

Gradoz, Julien

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INTRODUCING A QUALITY VARIABLE IN
THE THEORY OF PRODUCT
DIFFERENTIATION: ABBOTT (1953)
AND DORFMAN AND STEINER
(1954).

BY JULIEN GRADOZ

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Introducing a Quality Variable in the Theory of Product Differentiation: Abbott (1953) and Dorfman and Steiner (1954).

Julien Gradoz, Center for the History of Political Economy, Duke University,
julien.gradoz@duke.edu

<https://orcid.org/0009-0005-1455-3073>

Abstract:

We trace the origins of the definition of the “quality variable” used in the theory of product differentiation, which states that quality is unanimously appreciated by consumers, entails a higher unit cost for firms, is unidimensional and continuous. We also document the transition from a diagrammatic treatment of the issue of product quality to an algebraic one in this theory. To do so, we examine two articles published a year apart in the *American Economic Review*: an article written by Lawrence Abbott (1953) and an article written by Robert Dorfman and Peter Otto Steiner (1954).

JEL codes: B21, D40

Keywords: Quality Variable, Product Differentiation, Lawrence Abbott, Robert Dorfman, Peter Otto Steiner

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In the theory of product differentiation, an important distinction is made between *vertical* and *horizontal* differentiation (Beath and Katsoulacos 1991, ix). Vertical differentiation is defined as a situation in which consumers similarly rank the products sold in the market according to their preference ordering. Horizontal differentiation is defined as a situation in which such common ranking does not exist. Vertical differentiation is traditionally

considered as *quality* differentiation while horizontal differentiation is considered as *variety* differentiation (Neven and Thisse 1989). In vertical differentiation models, quality differentiation is often examined through the introduction of a “quality variable” in the consumers’ utility function or the firms’ profit function (Tirole 1988; Wauthy 1996). Most of the time, this variable is assumed to be unidimensional, continuous, and such that a product associated with a higher value for this variable is preferred by all consumers and entails a higher unit cost for firms.

The association between “quality” and vertical differentiation, as well as the origins of the assumptions associated with the quality variable in vertical differentiation models, have hardly been questioned. We propose to examine two articles published a year apart in the *American Economic Review* to highlight that this association is arbitrary and that these assumptions were initially chosen to describe a specific situation, implying that there is no real justification for why this definition is still used today. The first article, titled “Vertical Equilibrium under Pure Quality Competition”, was written by Lawrence Abbott (1953b). In this article, Abbott examines quality competition between firms through a partial equilibrium analysis, assuming a uniform and unchanging price in the market. He labels this situation as “pure quality competition”. He proposes a broad definition of quality, as it encompasses all the characteristics that allow for the identification of products. Abbott then distinguishes three forms of quality variability in the market: Vertical, horizontal and innovational. After discussing this typology, he constructs a model that exclusively focuses on vertical quality variability for the sake of simplicity. This is a diagrammatic model, which means Abbott does not introduce a quality variable. Furthermore, to represent quality on a two-dimensional graph, Abbott assumes that quality is unidimensional, continuous, and that a product associated with a higher value for this variable is preferred by all consumers and entails a higher unit cost for firms.

The following year, Robert Dorfman and Peter Otto Steiner published “Optimal Advertising and Optimal Quality” (1954). Nowadays, this article is primarily known for its theorem determining the optimal level of advertising chosen by a firm in market equilibrium and is considered a pioneer in the economics of advertising (Bagwell 2007). However, as suggested by its title, this article also examines the role of quality competition in the market. Notably, Dorfman and Steiner present their article as a *generalization* and a *simplification* of

Abbott's model. They consider it a generalization because it examines both quality and advertising competition, while Abbott's model focuses on quality competition. They consider it a simplification because they *algebraically translate* Abbott's model, a method they judge more convenient. In the 1950s, there were many translations from diagrams to algebra (e.g., Yeager 1954), or from algebra to diagrams (e.g., Dorfman 1953; Samuelson 1955). These translations reflect the doubts of that time about the "right" way to present results to other economists and students, which sparked many debates. These debates saw proponents of diagrams opposing proponents of algebra, with the latter gradually prevailing during the second half of the 20th century. As a consequence of this algebraic translation, Dorfman and Steiner introduce a quality variable into their model (in the firm's profit function), which is associated with the assumptions chosen by Abbott.

We make three contributions in this article. First, we demonstrate that the distinction between vertical and horizontal differentiation, which is often attributed to Kelvin Lancaster in the book *Variety, Equity and Efficiency* (1979), was already proposed by Abbott more than twenty-five years earlier. Second, we trace the origin of the definition of the quality variable that is still used in the contemporary theory of product differentiation, showing that this definition has no real analytical foundation and was adopted in a specific context. Finally, we document the transition from a diagrammatic treatment of quality variability to an algebraic one, and also the transition from an encompassing view of quality to a very restrictive one. To the best of our knowledge, the only contribution that has examined Abbott's article and connected it with Dorfman and Steiner's article is the book *Variable Quality in Consumer Theory* written by William Wadman (2015). However, Wadman dedicates only a few pages to these two articles and does not examine, *per se*, the introduction of a quality variable in the theory of product differentiation.

The article is structured as follows. The first section presents Abbott's article, his concept of "pure quality competition", his typology of the different forms of quality variability and his model, which focuses on vertical quality variability. The second section presents Dorfman and Steiner's article, their generalization and algebraic translation of Abbott's model and their definition of the quality variable. The third section concludes.

