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Critical dimensions in the empirical measurement of common shareholding

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Full length article

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ABSTRACT

The debate on common shareholding and its potential antitrust effects is currently on the agenda of major institutions worldwide. Discussions point to the need for improved empirical quantification of this phenomena. This work presents a flexible, multifaceted statistical framework for a set of new common shareholding indicators, covering both firm and investor perspectives, which can be adopted under different economic models. Many indices currently used in the literature fall within this framework as special cases. Aggregation at market level yields suitable industry-level indicators, providing policymakers with tools to evaluate the extent of common ownership in strategic markets. The indices are tested using firm-level data for European Mobile Network Operators in 2007–2021, showing a sector with concentrated ownership under large corporate groups, but also the presence of institutional investors with extensive ownership across the major firms.

1. Introduction

In December 2017, the OECD organised in Paris a Competition Policy Roundtable to discuss the rise of the relatively new phenomenon of common ownership.¹ According to this report, the previous ten years had been characterised by a “*rapid growth in passively-managed investment funds, [which] has had a significant impact on the ownership structure of large firms in several industries*”. Common ownership, or the simultaneous ownership of shares in many firms active in the same market, was also on the radar of other industry watchdogs and stakeholders.² Subsequently in May 2018, the European Corporate Governance Institute (ECGI) dedicated a focus panel of its Annual Members’ Meeting to “Common Ownership: Antitrust Meets Corporate Governance”. The ECGI Event Report³ noted attendants raised concerns regarding “*potential collusion between competing firms having the same shareholders*”. The

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¹ The report is titled “Common ownership by institutional investors and its impact on competition” and is accessible at <http://www.oecd.org/daf/competition/common-ownership-and-its-impact-on-competition.htm>. This Roundtable built on a previous one held in 2008 entitled “Minority Shareholdings and Interlocking Directorates”.

² In the literature, common shareholders are mostly known as “common owners”. The term “common owners” can be somehow misleading, as these investors do not actually own companies, they rather own (usually small) participations in many companies. The two terms will be used interchangeably in this paper.

³ https://ecgi.global/sites/default/files/events/2018_annual_members_meeting.pdf

same year, the “Viewpoint” of the International Corporate Governance Network issued in October (see ICGN, 2018), and a public hearing organised by the Federal Trade Commission (FTC) in December in the US⁴ concluded that the impact of common ownership on competition required further understanding and analysis.

These kind of debates are now becoming more common among policymakers. Although traditionally common ownership has not been seen as an antitrust issue, in recent years researchers and policy makers have started to consider its potential anticompetitive effects. Following seminal work on anticompetitive effects of common ownership among U.S airlines (Azar et al., 2018), numerous empirical studies have analysed the impact in specific sectors, among which agrifood (Clapp, 2019; Backus et al., 2021a; Torshizi and Clapp, 2021), airlines (Kennedy et al., 2017; Azar et al., 2018; Schmalz, 2018; Dennis et al., 2022), banking (Schmalz, 2018; Azar et al., 2021), energy (Argentesi et al., 2021) and pharmaceuticals (Newham et al., 2018); Banal-Estañol et al. 2021; (Xie, 2021)). These various studies underline the relevance of common ownership, but also highlight empirical challenges and differing evidence regarding the effects of common shareholding on competition. In 2021, the Antitrust Bulletin dedicated its March special issue to the topic of common ownership and its anticompetitive effects. In this issue, Schmalz (2021) presents a comprehensive survey of recent studies on this topic, concluding that empirical evidence has confirmed anticompetitive effects and that the study of the economic channels implementing anticompetitive incentives has advanced. Along this line, Tzanaki (2022) looks from a corporate governance perspective at varieties and mechanisms of common shareholding, and at the plausibility of common owners’ anticompetitive strategies, discussing possible policy implications.

Debate on the relevant methodological topics is ongoing. At the 2020 Association of Competition Economics special panel session on common ownership, top scholars debated at length the question of empirical measurement. They highlighted that research dealing with the empirical quantification of common shareholding is at an advanced stage, but a framework for quantifying common ownership remains a key objective. Few recent scholarly articles propose tentative solutions to the measurement challenge, such as profit weights (Backus et al., 2019, 2021a,b,c); (Antón et al., 2023), model-based measures (Gilje et al., 2020), or data-derived measures (He and Huang, 2017).⁵ Schmalz (2021) highlights that any proposed measure has an intrinsic synthetic nature, which will capture in turn different aspects of an industry or market, and that therefore one single best measure of common ownership does not exist, encouraging researchers to consider the relevant economic context when adopting a certain measure.

This paper contributes to the quantification of common ownership with a flexible, multidimensional framework that can be adopted within different economics models for studying the impact on market outcomes. Many indices currently used in the literature fall within this framework as special cases. This new framework exploits solely the ownership links between market actors through indices based on sparse matrix theory and network analysis, avoiding the shortcomings of other common ownership measures, such as subjective assumptions about control weights or the computation of market shares.⁶ The indices explore both the firm’s and the investor’s perspective, considering interactions between the two but also within peer groups. There are a number of useful applications for this proposed framework, which are illustrated here through a real-data example for the Mobile Telecoms market in the EU.⁷

In the first part of this paper (Section 2), we review the current knowledge, together with its growing critiques, and identify the main measurement issues to be tackled. In Sections 3 and 4 a series of new indices of common shareholding are proposed, based on balance sheet and ownership firm-level data, under a unifying statistical framework with detailed mathematical properties. Many of the indices currently used in the literature can be identified as special cases falling within this framework. The new indices cover both the firm and the investor’s perspectives, and are then aggregated to obtain suitable industry-level measurements of common ownership. Section 5 goes over the application of these new indices in the context of common ownership. Finally, Section 6 presents an empirical application of the proposed measures using firm-level financial and ownership data for Mobile Network Operators (MNOs) active in Europe over the period 2007–2021. The last Section concludes.

2. Measuring common ownership

All debates mentioned earlier point to the increased need for developing sound measures of common ownership and of its potential impacts. This Section reviews some of the measurement approaches used in past literature, together with their main drawbacks, and recent developments in this area.

Table 1 summarises the main measures outlined below, together with their main limitations.

The most popular tool used to assess the effects of common shareholding was, until recently, the so-called Modified Herfindahl–Hirschman Index (MHHI), a market-level indicator that captures the distortion introduced in market competition by the presence of common shareholders. It does so by correcting the Herfindahl–Hirschman Index (HHI) of competition according to the ownership and control shares of common shareholders in competing companies. The MHHI however presents several drawbacks. Some relate to the computation of the MHHI itself — it requires computing the market shares of the firms, and also control weights of shareholders in firms which are difficult to determine in practice. In addition, the equations generally used to compute the index in the empirical

⁴ US FTC public hearing on “Common ownership”, 6 December 2018, <https://www.ftc.gov/news-events/events-calendar/ftc-hearing-8-competition-consumer-protection-21st-century>.

⁵ Section 2 below discusses more in detail these measures.

⁶ The statistical framework was initially developed in the European Commission report “Common Shareholding in Europe” (Rosati et al., 2020), where it was illustrated through a simplified market example. The present work largely draws from Rosati et al. (2020).

⁷ Rosati et al. (2022a) present an application of the new indices proposed here to test the possible effects of common ownership on competitiveness in the EU beverages industry.

Table 1
Summary of common ownership (CO) measures.

Index	Type of measure	Description	Limitations
MHHI	market-level concentration measure	competition measure; direct policy interpretation	market and not firm-level; needs the calculation of market shares and control weights; not a measure of CO; misspecification and endogeneity issues in empirical applications
Profit weights (Backus et al.)	model-based; firm-level measure	define weight a firm puts on competitors' profits	need assumptions on control weights
GGL (Gilje et al.)	model-based; firm-level measure	measures managers' incentives shifts due to CO	rules out strategic interactions of firms; not suitable for some industries
Descriptive measures (He and Huang)	data-driven; market- and firm-level	firm-level explanatory variables used to model effects of CO	not model-based
Our approach	data-driven; market-, firm- and investor-level	multidimensional framework; includes most above measures as special cases; measures distortions and links created by CO at firm–firm, investor–investor, firm–investor levels; measures market structures due to CO; measures network effects	not model-based

literature suffer from a misspecification problem, which may generate a (positive) correlation between price and the measure of common shareholding, even in the absence of a causal effect of common shareholding on price (O'Brien and Waehrer, 2017). Finally, some of the factors that drive prices may also affect institutional investors' stock purchasing decisions, and consequently the financial shares of investors, which then become endogenous (Kennedy et al., 2017; O'Brien and Waehrer, 2017). More importantly, it fails to measure directly the extent of common shareholding itself.

More recently, Backus, Conlon and Sinkinson introduced the “profit weights” (see Backus et al., 2019, 2021a,b,c), a measure representing the weight a firm puts on its competitors' profits. These weights arise within a firm's objective function under common ownership, where the firm maximises a combination of its own profits together with its competitors' profits, duly weighted by the proposed weights. In the authors' own words the “*profit weights [...] are the channels through which common ownership [...] affects firm behaviour*”. Similarly to the MHHI, the profit weights need for their calculation the choice of control weights (Pareto weights) of shareholders on firms, representing the influence of the investors on firm decisions. The typical choice throughout the literature has been one of proportional control (where the control weights equal the ownership shares), but other alternatives are possible. The profit weights have been used in some recent empirical studies such as Antón et al. (2023) and Boller and Scott Morton (2020).

Gilje et al. (2020) derive a bi-directional, pair-level measure of common ownership aiming at capturing the extent to which common ownership shifts managers' incentives to internalise externalities. The index accounts for the shares held by common investors in two competing companies, as well as for the relative weight of each firm in the investors' portfolios. The measure is based on a model where assumptions are made on how managers deal with externalities imposed on one another by commonly-owned firms, specifying a function capturing how attention is allocated across portfolio companies. However, as pointed out by Schmalz (2021), the measure is not suitable for capturing the competitive effects of common ownership in certain industries, given that the model underlying this measure rules out strategic interactions of firms.

As alternatives, other studies have generally limited the measurement of common shareholding to a small set of descriptive measures. Examples are the proportion of common shareholders among all the investors present in a market; the proportion of firms that are cross-held by a common holder, at a certain level of ownership; the number of competitors linked through a common shareholder; the proportion of a firm's shares held by common shareholders, or still the shares held by common shareholders in a firm's competitors, and so on (see for instance, He and Huang, 2017). Such descriptive measures have been used as firm-level explanatory variables in models trying to capture the effect of common shareholding on markets, together with other measures capturing the corporate ownership structure, such as the proportion of atomistic shareholders of a firm.

However, several other aspects of investors' behaviour and of portfolios' composition can help draw a more precise picture of the phenomenon. The same applies to the analysis of the firms' shareholding structures, which can reveal interesting patterns of overlap in a given market. Focusing on the behaviour of individual investors, we consider several dimensions of interest. A general overview of the degree of connectedness of a market due to the presence of common shareholders is a starting point, but the investment decisions are driven by a variety of objectives, which determine not only how many and which firms to include in an investor's portfolio, but also the amount to be held in each of the chosen companies. The distribution of investments within a portfolio can also vary, being more or less concentrated around few players rather than equally spread across all chosen firms, revealing different shareholders' strategies. Another aspect of interest is the comparison of portfolios of concurrent investors. This allows for the analysis of possible market-level structures, in particular considering whether a market is split into segments allotted to different stakeholders or – on the opposite side – total access to any company is available to all potential investors. Finally, the

consideration of the shareholders' type (such as industrial company, financial company, public authority, individual, etc.) is also of interest to investigate possible differentiation of investments across certain groups.

A complementary perspective of the one proposed above regards the presence of common shareholders across individual firms' ownership structure. Here, the objective becomes the study of the shareholder structure of a given firm, and the assessment of the degree of overlap with other competitors' ownership information. The stronger the similarity of the shareholding structures of competitors, the stronger the potential distortion in competition due to common investors. It is crucial then to assess the strength of the firm–firm links induced by common owners. Furthermore, the degree of overlap will be contingent on the shareholders considered for a given firm, and will vary based on the threshold of equity considered in the analysis.

