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Longevity trend in Germany

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Abstract

In Germany, a trend for decreasing mortality probabilities has been observed in the last 50 years, yielding an increasing life expectancy. The German Actuarial Association DAV offers a standard method for modeling this longevity trend in calculations concerning life insurance by using the life table DAV 2004R. In this note it is investigated, whether or to which extent the longevity function of the DAV 2004R can be used for calculating the expected total number of deaths in Germany.

Keywords Life tables · Longevity factors · Mortality decrease · Expected number of deaths

1 Introduction

One of the key models in life insurance is the binomial model for calculating the number of deaths in a given population. The key ingredients for this model are a population table and a life table. A population table contains the number $l_{x,t}$ of x year old males¹ at January 1st in year t , and $l_{y,t}$, the number of y year old females at January 1st in year t . A life table contains the mortality probabilities $q_{x,t}$ (for males) and $q_{y,t}$ (for females) for year t and age x , resp. y .

The population size is a given data set, but life tables which fit the population have to be developed and are not a given information. In 2004, the German Actuarial Association DAV calculated a new life table DAV 2004R for usage in life insurances. This life table takes into account, that a trend of decreasing mortality probabilities in Germany—and in most western countries—has been observed for more than 100 years. This trend is among others due to improved health and working conditions, and medical progress. Clear exceptions of such a trend are external ‘shocks’ like the first and second world war, and territorial changes. For example, in Germany the trend was

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¹ Here we use the actuarial standard notation that x always indicates ‘male’ and y indicates ‘female’.

different in the eastern and western part of Germany, and the reunion also constitutes some kind of external shock, because in this moment the life tables in both parts of Germany had to be merged to a joint life table. For more information on the historical development and the construction of life tables we refer to [9].

In this note we concentrate on mortality probabilities and the longevity trend in the Federal Republic of Germany in the years since 2010. The life table DAV 2004R of the German Actuarial Association gives base mortality probabilities q_{x,t_0} and q_{y,t_0} for $t_0 = 2004$ and longevity functions which model the future mortality decrease. On May 10, 2023 the DAV confirmed that the mortality probabilities and the longevity functions of the life table DAV 2004R can still be used as a basis in calculations in life insurance.

It is the aim of this short note to check to which extent a longevity trend was still visible in the years 2010–2019, and in particular, if the trend given by the DAV 2004R, can be used as a basis for a longevity trend of the whole German population since 2010, because the DAV 2004R life table is tailor made for life annuities and pensions funds.

Recent interest in this question comes from the problem to estimate the excess mortality, or the mortality deficit, in the pandemic years 2020–2022, see e.g. [1–5, 10–13]. In particular, there have been discussions whether a longevity trend still has to be taken into account, when computing the expected number of deaths for the pandemic years from the life table 2017/19 published by the Federal Statistical Office of Germany [6]. This life table and the population tables of the Federal Statistical Office of Germany used in this note, take into account all inhabitants in Germany and not only German citizens. The results of this note show, that a longevity trend still has to be taken into account, and that the DAV 2004R table can be a useful base for such investigations.

2 Mortality probabilities

As is standard in actuarial science, generation life tables are used to calculate the expected number of deaths in a population. To develop generation life tables, historical life tables are used to estimate the longevity trend.

Denote by $q_{x,t}$, resp. $q_{y,t}$ the mortality probability for an x year old male, resp. female in year t . Generation life tables observe the development of $q_{x,t}$, resp. $q_{y,t}$ over a long period, roughly 100 years, smoothen the existing data, and estimate the long term behaviour of the mortality probabilities. The DAV models the mortality probabilities as a function of age $x, y \in \mathbb{N}$ and time $t, t_0 \in \mathbb{N}$ by

$$q_{x,t} = q_{x,t_0} e^{-G_m(x;t,t_0)}, \quad q_{y,t} = q_{y,t_0} e^{-G_f(y;t,t_0)} \quad (1)$$

with base mortality probabilities q_{x,t_0}, q_{y,t_0} in year t_0 and some longevity trend functions $G_m(x; t, t_0)$ for the male population, and $G_f(y; t, t_0)$ for the female population, whose values are determined using historical data.