1 Abbott's Article (1953): An Encompassing View of Quality, then Restricted to a Specific Definition due to Modeling Constraints

In this section, we first present the concept of “pure quality competition” as proposed by Lawrence Abbott. We then detail the three forms of quality variability he identifies and demonstrate that the distinction between horizontal and vertical differentiation was already proposed in his article, more than twenty-five years before Lancaster (1979). Finally, we examine the model he constructs, which is presented diagrammatically to the readers, and in which he adopts a restrictive definition of product quality due to modeling constraints.

Let us briefly present Lawrence Abbott. According to the *Marquis Who's Who in the World* (6th edition, 1982-1983), he was born in Cornwall in 1902. He obtained a Master of Arts from Columbia University in 1945 and a PhD from Columbia University in 1951. His dissertation was titled *The Theory of Quality Competition* and was supervised by John Maurice Clark. Therefore, he obtained his PhD when he was forty-nine years old. Before his PhD, he worked in the advertising industry (1924-1933) and as a member of the NBC writing staff program department (1934-1942). He then became a faculty member at Hotchkiss School (1943-1947), instructor of economics at Columbia University (1947-1951), associate professor of economics at Mount Holyoke College (1951-1953), associate professor then professor of economics at Union College (1953-1968) and he retired in 1968. In parallel with his academic career, he held various positions in the *World Federalist Association*.¹ He died in 1985. Regarding his publications, he wrote three books about music (1939; 1940; 1941), the 1953 article we examine in this section, which later became a book (1955), a manual titled *Economics and the Modern World* (1960), the book *World Federalism: What? Why? How?* (1975) and several short pieces and comments (1953a; 1956; 1962).

1.1 The Concept of “Pure Quality Competition”

In the Marshallian tradition of partial equilibrium analysis, it is assumed that the products sold in the market are *homogenous* (i.e., perfect substitutes), implying that

¹ This association, and more generally the *world federalist movement*, aims at promoting the establishment of a world government to reduce the occurrence of wars, notably atomic wars (Baratta 1989).

competition between firms is restricted to price competition. This situation is generally referred to as “pure price competition”. This simplification allows economists to obtain the “clearest possible picture of the price variable” (Abbott 1953b, 827). The reverse situation could also be imagined, in which price is homogenous in the market (i.e., uniform and unchanging) and products are differentiated, implying that competition between firms is restricted to “quality competition”. Abbott suggests referring to this situation as “pure quality competition”. He defines quality as follows:

The word *quality* will be used in its broadest sense, to describe any or all of the various qualitative characteristics of a physical product or service, or combination of the two, offered for sale. “Quality” thus includes materials, design, style, location of a retail outlet—in short, any and every qualitative attribute. Quality is therefore a multidimensional variable—a compound of numerous elements (e.g., in a necktie: size, shape, type of construction, pattern, color scheme, material, texture, durability, resistance to wrinkling, color fastness), each of which is variable (Abbott 1953b, 827).²

Abbott points out that the economic analysis of his time had mainly focused on pure price competition, thereby neglecting to explore pure quality competition. In fact, “pure price competition” is often referred to as “pure competition”, a situation which obscures another dimension of pure competition. As a consequence, Abbott suggests shedding light on the role of quality competition in the market, even though he acknowledges the existence of prior works that have examined this issue (Hotelling 1929; Chamberlin 1933; Smithies 1940; Brems 1951). He notably mentions an article written by Peter Otto Steiner (1952) which examines the choice of programs in the broadcasting industry, the same Peter Otto Steiner who would algebraically translate Abbott’s model in 1954 (see the second section).

² This definition is close to Edward Hastings Chamberlin’s definition of *product differentiation*: “A general class of product is differentiated if any significant basis exists for distinguishing the goods (or service) of one seller from those of another. Such a basis may be real or fancied, so long as it is of any importance whatever to buyers, and leads to a preference for one variety of the product over another [...] Differentiation may be based upon certain characteristics of the product itself, such as exclusive patented features; trademarks, trade names; peculiarities of the package or container, if any; or singularity in quality, design, color, or style. It may also exist with respect to the conditions surrounding its sale” (1933, 56). A notable difference between Abbott and Chamberlin is the role of “fancied” differences between products, created, for instance, by advertising, which play a significant role in Chamberlin’s theory, and which are set aside by Abbott, who focuses on products’ characteristics. Wadman (2015, 18) also highlights this difference, and notices that introducing the adjective “qualitative” in the definition of quality implies *circularity* in the definition of this concept.

While pure quality competition is a simplification that enables economists to understand the specific role of quality equilibrating adjustments in the market, Abbott also judges that pure quality competition corresponds to a “substantial fraction of today’s economy” (1953b, 827). However, he does not provide examples of markets characterized by pure quality competition. He only mentions that pure quality competition corresponds to the extreme case of a market characterized by *price sluggishness*, and therefore that pure quality competition is a useful approximation to examine this situation. In later works studying pure quality competition (e.g., Brems 1960; Schmalensee 1978; Horstmann and Slivinski 1985; Falkinger 1992), a similar argument is often proposed: They acknowledge that pure quality competition is a simplification, useful for theoretical purposes, while also considering that pure quality competition describes real-world situations. Dennis Smallwood and John Conlisk (1979) offer a clear illustration of this idea, and provide examples of pure quality competition:

The reasons for modeling quality variation at a given price are analogous to the reasons for modeling price variation at a given quality. Such simplified models are steps toward a more general model; and, for some markets, the simplification is appropriate. Quality variation at a given price might occur if price is fixed by law, if oligopolists have an implicit agreement to engage only in nonprice competition, or if consumer information is good about price but bad about quality (Smallwood and Conlisk 1979, 4).

