambda(t)\right]$$

The time cost of investing in knowledge; the LHS of (11-iv), equals the increase in the shadow value due to accumulated knowledge (if no good threshold should occur immediately). In addition we have the "expected marginal productivity of knowledge", for an increasing hazard rate $\gamma(M)$, adjusted for the gain from shortening the period over which the expected conditional (pre-threshold) welfare flow, $\lambda(t)$, will be reaped. Investing in R&D will make it more likely to reach the good continuation regime sooner, while reaping a gain by giving up the expected pre-threshold conditional welfare flow λ at an earlier point in time. The incentive to invest will be weaker the less likely it is to have a breakthrough and the more likely it is to hit the bad threshold.

Since we have introduced the conditional shadow prices, we might get more insight into the trade-offs if we consider the optimality conditions as decision rules. The question then is "what to do at some point in time given that no threshold has been hit?"

4. An optimal strategy

Along with (11i-ii), an optimal strategy has to be supported by the following conditions on the dynamics of the conditional shadow prices $\pi(t)$ and Q(t). Combining (11-iii) and (11-iv), with (9), we get:

(12)
$$\frac{\dot{\pi}(t)}{\pi(t)} = \frac{\dot{p}(t)}{p(t)} + \frac{P'(t)}{1 - P(t)} = r - \frac{1}{\pi(t)} \Big[\eta(z)\lambda - \eta'(z)xJ \Big] + \eta(z)x + \gamma(M)h(m)$$

(13)
$$\frac{\dot{Q}(t)}{Q(t)} = \frac{\dot{q}(t)}{q(t)} + \frac{P'(t)}{1 - P(t)} = r - \frac{1}{Q(t)} \Big[\gamma'(M)h(m)W - \gamma(M)\lambda \Big] + \eta(z)x + \gamma(M)h(m)$$

Define a risk-augmented utility discount rate as the pure rate of time preference, r, augmented by the hazard rate for $\min[T_1, T_2]$; cf. (9), as given by:⁷

(14)
$$\rho(x, m, z, M; r, t) := r + \frac{P'}{1 - P} = r + \eta(z)x + \gamma(M)h(m)$$

Using (14) in (13) and (14), we can rewrite the conditional no-arbitrage conditions as:

$$(12 - iii)' \quad \rho(x, m, z, M; r, t) - \frac{\dot{\pi}(t)}{\pi(t)} = \frac{1}{\pi} \Big[\eta(z)\lambda - \eta'(z)xJ \Big]$$

$$(12 - iv)' \quad \rho(x, m, z, M; r, t) - \frac{\dot{Q}(t)}{Q(t)} = \frac{1}{Q} \Big[\gamma'(M) h(m) W - \gamma(M) \lambda \Big]$$

We can interpret these as saying that the required rate of risk-augmented return in each capital category should be equal to their respective conditional expected marginal rates of return. This risk-adjusted utility discount rate is for short written as $\rho(t)$ later. On this background we should now be able to discuss some of the properties of an optimal program or strategy for our global economy. However, because the various hazard rates enter the no-arbitrage conditions in a rather complex way, we cannot expect to reach unambiguous conclusions about the character of the optimal strategy. But some conclusions can be reached as shown in the next section.

But before we take a closer look at the strategies, let us try to see how the optimal tax rule from the one-threshold case in (11-i)' will be modified if the planner

⁷ This can be regarded as an extension of a result derived by Yaari (1965).

has an opportunity to undertake a counteracting risky R&D program of the type introduced above. To have a simple comparison, let us assume that the expected continuation welfare in the catastrophic scenario is normalized to zero, i.e. J = 0. In that case we can provide a characterization of the shadow cost p(t); and hence $\pi(t)$.