Because of the historical development up to 2004, the DAV life table DAV 2004R [9] distinguishes between a higher short-term trend and a lower long-term trend. Nearly

twenty years have passed since the construction of the life table DAV 2004R. Therefore we just use the long term trend in our analysis. The DAV defines these long-term trends via linear longevity functions

$$G_m(x; t, t_0) = (t - t_0)F_m(x), \quad G_f(y; t, t_0) = (t - t_0)F_f(y), \quad (2)$$

with longevity factors $F_m(x), F_f(y) \geq 0$. For $x, y \in \mathbb{N}$ these factors are computed from historical data by the method of Whittaker–Henderson, which consists in a logarithmic regression in time t and a spline interpolation for smoothing the results between different ages x, y . For more details on the method how to construct these longevity trends we refer to the detailed explanations in [9], the resulting values for $F_m(x), F_f(y)$ are stated in the DAV 2004R—table [9, p. 61, ‘Zieltrend 2. Ordnung’].

In principle, one would like to use these longevity functions of the DAV 2004R life table directly for calculating mortality probabilities for the German population. Yet one should keep in mind that the life tables DAV 2004R and the longevity factors DAV 2004R are tailor made for life annuities and pensions funds. If one is interested in predictions concerning the whole German population, the question how to adapt the longevity factors of the DAV 2004R to fit for the whole population is a core problem and the main question of this note.

To answer this question we compare the longevity trends of the DAV 2004R to the observed trend in the mortality probabilities $\hat{q}_{x,t}, \hat{q}_{y,t}$ of the life tables 2009/11, ..., 2017/19 of the pre-pandemic years of the Federal Statistical Office of Germany [6] for the age $x, y = 0, \dots, 100$. In principle it would be desirable to use life tables and population tables at least up to age 110 but these data are not available.

3 Global multipliers

In a first attempt we keep the structure of the longevity factors $F_m(x), F_f(y)$ and ask for optimal multipliers $\lambda_{m,\min}, \lambda_{f,\min}$ which decrease or increase the longevity factors $F_m(x), F_f(y)$ globally to best fit the mortality decrease of the last years since 2011, i.e. we replace (1) and (2) by

$$\begin{aligned} \hat{q}_{x,t} &\approx q_{x,t} = q_{x,t_0} e^{-\lambda_{m,\min}(t-t_0)F_m(x)} = e^{c_x - \lambda_{m,\min}(t-t_0)F_m(x)}, \\ \hat{q}_{y,t} &\approx q_{y,t} = q_{y,t_0} e^{-\lambda_{f,\min}(t-t_0)F_f(y)} = e^{c_y - \lambda_{f,\min}(t-t_0)F_f(y)} \end{aligned} \quad (3)$$

with $t_0 = 2019$ and $\lambda_{m,\min}, \lambda_{f,\min}$ independent of x, y . The observed mortality probabilities $\hat{q}_{x,t}, \hat{q}_{y,t}$ are taken from the life tables 2009/11, ..., 2017/19 of the Federal Statistical Office of Germany [6], $t = 2011, \dots, 2019$.

We take logarithm on both sides of (3) and aim to minimize the function

$$f(c_x, \lambda_m) = \sum_{x=0}^{100} \sum_{t=2011}^{2019} w_{x,t} (\ln \hat{q}_{x,t} - c_x + \lambda_m(t - 2019)F_m(x))^2, \quad (4)$$

in c_x, λ_m , and analogously $f(c_y, \lambda_f)$ in c_y, λ_f , with certain given weights $w_{x,t}, w_{y,t}$. In this section we are only interested in the optimal choice of λ_m and λ_f which we denote by $\lambda_{m,\min}$ and $\lambda_{f,\min}$. We set

$$\overline{\ln \hat{q}_{x,\cdot}} = \frac{\sum_{t=2011}^{2019} w_{x,t} \ln \hat{q}_{x,t}}{\sum_{t=2011}^{2019} w_{x,t}}, \quad \bar{t}_x = \frac{\sum_{t=2011}^{2019} w_{x,t} t}{\sum_{t=2011}^{2019} w_{x,t}}. \quad (5)$$