We consider in the following some methodological strategies to construct indices of the extent common ownership, which capture the various aspects mentioned above.

2.1. Sparsity and networks methods in the context of common ownership

A starting point for the analysis of common ownership is the matrix representation of a given market. The utility of this representation is evident when the analysis shifts into a more formal framework where the properties of these matrices can be leveraged to compute the relevant indices.

A simplified representation of the ownership structure of a market can be obtained through a table, where – for instance – each row corresponds to a shareholder and each column to a firm:

	F1	F2	...
SH1	e_{11}	e_{12}	...
SH2	e_{21}	e_{22}	...
...

The elements e_{ij} of the table can either report the corresponding ownership share, in which case we can name it **ownership matrix (OM)**, or simply report a value of one if a link exists between a firm and an owner, zero otherwise (**relation matrix - RM**). The empirical structure of this matrix in the ownership context plays a relevant role in the choice of appropriate statistical techniques for its analysis. In fact, the number of investors is typically way larger than the number of firms, with an average shareholding structure easily presenting dozens of owners; on the other hand, the large majority of shareholders only invest in one firm, hence displaying no link to the remaining competitors.⁸ This gives rise to a very large matrix where the majority of the elements are zero.

Several different statistical techniques that extract patterns from given matrices are available, both for the case of numerical and of binary (relational) matrices. Such techniques allow for the identification of matrices' characteristics, as well as for the calculation of indices quantifying specific aspects of the relationships represented in the matrix, and are therefore a valuable starting point for the analysis of CO. Given the multiplicity of possible matrix aspects to be considered, we shall analyse in this work the measures related to the concept of sparsity of a matrix, which is more directly linked to the CO problem. In fact, the concept of sparsity has to do with the representation of a phenomenon where only a small number of coefficients contain a large proportion of the total information, the remaining elements of the representation being negligible, in most cases considered just noise. In matrix language, a sparse matrix or vector is such that most of its elements are zero, just like the typical empirical structure of a market represented through the ownership and relation matrices — as just noted above.

The distance or similarity between matrices will also be analysed. The similarity between matrices is defined according to a specific metric used to determine the distance between two given matrices. If one of the two matrices is a benchmark – for example the most sparse matrix in a specific context – the distance or similarity measure can be used to identify the degree of a certain phenomenon with respect to the given benchmark. In the study of CO, a specific benchmark matrix can be easily defined, representing for instance absence of CO, rather than total interconnection between owners and firms, or any other market structure of interest.

Finally, network methods will also be considered, applied separately to the network of investors and to the network of firms. The assessment of the strength of the links existing in these two networks will be performed applying the standard network indices, but also using some of the matrix methods mentioned above, applied to the matrix representation of the network links.

In the following sections, the aforementioned concepts and statistical methods will be reviewed, and their relevance in the context of CO measurement analysed.

3. Sparsity

The concept of sparsity is often linked to definitions of inequality or diversity of the distribution of a phenomenon in a population of size N , say (c_1, \dots, c_N) . Although the literature presents different interpretations and measures of sparsity,⁹ a common agreement is that a distribution with all its information concentrated in one coefficient, and all other zero, is the most sparse. On the other

⁸ For example, in the EU Mobile Telecoms industry studied in Section 6, more than two-thirds of the investors are “single owners” i.e. hold stakes only of one of the firms active in this industry, consistently throughout the period of observation. This is in line with what observed in other EU industries, such as Oil&Gas and Electricity (Rosati et al., 2022b), or Beverages (Rosati et al., 2022a), but also with empirical findings for the US listed firms (He and Huang, 2017).

⁹ A comprehensive review can be found in Hurley and Rickard (2009).

Table 2
Some common sparsity measures and their properties.

No.	Measure	Definition	RH	Sc	RT	Cl	BG	Ba
1.	ℓ^0/N	$\#\{k : c_k = 0\}/N$		✓		(✓)		✓
2.	ℓ^0_e/N	$\#\{k : c_k \leq \epsilon\}/N$				(✓)		✓
3.	$-\ell^1/N$	$-\frac{1}{N} \sum_k c_k$			✓	(✓)		(✓)
4.	$-\ell^p/N$	$-\frac{1}{N} (\sum_k c_k^p)^{1/p}, \quad 0 < p < 1$	✓		✓			(✓)
5.	ℓ^2/ℓ^1	$\sqrt{\sum_k c_k^2} / (\sum_k c_k)$	✓	✓			✓	
6.	$-\log/N$	$-\frac{1}{N} \sum_k \log(1 + c_k^2)$			✓	(✓)		(✓)
7.	$N\kappa_4$	$N \sum_k c_k^4 / (\sum_k c_k^2)^2$		✓	✓	(✓)	✓	(✓)
8.	Hoyer	$\frac{1}{\sqrt{N-1}} \left(\sqrt{N} - \frac{\sum_k c_k}{\sqrt{\sum_k c_k^2}} \right)$	✓	✓	✓		✓	✓
9.	pq -mean $p < q$	$-\left(\frac{1}{N} \sum_k c_k^p \right)^{1/p} / \left(\frac{1}{N} \sum_k c_k^q \right)^{1/q}$	$p \leq 1, q > 1$	✓	✓	✓	✓	✓
10.	Gini	$1 - \frac{2}{N \sum c_k} \sum_{k=1}^N (N - k + \frac{1}{2}) c_{(k)}$	✓	✓	✓	✓	✓	✓

Notes: Properties as presented earlier: RH = Robin Hood; Sc = Scaling; RT = Rising Tide; Cl = Cloning; BG = Bill Gates; Ba = Babies. Properties in brackets are only valid for the normalised version of the measures.

hand, there is agreement that the least sparse distribution is found when the information is evenly spread over all coefficients. In the following, this will be the reference definition of sparsity. In the case of corporate ownership, a sparse investment behaviour would correspond generically to a portfolio with high concentration of ownership in few firms, or, from a firm's perspective, a shareholding structure with few shareholders owning large stakes. Low sparsity would be observed, instead, in case of more widespread investments across the market, typically with minority participation.¹⁰

3.1. Some common sparsity indices

There is a large set of sparsity measures in the literature, coming mainly from the fields of signal processing and information theory. The most common are the Kurtosis, the Gini Index, the Hoyer measure, the pq -means, the ℓ_p norms and their combinations. These and other measures are discussed, for instance, in Karvanen and Cichocki (2003), Rickard and Fallon (2004), Hurley and Rickard (2009), Zonoobi et al. (2011), Pastor et al. (2013, 2015).

The choice between alternative measures can be motivated by their mathematical properties, which represent minimum criteria a “good” measure of sparsity should satisfy. Six main properties are recognised to be desirable for a sparsity measure¹¹: the Dalton's Laws (“Robin Hood”, scaling, “rising tide” and “cloning”), the “Bill Gates” and the “Babies” properties. Their aim is to guarantee that a sparsity index goes in the right direction when a change occurs in the underlying distribution. The properties are described in detail in Table A.1 in Appendix A, where their meaning and relevance in the context of common ownership is also discussed.

Table 2 presents some among the most common sparsity measures with their properties. Again, the coefficients of the distribution under analysis are denoted by c_k , $k = 1 \dots, N$, while their ordered set is indicated by $c_{(k)}$ (in increasing order). Measures 1–4 and 6–7 are presented in their normalised version, which accounts for the length N of the vector representing the distribution.

The measures in Table 2 do not present a unified notation in the literature; we follow here the suggestions of Hurley and Rickard (2009), where some measures have been modified – either with a minus sign or by changing the direction of some inequality – in order to obtain an homogeneous interpretation in the sense that an increase in sparsity yields a positive increase in the corresponding measure. In particular, notice that measure 1 is usually defined in the literature as the count of non-zero values, but this would go in the opposite direction, increasing when sparsity decreases. For this reason here the definition is reversed, counting the number of zeros instead. For all these measures, the less sparse the distribution, the smaller the value of the index, the value increasing with sparsity.

As we can see from Table 2, most measures fulfil many of the presented properties, if not all. Karvanen and Cichocki (2003) compare measures 1–4 and 6–7. Quére and Frélicot (2012) compare 7 and 8 through a set of simulations on binary (0–1) distributions, testing their performance in the context of fuzzy partitions. Measure 10 is considered, among others, in Rickard and Fallon (2004) and Zonoobi et al. (2011). Hurley and Rickard (2009) and Pastor et al. (2015) propose more comprehensive accounts of original and normalised measures, other additional measures and properties not discussed here, as well as proofs of the fulfilment of the respective properties.

¹⁰ A more detailed analysis of the application of sparsity concepts to corporate ownership is discussed in Appendix A.

¹¹ See for example Hurley and Rickard (2009).

Measures 1 and 2 simply compute the proportion of zero or negligible elements of the distribution; the higher the proportion, the more concentrated the distribution, i.e. the higher the sparsity. In measure 6, any zero coefficient gives a zero log value in the summation, contributing towards a smaller total; the minus sign reverts the direction of this effect, so that the more the null coefficients in the sum, the higher the sparsity measure. Measure 7 is based on the Kurtosis coefficient κ_4 – named by analogy to the measure of peakedness of a probability distribution – while measure 10 is the well-known Gini Coefficient of inequality.

Measures 3–5 and 8–9 are based on ℓ^p -type norms, which are sums of the coefficients each raised to a certain power p , sum that in turn is raised to the power $1/p$ in order to go back to the original scale of the coefficients. The ℓ^p -type norms do not have in general an intuitive interpretation, except for the ℓ^1 norm which is simply the average. However, a special note should be devoted to two measures, namely ℓ^0 and ℓ^2 , from which several other measures in the table are derived.

The ℓ^0 index is the base for the popular *density* measure, giving the proportion of non-zero elements of a vector. The density concept is complementary to sparseness, and has several applications both to matrices and to networks, which will be discussed later. Measure 2 restricts the attention to relevant coefficients only, thus ignoring all those of a negligible size. A corrected density index can be proposed according to this more restrictive exclusion criterion, only considering values above a certain threshold.

The ℓ^2 norm is also known as *Euclidean norm*, and for vector $C = (c_1, \dots, c_N)$ of coefficients is given by:

$$\ell^2(C) = \sqrt{\sum_{k=1}^N c_k^2}$$

This is one of the most popular norms in several fields of application; it is commonly used to compute the “length” of vectors of size N , since it corresponds to the distance from the origin to the point C in an N -dimensional space. The vectors presenting larger coefficients will have a larger ℓ^2 norm, showing that they are further away from the origin (the null vector). Based on this norm, several distance measures have been developed, especially in error minimisation contexts, such as the *least squares* criterion (which minimises the square of the ℓ^2 norm), or the *mean square error* measurement (divides by N the square of the ℓ^2 norm).

Notice that the ℓ^2 norm gives higher weight to larger coefficients, by squaring their values, contrary to the simple average ℓ^1 . For this reason, the derived measure number 5, given by the ratio of ℓ^2 to ℓ^1 , can be seen as assessing the relative weight of larger coefficients over the coefficients’ total, hence showing larger values for more concentrated distributions, i.e. for higher sparsity. For example, considering three distributions with same total value and increasing concentration – say (2, 2, 2), (4, 1, 1) and (5, 1, 0) – measure ℓ^2/ℓ^1 takes values 0.58, 0.71 and 0.85 respectively. In the sparsest case, i.e. (6, 0, 0), we get $\ell^2/\ell^1 = 1$.

3.1.1. Extension of sparsity measures to ownership and relation matrices

Sparsity measures can be of particular interest in the framework of CO when considering a given investor and the relationship with all the firms in its portfolio. Referring to the matrix representation of a market introduced in Section 2.1, the dimension of interest is the so called row-wise sparsity,¹² given that each row in the matrix corresponds to an investor’s portfolio, reporting the full shareholder-firm relationships. If we look at a row, the sparsest case occurs when an owner only holds shares of one firm in the market (whatever the percentage), the minimum sparsity being achieved when the owner owns shares in all firms in the market, at a constant percentage, i.e. does not show preference for any firm in particular. This also represents the most extreme form of common ownership, an investor reaching all competitors active in a market.

