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Efficient Education Policy – A Second-Order Elasticity Rule

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Efficient Education Policy – A Second-Order Elasticity Rule

Abstract

Assuming a two-period model with endogenous choices of labour, education, and saving, efficient education policy is characterized for a Ramsey-like scenario in which the government is constrained to use linear instruments. It is shown that education should be effectively subsidized if, and only if, the elasticity of the earnings function is increasing in education. The strength of second-best subsidization increases in the elasticity of the earnings function. This second-order elasticity rule extends the well-known Ramsey-Boiteux Inverse Elasticity Rule.

JEL-Code: H21, I28, J24.

Keywords: endogenous choice of education, labour, and saving, second-best efficient taxation, linear instruments, finite periods, Ramsey's Rule, Inverse Elasticity Rule.

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1. Introduction

The Inverse Elasticity Rule states that optimally chosen commodity tax rates should be inversely related to the price elasticities of demand. The Rule is usually attributed to Ramsey (1927) although a structurally equivalent formula has been derived independently by Boiteux (1956) for the problem of optimal monopoly pricing. The assumptions underlying optimal taxation in Ramsey's tradition may well be criticized from a conceptual point of view. Still, there will be no introductory course in optimal taxation that would not mention the Rule. This paper demonstrates how the Rule has to be extended if it is to guide efficient education policy in Ramsey's tradition. It is shown that the standard role taken by the elasticity of demand or supply has to be replaced with the elasticity of elasticity – also called second-order elasticity in what follows – if the Rule is to apply to education. The extended Rule calls for subsidizing education effectively if, and only if, the elasticity of the earnings function is increasing in education. In particular, the strength of subsidization should increase in the function's second-order elasticity.

The paper is structured as follows. Section 2 sets up the model of a representative taxpayer. Section 3 derives the extended Elasticity Rule for education. Section 4 provides the proof.

2. A representative-household model

The model is taken from Richter (2009). It assumes a representative taxpayer living for two periods and deriving strictly increasing utility U from consumption C_i and strictly decreasing disutility from non-leisure time L_i in periods $i=1,2$. $U=U(C_1, C_2, L_1, L_2)$ is strictly quasi-concave. L_2 is identical with second-period labour supply. By contrast, only $L_1 - E$ is time spent in the market, while E is time spent on education. First-period labour supply earns a constant wage rate ω_1 ; the return to second-period labour depends on the amount of education. It is paid $\omega_2 H(E)$, where ω_2 is constant while the earnings function $H(E)$ displays positive but diminishing returns, $H' > 0 > H''$. The quantity L_2 is interpreted as qualified labour. Likewise, the quantities $L_1 - E$ and L_1 are interpreted as nonqualified labour and nonqualified non-leisure, respectively. Education causes an opportunity cost in forgone earnings and a monetary cost of tuition. Both costs are assumed to be linear in time. The cost of foregone earnings is denoted by $\omega_1 E$ and the cost of tuition is denoted by ϕE . The share of first-period income that is spent neither on education nor on consumption is saved:

$$S = \omega_1(L_1 - E) - \varphi E - C_1 = \omega_1 L_1 - (\omega_1 + \varphi)E - C_1 \quad (1)$$

By way of normalization, the price of consumption is set equal to one. The gross rate of return to saving is denoted by ρ . Second-period consumption is constrained by income earned:

$$C_2 = \rho S + \omega_2 H(E) L_2 \quad (2)$$

Substituting for S in (1) and (2) yields the lifetime budget constraint:

$$C_1 + C_2 / \rho = \omega_1 L_1 + \omega_2 H(E) L_2 / \rho - (\varphi + \omega_1)E \quad (3)$$

The sole objective of this note is to extend the Inverse Elasticity Rule to education. For this purpose consideration is restricted to utility functions which are quasi linear in first-period consumption and additive in periodic sub-utilities:

$$U(C_1, C_2, L_1, L_2) = C_1 - V(L_1) + U(C_2, L_2) \quad (4)$$

The function V is strictly increasing and strictly convex. The representative taxpayer maximizes (4) in C_1, C_2, L_1, L_2 , and E subject to (3). In what follows it is assumed that this constrained maximization is well behaved. This means that there exists a unique solution which is differentiable in $\omega_1, \omega_2, \rho, \varphi$. The first-order conditions are

$$\omega_1 = V'(L_1), \quad \rho = 1/U_C, \quad (5)$$

$$U_L + \omega_2 H(E) / \rho = 0, \quad \omega_2 H'(E) L_2 / \rho = \omega_1 + \varphi. \quad (6)$$