To minimize (4) in c_x , we set $\frac{\partial}{\partial c_x} f(c_x, \lambda_m) = 0$. This yields $c_{x,\min} = \overline{\ln \hat{q}_{x,\cdot}} + \lambda_m(\bar{t}_x - 2019)F_m(x)$ and an analogous formula for $c_{y,\min}$. In a second step we minimize $f(c_{x,\min}, \lambda_m)$ with respect to λ_m , i.e. we set $\frac{\partial}{\partial \lambda_m} f(c_{x,\min}, \lambda_m) = 0$, which gives the optimal value

$$\lambda_{m,\min} = - \frac{\sum_{x=0}^{100} \left(\sum_{t=2011}^{2019} w_{x,t} (\ln \hat{q}_{x,t} - \overline{\ln \hat{q}_{x,\cdot}})(t - \bar{t}_x) \right) F_m(x)}{\sum_{x=0}^{100} \sum_{t=2011}^{2019} w_{x,t} (t - \bar{t}_x)^2 F_m(x)^2} \quad (6)$$

and an analogous formula for $\lambda_{f,\min}$.

There are two natural choices of the weights. First we could simply set $w_{x,t} = w_{y,t} = 1$ and thus ask for multipliers $\lambda_{m,\min}, \lambda_{f,\min}$ which minimize the sum of the relative errors in (3) in this unweighted case.

A second more natural possibility from a stochastic point of view is to normalize the random variables $\ln \hat{q}_{x,t}$ by taking into account the standard deviation of $\ln \hat{q}_{x,t}$. As usual in actuarial mathematics, let us denote by $l_{x,t}$ the number of x year old male at January 1st in year t , and by $l_{y,t}$ the number of y year old female at January 1st in year t . Then the number of deaths of x year old males in year t is a binomially distributed random variable $D_{x,t}$ with parameters $l_{x,t}$ and $q_{x,t}$, the mortality probability, and analogously the number $D_{y,t}$ of deaths of y year old females in year t is binomially distributed with parameters $l_{y,t}$ and $q_{y,t}$. The observed mortality probabilities $\hat{q}_{x,t}$ and $\hat{q}_{y,t}$ are realizations of $D_{x,t}/l_{x,t}$, resp. $D_{y,t}/l_{y,t}$. We show in the appendix, Eq. (24), that the logarithmic variance of $\hat{q}_{x,t}$ is given by

$$\mathbb{V} \ln \frac{D_{x,t}}{l_{x,t}} = \mathbb{V} \ln D_{x,t} = \frac{1 - q_{x,t}}{l_{x,t} q_{x,t}} + O\left(\frac{1}{l_{x,t}^2 q_{x,t}^2}\right).$$

Hence we put

$$w_{x,t} = \frac{l_{x,t} \hat{q}_{x,t}}{1 - \hat{q}_{x,t}} \approx \frac{1}{\mathbb{V} D_{x,t}} \quad (7)$$

and derive in this variance-weighted case the optimal multipliers $\lambda_{m,\min}^w, \lambda_{f,\min}^w$.

Assume, that $\ln \hat{q}_{x,t}$ and $\ln \hat{q}_{y,t}$ are realizations of uncorrelated random variables. Then it is well known that $\lambda_{m,\min}$ and $\lambda_{f,\min}$ are unbiased estimators for their

expectations, and the empirical variances

$$\mathbb{V}\lambda_{m,\min} = \frac{\sum_{x=0}^{100} \sum_{t=2011}^{2019} w_{x,t} \left((\ln \hat{q}_{x,t} - \overline{\ln \hat{q}_{x,\cdot}}) + \lambda_{m,\min}(t - \bar{t}_x) F_m(x) \right)^2}{(n-2) \sum_{x=0}^{100} \sum_{t=2011}^{2019} w_{x,t} (t - \bar{t}_x)^2 F_m(x)^2} \quad (8)$$

(resp. an analogous formula for $\mathbb{V}\lambda_{f,\min}$) are unbiased estimators for the true underlying variances. Here n is the number of summands in the double sum, $n = 909$.