From (11-iii), and with the transversality condition $\lim_{t\to\infty} e^{-rt} p(t) = 0$, we can write $p(t) = \int_{t}^{\infty} e^{-r(\tau-t)} (1 - G(M(\tau))F'(z(\tau))\lambda(\tau)d\tau, \text{ so that}$

(15)
$$\pi(t) \coloneqq \frac{p(t)}{1 - P(t)} = \int_{t}^{\infty} e^{-r(\tau - t)} \frac{1 - G(M(\tau))}{1 - G(M(t))} \eta(z(\tau)) x(\tau) \frac{\lambda(\tau)}{x(\tau)} d\tau$$

Then the optimal price of fossil fuel can be expressed in a similar way as in (11-i)', but with a modified tax element. The optimal tax rate is the part added to the marginal cost a, below as

(16)
$$f'(x(t)) = a + \frac{1}{U'(c(t))} \int_{t}^{\infty} e^{-r(\tau-t)} \frac{1 - G(M(\tau))}{1 - G(M(t))} \eta(z(\tau)) x(\tau) \frac{\lambda(\tau)}{x(\tau)} d\tau$$

The tax rate has a similar interpretation as the one given in relation to (11-i)', but the "probability weight" for the bad threshold is now modified by "the odds ratio", $\frac{1-G(M(\tau))}{1-G(M(t))} \leq 1 \text{ for } \tau \geq t \text{ . If knowledge is increasing over time as long as no}$

threshold has been hit at time t, this term is expected to decline as the M-sequence is increasing. The lower tax on fossil fuel will, cet.par., motivate the producers to increase output, leading to higher emissions and hence a shift in the conditional probability weight related to the bad threshold. If for instance the hazard rate $\eta(z)$ is increasing, there is a counteracting effect on the tax rate, which might lead to a higher tax rate. Also, the closer we are, in a probabilistic sense, to hit the good threshold, the lower is the tax rate, which reflects a "less critical" situation; emissions of CO2 are considered less dangerous. Here we are facing a delicate balancing problem. Because when knowledge has been accumulated to an extent so that we are close to reaching the good threshold, the tax rate is reduced. However, this implies more emissions which can extend the period before hitting the good threshold, or worse, hit the ugly outcome earlier. Therefore, the global planner must be aware of this switching possibility. The moral is: when the prospects seem so good that one should be tempted to celebrate the soon arrival of the good outcome, one must be careful.

5. Some tentative strategies: A discussion

Consider therefore the necessary conditions above written as:

$$(17-i) \quad \omega(t) \coloneqq U'(c(t)) \cdot \left[f'(x(t)) - a\right] = \pi(t) - \eta(z(t)) \cdot J \Rightarrow \dot{\omega}(t) = \dot{\pi}(t) - \eta'(z(t)) \cdot x(t) \cdot J$$

where $\omega(t)$ is the utility valuation of the increased consumption from a marginal increase in the use of fossil fuel, for a given R&D input, when the conditional shadow cost $\pi(t)$ must obey

$$(17 - iii) \quad \dot{\pi}(t) = \rho(t)\pi(t) - \left[\lambda(t)\eta(z(t)) - \eta'(z(t))x(t)J\right]$$

In the following we will assume that $\pi(t) > \eta(z(t))J \Rightarrow f'(x) - a > 0 \Leftrightarrow \omega(t) > 0$.

The optimal input into knowledge production, must obey the following condition, with $B(t) \coloneqq \frac{U'(c(t))}{h'(m(t))}$, and can be regarded as the opportunity cost or utility

valuation of the resources required for having an additional unit of knowledge, or simply, the marginal willingness (in units of utility) for knowledge:

$$(17 - ii) \quad B(t) = Q(t) + \gamma(M(t)) \cdot W \Rightarrow \frac{d}{dt} \left| \frac{U'(c(t))}{h'(m(t))} \right| := \dot{B}(t) = \dot{Q}(t) + \gamma'(M(t))h(m(t))W$$

when the dynamics of the conditional shadow value Q(t), must obey:

$$(17 - iv) \quad \dot{Q}(t) = \rho(t)Q(t) - \left[\gamma'(M(t))h(m(t))W - \gamma(M(t))\lambda(t)\right]$$

Combining these conditions, we get the central relationships underlying the trade-offs of the problem:

(18) (i)
$$\omega(t) = \pi(t) - \eta(z(t)) \cdot J$$
 and (ii) $\left[-\frac{\dot{\omega}(t)}{\omega(t)}\right] = \frac{\eta(z(t))\lambda(t) - \rho(t)\pi(t)}{\omega(t)}$

(19) (i) $B(t) = Q(t) + \gamma(M(t)) \cdot W$ and (ii) $\dot{B}(t) = \rho(t)Q(t) + \gamma(M(t))\lambda(t)$