Consider now the application of the sparsity measures to either the OM or RM matrix, say X , whose elements shall be denoted by x_{ij} , where again index $i = 1 \dots I$ spans the rows i.e. the owners, and index $j = 1 \dots J$ the firms in the columns. Recall that the elements represent, respectively, either the ownership share or the presence/absence of a owner–firm link.

Any of the sparsity measures of Table 2 can be applied to each row i of X , and then aggregated across rows according to some criterion. Denoting generically by $S(x_i)$ a sparsity measure applied to row i (i.e. to elements x_{ij} , with fixed i), an overall measure of row-wise sparsity for matrix X is given by $S(X)$ defined as follows:

$$S(X) = \sum_{i=1}^I S(x_i)$$

This type of aggregation is mentioned in Rickard and Fallon (2004) as a common measure of matrix sparsity, however other ways of summarising rows information can be considered.

Table 3 proposes some alternative aggregation methods of row sparsity indices $S(x_i)$, discussing the interpretation of the resulting matrix measures. Besides the sum of row indices, the average across rows is considered, as well as the median and other relevant percentiles; as alternatives, the maximum or minimum row sparsity are also of interest, representing, respectively, the value of sparsity corresponding to the owner with most concentrated investments (in the limit *not* a common owner), and to the most “democratic” owner, investing more equally across firms in the market (in principle a common owner). In general, low values of $S(X)$ raise concerns in the CO context, showing more evenly spread investments of owners across firms in the market, going in the direction of CO.

Among the possible row-sparsity measures that can be used to construct the matrix indices, measure 5 from Table 2 will be used to discuss the application to the OM and RM matrices, the remaining measures presenting in general analogous interpretations. The special case of measure 1, given its links with other fields in the literature, will be discussed separately below.

¹² See more details in Appendix A.

Table 3
Matrix sparsity measures $S(X)$ constructed applying different row aggregation criteria.

No.	Criterion	$S(X)$	Interpretation
1.	Sum	$\sum_{i=1}^I S(x_i)$	Total owners' sparsity. A high value denotes high concentration in the investment behaviour, i.e. owners tend to hold shares of few firms. A low value denotes tendency of owners to distribute investments across firms.
2.	Average	$\frac{1}{I} \sum_{i=1}^I S(x_i)$	Average owners' sparsity. Same as above, but normalised by I , the number of owners in the market.
3.	Median	Med $S(x_i)$	Median sparsity. 50% of owners have an investment behaviour with sparsity lower than $S(X)$. If $S(X)$ is high, then owners do not diversify much investments; if low, there is stronger tendency for CO.
4.	p^{th} percentile	$Q_p S(x_i)$	Same as above, with now $p\%$ of owners having investments with sparsity lower than $S(X)$. Threshold that determines the degree of sparsity of the $p\%$ most "democratic" owners.
5.	Minimum	$\min_i S(x_i)$	Sparsity of most "democratic" market owner, holding a very similar proportion of shares in all market firms.
6.	Maximum	$\max_i S(x_i)$	Sparsity of most "unequal" market owner, holding very different proportion of shares across market firms, in the limit having invested only in one firm.

Measure 5 is the ℓ^2/ℓ^1 sparsity index; the expression applied to one row of the OM or RM is given by:

$$S(x_i) = \frac{\ell^2(x_i)}{\ell^1(x_i)} = \frac{\sqrt{\sum_{j=1}^J x_{ij}^2}}{\sum_{j=1}^J x_{ij}}$$

The general meaning of this measure was discussed in Section 3.1; in the CO application, the ℓ^2 measure considers one owner i at a time, sums the squares of the shares the owner holds in each subsidiary j in the market – giving more weight to larger shares – and finally takes the square root of the total. This measures the (Euclidean) "distance" of the owner's investment behaviour from the "null" owner, i.e. an owner that holds zero (or negligible) shares in all firms in the market. The ratio of ℓ^2 to the ℓ^1 measure (simple sum of the shares held by the owner in all its subsidiaries), rescales such distance according to the owner's total investment, giving a relative measure of how concentrated is the owner's investment behaviour across the market. Such individual behaviour can then aggregated according to any method proposed in Table 3.

A little more attention should be devoted to measure 1, whose link to the vector density measure was discussed in Section 3.1. If we choose $S(x_i) = \ell^0/J$ i.e. the proportion of null elements of row i , then the average aggregation criterion would give:

$$S(X) = \frac{1}{I} \sum_{i=1}^I S(x_i) = \frac{1}{I} \sum_{i=1}^I \frac{\#\{j : x_{ij} = 0\}}{J} = \frac{\#\{(i, j) : x_{ij} = 0\}}{IJ}$$

This is the overall proportion of null elements of the X matrix, the complement of the well-known *matrix density* index – an index very widely used in matrix analysis and also in the networks literature – i.e. the proportion of non-zero elements of the matrix:

$$\text{density}(C) = \frac{\#\{(i, j) : x_{ij} \neq 0\}}{IJ}$$

A thorough discussion of this index in the analysis of networks will follow in Section 4.2.

In the ownership application, however, all lines of the OM or RM must have at least one non-zero element, i.e. each owner is included in the matrix if and only if it owns at least one firm in the market. Therefore, the sparsity index for each row should compute the proportion of null row elements excluding those that are structurally non-zero. i.e. should take the expression $S(x_i) = \ell^0/(J-1)$, otherwise the row index would never reach the maximum of one in the case of maximum sparsity (absence of CO). This yields the following CO-corrected matrix sparsity measure:

$$S_{\text{CO}}(X) = \frac{\#\{(i, j) : x_{ij} = 0\}}{I(J-1)}$$

For the same reason, a CO variation of the matrix density index should also be considered as follows:

$$\text{density}_{\text{CO}}(X) = \frac{\#\{(i, j) : x_{ij} \neq 0\} - I}{I(J-1)}$$

Table 4
Some common matrix norms.

No.	Norm	Definition — Single matrix	Definition — $d(A, B)$
1.	ℓ^p -type	$(\sum_i \sum_j c_{ij} ^p)^{1/p}$	$(\sum_i \sum_j a_{ij} - b_{ij} ^p)^{1/p}$
2.	$L_{2,1}$	$\sum_i \sqrt{\sum_j c_{ij} ^2}$	$\sum_i \sqrt{\sum_j a_{ij} - b_{ij} ^2}$
3.	Frobenius	$\sqrt{\sum_i \sum_j c_{ij} ^2}$	$\sqrt{\sum_i \sum_j a_{ij} - b_{ij} ^2}$
4.	$L_{p,q}$	$(\sum_i (\sum_j c_{ij} ^p)^{q/p})^{1/q}$	$(\sum_i (\sum_j a_{ij} - b_{ij} ^p)^{q/p})^{1/q}$

Notes: The elements of the reference matrix are denoted by p_{ij} , $i = 1 \dots I$, $j = 1 \dots J$, where $p = a, b$ or c , according to the case. In the third column norms applied to a pair of matrices A and B in order to determine their distance $d(A, B)$.

which computes the proportion of non-zero elements only among the entries in the matrix which can actually be zero, thus excluding the cases that are bound to be non-zero. This modified density index will reach the minimum value of zero in case of absence of common owners, hence allowing to have a zero-density benchmark for a market with only single owners.

3.2. Similarity measures for comparison to benchmark scenarios

A possible alternative approach to the measurement of the extent of CO, as mentioned earlier, is the comparison of the observed market structure with an hypothetical benchmark scenario of interest. As long as it is possible to express the desired benchmark through a specific matrix, then the comparison with the benchmark can be performed through the computation of a similarity measure between the benchmark matrix and the matrix representing the actual market.

Matrix similarity measures are intended to assess the “distance” $d(A, B)$ between two matrices A and B , with identical number of rows and columns, not necessarily square. Similarity measures are usually calculated entry-wise, that is the elements of the two matrices in the same position are compared (usually calculating differences), and these values are then aggregated through a matrix norm, i.e. a matrix measure similar to the vector measures considered earlier.

In order to compute a matrix norm, the matrix is treated as a long vector, where the columns (or rows) have been stacked together; the norms treat a $I \times J$ matrix as a vector of size IJ , and apply any vector measure to it. Among the most popular, we have the ℓ^p -type norms, the Frobenius norm and the more general family of $L_{p,q}$ -norms, whose formulas are given in Table 4. The structure of these norms is very similar to the sparsity measures presented earlier, the main differences being that summation is now performed over the two row and column indices, i and j respectively, to span all elements of the matrix.

Both the $L_{2,1}$ and the Frobenius norm are particular cases of the family of $L_{p,q}$ norms, since the Frobenius is actually an $L_{2,2}$ norm. The $L_{2,1}$ norm is a popular error function used in robust data analysis and in sparse coding, given that the error for each data point (the matrix row in this case) is not raised to any power, but simply summed over all points (rows).

The Frobenius norm is also rather popular, as being the Euclidean norm on the $I \times J$ space of the matrix elements. Both norms are invariant under rotations of the row and/or of the columns, i.e. the order in which the firms and the owners are arranged into the matrix is irrelevant. For a fuller account of matrix theory and applications, see for instance Zhang (2017), or Boyd and Vandenberghe (2018).

For the calculation of the distance between matrices, the chosen norm is not applied to the original matrices, but to the transformed matrix (of differences). In the right column of Table 4, the norms presented above are used for the measurement of matrices similarity, based on the matrix of differences $A - B$.

If we wish to use the density index (or its variation presented earlier) in the case of comparison of matrices, we can either compute the densities of the two matrices and then compare them, or compute directly the density of the matrix of differences. Since the density is not a linear operator, in general $\text{Density}(A) - \text{Density}(B) \neq \text{Density}(A - B)$ (unless $A = B$, in which case they are both null). In the first computation on the left-hand side, we assess the difference in sparsity between A and B ; if the difference is positive, then matrix A is more dense i.e. has more non-zero elements than B , the reverse applying in case of a negative value. On the other hand, the density of $A - B$ will always be a positive value, computing the proportion of non-zero values of the difference matrix, i.e. the proportion of cases for which the elements of A and B are not equal. A small value indicates that the two matrices coincide in most entries, while a value close to one means that most of the elements of the two matrices are different, hence producing non-zero entries in the difference matrix.

4. Network methods

Social network analysis studies the empirical structure of social relations and associations that may be expressed in network form. It can therefore be applied to the analysis of the corporate ownership structure of a market, which can be easily represented through a network of relationships between owners and firms. A light introduction to social networks can be found in Borgatti et al. (2009), while König and Battiston (2009) present the main features of economic networks. For a comprehensive account of social networks analysis see, for instance, Scott (2017) or Borgatti et al. (2018).

A social network is constituted by a set of nodes (or actors), and a set of ties (or links) that connect pairs of nodes. The nodes can be all of the same type, for example when studying relationships between individuals, giving rise to a so-called *one-mode network*.

However, in many cases the interest is in studying the links between actors of different nature, such as individuals and organisations (*affiliation networks*). These kinds of data present several facets of interactions, considering that – for example – belonging to the same organisation or club creates links between individuals, but on the other hand common members can also create mutual influence among organisations. This kind of data can be represented through a *two-mode network*, also known as *bipartite network*. Two interesting applications of bipartite network methods are developed in the empirical studies by Bonaccorsi and Giuri (2001) and by Robins and Alexander (2004), where network indices are utilised for the characterisation of industry dynamics and corporate interlocking, respectively.

The network nodes and ties can be represented through a matrix, where rows and columns refer to the same set of entities (one-mode matrix), or two distinct groups of actors, such as individuals and organisations (two-mode matrices), in this case the nodes from one group being arranged on the rows and those from the other group on the columns. The entries of the matrices represent the presence (or absence) of a link between the nodes; in the case of a two-mode network, a tie exists only between one entity of one set and one entity of the other set, but not between entities belonging to the same set.

