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The Inherent Benefit of Monetary Unions*

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Abstract

The desirability of flexible exchange rates is a central tenet in international macroeconomics. We show that, with forward-looking staggered pricing, this result crucially depends on the monetary authority's ability to commit. Under full commitment, flexible exchange rates generally dominate a monetary union (or fixed exchange rate) regime. Under discretion, this result is overturned: a monetary union dominates flexible exchange rates. By fixing the nominal exchange rate, a benevolent monetary authority finds it welfare improving to tradeoff flexibility in the adjustment of the terms of trade in order to improve on its ability to manage the private sector's expectations. Thus, inertia in the terms of trade (induced by a fixed exchange rate) is a *cost* under commitment, whereas it is a *benefit* under discretion, for it acts like a commitment device. The higher the incidence of asymmetric inefficient shocks, and/or the higher the degree of nominal price rigidity, the higher the inherent benefit of monetary unions, in stark contrast with the traditional optimum currency area theory.

Keywords: monetary union, flexible exchange rates, commitment, discretion, welfare losses, nominal rigidities.

JEL Classification No.: E52, F33, F41

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1 Introduction

A central tenet in international macroeconomics is that flexible exchange rates are desirable because they compensate for the inertia in nominal prices, thereby easing the necessary adjustment in the terms of trade in response to asymmetric disturbances (Friedman 1953, Corsetti, Dedola and Leduc 2011, Farhi and Werning 2017).¹ This argument, which implies that fixed exchange rates are inherently costly, has recently gained renewed interest in light of the observed divergence in macroeconomic performance between the periphery and the core of the European Monetary Union after the 2008-11 crisis.

In this paper we revisit the classic dichotomy between flexible exchange rates and monetary unions, within the context of a baseline two-country dynamic New Keynesian model, the workhorse paradigm of the recent optimal monetary policy literature in open economies (Devereux and Engel 2003, Benigno and Benigno 2003, Corsetti and Pesenti 2001).

The key insight of our analysis is that the desirability of flexible exchange rates relative to monetary unions (or, generally, fixed exchange rates) crucially depends on the (in)ability of the monetary authority to commit. If, somewhat unrealistically, the monetary authority can fully commit, flexible exchange rates always implement the constrained efficient allocation. If, however, the monetary authority can only choose its course of action period by period (i.e., it acts under discretion), the previous result is overturned: a monetary union generally dominates flexible exchange rates.

The intuition for the desirability of flexible exchange rates under commitment is well understood. In a baseline New Keynesian model, characterized by forward-looking staggered prices (Woodford 2003, Galí 2015), the monetary authority's inability to commit

¹The recent New Keynesian optimal monetary policy literature in open economies has revisited this argument, arguing that, in the presence of local currency price stability of imports, full flexibility of the nominal exchange rate is generally not the welfare maximizing policy (Devereux and Engel 2003, Engel 2011, Corsetti, Dedola and Leduc 2011). Here we purposely abstract from issues related to local currency price stability of imports.

typically results in a stabilization bias, i.e., a suboptimal policy response to those disturbances that drive a wedge between the welfare-efficient and the natural level of output (so called inefficient or “cost-push” shocks). Gains from commitment arise from policy-induced inertia in inflation, which, in turn, improves the monetary authority’s management of the private sector’s expectations. The above argument supports the following proposition: under flexible exchange rates, if the monetary authority can commit, there is no tradeoff between the optimal management of (inflation) expectations and the efficient adjustment of international relative prices in response to asymmetric disturbances.

Our analysis focuses on the case of *lack of commitment*. We show that a fixed exchange rate induces a “commitment-like” inertia in the behavior of the terms of trade and inflation, which mitigates the stabilization bias. As a result, if a credible policy commitment is not feasible, the monetary authority finds it welfare-improving, in a monetary union, to trade off some flexibility in the adjustment of the terms of trade in order to improve on its ability to manage expectations. This is what we label as the *inherent benefit* of monetary unions.

To better understand this point, it is instructive to recall that, in a two-country setting with nominal rigidities (and under the assumption of cross-country symmetry), average welfare losses depend not only on the variability of *average* inflation (in a way similar to its closed economy analog), but also on its cross-country *composition*. Thus, inertia in the terms of trade, induced by fixed exchange rates, translates, under discretion, into welfare-enhancing inertia of relative inflation, i.e., precisely of what measures the cross-country composition of inflation. More generally, under discretion, the expectation-management gain stemming from inertia in the terms of trade can outweigh the cost of inefficient adjustment of relative prices, thereby making a monetary union welfare dominant relative to flexible exchange rates. In a nutshell, inertia in the terms of trade, induced by fixed exchange rates, is a *cost* under commitment - because it does not compensate for the underlying stickiness in nominal prices; whereas it is a *benefit* under discretion - because

it acts as a commitment device, thereby improving on the policymaker’s ability to manage expectations. This result holds for a large range of parameter values, and is especially sharp under three configurations: a sufficiently high degree of nominal price rigidity; a relatively high incidence of *asymmetric* inefficient vs. efficient shocks; and a sufficiently high degree of substitutability in internationally traded goods. Noticeably all three conditions stand in stark contrast with the traditional Optimum Currency Area (OCA) literature (Mundell, 1961; McKinnon, 1963; Kenen, 1969).

Interestingly, while the inertia in the terms of trade has been recognized before as a typical feature of a monetary union (Benigno 2004, Pappa 2004), it was *solely* regarded as a distortion of that regime. Relatedly, Farhi and Werning (2017) emphasize that the inefficiency at the heart of *any* monetary union, and regardless of the underlying degree of completeness in international financial markets, is a structural “lack of insurance”, which stems precisely from the suboptimal adjustment in the terms of trade that results from the combination of nominal price rigidity and lack of nominal exchange rate flexibility. Unlike those contributions, we wish instead to emphasize that, in a monetary union, the inertial behavior of the terms of trade can be turned to policymakers’ advantage when the latter lack the ability to commit.

Finally, it is worth noting that, throughout the paper, we assume that a benevolent monetary authority aims at maximizing world welfare. In a flexible exchange rate regime, in particular, this corresponds to assuming that the monetary authorities of both countries conduct policy under cooperation. This not only implies that the benchmark regime we compare the monetary union to is a very strong one. It also implies that the inherent benefit of a monetary union described in this paper cannot be related to gains from cooperation, a benefit which is frequently associated with monetary unions in the literature.

Related literature Our paper relates to a large literature analyzing optimal monetary policy in an international setting, and within the context of dynamic optimizing New Keynesian models. Corsetti, Dedola and Leduc (2011) thoroughly survey that literature. A pillar of this research program (a sort of flexible exchange rates *manifesto*) is that under the assumption of (i) cross-country risk sharing, (ii) complete pass-through of exchange rates to import prices, and (iii) full commitment, flexible exchange rates implement the welfare-maximizing policy. The existing literature has typically explored the implications of relaxing (i) and (ii) in order to (re-)assess the desirability of fixed vs. flexible exchange rates (see, e.g., Devereux and Engel 2003, Corsetti, Dedola and Leduc 2011). Our paper differs from the previous ones in that it focuses on the role of relaxing (iii) in determining the desirability of monetary unions vs. flexible exchange rates.

Monacelli (2004), Soffritti and Zanetti (2008), and Groll (2013) are earlier studies of the properties of flexible vs. fixed exchange rates in a New Keynesian open economy model, showing that, with lack of commitment, the classic ranking between flexible and fixed exchange rates can be reversed. The key difference in our paper is that, in order to assess the relative desirability of the two regimes, we frame the analysis within a fully choice-theoretic environment as opposed to relying on ad hoc policy objective functions and/or Taylor-type rules. Benigno (2004) studies optimal monetary policy in a currency area, but under the maintained assumption that the monetary authority can commit, and with no comparison between flexible and fixed exchange rate regimes. Our central focus here is instead on the case of lack of commitment and on the relative desirability of the two regimes. Corsetti, Kuester and Muller (2013) compare the transmission of fiscal disturbances under flexible vs fixed exchange rates (described by simple feedback rules) and highlight the role of nominal anchor played by fixed exchange rates. Cook and Devereux (2016) point out the desirability of fixed exchange rates (or monetary unions) when asymmetric shocks hit a country at the zero lower bound. Our paper shows that, with lack of commitment, the desirability of monetary unions (or fixed exchange rates)

holds also in “normal” times, regardless of the occurrence of the zero lower bound.

Our paper is also related to a literature, exemplified by Alesina and Barro (2002), which emphasizes that countries, when they lack commitment, may generally benefit from monetary unification. Our paper differs from that strand of the literature in at least two ways. First, the commitment gain in Alesina and Barro (2002) derives from the removal of a typical average inflation bias, whereas the commitment gains from participating to a monetary union arise, in our setup, due to the improved ability of policy to respond to shocks, even in the absence of any source of average inflation bias. Second, and most importantly, the benefit, in Alesina and Barro (2002), of eliminating an average inflation bias is not inherent to a monetary union because it is only obtained if the monetary policy authority after monetary unification is more credible than the one before monetary unification. By contrast, the benefit described in our paper is *inherent* to a monetary union because it is obtained *even if* the monetary policy authority after monetary unification suffers from the same lack of commitment as before unification. So, in our case, two countries that, ex ante, suffer from a lack of commitment gain by establishing a monetary union even if the new common monetary policy authority suffers from the same lack of commitment.

Rethinking Optimum Currency Area theory Our argument is also related to Chari et al. (2015) in that it stresses the role of inefficient asymmetric shocks (i.e., the ones that, in the New Keynesian jargon, break the “divine coincidence”) in determining the desirability to form a monetary union. This is in stark contrast to the standard OCA literature stressing the commonality of shocks. Thus, the single monetary policy is a commitment device not to suboptimally react to inefficient country-specific shocks. However, the mechanism highlighted in our paper is completely different from the one in Chari et al. (2015). Our mechanism relies on the endogenous inertia that characterizes the *dynamic* behavior of the terms of trade in a monetary union when the economic

environment features forward-looking price setting (modeled a la Calvo in our case). This differs from the setup in Chari et al. (2015), who assume preset prices, therefore ruling out the possibility of inertial behavior in the terms of trade by construction.

Our finding that countries benefit from forming a monetary union when prices are relatively sticky but not when prices are relatively flexible also stands in stark contrast to Friedman (1953)'s case for flexible exchange rates and - once again - to the predictions of the traditional OCA theory. The most important reason for this discrepancy is the fact that, in New Keynesian models, and unlike the traditional OCA literature, expectations are treated as endogenous. Since the inherent benefit of monetary unions works through expectations, this channel is naturally missing in models without such an expectational feedback mechanism. Note that Carré and Collard (2003), Dellas and Tavlas (2005), and Galí and Monacelli (2016) equally cast doubt on the prediction of the traditional OCA theory that countries are worse off by forming a monetary union when nominal rigidities are present.

2 A two-country model

We describe a baseline two-country model characterized by full financial integration, monopolistic competitive markets and nominal price rigidity (Benigno 2004, Corsetti, Dedola and Leduc 2011). Henceforth we refer to the two countries as Home and Foreign, having measure n and $(1 - n)$ respectively. The total mass of households in the world economy is therefore equal to 1.

2.1 Domestic households

Consumption preferences in Home are described by the following composite index of domestic and imported bundles of goods (Faia and Monacelli 2008, De Paoli 2009):

$$C_t \equiv [(1 - \gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \quad (1)$$

where $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\gamma \equiv (1 - n)\alpha$ denotes the weight of imported goods in the Home consumption basket. This weight depends on $(1 - n)$, the relative size of Foreign, and on α , the degree of trade openness of Home. In an analogous manner, preferences in Foreign can be described as:

$$C_t^* \equiv [(1 - \gamma^*)^{\frac{1}{\eta}} C_{F,t}^{*\frac{\eta-1}{\eta}} + (\gamma^*)^{\frac{1}{\eta}} C_{H,t}^{*\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \quad (2)$$

where $\gamma^* \equiv n \alpha^*$.

Each consumption bundle $C_{H,t}$ and $C_{F,t}$ is composed of imperfectly substitutable varieties (with elasticity of substitution $\varepsilon > 1$). Optimal allocation of expenditure within each variety of goods yields:

$$C_{H,t}(i) = \frac{1}{n} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad ; \quad C_{F,t}(i) = \frac{1}{1 - n} \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \quad (3)$$

where $C_{H,t} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{F,t} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$.

Optimal allocation of expenditure between domestic and foreign bundles yields:

$$C_{H,t} = (1 - \gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \quad (4)$$

where

$$P_t \equiv [(1 - \gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}} \quad (5)$$

is the CPI index.

A generic household in Home derives utility from consumption and disutility from the production of a continuum of differentiated products indexed by $i \in [0, n]$:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t) - \frac{1}{n} \int_0^n V(Y_t(i), Z_{Y,t}) di \right\} \quad (6)$$

where $Z_{Y,t}$ is a productivity disturbance. To insure their consumption pattern against random shocks at time t households spend $\nu_{t+1,t} B_{t+1}$ in nominal state contingent securities, where $\nu_{t,t+1} \equiv \nu(h^{t+1}|h^t)$ is the period- t price of a claim to one unit of domestic

currency in state h^{t+1} divided by the probability of occurrence of that state. Each asset in the portfolio B_{t+1} pays one unit of domestic currency at time $t + 1$ and in state h^{t+1} .

By considering the optimal expenditure conditions (3) and (4), the sequence of budget constraints assumes the following form:

$$P_t C_t + \sum_{h^{t+1}} \nu_{t+1,t} B_{t+1} \leq B_t + \frac{1 - \tau_{H,t}}{n} \int_0^n P_t(i) Y_t(i) di + T_t \quad (7)$$

where $\tau_{H,t}$ is a country-specific tax on firms' profits, and T_t denotes lump-sum transfers (or taxes).

2.2 Risk sharing, the real exchange rate and demand in Foreign

We assume throughout that the law of one price holds, implying that $P_{H,t}(i) = \mathcal{E}_t P_{H,t}^*(i)$ and $P_{F,t}(i) = \mathcal{E}_t P_{F,t}^*(i)$ for all $i \in [0, 1]$, where \mathcal{E}_t is the *nominal* exchange rate, i.e., the price of foreign currency in terms of home currency, and $P_{F,t}^*(i)$ is the price of foreign good i denominated in foreign currency. Importantly, the law of one price does not necessarily imply that purchasing power parity (PPP) holds, unless we make the further restrictive assumption of absence of home bias in consumption.

Under complete markets for state contingent assets, the efficiency condition for bonds' holdings by residents in Foreign reads:

$$\beta \frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}} \frac{U_{c,t+1}^*}{U_{c,t}^*} = \nu_{t,t+1} \quad (8)$$

Taking conditional expectations of (8) and defining the foreign nominal interest rate

$(1 + i_t^*) \equiv \left(\mathbb{E}_t \left\{ \nu_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\} \right)^{-1}$ one can write:

$$(1 + i_t^*) = \left[\beta \mathbb{E}_t \left\{ \frac{P_t^*}{P_{t+1}^*} \frac{U_{c,t+1}^*}{U_{c,t}^*} \right\} \right]^{-1} \quad (9)$$

Foreign demand for domestic variety i must satisfy:

$$\begin{aligned}
C_{H,t}^*(i) &= \frac{1}{n} \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\varepsilon} C_{H,t}^* \\
&= \frac{1}{n} \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*} \right)^{-\varepsilon} \gamma^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^*
\end{aligned} \tag{10}$$

210 **Terms of trade and the real exchange rate** The terms of trade is the relative
211 price of imported goods:

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{11}$$

212 while the real exchange rate is defined as $Q_t = \mathcal{E}_t P_t^* / P_t$. Using equation (5), the terms of
213 trade can be related to the CPI-PPI ratio as follows:

$$\frac{P_t}{P_{H,t}} = [(1 - \gamma) + \gamma S_t^{1-\eta}]^{\frac{1}{1-\eta}} \equiv \mathcal{T}(S_t), \tag{12}$$

214 with $\mathcal{T}_{s,t} \equiv \partial \mathcal{T}(S_t) / \partial S_t > 0$.

215 The terms of trade and the real exchange rate are linked through the following expres-
216 sion:

$$\begin{aligned}
Q_t &= S_t \frac{P_t^*}{P_{F,t}^*} \left(\frac{P_t}{P_{H,t}} \right)^{-1} \\
&= S_t \frac{\mathcal{T}^*(S_t)}{\mathcal{T}(S_t)} \equiv q(S_t),
\end{aligned} \tag{13}$$

217 where

$$\frac{P_t^*}{P_{F,t}^*} = [(1 - \gamma^*) + \gamma^* S_t^{\eta-1}]^{\frac{1}{1-\eta}} \equiv \mathcal{T}^*(S_t). \tag{14}$$

Deviations from purchasing power parity (PPP) By using (12), (13) and (14)

one can write:

$$Q_t = q(S_t) = \left(\frac{\gamma^* + (1 - \gamma^*) S_t^{1-\eta}}{(1 - \gamma) + \gamma S_t^{1-\eta}} \right)^{\frac{1}{1-\eta}}. \quad (15)$$

Notice that if $\gamma = \gamma^* = 1/2$ it follows immediately that $Q_t = 1$ (i.e., PPP holds at all times), regardless of the equilibrium value of S_t .