Result 1. *In the unweighted case, the best-fit multiplier λ_m for the male population, and λ_f for the female population given by (6), which reduce the longevity factors of the DAV 2004R to best suit the German life tables 2009/11–2017/19 are*

$$\lambda_{m,\min} = 0.94 \quad \text{and} \quad \lambda_{f,\min} = 0.61. \quad (9)$$

The empirical standard deviations given by (8) are

$$\sigma(\lambda_{m,\min}) = 0.033 \quad \text{and} \quad \sigma(\lambda_{f,\min}) = 0.037. \quad (10)$$

Result 2. *In the variance-weighted case, the best-fit multiplier λ_m^w for the male population, and λ_f^w for the female population given by (6), which reduce the longevity factors of the DAV 2004R to best suit the German life tables 2009/11–2017/19 are*

$$\lambda_{m,\min}^w = 0.75 \quad \text{and} \quad \lambda_{f,\min}^w = 0.60. \quad (11)$$

The empirical standard deviations given by (8) are

$$\sigma(\lambda_{m,\min}^w) = 0.024 \quad \text{and} \quad \sigma(\lambda_{f,\min}^w) = 0.021. \quad (12)$$

Three remarks are in order:

- Because the observed number of deaths $\hat{d}_{x,t}$ is equal to $l_{x,t} \hat{q}_{x,t}$, the above approach equivalently minimizes the logarithmic difference between the observed deaths $\hat{d}_{x,t}$ and the expected deaths $\mathbb{E}D_{x,t} = l_{x,t} q_{x,t}$, and analogously for $\hat{d}_{y,t}$ and $\mathbb{E}D_{y,t}$.
- We could also compute a gender-independent optimal multiplier λ for the whole population by summing in (4) over both genders x, y ; this results in the unweighted case in a global multiplier

$$\lambda_{\min} = 0.76$$

and in the variance-weighted case in a global multiplier

$$\lambda_{\min}^w = 0.67. \quad (13)$$

- Modifying the weights to $\tilde{w}_{x,t} = \left(1 + \frac{t-2011}{8}\right) w_{x,t}$ puts more weight to more recent years, thus multiplying the weights $w_{x,t}$ given in (7) in year 2011 by one,

and increasing this until the weights in year 2019 are multiplied by a factor two. This results in male, resp. female multipliers

$$\lambda_{m,\min}^{\tilde{w}} = 0.74 \text{ and } \lambda_{f,\min}^{\tilde{w}} = 0.60.$$

and in a global multiplier

$$\lambda_{\min}^{\tilde{w}} = 0.66.$$

- Assuming that the sum occurring in (6) allows for normal approximation, (empirical) confidence intervals for $\lambda_{m,\min}$, $\lambda_{f,\min}$, $\lambda_{m,\min}^w$ and $\lambda_{f,\min}^w$ can be constructed by taking twice the empirical standard deviations in (10), resp. (12) around the estimates (9), resp. (11).
- Clearly, our approach does not yield a new generation life table. In fact, we just check to which extent a longevity trend should be taken into account, in particular a modified longevity trend from the DAV 2004R table, or whether the longevity trend has already vanished in Germany.