In the study of CO, the structure of the market yields a natural division of the actors into the two groups of firms and owners, whose links have been represented through matrices OM and RM defined in Section 2.1. This structure corresponds precisely to the concept of two-mode data referred above, and to a two-mode matrix representation¹³; therefore in the following our attention will focus on two-mode network methods. For the sake of simplicity, the initial presentation of the methods will be based on presence/absence of ties, i.e. on the RM, and not on the complete OM. The actual value (weight) of the tie will be considered later, with reference to the weighted networks literature.

The network analysis of two-mode data can be performed according to two different approaches. The first is a *unimodal* or separate approach, which projects the data into one-mode matrices and then looks at relationships within one group of actors at a time; the second approach is *bimodal* or joint, where the two groups of actors are analysed together. The one-mode projection matrices represent the overlaps between actors belonging to the same group induced by a common link with an actor of the other group. For instance, in a typical market where the two groups represent firms and shareholders, the one-mode projection matrices will contain in one case the number of owners common to each pair of firms, and in the other number of firms common to each pair of owners.

4.1. Alternative representations for one-mode matrices

Borgatti (2012) presents alternative representations of the one-mode projections, leading to matrices computed according to different methods that re-scale the mere counts of overlaps.

Two of such alternatives are discussed below, and can be considered in the analysis of CO. For the sake of the example, the alternative methods will be presented here with reference to the projection matrix of the owners' group, analogous reasoning holding for the firms' projection matrix. Besides the suggestions in Borgatti (2012), some further alternatives for binary matrices are also considered below.

Following the notation in Borgatti (2012), let X be the original relation matrix (RM) (also referred to as *incidence matrix*), and let $P = XX'$ be the projection matrix into the rows' space (owners); denote by p_{ij} the corresponding elements of P , counting the number of ties between owners i and j . The maximum number of ties between two owners is J , i.e. the total number of firms in the market.

(1) A first alternative to the projection in P is given by the matrix of Pearson correlations among the rows of X , say R , whose entries r_{ij} are related to the p_{ij} as follows:

$$r_{ij} = \frac{p_{ij}/J - u_i u_j}{s_i s_j}$$

Here p_{ij}/J represents the proportion of common ties between two owners, relative to the maximum possible number of ties, while the u and s elements are used to standardise the previous measure and represent, respectively, the mean and standard deviation of the corresponding row of X . The row mean u can be interpreted as the proportion of links each owner has with firms in the market, with respect to the total number of firms.

The Pearson correlation coefficient varies between -1 and 1 , where the two values indicate, respectively, maximum negative and maximum positive association between the ownership choices of the two owners. A value of zero denotes absence of association between the two behaviours. Notice that in case of zero ties existing between two owners (i.e. $p_{ij} = 0$), the value of r_{ij} will always be negative, indicating that the two owners go in opposite directions in their choices, given that they do own some firms, but never the same ones. The analysis of this matrix, and of the signs of its elements, can reveal further features of the interaction of owners in the market.

(2) The second alternative to projection matrix P proposed by Borgatti (2012) is a matrix, say K , whose elements are given by the Jaccard coefficients k_{ij} defined below. Such coefficients are obtained starting from a cross-tabulation of row i and j of the

¹³ Although a common representation of networks is given by graphs, the matrix representation is preferred in the present approach, given the connection to the matrix methods presented in Section 3, which will be made explicit later.

original relation matrix X (shown below right), which shows the number of firms owned by both owners (a), the number not owned by any of them (d), and the number owned by one but not the other owner (b and c respectively):

$$k_{ij} = \frac{a}{a+b+c}$$

		Row j	
		1	0
Row i	1	a	b
	0	c	d

The Jaccard coefficient measures the degree of association between the choices of the two owners, since it computes the proportion of firms owned by both owners over the total number of firms owned by any of the two.

The coefficient varies between a minimum of zero (there is no firm in the market that is owned by both owners, i.e. $a = 0$), and a maximum of one (all firms owned by owner i are also held by owner j and vice-versa ($b = c = 0$). Contrary to the case of matrix R , the elements of K cannot take up negative values, although the qualitative picture of the association of owners' behaviour is similar. Incidentally, notice that element a of the contingency table for owners i and j displayed above corresponds to the previously defined entry p_{ij} of matrix P , so the k_{ij} coefficients are again a normalisation of the values of P .

Besides the coefficients presented in Borgatti (2012), many other measures of similarity or association between pairs of rows/columns are available in the case of a binary matrix, like the RM analysed above. In general, they are based on the contingency table representation introduced in alternative (2) above, like for instance the Goodman and Kruskal's gamma, or its special case called Yule's Q association coefficient, defined as:

$$Q = \frac{ad - bc}{ad + bc}$$

where elements a, \dots, d are those from the contingency table displayed above.

For this binary case, the Pearson correlation coefficient introduced in alternative (1) has itself a simplified expression, called Pearson's Φ coefficient:

$$\phi = \frac{ad - bc}{\sqrt{(a+c)(b+d)(a+b)(c+d)}}$$

again based on elements a, \dots, d as above. For more association measures for 2×2 contingency table, see for instance Bernard (2012).

4.2. Network measures

The previous sections proposed an overview of various matrix representations that can originate from the initial relation matrix of a network, both in the unimodal and bimodal approaches to bipartite networks. From any of the matrices presented above, different measures can be calculated in order to capture certain network characteristics. For the sake of simplicity, the measures will be presented and interpreted here with reference to either incidence matrices or projections matrices based on counts of links between nodes. Applications to alternative representations of projection matrices – such as the matrices based on Pearson coefficients or other nodes' association measures presented in 4.1 – are also possible, but will not be discussed thoroughly in the present note. A first simple example is given in Section 4.3 below, where a new network index is proposed for the Pearson coefficients' matrix, drawing from the measures presented earlier in the sparsity context.

In this section, two sets of measures taken from the network literature will be presented — respectively *cohesion* and *centrality* measures.¹⁴ The general purpose of centrality and cohesion measures is on the one hand studying the importance of a node based on its position, and on the other hand assessing the extent of connectedness of the network, respectively. In the CO measurement, the focus is a global evaluation of the strength of the ties in the whole network induced by CO, rather than the analysis of the importance of single actors, or their individual contribution to the cohesiveness of the network. For this reason, cohesion measures arise as a natural candidate in this context, and their possible application to the CO problem is straightforward. However, some centrality measures can also be applied to the CO study, as long as they are interpreted as a global characteristic of the network, and not used to assess individual performance of actors. Below the two most common measures of cohesion and centrality are presented, *density* and *degree*, respectively, and their interpretation in the CO context is discussed. The connection of each measure to the previous matrix methods is also highlighted and commented upon.

Network cohesion

The most popular network cohesion measure is the so-called *density*, defined as the proportion of actual links known in the network over the theoretical number of possible relationships. This coincides with the concept introduced in Section 3.1.1. This measure evaluates the degree of connectedness between the nodes of a network, and consequently the extent to which different paths facilitate circulation of information among network's participants. The minimum value of the density is zero, corresponding

¹⁴ For a thorough account of network measures, see for instance the aforementioned reference works of Scott (2017) or Borgatti et al. (2018). A specific discussion of network measures for large two-mode networks can be found in Latapy et al. (2008).

to a totally disconnected network, while the maximum of one represents a network where all possible links are actually present. A low value corresponds to a sparse network, where information flows are weaker, since nodes are less cohesive. As the number of nodes increases, the density tends to decrease, given that in a larger network the number of possible links quickly increases.

When a matrix representation of the network is adopted, as in our approach, the density can be computed with respect to different matrices, obtaining different insights and interpretations of a network's properties. For example, in the case of a bipartite network the density index can be calculated with respect either to the original incidence matrix X , taking into account the direct (one-step) links between the two groups, or also be applied to the one-mode projection matrices, evaluating the density of the (two-step) links induced within each group by the connections in X . Notice that, even if the X matrix is rather sparse – and hence presents low density – it is sufficient to have one element of a group having links to all members of the second group to obtain a totally (two-step) connected second group, i.e. only one common shareholder that has ownership in all firms would yield a sparse bipartite (X) network but a fully connected firm group.

The calculation of the density index of a network matrix will differ in the case of one-mode and of two-mode matrices, as discussed in the following.

In a *one-mode matrix* – say P – all self-links should be ignored, i.e. all elements on the diagonal are excluded. It follows that the density for a one-mode matrix with n nodes is given by a corrected version of the expression introduced in Section 3.1.1, namely:

$$d_{1M} = \frac{\#\{(i, j), i \neq j : p_{ij} \neq 0\}}{n(n-1)}$$

In terms of a *two-mode matrix* in the CO context, the modified density index proposed in Section 3.1.1 should be adopted. Assuming without loss of generality that $I \geq J$, i.e. that there are more owners than firms, then the minimum possible number of ties in the incidence matrix X equals I – the number of owners. Hence we subtract from the recorded number of ties the number of owners, in order to attain the desired minimum value of zero in case of absence of CO. Therefore, the density of X is given by:

$$d_{2M} = \frac{\#\{(i, j) : x_{ij} \neq 0\} - I}{I(J-1)}$$

Node centrality

An alternative common network characteristic is the *degree* of a node – which is a centrality measure – given by the number of connections or ties that a node has to other nodes. The degree can be normalised dividing by the maximum number of possible ties. The row and column totals of the incidence matrix X of a bipartite network correspond exactly to the number of connections of the nodes in the two groups, i.e. to the degree of each node.

If we denote by d_i and d_j the degree of owner i or firm j of X , respectively, then the normalised degree of the nodes in the bipartite network is given by:

$$d_i^* = \frac{d_i}{J} \quad \text{and} \quad d_j^* = \frac{d_j}{I}$$

The actor (firm or owner) presenting highest degree is considered to be the *most central actor*.

Although the degree in itself is usually computed as a measure of the centrality of one node, in order to identify the most important actors, this measure can be useful also in identifying general patterns in the network. This can be achieved by analysing the distribution of the degree over the whole network, called the *degree distribution*. In a bipartite network, the degree distribution of each group of actors should be analysed separately, giving rise to two different degree distributions, for example one for the firms and one for the owners. A high degree in one distribution does not necessarily correspond to a high value in the other. For example, a market can present complete absence of common ownership, so that all owners have degree one, but each firm can have a large number of owners, therefore showing large values for the firms' degree distribution. The average degree for each group of actors can be a useful indicator of the level of connectedness of the network. Further details about the degree distribution, as well as other degree measures such as average or maximum degree, can be found, for example, in Newman (2003).

4.3. Other matrix summary indices

Additional indices coming from the matrix literature presented in Section 3 can be considered in the study of networks. The main advantage of such indices is that they can be applied to any kind of matrix, i.e. not only to the relation matrix recording presence or absence of owner–firm links, but also to its derived matrices.

For example, if we consider the alternative matrix representations for one-mode groups – such as the case of the matrix R of Pearson correlation coefficients between actors r_{ij} – interest could lie only on non-zero correlations that pass a specific threshold, rather than considering all non-zero elements. In this case, an alternative definition of density based on the sparsity measure ℓ_ϵ^0 (see measure 2 in Table 2) could be considered, giving rise to the following one-mode index:

$$d'_{1M} = \frac{\#\{(i, j), i \neq j : r_{ij} \geq \epsilon\}}{n(n-1)}$$

This index evaluates the proportion of actors in the group presenting a “high” correlation in their investment behaviour (or in their ownership structure), where threshold ϵ will determine the relevant correlation strength.

Additionally, notice that matrix or sparsity indices can also be applied to the full ownership matrix, taking into account the actual shares involved in each ownership link. For example, the above modified density could be applied to a matrix with Pearson correlations based on the full ownership information in OM, or to a matrix whose elements are given by the Euclidean distance between the rows (or columns) of the OM.