Risk Sharing Under full international risk sharing, one can combine (8) with the corresponding condition for Home and obtain, after iteration, the following condition linking the real exchange rate to the ratio of the marginal utilities of consumption:

$$\omega_0 \frac{U_{c,t}^*}{U_{c,t}} = \frac{\mathcal{E}_t P_t^*}{P_t} = q(S_t), \quad (16)$$

where ω_0 is a constant that depends on initial conditions, and can be normalized to 1.

2.3 Price setting

Each domestic producer can revise its price at random intervals (Calvo 1983). Let $(1 - \theta_H)$ be the probability that a firm can reoptimize its price at any given time t , and $\bar{P}_{H,t}$ the optimally chosen price at time t . Each producer maximizes expected discounted profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_H)^k \nu_{t,t+k} [\lambda_{t+k} (1 - \tau_{H,t+k}) \bar{P}_{H,t}(i) Y_{t+k|t}(i) - V(Y_{t+k|t}(i), Z_{Y,t+k})] \quad (17)$$

where (from equilibrium) $\nu_{t,t+k} = \beta^k \frac{U_{c,t+k} P_t}{U_{c,t} P_{t+k}}$ is the stochastic discount factor, $\lambda_{t+k} = \frac{U_{c,t+k}}{P_{t+k}}$ is the marginal utility of nominal revenues, and $Y_{t+k|t}$ is total demand for variety i faced by a firm that last reset its price at time t .

The first order condition yields the optimal price

$$\bar{P}_{H,t}(i) = \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_H)^k \nu_{t,t+k} \mathcal{M}_{H,t+k} V_y(Y_{t+k|t}(i), Z_{Y,t+k}) Y_{t+k|t}(i)}{\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_H)^k \nu_{t,t+k} \lambda_{t+k} Y_{t+k|t}(i)}, \quad (18)$$

where V_y is the marginal disutility from producing output $Y(i)$ and

$$\mathcal{M}_{H,t+k} = \frac{\varepsilon}{(\varepsilon - 1)(1 - \tau_{H,t+k})}$$

denotes the tax-adjusted optimal markup. We assume that, in order to neutralize the market power distortion in the steady state, $\tau_H = -(\varepsilon - 1)^{-1} \equiv \tilde{\tau}_H$. By construction, then, any deviation of $\tau_{H,t}$ from $\tilde{\tau}_H$ is an exogenous stochastic source of inefficiency.

Markup shocks We assume that the markup follows the process (in logs):

$$\log \mathcal{M}_{i,t} \equiv \mu_{i,t} = \rho_\mu \mu_{i,t-1} + \epsilon_{i,t}^\mu \quad (i = H, F) \quad (19)$$

where $\epsilon_{i,t}^\mu$ is an iid random disturbance, with mean zero and variance σ_{μ_i} .

In any given period, the price from the previous period remains effective for a fraction θ_H of producers. The optimal relative price $\bar{P}_{H,t}/P_{H,t}$ follows:

$$1 = \theta_H \left(\frac{P_{H,t}}{P_{H,t-1}} \right)^{\varepsilon-1} + (1 - \theta_H) \left(\frac{\bar{P}_{H,t}}{P_{H,t}} \right)^{1-\varepsilon}. \quad (20)$$

3 Equilibrium

Below we describe the relevant set of equilibrium conditions in log-linearized form (denoted by lower case letters) and for each exchange rate regime - flexible exchange rate and monetary union - respectively (see also Appendix A). In the expressions below, variables with a superscript (e.g., \bar{x}_t) refer to the corresponding values under the first-best or efficient allocation, characterized by flexible prices and the absence of markup shocks (see Appendix B).

3.1 Flexible exchange rates

For a given specification of the two policy instruments $\{i_t, i_t^*\}$, an equilibrium under *flexible* exchange rates is a set of endogenous processes $\{y_t, c_t, \pi_{H,t}, s_t, y_t^*, c_t^*, \pi_{F,t}^*\}$

251 and exogenous processes $\{\mu_{j,t}, j = H, F\}$ satisfying the following set of conditions:

252 - *Aggregate demand*

$$\mathbb{E}_t c_{t+1} = c_t + \sigma^{-1} (i_t - \mathbb{E}_t \pi_{H,t+1} - \gamma \Delta s_{t+1}) \quad (21)$$

$$\mathbb{E}_t c_{t+1}^* = c_t^* + \sigma^{-1} (i_t^* - \mathbb{E}_t \pi_{F,t+1}^* + \gamma^* \Delta s_{t+1}) \quad (22)$$

253 - *Market clearing*

$$y_t = (1 - \gamma) c_t + \gamma c_t^* + \gamma(2 - \gamma - \gamma^*) \eta s_t \quad (23)$$

$$y_t^* = \gamma^* c_t + (1 - \gamma^*) c_t^* - \gamma^*(2 - \gamma - \gamma^*) \eta s_t \quad (24)$$

254 - *Risk sharing*

$$(1 - \gamma - \gamma^*) s_t = \sigma (c_t - c_t^*) \quad (25)$$

255 - *Aggregate supply*

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + (\sigma + \zeta) \kappa (y_t - \bar{y}_t) - \gamma(2 - \gamma - \gamma^*) (\sigma \eta - 1) \kappa (s_t - \bar{s}_t) + \kappa \mu_{H,t} \quad (26)$$

$$\pi_{F,t}^* = \beta \mathbb{E}_t \pi_{F,t+1}^* + (\sigma + \zeta) \kappa^* (y_t^* - \bar{y}_t^*) + \gamma^*(2 - \gamma - \gamma^*) (\sigma \eta - 1) \kappa^* (s_t - \bar{s}_t) + \kappa^* \mu_{F,t}, \quad (27)$$

where

$$\begin{aligned} \pi_{i,t} &\equiv p_{i,t} - p_{i,t-1}, i = H, F \\ \kappa &\equiv \frac{(1 - \theta_H \beta)(1 - \theta_H)}{\theta_H(1 + \varepsilon \zeta)}; \kappa^* \equiv \frac{(1 - \theta_F \beta)(1 - \theta_F)}{\theta_F(1 + \varepsilon \zeta)}. \\ \sigma &\equiv -\frac{U_{cc} C}{U_c}; \zeta \equiv \frac{V_{yy} Y}{V_y}, \end{aligned}$$

256 with σ and ζ assumed equal in both countries.

Notice that the equilibrium characterization (21)-(27) does not feature the nominal depreciation rate, $\Delta e_t \equiv e_t - e_{t-1}$ (with $e_t \equiv \log \mathcal{E}_t$). The equilibrium path of the latter, in fact, can be derived residually from the one of the terms trade. Given $\{\pi_{j,t}\}_{t=0}^{\infty}$ and $\{s_t\}_{t=0}^{\infty}$ from above, one can derive $\{\Delta e_t\}_{t=0}^{\infty}$ using the expression

$$\Delta e_t = \Delta s_t + \pi_{H,t} - \pi_{F,t}^*, \quad (28)$$

holding for all t .²

It is useful, in order to eliminate c_t and c_t^* , to combine (23), (24), and (25), to obtain the following equilibrium condition linking the terms of trade to (cross-country) relative output:

$$\Gamma s_t = \sigma (y_t - y_t^*). \quad (29)$$

where $\Gamma \equiv (\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2 > 0$.

Equation (29) indicates that a rise of domestic output above foreign output requires, in equilibrium, a depreciation of the domestic terms trade. This is the result of two effects: first, holding relative consumption constant, higher output of domestic goods exert a downward pressure on domestic prices; second, since higher domestic output translates, at least in part, into higher relative consumption, this requires a real depreciation to allow for risk sharing, i.e., part of the higher consumption should be shared by foreign households via an increase in their real purchasing power.

3.2 Monetary union

There are two main differences that characterize the equilibrium under a monetary union relative to the case of flexible exchange rates. First, the law of motion (28) can no longer

²We assume throughout that the initial price levels, $p_{j,-1}$, are given and all equal to 1. In a steady state with balanced trade, we also have $s = s_{-1} = 1$. These conditions combined allow to pin down e_{-1} , the initial nominal exchange rate level. Combining the latter with the equilibrium path $\{\Delta e_t\}_{t=0}^{\infty}$ allows to derive $\{e_t\}_{t=0}^{\infty}$.

play a mere residual role in pinning down the equilibrium path of the nominal exchange rate. That expression is a necessary cross-equation restriction in the minimal set of equilibrium conditions, so that the fixed exchange rate condition $e_t = 0$ is explicitly accounted for. Second, given that a single monetary authority sets the common policy instrument, henceforth labeled i_t^{MU} , either one of equations (21) and (22) becomes irrelevant for the minimal specification of the equilibrium.

Hence, for a given specification of the policy instrument $\{i_t^{MU}\}$, an equilibrium under a *monetary union* is a set of endogenous processes $\{y_t, c_t, \pi_{H,t}, s_t, y_t^*, c_t^*, \pi_{F,t}^*\}$ and exogenous processes $\{\mu_{j,t}, j = H, F\}$ satisfying the same conditions (23)-(27) along with:

$$\mathbb{E}_t c_{t+1} = c_t + \sigma^{-1} (i_t^{MU} - \mathbb{E}_t \pi_{H,t+1} - \gamma \Delta s_{t+1}), \quad (30)$$

and the implied law of motion:

$$\Delta s_t = \pi_{F,t}^* - \pi_{H,t}. \quad (31)$$

Finally, notice that equation (29) holds irrespective of the underlying exchange rate regime, and is therefore valid also in the monetary union case.

4 Welfare objective

Under both regimes, we assume that a benevolent monetary authority aims at maximizing world welfare. In the flexible exchange rate regime, in particular, this corresponds to assuming that the monetary authorities of both countries conduct policy under cooperation. This not only implies that the benchmark regime we compare the monetary union to is a very strong one. It also implies that the inherent benefit of a monetary union described below cannot be related to gains from cooperation, a benefit which is frequently associated with monetary unions.

As already well understood in the literature (Galí 2015, Corsetti, Dedola and Leduc

297 2011), a case of particular interest arises when any (real) inefficiency possibly associated
 298 with the flexible price allocation is assumed not to affect the steady state. This is achieved
 299 by means of setting the lump sum tax τ_i in order to offset the distortion associated with
 300 market power in the goods markets:

$$\tau_i = -\frac{1}{\varepsilon - 1} \equiv \tilde{\tau}_i < 0 \rightarrow \mathcal{M}_i = 1. \quad (32)$$

301 In Appendix C we show that, under this assumption, and the additional condition that
 302 the degree of trade openness is symmetric across countries ($\alpha = \alpha^*$),³ the welfare losses
 303 experienced by households in the world economy, appropriately weighted by country size,
 304 are, up to second order, given by:

$$\mathbb{W}_t \equiv \sum_{t=0}^{\infty} \beta^t \mathbb{V}_t \quad (33)$$

$$\begin{aligned} \mathbb{V}_t \equiv & \frac{(\sigma + \zeta)}{2} [n(y_t - \bar{y}_t)^2 + (1 - n)(y_t^* - \bar{y}_t^*)^2] \\ & - \frac{n\Phi_s}{2} (s_t - \bar{s}_t)^2 + \frac{\varepsilon}{2} \left[\frac{n}{\kappa} \pi_{H,t}^2 + \frac{(1 - n)}{\kappa^*} \pi_{F,t}^{*2} \right], \end{aligned} \quad (34)$$

305 where $\Phi_s \equiv \frac{\sigma\eta-1}{\sigma} \gamma(2 - \gamma - \gamma^*)\Gamma$.

306 Hence welfare losses depend on the deviation of output from its natural level in each
 307 country (which also corresponds to the efficient level given the assumption in (32)), the
 308 deviation of the terms of trade from its natural level, and the deviations of domestic infla-
 309 tion (in each country) from its efficient level of zero. Taking unconditional expectations
 310 of (33), and letting $\beta \rightarrow 1$, we can express unconditional welfare losses (i.e., welfare losses
 311 in an average period) as a linear combination of the variances of each argument featured
 312 in (34):

³Notice that this further implies

$$n\gamma = (1 - n)\gamma^*.$$

$$\begin{aligned} \overline{\mathbb{W}} \equiv & \frac{(\sigma + \zeta)}{2} [n \operatorname{var}(y_t - \bar{y}_t) + (1 - n) \operatorname{var}(y_t^* - \bar{y}_t^*)] \\ & - \frac{n\Phi_s}{2} \operatorname{var}(s_t - \bar{s}_t) + \frac{\varepsilon}{2} \left[\frac{n}{\kappa} \operatorname{var} \pi_{H,t} + \frac{(1 - n)}{\kappa^*} \operatorname{var} \pi_{F,t}^* \right]. \end{aligned} \quad (35)$$

Notice that $\operatorname{sign}(\Phi_s)$, and therefore the contribution to welfare losses stemming from the variability in the terms of trade gap, depends on the assumption on the inverse elasticity of intertemporal substitution σ and the trade elasticity η , with $\sigma\eta > (<)1$ implying $\operatorname{sign}(\Phi_s) > (<)0$.

5 Optimal monetary policy

Next we turn to the central theme of the paper: the characterization of optimal monetary policy under two alternative regimes, flexible exchange rates and monetary union. For each regime we study two polar cases, depending on the underlying assumption about the ability of the monetary authority to commit. We are particularly interested in studying the case of a Markov perfect equilibrium (henceforth labeled *discretion*) in which, under either regime, the monetary authority cannot credibly commit to any future course of actions.

Constrained efficiency A standard constrained-efficiency approach to optimal policy prescribes that a social planner maximizes world welfare (33) subject to the relevant constraints that characterize the competitive equilibrium, i.e., (21)-(27) under flexible exchange rates, and (23)-(27) together with (30) and (31) in the monetary union case. The optimal policy problem, however, can be characterized in terms of a less constrained problem, under both exchange rate regimes, as we show below.⁴

⁴Put differently, in this setup, in order to characterize the constrained efficient allocation (either under flexible exchange rates or a monetary union) it is sufficient to solve a planner problem with a number of constraints which is smaller than the whole set of optimality conditions that describe the competitive equilibrium.

5.1 Flexible exchange rates

Optimal cooperative policy under flexible exchange rates requires solving the following problem:

$$\begin{aligned} \max \quad & \mathbb{W}_t \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{V}_t \\ \text{s.t.} \quad & (26), (27), (29). \end{aligned} \tag{36}$$

For a given specification of the exogenous processes $\{\mu_{j,t}\}$, $j = H, F$, a *flexible-exchange rate* equilibrium under the optimal policy consists of a vector $\{\pi_{H,t}, \pi_{F,t}^*, y_t, s_t, y_t^*\}^{OPT,FLEX}$ solving (36). One can then use (21), (22), (23), and (24) to residually obtain $\{c_t, c_t^*, i_t, i_t^*\}^{OPT,FLEX}$.

Commitment Under commitment, and flexible exchange rates, the centralized planner is able to choose a path for current and future values of the vector $\{\pi_{H,t}, \pi_{F,t}^*, y_t, s_t, y_t^*\}_{t=0}^{\infty}$ in order to maximize \mathbb{W}_t subject to an infinite sequence of constraints given by (26), (27), (29). Appendix D specifies the related dynamic Lagrangian problem. In the same Appendix D we show that, under commitment, the system of equations describing the equilibrium evolution under the optimal policy is given by (26), (27), (29) together with the following targeting rules:

$$(y_t - \bar{y}_t) - (y_{t-1} - \bar{y}_{t-1}) + \varepsilon \pi_{H,t} = 0 \tag{37}$$

$$(y_t^* - \bar{y}_t^*) - (y_{t-1}^* - \bar{y}_{t-1}^*) + \varepsilon \pi_{F,t}^* = 0. \tag{38}$$

Discretion Under discretion, and flexible exchange rates, the decisions of the centralized planner at time t do not bind at any future date. Given that the constraints (26), (27), (29) do not feature any endogenous state variable, though, the planner solves an infinite sequence of static problems of the form:

$$\max \mathbb{V}_t + \mathcal{F}_t \quad (39)$$

348 subject to the sequence of constraints (expressed in gaps form) given by:

$$\pi_{H,t} = (\sigma + \zeta) \kappa (y_t - \bar{y}_t) - \gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1) \kappa (s_t - \bar{s}_t) + f_t$$

$$\pi_{F,t}^* = (\sigma + \zeta) \kappa^* (y_t^* - \bar{y}_t^*) + \gamma^*(2 - \gamma - \gamma^*)(\sigma\eta - 1) \kappa^* (s_t - \bar{s}_t) + f_t^*$$

$$\Gamma (s_t - \bar{s}_t) = \sigma [(y_t - \bar{y}_t) + (y_t^* - \bar{y}_t^*)]$$

349 In the above expressions \mathcal{F}_t , f_t , and f_t^* are terms taken as given by the policymaker, and

350 are respectively equal to:

$$\mathcal{F}_t \equiv \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \mathbb{V}_t,$$

351

$$f_t \equiv \kappa \mu_{H,t} + \beta \mathbb{E}_t \pi_{H,t+1}, \quad (40)$$

$$f_t^* \equiv \kappa^* \mu_{F,t} + \beta \mathbb{E}_t \pi_{F,t+1}^*.$$

352 In Appendix D we show that the complete system of equations that describe the
 353 evolution of the welfare-relevant variables is given by (26), (27), (29), together with the
 354 following (static) targeting rules for each time t:

$$(y_t - \bar{y}_t) + \varepsilon \pi_{H,t} = 0, \quad (41)$$

$$(y_t^* - \bar{y}_t^*) + \varepsilon \pi_{F,t}^* = 0. \quad (42)$$

5.2 Monetary union

Relative to (36) under flexible exchange rates, optimal cooperative policy under a monetary union requires solving the more constrained optimization problem⁵:

$$\begin{aligned} \max \quad & \mathbb{W}_t \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathbb{V}_t \\ \text{s.t.} \quad & (26), (27), (29), (31) . \end{aligned} \tag{43}$$

For a given specification of the exogenous processes $\{\mu_{j,t}\}$, $j = H, F$, a *monetary union* equilibrium under the optimal policy consists of a vector $\{\pi_{H,t}, \pi_{F,t}^*, y_t, s_t, y_t^*\}^{OPT,MU}$ solving (43). One can then use (23), (24) and (30) to residually obtain $\{c_t, c_t^*, i_t^{MU}\}^{OPT,MU}$.