In Appendix C we have shown how $\dot{\omega}(t) \coloneqq \frac{d\omega(t)}{dt}$ and $\dot{B}(t) \coloneqq \frac{dB(t)}{dt}$ depend

on the primitives of the model, as given by

(20)
$$\dot{\omega}(t) = \left[U''(f'-a) + U' \frac{f''}{f'-a} \right] \dot{c}(t) + U' \frac{f''}{f'-a} \dot{m}(t)$$

(21)
$$\dot{B}(t) = \left[\frac{c U''(c)}{U'(c)} \frac{\dot{c}(t)}{c(t)} - \frac{mh''(m)}{h'(m)} \frac{\dot{m}(t)}{m(t)} \right] \cdot B(t)$$

One observation, from (18-ii) is a similarity between this condition and the well-known *Ramsey rule*. If we put J = 0 so that $\omega(t) = \pi(t)$ as in (15), we have that (18-ii) can be expressed as saying that the relative rate of decline of the (extended) marginal utility of consumption, $\omega(t)$, from using more fossil fuel, should be equal to the difference between the conditional expected marginal rate of return to environmental capital and the risk-augmented social rate of time preference; expressed as

(22)
$$\left(-\frac{\dot{\omega}(t)}{\omega(t)}\right) = \eta(z(t))\frac{\lambda(t)}{\omega(t)} - \rho(t)$$
 A modified Ramsey Rule

A second observation, from (19), is that the conditional expected marginal utility valuation of knowledge, B(t), *must be increasing*. Hence, from (21), we then have that a sufficient, but not a necessary, condition for B(t) to be increasing is the combination ($\dot{c} \leq 0 \land \dot{m} > 0$); i.e. non-increasing consumption path and increasing

R&D investment. However, if $\dot{m}(t) = 0$, then it is necessary (and sufficient as well) that c(t) is strictly declining over time. Because we cannot state precisely how the marginal utility valuation of using fossil fuel for consumption, $\omega(t)$, will behave over time; cf. (18i-ii) we should, however, from using (20) and (21) be able to say something about *what combinations of conditional time paths*, as characterized by (\dot{c}, \dot{m}) or (\dot{c}, \dot{x}) , *are compatible with the dynamics* of (ω, B) .

We can therefore exclude the following four, out of a total of nine combinations, $(\dot{c} \ge 0 \land \dot{m} \le 0) \Rightarrow \dot{B} \le 0$ immediately. However, if we in addition restrict attention to the time paths with B(t) being *strictly increasing*, we have three remaining feasible combinations:

(i)
$$(\dot{c}(t) = 0 \land \dot{m}(t) > 0) \Rightarrow \dot{x}(t) > 0 \land \dot{\omega}(t) < 0 \land \dot{B}(t) > 0$$

(*ii*)
$$(\dot{c}(t) < 0 \land \dot{m}(t) > 0) \Rightarrow \dot{B}(t) > 0$$
 (but ambiguous sign of $\dot{\omega}(t)$ as well as $\dot{x}(t)$)

$$(iii) \quad (\dot{c}(t) < 0 \land \dot{m}(t) = 0) \Rightarrow \dot{x}(t) < 0 \land \dot{\omega}(t) > 0 \land \dot{B}(t) > 0$$

Note that because both $\lambda(t)$ and $\rho(t)$ will include the hazard rates as well as the control variables, we cannot state precisely how the relative rate of decline of the extended marginal utility of consumption will be affected by higher hazard rates. However, this is not the case for the dynamics of B(t). The higher the hazard rates are and the higher is the pre-threshold expected welfare flow, the higher is $\dot{B}(t)$, and the stronger is the incentive to invest in knowledge production.

A third observation is then that the conditional optimal consumption path must be non-increasing! "Current generations", prior to any continuation regime, must pay the price of reducing the likelihood for a climate change by not increasing the rate of consumption. On the other hand, whether the likelihood for a technological breakthrough will increase over time, depends on the character of the fossil-based output activity, as well as both probability distributions, which follows from the behavior of the willingness to pay for fossil fuel or the sign of $\dot{\omega}(t)$. As seen from (18ii) it is almost impossible to identify under what restrictions or assumptions about the hazard rates this marginal willingness will be increasing or decreasing. To get more precise conclusions we have to specify the functional forms and the probability distributions used in the model in more detail.