3. Second-best policy

The government faces the need to raise an exogenous amount of revenue G . Four linear tax instruments are available, each of which is distorting. The taxes are levied on period i 's labour income, on the return to saving, and on the cost of tuition. They are modelled implicitly as the difference between prices before and after tax. The prices after taxes and subsidies are endogenous and denoted by $\omega_1, \omega_2, \rho, \varphi$. The prices before taxes and subsidies are exogenous and denoted by w_1, w_2, r, f . The tax on period i 's labour income is modelled by $w_i - \omega_i$, the tax on capital income by $r - \rho$, and the tax on the cost of tuition by $\varphi - f$. It goes without saying that each tax can well take on a negative value so that it is effectively a subsidy. Government's net revenue has to balance the budget:

$$(w_1 - \omega_1)(L_1 - E) + (\varphi - f)E + [(w_2 - \omega_2)H(E)L_2 + (r - \rho)S] / r$$

$$\stackrel{(2)}{=} (w_1 - \omega_1)L_1 + [(\varphi + \omega_1) - (f + w_1)]E + \left[\frac{w_2}{r} - \frac{\omega_2}{\rho}\right]HL_2 + \left[\frac{1}{\rho} - \frac{1}{r}\right]C_2 = G. \quad (7)$$

The planner maximizes the representative taxpayer's utility (4) in quantities C_1, C_2, L_1, L_2, E, S and prices $\omega_1, \omega_2, \rho, \varphi$ subject to the behavioural constraints, (1), (3), (5), (6), and the budget constraint (7). Assume that the planner's maximization is well behaved. The sole objective of this note is to characterize efficient policy for education in relation to the taxation of nonqualified labour. Efficiency is characterized in terms of wedges. Denote by

$$\Delta_{L_1} \equiv \frac{w_1 - \omega_1}{\omega_1} \quad \text{the tax wedge, and by} \quad (8)$$

$$\Delta_E \equiv \frac{w_2 H' L_2 / r - f - w_1}{\varphi + \omega_1} \stackrel{(6)}{=} \frac{w_2 / r}{\omega_2 / \rho} - \frac{f + w_1}{\varphi + \omega_1} \quad \text{the wedge on education.} \quad (9)$$

According to (9), the wedge on education equals the difference between two ratios. The first ratio relates present returns before and after taxes and subsidies and the second ratio relates costs before and after taxes and subsidies. Hence the wedge vanishes if the ratio in returns equals the ratio in costs. Let us speak of effective subsidization if Δ_E is negative. According to (9), a negative value of Δ_E is the combined result of all four policy instruments. Effective subsidization is clearly reached by the statutory subsidization of the cost of tuition. This is however not the only way of reducing Δ_E . Other effective means are (i) increasing the tax on nonqualified labour and thus reducing the opportunity cost of education, (ii) reducing the tax on qualified labour and thus increasing the return to education, and finally (iii) taxing saving and thus increasing the return to education.

Denote by

$$\nu_1 \equiv L_1 V'' / V' > 0 \quad \text{the elasticity of marginal disutility of nonqualified labour, i.e.}$$

the inverse of the wage elasticity. Set

$$\eta \equiv EH' / H \quad \text{the elasticity and}$$

$$\eta_\eta \equiv E\eta' / \eta \quad \text{the second-order elasticity of the earnings function.}$$