Commitment In a monetary union, if the policy authority can commit, the centralized planner is able to choose a path as of time $t = 0$ for current and future values of the vector $\{\pi_{H,t}, \pi_{F,t}^*, y_t, s_t, y_t^*\}_{t=0}^{\infty}$ in order to maximize \mathbb{W}_t subject to an infinite sequence of constraints given by (26), (27), (29), and (31). In Appendix D we present the general system of first order optimality conditions.

Discretion Under discretion, the monetary union central planner's policy problem differs substantially from the one under flexible exchange rates. The reason lies in the nature of constraint (31), which features an endogenous state variable. Hence, even under discretion, the nature of the policy problem is inherently *dynamic*: decisions that affect current values of inflation, output gap and the terms of trade, also affect the future values of the same variables.⁶ The dynamic nature of the optimal policy problem under discretion

⁵Given the political implications, and the implied costs of withdrawal, the commitment to a monetary union should imply “tying one’s hands” even more strongly than in a fixed exchange rate regime. Our model though is not suited to capture this further commitment gain. In order to do it in a satisfactory way, the choice of participating to a monetary union should be made endogenous.

⁶See Svensson (1999) and Vestin (2006) for the computation of a Markov perfect equilibrium in a linear quadratic optimal policy problem where inflation features endogenous inertia. See Currie and Levin (1993) for a general treatment of linear quadratic problems in the presence of endogenous state variables which can be influenced by policy.

is a genuine feature of a monetary union, and lies at the heart of our central argument whereby, under discretion, a monetary union can be welfare superior to a flexible exchange rate regime.

Therefore, in a monetary union and under discretion, the centralized planner's problem can be written in terms of the value function defined by

$$\mathcal{V}_t(s_{t-1}, \mu_{H,t}, \mu_{F,t}) = \max_{\{\pi_{H,t}, \pi_{F,t}^*, y_t, s_t, y_t^*\}} \{\mathbb{V}_t + \beta \mathbb{E}_t \mathcal{V}_{t+1}(s_t, \mu_{H,t+1}, \mu_{F,t+1})\} \quad (44)$$

subject to

$$(26), (27), (29), (31).$$

We solve the problem in (44) by searching for the existence of a Markov-perfect equilibrium.⁷ This is a concept of subgame perfect equilibrium in which the policymaker acts as a Stackelberg leader, whereas the private agents and the future policymakers act as followers. The characterization of an algorithm to solve for a Markov perfect equilibrium of this class of problems is given in Dennis (2007), which generalizes previous contributions in Backus and Driffill (1986), and Oudiz and Sachs (1985).

6 Symmetric nominal price rigidity

A useful benchmark arises in the special case of $\kappa = \kappa^*$. Under our maintained assumption that parameters ε and ζ are equal across countries, that special case obtains when the degree of nominal price rigidity is identical across countries, $\theta_H = \theta_F$.

Let

$$\mathcal{Y}_t \equiv n(y_t - \bar{y}_t) + (1 - n)(y_t^* - \bar{y}_t^*); \quad \pi_t \equiv n\pi_{H,t} + (1 - n)\pi_{F,t}^* \quad (45)$$

denote respectively the *average* output gap and inflation rate in the world economy (or monetary union), and

⁷As it is well-known other time-consistent equilibria, outside the class of Markov-perfect Stackelberg-Nash, might exist.

$$\tilde{\mathcal{Y}}_t \equiv y_t - y_t^*; \quad \tilde{\pi}_t \equiv \pi_{H,t} - \pi_{F,t}^*, \quad (46)$$

denote the *relative* output gap and inflation rate respectively.

Then, equation (35) can be written as:

$$\begin{aligned} \overline{\mathbb{W}}|_{\kappa=\kappa^*} \equiv & \frac{(\sigma + \zeta)}{2} \left[\underbrace{\text{var}(\mathcal{Y}_t)}_{\text{average area-wide output gap}} + n(1-n) \underbrace{\text{var}(\tilde{\mathcal{Y}}_t)}_{\text{composition of output gap}} \right] \\ & - \frac{n\Phi_s}{2} \text{var}(s_t - \bar{s}_t) + \frac{\varepsilon}{2\kappa} \left[\underbrace{\text{var}(\pi_t)}_{\text{average area-wide inflation}} + n(1-n) \underbrace{\text{var}(\tilde{\pi}_t)}_{\text{composition of inflation}} \right]. \end{aligned} \quad (47)$$

Hence we see that, in addition to variations in the terms of trade gap, welfare losses depend on both the *average* level and the *composition* of the area-wide output gap and inflation rate. We will show below that, in the assumed case of $\kappa = \kappa^*$, both the average output gap and average inflation are identical across exchange rate regimes. Therefore, the welfare ranking will crucially depend on how monetary policy shapes the composition of the average output gap and inflation under alternative regimes.

In the same special case of symmetric price stickiness, the equilibrium relationship between relative inflation and the terms of trade can be conveniently characterized analytically. We express that relationship in the following proposition.

Proposition 1. *Under the assumption $\kappa = \kappa^*$ (symmetric price stickiness), current relative inflation, $\tilde{\pi}_t$, is a function of its expected future value, of the current terms of trade gap, and of the relative markup shock, according to the following equation:*

$$\tilde{\pi}_t = \Omega(s_t - \bar{s}_t) + \kappa\tilde{\mu}_t + \beta\mathbb{E}_t\tilde{\pi}_{t+1}, \quad (48)$$

where $\Omega \equiv \frac{(\gamma+\gamma^*)(2-\gamma-\gamma^*)(\sigma\eta-1)\zeta+(\sigma+\zeta)}{\sigma}\kappa > 0$, and $\tilde{\mu}_t \equiv \mu_{H,t} - \mu_{F,t}$.

In order to derive expression (48), it suffices to combine (26), (27) and (29). Next, integrating equation (48) forward, and recalling (19), we obtain:

$$\tilde{\pi}_t = \Omega \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j (s_{t+j} - \bar{s}_{t+j}) \right\} + \frac{\kappa}{1 - \beta\rho_\mu} \tilde{\mu}_t, \quad (49)$$

where we have assumed that $\lim_{j \rightarrow \infty} \mathbb{E}_t \{ \beta^j (s_{t+j} - \bar{s}_{t+j}) \} = 0$.

Equation (49) shows that relative inflation depends on the current and expected future values of the terms of trade gap (as well as of the relative markup shocks). Hence, both the volatility and the persistence of the terms of trade gap contribute to the volatility of relative inflation, and therefore to welfare losses. Equivalently, by recalling equation (29), the volatility of relative inflation depends on the volatility and the persistence of the relative output gap.

6.1 An irrelevance result

Under the specified assumption of symmetric degree of nominal price rigidity across countries ($\kappa = \kappa^*$), it is useful to first derive a set of analytical results. The first result concerns the dynamic evolution of the terms of trade in a monetary union. We show that, in a monetary union, the equilibrium behavior of the terms of trade is independent of the monetary policy authority's ability to commit. This result will provide a useful benchmark.

Proposition 2. *Suppose $\kappa = \kappa^*$. Then in a monetary union the equilibrium behavior of relative inflation, $\tilde{\pi}_t$, and of the terms of trade, s_t , is independent of the monetary policy authority's ability to commit.*

To understand Proposition 2, consider first the law of motion for the terms of trade, which holds under a fixed nominal exchange rate:

$$s_t = s_{t-1} - \tilde{\pi}_t. \quad (50)$$

Notice that, for any given exogenous process for the relative markup shock $\{\tilde{\mu}_t\}$, the pair of equations (48) and (50) is sufficient to determine an equilibrium in the two endogenous variables $\tilde{\pi}_t$ and s_t , regardless of whether the behavior of monetary policy is specified to be under commitment or discretion.⁸

Taking advantage of the previous result, we show next that the exchange rate regime (whether flexible exchange rates or monetary union) is irrelevant for the equilibrium behavior of average inflation π_t and of the average output gap \mathcal{Y}_t . This is our irrelevance result. The fact that, under specific conditions of symmetry, area-wide average inflation and output gap behave identically under both flexible exchange rates and a monetary union is important to highlight that the difference in welfare losses across the two regimes lies precisely in the behavior of the terms of trade and, consequently, of relative inflation. Under those conditions, therefore, it is the *composition* of inflation across countries which lies at the heart of the differences in welfare losses. We formalize this argument in the following proposition.

Proposition 3. *Assume $\alpha = \alpha^*$. Then if $\kappa = \kappa^*$, the underlying exchange rate regime is irrelevant for the equilibrium behavior of the area-wide average inflation π_t and output gap \mathcal{Y}_t .*

To prove Proposition 3, notice first that combining (26) and (27), for both exchange rate regimes (flexible exchange rates and monetary union), and irrespective of whether monetary policy is conducted under commitment or discretion, one can write the following area-wide average expression for aggregate supply:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + (\sigma + \zeta) \kappa \mathcal{Y}_t + \kappa \mu_t \quad (51)$$

where $\mu_t \equiv n\mu_{H,t} + (1 - n)\mu_{F,t}$. The above equation has the flavor of a closed-economy New Keynesian Phillips curve, relating area-wide current inflation to its future values, to

⁸See also Benigno (2004) on this point.

the area-wide output gap and to an average cost-push shock term.

Consider, first, the case of commitment. Under flexible exchange rates, taking a weighted average of (37) and (38), and recalling the definitions of average output gap and inflation given in (45), one can characterize optimal policy in terms of the following targeting rule involving only the union-wide average output gap and inflation:

$$(\mathcal{Y}_t - \mathcal{Y}_{t-1}) + \varepsilon\pi_t = 0 \quad (52)$$

On the other hand, we show in Appendix D that, under the particular assumption of symmetric nominal price stickiness ($\kappa = \kappa^*$), a condition identical to (52) describes optimal policy also in a monetary union.⁹ We have therefore shown that, under commitment, and irrespective of the underlying exchange rate regime, an equilibrium in the area-wide average variables is a pair $\{\pi_t, \mathcal{Y}_t\}$ solving (51) and (52) for any given process $\{\mu_t\}$.

We turn next to the case of discretion. To start with, notice that, under flexible exchange rates, one can take a weighted average of (41) and (42), to obtain the targeting rule

$$\mathcal{Y}_t + \varepsilon\pi_t = 0 \quad (53)$$

What remains to be shown is that also under a monetary union it is possible to derive an optimality condition like (53). Recall our result in Proposition 2 whereby, in the assumed case of symmetric price rigidity, the equilibrium behavior of the terms of trade is given by the pair of equations (48) and (50). Therefore, and relative to the specification in (44), the optimal policy problem under discretion can be written in terms of a less constrained problem which can abstract from equation (50). The resulting optimization problem does not feature any endogenous state variable. The targeting rule that can be

⁹This implies that, in a monetary union, and conditional on $\kappa = \kappa^*$, the system of equations that describe the evolution of the welfare-relevant variables consists of (26), (27), (29), (31) together with the targeting rule (52).

derived is then identical to (53). This completes our proof of Proposition 3. The essence of this proposition is that, under conditions of symmetry, average inflation π_t and average output gap \mathcal{Y}_t , somewhat similar to a closed economy environment, are unaffected by the underlying exchange rate regime.

7 Inertia in the terms of trade: a commitment device

In this section we show that the key difference between the flexible exchange rate regime and the monetary union regime lies in the different equilibrium behavior of the terms of trade. We can illustrate this case analytically holding constant the symmetry assumption $\kappa = \kappa^*$.

Notice, first, that under all regimes, and irrespective of whether policy is conducted under commitment or discretion, equation (48) holds. We reproduce it here for convenience:

$$\tilde{\pi}_t = \Omega (s_t - \bar{s}_t) + \kappa \tilde{\mu}_t + \beta \mathbb{E}_t \tilde{\pi}_{t+1} \quad (54)$$

Flexible exchange rates Consider the case of flexible exchange rates. Under commitment, and the assumption $\kappa = \kappa^*$, combining (37) and (38) with (29) yields:

$$\varepsilon \tilde{\pi}_t = -\frac{\Gamma}{\sigma} [(s_t - \bar{s}_t) - (s_{t-1} - \bar{s}_{t-1})]. \quad (55)$$

Combining (54) with (55) to eliminate $\tilde{\pi}_t$ yields the following second-order stochastic difference equation for the terms of trade (note that $\bar{s}_t = 0$ if markup shocks are the only shocks):

$$\beta \mathbb{E}_t s_{t+1} - \left(1 + \beta + \frac{\sigma \varepsilon \Omega}{\Gamma}\right) s_t + s_{t-1} = \frac{\kappa \sigma \varepsilon}{\Gamma} \tilde{\mu}_t. \quad (56)$$

The above equation has a unique stationary representation expressing current s_t as a linear function of lagged s_{t-1} and $\tilde{\mu}_t$. Hence, under commitment, and even in the presence

of purely iid markup disturbances, the terms of trade feature an endogenous degree of persistence.

Under discretion, and once again assuming $\kappa = \kappa^*$, combining (41) and (42) with (29) yields:

$$\varepsilon \tilde{\pi}_t = -\frac{\Gamma}{\sigma} (s_t - \bar{s}_t). \quad (57)$$

Combining (57) with (54) to eliminate $\tilde{\pi}_t$ yields the following first-order stochastic difference equation for the terms of trade :

$$\beta \mathbb{E}_t s_{t+1} - \left(1 + \frac{\varepsilon \sigma \Omega}{\Gamma}\right) s_t = \frac{\kappa \varepsilon \sigma}{\Gamma} \tilde{\mu}_t. \quad (58)$$

The above equation has a unique stationary solution, expressing the terms of trade as a purely forward-looking variable and as a function of the (relative) markup shocks. The main insight stemming from (58), and in stark contrast to the case under commitment, is that under discretion the terms of trade do not feature any degree of endogenous persistence.

To summarize, in a flexible-exchange rate equilibrium under the optimal policy, the dynamic properties of the terms of trade depend on the ability of the monetary authority to commit. Under discretion, the terms of trade are exogenously persistent (i.e., only to the extent that the markup shocks are persistent); under commitment, and regardless of the stochastic properties of the markup shocks, the terms of trade feature endogenous inertia.

Monetary union A genuine feature of a monetary union, conversely, is that the terms of trade feature endogenous inertia. This result follows as a corollary of Proposition 2 illustrated earlier.

Corollary 1. *Suppose $\kappa = \kappa^*$. Then, in a monetary union, and regardless of the stochastic properties of the markup shocks, the equilibrium behavior of the terms of trade*

511 can be characterized by the following second-order stochastic difference equation:

$$\beta \mathbb{E}_t s_{t+1} - (1 + \beta + \Omega) s_t + s_{t-1} = \kappa \tilde{\mu}_t. \quad (59)$$

512 Representation (59) follows immediately by combining equation (48) and (50) to elim-
 513 inate $\tilde{\pi}_t$. Similar to (56), equation (59) has a unique stationary representation expressing
 514 current s_t as a linear function of lagged s_{t-1} and $\tilde{\mu}_t$. Hence, in a monetary union, the terms
 515 of trade feature endogenous persistence, a key feature shared with a flexible exchange rate
 516 regime under commitment. As a result, in a monetary union, and *irrespective* of whether
 517 or not the policy authority can commit, the terms of trade are intrinsically inertial. The
 518 intuition for this result is simple: the lack of nominal exchange rate flexibility combined
 519 with nominal price rigidity.

520 Notice that, while the inertia in the terms of trade in the context of a monetary
 521 union has been recognized before (Benigno 2004, Pappa 2004), it was typically regarded
 522 as a distortion of that regime. Relatedly, Farhi and Werning (2017) emphasize that the
 523 inefficiency at the heart of *any* monetary union, and regardless of the underlying degree of
 524 completeness in international financial markets, is a structural “lack of insurance”, which
 525 stems precisely from the suboptimal adjustment in the terms of trade that results from
 526 the combination of nominal price rigidity and lack of nominal exchange rate flexibility.