4.4. Modified indices for weighted networks

Some of the previous indices have been modified in the context of *weighted networks*, i.e. when the ties between nodes are characterised by a weight representing the strength of the tie, rather than a simple binary indicator of presence/absence. Such weighted representation of the network resembles the representation of corporate ownership through the full ownership information of matrix OM, hence some additional indices of the market structure can be derived from this literature. The popular *node strength* index, extension of the node degree presented above, will be discussed here. For further indices see for instance [Egghe and Rousseau \(2003\)](#), [Antoniou and Tsompa \(2008\)](#) or [Opsahl et al. \(2010\)](#).

The node strength is defined as the sum of the weights of all ties stemming from one node, i.e. the total strength of all relationships of that node. In the simple (unweighted) network all ties have weight one, therefore the node strength simply counts the number of ties per node, i.e. the node degree as defined earlier. If w_{ij} represent the strength of the relationship between node i and node j , the node strength is obtained, respectively, as:

$$s_i = \sum_{j=1}^J w_{ij} \quad \text{and} \quad s_j = \sum_{i=1}^I w_{ij}$$

corresponding to the row and column total of the weighted network matrix representation. The definition is valid both for a one-mode and a two-node network.

Given that the information about the total strength of the ties is complementary to the number of ties, both should be accounted for in the analysis of a node's centrality. [Opsahl et al. \(2010\)](#) propose a degree centrality measure that combines both aspects, given by:

$$C_i^\alpha = d_i \left(\frac{s_i}{d_i} \right)^\alpha$$

where d_i is the degree of node i and s_i the degree strength. The exponent α is a positive tuning parameter, whose value is chosen according to the research objectives. If $0 < \alpha < 1$, a high degree is taken as favourable, while the opposite occurs if $\alpha > 1$. Further details about this measure can be found in [Opsahl et al. \(2010\)](#).

5. A set of new common ownership indices

A series of alternative indices of common shareholding are now proposed, based on the two methods presented in Sections 3–4. The chosen indicators are constructed on participation shares alone, considered as the main expression of a shareholder's influence on a firm. The first set of methods looks at the shareholder-firm relationships, i.e. is based on the analysis of each shareholders' portfolio, then aggregated at market level according to different criteria. The second one, instead, analyses only links between agents of the same type, i.e. investor-investor or firm-firm, looking separately at the respective networks; this entails looking at similarities between portfolios of pairs of shareholders, or at overlaps of shareholders structures for pairs of firms. The relationships in these last two groups are not direct, but induced by the presence of an external agent making a connection – a common shareholder linking two firms, or a firm present in two investors' portfolios, creating an overlap. The market level aggregation of these last indicators gives rise to network indices, assessing the strength of connectedness of the networks of shareholders or firms, respectively. The main features of the new indices are presented below, while the next Section applies them to a real-data example for EU Mobile Telecoms.

These indices offer several advantages compared to previous measures, such as the MHHI or profit weight measures. Both of these previous measures require determining the values of given coefficients (or control weights for the case of the profit weight measure), typically determined based on certain assumptions or on economic theory (for example, “proportional control” assumed in [Backus et al., 2021a](#)). Our indices however do not require the specification or calibration of any coefficients, and the consequent assumptions upon which the values of such coefficients are set. Instead they have the advantage that they can be computed using firm level balance sheet data alone. In addition, alternative common ownership indicators such as the density in [Azar \(2012\)](#) or the descriptive measures in [He and Huang \(2017\)](#) have a strong focus on the ownership structure of firms and possible shareholder overlaps with competitors. In this paper we contribute to the literature by proposing additional owner-oriented common shareholding indicators, which include an extra dimension and allow for a more complete measurement of common ownership. This is done by analysing the portfolio structure of investors, and the overlap between investment strategies of different owners active in a sector, highlighting whether the common shareholders tend to target the same companies.

In addition, following examples in the literature (see for instance [Seldeslachts et al., 2017](#), or [Fichtner et al., 2017](#)), some new indicators based on the number of block-holdings have been added, for different definitions of blocks (minimum 1%, 3%, 5% and 10% of shares held).

Some previous shareholders' indicators have also been aggregated at market level in new ways. For example, the joint market shares of the ‘Big Three’ portfolios (BlackRock, State Street and Vanguard) have been calculated based on Total Assets (TOAS) or Market Capitalisation (MKT CAP).

The construction and meaning of the indicators are summarised in [Tables B.1 and B.2](#) in the Appendix, and their interpretation in the context of the present study is given in the next Sections.

5.1. Shareholder-firm relationships

The size of an investor's portfolio is an indicator of the potential spread of connections induced in the commonly held firms. Its intensity, given by the amount of stakes held, measures still another dimension of the links. The shareholder-firm relationships can be combined into a series of indices, capturing different aspects of portfolio size and intensity, which reflect alternative measurements of the extent and strength of common shareholding induced in a market by the portfolio. The indices proposed below are derived from the sparsity measures no. 1 and no. 5 from Table 2, namely measures ℓ^0 and ℓ^2/ℓ^1 , from which row-wise density and uniformity indices are obtained, respectively. As discussed earlier, the row-wise application of the indices corresponds to the analysis of a portfolio at a time, subsequently aggregated to obtain market-level indicators. The interpretation of such measures in the common ownership context was already discussed in Section 3.1.1.

We consider various types of **density** of investments, representing the share of the market in the hands of a specific investor, according to different definitions. The density can be based on the headcount of firms in a portfolio, or on their value, expressed in terms of some financial indicator of firm size. In this study, value-based densities are computed according to the total assets (TOAS), available for most firms, or to the market capitalisation (MKT CAP) — only for listed firms. Moreover, the density can also be weighted by the actual participation shares held by the shareholder in each firm, giving rise to a weighted density. By using the TOAS or the MKT CAP, these indices account for differences in player sizes (for example, the MHHI measure uses market shares as size weights, accounting for size in a different way). All-together, this originates five different indicators as reported in Table B.1. Shareholders scoring exceptional values in the above indicators (so-called *top investors*) are deemed to dominate the market in some respect, and therefore are subject to further analysis.

Another index describing the type of investment strategy is the so-called **uniformity** index, which is based on the ℓ^2/ℓ^1 measure. The index takes the shares held by an investor in each company in the market, and assesses the relative weight of larger participation shares over the total shares in the investor's portfolio, showing smaller values for more concentrated distributions. A value of the index closer to one denotes an even distribution of investments within the portfolio (uniformity). In the limit, if the investor only holds shares in one company, and zero shares in all remaining companies, the uniformity index will have a value of zero. On the other hand, when an investor holds participation in all firms of the given market, and all with equal shares ("democratic" investor), the index attains its highest value, equal to one. The smaller the values of the index, the more concentrated the investment strategy of a shareholder, i.e. the more the shareholder discriminates between different companies and chooses only some, typically with large participation shares. A large value of uniformity instead generally corresponds to many firms, all held with small shares.

The distribution of the above indices across the population of shareholders characterises the market ownership structure, identifying for instance the **average, median or maximum density**, or some **percentiles** of interest. Such values, and their evolution over time, will define the market dynamics of common ownership, leading to the subsequent analysis of specific cases of interest.

5.2. Networks of firms and of shareholders

The projection of the links between firms and shareholders into the one-mode networks of shareholders or firms allows us to implement network indices revealing the interconnectedness of portfolios or of shareholding structures. In order to quantify in a meaningful way the strength of the links in each network, we choose to use the alternative representation of the one-mode matrices based on the Pearson correlation coefficients presented in Section 4.1. To such correlation matrix we apply in turn (i) the network cohesion measures presented in Section 4.2, looking at the non-zero elements, and (ii) the additional indices obtained in Section 4.3 through the application of sparsity methods to the network matrix representations, looking at elements exceeding a certain threshold of interest.

In the analysis of the *shareholder-shareholder relationships*, the **correlation** between pairs of portfolios reveals the level of overlap between two shareholders' investment strategies. Several market indices can be obtained from the above, such as the **proportion of non-zero correlations**, measuring the *overall degree of overlap* of portfolios; this proportion is meaningful mainly for shareholders with high-density portfolios, which include a considerable number of firms in the market. The **proportion of highly correlated high-density portfolios** is a signal of the degree of overlap in the investment behaviour of top shareholders.

Similarly, the analysis of the *firm-firm relationships* induced by common shareholders assesses the degree of overlap of a firm's shareholding structure with other competitors' ownership information, computed through their **correlation**. The higher the correlation between two firms, the greater the similarity of the respective shareholders' structures, both in terms of the identity of the shareholders and of the quantities of participation they hold in each competitor. In a case where two firms are not cross-held, the correlation between their respective ownership structures is zero, therefore the **proportion of non-zero correlations** is again a useful indicator of the degree of connectedness of the firms' network. In order to consider only relevant stakeholders, the correlation can be calculated only taking into consideration blocks of holdings — for example imposing a threshold of minimum 5%.¹⁵

Although the formula of the correlation indicator for the firms' network is slightly different, its nature and interpretation is very similar to the "profit weights" of Backus et al. (2021b). In fact, under the hypothesis of proportional control the profit weights amount to the calculation of the cosine of the angle between the vectors representing the shareholding structure of the two firms (cosine similarity or "overlapping ownership"), rescaled by an index of the relative concentration of the shareholding structure of

¹⁵ In this case, the index is identical to a common shareholding measure presented in Chapter 6 of Azar (2012), based on the density of the network of firms, which computes the proportion of firm-firm connections existing between pairs of competitors in a given market, due to the presence of a common block-holder.

the two firms (“relative HHI”).¹⁶ The cosine similarity approaches the correlation when the average amount of shares held by all investors in a specific firm gets close to zero. This is often the case considered that empirically firms have few dozens of shareholders, out of the thousands of investors present in each market, hence most of the holdings are actually zero. Moreover, for the main relevant players of concentrated markets we usually observe a rather dispersed shareholding structure, giving rise to a “relative HHI” component close to one. It follows that empirically the correlation indicator and the “profit weights” will in general be very close, at least for the largest players.

5.3. Indices based on block-holdings

A final note regards the **intensity of the investments**, i.e. specific thresholds for the level of participation. The existing empirical studies on common shareholding mainly consider block-holding as defined by thresholds of 5%. This is mostly due to data availability and not due to a specific economic meaning of the chosen value. This study experiments with **new thresholds** in the definition of block-holders, namely using minimum equity investments of 1, 3, 5, and 10 percent, following some recent studies. For example, Fichtner et al. (2017) presents the number of **block-holdings of top global investors** for holdings of minimum 3%, 5% and 10% respectively, highlighting specifically results for the ‘Big Three’ — BlackRock, State Street and Vanguard. Similarly, Seldeslachts et al. (2017) present the number of block-holdings at 1% of selected institutional investors in German listed companies.

In accordance with these studies, we propose the calculation of the number of block-holdings for the ‘Big Three’ in each market under analysis for 1%, 3% and 5% levels; we add the proportion of such block-holdings relative to the total portfolio of the investors in each industry. Finally, we propose to compute the total number of block-holdings at 1%, 3%, 5% and 10% for all investors in each market. These indices reveal interesting patterns of industry-specific intensities of investment by shareholders.

5.4. Firm-level controls

Several controls can be constructed from the ownership data to explicitly account for heterogeneity of firm-level characteristics. Some examples are as follows:

- **Shares of top n owners:** total shares held by the n recorded shareholders with largest direct participation (excluding aggregated shareholders);
- **Shares owned by Big3:** total shares held jointly by BlackRock, State Street and Vanguard;
- **Shareholders’ HHI or ℓ^2/ℓ^1 :** index of concentration of the shareholding structure of a firm;
- **Firm’s integration:** degree of integration of a firm with the other firms in the market due to common owners (average correlation between shareholding structure of a firm with competitors).

The link between shareholding structure and corporate performance has been investigated in the past, for example in Demsetz and Villalonga (2001); in such work, based on 511 US firms from all sectors of economy observed in 1976–1980, no significant relation is found, mainly due to the presence of diffuse ownership. Nevertheless the period of observation is rather back in time, hence we believe that the issue is still worth investigating in empirical studies covering more recent periods of time, where in general an increase of common ownership has been observed. The indicators suggested above are two indices of concentration of ownership (Shareholders’ HHI and Shares of top n owners), and an index of presence of some main institutional investors among the shareholders (total shares held jointly by the “Big Three”). Similar indices have been investigated in Seldeslachts et al. (2017) and in Fichtner et al. (2017), among others.