1.2 Vertical, Horizontal and Innovational Quality Variability

To clarify the nature of quality competition in the market, Abbott distinguishes three forms of quality variability: Vertical, horizontal and innovational. Vertical quality variability corresponds to a situation in which the evolution of one or several characteristics of the product is considered to result in a “better” or “worse” product according to all consumers. In the case of a “better” product, it entails a higher production cost for firms. For instance, a product with a higher life expectancy is considered “better” according to all consumers and is more expensive to produce. As another illustration, Abbott suggests that “the simplest kind of vertical variation is a change in the size of the unit” (1953b, 828). For instance, if a bag of flour goes from 250 grams to 500 grams, then *ceteris paribus* all consumers will prefer the

500 grams bag (assuming *non-satiation* of consumers³). In contemporary vocabulary of the theory of product differentiation, this particular dimension of vertical quality variability is referred to as the “simple repackaging approach” (Fisher and Shell 1971; Deaton and Muellbauer 1980).

Horizontal quality variability corresponds to a situation in which consumers disagree about the desirability of the evolution of one or several characteristics of the product and there is no clear relationship between the evolution of the product and the evolution of the production cost. For instance, some consumers may prefer to have a green car while other consumers may prefer to have a red car and there is no clear relationship between the choice of a specific color and the production cost of the car. As underlined by Abbott:

With differences of this sort we may properly speak of one quality being “more suitable” or “more appealing” than another, but such a statement is meaningful only if made with reference to a particular buyer or group of buyers (1953b, 829).

In contemporary vocabulary of the theory of product differentiation, we would say that each consumer or group of consumers have an “ideal variety” and that the actual varieties offered in the market are more or less good substitutes for this ideal variety. In this situation, if price is uniform and unchanging in the market, then all firms producing a variety that is the one closest to the ideal variety of a consumer or group of consumers will obtain a positive market share, given that the consumer or group of consumers have the sufficient resources to purchase the variety (Beath and Katsoulacos 1991, 109).

Finally, innovational quality variability corresponds to a situation in which the evolution of one or several characteristics of the product is considered to result in a “better” or “worse” product according to all consumers. Unlike vertical quality variability, in the case of a “better” product, it “either costs no more to produce or is well worth whatever additional cost is involved, so that the older quality must eventually become obsolete” (Abbott 1953b, 829). For instance, the computing capacities of computers have continuously increased since the 1970s, while the production cost per unit of computing capacity has decreased (Greenstein 1997, 330). Moreover, it is reasonable to assume that consumers prefer more powerful computers. Therefore, the innovational quality variability that characterizes

³ Abbott discusses the limitations of this assumption before adopting it in his model (1953b, 833).

the computer industry could explain the rapid obsolescence of computers (Whelan 2002). In contemporary vocabulary of the theory of product differentiation, innovational quality variability is sometimes labeled as “drastic innovation”, a situation that “occurs when the new product’s quality is much higher than that of the old product, without a substantially higher cost” (Greenstein and Ramey 1998, 290).

Therefore, several notions commonly used in the contemporary theory of product differentiation were already discussed by Abbott in 1953, while Abbott’s article is almost never mentioned, and the origin of these notions is often associated with later works. More fundamentally, we argue that the distinction between horizontal and vertical quality variability proposed by Abbott corresponds to the nowadays commonly used distinction between horizontal and vertical differentiation. For instance, in their textbook devoted to the theory of product differentiation, John Beath and Yannis Katsoulacos (1991) propose the following definition of vertical and horizontal differentiation:

The distinguishing property [of horizontal differentiation] is that, if such products are offered at the same price, consumers, if asked to do so, would *rank them differently*. [...] It is in this sense that the equilibrium might be one of product *variety*. However, an equally important aspect of product differentiation is *quality*, the idea that some goods are just of a higher specification than others and that it is this that is the source of their higher valuation. [...] Products are said to be “vertically” differentiated if, when offered at the same price, *all consumers choose to purchase the same one*: that of highest quality (Beath and Katsoulacos 1991, 109).

Based on this definition, vertical differentiation is characterized by the existence of a products’ ranking shared by all consumers according to their preference ordering, a property that is not verified in a situation of horizontal differentiation. This is the definition proposed by Abbott. A consequence of this common ranking is that if all products are sold at the same price, demand will concentrate on the product ranked in first position by all consumers. Abbott is essentially saying the same thing when he states that “an upward (or downward) vertical change in quality unaccompanied by a change in price gives the buyer more (or less) for his money than before” (1953b, 828). Therefore, if we assume non-satiety of consumers and a homogenous price in the market, then all consumers will choose the product with the

highest vertical quality (all other things being equal). Many works associate the distinction between horizontal and vertical differentiation with Kelvin Lancaster, in the book *Variety, Equity and Efficiency*, published in 1979 (e.g., Gabszewicz and Thisse 1986, 160; Shaked and Sutton 1987, 137; Ziss 1993, 523; Dos Santos Ferreira and Thisse 1996, 486; Herweg 2012, 3). However, we have demonstrated that this distinction was already proposed more than twenty-five years earlier in Abbott's article.⁴

Abbott justifies the elaboration of his typology by noting that prior research had mainly focused on vertical quality variability, which had the consequence of overlooking the role of horizontal quality variability in the market (1953b, 829). Therefore, he presents his typology as a way to clarify the different forms of quality variability in the market and to shed light on horizontal quality variability. As we will show in the next sub-section, this remark is somewhat paradoxical, as Abbott constructs a model that exclusively focuses on vertical quality variability, thereby occulting the role of horizontal quality variability in the market. Finally, Abbott acknowledges that the three categories of his typology are *ideal types* (1953b, 829) and that quality variability most often combines horizontal, vertical and innovational aspects. However, for a long time after the publication of Abbott's article, the theory of product differentiation continued to separately study these three ideal types. It is in the 1990s that economists began to propose models which combined horizontal and vertical differentiation (Neven and Thisse 1989; Irmen and Thisse 1998).