527 Unlike those previous contributions, which are centered on the inherent *inefficiency*
 528 of a monetary union, we wish to show that the inertia in the terms of trade can be an
 529 advantage for policy, rather than a constraint, depending on whether or not the monetary
 530 authority can commit. Thus, under commitment, inertia in the terms of trade is *always* a
 531 cost. Under discretion, however, inertia in the terms of trade can generally be *beneficial*:
 532 it allows the policy authority to trade-off efficiency in the response to asymmetric shocks
 533 (the aggregate demand stabilization cost) in order to gain in terms of management of
 534 inflation expectations. We turn to clarifying this point below.

7.1 Dynamics

In this section, we study the equilibrium dynamics under flexible exchange rates vis-a-vis a monetary union depending on the ability of the monetary authority to commit. We first describe the numerical calibration employed in our exercises.

Calibration We resort to the following calibration. The baseline parameter values are displayed in Table 1. A value of 0.99 for the discount factor β implies a steady-state real interest rate of around 4.1 percent annually. A value of 7.66 for the elasticity of substitution between differentiated goods ε implies a steady-state markup of prices over marginal costs of 15 percent. The trade elasticity of substitution η is calibrated to 2, which implies non-negligible degree of substitutability between domestic and foreign goods. Given the well-known uncertainty in the literature about the value of η , we perform robustness exercises below. A value of 0.75 for the probability of not being able to reset the price θ_i implies an average duration of price contracts of four quarters, consistent with much of the empirical evidence based on micro data. Both the degree of trade openness α and the relative size of the Home country n are calibrated to 0.5. These values imply a steady-state share of home-produced goods in the consumption basket, γ and γ^* , of 0.75. Note that the share of home-produced goods is symmetric across countries only under symmetric country size ($n = 0.5$). Asymmetries in country size ($n \neq 0.5$) will lead to asymmetries in the share of home-produced goods ($\gamma \neq \gamma^*$).

554

Table 1. Baseline calibration

Parameter	Description	Value/Target
β	Discount factor	0.99
σ	Inverse elasticity of intertemporal substitution	1
ζ	Inverse elasticity of producing differentiated good	0.67
n	Relative size of Home country	0.5
α	Degree of trade openness	0.5
ε	Elasticity of substitution btw. differentiated goods	7.66
η	Elasticity of substitution btw. Home and Foreign goods	2
θ_i	Probability of not being able to reset price in country $i = H, F$	0.75
ρ_μ	Persistence of markup shock in country $i = H, F$	0.9
σ_{μ_i}	Variance innovation markup process in country $i = H, F$	1

556 Unless otherwise stated, we assume that all structural parameters indicated above have
557 identical values across countries, including the variance of the innovation to the markup
558 process, which are also assumed to be uncorrelated across countries. This implies that, in
559 our baseline calibration, the symmetry assumptions $\kappa = \kappa^*$ and $n\gamma = (1 - n)\gamma^*$ both hold.

560 **Impulse responses** Figure 1 shows impulse responses of relative inflation and terms
561 of trade gap to a markup shock in Home under the assumption of commitment (the
562 behavior of the relative output gap is isomorphic to the one of the terms of trade, so it was
563 omitted for the sake of clarity). For each panel, the case of flexible exchange rates (dashed
564 line) is contrasted to the one of a monetary union (solid line). Qualitatively, the behavior
565 of both variables is similar under the two policy regimes: this is an implication of our
566 results derived above, and expressed in particular in equations (56) and (59), which show
567 that, under commitment, the terms of trade exhibit inertia under both flexible exchange
568 rates and a monetary union. In both cases, a rise in the Home markup generates a
569 typical “cost-push driven” tradeoff between higher relative inflation and lower output gap
570 (and/or appreciated terms of trade).¹⁰ In the case of flexible exchange rates, however,

¹⁰See Clarida et al. (1999) for the seminal analysis of so called cost-push shocks in a closed economy environment.

the more pronounced terms of trade appreciation, made feasible by the flexibility of the nominal exchange rate, restrains the response of relative inflation when compared to the case of a monetary union: under both regimes, in fact, relative inflation is a function of current and expected future movements in the terms of trade via equation (49). Overall, this effect is welfare increasing, and lies at the heart of the widely accepted optimality of flexible exchange rate regimes relative to monetary unions. Conversely, the gap between the response of the terms of trade under the two regimes is a measure of the inefficient adjustment of international relative prices under a monetary union. That inefficiency has been emphasized as the key one leading to a structural lack of insurance characterizing any monetary union (see e.g., Farhi and Werning 2017).

Figure 2 reports the impulse responses of the terms of trade gap and relative inflation in the case of discretion, which is the case of particular interest for our purposes. Once again solid lines indicate the response under a monetary union, whereas dashed lines indicate the responses in the case of flexible exchange rates.

Two results are worth emphasizing. First, notice that the terms of trade feature an inertial behavior only in the case of a monetary union, as (once again) indicated by equation (59). Under flexible exchange rates, the terms of trade appreciate sharply, and follow a Markov-type path afterwards: in practice they are the mirror image of the autoregressive exogenous markup process. Under a monetary union, and due to the inflexibility of the nominal exchange rate, the appreciation of the terms of trade is muted in the short run, but builds up afterwards. It is as if the monetary authority, in the current period, generated expectations of a future, more pronounced appreciation of the terms of trade (relative to the case of flexible exchange rates). The inertial behavior of the terms of trade in the case of a monetary union is reflected in the behavior of relative inflation. Since relative inflation, via equation (49), is a function of both the current and expected future terms of trade, the expectations of a more prolonged future real appreciation restrain the short-run increase in relative inflation under a monetary union. Overall, inflation is

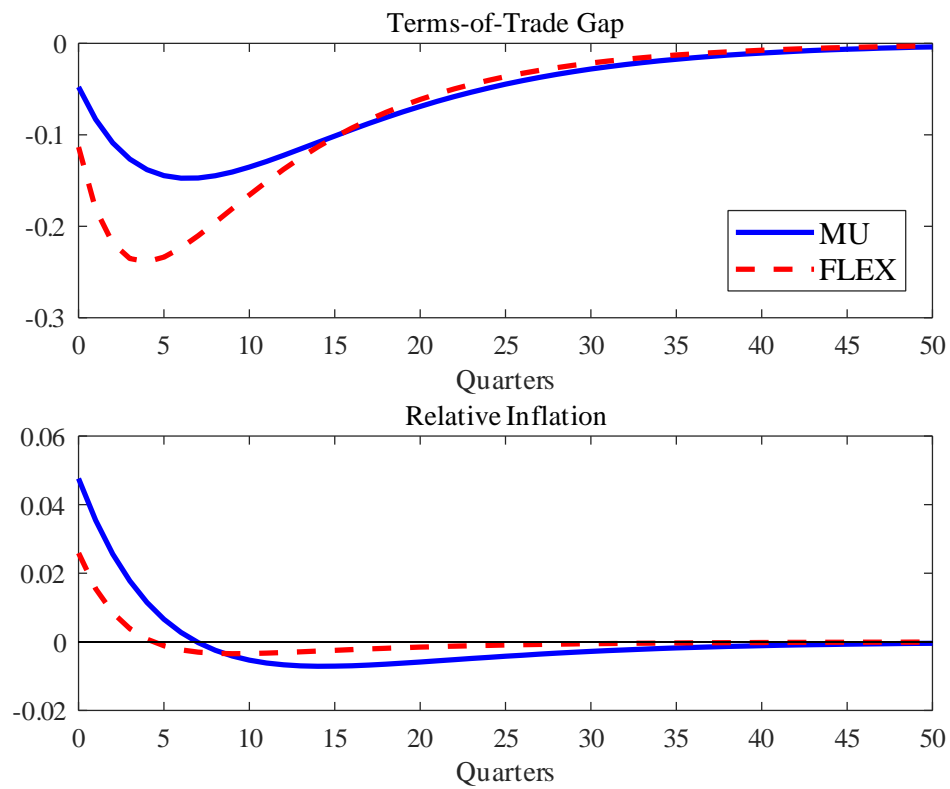


Figure 1: Impulse responses to a positive markup shock in Home under *commitment*: monetary union (solid) vs. flexible exchange rates (dashed). Note: % deviations from steady state.

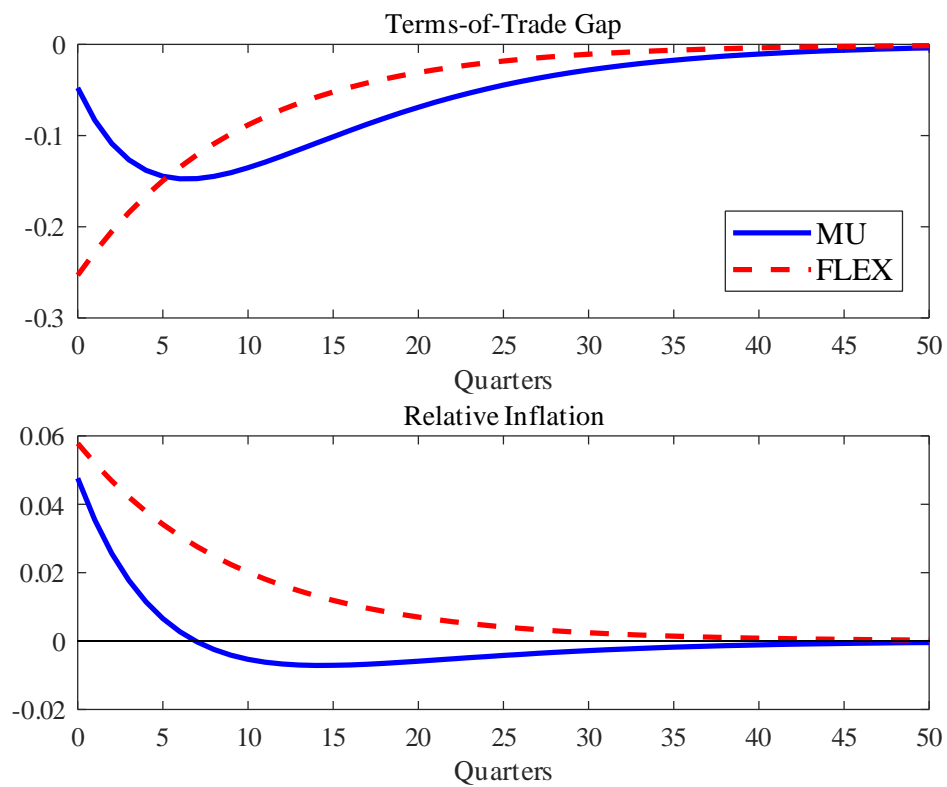


Figure 2: Impulse responses to a positive markup shock in Home under *discretion*: monetary union (solid) vs. flexible exchange rates (dashed). Note: % deviations from steady state.

more stable in a monetary union. We will show below that this inherent benefit is critical in generating a welfare improvement in the monetary union case with respect to flexible exchange rates.

8 The (un)desirability of a monetary union

In this section we conduct a thorough comparison of the welfare properties of each exchange rate regime, depending on the ability of the monetary authority to commit.

Figure 3 depicts our key result. It reports the *difference* in (area-wide) welfare losses, $\bar{W}_{MU} - \bar{W}_{FLEX}$, between a monetary union and a flexible exchange rate regime, as a function of the underlying degree of nominal rigidity assumed equal across countries ($\theta_H = \theta_F$), and separately for discretion and commitment. Positive values of the welfare loss difference, therefore, indicate that a monetary union entails higher welfare costs than flexible exchange rates.

The dashed line is illustrative of the standard consensus: under commitment, and regardless of the underlying degree of nominal rigidities, a monetary union always entails higher welfare losses relative to flexible exchange rates. This is a plain application of the classic Friedman dictum, whereby, under nominal rigidities, flexible exchange rates compensate for the inertial behavior in goods prices, thereby allowing the economy to replicate the constrained-efficient response of the terms of trade.¹¹

However, under discretion, the consensus result is overturned (solid line): a monetary union now entails lower welfare losses than flexible exchange rates. Only for very low degrees of price stickiness, well outside the range of plausible empirical estimates, a monetary union entails higher welfare losses.¹²

¹¹It is worth recalling, however, that the constrained efficient allocation under flexible exchange rates still differs from the first-best flexible price allocation, due to the presence of markup (i.e., inefficient) shocks.

¹²See Nakamura and Steinsson (2013) for a review of the available empirical micro evidence on the degree of nominal price rigidity.

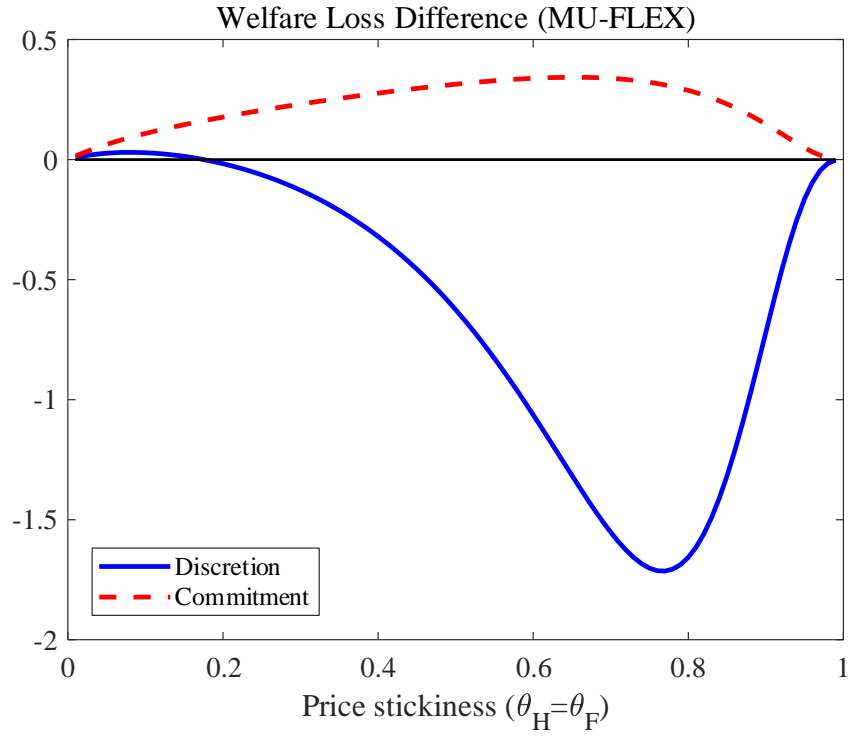


Figure 3: Welfare loss difference $\bar{W}_{MU} - \bar{W}_{FLEX}$ as a function of the degree of price stickiness under discretion and commitment. Note: a positive value indicates that flexible exchange rates dominate a monetary union.

Before looking at the factors driving these key results, two further observations are worth noticing. First, and at least under our baseline calibration, the welfare gain of a monetary union over flexible exchange rates under discretion is greater than the welfare gain of flexible exchange rates over a monetary union under commitment. Second, the welfare loss difference features a U-shaped, non-monotonic, relationship with the degree of price stickiness. The intuition for the non-monotonicity can be easily grasped by looking at the two extreme cases of full price flexibility ($\theta_H = \theta_F = 0$) and full price rigidity ($\theta_H = \theta_F = 1$). Under full price flexibility, the underlying exchange rate regime is irrelevant, because the flexibility of prices eventually compensates for the inertia in the nominal exchange rate; conversely, full price rigidity entails that inflation is always perfectly constant in equilibrium (and starting from a steady state in which the inflation rate is in line with the assumed target of zero), implying no welfare losses irrespective of commitment or discretion.

To inspect the mechanism more closely, Figure 4 depicts, under commitment, the contribution of the relevant components to the absolute welfare loss: relative inflation, the relative output gap, and the terms of trade gap.¹³ In each panel, the case of a monetary union (solid line) is contrasted to the one of flexible exchange rates (dashed line). The figure features a breakdown of the main factors that contribute to the relatively higher welfare cost of monetary unions: first, and foremost, a higher volatility in relative inflation (which increases the overall welfare losses); second, a lower volatility of the terms of trade gap (which, under the assumption $\sigma\eta > 1$, and therefore $\Phi_s > 0$, contributes in decreasing welfare losses relatively less). Both terms, in Figure 4, are appropriately weighted by the structural coefficients featured in (47). Clearly, moving from flexible exchange rates to a monetary union entails costs (a higher volatility of relative inflation and a lower volatility of the terms of trade gap, the latter being a cost under the assumption $\sigma\eta > 1$), but also

¹³Recall that under the assumption of symmetric price stickiness, both the area-wide average inflation rate and output gap are independent of the underlying exchange rate regime (see Proposition 3). Therefore, in Figure 4 we do not report their contribution to welfare losses.

benefits (a lower output gap volatility). As the top left panel shows, under our baseline parameterization, the costs uniformly outweigh the benefits, making a monetary union invariably more welfare costly, regardless of the degree of nominal price rigidity.