The last control variable accounts for the cross-links in shareholding structures between a firm and its competitors, a measure of the firm’s integration in the market through its owners. Although the measure is computed in a slightly different manner, the logic behind this indicator is similar to that of Backus et al. (2021b) – as discussed earlier – where “profit weights” are computed for each pair of firms, starting from their shareholding structures. Such weights give rise to “cooperation matrices” reflecting the interactions of pairs of firms generated by their ownership overlaps. The average “profit weights” on competitors, a measure of firm’s integration similar to ours, is used in the cited works by Backus et al. and in Antón et al. (2023) as the leading common ownership indicator.

Rosati et al. (2022a) use the firm’s integration index and the other controls proposed above to test the possible effects of common ownership on competitiveness in the EU beverages industry.

6. Empirical application — Mobile Network Operators sector

In this section, we illustrate the common ownership indices outlined in the previous sections by calculating them using real firm-level data obtained from the Orbis dataset.¹⁷ The sample used in this application includes 105 active MNOs observed between

¹⁶ For details about this decomposition of the “profit weights”, see Backus et al. (2021c).

¹⁷ The Orbis dataset is published by Bureau Van Dijk, a Moody’s Analytics Company (see Bureau van Dijk, 2023a and Bureau van Dijk, 2023b). Starting from the Orbis raw data, a specific dataset was constructed on purpose for this empirical analysis. The dataset underwent a thorough set of cleaning procedures, illustrated in detail in Rosati et al. (2020) - Appendix A. From the point of view of the measurement of common ownership, both direct and indirect levels of shareholding were considered, given that both represent a link between the shareholder and the owned firm.

Table 5
Summary statistics for firms active in the EU28 in the MNOs sector between 2007 and 2021.

Year	Number Firms	Number (%) firms Cross-held by BH	Number of SH	Number (%) Common SH	Number (%) Single SH
2007	84	61 (72.62)	640	188 (29.38)	452 (70.63)
2008	87	64 (73.56)	652	193 (29.60)	459 (70.40)
2009	90	63 (70.00)	662	187 (28.25)	475 (71.75)
2010	91	67 (73.63)	717	194 (27.06)	523 (72.94)
2011	94	65 (69.15)	639	191 (29.89)	448 (70.11)
2012	95	63 (66.32)	657	169 (25.72)	488 (74.28)
2013	94	67 (71.28)	645	185 (28.68)	460 (71.32)
2014	94	67 (71.28)	617	177 (28.69)	440 (71.31)
2015	96	72 (75.00)	642	205 (31.93)	437 (68.07)
2016	100	76 (76.00)	616	209 (33.93)	407 (66.07)
2017	100	78 (78.00)	593	188 (31.70)	405 (68.30)
2018	100	79 (79.00)	595	179 (30.08)	416 (69.92)
2019	96	75 (78.13)	712	185 (25.98)	527 (74.02)
2020	95	71 (74.74)	692	164 (23.70)	528 (76.30)
2021	95	73 (76.84)	500	139 (27.80)	361 (72.20)

Notes: Percentage of firms cross-held by block-holders (BH) at minimum 5%. Percentage of common shareholders = Percentage of shareholders with more than one firm in their portfolio. Single owners = Those holding shares only of one firm.

2007 and 2021.¹⁸ All of these operators are based in Europe, 97% being registered in the EU28, while the remaining 3% (i.e. 3 firms) are registered in Norway.

Table 5 presents the summary figures for the shareholder structure of the industry over the period of observation. The number of firms in this industry has been increasing slowly, stabilising just under 100 in 2021. The sample includes an average of 642 shareholders per year. The proportion of common owners has been rather steady around a mean of 29% throughout the period. The percentage of MNOs that share block holders with competitors has increased between 2007 and 2021 to almost 77% of firms. By definition, two firms are cross-held by a block-holder if the common investor holds at least 5% in both firms. The empirical evidence for our sample implies that the vast majority of firms are linked to at least one other company in the market through a substantial amount of shares in the hand of a common owner. This is mainly due to the peculiar structure of this market, where most companies belong to large corporate groups such as Vodafone or Orange, where all subsidiaries share a common parent. In fact, these figures are markedly higher than EU and US indicators calculated on a variety of markets for the same period in Rosati et al. (2020) and He and Huang (2017), respectively, reporting proportions of block cross-held firms well below 70%. Notice that institutional investors hold very large portfolios in this market, as will be detailed later, and hence are responsible for linking a substantial number of competitors; however, the shares held are usually below the block threshold of 5% considered here, so their holdings do not influence this indicator.

6.1. Main players

Since most of the MNOs included in our sample are registered in the EU28, there are only few large players from outside (Norway). Therefore, in the analysis of the main firms active in this industry ten EU28-based companies and only two non-EU28 corporations will be considered. Table 6 displays summary information for the top 10 largest enterprises in EU28 and top 2 outside EU28, by country of registration. The firms are ranked according to the value of their total assets (TOAS, in Bln €) in 2021. Note that some of the top firms are controlled subsidiaries; in these cases, the ownership information refers to the mother companies (see “Controlled by” column).

As a single investor, BlackRock dominates the MNOs industry, with rather large shares in some of the top players. This behaviour is in line with other empirical studies of institutional investors in strategic markets, which has in turn been associated to anticompetitive behaviour.¹⁹ Vanguard and Norway lag behind, with relatively small participations, while State Street plays only a minor role. France is also present with small shares in only a handful of top competitors. To the best of our knowledge, this paper also represents the first instance where the role of sovereign investments funds in common ownership is methodically quantified. The additional association to anticompetitive behaviour of sovereign funds could be an interesting topic for further research.

6.2. Density and uniformity

Table 7 now presents summary statistics for the density-based and uniformity indicators calculated on 2021 data for MNOs.²⁰ These statistics represent industry-level empirical estimates for the presence of common ownership. The density index indicates that

¹⁸ Most of the companies (82%) classify their core activity within the NACE Division “61: Telecommunications”. The other 18% of firms are scattered across different activities (e.g. Retail, Postal and Courier Activities and Head Offices).

¹⁹ See for example Azar et al. (2018), Banal-Estañol et al. (2021), and Azar et al. (2021) which identify Blackrock as one of the key common shareholders in the airlines, pharmaceuticals, and banking sectors, respectively.

²⁰ Tables displaying the full set of matrix indices calculated for all years between 2007 and 2021 are available upon request from the corresponding author.

Table 6

Largest firms of MNOs sector and selected ownership data on their main common owners.

	Country	TOAS, 2021 (Bln €)		Black rock	France	Norway	State street	Vanguard
Registered in EU28			Controlled by	Shares held in 2021 (%)				
VODAFONE GROUP	UK	153.95		9.11	0.13	3.09	2.40	3.77
ORANGE	FR	99.46		4.87	29.41	1.65	0.27	2.00
TELECOM ITALIA	IT	57.48		1.55		0.75	0.21	1.34
TELIA COMPANYY	SE	20.25		3.00		1.12	0.18	1.90
WIND TRE	IT	16.41	CK HUTCHISON HOLD. (KY)	4.90		1.07	0.91	2.04
EE LIMITED	UK	13.60	BT GROUP PLC (UK)	4.55		2.99	2.16	3.31
KONINKLIJKE	NL	12.74		4.84		2.80	0.38	2.20
O2 HOLDINGS LTD	UK	12.58	(50%) LIBERTY GLOBAL (UK)	3.49		0.92	1.50	2.02
			(50%) TELEFONICA (ES)	2.90		1.89		2.49
TELENET GROUP	BE	9.76	TELENET GROUP HOLD. (BE)	0.64		0.99	0.14	1.13
PROXIMUS SA	BE	9.23		4.88		1.14	0.15	1.17
TELEFONICA DE	DE	7.59		1.78		0.90	0.10	0.64
Registered outside EU28			Controlled by	Shares held in 2021 (%)				
TELENOR ASA	NO	20.03		1.60		53.97	0.80	1.25
TELIA NORGE AS	NO	3.76	TELIA COMPANY AB (SE)	3.00		1.12	0.18	1.90

Notes: Direct or indirect participation shares (%) held in 2021 are displayed.

Table 7

Summary statistics for MNOs sector common shareholding indices (year 2021).

Index	Mean	75th percentile	99th percentile	Maximum
Density	2.05	2.11	12.11	17.89
Uniformity	11.45	19.19	74.78	76.54
TOAS density	12.67	26.40	67.25	68.48
TOAS weighted density	0.25	0.15	3.68	5.73
MKT CAP density	16.35	28.04	96.61	98.98
MKT CAP weighted density	0.20	0.08	4.22	8.77

Notes: Percentage points displayed, index values between 0 and 100.

the MNOs investors tend to have small portfolios, including at most 18% of the firms in the market. The top 1% of largest portfolios are actually rather smaller, holding shares in just above 12% of the market firms. As noted earlier, this is due to the presence, in this industry, of large groups mothered by Telecom giants such as Vodafone, Orange or Deutsche Telekom, which have little if no overlap in their portfolios, and have basically partitioned the set of firms active in this industry into independent domains. From this perspective of the shareholders then, common ownership in 2021 remained a relatively contained phenomenon. This is also confirmed by overall lower levels of uniformity indices,²¹ where the maximum did not reach 77% in 2021, showing a higher concentration of investment strategy. This suggests that investors are less “democratic” and tend to target higher participations in a smaller set of firms, rather than widespread low investment across all market.

In terms of value-based indices, there are evident differences in level between total assets and market capitalisation results, given that the set of listed firms in this industry is rather reduced, and therefore the second set of indices is computed on a very restricted part of the market. We see that the top 1% of investors hold stakes in firms that represent almost 68% of the Total Assets of the industry, showing that such top investors tend to privilege the largest enterprises. Given the high participation shares in this market, the weighted TOAS index reaches high values, with the top investor holding 5.73% of the market total assets through shares. A similar picture is found for listed firms, where the largest portfolios hold stakes in firms representing basically the entire market capitalisation (almost 99%), and through their participations control about 9% of the market value. Unlike the unweighted indices, these industry measures imply a degree of common ownership that could present anticompetitive concerns, as a few top investors exert influence over a large portion of the market assets.

6.3. Common ownership indices for top investors

Table 8 presents the density indices for the top investors engaged in common ownership in the MNO sector in 2021. These are mainly large Telecom groups, with several subsidiaries in different countries. For example, Orange holds 2.38% of the market Total Assets through virtually full ownership of its subsidiaries in Belgium, Poland, Romania, Slovakia or Spain. Similarly, Deutsche Telekom holds 1.89% of the Total Asset of the industry through control of firms in Austria, Check Republic, Croatia, Hungary, Poland and Slovakia.

²¹ Compared to other markets. See for example Rosati et al. (2020).

Table 8

Top investors in MNOs sector (year 2021).

