

A Service of

ZBW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Ponthiere, Gregory

Working Paper Higher Education Subsidies and the Universal Insurance against a Short Life

GLO Discussion Paper, No. 1528

Provided in Cooperation with: Global Labor Organization (GLO)

Suggested Citation: Ponthiere, Gregory (2024) : Higher Education Subsidies and the Universal Insurance against a Short Life, GLO Discussion Paper, No. 1528, Global Labor Organization (GLO), Essen

This Version is available at: https://hdl.handle.net/10419/306681

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



WWW.ECONSTOR.EU

Higher Education Subsidies and the Universal Insurance against a Short Life

Gregory Ponthiere*

November 25, 2024

Abstract

This paper examines the potential role of higher education subsidies as an insurance device against the risk of having a short life, that is, as a device reducing the variance in lifetime well-being due to unequal longevities. We use a two-period dynamic OLG economy with human capital and risky lifetime to study the impact of a subsidy on higher education (financed by taxing labor earnings at older ages) on the distribution of lifetime well-being between long-lived and short-lived individuals. It is shown that, whereas the subsidy on higher education improves necessarily the lot of short-lived individuals in comparison to the laissez-faire, it is only when the subsidy is higher than a critical threshold that this reduces inequalities in lifetime well-being between long-lived and short-lived individuals. Whether one adopts the utilitarian or the ex post egalitarian social welfare function, the optimal subsidy on higher education lies above the critical threshold, but is larger under the latter social objective.

Keywords: higher education, mortality risk, insurance, longevity, human capital.

JEL classification codes: I25, I28, I31, J17.

^{*}Ecole normale supérieure de Rennes (ENS-Rennes), CREM (UMR 6211) and Global Labor Organization (GLO). Address: ENS-Rennes, Campus de Ker Lann, 11 avenue Robert Schumann, 35170 Bruz, France. E-mail: gregory.ponthiere@ens-rennes.fr

1 Introduction

In the recent decades, economists have paid much attention to the relation between mortality and education. In particular, economists have, following the pioneer work of Ben-Porath (1967), studied the effects of life expectancy on education choices. The so-called Ben-Porath effect states that, when life expectancy goes up, lifetime returns from education increase, which makes individuals invest more in education. This horizon effect has become a key ingredient in theoretical models studying interactions between economic development and demography, as in Boucekkine et al (2002, 2003), Blackburn and Cipriani (2002), Chakraborty (2004), Cervelatti and Sunde (2005, 2011, 2013), Soares (2005), Lorentzen et al (2008), de la Croix (2009), Ludwig and Vogel (2010) and de la Croix and Licandro (2013).¹ Econometric analyses have produced mixed results concerning the empirical validity of the Ben-Porath hypothesis.²

While the Ben-Porath effect has attracted much attention, this particular channel does not exhaust all relevant relations between mortality and education. Another key issue is the potential role of education as a determinant of lifetime well-being inequalities between individuals having unequal longevities. Do investments in education act as an insurance device reducing well-being volatility due to unequal longevities, or, on the contrary, do they exacerbate welfare losses due to a premature death? The answer is not independent from the Ben-Porath effect: it depends on the extent to which lifetime returns from education are higher for individuals who live longer. However, examining the effect of education choices on inequalities in lifetime well-being between short-lived and long-lived persons requires a change of perspective: while the study of the Ben-Porath effect relies on the ex ante perspective of a person who ignores how long she will live, the study of the distributional implications of education choices under unequal longevities requires to adopt an ex post view, that is, to consider lives once these are complete.

Studying the relation between education and inequalities in lifetime wellbeing between long-lived and short-lived individuals matters from the perspective of constructing a new "pilar" of the Welfare State: *a universal insurance against the risk of having a short life.* Given the high and persistent risk about the age at death, and given the substantial welfare losses due to a premature

¹Other theoretical papers studying the Ben-Porath effect include Leker and Ponthiere (2015), Nishimura et al (2018) and Leukhina and Vandenbroucke (2022).

²Bloom et al (2004) showed that a 5-year increase in life expectancy generates a 21 % rise of the GDP growth rate. Similar results were found in Bloom and Sachs (1998), Bloom and Williamson (1998) and Gallup and Sachs (2001). On the contrary, Hazan (2009) argued that the Ben-Porath effect holds only if there is a rise in the number of years of occupation, which was not observed. Accemoglu and Johnson (2007) found, using an instrumental variable method, that life expectancy does not affect GDP growth. That study was criticized by de la Croix et al (2009) who find that longevity increases account for 20 % of the rise in education over the last two centuries in Sweden, and by Cervelatti and Sunde (2011), who find evidence that life expectancy increases GDP growth in countries having accomplished their fertility transition. Acemoglu and Johnson's approach was also criticized by Bloom et al (2014) on the ground that they neglect the impact of initial conditions. Using long-run cohort data, Cohen and Leker (2016) also find support for the Ben-Porath effect.

death, basic concerns for social justice justify the construction of this new pilar of social insurance, in order to minimize the well-being deprivation faced by the unlucky short-lived persons. Due to the difficulty to compensate victims of a premature death *ex post* (that is, once individual longevities are known), such a policy must take place *ex ante* and rely on age-based statistical discrimination favoring all the young (Fleurbaey et al 2014). The idea is to reallocate "good things of life" at young ages, so as to improve the situation of all young persons, including the one of the (unidentified) persons who will die prematurely. Statistical discrimination strategies include taxing savings so as to encourage consumption early in life (Fleurbaey et al 2014), postponing the final retirement age so as to transfer more resources to the young (Fleurbaey et al 2016), or providing a period of retirement to all young adults (Ponthiere 2020, 2023).

The goal of this paper is to examine the potential role of education policies as an insurance device against the risk of having a short life, that is, as a device that reduces the variance in lifetime well-being due to unequal longevities. In particular, we will study the implications of a subsidy on higher education on the distribution of well-being between long-lived and short-lived persons. Can a subsidy on higher education reduce the welfare loss due to a premature death? Under which conditions can this serve as an insurance against a short life?

These questions cannot be answered *a priori*, because higher education affects well-being inequalities between the long-lived and the short-lived through various channels. The crux of the problem lies in the mixed nature of higher education. On the one hand, higher education is an *investment good*, that is, a kind of effort made at the young age in order to improve one's situation at older ages of life. Unfortunately, a premature death prevents the person from benefiting from the positive effects of that investment. Therefore, if higher education is taken as a pure investment good, encouraging it is regressive from the perspective of lifetime well-being inequalities due to unequal longevities. Subsidizing higher education encourages a "forward-looking" investment (like savings), which makes short-lived persons sacrifice their limited lifetime for the sake of a future that they will not enjoy.³ On the other hand, higher education is also valuable in itself, i.e., a *consumption good*. From that perspective, encouraging higher education may have opposite effects on well-being inequalities between the long-lived and the short-lived. Indeed, higher education usually takes place when adults are young. Hence, if the compensation for a premature death requires to increase consumption opportunities at the young age, subsidizing higher education (by taxing labor earnings at older ages) may serve as an insurance device against the risk of having a short life.

Higher education being both a consumption good and an investment good, it is hard to see *a priori* to what extent it should be encouraged as an insurantial device reducing the variance of lifetime well-being due to unequal lifetime. The goal of this paper is to examine the relation between higher education subsidies and inequalities in lifetime well-being between long-lived and short-lived indi-

 $^{^3 \}rm See$ Fleurbaey et al (2014) on the detrimental effect of savings on the realized lifetime well-being of short-lived persons.

viduals. To what extent does a higher education subsidy increase or decrease the well-being loss due to a premature death? What would be the optimal level of the subsidy on higher education under unequal lifetimes?

In order to answer these questions, the present paper studies a two-period dynamic OLG model with human capital and risky lifetime. In that Ben-Porath economy, individuals decide how much time they invest in higher education at the young age, without knowing how long they will live. We study how a subsidy on higher education would affect education choices, and, then, inequalities in realized lifetime well-being between long-lived and short-lived persons. That policy is financed by taxing labor earnings of older adults.

Anticipating on our results, we first show that, whereas the subsidy on higher education improves necessarily the lot of short-lived individuals in comparison to the stationary equilibrium prevailing at the laissez-faire, it is only when the subsidy is higher than a critical threshold that this reduces inequalities in lifetime well-being between long-lived and short-lived individuals at the stationary equilibrium. This critical threshold is shown to depend on the structure of preferences, and to be decreasing in the pure taste for higher education, and increasing in the pure taste for leisure time. On the normative side, we show that, whether one adopts the utilitarian social welfare function or the ex post egalitarian social welfare function, the optimal subsidy on higher education lies above the critical threshold, but is larger under the latter social objective.

As such, this paper highlights the somewhat mixed role of higher education subsidies as an insurance against the risk of having a short life: when these subsidies are low, they contribute not to weaken, but to *exacerbate* inequalities in realized lifetime well-being between long-lived and short-lived persons. It is only when the subsidy on higher education is sufficiently large that it can serve as an insurance device against the risk of having a short life. Interestingly, what we mean by "sufficiently large" depends on the structure of preferences, that is, the extent to which higher education is regarded also as a consumption good. When higher education is purely instrumental (i.e., a pure investment good), the critical threshold is larger, which makes it more difficult for educational policies to act as an insurance device against the risk of having a short life. On the contrary, when higher education is enjoyed by the young for its own sake, the critical threshold is lower, and then even low subsidy rates can achieve to reduce inequalities in lifetime well-being between the long-lived and the short-lived.

This paper lies at the intersection of three literatures. First, it contributes to the literature dedicated to the theoretical study of Ben-Porath economies, such as Boucekkine et al (2002, 2003), Blackburn and Cipriani (2002), Chakraborty (2004), Cervelatti and Sunde (2005, 2011), Soares (2005), Lorentzen et al (2008), de la Croix (2009), Ludwig and Vogel (2010), de la Croix and Licandro (2013), Leker and Ponthiere (2015), Nishimura et al (2018) and Leukhina and Vandenbroucke (2022). Our specific contribution to that literature is a shift of perspective: instead of considering, from an *ex ante* perspective, the relation between survival conditions and education choices, this paper examines how education choices affect inequalities in realized (*ex post*) lifetime well-being under unequal lifetimes. Second, the paper is also linked to the literature examining the distributional effects of higher education subsidies. That literature studied, for instance, how higher education subsidies transfer resources from unskilled workers to skilled workers (Hansen and Weisbrod 1969, Fernandez and Rogerson 1995, Garcia-Penalosa and Walde 2000). It also studied the effects on higher education policies on intergenerational inequalities (Barham et al 1995, Fender and Wang 2003, Abbott et al 2019). Here the contribution of the present paper is to consider other distributional effects of higher education policies, concerning well-being inequalities between long-lived and short-lived persons. Third, the present paper is also linked to the literature dedicated to the construction of a universal insurance scheme against the risk of a short life, such as Fleurbaey et al (2014, 2016) and Ponthiere (2020, 2023, 2024). Our contribution is here to examine the conditions under which a particular policy instrument, i.e., a subsidy on higher education, could serve as an insurance device reducing the variance of lifetime well-being due to mortality risk.

The remaining of this paper is organized as follows. Section 2 presents the model and examines the existence, uniqueness and stability of the stationary equilibrium at the laissez-faire. Then, Section 3 studies how the introduction of a subsidy on higher education affects individual education choices, as well as the long-run dynamics. Section 4 examines the implications of the subsidy on higher education for the size of inequalities in realized lifetime well-being between long-lived and short-lived persons. Then, Section 5 studies the optimal subsidy on higher education under the utilitarian and the expost egalitarian social welfare functions. Concluding remarks are left to Section 6.

2 The model

Let us consider a two-period overlapping generations economy. Time goes from t = 0 to $t = +\infty$. Each period of time has a length normalized to unity.⁴

The first period is young adulthood, during which each individual in (young adult) cohort t invests e_t units of time in higher education ($e_t \in [0, 1]$), supplies ℓ_t^y units of time in labor ($\ell_t^y \in [0, 1]$), enjoys free time during the remaining time $1 - e_t - \ell_t^y$ and has one child.⁵ The superscript y refers to young adults.

The second period is mature adulthood, during which each individual in (young adult) cohort t supplies ℓ_{t+1}^m units of time in labor ($\ell_{t+1}^m \in [0, 1]$) and enjoys free time during the remaining time $1 - \ell_{t+1}^m$. The superscript m refers to mature adults. The second period is reached with a probability $\pi \in [0, 1[$, which is here taken to be exogenous.⁶

⁴As usual in the OLG literature, the childhood period is left implicit here.

⁵For simplicity, there is no savings: individuals consume what they produce in each period (net of education costs). This assumption is standard in dynamic models of human capital accumulation (see Blackburn and Cipriani 2002).