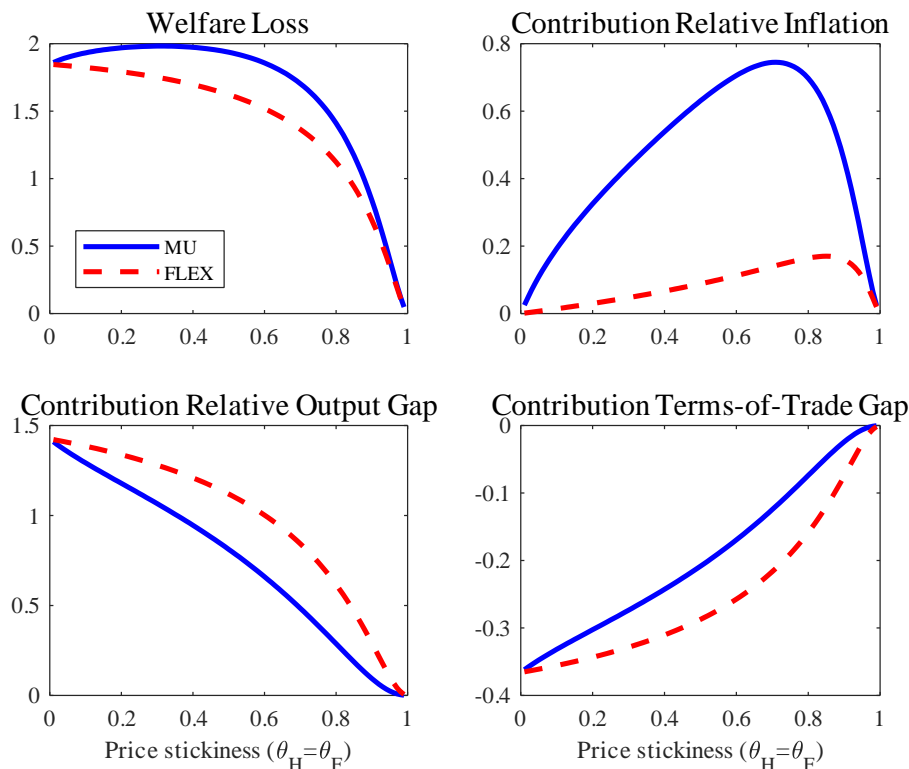


Figure 4: Average welfare loss and its components under *commitment*: monetary union (solid) vs. flexible exchange rates (dashed).

By contrast, Figure 5 shows the case of discretion. As shown in the top left panel, a monetary union entails now a lower welfare loss relative to a regime of flexible exchange rates, except for very low degrees of price rigidity. The main driver of this result, as clearly illustrated in the top right panel, is a lower volatility in the relative inflation term. This stems precisely from the inertia in the terms of trade induced by the fixed exchange rate. Noticeably, the welfare advantage of being in a monetary union relative to a regime of flexible exchange rates is non-monotonic in the degree of price rigidity, and reaches a

655 peak around the value for price stickiness (0.75) assumed in our baseline calibration (and
 656 allegedly in line with most empirical evidence, especially for the Eurozone¹⁴). The reason
 657 for the non-monotonicity is as follows. The costs, associated with discretionary policy, of
 658 taking inflation expectations as given are increasing in the degree of price rigidity because,
 659 as the latter increases, price setters become more and more forward-looking. In the limit,
 660 however, when prices are perfectly rigid, the gains associated to a better management of
 inflation expectations disappear, because a cost-push shock becomes ineffective.

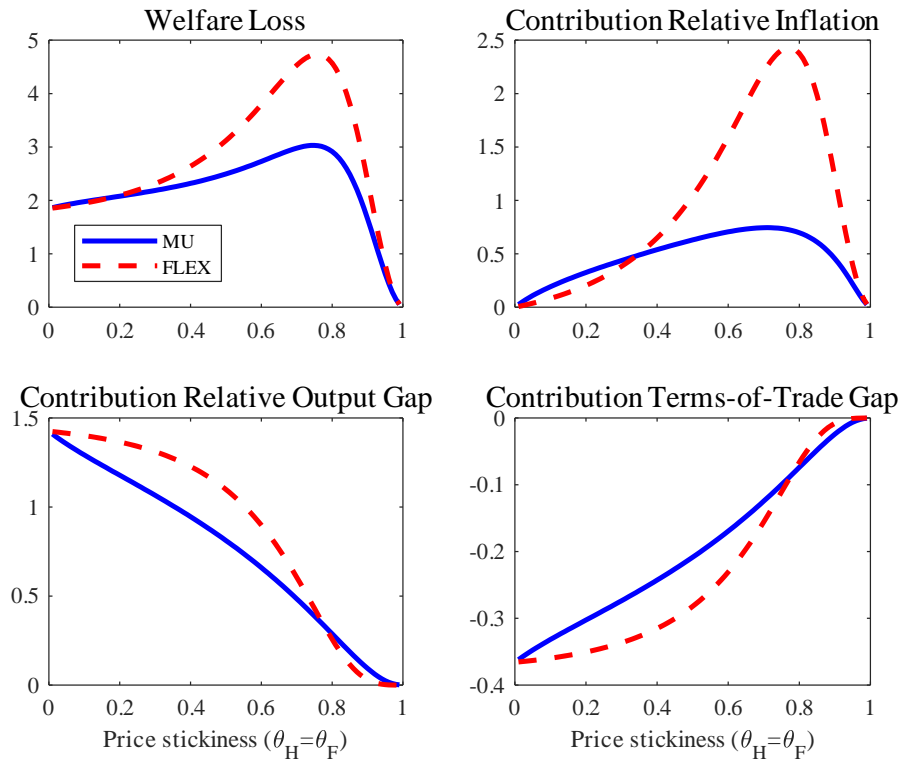


Figure 5: Average welfare loss and its components under *discretion*: monetary union (solid) vs. flexible exchange rates (dashed).

661

¹⁴See Dhyne et al. (2005).

9 Asymmetric nominal price rigidity

So far we have worked under the assumption of cross-country symmetry in the degree of price stickiness ($\kappa = \kappa^*$). This assumption has the important implication that it makes the policy maker's ability to commit irrelevant for the determination of the equilibrium behavior of the terms of trade (see Proposition 2 and Corollary 1). Therefore, under discretion, the inherent benefit of a monetary union (relative to flexible exchange rates) arises exclusively from the endogenous inertia that characterizes the terms of trade under that exchange rate regime.

The general case of asymmetric price stickiness is particularly interesting to study because in that case the characterization of a Markov perfect equilibrium requires, also under discretion, the solution of a fully dynamic recursive problem (as formalized in 44).¹⁵ Figure 6 depicts the effects on the welfare loss difference, $\bar{W}_{MU} - \bar{W}_{FLEX}$, of varying (only) the degree of *domestic* price stickiness, while holding constant θ_F , the degree of price stickiness in Foreign. This is shown for several cases, corresponding to alternative values of θ_F . Clearly, with the exception of limiting cases in which the degree of price stickiness is extremely low in both countries (although not necessarily equal), the welfare loss difference takes invariably a negative value, implying a lower welfare loss under a monetary union. We therefore conclude that asymmetries in the degree of price stickiness are not, to any important degree, relevant for the welfare ranking between the two monetary regimes. They only affect the size of the welfare gain of the monetary union regime. Notice also that the maximum welfare gain from a monetary union (i.e., the lowest value in the welfare loss difference) is not necessarily achieved under a symmetric degree of price stickiness.

¹⁵We solve the problem specified in (44) by resorting to the algorithm developed by Dennis (2007) and implemented in Dynare (Adjemian et al. 2011).

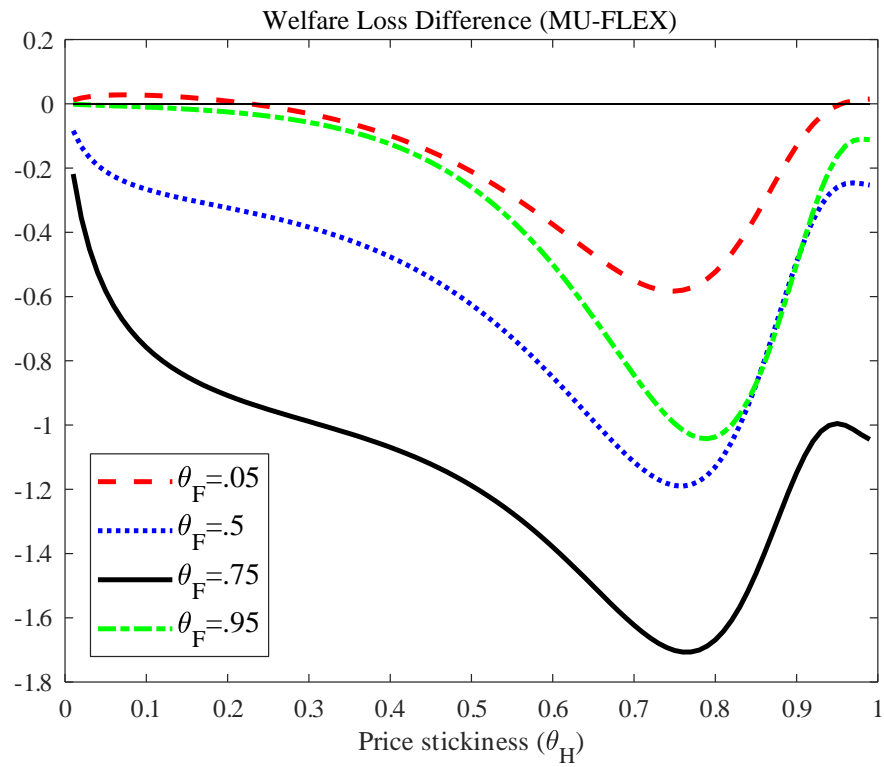


Figure 6: Effect of varying domestic price stickiness on the welfare loss difference $\overline{\mathbb{W}}_{MU} - \overline{\mathbb{W}}_{FLEX}$ under *discretion* and for alternative foreign price stickiness.

10 Robustness

In this section, we assess the robustness of our main result (i.e., the desirability of a monetary union under discretion) to variations in some key model parameters.

Efficient vs. inefficient shocks So far, the analysis was exclusively based on the presence of (asymmetric) markup shocks. These inefficient shocks create fundamental tradeoffs for monetary stabilization policy and, together with forward-looking staggered pricing, give rise to gains from commitment (Woodford 2003, Clarida et al. 1999). Under efficient shocks, like productivity shocks, and under our maintained assumptions of producer currency pricing and complete asset markets across countries, there are no tradeoffs for stabilization policy - the “divine coincidence” case of Blanchard and Galí (2007). Therefore, there are also no gains from commitment, or commitment-like features such as inertia in the terms of trade (as emphasized here).

Figure 7 shows the welfare loss difference, under discretion, conditional on alternative values of the volatility of productivity shocks ($z_{Y,t}, z_{Y,t}^*$) and of the markup shocks $\mu_{j,t}$. The black line shows our baseline result (i.e. no productivity shocks). Clearly, and as expected, the more important productivity shocks become relative to markup shocks, the smaller are the gains of a monetary union and the higher the minimal degree of price stickiness needed to make a monetary union more desirable relative to flexible exchange rates. If the volatility of productivity shocks is twice as large as the one of markup shocks (green line), a monetary union is no longer beneficial, irrespective of the degree of price stickiness.

Persistence of markup shocks Next, we look at how price stickiness affects the welfare loss difference under alternative assumptions on the persistence of the markup shock ρ_μ . In what follows, we return to the baseline assumption of a symmetric degree of price stickiness across countries. Figure 8 shows that a monetary union continues to

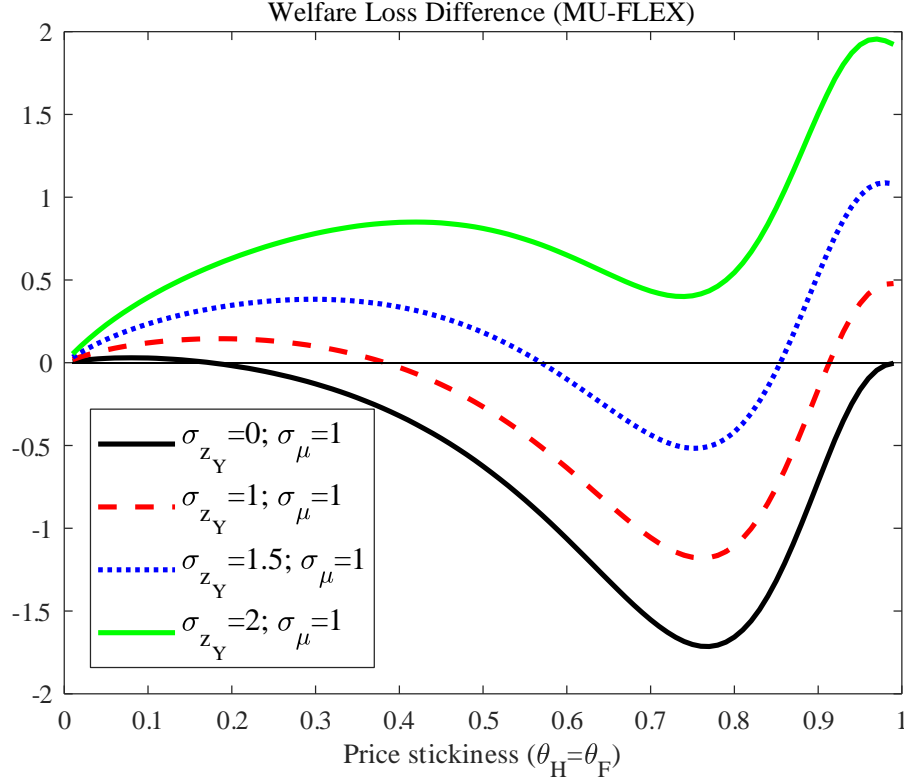


Figure 7: Effect of varying domestic price stickiness on the welfare loss difference $\bar{\mathbb{W}}_{MU} - \bar{\mathbb{W}}_{FLEX}$ under *discretion* and for alternative values of the volatility of productivity shocks ($z_{Y,t}, z_{Y,t}^*$) and of the markup shocks $\mu_{j,t}$.

be desirable for plausible estimates of the degree of price stickiness, though the size of the welfare gain decreases rapidly as the shock persistence decreases. Notice, however, that the lower the degree of persistence of the markup shock, the higher the minimal degree of price stickiness needed to make a monetary union more desirable relative to flexible exchange rates. The role played by the shock persistence is intuitive, given that the inherent benefit of a monetary union is due to the inertia in the terms of trade and its stabilizing effect on inflation expectations. As the shock becomes more and more short-lived, the importance of expectations in general decreases. As a result, the benefit of stabilizing inflation expectations tends to vanish.

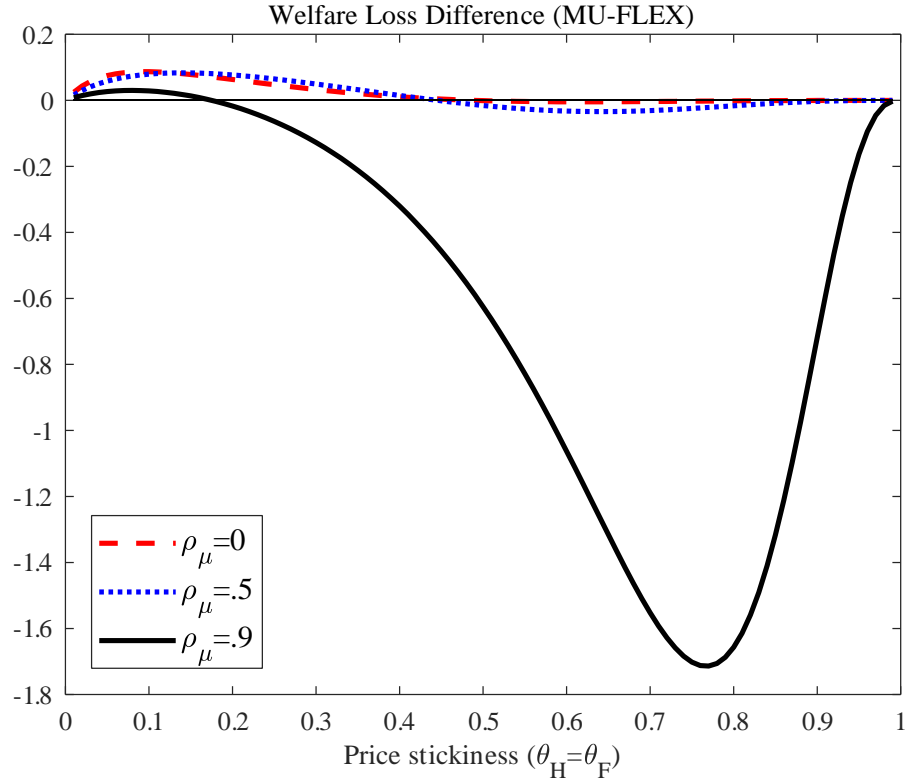


Figure 8: Effect of varying domestic price stickiness on the welfare loss difference $\overline{\mathbb{W}}_{MU} - \overline{\mathbb{W}}_{FLEX}$ under *discretion* and for alternative degrees of persistence of the markup shock.