4 Age-dependent multipliers

The global multipliers described in Sect. 3 just yield a global picture and show, to which extent the total longevity trend of the life table DAV2044R is still applicable. In this section we investigate in more detail what happens for each age separately. This will indicate whether the structure of the longevity factors of the DAV 2004R table fits the actual evolution of the last ten years, or if the trend has evolved for different age groups in different directions. For this assume now that

$$\hat{q}_{x,t} \approx q_{x,t} = q_{x,b} e^{-\lambda_{m,\min}(x)(t-t_0)F_m(x)}, \quad \hat{q}_{y,t} \approx q_{y,t} = q_{y,b} e^{-\lambda_{f,\min}(y)(t-t_0)F_f(y)},$$

with $t_0 = 2019$ for suitable base mortality probabilities $q_{x,b}$, $q_{y,b}$ and age-dependent multipliers $\lambda_{m,\min}(x)$, $\lambda_{f,\min}(y)$. We minimize for fixed x , $y \in \{0, \dots, 100\}$,

$$f(q, \lambda) = \sum_{t=2011}^{2019} w_{x,t} (\ln \hat{q}_{x,t} - \ln q_x + \lambda_x(t - 2019)F_m(x))^2$$

in q_x , λ_x , resp. the analogous expression for $f(q_y, \lambda_y)$ in q_y , λ_y . Here we use the weights given in (7). The same computation as in the previous section yields for the optimal base mortality probabilities $q_{x,b} := q_{x,\min}$ the values

$$\ln q_{x,b} = \overline{\ln \hat{q}_{x,t}} + \lambda_x(\bar{t}_x - 2019)F_m(x) \quad (14)$$

with $\overline{\ln \hat{q}_{x,t}}$, \bar{t}_x given in (5), and an analogous formula for $\ln q_{y,b}$. Minimizing in a second step with respect to λ_x gives the variance-weighted optimal longevity

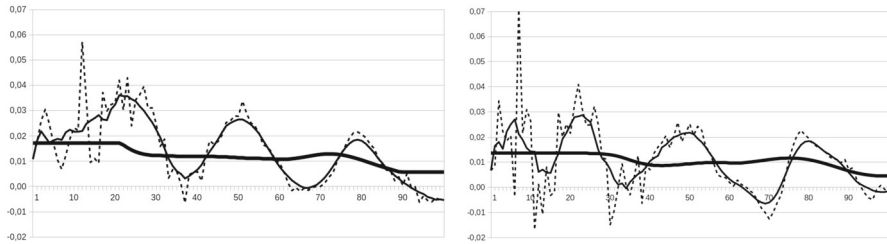


Fig. 1 Individual male (left) and female (right) multipliers: The dotted curve shows the strongly fluctuating age-dependent longevity factors $\lambda_{m,\min}^w(x)F_m(x)$, resp. $\lambda_{f,\min}^w(y)F_f(y)$, the fine black curve shows the smoothed longevity factors $\bar{\lambda}_{m,\min}^w(x)F_m(x)$, $\bar{\lambda}_{f,\min}^w(y)F_f(y)$, and the thick black line $\lambda_{m,\min}^w F_m(x)$, resp. $\lambda_{f,\min}^w F_f(y)$ —and thus the global form of the DAV longevity factors

multipliers

$$\lambda_{m,\min}^w(x) = -\frac{\sum_{t=2011}^{2019} w_{x,t}(\ln \hat{q}_{x,t} - \overline{\ln \hat{q}_{x,\cdot}})(t - \bar{t}_x)}{\sum_{t=2011}^{2019} w_{x,t}(t - \bar{t}_x)^2 F_m(x)} \quad (15)$$

and an analogous formula for $\lambda_{f,\min}^w(y)$. As can be seen in the following Fig. 1, these age-dependent multipliers strongly fluctuate around the global multipliers (and thus the modified longevity factors $\lambda_{m,\min}^w(x)F_m(x)$, $\lambda_{f,\min}^w(y)F_f(y)$ around the gender-dependent modified long-term trends $\lambda_{m,\min}^w F_m(x)$, $\lambda_{f,\min}^w F_f(y)$ of the DAV 2004R). In Fig. 1 we also show a smoothing average $\bar{\lambda}_{m,\min}^w(x) = \frac{1}{9}(\lambda_{m,\min}^w(x-4) + \dots + \lambda_{m,\min}^w(x+4))$, and analogously $\bar{\lambda}_{f,\min}^w(y)$. Again we want to stress that even this smoothing operation does not yield a properly calculated generation life table, since we do not use e.g. the Whittaker–Henderson method for smoothing and interpolating historical data with splines.