1.3 A Model of Vertical Equilibrium

After presenting his typology, Abbott constructs a model in which he examines equilibrating adjustments in the market in a situation of pure quality competition. Abbott excludes innovational quality variability because taking it into account would raise "enormous analytical difficulties" (Abbott 1953b, 830). He regrets this choice, as he believes that innovational quality variability represents the majority of situations involving quality variability in the market. However, he does not provide examples or arguments to support his assertion.

⁴ Lancaster does not mention Abbott's article in his book.

Regarding horizontal quality variability, Abbott does not exclude it. However, it is considered as being *exogenous*. Specifically, Abbott assumes that in the absence of vertical differentiation, firms are horizontally differentiated in such a way that market shares are equally distributed among firms. Moreover, when a new firm enters the market, the market shares are redistributed such that equal distribution is still observed after the entry. Horizontal quality variability is introduced in this model solely to ensure that the market is not monopolized. As a matter of fact, according to the definition of vertical differentiation, in a situation of uniform and unchanging price, demand would concentrate on a single firm unless horizontal differentiation is also introduced. Abbott implicitly adopts the *uniformity assumption* proposed by Chamberlin (1933) to study monopolistic competition, which states that “both demand and cost curves for all the ‘products’ are uniform throughout the group [of firms]” (Chamberlin 1933, 82). Chamberlin considered this assumption to be “heroic” (*ibid.*) and Abbott concedes that it would have been preferable to endogenously determine the level of horizontal quality variability in the market rather than assuming an equal distribution of the market shares among firms. Regarding vertical quality variability, on which Abbott focuses, he assumes that:

[It] will be restricted to a single dimension, and conceived to be of such a character that the possible varieties can be arranged in order, in an “array” or “spectrum”. More specifically, it will be assumed that vertical quality is variable in only one respect, is continuously variable, and is such that for every level of quality there is a different, and only one, corresponding cost function (1953b, 831).

Therefore, even though Abbott previously acknowledged that quality was multidimensional, that some aspects of quality could not be measured and that quality variability could be a combination of vertical, horizontal and innovational aspects, his model focuses on a specific situation, in which quality is vertical, unidimensional, measurable and continuous. Consequently, he moves from an encompassing view of “quality” to a very restrictive one. According to him, “this restriction enables us to depict quality levels graphically by drawing their corresponding cost curves” (1953b, 831). He acknowledges that this definition is very restrictive, but its adoption can be explained by modeling constraints related to the possibility of representing quality on the axes of a two-dimensional graph.

Even though Abbott does not explicitly refer to Chamberlin, Chamberlin wrote something similar in *The Theory of Monopolistic Competition*:

“Product” variations are very often qualitative, rather than quantitative, and in this case cannot be measured along an axis and displayed in a single diagram (Chamberlin 1933, 79).

While Chamberlin made this brief remark in his book before moving on, Abbott identifies the conditions that ensure the representability of quality variability on a diagram. Chamberlin elaborated this idea in the article “The Product as an Economic Variable” (1953), published the same year as Abbott’s article, which is not mentioned by Abbott. In the subsequent presentation of his model, Abbott introduces numerous assumptions that we do not detail, concerning perfect information, absence of collusion, monoproducer firms, or the existence of instantaneous adjustments in the market. We focus on the assumptions related to demand and production cost. Regarding demand, even though Abbott proposes a nuanced discussion concerning the relation between vertical quality and demand, notably concerning the possible substitution between vertical quality and quantity, he ultimately assumes that demand increases with vertical quality. Moreover, he assumes that all firms choose the same level of vertical quality, implying that only one level of vertical quality is offered in the market. In other words, when a firm changes the vertical quality of its product, other firms immediately imitate it. Regarding production cost, even though Abbott proposes a nuanced discussion concerning the relation between vertical quality and production cost, he ultimately assumes that vertical quality exclusively impacts the unit cost of the firms and that the unit cost is increasing with vertical quality (1953b, 837). We have shown that this is the definition of product quality used nowadays in many works of the theory of product differentiation. This definition was proposed in a specific context, related to the desire to represent quality on a two-dimensional graph. It is very restrictive and lacks a solid analytical foundation. It is worth noting that Abbott’s article has been largely forgotten. In the few contemporary works that seek to trace the origin of this definition, it is systematically associated with an article by Dorfman and Steiner from 1954, which is the focus of the next section. Revisiting Abbott’s article is therefore an opportunity to highlight his earlier contribution and, more importantly, to discuss the specific context in which this definition was adopted, raising questions about the relevance of continuing to use this definition.