Trade elasticity of substitution

Figure 9 displays the effect on the welfare loss difference of varying the trade elasticity of substitution η . The relationship is clear-cut: The lower the degree of substitutability of internationally traded goods, the higher the minimal degree of price stickiness needed to make a monetary union more desirable relative to flexible exchange rates, and the smaller is the corresponding welfare gain. As the internationally traded goods become less substitutable for consumers in both countries, the terms of trade become less and less important for price setters and their expectations, due to a decreasing expenditure switching effect. In the extreme case of zero substitutability ($\eta = 0$), households consume the goods in fixed proportions, irrespective of changes in their relative price. In this case, the benefit of policy-induced inertia in the terms of trade ceases to exist. By contrast, if the elasticity of substitution increases, the terms of trade become more and more important for price setters due to the increasing expenditure switching effect. As a result, the beneficial effect of terms-of-trade inertia on inflation expectations, and ultimately on inflation, tends to increase. Lastly, notice that, given $\sigma = 1$, Figure 9 displays cases where, respectively, $\sigma\eta < 1$, $\sigma\eta = 1$, and $\sigma\eta > 1$. It therefore becomes clear that this condition, which plays a crucial role in shaping the cross-border spillovers of shocks, does not play an important role *per se* in shaping the welfare implications of monetary unification.

11 Conclusions

We have studied a classic issue in international monetary economics, namely whether, as originally argued by Friedman (1953), the presence of nominal price rigidity makes an unequivocal (welfare) case in favor of flexible exchange rates. We have shown that the answer to this question hinges crucially on the monetary authority's ability to commit. When the monetary authority lacks commitment, a regime of fixed exchange rates (or monetary union) generally welfare dominates one of flexible exchange rates. This result is in stark contrast with the general consensus whereby the participation to a monetary union en-

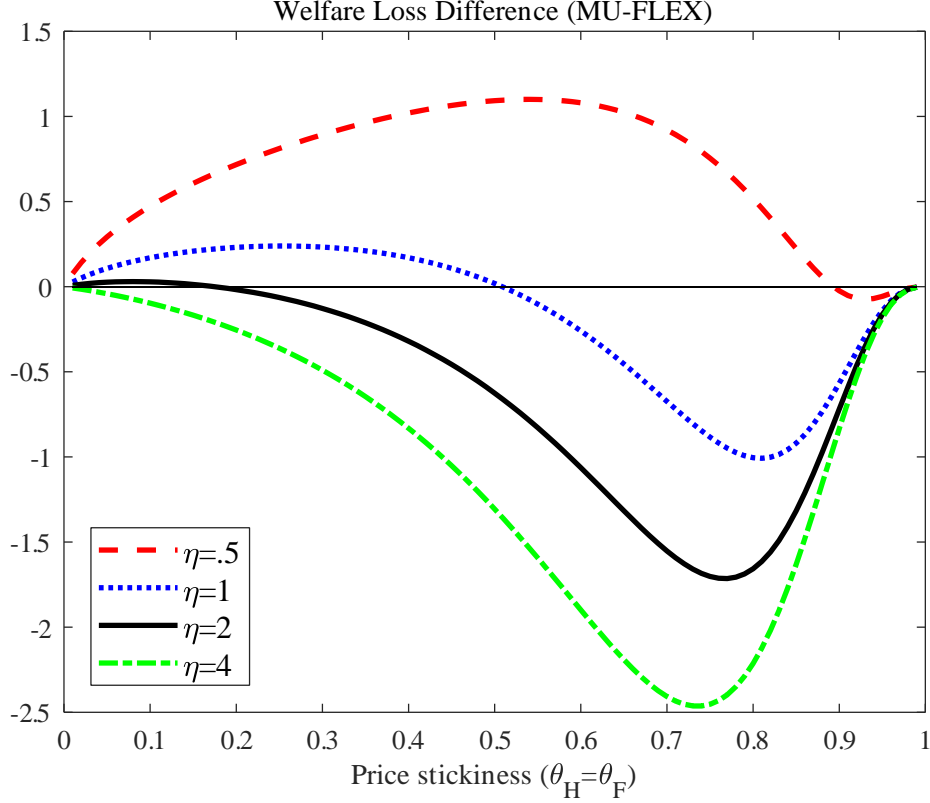


Figure 9: Effect of varying domestic price stickiness on the welfare loss difference $\bar{W}_{MU} - \bar{W}_{FLEX}$ under *discretion* and for alternative values of the trade elasticity of substitution η .

745 tails a genuine inefficiency, in that it precludes the efficient adjustment of international
 746 relative prices in response to asymmetric shocks. Our analysis shows that such an inef-
 747 ficiency typically makes a monetary union welfare dominated only under the admittedly
 748 extreme assumption of full commitment by the monetary policy authority. Importantly,
 749 we have shown that, when the monetary policy authority lacks commitment, the “inherent
 750 benefit” of a monetary union is higher the higher the incidence of asymmetric inefficient
 751 shocks, and/or the higher the degree of nominal price rigidity, in stark contrast with both
 752 the traditional OCA theory and the classic Friedman (1953) argument. Although both
 753 polar cases of full commitment and discretion are somewhat extreme, this result highlights

754 the importance of focusing on the policy credibility dimension when employing dynamic
755 microfounded models to assess the desirability of alternative exchange rate regimes.

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846 **A Market clearing conditions in the full model**

847 Market clearing for domestic variety i must satisfy:

$$Y_t(i) = n C_{H,t}(i) + (1-n) C_{H,t}^*(i) \quad (\text{A.1})$$

$$\begin{aligned} &= \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left[(1-\gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \frac{(1-n)}{n} \gamma^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \right] \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1-\gamma) C_t + \frac{(1-n)}{n} \gamma^* Q_t^\eta C_t^* \right] \end{aligned} \quad (\text{A.2})$$

848 Market clearing for foreign variety i must satisfy:

$$Y_t^*(i) = n C_{F,t}(i) + (1-n) C_{F,t}^*(i) \quad (\text{A.3})$$

$$= \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} \left[\frac{n}{1-n} \gamma C_t + (1-\gamma^*) Q_t^\eta C_t^* \right]$$

Inserting (A.1) and (A.3) into the following two equations, respectively

$$\begin{aligned} Y_t &= \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ Y_t^* &= \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 Y_t^*(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \end{aligned} \quad (\text{A.4})$$

yields aggregate demand in each country:

$$\begin{aligned} Y_t &= \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1-\gamma) C_t + \frac{1-n}{n} \gamma^* Q_t^\eta C_t^* \right] \\ Y_t^* &= \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} \left[\frac{n}{1-n} \gamma C_t + (1-\gamma^*) Q_t^\eta C_t^* \right]. \end{aligned} \quad (\text{A.5})$$

In the particular case of a *symmetric degree of trade openness* across countries ($\alpha = \alpha^*$), we can write aggregate demand in each country as:

$$\begin{aligned} Y_t &= \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} [(1-\gamma) C_t + \gamma Q_t^\eta C_t^*] \\ Y_t^* &= \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} [\gamma^* C_t + (1-\gamma^*) Q_t^\eta C_t^*]. \end{aligned} \quad (\text{A.6})$$

Assuming $C = C^*$ in steady state, log-linearization of the previous two equations yields

$$y_t = \gamma\eta s_t + (1 - \gamma)c_t + \gamma c_t^* + \gamma\eta q_t \quad (\text{A.7})$$

$$y_t^* = -(1 - \gamma)\eta s_t + \gamma^* c_t + (1 - \gamma^*)c_t^* + (1 - \gamma^*)\eta q_t.$$

849 Inserting the log-linearized version of (15), which is given by

$$q_t = (1 - \gamma - \gamma^*)s_t, \quad (\text{A.8})$$

to eliminate q_t yields:

$$y_t = (1 - \gamma)c_t + \gamma c_t^* + \gamma(2 - \gamma - \gamma^*)\eta s_t \quad (\text{A.9})$$

$$y_t^* = \gamma^* c_t + (1 - \gamma^*)c_t^* - \gamma^*(2 - \gamma - \gamma^*)\eta s_t. \quad (\text{A.10})$$

850 B Efficient allocation

851 The first-best or efficient allocation describes the equilibrium in which prices are fully
 852 flexible and in which markups are neutralized at all times with an appropriate subsidy
 853 ($\mu_{j,t} = 0$). This efficient allocation provides a useful benchmark in order to assess the
 854 welfare implications of the two exchange rate regimes. Again, we assume a symmetric
 855 degree of trade openness, i.e., $\alpha = \alpha^*$.

856 Log-linearizing the risk sharing condition (16) yields¹⁶

$$\bar{q}_t = \sigma(\bar{c}_t - \bar{c}_t^*) + (z_{C,t}^* - z_{C,t}). \quad (\text{B.1})$$

857 Combining (A.9), (B.1) and (A.8) yields

$$\sigma\bar{y}_t = \sigma\bar{c}_t + \gamma[(2 - \gamma - \gamma^*)(\sigma\eta - 1) + 1]\bar{s}_t + \gamma(z_{C,t}^* - z_{C,t}). \quad (\text{B.2})$$

858 Log-linearizing the optimal pricing equation (under flexible prices) in Home yields:

$$\zeta\bar{y}_t = -\gamma\bar{s}_t - \sigma\bar{c}_t + z_{C,t} + \zeta z_{Y,t}. \quad (\text{B.3})$$

¹⁶The following equations contain consumption preference shocks $z_{C,t}$ and $z_{C,t}^*$ that we have abstracted from in the main body of the paper; see also the derivation of the welfare loss function.

Combining the previous two equations to eliminate \bar{c}_t yields:

$$(\sigma + \zeta)\bar{y}_t = \gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1)\bar{s}_t - \gamma(z_{C,t} - z_{C,t}^*) + z_{C,t} + \zeta z_{Y,t}. \quad (\text{B.4})$$

The corresponding equation for country F is derived in a completely analogous way. Thus,

$$(\sigma + \zeta)\bar{y}_t^* = -\gamma^*(2 - \gamma - \gamma^*)(\sigma\eta - 1)\bar{s}_t + \gamma^*(z_{C,t} - z_{C,t}^*) + z_{C,t}^* + \zeta z_{Y,t}^*. \quad (\text{B.5})$$

The efficient terms of trade are given by

$$\left[(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2 \right] \bar{s}_t = \sigma (\bar{y}_t - \bar{y}_t^*) - (1 - \gamma - \gamma^*) (z_{C,t} - z_{C,t}^*), \quad (\text{B.6})$$

which is obtained by subtracting the country-specific aggregate demand equations (A.9) and (A.10) from each other and by using the risk sharing condition (B.1) and equation (A.8) to eliminate country-specific consumption and the real exchange rate.

C Welfare loss function

The derivation of the quadratic welfare loss function follows Corsetti, Dedola, and Leduc (2011). The period utility of agents living in country H is given by

$$\mathcal{W}_t = U(C_t, Z_{C,t}) - \frac{1}{n} \int_0^n V(Y_t(i), Z_{Y,t}) di, \quad (\text{C.1})$$

where

$$U(C_t, Z_{C,t}) = Z_{C,t} \frac{C_t^{1-\sigma} - 1}{1 - \sigma} \quad (\text{C.2})$$

$$V(Y_t(i), Z_{Y,t}) = Z_{Y,t}^{-\zeta} \frac{Y_t(i)^{1+\zeta}}{1 + \zeta}, \quad (\text{C.3})$$

where $Z_{C,t}$ and $Z_{Y,t}$ are shocks to consumption preferences and to productivity, respectively. A second-order approximation of $U(C_t, Z_{C,t})$ yields

$$U(C_t, Z_{C,t}) = U_C C \left[c_t + \frac{1 - \sigma}{2} c_t^2 + c_t z_{C,t} \right] + t.i.p. + O(\|Z\|^3), \quad (\text{C.4})$$

where the term *t.i.p.* collects all the terms that are independent of monetary policy as well as independent of the exchange rate regime and the term $O(\|Z\|^3)$ groups all the terms that are of third or higher order in the deviations of the various variables from their steady state.

A second-order approximation of $V(Y_t(i), Z_{Y,t})$ yields

$$V(Y_t(i), Z_{Y,t}) = V_Y Y \left[y_t(i) + \frac{1+\zeta}{2} y_t(i)^2 - \zeta y_t(i) z_{Y,t} \right] + t.i.p. + O(\|Z\|^3). \quad (C.5)$$

Under the assumption that $\alpha = \alpha^*$ and that in the steady state $C = C^*$, it follows that $S = Q = 1$ and $Y = C$ in the steady state. Under the additional assumption that the steady state is efficient, i.e., an appropriate subsidy τ^H eliminates the distortion due to monopolistic competition, the efficiency of the flexible price allocation implies

$$V_Y Y = U_C C. \quad (C.6)$$

Integrating (C.5) over the differentiated goods i yields

$$\begin{aligned} \frac{1}{n} \int_0^n V(Y_t(i), Z_{Y,t}) di = & U_C C \left[y_t + \frac{1+\zeta}{2} y_t^2 - \zeta y_t z_{Y,t} + \frac{1}{2} (\varepsilon^{-1} + \zeta) {}_i y_t(i) \right] \\ & + t.i.p. + O(\|Z\|^3), \end{aligned} \quad (C.7)$$

where we used

$${}_i y_t(i) = \mathbb{E}_i y_t(i)^2 - [\mathbb{E}_i y_t(i)]^2 \quad (C.8)$$

to eliminate $\mathbb{E}_i y_t(i)^2$ and the second-order approximation of y_t

$$y_t = \mathbb{E}_i y_t(i) + \frac{1}{2} \frac{\varepsilon - 1}{\varepsilon} {}_i y_t(i) + O(\|Z\|^3) \quad (C.9)$$

to eliminate $\mathbb{E}_i y_t(i)$.

Inserting (C.4) and (C.7) into (C.1) yields

$$\begin{aligned} \mathcal{W}_t = & U_C C \left[c_t + \frac{1-\sigma}{2} c_t^2 + c_t z_{C,t} - y_t - \frac{1+\zeta}{2} y_t^2 + \zeta y_t z_{Y,t} - \frac{1}{2} (\varepsilon^{-1} + \zeta) {}_i y_t(i) \right] \\ & + t.i.p. + O(\|Z\|^3). \end{aligned} \quad (C.10)$$

882 The log-linear expression of (A.1) implies that

$${}_i y_t(i) = \varepsilon_i^2 p_{H,t}(i). \quad (\text{C.11})$$

883 And completely analogous to Woodford (2003)

$$\sum_{k=0}^{\infty} \beta^k p_{H,t+k}(i) = \frac{\theta^H}{(1 - \theta^H \beta)(1 - \theta^H)} \sum_{k=0}^{\infty} \beta^k \pi_{H,t+k}^2. \quad (\text{C.12})$$

884 Using the previous two relationships to eliminate ${}_i y_t(i)$ yields

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \mathcal{W}_{t+k} = U_C C \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \mathcal{V}_{t+k} \quad (\text{C.13})$$

where

$$\begin{aligned} \mathcal{V}_t = & c_t - y_t + \left(\frac{1 - \sigma}{2} c_t + z_{C,t} \right) c_t - \left(\frac{1 + \zeta}{2} y_t - \zeta z_{Y,t} \right) y_t \\ & - \frac{1}{2} \varepsilon (1 + \varepsilon \zeta) \frac{\theta^H}{(1 - \theta^H \beta)(1 - \theta^H)} \pi_{H,t}^2 + t.i.p. + O(\|Z\|^3). \end{aligned} \quad (\text{C.14})$$

885 The previous steps can be repeated completely analogously to obtain the corresponding
886 expression for country F :

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \mathcal{W}_{t+k}^* = U_C C \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \mathcal{V}_{t+k}^* \quad (\text{C.15})$$

where

$$\begin{aligned} \mathcal{V}_t^* = & c_t^* - y_t^* + \left(\frac{1 - \sigma}{2} c_t^* + z_{C,t}^* \right) c_t^* - \left(\frac{1 + \zeta}{2} y_t^* - \zeta z_{Y,t}^* \right) y_t^* \\ & - \frac{1}{2} \varepsilon (1 + \varepsilon \zeta) \frac{\theta^F}{(1 - \theta^F \beta)(1 - \theta^F)} \pi_{F,t}^{*2} + t.i.p. + O(\|Z\|^3). \end{aligned} \quad (\text{C.16})$$

The world welfare loss function is given by the weighted average of the country-specific

welfare loss functions:

$$\begin{aligned}
\mathbb{V}_t &= n\mathcal{V}_t + (1-n)\mathcal{V}_t^* \\
&= nc_t + (1-n)c_t^* - ny_t - (1-n)y_t^* \\
&\quad + n\left(\frac{1-\sigma}{2}c_t + z_{C,t}\right)c_t + (1-n)\left(\frac{1-\sigma}{2}c_t^* + z_{C,t}^*\right)c_t^* \\
&\quad - n\left(\frac{1+\zeta}{2}y_t - \zeta z_{Y,t}\right)y_t - (1-n)\left(\frac{1+\zeta}{2}y_t^* - \zeta z_{Y,t}^*\right)y_t^* \\
&\quad - \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\
&\quad - \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\
&\quad + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.17}$$