SH name	SH country	No. Subs.	Density	TOAS density	W TOAS den.	Largest subsidiaries and shares held (%)
FRANCE	FR	9	9.47	46.39	5.73	ORANGE (29.41); VODAFONE GROUP (0.13); ORANGE ESPAGNE (25.01); ORANGE POLSKA (25.01); ORANGE BELGIUM (25.01); ORANGE ROMANIA (25.01); ORANGE SLOVENSKO (25.01)
DEUTSCHE BANK	DE	12	12.63	67.46	3.82	VODAFONE GROUP (13.68); ORANGE (0.48); TELECOM ITALIA (0.17); TELIA COMP (0.16); TELENOR (1.62); KONINKLIJKE (0.64); PROXIMUS (0.63)
CK-HUTCHISON HOLDING	KY	6	6.32	3.89	3.82	WIND TRE (98); HUTCHISON DREI AUSTRIA (100); THREE IRELAND (98); CK HUTCHISON UK (98); HI3G ACCESS AB (98); HI3G DENMARK APS (98)
BLACKROCK	US	12	12.63	67.46	3.79	VODAFONE GROUP (9.11); ORANGE (4.87); TELECOM ITALIA (1.55); TELIA COMP (3.00); TELENOR (1.6); KONINKLIJKE (4.84); PROXIMUS (4.88)
TELEFONICA	ES	3	3.16	4.14	3.74	O2 HOLDINGS LTD (100); TELEFONICA DEUTSCHLAND (69.93); TELEFONICA MOVILES (100)
NORWAY	NO	17	17.89	68.48	3.35	TELENOR (53.97); VODAFONE GROUP (3.09); KONINKLIJKE (2.80); ORANGE (1.65); TELECOM ITALIA (0.75); TELIA COMP (1.12); PROXIMUS (1.14)
ORANGE	FR	7	7.37	2.94	2.38	ORANGE ESPAGNE (100); ORANGE POLSKA (50.67); ORANGE BELGIUM (78.32); ORANGE ROMANIA (99.20); ORANGE SLOVENSKO (100); ORANGE ROMANIA COMM. (53.58); ORANGE COMM. LUX. (78.32)
VODAFONE GROUP	UK	9	9.47	2.41	2.22	VODAFONE ITALIA (100); VODAFONE PORTUGAL (38.62); VODAFONE MAGYARORSZAG (100); VODAFONE CZECH REPUBLIC (100); VODAFONE ROMANIA (100); VODAFONE LIBERTEL (50); VODAFONE ENTERPRISE GERMANY (100)
SWEDEN	SE	15	15.79	58.73	1.97	TELIA COMP AB (39.50); VODAFONE GROUP (0.18); ORANGE (0.27); TELENOR (0.13); KONINKLIJKE (0.23); PROXIMUS (0.19); TELEFONICA DEUTSCHLAND HOLDING (0.12)
TELIA COMP	SE	6	6.32	1.94	1.90	TELIA SVERIGE (100); TELIA FINLAND (100); TELIA NORGE (100); TELIA LIETUVA (88.2); TELIA EESTI (100); LATVIJAS MOBILAIS TELEFONS (50.01)
DEUTSCHE TELEKOM	DE	8	8.42	2.40	1.89	MAGYAR TELEKOM (40.77); T-MOBILE POLSKA (100); T-MOBILE AUSTRIA (100); HT-HRVATSKI TELEKOMUNIKACIJE (52.17); SLOVAK TELEKOM (100); T-MOBILE CZECH REP. (100); TELEKOM DEUTSCHLAND (100)
VANGUARD	US	12	12.63	67.46	1.68	VODAFONE GROUP (3.77); ORANGE (2.00); TELECOM ITALIA (1.34); TELIA COMP (1.90); TELENOR (1.25); KONINKLIJKE (2.20); PROXIMUS (1.17)

Notes: SH country: Country of shareholder. Last column displays the firms with largest total assets in portfolio and respective quantity of shares held.

Rather important are also the States, with France and Norway ranking amongst the top investors, holding the largest percentages of the market Total Assets, with 5.73% and 3.35% respectively. As for the Funds, only BlackRock has large enough participation (i.e. 3.79% of the market Total Assets) to appear in the top positions, while Vanguard is left behind with a lower, yet still sizeable, share of Total Assets held (about 1.68%).

While our indices do document a degree of common ownership among state investors (such as France, Norway and Sweden), there is still no evidence in the literature of governance related channels which tie the presence of common ownership to anticompetitive behaviour.

6.4. Network indices

The results for shareholders' and firms' networks in the MNOs sector are reported in [Tables 9](#) and [10](#), respectively. There are strong interconnections between actors of the same type. The proportion of pairs of portfolios that are linked through commonly held firms (i.e. proportion of non-zero correlations in [Table 9](#)) has been oscillating over time between 4 and 5%, with a slight decline in the last few years. On the other hand, among the large²² connected portfolios the links are rather strong, having observed

²² Portfolios in this sector are considered large if they hold stakes in at least 10% of the market's firms.

Table 9
Shareholders' network indices for MNOs sector.

Year	Total No. Common SH	Prop. Non-zero Correlations	No. High Corr. High Dens.	Prop. High Corr. High Dens.
2007	188	4.57	21	6.46
2008	193	5.02	35	6.34
2009	187	4.37	38	7.25
2010	194	4.24	14	4.07
2011	191	5.27	19	6.33
2012	169	4.23	12	4.08
2013	185	4.51	9	4.46
2014	177	4.72	14	5.74
2015	205	5.05	22	7.97
2016	209	5.28	20	6.67
2017	188	4.73	14	6.67
2018	179	4.30	12	7.84
2019	185	3.26	12	7.02
2020	164	2.72	19	13.97
2021	139	3.06	21	23.08

Notes: High-density portfolios are considered as such if the densities are higher than 10%, i.e. if they hold stakes in at least 10% of the market's firms.

Table 10
Firms' network indices for MNOs sector.

Year	Total No. firms cross-held by BH	Prop. Non-zero correlations	No. of high correlations	Prop. of high correlations
2007	61	3.96	73	52.90
2008	64	4.49	99	58.93
2009	63	4.39	99	56.25
2010	67	4.37	98	54.75
2011	65	3.84	88	52.38
2012	63	4.32	105	54.40
2013	67	4.12	97	53.89
2014	67	4.37	87	45.55
2015	72	3.84	82	46.86
2016	76	3.31	88	53.66
2017	78	3.25	95	59.01
2018	79	3.41	101	59.76
2019	75	3.55	86	53.09
2020	71	3.85	85	49.42
2021	73	3.54	83	52.53

Notes: Block-holders (BH) hold at least 5% of shares.

an increasing proportion of highly correlated investors, on average about 7% of owners showing almost coincident investment strategies, with portfolio overlaps of 80% or more.

The firms' network is also highly interconnected through block holders, with between 3.5% and 4% of pairs of firms connected through some common shareholder. More importantly, steadily over half of these linked portfolios present a correlation of more than 80%. Regardless, this very high fraction of highly correlated ownership structures is probably the result of the presence of clusters of firms belonging to the large Telecom groups, which are almost all wholly owned or at least controlled by the parent, showing therefore very similar shareholders' structures. Again, compared to other sectors like Oli&Gas, Electricity or Beverages (see Rosati et al., 2020), the MNOs show stronger network interconnections both between firms and between shareholders.

As mentioned earlier, the source of the quantified links between firms is relevant for competition concerns. If the large share of highly correlated ownership among firms derives from corporate groups, it would not necessarily indicate anticompetitive behaviour stemming from common ownership as the empirical literature typically finds. The indices proposed provide insight into these different dimensions of industry links.

7. Conclusions

The existence of common shareholders among competing companies in a given industry has raised the concern of academics and policy makers worldwide, due to its possible effects on market efficiency and competition. The main empirical works investigating this issue cover only a limited number of industries, and a more general effect on the economy has yet to be analysed. This is in part because the measures used so far to assess the effects of common ownership in a given market have been subject to criticism from several scholars, raising the need for the development of a sound measurement framework. Besides specific technical shortcomings of the known metrics, critics underline that each measure will capture different aspects of an industry or market, and encourage researchers to consider the relevant economic context when adopting a certain measure. The absence of a single one-size-fit-all

“best” measure of common ownership points towards a plural approach, where a set of indices are able, together, to capture the complexity of the phenomenon.

This work considered some novel multifaceted methodological strategy to be applied to the measurement of common ownership. The methods are taken in part from the theory of sparse matrices and in part from social network analysis, and are adapted to reflect the need for proper measurement of the phenomenon of common shareholding. From sparsity theory, a series of concentration indicators are derived for the analysis of a shareholders’ portfolio, describing its investment behaviour both at industry level and at investors’ level. Value-based indices are also proposed, where the size of the firms held in each portfolio is taken into account. The strength of the relationships existing within the groups of firms and of investors is also studied, based on purpose-built network indices. The network analysis looks, in turn, at similarities between portfolios of pairs of shareholders, or at overlaps of shareholders structures for pairs of firms. The proposed methodologies can be applied both to the set of owner–firm relations induced by corporate ownership, as well as to the full information about the amount of participation shares. Comparison with given benchmarks such as the case of absence of common ownerships are also considered. Because the framework proposed here abstracts from previous, theory-based approaches, there are limitations to what the measurements can say about the transmission mechanisms of common ownership trends to anticompetitive behaviour. We argue however that their model-free nature makes them more objective and compatible with a range of different economic models, as they do not require a number of the assumptions made by previous metrics. As a matter of fact, many of the indicators currently used in the literature are identified as special cases falling within this framework.

The measures are tested using firm-level data for European Mobile Network Operators between 2007–2019, with special attention dedicated to the role of the largest investors. Results show that investors’ portfolios on average in this industry are rather specialised, due to the presence of large corporate groups controlling country-specific subsidiaries, such as Vodafone. However, certain institutional investors typically associated with common-ownership-related anticompetitive concerns, such as Blackrock, present a share of firms held and a total asset share comparable to these corporate groups. Moreover, the subsample of large investors with extended interests (again generally corresponding to institutional investors), shows a rather strong network connection with highly-correlated portfolios. This is not the case, instead, for the corporate groups, which present non-overlapping portfolios, creating clusters of ownership segmenting the market. This highlights the importance of a holistic approach to common shareholding as proposed here.

The framework proposed in this paper, based on directly observed linkages between firms and investors provides a data-driven measure for common ownership. Its multidimensional approach helped disentangle the differential behaviour of common owners of different nature, such as the parents of corporate groups as opposed to institutional investors. The policy implications in terms of possible anti-competitive behaviour of these investors are distinct, so the approach provides policy makers with a more complete picture of the market distortions introduced by different varieties of common owners. The empirical application proved useful also in highlighting new relevant trends for research. In particular, our findings underscored the growing role of states as common shareholders, something which merits closer attention in future research.

CRedit authorship contribution statement

Nicoletta Rosati: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Software, Supervision, Validation, Writing – original draft, Writing – review & editing. **Pietro Bompreszi:** Data curation, Formal analysis, Investigation, Software, Validation, Writing – original draft, Writing – review & editing. **Maria Martinez Cillero:** Data curation, Formal analysis, Investigation, Software, Validation, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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Appendix A. Application of the sparsity framework to common ownership

In order to apply the sparsity concept to the CO problem, and in particular to the two matrices defined in Section 2.1 (the ownership matrix OM and the relation matrix RM), it is necessary to identify the most and least sparse scenarios and their meaning.

The main issue to be considered is the applicability of the sparsity concept to a matrix, since the concept itself, as well as the measures, were initially developed in relation to a vector of coefficients — representing the distribution of wealth.

The extension of a vector measurement to a matrix can be done in three different ways, which in turn give rise to different benchmark scenarios, as discussed below. Let index $i = 1, \dots, I$ denote the owners, and $j = 1, \dots, J$ the firms. The sparsity of a given ownership structure can be studied looking at the following dimensions in the OM and RM:

(1) *column dimension: sparsity of the shareholders' distribution for a given firm.*

In the analysis of a column, the maximum sparsity is achieved when a given firm j presents only one owner who holds 100% of the shares, the least sparse distribution corresponding instead to having n_j shareholders, each with a proportion of $1/n_j$ of the shares. For the RM, these cases correspond, respectively, to a vector with one unit element and all the rest zero, or to a vector of all ones.

(2) *row dimension: sparsity of the investment distribution of a given owner into the firms constituting the market.*

If we look at a row, the sparsest case occurs when an owner only holds shares of one firm in the market (whatever the percentage), the minimum sparsity being achieved when the owner owns shares in all firms in the market, at a constant percentage (does not show preference for any firm in particular).

(3) *overall dimension: sparsity of the firm–owner links present in the market.*

In the overall analysis of the matrices we can define maximum sparsity either of all the rows (one firm only per each owner, for all owners; one firm can have more owners, $I \geq J$), or of all the columns (one owner only per each firm, for all firms; the owner can be common to other firms, $I \leq J$), or of both at the same time (square diagonal matrix, one owner only per each firm and one firm only per each owner, $I = J$).