The three cases specified above, (*i*)-(*iii*), describe various strategies. We can distinguish between an *optimistic* strategy (the one in (*i*)), and a *pessimistic* one in (*iii*), if we leave out the intermediate case with ambiguous sign on both $\dot{\omega}(t)$ and $\dot{x}(t)$. The optimistic strategy, characterized by constant consumption, increasing R&D investment and hence an increasing output path, will be adopted if $\rho(t)\pi(t) < \eta(z(t))\lambda(t)$, with "more emphasis" being put on increasing the likelihood for hitting the good threshold first. This case might be classified as the "good regime". On the other hand, the pessimistic strategy will be adopted if $\rho(t)\pi(t) > \eta(z(t))\lambda(t)$. In that case the global planner is more worried about entering "the ugly regime" first, by not making it more likely to hit the bad threshold. This is accomplished by lowering output over time with less emissions. Because investment in knowledge requires the input of the fossil-fuel based output, the probability of hitting the bad threshold will increase too much if output itself should not be reduced over time. If the planner should increase the rate of R&D investment, consumption must be reduced at an even higher rate. To avoid a catastrophic outcome or prevent the occurrence of the ugly regime, consumption must be reduced over time to release resources for investment in knowledge production without having a higher output. Current generations must therefore bear the cost of a future catastrophe by lowering consumption to reduce the likelihood of a catastrophic outcome, while at the same time giving room for an increase in the likelihood of a technological breakthrough.

6. Concluding remarks

A society with zero net emissions will require changes in the energy mix, new products and changes in existing industrial processes. *Hydrogen* as an energy carrier is a solution in sectors that are difficult to decarbonize, such as industrial processes, long-distance transport and shipping – sectors that have few other options. In order to develop the hydrogen solution, one needs to develop the value chain for low-carbon hydrogen using fossil fuels like oil, natural gas or coal. Moreover, low-carbon hydrogen depends on storage capacity for CO2. The geology of the North Sea basin is very suitable for CO2 storage and has an enormous storage capacity estimated to store Europe's current emissions for almost 25 years. These hydrogen projects have started in Europe and are in an early phase and are subject to uncertainty. Note that to develop a new clean technology it is necessary to use energy inputs. In the current state of the world, that means the use of fossil fuels, as was assumed in the model.

To implement the desired outcome, we must see how an unregulated market outcome must be corrected to provide incentives for realizing an optimal outcome over time. Within the present context there are two issues: First, *the global stock externality* caused by emissions of GHGs and the subsequent impact on the future expected cost of hitting the ugly or bad threshold, must be internalized. The producers must therefore face the true social marginal cost of using fossil energy, as the one outlined in (16). The output must be produced at the desired scale, according to (11-i). This outcome can be implemented at any point in time, as long as no threshold has been hit, by imposing a *carbon tax*, or a "*precautionary tax on fossil fuel*", or a "*Pigouvian hazard tax*", related to the magnitude of the conditional shadow cost of the stock of GHGs, $\pi(t)$. But such a tax alone will not be sufficient for implementing an optimal outcome. The second issue is related to the allocation of output between consumption and investment that must obey the allocation rule (11-ii). The consumption decision must be based on the opportunity cost of consumption, i.e., the value of the lost investment opportunities or the corresponding prospects of

having a technological breakthrough, while taking into account that knowledge itself is a *global public good*. Hence, to implement the optimal investment in knowledge production, the global planner should subsidize R&D investment according to the conditional marginal valuation of knowledge, B(t).

The implementation of an optimal strategy requires a highly centralized authority, say a global organization like UN, because the optimal strategy is rather "information-intensive". Such a global organization must be equipped with enforcement power; a sufficient set of instruments, as well as "coercive" power or authority to "force" individual nations to abide by the rules set by the planner. An important task is the need to update continuously the hazard rates for hitting the two competing stochastic thresholds. This updating enables the global organization to implement continuously the instruments required for an optimal solution. The instruments needed are similar to what Acemoglu et al. (2012) proposed, but within our context this is embedded in a fully stochastic model with two competing thresholds, one good and one bad. First, every nation should face the global tax on emissions, as suggested above; a tax that is revised according to new information. The corresponding tax revenues should, along with national contributions say ε % of GDP, finance a global fund used to subsidize knowledge production around the world in the desired scale, with a subsidy being revised according to new information as well.

Even though this proposal might be regarded as rather naïve or idealistic, the global climate scenarios are rather unpleasant and will require collective actions beyond what we have seen being done so far. However, the cost of inaction might be too high with no opportunity for regret if the ugly outcome should be realized.