⁶Note that one could endogenize π by making it a function of human capital $\pi_t = \pi (h_t)$, as in Blackburn and Cipriani (2002), Cervelatti and Sunde (2005, 2011) and Leker and Ponthiere (2015). But since the goal of this paper is not to study interactions between demographic and economic dynamics, we prefer to keep π as a parameter, for the sake of simplicity.

Production Output is obtained by means of labor and human capital. For simplicity, the production function is linear:

$$y_t^i = Ah_t \ell_t^i \text{ for } i \in \{y, m\}$$

$$\tag{1}$$

where A > 0 is a productivity parameter, while h_t is the level of human capital prevailing at time t.

Human capital accumulation Each young adult at period t inherits a level of human capital stock from his parent. He is thus endowed with h_t units of human capital, which are the outcome of human capital accumulation through past cohorts' education efforts. Human capital accumulates over time according to the following dynamic law:

$$h_{t+1} = Bh_t^{1-\sigma} e_t^{\sigma} \tag{2}$$

where B > 0 is a technical accumulation parameter, while $\sigma > 0$ is the elasticity of human capital in t + 1 with respect to the higher education at time t.

Preferences Preferences take a log-linear form. The expected lifetime well-being of an individual in cohort t is:

$$\log\left(c_{t}^{y}\right) + \alpha\log\left(e_{t}\right) + \gamma\log\left(\tilde{\ell}_{t}^{y}\right) + \pi\beta\left[\log\left(c_{t+1}^{m}\right) + \gamma\log\left(\tilde{\ell}_{t+1}^{m}\right)\right] + (1-\pi)\Omega \quad (3)$$

where c_t^y is consumption at the young age for a young adult at time t, while c_{t+1}^m is consumption at the mature age for a mature adult at period t+1. $\tilde{\ell}_t^y$ is the leisure time at young adulthood, whereas $\tilde{\ell}_{t+1}^m$ is the leisure time at mature adulthood. The parameter $\alpha \geq 0$ captures the utility of higher education as a consumption good. The parameter β reflects the individual's time preferences. The parameter $\gamma > 0$ captures the utility of leisure time.

The parameter $\Omega \leq 0$ accounts for the pure disutility of a premature death, that is, the utility loss due to an early death independently from the opportunity cost of dying early (i.e., the utility of lost consumption and leisure). If the badness of an early death is reduced to the opportunity cost of dying early, in line with the Deprivation Account (Nagel 1979), then $\Omega = 0$. However, if the badness of a premature death cannot be reduced to the deprivation it causes, for instance because of Insult factors (Kamm 1993), then $\Omega < 0$. We assume that the pure disutility of a premature death is finite: $\Omega > -\infty$.

Constraints At the young age, individuals obtain some earnings from labor, but must pay a unit cost $\mu > 0$ for each unit of higher education e_t . Consumption at young age is given by:

$$c_t^y = y_t^y - \mu e_t = Ah_t \ell_t^y - \mu e_t \tag{4}$$

The time constraint at the young age is:

$$1 = \ell_t^y + \tilde{\ell}_t^y + e_t \implies \ell_t^y = 1 - \tilde{\ell}_t^y - e_t \tag{5}$$

At the mature age, individuals obtain earnings from their labor, which depend on past education investment. Consumption at the mature age is:

$$c_{t+1}^m = y_{t+1}^m = Ah_{t+1}\ell_{t+1}^m = A\underbrace{Bh_t^{1-\sigma}e_t^{\sigma}}_{h_{t+1}}\ell_{t+1}^m \tag{6}$$

By investing in higher education, the young individual increases his future productivity, and, hence, his future consumption.

The time constraint at the mature age is:

$$1 = \ell_{t+1}^m + \hat{\ell}_{t+1}^m \implies \ell_{t+1}^m = 1 - \hat{\ell}_{t+1}^m \tag{7}$$

Temporary equilibrium Individuals in the cohort t choose higher education e_t , young-age leisure $\tilde{\ell}_t^y$ and mature-age leisure $\tilde{\ell}_{t+1}^m$ to maximize expected lifetime well-being subject to budget constraints and time constraints, and subject to the accumulation law for human capital:

$$\max_{e_t, \tilde{\ell}_t^y, \tilde{\ell}_{t+1}^m} \quad \log\left(Ah_t(1 - e_t - \tilde{\ell}_t^y) - \mu e_t\right) + \alpha \log\left(e_t\right) + \gamma \log\left(\tilde{\ell}_t^y\right) \\ + \pi \beta \left[\log\left(ABh_t^{1 - \sigma} e_t^{\sigma}(1 - \tilde{\ell}_{t+1}^m)\right) + \gamma \log\left(\tilde{\ell}_{t+1}^m\right)\right] + (1 - \pi)\Omega$$

The first-order condition (FOC) for an interior optimal education e_t is:

$$\frac{\alpha}{e_t} + \frac{\pi\beta\sigma}{e_t} = \frac{Ah_t + \mu}{Ah_t(1 - e_t - \tilde{\ell}_t^y) - \mu e_t}$$
(8)

The left-hand-side (LHS) is the marginal utility gain from higher education. The first term of the LHS is the marginal utility gain from higher education taken as a consumption good, whereas the second term of the LHS is the marginal utility gain from higher consumption at the mature age thanks to more education (education being here taken as an investment good). That second effect depends on the survival probability π . This is the well-known Ben-Porath effect (Ben Porath 1967): better survival conditions contribute, by expanding the life horizon of the person, to encourage education. The right-hand-side (RHS) is the marginal utility loss from reduced consumption at the young age because of time spent on higher education. The cost of education includes two components: the opportunity cost of not working and the monetary cost of education.

The FOC for interior optimal leisure at the young age $\hat{\ell}_t^y$ is:

$$\frac{Ah_t}{Ah_t(1-e_t-\tilde{\ell}_t^y)-\mu e_t} = \frac{\gamma}{\tilde{\ell}_t^y} \tag{9}$$

The LHS is the marginal utility loss from less consumption at the young age due to less labor. The RHS is the marginal utility gain thanks to more free time.

The FOC for interior optimal leisure at the mature age $\tilde{\ell}_{t+1}^m$ is:

$$\frac{1}{1 - \tilde{\ell}_{t+1}^m} = \frac{\gamma}{\tilde{\ell}_{t+1}^m} \implies \tilde{\ell}_{t+1}^m = \frac{\gamma}{1 + \gamma}$$
(10)

Lemma 1 At the temporary equilibrium at period t, higher education, youngage leisure, young-age labor, mature-age leisure and mature-age labor are:

$$\begin{split} e_t &= \frac{\left(\alpha + \pi\beta\sigma\right)Ah_t}{\left(1 + \gamma + \alpha + \pi\beta\sigma\right)\left(Ah_t + \mu\right)} < 1,\\ \tilde{\ell}^y_t &= \frac{\gamma}{1 + \gamma + \alpha + \pi\beta\sigma} < 1,\\ \ell^y_t &= \frac{\left(1 + \alpha + \pi\beta\sigma\right)\mu + Ah_t}{\left(1 + \gamma + \alpha + \pi\beta\sigma\right)\left(Ah_t + \mu\right)} < 1,\\ \tilde{\ell}^m_t &= \frac{\gamma}{1 + \gamma} < 1 \text{ and } \ell^m_t = \frac{1}{1 + \gamma} < 1. \end{split}$$

Proof. See the Appendix. \blacksquare

Higher education is increasing in the level of human capital h_t . It is also increasing in the survival probability π , in line with the Ben-Porath effect, and increasing in preference parameters α and β . However, education is decreasing in the unit cost μ , and in the pure preference for leisure γ .

Stationary equilibrium Substituting for optimal higher education in the equation for human capital accumulation, we obtain:

$$h_{t+1} = \frac{B\Theta h_t}{(Ah_t + \mu)^{\sigma}} \equiv G(h_t) \tag{11}$$

where $\Theta \equiv \left[\frac{(\alpha+\pi\beta\sigma)A}{(1+\gamma+\alpha+\pi\beta\sigma)}\right]^{\sigma} > 0$. Studying the properties of the transition function $G(h_t)$ leads us to the following results.

Proposition 1 If $\mu \geq (B\Theta)^{\frac{1}{\sigma}}$, the laissez-faire economy admits a unique locally stable stationary equilibrium $\bar{h}^1 = 0$. If $\mu < (B\Theta)^{\frac{1}{\sigma}}$, the laissez-faire economy admits two stationary equilibria $\bar{h}^1 = 0$ and $\bar{h}^2 = \frac{(B\Theta)^{\frac{1}{\sigma}} - \mu}{A}$. The stationary equilibrium \bar{h}^1 is unstable, and the stationary equilibrium \bar{h}^2 is locally stable.

Proof. See the Appendix.

The condition $\mu < (B\Theta)^{\frac{1}{\sigma}}$ can be interpreted as a requirement that the cost of higher education is not too large, or, alternatively, as a requirement that the accumulation of human capital is sufficiently strong. When that condition is not satisfied, the economy is inevitably trapped in extreme poverty, and any attempt of economic take-off will be unsuccessful. The reason is that the transition function $G(h_t)$ is then always below the 45° line, so that the economy is mechanically attracted towards $\bar{h}^1 = 0$. That case is illustrated on Figure 1a. On the contrary, when $\mu < (B\Theta)^{\frac{1}{\sigma}}$, the transition function $G(h_t)$ first starts above the 45° line, and, then, falls below it. Hence the economy will, from any strictly positive initial human capital stock $h_0 > 0$, converge asymptotically towards the stationary equilibrium \bar{h}^2 . The level of \bar{h}^2 is decreasing in the monetary cost of higher education μ . That second case is illustrated on Figure 1.b.



Figure 1a: Existence of a unique Figure 1b: Existence of two stationary stationary equilibrium. equilibria.

Well-being inequalities at the stationary equilibrium Let us now compare the well-being of long-lived and short-lived persons at the steady-state.

Proposition 2 If $\mu \geq (B\Theta)^{\frac{1}{\sigma}}$, long-lived individuals are, at \bar{h}^1 , worse-off than short-lived individuals. If $\mu < (B\Theta)^{\frac{1}{\sigma}}$, long-lived individuals are, at \bar{h}^2 , better off than short-lived individuals if and only if:

$$\beta \left[\log \left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu}{1 + \gamma} \right) + \gamma \log \left(\frac{\gamma}{1 + \gamma} \right) \right] > \Omega$$

Proof. The first part of the proof follows from $\log(0) = -\infty < \Omega$. The second part of the proof follows from substituting for steady-state human capital and mature labor in the second-period utility at mature adulthood.

When higher education was too costly to allow for the take-off of the economy, long-lived persons were unambiguously worse-off than short-lived persons, because they had to face extreme poverty over a longer time period. Once the cost of higher education is lower, and allows for economic take-off, whether or not the long-lived are better off than short-lived individuals depends on the structure of preference parameters α , β , γ and Ω , and on technical parameters A, B and σ . In advanced economies, the condition of Proposition 2 will be satisfied, making long-lived individuals better off than short-lived persons.

3 A subsidy on higher education

Whereas subsidizing higher education is often interpreted as a policy that aims at forstering human capital accumulation and growth, one can also interpret that policy as a particular form of age-based statistical discrimination policy favouring the young, which amounts, *in fine*, to allocate more resources to the young. As such, that policy is a candidate for serving as an insurance device against the risk of having a short life (Fleurbaey et al 2014).

Let us now suppose that higher education is subsidized at a rate $s \in [0, 1]$. The net monetary cost of higher education then becomes equal to $\mu(1-s)e_t$, so that young-age consumption is:

$$c_t = y_t^y - \mu(1-s)e_t = Ah_t \ell_t^y - \mu(1-s)e_t$$
(12)

When the subsidy is financed by taxing mature-age labor income at a rate τ_t , mature-age consumption at time t is:

$$c_t^m = y_t^m (1 - \tau_t) = A h_t \ell_t^m (1 - \tau_t)$$
(13)

Assuming that the government's budget is balanced, we have, at time t:

$$\mu se_t = \pi \tau_t A h_t \ell_t^m \tag{14}$$

The LHS is the total cost of subsidizing higher education for cohort t. The RHS is the fiscal revenue obtained by taxing mature-age labor earnings of cohort t-1 at a rate τ_t . This is weighted by the survival probability to mature age π .