C.1 Eliminating the linear terms

The world resource constraint expressed in the currency of country H is given by

$$nP_t C_t + (1-n)\mathcal{E}_t P_t^* C_t^* = nP_{H,t} Y_t + (1-n)\underbrace{\mathcal{E}_t P_{F,t}^*}_{=P_{F,t}} Y_t^* \tag{C.18}$$

Dividing both sides by P_t yields

$$nC_t + (1-n)\underbrace{\frac{\mathcal{E}_t P_t^*}{P_t}}_{=Q_t} C_t^* = n\frac{P_{H,t}}{P_t} Y_t + (1-n)\frac{P_{F,t}}{P_t} Y_t^* \tag{C.19}$$

Note that

$$\frac{P_{H,t}}{P_t} = [1 - \gamma + \gamma S_t^{1-\eta}]^{-\frac{1}{1-\eta}} \tag{C.20}$$

and

$$\frac{P_{F,t}}{P_t} = \frac{P_{F,t}}{\mathcal{E}_t P_t^*} \frac{\mathcal{E}_t P_t^*}{P_t} = [\gamma^* S_t^{\eta-1} + 1 - \gamma^*]^{-\frac{1}{1-\eta}} Q_t. \tag{C.21}$$

Thus, the world resource constraint can be written as follows:

$$\begin{aligned}
nC_t + (1-n)Q_t C_t^* &= n[1 - \gamma + \gamma S_t^{1-\eta}]^{-\frac{1}{1-\eta}} Y_t \\
&\quad + (1-n)[\gamma^* S_t^{\eta-1} + 1 - \gamma^*]^{-\frac{1}{1-\eta}} Q_t Y_t^*.
\end{aligned} \tag{C.22}$$

The second-order approximation of this expression yields

$$\begin{aligned}
& nc_t + (1-n)c_t^* - ny_t - (1-n)y_t^* \\
&= -\frac{1}{2} \left(nc_t^2 + (1-n)c_t^{*2} \right) + \frac{1}{2} \left(ny_t^2 + (1-n)y_t^{*2} \right) \\
&\quad + (1-n)(y_t^* - c_t^*)q_t - n\gamma(y_t - y_t^*)s_t \\
&\quad + \frac{1}{2}n\gamma(2-\gamma-\gamma^*)\eta s_t^2 \\
&\quad + t.i.p. + O(\|Z\|^3),
\end{aligned} \tag{C.23}$$

892 where we have used the fact that, under $\alpha = \alpha^*$:

$$n\gamma = (1-n)\gamma^*. \tag{C.24}$$

Note that this equality is frequently used in the subsequent manipulations as well. Inserting (C.23) into (C.17) to eliminate the linear terms yields

$$\begin{aligned}
\mathbb{V}_t &= -n \left(\frac{\sigma}{2}c_t - z_{C,t} \right) c_t - (1-n) \left(\frac{\sigma}{2}c_t^* - z_{C,t}^* \right) c_t^* \\
&\quad + (1-n)(y_t^* - c_t^*)q_t + \frac{1}{2}\eta n\gamma(2-\gamma-\gamma^*)s_t^2 - n\gamma(y_t - y_t^*)s_t \\
&\quad - n \left(\frac{\zeta}{2}y_t - \zeta z_{Y,t} \right) y_t - (1-n) \left(\frac{\zeta}{2}y_t^* - \zeta z_{Y,t}^* \right) y_t^* \\
&\quad - \frac{1}{2}n\varepsilon(1+\varepsilon\zeta) \frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)} \pi_{H,t}^2 \\
&\quad - \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta) \frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)} \pi_{F,t}^{*2} \\
&\quad + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.25}$$

C.2 Further manipulations

Add and subtract $\frac{1}{2}n\gamma s_t y_t$ and $\frac{1}{2}(1-n)\gamma^* s_t y_t^*$ to obtain

$$\begin{aligned}
\mathbb{V}_t = & -n \left(\frac{\sigma}{2} c_t - z_{C,t} \right) c_t - (1-n) \left(\frac{\sigma}{2} c_t^* - z_{C,t}^* \right) c_t^* \\
& + (1-n) (y_t^* - c_t^*) q_t + \frac{1}{2} \eta n \gamma (2 - \gamma - \gamma^*) s_t^2 - \frac{1}{2} n \gamma (y_t - y_t^*) s_t \\
& - n \left(\frac{\zeta}{2} y_t - \zeta z_{Y,t} + \frac{1}{2} \gamma s_t \right) y_t - (1-n) \left(\frac{\zeta}{2} y_t^* - \zeta z_{Y,t}^* - \frac{1}{2} \gamma^* s_t \right) y_t^* \\
& - \frac{1}{2} n \varepsilon (1 + \varepsilon \zeta) \frac{\theta^H}{(1 - \theta^H \beta)(1 - \theta^H)} \pi_{H,t}^2 \\
& - \frac{1}{2} (1-n) \varepsilon (1 + \varepsilon \zeta) \frac{\theta^F}{(1 - \theta^F \beta)(1 - \theta^F)} \pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.26}$$

Add and subtract $n \left(\frac{\sigma}{2} c_t - z_{C,t} \right) y_t$ and $(1-n) \left(\frac{\sigma}{2} c_t^* - z_{C,t}^* \right) y_t^*$ to obtain

$$\begin{aligned}
\mathbb{V}_t = & -n \left(\frac{\sigma}{2} c_t - z_{C,t} \right) (c_t - y_t) - (1-n) \left(\frac{\sigma}{2} c_t^* - z_{C,t}^* + q_t \right) (c_t^* - y_t^*) \\
& + \frac{1}{2} \eta n \gamma (2 - \gamma - \gamma^*) s_t^2 - \frac{1}{2} n \gamma (y_t - y_t^*) s_t \\
& - n \left(\frac{\sigma}{2} c_t - z_{C,t} + \frac{\zeta}{2} y_t - \zeta z_{Y,t} + \frac{1}{2} \gamma s_t \right) y_t \\
& - (1-n) \left(\frac{\sigma}{2} c_t^* - z_{C,t}^* + \frac{\zeta}{2} y_t^* - \zeta z_{Y,t}^* - \frac{1}{2} \gamma^* s_t \right) y_t^* \\
& - \frac{1}{2} n \varepsilon (1 + \varepsilon \zeta) \frac{\theta^H}{(1 - \theta^H \beta)(1 - \theta^H)} \pi_{H,t}^2 \\
& - \frac{1}{2} (1-n) \varepsilon (1 + \varepsilon \zeta) \frac{\theta^F}{(1 - \theta^F \beta)(1 - \theta^F)} \pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.27}$$

The term in parentheses in front of y_t and y_t^* in the third and fourth line, respectively,

can be related to the output gap and the terms of trade gap. Rearranging (B.2), which

also holds under sticky prices, yields

$$\gamma (z_{C,t} - z_{C,t}^*) = \sigma (c_t - y_t) + \gamma [(2 - \gamma - \gamma^*)(\sigma \eta - 1) + 1] s_t. \tag{C.28}$$

897 Inserting this into (B.4) yields

$$\sigma c_t - z_{C,t} + \zeta (y_t - z_{Y,t}) + \gamma s_t = (\sigma + \zeta) (y_t - \bar{y}_t) - \gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1) (s_t - \bar{s}_t). \quad (\text{C.29})$$

This equation can be rewritten as follows:

$$\begin{aligned} & \frac{\sigma}{2} c_t - z_{C,t} + \frac{\zeta}{2} y_t - \zeta z_{Y,t} + \frac{1}{2} \gamma s_t \\ &= (\sigma + \zeta) \left(\frac{1}{2} y_t - \bar{y}_t \right) - \gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1) \left(\frac{1}{2} s_t - \bar{s}_t \right) - \frac{1}{2} \gamma (z_{C,t} - z_{C,t}^*). \end{aligned} \quad (\text{C.30})$$

Repeating the same steps, the foreign analog to this equation is given by

$$\begin{aligned} & \frac{\sigma}{2} c_t^* - z_{C,t}^* + \frac{\zeta}{2} y_t^* - \zeta z_{Y,t}^* - \frac{1}{2} \gamma^* s_t \\ &= (\sigma + \zeta) \left(\frac{1}{2} y_t^* - \bar{y}_t^* \right) + \gamma^*(2 - \gamma - \gamma^*)(\sigma\eta - 1) \left(\frac{1}{2} s_t - \bar{s}_t \right) + \frac{1}{2} \gamma^* (z_{C,t} - z_{C,t}^*). \end{aligned} \quad (\text{C.31})$$

The left-hand side of the previous two equations corresponds to the terms in parentheses in front of y_t and y_t^* in (C.27). Accordingly, substitution yields

$$\begin{aligned} \mathbb{V}_t = & -n \left(\frac{\sigma}{2} c_t - z_{C,t} \right) (c_t - y_t) - (1 - n) \left(\frac{\sigma}{2} c_t^* - z_{C,t}^* + q_t \right) (c_t^* - y_t^*) \\ & - \frac{1}{2} n \gamma [s_t - (z_{C,t} - z_{C,t}^*)] (y_t - y_t^*) + \frac{1}{2} \eta n \gamma (2 - \gamma - \gamma^*) s_t^2 \\ & + n \gamma (2 - \gamma - \gamma^*)(\sigma\eta - 1) \left(\frac{1}{2} s_t - \bar{s}_t \right) (y_t - y_t^*) \\ & - n(\sigma + \zeta) \left(\frac{1}{2} y_t - \bar{y}_t \right) y_t - (1 - n)(\sigma + \zeta) \left(\frac{1}{2} y_t^* - \bar{y}_t^* \right) y_t^* \\ & - \frac{1}{2} n \varepsilon (1 + \varepsilon \zeta) \frac{\theta^H}{(1 - \theta^H \beta)(1 - \theta^H)} \pi_{H,t}^2 \\ & - \frac{1}{2} (1 - n) \varepsilon (1 + \varepsilon \zeta) \frac{\theta^F}{(1 - \theta^F \beta)(1 - \theta^F)} \pi_{F,t}^{*2} \\ & + t.i.p. + O(\|Z\|^3). \end{aligned} \quad (\text{C.32})$$

898 Combining the first-order approximation of the world resource constraint (C.22)

$$n c_t + (1 - n) c_t^* = n y_t + (1 - n) y_t^* \quad (\text{C.33})$$

899 with the risk-sharing condition

$$q_t = \sigma (c_t - c_t^*) + (z_{C,t}^* - z_{C,t}) \quad (\text{C.34})$$

900 to eliminate c_t^* yields

$$-(c_t - y_t) = (1 - n) \left[y_t - y_t^* - \frac{1}{\sigma} (q_t + z_{C,t} - z_{C,t}^*) \right]. \quad (\text{C.35})$$

901 The foreign analog is given by

$$(c_t^* - y_t^*) = n \left[y_t - y_t^* - \frac{1}{\sigma} (q_t + z_{C,t} - z_{C,t}^*) \right]. \quad (\text{C.36})$$

Inserting the previous two equations into (C.32) yields

$$\begin{aligned} \mathbb{V}_t = & n(1 - n) \left[\frac{\sigma}{2} (c_t - c_t^*) - q_t - (z_{C,t} - z_{C,t}^*) \right] \left[y_t - y_t^* - \frac{1}{\sigma} (q_t + z_{C,t} - z_{C,t}^*) \right] \\ & - \frac{1}{2} n \gamma [s_t - (z_{C,t} - z_{C,t}^*)] (y_t - y_t^*) + \frac{1}{2} \eta n \gamma (2 - \gamma - \gamma^*) s_t^2 \\ & + n \gamma (2 - \gamma - \gamma^*) (\sigma \eta - 1) \left(\frac{1}{2} s_t - \bar{s}_t \right) (y_t - y_t^*) \\ & - n(\sigma + \zeta) \left(\frac{1}{2} y_t - \bar{y}_t \right) y_t - (1 - n)(\sigma + \zeta) \left(\frac{1}{2} y_t^* - \bar{y}_t^* \right) y_t^* \\ & - \frac{1}{2} n \varepsilon (1 + \varepsilon \zeta) \frac{\theta^H}{(1 - \theta^H \beta)(1 - \theta^H)} \pi_{H,t}^2 \\ & - \frac{1}{2} (1 - n) \varepsilon (1 + \varepsilon \zeta) \frac{\theta^F}{(1 - \theta^F \beta)(1 - \theta^F)} \pi_{F,t}^{*2} \\ & + t.i.p. + O(\|Z\|^3). \end{aligned} \quad (\text{C.37})$$

Using the risk-sharing condition to eliminate the consumption differential $(c_t - c_t^*)$ and

rearranging the resulting first line yields

$$\begin{aligned}
\mathbb{V}_t = & -\frac{1}{2}n(1-n) (q_t + z_{C,t} - z_{C,t}^*) (y_t - y_t^*) + \frac{1}{2}n(1-n) \frac{1}{\sigma} (q_t + z_{C,t} - z_{C,t}^*)^2 \\
& - \frac{1}{2}n\gamma [s_t - (z_{C,t} - z_{C,t}^*)] (y_t - y_t^*) + \frac{1}{2}\eta n\gamma(2 - \gamma - \gamma^*) s_t^2 \\
& + n\gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1) \left(\frac{1}{2}s_t - \bar{s}_t \right) (y_t - y_t^*) \\
& - n(\sigma + \zeta) \left(\frac{1}{2}y_t - \bar{y}_t \right) y_t - (1-n)(\sigma + \zeta) \left(\frac{1}{2}y_t^* - \bar{y}_t^* \right) y_t^* \\
& - \frac{1}{2}n\varepsilon(1 + \varepsilon\zeta) \frac{\theta^H}{(1 - \theta^H\beta)(1 - \theta^H)} \pi_{H,t}^2 \\
& - \frac{1}{2}(1-n)\varepsilon(1 + \varepsilon\zeta) \frac{\theta^F}{(1 - \theta^F\beta)(1 - \theta^F)} \pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.38}$$

Collecting the terms in the output differential $(y_t - y_t^*)$ in the first two lines yields

$$\begin{aligned}
\mathbb{V}_t = & \frac{1}{2}n(1-n) \frac{1}{\sigma} (q_t + z_{C,t} - z_{C,t}^*)^2 + \frac{1}{2}\eta n\gamma(2 - \gamma - \gamma^*) s_t^2 \\
& - \frac{1}{2}n(1-n) \left[\left(1 - \frac{\gamma}{1-n} \right) (z_{C,t} - z_{C,t}^*) + s_t \right] (y_t - y_t^*) \\
& + n\gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1) \left(\frac{1}{2}s_t - \bar{s}_t \right) (y_t - y_t^*) \\
& - n(\sigma + \zeta) \left(\frac{1}{2}y_t - \bar{y}_t \right) y_t - (1-n)(\sigma + \zeta) \left(\frac{1}{2}y_t^* - \bar{y}_t^* \right) y_t^* \\
& - \frac{1}{2}n\varepsilon(1 + \varepsilon\zeta) \frac{\theta^H}{(1 - \theta^H\beta)(1 - \theta^H)} \pi_{H,t}^2 \\
& - \frac{1}{2}(1-n)\varepsilon(1 + \varepsilon\zeta) \frac{\theta^F}{(1 - \theta^F\beta)(1 - \theta^F)} \pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.39}$$

Using (B.6), which also holds under sticky prices, to eliminate the output differential

$(y_t - y_t^*)$ yields

$$\begin{aligned}
\mathbb{V}_t = & \frac{1}{2}n(1-n)\frac{1}{\sigma} (q_t + z_{C,t} - z_{C,t}^*)^2 - \frac{1}{2}n(1-n)\frac{1-\gamma-\gamma^*}{\sigma} (z_{C,t} - z_{C,t}^*) s_t \\
& - \frac{1}{2}n(1-n) \left(1 - \frac{\gamma}{1-n}\right) \frac{(\gamma + \gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2}{\sigma} (z_{C,t} - z_{C,t}^*) s_t \\
& + n\gamma(2-\gamma-\gamma^*)(\sigma\eta-1)\frac{1-\gamma-\gamma^*}{\sigma} (z_{C,t} - z_{C,t}^*) \left(\frac{1}{2}s_t - \bar{s}_t\right) \\
& + \frac{1}{2}n \left[\eta\gamma(2-\gamma-\gamma^*) - (1-n) \frac{(\gamma + \gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2}{\sigma} \right] s_t^2 \\
& + n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma} [(\gamma + \gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2] \left(\frac{1}{2}s_t - \bar{s}_t\right) s_t \\
& - n(\sigma + \zeta) \left(\frac{1}{2}y_t - \bar{y}_t\right) y_t - (1-n)(\sigma + \zeta) \left(\frac{1}{2}y_t^* - \bar{y}_t^*\right) y_t^* \\
& - \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\
& - \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.40}$$