Defining maximum sparsity of a matrix looking at the column dimension would consider only the shareholders' structure of a given firm, and not the inter-linking between firms caused by common owners, so does not correspond to the primary objective of our study. However, this approach can be used to assess other market characteristics in a later stage. The last approach (joint maximum sparsity of both rows and columns) again imposes the column maximum sparsity, which is not of central interest at this stage, so it will be left for later analyses.

It follows that the row-wise approach for the definition of overall maximum sparsity of a matrix seems more appropriate for our case, as being directly connected to the study of CO. In fact, in this approach the most sparse matrix corresponds to the absence of common owners, i.e. each row only presents one non-zero element, while the opposite happens when each row is completely filled with positive and equal values, that is each owner being linked to all firms in the economy with equal shares (least sparse scenario). Notice that this last case, being repeated for all owners, implies that the shareholder structure of all firms is identical. The relative weight of each shareholder in a firm is not constrained to a specific value, however it will depend on its financial capacity.

Properties of sparsity measures

Table A.1 presents the definition and interpretation of the most common properties of sparsity indices.

Desirable criteria that a sparsity measure should fulfil are, among others, the so-called “Robin Hood” property, the scaling or homogeneity, the “rising tide” property, and the “cloning” property. These four criteria were initially proposed by Dalton (1920) in the financial setting of inequality measurement of wealth distribution, but are nowadays generally recognised to be minimum criteria a “good” measure of sparsity should satisfy, being referred to as “Dalton's Laws”.

More recently, Rickard and Fallon (2004) add two extra properties, named “Bill Gates” and “Babies”, respectively. Additional axioms and attributes, whose analysis goes beyond the scope of this note, can be found in Pastor et al. (2013, 2015).

More details about the above properties can be found in Hurley and Rickard (2009); the paper also presents sixteen sparsity measures and prove which of them satisfy which properties. The main result is that only two measures satisfy all six properties, namely the Gini Index and the pq -means (see definitions in Table 2), while all remaining measures satisfy some, but not all of them.

Properties revisited in the context of common ownership

Given that the properties were initially considered in a welfare inequality context, they need to be critically reassessed in the CO framework, specifically in the sense discussed above, i.e. looking at row-wise sparsity (increase in sparsity = decrease of the extent of CO; decrease in sparsity = stronger presence of CO). Each property will be analysed initially looking at a change in behaviour of one single owner, subsequently extending the same behaviour to all owners in the market (if meaningful) in order to assess the overall effect on the OM and RM. This exercise will also help exclude sparsity measures that are inappropriate for our purpose. The revised properties are those presented in Table A.1. Summarising, all six properties of sparseness measures are overall still meaningful in the context of CO, although the scaling and cloning properties show some limitations when applied to the OM, being less problematic in the RM case.

Robin Hood: in case an owner holding a large share in a firm decides to divest, redirecting the investment towards other firms where it holds smaller (or zero) shares, the measure captures the change as a decrease in sparsity, i.e. a movement from absence or low level of CO towards a higher level of CO. The same applies if all owners change their behaviour in an analogous way: the measure would detect a reduction in sparsity. This is a desirable property as it goes in the direction of the effect we would like to measure. Therefore sparsity measures that fulfil the Robin Hood property are appropriate in our context.

Table A.1
Properties of sparsity measures.

No.	Property	Definition	Description
1.	Robin Hood	A “fair” readjustment of coefficients decreases sparsity	Taking some amount from the larger coefficients and “redistributing” towards smaller coefficients yields a less concentrated i.e. more equal distribution.
2.	Scaling	A change in scale of the coefficients does not change sparsity	Multiplying wealth of all units by an equal factor does not change the level of (in)equality of the distribution.
3.	Rising Tide	An identical increase in all coefficients decreases sparsity	Adding a fixed amount to each coefficient reduces the relative difference between large and small values, i.e. yields a more equal distribution.
4.	Cloning	Doubling the number of coefficients by cloning the same values does not change sparsity	If a second population is cloned, reproducing the same values as the first one, then the level of (in)equality of the merged populations equals that of the initial population.
5.	Bill Gates	A significant increase in one coefficient increases sparsity	When one coefficient becomes very large, the level of inequality increases.
6.	Babies	The addition of extra zero coefficients increases sparsity	Adding individuals with zero wealth to a population increases inequality, by concentrating the (positive) total wealth in the hands of a smaller share of individuals.

Scaling: If an owner, say, doubles the ownership shares held in all its subsidiaries, the measure does not detect a change in sparsity, i.e. the extent of CO is considered unchanged. For a single row, this change in investment does not alter the presence of CO, since the number of existing links is unchanged. However, given that the strength of the links increased, and that the column sum of the shares is constrained to be $\leq 100\%$, this implies a decrease of the strength of the links other owners have with the same subsidiaries. Such decrease in the limit could even lead to a zero share, therefore eliminating an existing link, and altering the sparsity. It follows that this property is somewhat controversial, especially in the case of the OM, which might suffer an (undetected) readjustment of all rows when one owner changes its investments. The property seems more acceptable for the RM, which in general will be unchanged under this scenario, although in an extreme case the actual number of links might be affected, as mentioned earlier. Therefore, a measure fulfilling this property raises some concerns, and should be tested further. The situation where all owners would double their shares is an impossible event given the above constraint on the column totals, therefore is not analysed.

Rising Tide: If an owner increases its ownership shares by $k > 0$ points in all firms in the market (even in those where it had previously no shares), then the sparsity decreases. In the limit, an owner that only owned shares in one firm, becomes common owner of all companies in the economy, so there is a change towards an increased level of CO. This property is in line with the dynamics of CO, therefore is acceptable for our study. If all owners made a similar change in investments, the RM would immediately become the sparsest one proposed earlier, therefore going again in the direction of this property; however, in the case of the OM we cannot add indefinitely extra shares to all elements, due to the column total constraint, so this case will not be contemplated.

Cloning: If the number of firms in the market doubles, and an owner invests in the new firms exactly the same amounts of shares held in the original set of firms, sparsity does not change. Duplicating (“cloning”) the initial vector of investments does not change their relative concentration, since the proportion of firms held by the owner is unchanged. However, the absolute number of firms linked by the owner doubled, introducing more inter-connections between firms. In the most extreme case, an owner who had invested only in one firm (and therefore who was not a common owner) will introduce a connection between two firms, and become a common owner. Therefore, measures with this property are acceptable if we seek to measure the relative extent of CO, but are less suitable for absolute measurements.

Bill Gates: If one owner increases largely its investment in one firm, sparsity increases. The owner will have to divest with respect to other firms held, in order to move funds massively to that specific firm. Therefore all remaining existing links of that owner will decrease in strength, some even reaching a zero value, i.e. some links may disappear. The same will happen at column level, as a larger share held by the owner under consideration makes all other shares of that specific firm decrease largely, again possibly causing some links of that firm with other owners to vanish. In both cases this implies an increase in sparsity, therefore the property is in line with the market dynamics and is acceptable for our analysis.

Babies: Adding an extra firm in the market whose owners are not common to any other firm increases sparsity. This amounts to adding a new column with the shareholders’ structure of the added firm, and some new lines corresponding to the new added owners, which were not present before in the market, since they are not common to any other firms. Therefore, the column will be filled with zeros, except for the last few elements containing the shares held by the new owners. This implies that each line corresponding to an existing owner will have an extra zero, hence increasing the sparsity of that line. On the other hand, since the new owners are not common to other firms, the degree of CO decreases. It follows that the market dynamics in this scenario goes in the direction predicted by the property, which therefore is admissible in the CO framework.

Appendix B. New measures of CO summary tables

See Tables B.1 and B.2.

Table B.1

Common shareholding indicators for shareholders' portfolios, and market level counterparts.

Indicator	Definition	Interpretation	Market level
Density	Nsubs/Nfirms	Number of firms in a shareholder's portfolio over total number of firms in the market. Represents the share of the market to which an investor has access through shareholding.	Average, median, maximum, percentiles
TOAS density	Total TOAS subs/Total market TOAS	Sum of the TOAS of all firms in a shareholder's portfolio over sum of TOAS of all firms in the market. Represents the relative weight of the firms chosen by a specific investor over the whole market.	Average, median, maximum, percentiles
TOAS weighted density	Sum weighted TOAS subs/Total market TOAS	Sum of the TOAS of firms in portfolio, each weighted by the actual ownership share of investor, over sum of TOAS of all firms in the market. Represents the actual share of TOAS of the market in the hand of an investor through the specific ownership shares held.	Average, median, maximum, percentiles, sum for 'Big 3'
MKT CAP density	Total MKT CAP subs/Total market MKT CAP	Sum of the MKT CAP of all listed firms in a shareholder's portfolio over sum of MKT CAP of all listed firms in the market. Interpretation is same as for TOAS density, but refers to market capitalisation. <i>Only listed firms in portfolio.</i>	Average, median, maximum, percentiles
MKT CAP weighted density	Sum weighted MKT CAP subs/Total market MKT CAP	Sum of the MKT CAP of listed firms in portfolio, each weighted by the actual ownership share of investor, over sum of MKT CAP of all listed firms in the market. Interpretation is same as for TOAS weighted density, but refers to market capitalisation. <i>Only listed firms in portfolio.</i>	Average, median, maximum, percentiles, sum for 'Big 3'
Uniformity	$1 - \frac{\sqrt{\sum \text{shares}^2}}{\sum \text{shares}}$	One minus the following ratio: (Square root of the) Sum of the squares of the shares in portfolio, over sum of all shares in portfolio. Index assesses the relative weight of larger participation shares over the shares total, showing smaller values for more concentrated distributions.	Average, median, maximum, percentiles
Number of Block-holdings	Number of holdings > $p\%$	Number of participations in portfolios with share value above a certain percentage p . Represents the number of more intensive investments of portfolio. Computed for $p = 3, 5, 10$.	Sum for 'Big 3', sum for all SH

Table B.2

Common shareholding indicators for shareholders' and firms' networks.

Shareholders' network			
Indicator	Definition	Interpretation	
Correlation	Pearson's ρ	Pearson's correlation coefficient ρ between pairs of portfolios. Reveals the level of overlap between two shareholders' investments.	
Proportion of non-zero correlations	$\frac{\text{No. non-zero } \rho}{n(n-1)/2}$	Number of non-zero correlations over total number of possible connections between pairs of portfolios. n is the total number of portfolios. Represents the proportion of existing links in the shareholders' network, i.e. the network's density.	
Number of highly correlated high-density portfolios	No. of $\rho > 80\%$	Number of correlations higher than 80% between pairs of large (high density) portfolios (holding more than 10% of the market's firms). Counts the number of very strong links among large shareholders' portfolios.	
Proportion of highly correlated high-density portfolios	$\frac{\text{No. } \rho > 80\%}{k(k-1)/2}$	Number of correlations higher than 80%, over total number of possible connections between pairs of large (high density) portfolios (holding more than 10% of the market's firms). k is the total number of high density portfolios. Represents the proportion of very strong links among large shareholders' portfolios, i.e. the degree of similarity of their investments.	
Firms' network			
Indicator	Definition	Interpretation	
Correlation	Pearson's ρ	Pearson's correlation coefficient ρ between pairs of SH structures of firms. Reveals the level of overlap between two firms' SH structures.	
Proportion of non-zero correlations	$\frac{\text{No. non-zero } \rho}{n(n-1)/2}$	Number of non-zero correlations over total number of possible connections between pairs of firms. n is the total number of firms. Represents the proportion of firms that are connected through some common shareholder, i.e. the firms' network density.	
Number of highly correlated SH structures	No. of $\rho > 80\%$	Number of correlations higher than 80% between pairs of firms. Counts the number of very strong links among firms' SH structures.	
Proportion of highly correlated SH structures	$\frac{\text{No. } \rho > 80\%}{k(k-1)/2}$	Number of correlations higher than 80%, over total number of non-zero connections between pairs of firms. k is the total number of connected firms. Represents the proportion of very strong links among connected firms, i.e. the degree of similarity of SH structure.	

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