Temporary equilibrium The problem of the young adult is now:

$$\max_{e_t, \tilde{\ell}_t^y, \tilde{\ell}_{t+1}^m} \log \left(Ah_t (1 - e_t - \tilde{\ell}_t^y) - \mu (1 - s)e_t \right) + \alpha \log (e_t) + \gamma \log \left(\tilde{\ell}_t^y \right)$$
$$+ \pi \beta \left[\log \left((1 - \tau_{t+1}) ABh_t^{1 - \sigma} e_t^{\sigma} (1 - \tilde{\ell}_{t+1}^m) \right) + \gamma \log \left(\tilde{\ell}_{t+1}^m \right) \right] + (1 - \pi) \Omega$$

The FOC for higher education is:

$$\frac{\alpha}{e_t} + \frac{\pi\beta\sigma}{e_t} = \frac{Ah_t + \mu(1-s)}{Ah_t(1-e_t - \tilde{\ell}_t^y) - \mu(1-s)e_t}$$
(15)

The only difference with respect to the laissez-faire concerns the RHS: the opportunity cost, in terms of consumption at the young age, of investing in higher education is now reduced. Note that, thanks to the log-linear form, the FOC for higher education does not depend directly on the rate of taxation of labor earnings at the mature age.

The FOC for optimal young-age leisure time is now:

$$\frac{Ah_t}{Ah_t(1-e_t-\tilde{\ell}_t^y)-\mu(1-s)e_t} = \frac{\gamma}{\tilde{\ell}_t^y}$$
(16)

The FOC for optimal mature-age leisure is:

$$\frac{1}{1-\tilde{\ell}_{t+1}^m} = \frac{\gamma}{\tilde{\ell}_{t+1}^m} \implies \tilde{\ell}_{t+1}^m = \frac{\gamma}{1+\gamma}$$
(17)

The following lemma summarizes our results.

Lemma 2 At the temporary equilibrium at period t when higher education is subsidized at a rate s, higher education, young-age leisure, young-age labor, mature-age leisure and mature-age labor are given by:

$$\begin{split} e_t^S &= \frac{\left(\alpha + \pi\beta\sigma\right)\left(Ah_t\right)}{\left(1 + \gamma + \alpha + \pi\beta\sigma\right)\left(Ah_t + \mu(1-s)\right)} > e_t, \\ \tilde{\ell}_t^{yS} &= \frac{\gamma}{1 + \gamma + \alpha + \pi\beta\sigma} = \tilde{\ell}_t^y, \\ \ell_t^{yS} &= \frac{\left(1 + \alpha + \pi\beta\sigma\right)\mu(1-s) + Ah_t}{\left(1 + \gamma + \alpha + \pi\beta\sigma\right)\left(Ah_t + \mu(1-s)\right)} < \ell_t^y, \\ \tilde{\ell}_t^{mS} &= \frac{\gamma}{1+\gamma} = \tilde{\ell}_t^m \text{ and } \ell_t^{mS} = \frac{1}{1+\gamma} = \ell_t^m. \end{split}$$

Proof. See the Appendix. \blacksquare

In comparison with the laissez-faire equilibrium, we see that, *ceteris paribus* (that is, for a given level of human capital), the temporary equilibrium involves now a larger amount of higher education, as well as a lower young-age labor. The subsidy makes higher education cheaper, which encourages individuals to invest more time in education, and to work less at the young age.

Stationary equilibrium At any period of time, the subsidy on higher education is financed by taxing labor earnings at the mature age. Assuming that the government's budget is balanced, we have:

$$\mu se_t = \pi \tau_t A h_t \ell_t^m \iff s = \frac{\pi \tau_t A h_t}{\mu e_t (1+\gamma)} \tag{18}$$

Substituting for this in the expression for optimal higher education, we have:

$$e_{t} = \frac{(\alpha + \pi\beta\sigma) (Ah_{t})}{(1 + \gamma + \alpha + \pi\beta\sigma) \left(Ah_{t} + \mu \left(\frac{\mu e_{t}(1 + \gamma) - \pi\tau_{t}Ah_{t}}{\mu e_{t}(1 + \gamma)}\right)\right)}$$

$$\iff e_{t} \left(Ah_{t} + \mu \left(\frac{\mu e_{t}(1 + \gamma) - \pi\tau_{t}Ah_{t}}{\mu e_{t}(1 + \gamma)}\right)\right) = \Theta^{\frac{1}{\sigma}}h_{t}$$

$$\iff e_{t} = \frac{\left(\Theta^{\frac{1}{\sigma}}(1 + \gamma) + \pi\tau_{t}A\right)h_{t}}{(Ah_{t} + \mu)(1 + \gamma)}$$
(19)

This expression gives us the level of higher education chosen by a young adult at period t as a function of the tax rate that prevails at time t. The level of higher education is increasing in the tax rate, since a higher tax rate implies also a higher rate of subsidy on education s, which encourages education.⁷

Financial feasibility of the education policy requires that the subsidy on higher education can be financed at any time period by taxing labor earnings at

⁷Note also that, in the laissez-faire, we have $\tau_t = 0$, so that: $e_t = \frac{\Theta_{\sigma h_t}^{\frac{1}{\sigma}}}{Ah_t + \mu}$ as in Section 2.

the mature age at a rate that is less than unity (in order to guarantee the nonnegativity of consumption at the mature age). Financial feasibility is obviously satisfied when h_t equals 0, because in that case individuals choose a zero level of higher education, so that the policy can then be trivially financed by a zero tax on labor earnings. In the more general case where $h_t > 0$, the financial feasibility of the policy requires that the tax rate allowing to balance revenues and expenditures is less than unity, that is:⁸

$$\tau_{t} = \frac{(1+\gamma) s\mu \left(\frac{(\alpha+\pi\beta\sigma) (Ah_{t})}{(1+\gamma+\alpha+\pi\beta\sigma) (Ah_{t}+\mu(1-s))}\right)}{\frac{e_{t}}{\pi Ah_{t}}} < 1$$
$$\iff s < \frac{\pi (1+\gamma+\alpha+\pi\beta\sigma)}{\mu \left[(1+\gamma) (\alpha+\pi\beta\sigma)+\pi (1+\gamma+\alpha+\pi\beta\sigma)\right]} (Ah_{t}+\mu) (20)$$

This is the general condition for financial feasibility of the education policy at any time t. Note that the RHS of the condition is *increasing* in h_t . Thus, the more developed the economy is, and the less constraining the financial feasibility condition is. When human capital accumulates itself monotonically over time, financial feasibility only requires that the above condition holds at h_0 . Indeed, given that the RHS of the condition is increasing in h_t , if human capital grows over time, satisfying the general condition for financial feasibility at t = 0 suffices to satisfy also the general condition for financial feasibility at all t > 0.

Lemma 3 Assume that the dynamics of human capital is monotonically increasing. Then financial feasibility of the education policy is satisfied at all periods $t = 0, ..., +\infty$ when:

$$h_0 > \frac{s\mu\left[(1+\gamma)\left(\alpha+\pi\beta\sigma\right)+\pi\left(1+\gamma+\alpha+\pi\beta\sigma\right)\right]}{A\pi\left(1+\gamma+\alpha+\pi\beta\sigma\right)} - \frac{\mu}{A}$$

Proof. The condition is obtained by rewriting the condition for financial feasibility at t = 0.

Financial feasibility requires sufficiently favorable initial conditions. The threshold depends on some key structural parameters of the economy. First of all, it is decreasing with the probability of survival to the mature age π . The intuition is that a low proportion of the population surviving to the mature age reduces the number of contributors to the system, and, hence, threatens the financial feasibility of subsidizing higher education. Financial feasibility depends also on the structure of preference parameters, and on the unitary cost of education μ . Throughout the remaining of the paper, we will suppose that initial conditions h_0 are such that this condition is satisfied.

⁸One can interpret the LHS of the condition as the level of the tax rate required to finance the total subsidy on higher education at period t (thus, given human capital stock h_t and the induced chosen education).

Substituting for higher education and for the tax rate in the accumulation equation for human capital yields:

$$h_{t+1} = B(h_t)^{1-\sigma} \left(\frac{\left(\Theta^{\frac{1}{\sigma}}(1+\gamma) + \pi \frac{s\mu\Theta^{\frac{1}{\sigma}}(1+\gamma)}{\pi A[(Ah_t+\mu)-s\mu]}A\right)h_t}{(Ah_t+\mu)(1+\gamma)} \right)^{\sigma}$$
$$h_{t+1} = \frac{B\Theta h_t}{[Ah_t+\mu(1-s)]^{\sigma}} \equiv H(h_t)$$
(21)

In comparison to the human capital accumulation process at the laissez-faire (where s = 0), the process is here strengthened by the presence of the subsidy on higher education, which increases the investment in higher education for a given human capital level. Proposition 3 summarizes our results.

Proposition 3 Assume $h_0 > \frac{s\mu[(1+\gamma)(\alpha+\pi\beta\sigma)+\pi(1+\gamma+\alpha+\pi\beta\sigma)]}{A\pi(1+\gamma+\alpha+\pi\beta\sigma)} - \frac{\mu}{A}$. If $\mu(1-s) \ge [B\Theta]^{\frac{1}{\sigma}}$, the economy admits a unique locally stable stationary equilibrium $\bar{h}^{1S} = 0$. If $\mu(1-s) < [B\Theta]^{\frac{1}{\sigma}}$, the economy admits two stationary equilibria:

$$\bar{h}^{1S} = 0 \ and \ \bar{h}^{2S} = \frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{A}$$

The stationary equilibrium \bar{h}^{1S} is unstable, and the stationary equilibrium \bar{h}^{2S} is locally stable.

Proof. See the Appendix.

In comparison to the laissez-faire (Proposition 1), the introduction of a subsidy on higher education affects the condition under which the economy can escape from poverty. The condition now concerns the *net* monetary cost of higher education $\mu(1-s)$ - instead of the gross cost μ -. Provided s is sufficiently large, it can make the economy escape from long-run poverty. Interestingly, in that case, the unique, locally stable strictly positive stationary equilibrium is increasing in the subsidy rate s.

Figures 2a and 2b illustrate a situation where a given economy is trapped in poverty at the laissez-faire (i.e., when s = 0), that is, we have $\mu \geq [B\Theta]^{\frac{1}{\sigma}}$ (Figure 2a), but, once a subsidy s > 0 on higher education is introduced, the economy can escape from poverty and converges towards a positive stationary equilibrium (that is, we then have $\mu(1-s) < [B\Theta]^{\frac{1}{\sigma}}$), even though all other structural parameters of the economy are left unchanged (Figure 2b).



Figure 2a: An economy trapped in Figure 2b: An economy escaping from poverty at the laissez-faire. Poverty thanks to the subsidy.

Let us now derive the main variables at the stationary equilibrium. Given that $\Theta \equiv \left[\frac{(\alpha + \pi\beta\sigma)A}{(1+\gamma+\alpha+\pi\beta\sigma)}\right]^{\sigma}$, we have, concerning the young age:

$$e^{S} = \frac{(\alpha + \pi\beta\sigma)\left((B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)\right)}{(1+\gamma+\alpha+\pi\beta\sigma)\left(B\Theta\right)^{\frac{1}{\sigma}}} = \frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{AB^{\frac{1}{\sigma}}}$$
(22)

$$\ell^{yS} = \frac{(\alpha + \pi\beta\sigma)\,\mu(1-s) + (B\Theta)^{\frac{1}{\sigma}}}{(1+\gamma+\alpha+\pi\beta\sigma)\,(B\Theta)^{\frac{1}{\sigma}}} = \frac{\mu(1-s)}{AB^{\frac{1}{\sigma}}} + \frac{1}{1+\gamma+\alpha+\pi\beta\sigma}(23)$$

$$c^{yS} = \frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{(1+\gamma+\alpha+\pi\beta\sigma)}$$
(24)

Regarding variables at the mature age, remind first that, if the government's budget is balanced, that is, if $\mu se = \pi \tau A h \ell^m$, we have: $1 - \tau = \frac{\pi A h \ell^m - \mu se}{\pi A h \ell^m}$, so that:

$$c^{mS} = \left(\frac{\pi Ah\ell^m - \mu se}{\pi Ah\ell^m}\right) Ah\ell^m = \left((B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)\right) \left(\frac{1}{1+\gamma} - \frac{\mu s}{\pi AB^{\frac{1}{\sigma}}}\right)$$
(25)

Mature-age consumption varies non-monotonically with the rate of subsidy s.

Lemma 4 Consider the economy where higher education is subsidized at a rate s. Assume that $\bar{h}^2 < \frac{\pi B^{\frac{1}{\sigma}}}{1+\gamma} < \bar{h}^2 + \frac{2\mu}{A}$ where $\bar{h}^2 = \frac{(B\Theta)^{\frac{1}{\sigma}} - \mu}{A}$ is the stationary equilibrium at the laissez-faire. There exists an interior critical threshold $\bar{s} = \frac{\pi A B^{\frac{1}{\sigma}}}{2\mu(1+\gamma)} - \frac{1}{2} \left(\frac{(B\Theta)^{\frac{1}{\sigma}}}{\mu} - 1 \right) \in]0,1[$ such that:

$$\frac{\partial c^{ms}}{\partial s} \stackrel{\geq}{=} 0 \iff s \stackrel{\leq}{=} \bar{s}$$

that is, consumption at the mature age is increasing in s when s is inferior to \bar{s} , and decreasing in s when s is superior to \bar{s} .