2 Dorfman and Steiner's Article (1954): Introducing a Quality Variable in the Theory of Product Differentiation

In 1954, Robert Dorfman and Peter Otto Steiner published “Optimal Advertising and Optimal Quality” in the *American Economic Review*. In this article, they propose a model that they present as a generalization and a simplification of Abbott's model. They consider it as a generalization because it examines both quality and advertising competition, while Abbott's model focuses solely on quality competition. They consider it as a simplification because they *algebraically translate* Abbott's model, a method they judge more convenient. In other words, they obtain the same results as Abbott, and additional ones, by using equations rather than diagrams. Consequently, they introduce a quality variable into their model (in the firm's profit function), and to obtain Abbott's results, they adopt the same definition of quality as him. However, presenting this model as a “simplification” is not straightforward, as many economists did not master algebra at that time. This choice could be linked to Robert Dorfman's active promotion of the use of algebra in economics during this period, a period characterized by numerous debates about the “right” way to present results to other economists and students. These debates featured proponents of diagrams opposing proponents of algebra, with the latter gradually prevailing during the second half of the 20th century. While Abbott's diagrammatic model followed the tradition of Chamberlin and many works published in the 1950s, the algebraic model of Dorfman and Steiner was a precursor to what would become the widespread method of studying product quality in the theory of product differentiation. Therefore, Dorfman and Steiner's article can be seen as embodying the transition from a diagrammatic to an algebraic treatment of quality.

Let us briefly present Robert Dorfman and Peter Otto Steiner. Robert Dorfman was born in New York City in 1916. According to his obituary published by *The Harvard Gazette* in 2002⁵, he obtained a Bachelor of Arts in mathematical statistics from Columbia College in 1936 and a Master of Arts in economics from Columbia University in 1937. He worked as a statistician for the federal government from 1939 to 1943 and served during World War II as an operations analyst for the U.S. Army Air Force. He received a PhD in 1950 from UC

⁵ <https://news.harvard.edu/gazette/story/2002/07/economist-dorfman-dies-at-85/> [Retrieved 02/08/2023].

Berkeley. His dissertation was titled *Applications of Linear Programming to the Theory of the Firm* and was supervised by William John Fellner and Robert Aaron Gordon. He was an associate professor at Berkeley (1950-1955) and a professor of economics at Harvard (1955-1987). He retired in 1987 and died in 2002. His notable publications include works on linear programming (1951; 1953; 1958), environmental economics (1965; 1972; 1977) and the history of economic thought (1989; 1991; 1995). Petter Otto Steiner was born in New York City in 1922. According to his obituary published by *Ann Arbor News* in 2010⁶, he obtained a PhD in 1950 from Harvard University. His dissertation was titled *Workable Competition in the Radio Broadcasting Industry* and was supervised by Edward Sagendorph Mason.⁷ He was a professor of economics at UC Berkeley (1949-1957), University of Wisconsin-Madison (1957-1968) and University of Michigan (1968-1991). He retired in 1991 and died in 2010. His notable publications include works on aged people (1954; 1957), the selection of projects by governments (1959; 1965) and diverse issues related to law and economics (1968; 1983; 1983).

2.1 From a Diagrammatic Treatment of Product Quality to an Algebraic One: Dorfman and the Promotion of the Use of Algebra in Economics

In the first paragraph of their article, Dorfman and Steiner position their contribution in relation to Abbott's article. They underline that "most of the conclusions obtained by Abbott and a number of other results of some interest can be derived more easily by approaching the problem of differentiated competition from a broader point of view than Abbott's, using rather simple analytic tools" (1954, 826). These "simple" analytic tools correspond to algebra (a multivariable profit function) and continuous optimization (the maximization of the profit function). Therefore, Dorfman and Steiner present their article as a *generalization* ("a number of other results") and a *simplification* ("derived more easily") of Abbott's article.

Concerning the second point, we prefer to present their article as an *algebraic translation* rather than a *simplification*. As a matter of fact, while their results could easily be derived by economists who have skills in algebra, this was far from being usual in the 1950s. Yann Giraud (2010) identified several elements supporting this idea. He first mentions the

⁶ <https://obits.mlive.com/us/obituaries/annarbor/name/peter-steiner-obituary?id=23168134> [Retrieved 02/08/2023].

⁷ We thank the staff of the Pusey Library (Harvard University) for this information.

“Graduate Education in Economics” report written by Howard R. Bowen in 1953 as a supplement to the *American Economic Review*, which points out that according to a sample of graduate professors in economics, only 2% of the PhD candidates in economics were considered “good” in mathematics at that time, 41% “fair” and 44% “poor”. Even though the definition of “mathematics” used by Bowen is unclear, it includes algebra (1953, 137). Giraud then mentions the article “The Pure Theory of Public Expenditure” (1954), written by Paul Anthony Samuelson, and published in *The Review of Economics and Statistics* the same year as Dorfman and Steiner’s article. This three-page article is an algebraic exposition of a condition concerning the optimal provision of public goods. When this article was published, Samuelson received harsh criticism (Enke 1955; Margolis 1955) for his use of algebra, and the following year, he published an article presenting his results diagrammatically (Samuelson 1955), suggesting that the use of algebra was uncommon among economists at that time, and that Samuelson had to “translate” (Giraud 2010, 188) his work to facilitate the communication with his peers. Finally, he mentions the database created by Roger Edward Backhouse (1998, 92), which consists of a sample of articles published in the *American Economic Review*, *The Journal of Political Economy*, and *The Quarterly Journal of Economics* between 1920 and 1960. This database highlights the low use of algebra in these articles, followed by an important increase starting in the mid-1950s.