The result shows that in fact different age groups developed in a different way: in some age groups the mortality decrease in the years 2010–2019 is even stronger than the longevity trend used by the DAV 2004R, whereas in other age groups the decrease already has stopped. Nevertheless we have to point out that this picture only reflects the behavior of the last pre-pandemic years 2011–2019, whereas the longevity factors of the DAV 2004R table are established to model the mortality development of the next centuries in Germany.

5 Results: the expected number of deaths

As already explained in Sect. 3, the simple binomial model in life insurance now assumes that the (unknown random) number of deaths $D_{x,t}$, $D_{y,t}$ is a binomial random variable with parameters $(l_{x,t}, q_{x,t})$, resp. $(l_{y,t}, q_{y,t})$, and with expectations $\mathbb{E}D_{x,t} = l_{x,t}q_{x,t}$, $\mathbb{E}D_{y,t} = l_{y,t}q_{y,t}$.

In this section we use a slightly more refined version of the binomial model, because in this simple model those individuals which have been of age $(x-1)$ at the beginning of year t , and died as x year olds are ignored. To take this into account, we follow

the procedure proposed by De Nicola et al. [3]. Roughly half of those people who die as x -year old have been $x - 1$ year old at the beginning of the year and died after their birthday. For them we use the smoothed mortality probability $\frac{1}{2}(q_{x-1,t} + q_{x,t})$. The other half of the x year old deaths belongs to the population of x year old at the beginning of the year. For them we use the smoothed mortality probability $\frac{1}{2}(q_{x,t} + q_{x+1,t})$. For more details see [3]. Putting things together, for $x = 0, \dots, 101$ the random number $D_{x,t}$ of deaths of age x in year t is the sum of two binomially distributed random variables and satisfies

$$\mathbb{E}D_{x,t} = \frac{l_{x-1,t}}{2} \frac{q_{x-1,t} + q_{x,t}}{2} + \frac{l_{x,t}}{2} \frac{q_{x,t} + q_{x+1,t}}{2}. \quad (16)$$

(For $x = 0$ we set $q_{-1,t} = q_{0,t}$, and $l_{-1,t} = l_{0,t+1}$ if available, and $l_{-1,t} = l_{0,t}$ else.) The same considerations lead to $\mathbb{E}D_{y,t}$. Finally summation gives the total expected number of deaths $\mathbb{E}D_t = \sum_{x=0}^{100} \mathbb{E}D_{x,t} + \sum_{y=0}^{100} \mathbb{E}D_{y,t}$ in year t . We are interested in the expected number of deaths in for the years 2020, 2021 and 2022. Because the year $t = 2020$ is a leap year, we add an additional day by multiplying the result by $\frac{366}{365}$.

The population data $l_{x,t}$ and $l_{y,t}$ are taken from the population table of the Federal Statistical Office of Germany [7].

First we state the result when using the most recent pre-pandemic life table 2017/19 for $\hat{q}_{x,2019}$, $\hat{q}_{y,2019}$, and the variance-weighted multiplier λ_{\min}^w from (13). Then (16) yields the expected number of deaths,

$$\mathbb{E}D_{2020} = 979,308, \quad \mathbb{E}D_{2021} = 985,199, \quad \mathbb{E}D_{2022} = 991,541. \quad (17)$$

Replacing the gender-independent multiplier by the male, resp. female multipliers $\lambda_{m,\min}^w$, $\lambda_{f,\min}^w$ from (11), the expected number of deaths are given by

$$\mathbb{E}D_{2020} = 979,247, \quad \mathbb{E}D_{2021} = 985,073, \quad \mathbb{E}D_{2022} = 991,351. \quad (18)$$

The differences to (17) are negligible.