Proof. See the Appendix.

In a static world, consumption at the mature age would be decreasing with the subsidy on higher education, because the funding of this subsidy requires to tax incomes at the mature age, and, hence, to reduce consumption. However, in a dynamic economy, another effect is at work: the subsidy on higher education encourages individuals to invest more time in education, which fosters human capital accumulation, and, hence, increases production possibilities. Lemma 4 states that, when the subsidy is lower than the threshold \bar{s} , this dynamic effect dominates the first effect, so that consumption at the mature age is increasing in the rate of subsidy s (despite the associated rise in earnings taxation). However, when the subsidy on higher education is higher than \bar{s} , this dynamic effect is dominated, so that a higher subsidy implies lower consumption at the mature age. This explains the existence of an inverted-U relation between the subsidy rate and consumption at the mature age.





Figure 3a illustrates the inverted-U relation between the rate of subsidy s on higher education (x axis) and the level of consumption at mature age at the stationary equilibrium (y axis). As shown on Figure 3a, the level of the threshold \bar{s} at which consumption at the mature age is maximized depends on preference parameters. If the pure utility from higher education goes up (i.e., a higher α), this pushes the consumption at the mature age up, and this reduces

also the threshold (since Θ is increasing in α). The intuition is that a higher pure taste for education makes individuals invest a lot of time in education even when the subsidy is low, so that the maximization of consumption at the mature age requires, *ceteris paribus*, a lower subsidy. Alternatively, as shown on Figure 3b, when the taste for leisure time increases (i.e., when γ is raised), this decreases consumption possibilities at the mature age. Moreover, the subsidy maximizing consumption at the mature age is then reduced.

4 Welfare inequalities reexamined

Let us now examine the welfare consequences of the subsidy on higher education. Consider first the case of prematurely dead persons. Their realized welfare at the (strictly positive) stationary equilibrium is:

$$U^{SL} = \log\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{1+\gamma+\alpha+\pi\beta\sigma}\right) + \alpha\log\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{AB^{\frac{1}{\sigma}}}\right) + \gamma\log\left(\frac{\gamma}{1+\gamma+\alpha+\pi\beta\sigma}\right) + \Omega$$
(26)

The first term, relative to the utility of young-age consumption, is increasing in the subsidy s. The second term, relative to the pure utility of higher education, is also increasing in s. The last term, relative to leisure time, is not affected by s. Hence, all in all, the prematurely dead persons are made better off by the subsidy on higher education.

Let us now consider the long-lived persons. Their total well-being is:

$$U^{LL} = \log\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{1+\gamma+\alpha+\pi\beta\sigma}\right) + \alpha\log\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{AB^{\frac{1}{\sigma}}}\right) + \gamma\log\left(\frac{\gamma}{1+\gamma+\alpha+\pi\beta\sigma}\right) + \beta\log\left(\left((B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)\right)\left(\frac{1}{1+\gamma} - \frac{\mu s}{\pi AB^{\frac{1}{\sigma}}}\right)\right) + \beta\gamma\log\left(\frac{\gamma}{1+\gamma}\right)$$
(27)

Given what we showed above, the young-age component of their well-being is increasing in s, while mature-age consumption is also increasing in s when $s < \bar{s}$, so that we have that the well-being of the long-lived is unambiguously increasing in the subsidy s when $s \leq \bar{s}$. However, due to adverse effects on well-being at young age and mature age, the subsidy has an ambiguous effect on total well-being once $s > \bar{s}$.

Regarding inequalities in lifetime well-being between the long-lived and the short-lived, i.e., $\Delta U \equiv U^{LL} - U^{SL}$, we have:

$$\Delta U = \beta \log \left(\left((B\Theta)^{\frac{1}{\sigma}} - \mu(1-s) \right) \left(\frac{1}{1+\gamma} - \frac{\mu s}{\pi A B^{\frac{1}{\sigma}}} \right) \right) + \beta \gamma \log \left(\frac{\gamma}{1+\gamma} \right) - \Omega$$
(28)

The welfare loss due to a premature death depends on the structure of preferences: the larger the pure disutility loss due to a premature death is (i.e., the smaller $\Omega \leq 0$ is), the larger the welfare loss due to a premature death is. However, it is possible to act on $U^{LL} - U^{SL}$ by playing on the subsidy on higher education s, which affects the opportunity cost of dying early. True, when $s < \bar{s}$, the subsidy increases consumption at mature age, and, hence, wellbeing inequalities between the long-lived and the short-lived ΔU . However, once $s > \bar{s}$, the subsidy decreases consumption at mature age, and, hence, reduces ΔU .

Proposition 4 Define the threshold $\bar{s} = \frac{\pi AB^{\frac{1}{\sigma}}}{2\mu(1+\gamma)} - \frac{1}{2}\left(\frac{(B\Theta)^{\frac{1}{\sigma}}}{\mu} - 1\right).$

- The realized lifetime well-being of prematurely dead persons is increasing in the rate of subsidy s on higher education.
- The realized lifetime well-being of long-lived persons is increasing in the rate of subsidy s on higher education when $s \leq \bar{s}$, but the subsidy has ambiguous effect on their total well-being when $s > \bar{s}$.
- Well-being inequalities between long-lived and short-lived persons increase with the subsidy s when s < s̄ and decrease with the subsidy when s > s̄.

Proof. See above.

Subsidizing higher education makes the prematurely dead persons better off with respect to the laissez-faire. From the perspective of the well-being of the short-lived, the higher the subsidy s is, the better off the short-lived are. This makes a subsidy on higher education a plausible candidate to contribute to improve the situation of the few - unidentified - persons who will turn out to die prematurely in the future.

However, when s is below the threshold \bar{s} , the subsidy on higher education improves also the situation of the long-lived individuals during the mature age, by increasing consumption at the mature age thanks to the associated human capital accumulation. Hence, when s is below the threshold, the subsidy on higher education improves the lot of long-lived persons even more than it improves the lot of short-lived persons, leading to a rise of well-being inequalities between the long-lived and the short-lived persons. It is only when s exceeds \bar{s} that a higher subsidy deteriorates the consumption at the mature age, and, hence, reduces well-being inequalities between the long-lived and the short-lived.

Thus, for a subsidy rate s that is below the critical threshold, subsidizing higher education has the effect of making the short-lived persons better off, while having, at the same time, the effect of increasing well-being inequalities between long-lived and short-lived persons. It is only when s is higher than the critical threshold that the subsidy on higher education contributes to reduce well-being inequalities between the long-lived and the short-lived.

Is it possible to obtain full insurance by means of the subsidy on higher education? The following proposition provides an answer to that question. **Proposition 5** There exists an interior policy (s^*, τ^*) with $0 < s^* < 1$ and $0 < \tau^* < 1$ that yields full insurance if and only if:

$$\beta \log \left((B\Theta)^{\frac{1}{\sigma}} \left(\frac{1}{1+\gamma} - \frac{\mu}{\pi A B^{\frac{1}{\sigma}}} \right) \right) + \beta \gamma \log \left(\frac{\gamma}{1+\gamma} \right) < \Omega$$

Proof. See the Appendix. \blacksquare

Given that $\Omega \leq 0$, this condition is unlikely to be satisfied in advanced economies, for which technical parameters A and B are large. It is thus unlikely that an interior subsidy on higher education could achieve full insurance.

5 Optimal subsidy on higher education

To examine the level at which the subsidy rate on higher education s should be fixed, we will rely on two distinct social welfare functions (SWF), and compute the level of s that maximizes the social objective under each of these.

The utilitarian SWF Let us first consider, as a normative benchmark, the standard utilitarian SWF, which is in the line of Bentham's (1789) Principle of the Largest Happiness for the Largest Number. Under the utilitarian SWF, the social planner selects the pair of instruments (s, τ) so as to maximize steadystate aggregate well-being (defined as the sum of lifetime well-being for shortlived and long-lived persons, with weights capturing their demographic size of the two groups), subject to the economy's resource constraint. The problem is:

$$\begin{split} \max_{s} \left[1 - \pi \right] \left[\begin{array}{c} \log \left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{1 + \gamma + \alpha + \pi \beta \sigma} \right) + \alpha \log \left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{AB^{\frac{1}{\sigma}}} \right) \\ + \gamma \log \left(\frac{\gamma}{1 + \gamma + \alpha + \pi \beta \sigma} \right) + \Omega \end{array} \right] \\ + \pi \left[\begin{array}{c} \log \left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{1 + \gamma + \alpha + \pi \beta \sigma} \right) + \alpha \log \left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{AB^{\frac{1}{\sigma}}} \right) + \gamma \log \left(\frac{\gamma}{1 + \gamma + \alpha + \pi \beta \sigma} \right) \\ + \beta \log \left(\left((B\Theta)^{\frac{1}{\sigma}} - \mu(1-s) \right) \left(\frac{1}{1 + \gamma} - \frac{\mu s}{\pi AB^{\frac{1}{\sigma}}} \right) \right) + \beta \gamma \log \left(\frac{\gamma}{1 + \gamma} \right) \end{split}$$

The FOC for optimal subsidy on higher education is, after simplifications:

$$\frac{\mu(1+\alpha)}{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)} + \pi\beta\mu \left[\frac{1}{1+\gamma} - \frac{2\mu s}{\pi AB^{\frac{1}{\sigma}}} - \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right] = 0$$
(29)

This can be rewritten as:

$$\frac{1+\alpha}{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)} = \pi\beta \left[\frac{2\mu s}{\pi AB^{\frac{1}{\sigma}}} - \frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right]$$
(30)

The LHS is the positive effect, at the margin, of subsidizing higher education on well-being at the young age, (through both increased consumption at the young age and increased time spent directly in higher education at the young age), for both the long-lived and the short-lived persons. The RHS is the marginal well-being variation at the mature age related to the induced variation in consumption at the mature age. As we have seen above, that second effect has an ambiguous sign: a higher subsidy on education fosters human capital accumulation, and, hence, consumption possibilities. But at the same time, this requires a higher tax rate on labor earnings at the mature age, which reduces consumption possibilities at the mature age.

Interestingly, if the subsidy rate is equal to the critical threshold $\bar{s} = \frac{\pi A B \frac{1}{\sigma}}{2\mu(1+\gamma)} - \frac{1}{2} \left(\frac{(B\Theta)^{\frac{1}{\sigma}}}{\mu} - 1 \right)$, the RHS vanishes to zero. However, if s is strictly above \bar{s} , the RHS is positive. Hence, given that the LHS of the FOC is strictly positive, the utilitarian optimum requires that the subsidy rate is strictly *above* the critical level \bar{s} . The economic intuition is that fixing s below \bar{s} would prevent the economy from benefiting from Pareto-improvements obtained by encouraging higher education. Further calculations allow us to derive the following result.

Proposition 6 Assume that:

$$\frac{(1+\alpha)}{(B\Theta)^{\frac{1}{\sigma}}-\mu} - \pi\beta \left[-\frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}}+\mu}{\pi AB^{\frac{1}{\sigma}}}\right] > 0$$

$$\frac{(1+\alpha)}{(B\Theta)^{\frac{1}{\sigma}}} - \pi\beta \left[\frac{2\mu}{\pi AB^{\frac{1}{\sigma}}} - \frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}}+\mu}{\pi AB^{\frac{1}{\sigma}}}\right] < 0$$

From a utilitarian perspective, there exists an unique interior optimal subsidy on higher education $s^U \in [0, 1[. s^U]$ is strictly above the critical threshold \bar{s} , and is given by the formula:

$$s^{U} = \frac{\begin{bmatrix} -\left[(1+\gamma)\beta\left[3(B\Theta)^{\frac{1}{\sigma}}-\mu\right]-\pi\beta AB^{\frac{1}{\sigma}}\right] \\ + \gamma^{2}\left[(B\Theta)^{\frac{2}{\sigma}}+9\mu^{2}-6(B\Theta)^{\frac{1}{\sigma}}\mu\right] \\ +\beta(1+\gamma)AB^{\frac{1}{\sigma}}\left[\pi\beta\left(2(B\Theta)^{\frac{1}{\sigma}}-6\mu+\pi AB^{\frac{1}{\sigma}}\right)+8(1+\alpha)(1+\gamma)\right] \\ \frac{4\beta\mu(1+\gamma)}{2\beta\mu(1+\gamma)} \end{bmatrix}}{2\beta\mu(1+\gamma)}$$

Proof. See the Appendix.