Appendix A.8

Define the event "a climate catastrophe", A_1 ; as well as the other event A_2 , "a technological breakthrough". Then the probability for the event A_1 to occur during a short interval [t, t + dt], with $A_1 = T_1 \in [t, t + dt] \cap T_2 \ge t + dt$, and similarly for A_2 , can be expressed, respectively, as:

$$(a-i) \quad \Pr(A_1)dt = \Pr \ T_1 \in \left[t, t+dt\right] \cap T_2 \ge t+dt = \Pr(T_1 \in \left[t, t+dt\right] \cdot \Pr(T_2 \ge t+dt))$$

and

$$(a-ii) \quad \Pr(A_2)dt = \Pr \ T_2 \in \left[t,t+dt\right] \cap T_1 \ge t+dt = \Pr(T_2 \in \left[t,t+dt\right] \cdot \Pr(T_1 \ge t+dt) = t+dt)$$

Divide through by dt and let $dt \rightarrow 0$, and, assuming that the functions are everywhere differentiable, we get,

$$(a - i)' \quad \Pr(A_1) = F'(z(t)) \cdot \dot{z}(t) \cdot (1 - G(M(t))) = F'(z(t)) \cdot x(t) \cdot (1 - G(M(t)))$$

and

$$(a - ii)' \quad \Pr(A_{\gamma}) = G'(M(t)) \cdot h(m(t)) \cdot (1 - F(z(t)))$$

It should be obvious that $Pr(A_1) + Pr(A_2) = P'(t)$; as being easily confirmed.

Therefore, the probability for the climate regime shift to occur, given the occurrence of a shift, as seen from ex ante, is

$$(a-iii) \quad \frac{\Pr(A_1)}{\Pr(A_1) + \Pr(A_2)} := \Gamma(z(t), M(t), x(t), m(t)),$$

and with probability

$$(a-iv) \quad \frac{\Pr(A_{\scriptscriptstyle 2})}{\Pr(A_{\scriptscriptstyle 1})+\Pr(A_{\scriptscriptstyle 2})}\coloneqq \Lambda(z(t),M(t),x(t),m(t))\,,$$

⁸ We use a formulation suggested by Lorentzen (2018).

the shift is triggered by a "technological breakthrough".

In addition, we introduce the non-negative hazard rates in their respective

state spaces, as
$$\gamma(M) \coloneqq \frac{G'(M)}{1 - G(M)}$$
, and $\eta(z) \coloneqq \frac{F'(z)}{1 - F(z)}$.

From (6), (7) and these hazard rates defined, we find the hazard rate for $Min[T_1, T_2]$, as

$$(a - v) \quad \frac{P'(t)}{1 - P(t)} = \eta(z(t))x(t) + \gamma(M(t))h(m(t))$$

Appendix **B**

We assume that a solution will exist and that it is unique. Then an optimal strategy prior to any continuation regime can be found from the Maximum Principle, with a present value Hamiltonian function defined as:

$$H(x, m, z, M, p, q, t) = e^{-rt} \left[(1 - P)U(f(x) - ax - m) + P'\Phi \right] - pe^{-rt}x + qe^{-rt}h(m)$$

where
$$P'(t)\Phi(t) = (1 - G(M(t))F'(z(t))x(t)J + (1 - F(z(t))G'(M(t))h(m(t))W$$
.

The control variables are (x,m), both non-negative. The clean input, y, equals zero prior to an invention, i.e., outside any continuation regime. The state variables are (z, M). We impose a set of current (unconditional) costate variables (p,q), with p as the shadow *cost* of the stock of GHGs (or (-p) being the shadow value of "environmental capital"), whereas q is the shadow value of knowledge, both in units of utility (the numéraire).