To understand why Dorfman and Steiner present their article as a “simplification,” which was far from obvious at the time, it is useful to briefly mention the relationship between Robert Dorfman and the use of algebra in economics (we do not have the biographical elements to do the same with Steiner). As previously mentioned, Dorfman studied mathematical statistics in his undergraduate degree. He also attended Harold Hotelling’s mathematical economics classes at Columbia University (Gaspard, Missemer, and Mueller 2024). One of his first articles, “The Detection of Defective Members of Large Populations” (1943), published in *The Annals of Mathematical Statistics*, is nowadays considered as an important contribution to a branch of statistics called *group testing* (McMahan, Tebbs, and Bilder 2012).⁸ He presents his doctoral dissertation as an “attempt to apply linear programming to the short-run behavior of the individual firm or entrepreneur

⁸ Likewise, “A Note on the δ -Method for Finding Variance Formulæ” (1938) is considered as one of the first use of the *delta method* in statistics (Ver Hoef 2012; Gorroochurn 2020).

and to assess its value for dealing with this problem” (Dorfman 1951, 5). Therefore, the theory of the firm is considered a “pretext” for studying the utility of linear programming in economics. When Dorfman presents the production process of the firm, he mentions that “in order to avoid the chance of ambiguity inherent in any verbal definition which deals with quantitative matters we shall express our formal definition of a process algebraically, borrowing, for this purpose, chemical notation” (*ibid.*, 14). Therefore, he presents algebra as a way to clarify his arguments. He presents the main findings of his doctoral dissertation in the article “Mathematical, or ‘Linear,’ Programming: A Nonmathematical Exposition” (1953), published in the *American Economic Review*. It starts with the following sentence: “This paper is intended to set forth the leading ideas of mathematical programming purged of the algebraic apparatus which has impeded their general acceptance and appreciation. This will be done by concentrating on the graphical representation of the method” (1953, 797). Therefore, like Samuelson in 1954, Dorfman had to translate his work from algebra to diagrams to facilitate communication with his peers. It is very unlikely that the skills of economists in algebra had radically improved in one year, and therefore presenting the algebraic translation of Abbott’s model as a “simplification” is not innocuous. At UC Berkeley, he taught “Mathematical Methods of Economics” and “Advanced Economic Theory” between 1951 and 1954 (Assaf 2022, 100).⁹ Later in his career, he actively promoted the use of mathematics in social sciences, notably as a staff member of the Social Science Research Council Summer Institutes on Mathematics in Social Sciences in 1957 and as a member of the Social Science Research Council Committee on Mathematics in Social Science Research between 1960 and 1964 (Orozco Espinel 2020, 33). At the time of his death, many tributes highlighted the role of mathematics in his career.¹⁰

The best illustration of Dorfman’s involvement in the debates concerning the use of algebra in economics is his participation to the 1954 issue of *The Review of Economics and*

⁹ “My principal responsibility when I joined Berkeley faculty was to introduce instruction in mathematical methods in economics” (Dorfman 1997, xv).

¹⁰ For instance: “Dorfman, according to his wife, Nancy, turned to mathematics in college as the closest substitute for poetry, after concluding that he did not have a future as a poet. His lifelong love of poetry and literature was reflected in the clarity and grace with which he was able to explain complex economics in simple language, widely remarked upon by his colleagues” (*The Harvard Gazette*, 2002). Likewise, “[Dorfman] was a leader in the introduction of mathematical methods to economics in the twentieth century” (*The Harvard Gazette*, 2012, <https://news.harvard.edu/gazette/story/2012/11/robert-dorfman/> [Retrieved 02/08/2024]).

Statistics devoted to this topic.¹¹ It is in the same issue that Samuelson published “The Pure Theory of Public Expenditure”. As Giraud (2010, 188) highlights, this was no accident, and refers to Pickhardt (2006) to point out that Samuelson’s article “is less a contribution to public expenditure than a demonstration that mathematics is a useful shorthand language of economic theorizing” (*ibid.*). As we will show, this observation also applies to Dorfman and Steiner’s article on product quality. Robert Dorfman’s article in this special issue is titled “A Catechism: Mathematics in Social Science” (1954). The article is a list of questions for which Dorfman proposes an answer. When he asks *Is mathematics necessary in social science?*, he answers that mathematics allows for the identification of problems that would not be possible to identify verbally, and to solve them (1954, 376). Therefore, mathematics is useful for solving specific economic problems and “there is little point in attacking these solutions because the bulk of the profession cannot understand them” (*ibid.*).¹² Considering the recent technical developments of general equilibrium theory, he mentions that: “As a matter of history, no one has succeeded in exploring the existence theorems and welfare implications of general equilibrium without a liberal application of fancy mathematics. Superficial treatments of this problem have led to fallacy, and fallacy, I feel, should be extirpated even at the cost of having to learn some mathematics” (*ibid.*). After acknowledging the utility of mathematics in economics, he asks *Should mathematical economists restate their theories in literary form?* Therefore, he addresses the issue of translation, and we must keep in mind that he had to translate his own work on linear programming the previous year. He answers:

¹¹ The issue consisted in inviting nine economists to respond to a two-page critical article by David Novick titled “Mathematics: Logic, Quantity, and Method”. Philip Mirowski describes this issue as follows: “In what one participant gleefully called a ‘slugfest,’ a relatively obscure economist, after penning a two-page plea for discussion of the limitations of some of the practices of postwar mathematical economists, was subjected to what by any estimation must appear the overkill of nine different economists piling on abuse and scorn. The identity of the economists tapped to discipline the poor sacrificial lamb named David Novick gives an even better idea of the imbalance of this supposed ‘debate’: Lawrence Klein, James Duesenberry, John Chipman, Jan Tinbergen, David Champenowne, Robert Solow, Robert Dorfman, Tjalling Koopmans, and Paul Samuelson” (2001, 396).

¹² In his answer to the question *Are mathematical results often unintelligible and subject to misunderstanding?* he emphasizes this point even more forcefully. “Alas, they are. This, of course, is Novick’s chief charge against mathematical economics, and it is a remarkable charge. I have many times had to argue that the church cannot be attacked because of the sins of the clergy, but this is one of the few times that I have seen the church roundly condemned for the shortcomings of the laity” (*ibid.*).