Proposition 6 states that utilitarianism recommends that higher education is subsidized. The intuition is that encouraging higher education allows to reach higher levels of human capital at the stationary equilibrium, and, hence, to enjoy larger consumption possibilities in the long run. While this rationale may justify any subsidy, the utilitarian SWF recommends a subsidy on higher education that exceeds the critical threshold \bar{s} . Indeed, we know that, for any subsidy *s* that is below \bar{s} , increasing *s* further would both increase well-being at the young age, and also increase well-being at the mature age, so that any further increase in *s* involves a Pareto-improvement. However, beyond \bar{s} , further increases in *s* will still raise well-being at the young age, but at the cost at reducing well-being at the mature age. Thus the utilitarian optimal subsidy on higher education lies somewhere between \bar{s} and the maximum subsidy $s^{max} = 1$. **The ex post egalitarian SWF** Consider now, as a guide for comparisons, the ex post egalitarian SWF, which gives absolute priority to the worst-off in realized terms, in line with Fleurbaey et al (2014). Assuming that the worst-off is, in realized terms, the short-lived, the problem is to select the level of s that maximizes the well-being of the short-lived, subject to the resource constraint, and subject to the constraint that the long-lived is not worse-off than the short-lived. That social planning problem can be written as:

$$\max_{s} \qquad \log\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{1+\gamma+\alpha+\pi\beta\sigma}\right) + \alpha\log\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{AB^{\frac{1}{\sigma}}}\right) + \gamma\log\left(\frac{\gamma}{1+\gamma+\alpha+\pi\beta\sigma}\right) + \Omega$$

s.t.
$$\beta \log \left(\left((B\Theta)^{\frac{1}{\sigma}} - \mu(1-s) \right) \left(\frac{1}{1+\gamma} - \frac{\mu s}{\pi A B^{\frac{1}{\sigma}}} \right) \right) + \beta \gamma \log \left(\frac{\gamma}{1+\gamma} \right) - \Omega = 0$$

This social planning problem can be rewritten by means of the Lagrangian:

$$\max_{s} \mathcal{L} = \log\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{1+\gamma+\alpha+\pi\beta\sigma}\right) + \alpha\log\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{AB^{\frac{1}{\sigma}}}\right) + \gamma\log\left(\frac{\gamma}{1+\gamma+\alpha+\pi\beta\sigma}\right) + \Omega$$
$$+\lambda\left[\beta\log\left(\left((B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)\right)\left(\frac{1}{1+\gamma} - \frac{\mu s}{\pi AB^{\frac{1}{\sigma}}}\right)\right) + \beta\gamma\log\left(\frac{\gamma}{1+\gamma}\right) - \Omega\right]$$

where λ the Lagrange multiplier associated to the egalitarian constraint.

The FOC for the optimal subsidy on higher education is:

$$\frac{\mu(1+\alpha)}{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)} + \lambda\beta\mu \left[\frac{1}{1+\gamma} - \frac{2\mu s}{\pi AB^{\frac{1}{\sigma}}} - \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right] = 0 \quad (31)$$

Note that this FOC is almost the same as the one under the utilitarian SWF, except that the second term on the LHS is not weighted by the survival probability π , but by the Lagrange multiplier λ , which captures the shadow value of relaxing the egalitarian constraint.

Two cases can arise regarding the form of the expost egalitarian optimum.

First, the egalitarian constraint is binding. We thus have $\lambda > 0$ and $U^{LL} = U^{SL}$. In that case, the subsidy s must be above the critical threshold \bar{s} . Indeed, when s equals \bar{s} , the second term of the LHS vanishes to 0, so that the FOC cannot be satisfied for such a value of s when the egalitarian constraint is binding. Similarly, for all levels of $s < \bar{s}$, the FOC cannot be satisfied when the egalitarian constraint is binding. Thus the expost egalitarian optimum requires $s > \bar{s}$, but for reasons that are different from the one prevailing under utilitarianism. Here the point is not to allow for all Pareto-improvements associated to higher education, but, rather, to satisfy the egalitarian constraint. Note that, in that case, the FOC can be rewritten as:

$$\frac{\mu(1+\alpha)}{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)} = \lambda\beta\mu \left[\frac{2\mu s}{\pi AB^{\frac{1}{\sigma}}} - \frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right]$$
(32)

The LHS is strictly decreasing in s, and since when $s > \bar{s}$, the RHS is increasing in s. Hence we have that, when $\lambda < \pi$, that is, when the shadow value of relaxing the egalitarian constraint is less than the survival probability to the mature age, the level of s that equalizes the LHS and the RHS must be higher than s^{U} , that is, the one that equalizes the terms:

$$\frac{\mu(1+\alpha)}{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)} = \pi\beta\mu \left[\frac{2\mu s}{\pi AB^{\frac{1}{\sigma}}} - \frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right]$$

Indeed, replacing π by $\lambda < \pi$ in the RHS lowers the RHS for any given s, so that the level of s that equalizes the LHS and the RHS must be higher, so that $s^E > s^U$. On the contrary, when $\lambda > \pi$, we have the opposite and $s^E < s^U$.

Second, the egalitarian constraint is not binding. We thus have $\lambda = 0$ and $U^{LL} > U^{SL}$. In that case, the second term on the LHS vanishes to zero, so that the FOC cannot be satisfied. The first term on the LHS is decreasing in s, so that the solution is to fix s at its maximal value $s^{\max} = 1$, that is, to have full subsidization of higher education expenditures.

Proposition 7 From an ex post egalitarian perspective, the optimal subsidy on higher education s^E is above the critical threshold \bar{s} . Two cases can arise: either the realized lifetime well-being of long-lived and short-lived persons can be equalized for an interior level of s, in which case we have $\bar{s} < s^U < s^E < 1$ when $\pi > \lambda$ and $\bar{s} < s^E < s^U < 1$ when $\pi < \lambda$; or the realized lifetime well-being of long-lived and short-lived persons cannot be equalized for an interior level of s, in which case we have $\bar{s} < s^U < s^E = 1$.

Proof. See the developments above.

In advanced economies, the condition for full equalization of lifetime wellbeing across long-lived and short-lived persons is unlikely to be satisfied for an interior subsidy s on higher education. It is thus likely that the expost egalitarian SWF will recommend a corner solution that consists of subsidizing higher education at a 100 % rate. This 100 % subsidy is, under mild conditions, above the optimal subsidy on higher education prevailing under utilitarianism. The economic intuition goes as follows. Under both utilitarianism and expost egalitarianism, encouraging higher education is desirable, because this fosters human capital accumulation and, hence, consumption possibilities. However, under utilitarianism this effect is (at least partly) counterbalanced by the associated fall in consumption at the mature age that takes place once the subsidy exceeds the critical level \bar{s} . This effect imposes, at some point, an upper limit on the subsidy from a utilitarian perspective. On the contrary, under the expost egalitarian SWF, reductions in consumption at the mature age are not problematic, as long as long-lived persons remain better off than short-lived ones. In advanced economies, that condition keeps on prevailing for all values of s in $[\bar{s}, 1]$, which leads to the result of full subsidization of higher education.

6 Conclusions

Higher education is both a consumption good and an investment good. Given this mixed nature of higher education, the effect of subsidizing higher education on inequalities in lifetime well-being due to longevity risk is ambiguous. Can a subsidy on higher education serve as an insurance device against the risk of having a short life? Under which conditions does a subsidy on higher education contribute to reduce the welfare loss due to a premature death?

In order to answer that question, this paper developed a two-period dynamic OLG model with human capital accumulation and risky lifetime. Our study of that Ben-Porath economy allowed us to demonstrate that, although a subsidy on higher education always improves the situation of short-lived persons, this does not necessarily contribute to reduce inequalities in lifetime well-being between the long-lived and the short-lived. Actually, when the subsidy is below some critical threshold, this contributes to expand these inequalities, and, hence, to increase the welfare loss due to a premature death. It is only if the subsidy on higher education exceeds some critical threshold that it contributes to reduce the welfare loss due to a premature death.

Importantly, this paper identifies the determinants of the critical threshold for the subsidy, above which subsidizing higher education contributes to decrease the welfare loss due to a premature death. That critical threshold has been shown to depend crucially on *the structure of preferences*. The threshold is lower when the pure preference for higher education is stronger, and is higher when the pure preference for leisure time is stronger. This indicates that the extent to which a given subsidy on higher education can serve as a social insurance scheme against the risk of having a short life may vary across cultures, depending on whether higher education is perceived as a pure means or also as an end in itself.

As such, this paper casts original light on the conditions under which education policies can serve as an insurance scheme against the risk of having a short life. Only generous subsidization programs can achieve the targets of *both* improving the situation of the unlucky short-lived and of reducing the welfare loss due to a premature death. Other education policies only improve the lot of the short-lived, without reducing the opportunity cost of dying early.

In the light of these results, it appears that some countries, where higher education is largely subsidized, such as France, do already provide - but *implicitly* - some indirect form of universal insurance against the risk of having a short life, by means of the large subsidization of higher education. On the contrary, countries where higher education is not (or little) subsidized do not provide such insurance. Obviously, these conclusions only hold *ceteris paribus*: countries may also differ regarding other policies that may affect the scope of the welfare loss due to a premature death.⁹ But it is good to know that subsidizing higher education can, if it is sufficiently generous, serve indirectly as a universal insurance scheme against the risk of having a short life.

 $^{^{9}}$ From that perspective, an important aspect of existing welfare states concerns their pension system, that is, their system of insurance against the risk of having a *long* life (see Ponthiere 2020, 2023).

7 References

Abbott, B., Gallipoli, G., Meghir, C., Violante, G.L., 2019. "Education policy and intergenerational transfers in equilibrium". *Journal of Political Economy*, 127, 2569-2624.

Acemoglu, D., Johnson, S., 2007. "Disease and Development: the Effect of Life Expectancy on Economic Growth", *Journal of Political Economy*, 115(6), 925-985.

Barham, V., Boadway, R., Marchand, M., Pestieau, P., 1995. "Education and the poverty trap". *European Economic Review*, 39, 1257-1275.

Ben-Porath, Y., 1967. "The Production of Human Capital and the Life-cycle of Earnings", Journal of Political Economy, 75(3), 352-365.

Bentham, J., 1789. An Introduction to the Principles of Morals and Legislation. Oxford, Clarendon Press.

Blackburn, K., Cipriani, J.P., 2002. "A Model of Longevity, Fertility and Growth", *Journal of Economic Dynamics and Control*, 26, 187-204.

Bloom, D., Canning, D., Fink, G., 2014. "Disease and Development Revisited", *Journal of Political Economy*, 122(6): 1355 - 1366.

Bloom, D., Canning, D., Sevilla, J., 2004. "The Effect of Health on Economic Growth: A Production Function Approach", *World Development*, 32(1), 1-13.

Bloom, D., Sachs, J., 1998. "Geography, Demography, and Economic Growth in Africa", Brookings Papers in Economic Activity, 2, 207-273.

Bloom, D., Williamson, J.G., 1998. "Demographic transitions and economic miracles in emerging Asia", World Bank Economic Review, 12, 419-455.

Boucekkine, R., de la Croix, D., Licandro, O., 2002. "Vintage Human Capital, Demographic Trends and Endogenous Growth", *Journal of Economic Theory*, 104(2), 340-375.

Boucekkine, R., D. de la Croix, and O. Licandro. 2003. "Early Mortality Declines at the Dawn of Modern Growth." *The Scandinavian Journal of Economics* 105 (3): 401–418.

Cervellati, M., Sunde, U., 2005. "Human Capital Formation, Life Expectancy and the Process of Development", *American Economic Review*, 95(5), 1653-1672.

Cervellati, M., Sunde, U., 2011. "Life Expectancy and Economic Growth: The Role of the Demographic Transition", *Journal of Economic Growth*, 16(2), 99-133.

Cervellati, M., Sunde, U., 2013. "Life Expectancy, Schooling, and Lifetime Labor Supply: Theory and Evidence Revisited". *Econometrica*, 81 (5), 2055-2086.

Chakraborty, S., 2004. "Endogenous Lifetime and Economic Growth", Journal of Economic Theory, 116(1), 119-137.

Cohen, D., and L. Leker. 2016. "Testing the Ben-Porath effect through the educational pattern of young cohorts". *Journal of Macroeconomics*, 48, 252-262.

de la Croix, D. 2009. Adult longevity and economic take-off from Malthus to Ben-Porath. In: N. Salvadori (ed.): *Institutional and Social Dynamics of Growth and Distribution*. Edward Elgard: 172-190.

de la Croix, D., Licandro, O., 2013. "'The Child is Father of the Man:' Implications for the Demographic Transition", *Economic Journal*, 123, 236-261.

de la Croix, D., Lindh, T., Malmberg, B., 2009. "Demographic Change and Economic Growth in Sweden: 1750-2050", *Journal of Macroeconomics*, 31(1), 132-148.