On using the Maximum Principle, we have that an optimal solution must obey the following conditions, when both control variables, x and m, are positive outside the continuation regimes:

$$(b-i) \qquad (1-P(t))U'(c(t)) \Big[f'(x(t)) - a \Big] + F'(z)(1-G(M))J - p(t) = 0 (b-ii) \qquad -(1-P(t))U'(c(t)) + \Big[G'(M)(1-F(z))W + q(t) \Big] h'(m(t)) = 0 (b-iii) \qquad \dot{p}(t) - rp(t) = -F'(z)(1-G(M))U(c) + F''(z)(1-G(M))xJ - F'(z)G'(M)h(m)W (b-iv) \qquad \dot{q}(t) - rq(t) = G'(M)(1-F(z))U(c) + G'(M)F'(z)xJ - (1-F(z))G''(M)h(m)W$$

along with the transversality conditions

$$(b-v) \quad \lim_{t \to \infty} e^{-rt} p(t) \ge 0, \\ \lim_{t \to \infty} e^{-rt} p(t) z(t) = 0 = \lim_{t \to \infty} e^{-rt} q(t) M(t), \\ \lim_{t \to \infty} e^{-rt} q(t) \ge 0$$

The ex ante condition (*b-i*) can then be expressed as:

$$(b-i)' \quad (1-P(t))U'(c(t))[f'(x(t)) - a] + \frac{d}{dx}[\Pr(A_1)J] = p(t)$$

Hence, along an optimal path and as long as no threshold has been hit, the unconditional expected marginal welfare from increasing the use of fossil fuels should be equal to the current shadow cost of the stocks of atmospheric GHGs.

Along an optimal path the holding of the various stocks must obey a set of noarbitrage or dynamic efficiency conditions. The stock equilibrium for GHGs, given the optimality of the other state variable, see (*b-iii*), can be rewritten as:

$$(b - iii)' rp(t) = \dot{p}(t) + F'(z(t))(1 - G(M(t))) \left[U(c(t)) + \gamma(M(t))h(m(t))W - \frac{F''(z(t))}{F'(z(t))}x(t)J \right]$$

However, the RHS of (*b-iii*)' can be simplified because we have:

$$\eta(z) = \frac{F'(z)}{1 - F(z)} \Rightarrow \eta'(z) = \frac{F'(z)}{1 - F(z)} \cdot \frac{F''(z)}{F'(z)} + \eta^2 = \eta \left[\eta + \frac{F''(z)}{F'(z)} \right] \Rightarrow \frac{F''(z)}{F'(z)} = \frac{\eta'(z)}{\eta(z)} - \eta(z)$$

The behavior of the hazard rate for accumulated stock of GHGs, $\eta(z)$, will depend on the degree of convexity of the distribution function F(z). One conjecture is that $\eta(z)$ is increasing for low values of z, for which the F – function is convex. For the hazard rate to decrease, the distribution function must be (highly) concave; most likely for higher values of z. This property will have implications for the design of an optimal policy. Using this in (*b-iii*)', yields (11-iii) in the text. (A similar property of the hazard rate $\gamma(M)$ is used to get (11-iv).)

Appendix C

From the definition of $\omega(t)$ in (17-i) we can derive (where, according to our assumptions, signs below the various terms are marked):

$$(c-i) \quad \dot{\omega}(t) = U''(c(t)) \Big[f'(x(t)) - a \Big] \dot{c}(t) + U'(c) f''(x) \dot{x} = \omega(t) \left[\underbrace{\frac{U''(c)c}{U'(c)}}_{-} \frac{\dot{c}(t)}{c(t)} + \underbrace{\frac{f''(x)x}{f'(x) - a}}_{-} \frac{\dot{x}(t)}{x(t)} \right]$$

where $(f'(x) - a)\dot{x}(t) = \dot{c}(t) + \dot{m}(t)$. From the definition of B(t), we have

$$(c-ii) \quad \dot{B}(t) = B(t) \left[\frac{cU''(c)}{U'(c)} \frac{\dot{c}(t)}{c(t)} - \frac{mh''(m)}{h'(m)} \frac{\dot{m}(t)}{m(t)} \right] \Rightarrow \frac{\dot{B}(t)}{B(t)} = \underbrace{\frac{cU''(c)}{U'(c)} \frac{\dot{c}(t)}{c(t)} - \frac{mh''(m)}{h'(m)} \frac{\dot{m}(t)}{m(t)}}_{+}$$

Inserting for $\dot{x}(t) = \frac{\dot{c}(t) + \dot{m}(t)}{f'(x(t)) - a}$ into $\dot{\omega}(t)$ in (*c*-*i*), we get

$$(c-i)' \quad \dot{\omega}(t) = \underbrace{\left[U''(f'-a) + U'\frac{f''}{f'-a} \right]}_{-} \dot{c}(t) + \underbrace{U'\frac{f''}{f'-a}}_{-} \dot{m}(t)$$

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