The approach, which best fits the data of the last years, uses the base mortality probabilities $q_{x,b}$ and $q_{y,b}$ from (14) and the age-dependent longevity multipliers (15). Note that here we optimized in such a way, that $\lambda_{m,\min}^w(x)F_m(x)$, $\lambda_{f,\min}^w(y)F_f(y)$ fits best to historical data. This model is thus independent of the DAV 2004R table, and yields

$$\mathbb{E}D_{2020} = 980,692, \quad \mathbb{E}D_{2021} = 987,453, \quad \mathbb{E}D_{2022} = 994,889. \quad (19)$$

These numbers are close to the preceding ones, and the use of longevity factors of the DAV 2004R table, modified by $\lambda_{m,\min}^w$, $\lambda_{f,\min}^w$ still seems to be a possibility to estimate future mortality even for the whole German population.

6 Conclusion

There are several papers and studies dealing with the computation of the excess mortality in Germany during the pandemic years. We only mention those, which use the actuarial method.

In the papers by De Nicola and Kauermann [3–5] no mortality decrease is used, i.e. they set $\lambda_{m,\min} = \lambda_{f,\min} = \lambda = 0$ in (3). In [3, 4] they use the life table 2017/19 and obtain the values

$$\mathbb{E}D_{2020} = 979,255 \text{ and } \mathbb{E}D_{2021} = 996,410. \quad (20)$$

In [5] they use a life table 2015/19 which yields

$$\mathbb{E}D_{2020} = 993,863 \text{ and } \mathbb{E}D_{2021} = 1,011,298. \quad (21)$$

In the study by Kuhbandner and Reitzner [11], a global multiplier $\lambda = 0.5$ in (3) is used, which yields the values

$$\mathbb{E}D_{2020} = 981,557, \mathbb{E}D_{2021} = 989,707, \text{ and } \mathbb{E}D_{2022} = 998,545. \quad (22)$$

All three studies mentioned above seem to underestimate the longevity. First, the results of this note show that globally mortality probabilities are still decreasing and it is necessary to take longevity factors into account for a precise prediction of the number of deaths. If a global multiplier is used to adjust the DAV 2004R table, then (13) implies that $\lambda_{\min}^w = 0.68$ is a more appropriate choice than $\lambda = 0.5$ as in [11]. Nevertheless, the results of [11] are closest to the results (19) obtained from the estimated base mortality probabilities and the individual longevity multipliers.

The obtained values for the expected number of deaths can be used to estimate the excess mortality during the pandemic years 2020, 2021 and 2022. For this, the expected values should be compared to the observed number of deaths \hat{d}_t published by the Federal Statistical Office of Germany [8] for the years $t = 2020, 2021, 2022$,

$$\hat{d}_{2020} = 985,572, \hat{d}_{2021} = 1,023,687, \hat{d}_{2022} = 1,066,341.$$

It should be pointed out, that the model described in this note does not aim to estimate the mortality during a pandemic situation, but to use historical data from 2010–2019 to estimate the expected mortality in the years 2020–2022 if there would have been *no* pandemic. The excess mortality $\hat{d}_{x,t} - \mathbb{E}D_{x,t}$ then is determined by the expected number of deaths given in (17)–(22) where as underlying mathematical model in principal the one proposed by the German Actuarial Association is used, with the modifications described above. Hence the results depend via $\mathbb{E}D_{x,t}$ on the details of the chosen model and parameters.

Yet the main results of this note show that a mortality decrease has been observed in the last pre-pandemic years and must be taken into account for estimating excess mortality—for example by using the DAV 2004R table with a suitable modification.

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Data Availability The data concerning German life tables and population tables of the Federal Statistical Office of Germany are available in [6] and [7], the longevity factors of the DAV 2004R are available in [9].

Declarations

Conflict of interest The author declares that no financial support was received from any organization for the submitted work. The author declares that he has no financial relationships at present or within the previous three years with any organizations that might have an interest in the submitted work.