Ehrlich, I., Lui, F., 1991. "Intergenerational Trade, Longevity, and Economic Growth", Journal of Political Economy, 99(5), 1029-59. Fender, J., Wang, P., 2003. "Educational policy in a credit constrained economy with skill heterogeneity". *International Economic Review*, 44, 939-964.

Fernandez, R., Rogerson, R., 1995. "On the political economy of education subsidies". *Review of Economic Studies*, 62, 249-262.

Fleurbaey, M., Leroux, M.L., Ponthiere, G., 2014. "Compensating the dead", *Journal of Mathematical Economics*, 51, 28-41.

Fleurbaey, M., Leroux, M.L., Pestieau, P., Ponthiere, G., 2016. "Fair retirement under risky lifetime", *International Economic Review*, 57, 177-210.

Gallup, J., and Sachs, J., 2001. "The economic burden of Malaria", American Journal of Tropical Medicine and Hygiene, 64, 85-96.

Garcia-Penalosa, C., Walde, K., 2000. "Efficiency and equity effects of subsidies to higher education". Oxford Economic Papers, 52, 702-722.

Hansen, W., Weisbrod, B., 1969. "The distribution of costs and direct benefits of public higher education: the case of California", *Journal of Human Resources*, 17, 6-191.

Hazan, M., 2009. "Longevity and Lifetime Labor Supply: Evidence and Implications", *Econometrica*, 77(6), 1829-1863.

Hazan, M., Zoabi, H., 2006. "Does longevity cause growth? A theoretical critique", Journal of Economic Growth, 11, 363-376.

Kamm, F. (1993). Morality, Mortality. Volume 1. Death and Whom to Save From It, Oxford University Press.

Leker, L., Ponthiere, G., 2015. "Education, Life Expectancy and Family Bargaining: the Ben-Porath Effect Revisited", *Education Economics*, 23 (4) 481-513.

Leukhina, O., Vandenbroucke, G., 2022. Lifetime work hours and the evolution of the gender wage gap. Federal Reserve Bank of St Louis, Working Paper 2022-25.

Lorentzen, P., J. McMillan, and R. Wacziarg. 2008. "Death and Development." *Journal of Economic Growth* 13 (2): 81–124.

Ludwig, A., Vogel, E., 2010. "Mortality, Fertility, Education and Capital Accumulation in a Simple OLG Economy", *Journal of Population Economics*, 23, 703–735.

Nagel, T. (1979). Mortal Questions. Cambridge University Press.

Nishimura, Y., Pestieau, P., Ponthiere, G., 2018. "Education choices, longevity and optimal policy in a Ben-Porath economy", *Mathematical Social Sciences*, 94, 65-81.

Ponthiere, G., 2020. "A theory of reverse retirement", *Journal of Public Economic Theory*, 22, 1618-1659.

Ponthiere, G., 2023. Allocating Pensions to Younger People. Towards a Social Insurance against a Short Life, Palgrave MacMillan.

Ponthiere, G., 2024. "Social insurance against a short life: Ante mortem versus post mortem policies", *Social Choice and Welfare*, forthcoming.

Soares, R., 2005. "Mortality Reductions, Educational Attainment and Fertility choice", *American Economic Review*, 95(3), 580-601.

8 Appendix

8.1 Proof of Lemma 1

The two FOCs are:

$$\frac{\alpha}{e_t} + \frac{\pi\beta\sigma}{e_t} = \frac{Ah_t + \mu}{Ah_t(1 - e_t - \tilde{\ell}_t^y) - \mu e_t}$$
$$\frac{Ah_t}{Ah_t(1 - e_t - \tilde{\ell}_t^y) - \mu e_t} = \frac{\gamma}{\tilde{\ell}_t^y}$$

From the second FOC, we have:

$$\tilde{\ell}_t^y A h_t = \gamma \left[A h_t (1 - e_t - \tilde{\ell}_t^y) - \mu e_t \right]$$
$$\implies \tilde{\ell}_t^y = \frac{\gamma \left[A h_t (1 - e_t) - \mu e_t \right]}{A h_t (1 + \gamma)}$$

Substituting this in the first FOC, we obtain:

$$\frac{\alpha + \pi\beta\sigma}{e_t} = \frac{Ah_t + \mu}{Ah_t \left(1 - e_t - \frac{\gamma[Ah_t(1 - e_t) - \mu e_t]}{Ah_t(1 + \gamma)}\right) - \mu e_t}$$
$$\frac{\alpha + \pi\beta\sigma}{e_t} = \frac{(Ah_t + \mu)(1 + \gamma)}{(1 - e_t)(Ah_t) - \mu e_t}$$

Hence we have:

$$\begin{aligned} (\alpha + \pi\beta\sigma) \left[(1 - e_t) \left(Ah_t\right) - \mu e_t \right] &= e_t \left(Ah_t + \mu\right) (1 + \gamma) \\ (\alpha + \pi\beta\sigma) \left(Ah_t\right) &= e_t \left[(Ah_t + \mu) \left(1 + \gamma\right) + \left(\alpha + \pi\beta\sigma\right) \left[Ah_t + \mu\right] \right] \\ &\implies e_t = \frac{(\alpha + \pi\beta\sigma) \left(Ah_t\right)}{(1 + \gamma + \alpha + \pi\beta\sigma) \left(Ah_t + \mu\right)} \end{aligned}$$

Hence, using the expression for optimal leisure at young age, we have:

$$\begin{split} \tilde{\ell}_t^y &= \frac{\gamma \left[Ah_t(1-e_t) - \mu e_t\right]}{Ah_t(1+\gamma)} \\ \tilde{\ell}_t^y &= \frac{\gamma \left[Ah_t \left(1 - \frac{(\alpha + \pi\beta\sigma)(Ah_t)}{(1+\gamma + \alpha + \pi\beta\sigma)(Ah_t+\mu)}\right) - \mu \frac{(\alpha + \pi\beta\sigma)(Ah_t)}{(1+\gamma + \alpha + \pi\beta\sigma)(Ah_t+\mu)}\right]}{Ah_t(1+\gamma)} \\ \tilde{\ell}_t^y &= \frac{\gamma}{1+\gamma + \alpha + \pi\beta\sigma} \end{split}$$

Thanks to the time constraint, the working time at the young age is:

$$\begin{split} \ell_t^y &= 1 - e_t - \tilde{\ell}_t^y \\ &= 1 - \frac{\left(\alpha + \pi\beta\sigma\right)\left(Ah_t\right)}{\left(1 + \gamma + \alpha + \pi\beta\sigma\right)\left(Ah_t + \mu\right)} - \frac{\gamma}{1 + \gamma + \alpha + \pi\beta\sigma} \\ &= \frac{\left(1 + \gamma + \alpha + \pi\beta\sigma\right)\left(Ah_t + \mu\right) - \left(\alpha + \pi\beta\sigma\right)\left(Ah_t\right) - \gamma\left(Ah_t + \mu\right)}{\left(1 + \gamma + \alpha + \pi\beta\sigma\right)\left(Ah_t + \mu\right)} \\ &= \frac{\left(1 + \alpha + \pi\beta\sigma\right)\mu + Ah_t}{\left(1 + \gamma + \alpha + \pi\beta\sigma\right)\mu + Ah_t\left(1 + \gamma + \alpha + \pi\beta\sigma\right)} < 1 \end{split}$$

8.2 Proof of Proposition 1

$$h_{t+1} \equiv G(h_t) = Bh_t \left(Ah_t + \mu\right)^{-\sigma} \Theta = \frac{B\Theta h_t}{\left(Ah_t + \mu\right)^{\sigma}}$$

We have: $G(0) = B0 (\mu)^{-\sigma} \Theta = 0$. Thus h = 0 is fixed point, that is, a stationary equilibrium.

We have:

$$G'(h_t) = \frac{B\Theta (Ah_t + \mu)^{\sigma} - B\Theta h_t \sigma (Ah_t + \mu)^{\sigma-1} A}{(Ah_t + \mu)^{2\sigma}}$$
$$= B\Theta \frac{(Ah_t(1 - \sigma) + \mu)}{(Ah_t + \mu)^{1+\sigma}} > 0$$

We thus have that $G'(h_t) > 0$.

Note that, at the stationary equilibrium $\bar{h}^1 = 0$, we have

$$G'(0) = \frac{B\Theta}{(\mu)^{\sigma}}$$

When $\mu \geq (B\Theta)^{\frac{1}{\sigma}}$, we have that $|G'(0)| \leq 1$, so that $\bar{h}^1 = 0$ is a stable stationary equilibrium. On the contrary, when $\mu < (B\Theta)^{\frac{1}{\sigma}}$, we have G'(0) > 1, so that $\bar{h}^1 = 0$ is an unstable stationary equilibrium.

We have also:

$$G''(h_t) = \frac{\begin{bmatrix} [B\Theta (A(1-\sigma))] (Ah_t + \mu)^{1+\sigma} \\ -[B\Theta (Ah_t(1-\sigma) + \mu)] (1+\sigma) (Ah_t + \mu)^{\sigma} A \end{bmatrix}}{(Ah_t + \mu)^{2+2\sigma}} = AB\Theta (Ah_t + \mu)^{\sigma} \frac{[-\sigma\mu - [(Ah_t(1-\sigma) + \mu)]\sigma]}{(Ah_t + \mu)^{2+2\sigma}} < 0$$

so that $G''(h_t) < 0$.

We have also:

$$\lim_{h \to 0} \frac{G(h_t)}{h_t} = \lim_{h \to 0} \frac{Bh_t \left(Ah_t + \mu\right)^{-\sigma} \Theta}{h_t} = \lim_{h \to 0} \frac{B\Theta}{\left(Ah_t + \mu\right)^{\sigma}} = \frac{B\Theta}{\left(\mu\right)^{\sigma}}$$

Suppose that $\mu < (B\Theta)^{\frac{1}{\sigma}}$. Then the transition function $G(h_t)$ is above the 45° line when h is close to 0. Suppose that $\mu \ge (B\Theta)^{\frac{1}{\sigma}}$. Then the transition function $G(h_t)$ is below the 45° line when h is close to 0.

We have also:

$$\lim_{h \to +\infty} \frac{G(h_t)}{h_t} = \lim_{h \to +\infty} \frac{Bh_t \left(Ah_t + \mu\right)^{-\sigma} \Theta}{h_t} = \lim_{h \to +\infty} \frac{B\Theta}{\left(Ah_t + \mu\right)^{\sigma}} = 0$$

Thus the transition function $G(h_t)$ is below the 45° line when h is extremely large.

In sum:

Under $\mu \geq (B\Theta)^{\frac{1}{\sigma}}$ the transition function $G(h_t)$ remains all along below the 45° line, so that 0 is the unique stationary equilibrium, which is also locally stable (see above).

Under $\mu < (B\Theta)^{\frac{1}{\sigma}}$, the transition function $G(h_t)$ is above the 45° line in the neighborhood of 0, and below the 45° line when h is extremely large. Thus, by continuity of $G(h_t)$, there must exist an intersection of $G(h_t)$ with the 45° line, that is, a level of $h_t > 0$ such that $G(h_t) = h_t$. Existence of a strictly positive stationary equilibrium is thus guaranteed.

The uniqueness of the strictly positive stationary equilibrium follows from the monotonicity and concavity of $G(h_t)$. Since $G'(h_t) > 0$ and $G''(h_t) < 0$, the intersection of $G(h_t)$ with the 45° line is unique at a strictly positive level.

Imposing $h_{t+1} = h_t = h$ yields:

$$h = \frac{B\Theta h}{\left(Ah + \mu\right)^{\sigma}}$$

If $h = \bar{h}^2 > 0$, this can be simplified to:

$$\bar{h}^2 = \frac{(B\Theta)^{\frac{1}{\sigma}} - \mu}{A} = \frac{B^{\frac{1}{\sigma}} \left[\frac{(\alpha + \pi\beta\sigma)A}{(1 + \gamma + \alpha + \pi\beta\sigma)}\right] - \mu}{A}$$

The positivity of stationary equilibrium requires $(B\Theta)^{\frac{1}{\sigma}} - \mu > 0$ or $\mu < (B\Theta)^{\frac{1}{\sigma}}$.

Regarding the local stability of the stationary equilibrium, this requires $|G'(\bar{h}^2)| < 1$. We have:

$$G'(\bar{h}^2) = B\Theta \frac{\left(A\left(\frac{(B\Theta)^{\frac{1}{\sigma}}-\mu}{A}\right)(1-\sigma)+\mu\right)}{\left(A\left(\frac{(B\Theta)^{\frac{1}{\sigma}}-\mu}{A}\right)+\mu\right)^{1+\sigma}}$$
$$= \frac{(B\Theta)^{\frac{1}{\sigma}}-\sigma\left[(B\Theta)^{\frac{1}{\sigma}}-\mu\right]}{(B\Theta)^{\frac{1}{\sigma}}} < 1$$

Thus $|G'(\bar{h}^2)| < 1$. Therefore, when it exists, the unique strictly positive stationary equilibrium \bar{h}^2 is locally stable.