Using $q_t = (1-\gamma-\gamma^*)s_t$ and collecting the terms in s_t^2 and $(z_{C,t} - z_{C,t}^*) s_t$, it is straightforward to show that the first four lines of the above expression are equal to $0 + t.i.p.$. As a result, the above expression simplifies to

$$\begin{aligned}
\mathbb{V}_t = & -n(\sigma + \zeta) \left(\frac{1}{2}y_t - \bar{y}_t\right) y_t - (1-n)(\sigma + \zeta) \left(\frac{1}{2}y_t^* - \bar{y}_t^*\right) y_t^* \\
& + n\gamma(2-\gamma-\gamma^*)\frac{\sigma\eta-1}{\sigma} [(\gamma + \gamma^*)(2-\gamma-\gamma^*)\sigma\eta + (1-\gamma-\gamma^*)^2] \left(\frac{1}{2}s_t - \bar{s}_t\right) s_t \\
& - \frac{1}{2}n\varepsilon(1+\varepsilon\zeta)\frac{\theta^H}{(1-\theta^H\beta)(1-\theta^H)}\pi_{H,t}^2 \\
& - \frac{1}{2}(1-n)\varepsilon(1+\varepsilon\zeta)\frac{\theta^F}{(1-\theta^F\beta)(1-\theta^F)}\pi_{F,t}^{*2} \\
& + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.41}$$

Note that for any given variable x_t

$$\begin{aligned}
\left(\frac{1}{2}x_t - \bar{x}_t\right)x_t &= \frac{1}{2}(x_t^2 - 2x_t\bar{x}_t) \\
&= \frac{1}{2}(x_t^2 - 2x_t\bar{x}_t + \bar{x}_t^2) - \frac{1}{2}\bar{x}_t^2 \\
&= \frac{1}{2}(x_t - \bar{x}_t)^2 + t.i.p.
\end{aligned} \tag{C.42}$$

Thus, the world welfare loss function can be rewritten in its final form:

$$\begin{aligned}
\mathbb{V}_t = & -\frac{1}{2} \left(n(\sigma + \zeta)(y_t - \bar{y}_t)^2 + (1-n)(\sigma + \zeta)(y_t^* - \bar{y}_t^*)^2 \right. \\
& - n\gamma(2 - \gamma - \gamma^*) \frac{\sigma\eta - 1}{\sigma} [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t)^2 \\
& + n\varepsilon(1 + \varepsilon\zeta) \frac{\theta^H}{(1 - \theta^H\beta)(1 - \theta^H)} \pi_{H,t}^2 \\
& \left. + (1-n)\varepsilon(1 + \varepsilon\zeta) \frac{\theta^F}{(1 - \theta^F\beta)(1 - \theta^F)} \pi_{F,t}^{*2} \right) \\
& + t.i.p. + O(\|Z\|^3).
\end{aligned} \tag{C.43}$$

D Derivation of targeting rules

In this section of the Appendix we specify the details of the optimal policy problem under each regime (flexible exchange rates and monetary union), distinguishing in each case between commitment and discretion.

D.1 Flexible exchange rates

Commitment Under commitment, the Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = & U_C C \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \left(n(\sigma + \zeta) (y_t - \bar{y}_t)^2 + (1-n)(\sigma + \zeta) (y_t^* - \bar{y}_t^*)^2 \right. \right. \\
& - n\gamma(2 - \gamma - \gamma^*) \frac{\sigma\eta - 1}{\sigma} [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t)^2 \\
& + n \frac{\varepsilon}{\kappa} \pi_{H,t}^2 + (1-n) \frac{\varepsilon}{\kappa^*} \pi_{F,t}^{*2} \Big) \\
& + n\varphi_{H,t} \left(\pi_{H,t} - (\sigma + \zeta)\kappa (y_t - \bar{y}_t) + \gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1)\kappa (s_t - \bar{s}_t) \right. \\
& \left. - \kappa\mu_{H,t} - \beta\mathbb{E}_t\pi_{H,t+1} \right) \\
& + (1-n)\varphi_{F,t}^* \left(\pi_{F,t}^* - (\sigma + \zeta)\kappa^* (y_t^* - \bar{y}_t^*) - \gamma^*(2 - \gamma - \gamma^*)(\sigma\eta - 1)\kappa^* (s_t - \bar{s}_t) \right. \\
& \left. - \kappa^*\mu_{F,t} - \beta\mathbb{E}_t\pi_{F,t+1}^* \right) \\
& \left. + \vartheta_t \left([(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t) - \sigma [(y_t - \bar{y}_t) - (y_t^* - \bar{y}_t^*)] \right) \right], \tag{D.1}
\end{aligned}$$

where $\varphi_{H,t}$, $\varphi_{F,t}^*$, and ϑ_t are the respective Lagrange multipliers. The first-order conditions with respect to $\pi_{H,t}$, $\pi_{F,t}^*$, y_t , y_t^* , and s_t are given by

$$0 = -\frac{\varepsilon}{\kappa} \pi_{H,t} + \varphi_{H,t} - \varphi_{H,t-1} \tag{D.2}$$

$$0 = -\frac{\varepsilon}{\kappa^*} \pi_{F,t}^* + \varphi_{F,t}^* - \varphi_{F,t-1}^* \tag{D.3}$$

$$0 = -n(\sigma + \zeta) (y_t - \bar{y}_t) - n(\sigma + \zeta)\kappa\varphi_{H,t} - \sigma\vartheta_t \tag{D.4}$$

$$0 = -(1-n)(\sigma + \zeta) (y_t^* - \bar{y}_t^*) - (1-n)(\sigma + \zeta)\kappa^*\varphi_{F,t}^* + \sigma\vartheta_t \tag{D.5}$$

$$\begin{aligned}
0 = & n\gamma(2 - \gamma - \gamma^*) \frac{\sigma\eta - 1}{\sigma} [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t) \\
& + n\gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1)(\kappa\varphi_{H,t} - \kappa^*\varphi_{F,t}^*) \\
& + [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] \vartheta_t. \tag{D.6}
\end{aligned}$$

Combining the first-order conditions to eliminate the Lagrange multipliers and using the gap version of (29) to eliminate the terms-of-trade gap yields the following two targeting rules (see also Corsetti, Dedola, and Leduc 2011):

$$0 = (y_t - \bar{y}_t) - (y_{t-1} - \bar{y}_{t-1}) + \varepsilon \pi_{H,t} \quad (\text{D.7})$$

$$0 = (y_t^* - \bar{y}_t^*) - (y_{t-1}^* - \bar{y}_{t-1}^*) + \varepsilon \pi_{F,t}^*. \quad (\text{D.8})$$

Discretion Under discretion, the Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \left(n(\sigma + \zeta) (y_t - \bar{y}_t)^2 + (1-n)(\sigma + \zeta) (y_t^* - \bar{y}_t^*)^2 \right. \\ & - n\gamma(2 - \gamma - \gamma^*) \frac{\sigma\eta - 1}{\sigma} [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t)^2 \\ & \left. + n \frac{\varepsilon}{\kappa} \pi_{H,t}^2 + (1-n) \frac{\varepsilon}{\kappa^*} \pi_{F,t}^{*2} \right) \\ & + n\varphi_{H,t} \left(\pi_{H,t} - (\sigma + \zeta)\kappa (y_t - \bar{y}_t) + \gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1)\kappa (s_t - \bar{s}_t) - f_t \right) \\ & + (1-n)\varphi_{F,t}^* \left(\pi_{F,t}^* - (\sigma + \zeta)\kappa^* (y_t^* - \bar{y}_t^*) - \gamma^*(2 - \gamma - \gamma^*)(\sigma\eta - 1)\kappa^* (s_t - \bar{s}_t) - f_t^* \right) \\ & + \vartheta_t \left([(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t) - \sigma [(y_t - \bar{y}_t) - (y_t^* - \bar{y}_t^*)] \right), \end{aligned} \quad (\text{D.9})$$

where $\varphi_{H,t}$, $\varphi_{F,t}^*$, and ϑ_t are the respective Lagrange multipliers and

$$f_t \equiv \kappa \mu_{H,t} + \beta \mathbb{E}_t \pi_{H,t+1} \quad (\text{D.10})$$

$$f_t^* \equiv \kappa^* \mu_{F,t} + \beta \mathbb{E}_t \pi_{F,t+1}^*. \quad (\text{D.11})$$

The first-order conditions with respect to $\pi_{H,t}$, $\pi_{F,t}^*$, y_t , y_t^* , and s_t are given by

$$0 = -\frac{\varepsilon}{\kappa}\pi_{H,t} + \varphi_{H,t} \quad (\text{D.12})$$

$$0 = -\frac{\varepsilon}{\kappa^*}\pi_{F,t}^* + \varphi_{F,t}^* \quad (\text{D.13})$$

$$0 = -n(\sigma + \zeta)(y_t - \bar{y}_t) - n(\sigma + \zeta)\kappa\varphi_{H,t} - \sigma\vartheta_t \quad (\text{D.14})$$

$$0 = -(1-n)(\sigma + \zeta)(y_t^* - \bar{y}_t^*) - (1-n)(\sigma + \zeta)\kappa^*\varphi_{F,t}^* + \sigma\vartheta_t \quad (\text{D.15})$$

$$\begin{aligned} 0 = & n\gamma(2 - \gamma - \gamma^*)\frac{\sigma\eta - 1}{\sigma} [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t) \\ & + n\gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1)(\kappa\varphi_{H,t} - \kappa^*\varphi_{F,t}^*) \\ & + [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] \vartheta_t. \end{aligned} \quad (\text{D.16})$$

Combining the first-order conditions to eliminate the Lagrange multipliers and using the gap version of (29) to eliminate the terms-of-trade gap yields the following two targeting rules:

$$0 = (y_t - \bar{y}_t) + \varepsilon\pi_{H,t} \quad (\text{D.17})$$

$$0 = (y_t^* - \bar{y}_t^*) + \varepsilon\pi_{F,t}^*. \quad (\text{D.18})$$

D.2 Monetary union

Commitment Under commitment, the Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = & U_C C \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \left(n(\sigma + \zeta) (y_t - \bar{y}_t)^2 + (1-n)(\sigma + \zeta) (y_t^* - \bar{y}_t^*)^2 \right. \right. \\
& - n\gamma(2 - \gamma - \gamma^*) \frac{\sigma\eta - 1}{\sigma} [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t)^2 \\
& + n \frac{\varepsilon}{\kappa} \pi_{H,t}^2 + (1-n) \frac{\varepsilon}{\kappa^*} \pi_{F,t}^{*2} \Big) \\
& + n\varphi_{H,t} \left(\pi_{H,t} - (\sigma + \zeta)\kappa (y_t - \bar{y}_t) + \gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1)\kappa (s_t - \bar{s}_t) \right. \\
& \left. \left. - \kappa\mu_{H,t} - \beta\mathbb{E}_t\pi_{H,t+1} \right) \right. \\
& + (1-n)\varphi_{F,t}^* \left(\pi_{F,t}^* - (\sigma + \zeta)\kappa^* (y_t^* - \bar{y}_t^*) - \gamma^*(2 - \gamma - \gamma^*)(\sigma\eta - 1)\kappa^* (s_t - \bar{s}_t) \right. \\
& \left. \left. - \kappa^*\mu_{F,t} - \beta\mathbb{E}_t\pi_{F,t+1}^* \right) \right. \\
& + \vartheta_t \left([(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t) - \sigma [(y_t - \bar{y}_t) - (y_t^* - \bar{y}_t^*)] \right) \\
& \left. + \psi_t \left(s_t - s_{t-1} - \pi_{F,t}^* + \pi_{H,t} \right) \right], \tag{D.19}
\end{aligned}$$

where $\varphi_{H,t}$, $\varphi_{F,t}^*$, ϑ_t , and ψ_t are the respective Lagrange multipliers. The first-order conditions with respect to $\pi_{H,t}$, $\pi_{F,t}^*$, y_t , y_t^* , and s_t are given by

$$0 = -n \frac{\varepsilon}{\kappa} \pi_{H,t} + n \varphi_{H,t} - n \varphi_{H,t-1} + \psi_t \quad (\text{D.20})$$

$$0 = - (1-n) \frac{\varepsilon}{\kappa^*} \pi_{F,t}^* + (1-n) \varphi_{F,t}^* - (1-n) \varphi_{F,t-1}^* - \psi_t \quad (\text{D.21})$$

$$0 = -n(\sigma + \zeta)(y_t - \bar{y}_t) - n(\sigma + \zeta)\kappa \varphi_{H,t} - \sigma \vartheta_t \quad (\text{D.22})$$

$$0 = - (1-n)(\sigma + \zeta)(y_t^* - \bar{y}_t^*) - (1-n)(\sigma + \zeta)\kappa^* \varphi_{F,t}^* + \sigma \vartheta_t \quad (\text{D.23})$$

$$\begin{aligned} 0 = & n\gamma(2 - \gamma - \gamma^*) \frac{\sigma\eta - 1}{\sigma} [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t) \\ & + n\gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1)(\kappa \varphi_{H,t} - \kappa^* \varphi_{F,t}^*) \\ & + [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] \vartheta_t \\ & + \psi_t - \beta \psi_{t+1}. \end{aligned} \quad (\text{D.24})$$

In the special case in which the degree of price stickiness is equal across countries ($\kappa = \kappa^*$), combining the first-order conditions to eliminate the Lagrange multipliers yields the following targeting rule:

$$\begin{aligned} 0 = & n [(y_t - \bar{y}_t) - (y_{t-1} - \bar{y}_{t-1})] + (1-n) [(y_t^* - \bar{y}_t^*) - (y_{t-1}^* - \bar{y}_{t-1}^*)] \\ & + \varepsilon (n\pi_{H,t} + (1-n)\pi_{F,t}^*). \end{aligned} \quad (\text{D.25})$$

Discretion Under discretion, and in the special case in which the degree of price stickiness is equal across countries ($\kappa = \kappa^*$), the terms of trade are independent of monetary policy (recall Proposition 2 in the main text). Therefore, the optimal policy problem under discretion can be written in terms of a less constrained problem, which can abstract from the terms-of-trade identity (50), and thus does not feature any endogenous

state variable. As a result, the Lagrangian can be written as follows:

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2} \left(n(\sigma + \zeta) (y_t - \bar{y}_t)^2 + (1-n)(\sigma + \zeta) (y_t^* - \bar{y}_t^*)^2 \right. \\
& - n\gamma(2 - \gamma - \gamma^*) \frac{\sigma\eta - 1}{\sigma} [(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t)^2 \\
& \left. + n\frac{\varepsilon}{\kappa}\pi_{H,t}^2 + (1-n)\frac{\varepsilon}{\kappa^*}\pi_{F,t}^{*2} \right) \\
& + n\varphi_{H,t} \left(\pi_{H,t} - (\sigma + \zeta)\kappa(y_t - \bar{y}_t) + \gamma(2 - \gamma - \gamma^*)(\sigma\eta - 1)\kappa(s_t - \bar{s}_t) - f_t \right) \\
& + (1-n)\varphi_{F,t}^* \left(\pi_{F,t}^* - (\sigma + \zeta)\kappa^*(y_t^* - \bar{y}_t^*) - \gamma^*(2 - \gamma - \gamma^*)(\sigma\eta - 1)\kappa^*(s_t - \bar{s}_t) - f_t^* \right) \\
& + \vartheta_t \left([(\gamma + \gamma^*)(2 - \gamma - \gamma^*)\sigma\eta + (1 - \gamma - \gamma^*)^2] (s_t - \bar{s}_t) - \sigma[(y_t - \bar{y}_t) - (y_t^* - \bar{y}_t^*)] \right),
\end{aligned} \tag{D.26}$$

where $\varphi_{H,t}$, $\varphi_{F,t}^*$, and ϑ_t are the respective Lagrange multipliers, and f_t and f_t^* are terms which are taken as given (as specified in the main text). The first-order conditions with respect to $\pi_{H,t}$, $\pi_{F,t}^*$, y_t , and y_t^* are given by

$$0 = -n\frac{\varepsilon}{\kappa}\pi_{H,t} + n\varphi_{H,t} + \psi_t \tag{D.27}$$

$$0 = -(1-n)\frac{\varepsilon}{\kappa^*}\pi_{F,t}^* + (1-n)\varphi_{F,t}^* - \psi_t \tag{D.28}$$

$$0 = -n(\sigma + \zeta)(y_t - \bar{y}_t) - n(\sigma + \zeta)\kappa\varphi_{H,t} - \sigma\vartheta_t \tag{D.29}$$

$$0 = -(1-n)(\sigma + \zeta)(y_t^* - \bar{y}_t^*) - (1-n)(\sigma + \zeta)\kappa^*\varphi_{F,t}^* + \sigma\vartheta_t. \tag{D.30}$$

908 Note that we do not derive the first-order condition with respect to s_t as this variable can-
909 not be influenced by monetary policy. Combining the first-order conditions to eliminate
910 the Lagrange multipliers yields the following targeting rule:

$$0 = n(y_t - \bar{y}_t) + (1-n)(y_t^* - \bar{y}_t^*) + \varepsilon \left(n\pi_{H,t} + (1-n)\pi_{F,t}^* \right). \tag{D.31}$$