Elasticity Rule for Education: If ω_1, ω_2 , and φ are optimally chosen, then

$$\frac{\Delta_E}{\Delta_{L_1}} = - \frac{\eta_\eta}{\nu_1}. \quad (10)$$

Note that the Rule holds even if the planner does not optimize with respect to ρ . Saving does not need to be taxed efficiently and yet education policy should respect (10). A straightforward implication is that education should not be distorted, $\Delta_E = 0$, if the elasticity of the earnings function, η , is constant. This is a well-known result also named the Education Efficiency Proposition by Richter (2009). The result has been derived before in more elaborate models with heterogeneous taxpayers by Bovenberg and Jacobs (2005) and Jacobs and Bovenberg (2008). Another implication of (10) is that education should be subsidized if the elasticity of the earnings function is increasing. Related results have been derived before by Braun (2009) and Richter (2009). What has been lacking so far is the simple elasticity formula (10) which allows one to characterize efficient subsidization by means of the second-order elasticity of the earnings function. For the purpose of interpreting (10) assume that nonqualified labour income is taxed, $\Delta_L > 0$, and that the elasticity of the earnings function is increasing. Then (10) suggests that the second-best tax on (nonqualified) labour should vary inversely with the wage elasticity. This is the known part of the Elasticity Rule. The innovative part is the suggestion that the effective subsidization of education should increase monotonically in the second-order elasticity of education.

One may be eager to learn how the recommendation to subsidize education effectively translates into specific tax and subsidy rates. Unfortunately, no simple formula exists for the Ramsey framework. As one may well conjecture, the efficient set of tax and subsidy rates strongly depends on the specification of the taxpayer's utility function. There is only one known robust result. This states that qualified labour should be less distorted than nonqualified labour (Richter, 2009). It holds for arbitrary utility and earnings functions.

4. Proof

The proof is fairly straightforward. Start by simplifying the planner's problem. Substitute ω_1 by V' and drop the constraint $\rho = 1/U_C$ because the proof does not rely on any derivatives with respect to ρ, C_2, L_2 . The restated planner's problem is:

$$\max [L_1 V'(L_1) + \omega_2 H(E) L_2 / \rho - (\varphi + V'(L_1)) E - V(L_1) + U(C_2, L_2) - C_2 / \rho] \quad (11)$$

in $\varphi, L_1, \omega_2, E$ subject to

$$U_L(C_2, L_2) + \omega_2 H(E) / \rho = 0, \quad (\lambda) \quad (12)$$

$$\omega_2 H'(E) L_2 / \rho = \varphi + V'(L_1), \quad (\mu) \quad (13)$$

$$\begin{aligned} (w_1 - V'(L_1))L_1 + [(\varphi + V'(L_1)) - (f + w_1)]E \\ + \left(\frac{w_2}{r} - \frac{\omega_2}{\rho}\right)H(E)L_2 + \left[\frac{1}{\rho} - \frac{1}{r}\right]C_2 = G. \quad (\gamma) \end{aligned} \quad (14)$$

The first-order conditions with respect to $\varphi, L_1, \omega_2, E$ are as follows:

$$\frac{\partial}{\partial \varphi}: \quad (\gamma - 1)E = \mu \quad (15)$$

$$\frac{\partial}{\partial L_1}: \quad \gamma(w_1 - V') \stackrel{(15)}{=} (\gamma - 1)L_1 V'' \quad (16)$$

$$\frac{\partial}{\partial \omega_2}: \quad \lambda \stackrel{(15)}{=} (\gamma - 1)(1 - \eta)L_2 \quad (17)$$

$$\begin{aligned} \frac{\partial}{\partial E}: \quad \gamma[f + w_1 - w_2 H'(L_2) / r] \stackrel{(15), (17)}{=} (\gamma - 1)[\varphi + V' - \eta \omega_2 H'(L_2) / \rho + \omega_2 H'' E L_2 / \rho] \\ \stackrel{(6)}{=} (\gamma - 1)\left[1 - \eta + \frac{H'' E}{H'}\right](\varphi + V') = (\gamma - 1) \frac{E \eta'}{\eta} (\varphi + \omega_1) \end{aligned} \quad (18)$$

The Lagrange multiplier γ is clearly positive. The multiplier γ even has to be greater than one if w_1 is to exceed $V' = \omega_1$. Dividing (16) through by V' and (18) through by $\varphi + \omega_1$ and dividing the resulting equation (18) through by the resulting equation (16) yields (10). \square

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