8.3 Proof of Lemma 2

The two FOCs are:

$$\frac{\alpha}{e_t} + \frac{\pi\beta\sigma}{e_t} = \frac{Ah_t + \mu(1-s)}{Ah_t(1-e_t - \tilde{\ell}_t^y) - \mu(1-s)e_t}$$
$$\frac{Ah_t}{Ah_t(1-e_t - \tilde{\ell}_t^y) - \mu(1-s)e_t} = \frac{\gamma}{\tilde{\ell}_t^y}$$

From the second FOC, we obtain:

$$\tilde{\ell}_t^y = \frac{\gamma \left[Ah_t(1-e_t) - \mu(1-s)e_t\right]}{Ah_t(1+\gamma)}$$

Substituting for this in the first FOC yields:

$$\begin{aligned} \frac{\alpha + \pi\beta\sigma}{e_t} &= \frac{Ah_t + \mu(1-s)}{Ah_t \left(1 - e_t - \frac{\gamma[Ah_t(1-e_t) - \mu(1-s)e_t]}{Ah_t(1+\gamma)}\right) - \mu(1-s)e_t} \\ \frac{\alpha + \pi\beta\sigma}{e_t} &= \frac{(1+\gamma)\left[Ah_t + \mu(1-s)\right]}{((1-e_t)Ah_t - \mu(1-s)e_t)} \\ (\alpha + \pi\beta\sigma)(Ah_t) &= e_t\left[(1+\gamma)\left[Ah_t + \mu(1-s)\right] + (\alpha + \pi\beta\sigma)(Ah_t + \mu(1-s))\right] \\ \implies e_t = \frac{(\alpha + \pi\beta\sigma)Ah_t}{(1+\gamma + \alpha + \pi\beta\sigma)(Ah_t + \mu(1-s))} \end{aligned}$$

Hence leisure time at the young age is:

$$\begin{split} \tilde{\ell}_t^y &= \frac{\gamma \left[Ah_t(1-e_t) - \mu(1-s)e_t\right]}{Ah_t(1+\gamma)}\\ \tilde{\ell}_t^y &= \frac{\gamma \left[\frac{(1+\gamma)(Ah_t) + (1+\gamma)(\mu(1-s))}{(1+\gamma+\alpha+\pi\beta\sigma)(Ah_t+\mu(1-s))}\right]}{(1+\gamma)}\\ &\implies \tilde{\ell}_t^y = \frac{\gamma}{1+\gamma+\alpha+\pi\beta\sigma}\end{split}$$

Hence young age labor supply is now:

$$\begin{split} \ell_t^y &= 1 - e_t - \tilde{\ell}_t^y \\ &= 1 - \frac{(\alpha + \pi\beta\sigma) Ah_t}{(1 + \gamma + \alpha + \pi\beta\sigma) (Ah_t + \mu(1 - s))} - \frac{\gamma [Ah_t + \mu(1 - s)]}{(1 + \gamma + \alpha + \pi\beta\sigma) (Ah_t + \mu(1 - s))} \\ &= \frac{(1 + \gamma + \alpha + \pi\beta\sigma) (Ah_t + \mu(1 - s)) - (\alpha + \pi\beta\sigma) Ah_t - \gamma [Ah_t + \mu(1 - s)]}{(1 + \gamma + \alpha + \pi\beta\sigma) (Ah_t + \mu(1 - s))} \\ &= \frac{(1 + \alpha + \pi\beta\sigma) (\mu(1 - s)) + Ah_t}{(1 + \gamma + \alpha + \pi\beta\sigma) (Ah_t + \mu(1 - s))} \end{split}$$

8.4 Proof of Proposition 3

We have:

$$h_{t+1} = \frac{B\Theta h_t}{\left[Ah_t + \mu(1-s)\right]^{\sigma}} \equiv H(h_t)$$

Note that when s = 0 (laissez-faire), $H(h_t)$ is reduced to: $\frac{B\Theta h_t}{[(Ah_t+\mu)]^{\sigma}}$. We have: H(0) = 0, so that $\bar{h}^{1S} = 0$ is a stationary equilibrium. Note that:

$$H'(h_t) = \frac{B\Theta [Ah_t + \mu(1-s)]^{\sigma} - B\Theta h_t \sigma [Ah_t + \mu(1-s)]^{\sigma-1} A}{[Ah_t + \mu(1-s)]^{2\sigma}}$$

= $B\Theta \frac{Ah_t(1-\sigma) + \mu(1-s)}{[Ah_t + \mu(1-s)]^{\sigma+1}} > 0$

Note that, at the stationary equilibrium $\bar{h}^{1S} = 0$, we have

$$H'(0) = \frac{B\Theta}{\left[\mu(1-s)\right]^{\sigma}}$$

When $\mu(1-s) \ge (B\Theta)^{\frac{1}{\sigma}}$, we have that $|H'(0)| \le 1$, so that $\bar{h}^{1S} = 0$ is a stable stationary equilibrium. On the contrary, when $\mu(1-s) < (B\Theta)^{\frac{1}{\sigma}}$, we have H'(0) > 1, so that $\bar{h}^{1S} = 0$ is an unstable stationary equilibrium.

We have also:

$$H''(h_t) = \frac{\begin{bmatrix} [B\Theta (A(1-\sigma))] (Ah_t + \mu(1-s))^{1+\sigma} \\ -[B\Theta (Ah_t(1-\sigma) + \mu(1-s))] (1+\sigma) (Ah_t + \mu(1-s))^{\sigma} A \end{bmatrix}}{(Ah_t + \mu(1-s))^{2+2\sigma}} = AB\Theta (Ah_t + \mu)^{\sigma} \frac{[-\sigma(1-s)\mu - [(Ah_t(1-\sigma) + \mu(1-s))]\sigma]}{(Ah_t + \mu(1-s))^{2+2\sigma}} < 0$$

so that $H''(h_t) < 0$.

We have also:

$$\lim_{h \to 0} \frac{H(h_t)}{h_t} = \lim_{h \to 0} \frac{B\Theta}{\left[Ah_t + \mu(1-s)\right]^{\sigma}} = \frac{B\Theta}{\left(\mu(1-s)\right)^{\sigma}}$$

Suppose that $\mu(1-s) < (B\Theta)^{\frac{1}{\sigma}}$. Then the transition function $H(h_t)$ is above the 45° line when h is close to 0. Suppose that $\mu(1-s) \ge (B\Theta)^{\frac{1}{\sigma}}$. Then the transition function $H(h_t)$ is below the 45° line when h is close to 0.

We have also:

$$\lim_{h \to +\infty} \frac{H(h_t)}{h_t} = \lim_{h \to +\infty} \frac{B\Theta}{[Ah_t + \mu(1-s)]^{\sigma}} = 0$$

Thus the transition function $H(h_t)$ is below the 45° line when h is extremely large.

In sum:

Under $\mu(1-s) \geq (B\Theta)^{\frac{1}{\sigma}}$ the transition function $H(h_t)$ remains all along below the 45° line, so that 0 is the unique stationary equilibrium, which is also locally stable (see above).

Under $\mu(1-s) < (B\Theta)^{\frac{1}{\sigma}}$, the transition function $H(h_t)$ is above the 45° line in the neighborhood of 0, and below the 45° line when h is extremely large. Thus, by continuity of $H(h_t)$, there must exist an intersection of $H(h_t)$ with the 45° line, that is, a level of $h_t > 0$ such that $H(h_t) = h_t$. Existence of a strictly positive stationary equilibrium is thus guaranteed.

The uniqueness of the strictly positive stationary equilibrium follows from the monotonicity and concavity of $H(h_t)$. Since $H'(h_t) > 0$ and $H''(h_t) < 0$, the intersection of $H(h_t)$ with the 45° line is unique at a strictly positive level. Imposing $h_{t+1} = h_t = h$ yields:

 $h = \frac{B\Theta h}{\left(Ah + \mu(1-s)\right)^{\sigma}}$

If $h = \bar{h}^{2S} > 0$, this can be simplified to:

$$\bar{h}^{2S} = \frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{A} = \frac{B^{\frac{1}{\sigma}} \left[\frac{(\alpha + \pi\beta\sigma)A}{(1+\gamma + \alpha + \pi\beta\sigma)} \right] - \mu}{A}$$

The positivity of stationary equilibrium requires $(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s) > 0$ or $\mu(1-s) < (B\Theta)^{\frac{1}{\sigma}}$.

Regarding the local stability of the stationary equilibrium, this requires $|H'(\bar{h}^{2S})| < 1$. We have:

$$\begin{split} H'(\bar{h}^{2S}) &= B\Theta \frac{\left(A\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{A}\right)(1-\sigma) + \mu(1-s)\right)}{\left(A\left(\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)}{A}\right) + \mu(1-s)\right)^{1+\sigma}} \\ &= \frac{(B\Theta)^{\frac{1}{\sigma}} - \sigma\left[(B\Theta)^{\frac{1}{\sigma}} - \mu(1-s)\right]}{(B\Theta)^{\frac{1}{\sigma}}} < 1 \end{split}$$

Thus $|H'(\bar{h}^{2S})| < 1$. Therefore, when it exists, the unique strictly positive stationary equilibrium \bar{h}^{2S} is locally stable.

8.5 Proof of Lemma 4

We have: $\frac{\partial c^{mS}}{\partial s} = \mu \left[\frac{1}{(1+\gamma)} - \frac{\mu s}{\pi A B^{\frac{1}{\sigma}}} \right] - \mu \frac{\left((B\Theta)^{\frac{1}{\sigma}} - \mu(1-s) \right)}{\pi A B^{\frac{1}{\sigma}}}$. The derivative $\frac{\partial c^{mS}}{\partial s}$ is positive iff:

$$\mu \left[\frac{1}{(1+\gamma)} - \frac{\mu s}{\pi A B^{\frac{1}{\sigma}}} \right] - \mu \frac{\left((B\Theta)^{\frac{1}{\sigma}} - \mu (1-s) \right)}{\pi A B^{\frac{1}{\sigma}}} > 0 \quad \Longleftrightarrow \quad \frac{1}{(1+\gamma)} > \frac{(B\Theta)^{\frac{1}{\sigma}} - \mu + 2\mu s}{\pi A B^{\frac{1}{\sigma}}}$$
$$\Leftrightarrow \quad \underbrace{\frac{\pi A B^{\frac{1}{\sigma}}}{2\mu (1+\gamma)} - \frac{(B\Theta)^{\frac{1}{\sigma}}}{2\mu} + \frac{1}{2}}_{=\bar{s}} > s$$

The threshold \bar{s} is positive iff:

$$\pi AB^{\frac{1}{\sigma}} \frac{1}{1+\gamma} - (B\Theta)^{\frac{1}{\sigma}} + \mu > 0 \iff \frac{\pi B^{\frac{1}{\sigma}}}{1+\gamma} > \qquad \underbrace{\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu}{A}}_{=}$$

steady-state \bar{h}^2 at laissez-faire

Hence the positivity condition is equivalent to: $\frac{\pi B^{\frac{1}{\sigma}}}{1+\gamma} > \bar{h}^2$. The threshold is less than 1 if and only if:

$$\frac{1}{2} \left[\frac{\pi A B^{\frac{1}{\sigma}} \frac{1}{1+\gamma} - (B\Theta)^{\frac{1}{\sigma}} + \mu}{\mu} \right] < 1 \iff \frac{\pi B^{\frac{1}{\sigma}}}{1+\gamma} < \underbrace{\frac{(B\Theta)^{\frac{1}{\sigma}} - \mu}{A}}_{\text{steady-state } \bar{h}^2 \text{ at laissez-faire}} + \frac{2\mu}{A}$$

Thus the interiority of the threshold $0 < \bar{s} < 1$ requires: $\bar{h}^2 < \frac{\pi B^{\frac{1}{\sigma}}}{1+\gamma} < \bar{h}^2 + \frac{2\mu}{A}$.

8.6 Proof of Proposition 5

Full insurance is obtained when $U^{LL} - U^{SL} = 0$, that is, when s satisfies:

$$\beta \log \left(\left(\left(B\Theta \right)^{\frac{1}{\sigma}} - \mu(1-s) \right) \left(\frac{1}{1+\gamma} - \frac{\mu s}{\pi A B^{\frac{1}{\sigma}}} \right) \right) + \beta \gamma \log \left(\frac{\gamma}{1+\gamma} \right) = \Omega$$

Is full insurance achievable with $s \in [0, 1]$? To answer this question, note that when $s > \bar{s}$, further increases in s reduce $U^{LL} - U^{SL}$ by reducing consumption at mature age.