Novick urges them to do so and Marshall gives the same advice. I must dissent. Practitioners of mathematical economics already have their hands full coping with some of the toughest problems which the science offers and it is unfair to impose on them the special problems of literary lucidity. [...] My very strong feeling is that any discussion, like this one, of the legitimacy of mathematical methods in economics is doomed to be fruitless. Time and tide will not be rolled back. The profession will not relinquish powerful instruments of reasoning (1954, 377).

Therefore, the same year that Dorfman and Steiner published their article on quality and advertising, Dorfman was actively promoting the use of mathematics (notably algebra) in economics and was encouraging mathematical economists not to bother trying to make their work intelligible to economists who do not have mathematical skills. Consequently, when Dorfman and Steiner present their algebraic translation of Abbott's model as a "simplification," we must consider the specific context of this publication, and we can understand this choice as an implicit way to promote the use of algebra in economics. This is particularly clear in the conclusion of their article. Rather than discussing quality or advertising, i.e., the topic of their article, they write (we reproduce the entire conclusion):

There are good grounds for doubting the economic significance of the whole business of writing down profit functions (or drawing curves) and finding points of zero partial derivatives (or graphical points of tangency). Such devices are merely aids to thinking about practical problems and it may be an uneconomical expenditure of effort to devote too much ingenuity to developing them. Yet such devices are aids to clear thought and, if sufficiently simple and flexible, they help us find implications, interrelationships, and sometimes contradictions which might escape notice without them. Such aids are particularly needed in the field of nonprice competition. We hope that the techniques suggested here will be of assistance in developing this field and bringing out its connections with the theory of price competition. The examples we have solved above are not only of importance in themselves but, we hope,

demonstrate the flexibility and convenience of the technique which we suggest (1954, 836).¹³

Their translation of Abbott's model is conceived as much as a contribution to the study of nonprice competition as an illustration of the virtues of using algebra in economics. Dorfman and Steiner hope that the technique they propose in their article will be adopted by other economists to study non-price competition. Retrospectively, we can affirm that their technique was largely adopted in the theory of product differentiation to study the issue of quality variability. This is why we consider Dorfman and Steiner's article to embody the transition from a diagrammatic treatment of the issue of quality variability to an algebraic one.¹⁴ This algebraic treatment notably resulted in the introduction of a "quality variable" in the theory of product differentiation.

2.2 Introducing a Quality Variable in the Theory of Product Differentiation

Dorfman and Steiner's article focuses on a firm that seeks to maximize its profit function according to three variables: The price of the product, its quality and the advertising budget. Four situations are examined by Dorfman and Steiner. First, the firm maximizes its profit function according to the price and the advertising budget. Second, the firm maximizes its profit function according to the price and the quality. Third, the firm maximizes its profit function according to the three variables. Finally, the firm maximizes its profit function according to the quality, assuming a uniform and unchanging price in the market. This corresponds to the "pure quality competition" situation examined by Abbott

¹³ They mention algebra and diagrams, but diagrams play virtually no role in their article. After having determined the algebraic solution to the maximization program of the firm, they introduce three diagrams to illustrate the optimal amount of advertising budget chosen by the firm for different values of the variables. They point out that these diagrams are "illustrative cases" and that they are proposed to the readers as they "may help bring out the significance of the result" (1954, 829). However, they acknowledge that curves have been "arbitrarily drawn" and therefore that determining which equilibrium is the more profitable for the firm "cannot be judged from these diagrams alone" (*ibid.*).

¹⁴ "Dorfman and Steiner (1954) joint paper on differentiated competition analyzed the problem using an optimizing firm framework, in contrast to the typical literature on the topic based in verbal reasoning and diagrammatic representation" (Assaf 2022, 99).

and this is why their model could be considered a generalization of Abbott's model.¹⁵ They define quality as:

Any aspect of a product, including the services included in the contract of sales, which influences the demand curve. The essential difference from advertising is that changes in quality enter into variable costs. Each conceivable quality will have a definite average cost curve, but there may be several different qualities with the same average cost curve. In this case we may assume that only that quality which has the most favorable demand curve will be given serious consideration. Thus we may assume that quality can be improved only at the expense of operating on a higher average cost curve. By quality improvement we mean any alteration in quality which shifts the demand curve to the right over the relevant range and raises the curve of average variable costs (1954, 831).

Quality is therefore considered as a "shift parameter" of the demand curve. The main difference between quality and advertising is that quality is considered as a unit cost, while advertising is considered as a fixed cost (1954, 826). This distinction between quality and advertising had a long-lasting influence on the theory of product differentiation and is still used nowadays (e.g., Anderson, De Palma, and Thisse 1992). In both cases, an increase in quality or advertising enhances the perceived value of the product to consumers and increases the production cost of the firm (with a difference between fixed cost and unit cost). Even if Dorfman and Steiner do not refer to the typology proposed by Abbott, they study a situation of vertical quality variability (Wadman 2015, 24). In their model, quality is taken into account through the introduction of a "quality variable" in the firm's profit function, which is assumed to be unidimensional, continuous and measured "in terms of horsepower, tensile strength, denier, etc." (1954, 832). Advertising is taken into account through the introduction of an "advertising budget variable", which is assumed to be unidimensional, continuous and measured according to the firms' expenditures allocated to it. To understand this difference in the choice of unit of measurement, we can quote Hans Brems (1957), who emphasized that:

¹⁵ Dorfman and Steiner focus on a single firm, while Abbott studies a group of firms. However, since Abbott adopts the uniformity assumption and assumes that firms imitate each other, the two situations are equivalent.