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7 Appendix: Moments of logarithmic binomial random variables

Let $D \sim \text{bin}(l, q)$ be a binomial random variable, i.e. $\mathbb{P}(D = k) = \binom{l}{k} q^k p^{l-k}$ with $p = 1 - q \in [0, 1]$. We are interested in the logarithmic variance $\mathbb{V} \ln D$, but there exists no closed form for this variance. Hence in this appendix we compute an approximation for large l . By Markov's inequality

$$\mathbb{P}\left(|D - lq| \geq \frac{1}{2}lq\right) \leq \frac{16\mathbb{E}(D - lq)^4}{(lq)^4}.$$

The first four centered moments of D are given by the expectation $\mathbb{E}(D - lq) = 0$, the variance $\mathbb{V} D = lqp$, $\mathbb{E}(D - lq)^3 = lqp(1 - 2q)$ and $\mathbb{E}(D - lq)^4 = lq(1 - q)(1 + (3l - 6)q(1 - q))$. We plug this fourth centered moment into Markov's inequality and obtain

$$\mathbb{P}\left(|D - lq| \geq \frac{1}{2}lq\right) \leq \frac{16(1 - q)(1 + (3l - 6)q(1 - q))}{(lq)^3} \leq \frac{16(1 + 3lq)}{(lq)^3} \leq \frac{64}{(lq)^2} \quad (23)$$

for $lq \geq 1$.

Also note that the Markov bound (23) implies that

$$\begin{aligned} \mathbb{E}\left((D - lq)\mathbb{1}(|D - lq| \leq \tfrac{1}{2}lq)\right) &= O((lq)^{-1}), \quad \mathbb{E}\left((D - lq)^2\mathbb{1}(|D - lq| \leq \tfrac{1}{2}lq)\right) \\ &= \mathbb{V} D + O(1). \end{aligned}$$

We start with the first moment. Because D is concentrated at its expectation a first naive approximation is $\mathbb{E} \ln D \approx \ln(lq)$. We make this precise.

For $x \in [-\frac{1}{2}, \frac{1}{2}]$ the Taylor series $\ln(1+x) = \sum_{k \geq 1} (-1)^{k+1} x^k / k$ is absolutely convergent, which implies that $\ln(1+x) = x - x^2/2 + O(x^3) = x + O(x^2)$. These estimates with $x = \frac{D-lq}{lq}$ yield for the logarithmic expectation

$$\begin{aligned} \mathbb{E} \ln D - \ln(lq) &= \mathbb{E} \left(\left(\frac{D-lq}{lq} - \frac{(D-lq)^2}{2(lq)^2} \right) \mathbb{1}(|D-lq| \leq \tfrac{1}{2}lq) \right) \\ &\quad + O \left(\frac{\mathbb{E}(D-lq)^3}{(lq)^3} + \frac{1}{(lq)^2} \right) \\ &= -\frac{\mathbb{V}D}{2(lq)^2} + O \left(\frac{1}{(lq)^2} \right) = -\frac{p}{2lq} + O \left(\frac{1}{(lq)^2} \right), \end{aligned}$$

and thus also

$$(\mathbb{E} \ln D - \ln(lq))^2 = O \left(\frac{1}{(lq)^2} \right).$$

For the second moment we obtain analogously

$$\begin{aligned} \mathbb{E}(\ln D - \ln(lq))^2 &= \mathbb{E} \left(\frac{D-lq}{lq} \mathbb{1}(|D-lq| \leq \tfrac{1}{2}lq) + O \left(\frac{(D-lq)^2}{l^2q^2} \right) \right)^2 + O \left(\frac{1}{(lq)^2} \right) \\ &= \mathbb{E} \left(\frac{(D-lq)^2}{l^2q^2} \mathbb{1}(|D-lq| \leq \tfrac{1}{2}lq) \right) + O \left(\frac{(D-lq)^3}{l^3q^3} + \frac{1}{(lq)^2} \right) \\ &= \frac{p}{lq} + O \left(\frac{1}{l^2q^2} \right). \end{aligned}$$

This implies that the logarithmic variance is given by

$$\mathbb{V} \ln D = \mathbb{E}(\ln D - \ln(lq))^2 - (\mathbb{E} \ln D - \ln(lq))^2 = \frac{p}{lq} + O \left(\frac{1}{l^2q^2} \right). \quad (24)$$

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