The conditions under which we can obtain full insurance can be derived by considering the entire policy range. The maximum higher education level is achieved when $s = s^{\max} = 1$. We then have:

$$e^{S\max} = \frac{(\Theta)^{\frac{1}{\sigma}}}{A} = \frac{\left[\frac{(\alpha+\pi\beta\sigma)A}{(1+\gamma+\alpha+\pi\beta\sigma)}\right]}{A} = \frac{(\alpha+\pi\beta\sigma)}{(1+\gamma+\alpha+\pi\beta\sigma)} < 1$$
$$\tau^{\max} = \frac{\mu\frac{(\Theta)^{\frac{1}{\sigma}}}{A}}{\pi A\frac{(B\Theta)^{\frac{1}{\sigma}}}{A}\frac{1}{1+\gamma}} = \frac{\mu(1+\gamma)}{\pi AB^{\frac{1}{\sigma}}}$$

We need $\tau^{\max} < 1$. Thus $\frac{\mu(1+\gamma)}{\pi AB^{\frac{1}{\sigma}}} < 1$, which requires $(1+\gamma) < \frac{\pi AB^{\frac{1}{\sigma}}}{\mu}$ or $\frac{1}{1+\gamma} > \frac{\mu}{\pi AB^{\frac{1}{\sigma}}}$. That condition is required for the non-negativity of consumption at mature age.

Substituting for these values in the above formula, we then obtain the welfare gap:

$$U^{LL} - U^{SL} = \beta \log \left(\left(B\Theta \right)^{\frac{1}{\sigma}} \left(\frac{1}{1+\gamma} - \frac{\mu}{\pi A B^{\frac{1}{\sigma}}} \right) \right) + \beta \gamma \log \left(\frac{\gamma}{1+\gamma} \right) - \Omega$$

Given the previous results, this welfare gap is the minimum achievable welfare gap between the long-lived and the short-lived.

Hence, we obtain that, since c^m is decreasing in s when $s > \bar{s}$, there exists an interior s that yields full insurance if and only if the minimum welfare gap is strictly negative, that is, if, when $s = s^{\max}$, we have:

$$\beta \log \left((B\Theta)^{\frac{1}{\sigma}} \left(\frac{1}{1+\gamma} - \frac{\mu}{\pi AB^{\frac{1}{\sigma}}} \right) \right) + \beta \gamma \log \left(\frac{\gamma}{1+\gamma} \right) < \Omega$$

That condition is necessary for an interior policy (s, τ) with 0 < s < 1 and $0 < \tau < 1$ that yields full insurance. If this is not satisfied, then $U^{LL} - U^{SL} \ge 0$ when $s = s^{\max}$, so that it implies that no interior $s < s^{\max}$ can bring $U^{LL} - U^{SL} = 0$. That condition is also sufficient for achieving full insurance, because reducing s below s^{\max} will increase consumption at mature age, and, hence, will make the welfare gap $U^{LL} - U^{SL} < 0$ converge progressively to 0.

8.7 **Proof of Proposition 6**

Regarding interiority conditions, remind that the FOC is:

_

$$\frac{1+\alpha}{(B\Theta)^{\frac{1}{\sigma}}-\mu(1-s)}-\pi\beta\left[\frac{2\mu s}{\pi AB^{\frac{1}{\sigma}}}-\frac{1}{1+\gamma}+\frac{(B\Theta)^{\frac{1}{\sigma}}+\mu}{\pi AB^{\frac{1}{\sigma}}}\right]=0$$

Given that the LHS is decreasing in s, the existence of a strictly positive subsidy $s^U > 0$ requires:

$$\frac{1+\alpha}{(B\Theta)^{\frac{1}{\sigma}}-\mu}-\pi\beta\left[-\frac{1}{1+\gamma}+\frac{(B\Theta)^{\frac{1}{\sigma}}+\mu}{\pi AB^{\frac{1}{\sigma}}}\right]>0$$

Regarding the interiority of $s^U < 1$, this requires:

$$\frac{1+\alpha}{(B\Theta)^{\frac{1}{\sigma}}} - \pi\beta \left[\frac{2\mu}{\pi AB^{\frac{1}{\sigma}}} - \frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right] < 0$$

Further calculations can allow us to derive the formula for the optimal subsidy s under utilitarianism. Indeed, from the FOC for optimal s under the utilitarian SWF, we have:

$$s^{2} \frac{2\beta\mu^{2}}{AB^{\frac{1}{\sigma}}} + s \left[\frac{2\mu\beta \left[(B\Theta)^{\frac{1}{\sigma}} - \mu \right]}{AB^{\frac{1}{\sigma}}} + \pi\beta\mu \left[-\frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}} \right] \right]$$
$$- \left[1 + \alpha + \pi\beta \left[(B\Theta)^{\frac{1}{\sigma}} \right] \left[\frac{1}{1+\gamma} - \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}} \right] + \pi\beta\mu \left[-\frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}} \right] \right]$$
$$= 0$$

Let us denote:

$$\begin{aligned} a &\equiv \frac{2\beta\mu^2}{AB^{\frac{1}{\sigma}}} \\ b &\equiv \frac{2\mu\beta\left[(B\Theta)^{\frac{1}{\sigma}} - \mu\right]}{AB^{\frac{1}{\sigma}}} + \pi\beta\mu\left[-\frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right] = \mu\beta\frac{(1+\gamma)\left[3(B\Theta)^{\frac{1}{\sigma}} - \mu\right] - \pi AB^{\frac{1}{\sigma}}}{AB^{\frac{1}{\sigma}}(1+\gamma)} \\ c &\equiv -\left[1+\alpha+\pi\beta\left(B\Theta\right)^{\frac{1}{\sigma}}\left[\frac{1}{1+\gamma} - \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right] + \pi\beta\mu\left[-\frac{1}{1+\gamma} + \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right]\right] \\ &= -\left[1+\alpha+\pi\beta\left[(B\Theta)^{\frac{1}{\sigma}} - \mu\right]\left[\frac{1}{1+\gamma} - \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}}\right]\right] \end{aligned}$$

We have thus:

$$\begin{split} \rho &= \left[\mu \beta \frac{(1+\gamma) \left[3(B\Theta)^{\frac{1}{\sigma}} - \mu \right] - \pi AB^{\frac{1}{\sigma}}}{AB^{\frac{1}{\sigma}}(1+\gamma)} \right]^2 + 8 \frac{\beta \mu^2}{AB^{\frac{1}{\sigma}}} \left[1 + \alpha + \pi \beta \left[(B\Theta)^{\frac{1}{\sigma}} - \mu \right] \left[\frac{1}{1+\gamma} - \frac{(B\Theta)^{\frac{1}{\sigma}} + \mu}{\pi AB^{\frac{1}{\sigma}}} \right] \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}} \right)^2 \left[3(B\Theta)^{\frac{1}{\sigma}} - \mu - \frac{\pi AB^{\frac{1}{\sigma}}}{(1+\gamma)} \right]^2 + \frac{8\beta^2 \mu^2 \left[\frac{(1+\alpha)AB^{\frac{1}{\sigma}}}{\beta} + \left[(B\Theta)^{\frac{1}{\sigma}} - \mu \right] \left[\frac{\pi AB^{\frac{1}{\sigma}}}{(1+\gamma)} - \left((B\Theta)^{\frac{1}{\sigma}} + \mu \right) \right] \right] \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}} \right)^2 \left[\left[3(B\Theta)^{\frac{1}{\sigma}} - \mu - \frac{\pi AB^{\frac{1}{\sigma}}}{(1+\gamma)} \right]^2 + 8 \left[\frac{(1+\alpha)AB^{\frac{1}{\sigma}}}{\beta} + \left[(B\Theta)^{\frac{1}{\sigma}} - \mu \right] \left[\frac{\pi AB^{\frac{1}{\sigma}}}{1+\gamma} - \left((B\Theta)^{\frac{1}{\sigma}} + \mu \right) \right] \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}} \right)^2 \left[\left(3(B\Theta)^{\frac{1}{\sigma}} - \mu - \frac{\pi AB^{\frac{1}{\sigma}}}{(1+\gamma)} \right)^2 - 2 \frac{(3(B\Theta)^{\frac{1}{\sigma}} - \mu)\pi AB^{\frac{1}{\sigma}}}{(1+\gamma)} \\ &+ 8 \left[\frac{(1+\alpha)AB^{\frac{1}{\sigma}}}{(1+\gamma)} - 8 \left((B\Theta)^{\frac{1}{\sigma}} - \mu \right] \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}} \right)^2 \left[\left(3(B\Theta)^{\frac{1}{\sigma}} - \mu \right)^2 + \left(\frac{\pi AB^{\frac{1}{\sigma}}}{(1+\gamma)} \right)^2 - 2 \frac{(3(B\Theta)^{\frac{1}{\sigma}} - \mu)\pi AB^{\frac{1}{\sigma}}}{(1+\gamma)} \\ &+ 8 \left[\frac{(1+\alpha)AB^{\frac{1}{\sigma}}}{(1+\gamma)} - 8 \left((B\Theta)^{\frac{1}{\sigma}} - \mu \right] \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}} \right)^2 \left[(B\Theta)^{\frac{2}{\sigma}} + 9\mu^2 - 6 (B\Theta)^{\frac{1}{\sigma}} + AB^{\frac{1}{\sigma}} \frac{\pi \beta}{(2(B\Theta)^{\frac{1}{\sigma}} - 6\mu + \pi AB^{\frac{1}{\sigma}}} \right) + 8 \left(1 + \alpha \right) \left(1 + \gamma \right) \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}\beta(1+\gamma)} \right)^2 \left[\beta^2 (1+\gamma)^2 \left[(B\Theta)^{\frac{2}{\sigma}} + 9\mu^2 - 6 (B\Theta)^{\frac{1}{\sigma}} - 6\mu + \pi AB^{\frac{1}{\sigma}} \right) + 8 \left(1 + \alpha \right) \left(1 + \gamma \right) \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}\beta(1+\gamma)} \right)^2 \left[\beta^2 (1+\gamma)^2 \left[(B\Theta)^{\frac{2}{\sigma}} + 9\mu^2 - 6 (B\Theta)^{\frac{1}{\sigma}} - 6\mu + \pi AB^{\frac{1}{\sigma}} \right) + 8 \left(1 + \alpha \right) \left(1 + \gamma \right) \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}\beta(1+\gamma)} \right)^2 \left[\beta^2 (1+\gamma)^2 \left[(B\Theta)^{\frac{2}{\sigma}} + 9\mu^2 - 6 (B\Theta)^{\frac{1}{\sigma}} - 6\mu + \pi AB^{\frac{1}{\sigma}} \right) + 8 \left(1 + \alpha \right) \left(1 + \gamma \right) \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}\beta(1+\gamma)} \right)^2 \left[\beta^2 (1+\gamma)^2 \left[(B\Theta)^{\frac{2}{\sigma}} + 9\mu^2 - 6 \left(\beta^2 (B\Theta)^{\frac{1}{\sigma}} - 6\mu + \pi AB^{\frac{1}{\sigma}} \right) + 8 \left(1 + \alpha \right) \left(1 + \gamma \right) \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}\beta(1+\gamma)} \right)^2 \left[\beta^2 (1+\gamma)^2 \left[(B\Theta)^{\frac{2}{\sigma}} + 9\mu^2 - 6 \left(\beta^2 (B\Theta)^{\frac{1}{\sigma}} \right) + 8 \left(1 + \alpha \right) \left(1 + \gamma \right) \right] \\ &= \left(\frac{\mu \beta}{AB^{\frac{1}{\sigma}}\beta($$

Excluding the negative root, we thus have that the optimal subsidy is, under utilitarianism, given by:

$$s^{U} = \frac{1}{4\beta\mu(1+\gamma)} \begin{bmatrix} -\left[(1+\gamma)\beta\left[3\left(B\Theta\right)^{\frac{1}{\sigma}} - \mu\right] - \pi\beta AB^{\frac{1}{\sigma}}\right] \\ + \sqrt[2]{\beta^{2}(1+\gamma)^{2}\left[(B\Theta)^{\frac{2}{\sigma}} + 9\mu^{2} - 6\left(B\Theta\right)^{\frac{1}{\sigma}}\mu\right]} \\ + \beta(1+\gamma)AB^{\frac{1}{\sigma}}\left[\pi\beta\left(2\left(B\Theta\right)^{\frac{1}{\sigma}} - 6\mu + \pi AB^{\frac{1}{\sigma}}\right) + 8\left(1+\alpha\right)\left(1+\gamma\right)\right] \end{bmatrix}$$