It has long been recognized that the nonprice variables in the theory of the firm are at least as important as the price variable. But the road to a satisfactory quantitative treatment of those variables has been blocked by the following dilemma. Some aspects of product quality and selling effort are thought of as being nonquantitative. As far as selling effort is concerned, this difficulty traditionally has been overcome by using not selling effort itself but selling-effort expenditure, which is always quantitative, as the variable to be optimized. If this were a satisfactory approach, it could obviously be used for nonquantitative aspects of product quality, too. But it is not a satisfactory approach for selling effort any more than for product quality, for selling effort and product quality are both multidimensional. Moreover, from the point of view of demand, alternative dimensions may be substitutional. For example, quantity sold might rise if the input of aluminum were to be substituted for the input of steel, or if television advertising were substituted for magazine advertising. It is not enough, then, to seek the optimal total expenditure. Somewhere in the firm, a decision-maker must know exactly how far to go in each particular dimension of quality and selling effort (1957, 105).¹⁶

Dorfman and Steiner adopt the following compromise: Advertising is measured through the firm's expenditures allocated to it, while quality is measured according to a product's characteristic. On the one hand, it is difficult to find a unit of measurement for advertising, which explains the choice of Dorfman and Steiner to focus on the firm's expenditures, despite the criticisms of Brems. On the other hand, if quality is restricted to a single measurable characteristic of the product, quality can be measured through the quantity of this characteristic. This is the assumption adopted by Dorfman and Steiner, which implies moving from a multidimensional conception of quality to a unidimensional one. This assumption was also adopted by Abbott. More generally, the assumptions associated with the quality variable by Dorfman and Steiner correspond to the algebraic translation of the assumptions chosen by Abbott in his diagrammatic exposition of pure (vertical) quality competition. These assumptions are still widely used, notably because they

¹⁶ Brems thanks Dorfman for the comments on a previous version of his article. We can note that they were colleagues at UC Berkeley (Assaf 2022). Moreover, Brems considers Abbott's and Dorfman and Steiner's articles as "prominent contributions" (1957, 106).

allow a straightforward integration of quality into the models of the theory of product differentiation.

In spite of the importance of firms' product choices, very little formal analysis of product quality determination was to be found in the literature until recently. One obvious reason for this persistent neglect is the apparent difficulty of employing mathematics to model decision-making in this dimension. While "price" and "quantity" can naturally be treated as scalars, it is far from obvious that any single mathematical representation of "quality" can serve for a broad spectrum of products. In their pioneering essay on advertising and quality, Dorfman and Steiner (1954) dealt with this issue by simply assuming the existence of some scalar quality measure, call it X , such that the larger is X the higher are unit costs and the more highly consumers value each unit of the end product. Models based on this assumption of a scalar quality measure are clearly not applicable to all commodities; there are many products, such as automobiles, for which such a measure cannot sensibly be assumed to exist. On the other hand, this assumption may be plausible as a first approximation in some contexts, and it is very convenient. Moreover, the ultimate value of this or any other assumption can only be judged in light of the usefulness of its implications (Schmalensee 1979, 177).

Since Dorfman and Steiner associate the quality variable with the same properties as the price and quantity variables, it becomes possible to apply the algebraic tools of economics to the issue of quality, such as optimization or elasticity calculations. As highlighted by Hayne Leland (1977, 127), Dorfman and Steiner's definition of the quality variable enabled the application of marginal analysis to the issue of quality. As Richard Schmalensee points out, this definition of the quality variable does not apply to many situations, but it has the advantage of being convenient and facilitates the study of non-price competition. Nowadays, this is the standard definition of the quality variable used in the theory of product differentiation (e.g., Tirole 1988; Beath and Katsoulacos 1991; Motta 1993; Wauthy 1996). However, most of the works which mobilize this definition forget that this is actually a specific conception of quality, among many others that had initially been identified by Abbott in his seminal contribution.

3 Conclusion

In this article, we trace the origins of the assumptions commonly associated with the quality variable in the theory of product differentiation, and the origins of the association between “quality” and vertical differentiation. To do so, we identified two articles that have played a central role in this regard. In the first article, Lawrence Abbott (1953b) provides a general discussion concerning the role of quality competition in the market and distinguishes three forms of quality variability: Vertical, horizontal, and innovational. He then constructs a diagrammatic model to study quality competition in the market, in which he arbitrarily focuses on vertical quality variability, moreover assuming that quality is unidimensional, exclusively impacts unit cost and is continuously measurable. This is a specific situation compared to all the situations that could have been studied by Abbott. He acknowledges this idea:

The writer is fully aware of the limited scope of the analysis just presented. The model studied illuminates only one small corner of the great arena of quality competition [...] By further restricting the analysis to a study of vertical variability of quality, we have neglected the important area of horizontal variability, in which decisions regarding quality affect the range of choice open to buyers, and the degree to which products conform to buyers’ needs and tastes and are suitable to the uses to which they are put (Abbott 1953b, 843).

The following year, Dorfman and Steiner (1954) proposed a model in which they algebraically translated Abbott’s model. In doing so, they introduced a quality variable into the firm’s profit function and defined this variable in order to describe the specific situation examined by Abbott. Dorfman and Steiner’s article has gained an important popularity in the theory of product differentiation, related with the increasing use of algebra in economics, and their definition of the quality variable became the standard definition of this variable in this theory. However, we have highlighted the contingency of this definition, and, by examining Abbott’s work, we hope to spark interest in studying the “great arena of quality competition” beyond vertical differentiation.

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