Li, Bingbing; Long, Yan

Working Paper

Output uncertainty mitigation in competitive markets

Center for Mathematical Economics Working Papers, No. 698

Provided in Cooperation with:
Center for Mathematical Economics (IMW), Bielefeld University

Center for Mathematical Economics
Working Papers

698

January 2024

Output Uncertainty Mitigation in Competitive Markets

Bingbing Li and Yan Long

Center for Mathematical Economics (IMW)
Bielefeld University
Universitätsstraße 25
D-33615 Bielefeld · Germany

e-mail: imw@uni-bielefeld.de
uni-bielefeld.de/zwe/imw/research/working-papers
ISSN: 0931-6558

Output uncertainty mitigation in competitive markets

Bingbing Li[*], Yan Long[†]

January 22, 2024

Abstract

Output uncertainty is a major concern for industries prone to exogenous, persistent and large fluctuations in output, such as agriculture, wind and solar power generation, while technology adoption aimed at mitigating output uncertainty can improve social welfare. This paper constructs a competitive market model with random output fluctuations to examine the scale of technology adoption at the long-term equilibrium and its comparison with the social optimum. We show that the First Welfare Theorem no longer holds in general, and depending on the characteristics of the demand function, the scale of technology adoption in the competitive market may be greater or less than the socially optimal scale.

Keywords: technology adoption, uncertainty mitigation, long-term equilibrium, non-optimal scale

JEL: D50, D61, D62

[*]School of Economics, Huazhong University of Science and Technology, Wuhan, China. Email: bbingli@hust.edu.cn.

[†]School of Economics, Huazhong University of Science and Technology, Wuhan, China. Email: ylong@hust.edu.cn. We are grateful to the Ministry of Education Project of Humanities and Social Sciences (Grant 21YJC790080) and the National Natural Science Foundation of China (Grant 7210030643) for funding. We thank Kaige Hu for the excellent RA work.

1 Introduction

Uncertainty is an inescapable reality that many industries have to consider in production activities. Ho (1989) divided production uncertainty into two categories: (i) environmental uncertainty and (ii) system uncertainty. Environmental uncertainty refers to uncertainties caused by factors outside the production process, such as demand uncertainty and uncertainty caused by suppliers. System uncertainty is related to uncertainties in the production process, such as uncertainty in operation yield, quality, and failure of the production system. This paper considers the latter category of uncertainty.

Specifically, this paper focuses on production activities that depend greatly on natural conditions such as weather and temperature, including agriculture, aquaculture, and the rapidly developing solar and wind power generation in recent years. The output uncertainty of such production activities has the following characteristics: (1) The uncertainty in output is determined by exogenous factors rather than the uncertainty in inputs such as raw materials; (2) The uncertainty in output is persistent rather than caused by accidental events such as sudden interruptions or inexperience; (3) The fluctuations in output caused by exogenous factors are large and are among the fundamental reasons determining the final output. With the intensification of global warming and the large-scale application of volatile renewable energy, such output uncertainty issues become increasingly severe.

While output uncertainty may cause huge negative impacts, there does exist a series of technologies that can reduce output fluctuations. For example, agriculture can use technologies such as precision irrigation and intelligent fertilization to reduce the impact of natural environmental factors such as climate and soil on output (Koundouri et al., 2006). In wind and solar power generation, pumped storage and electrochemical energy storage are effective ways to mitigate output fluctuations (Schmidt et al., 2017).

However, whether firms in a competitive market have enough incentives to adopt these technologies is still an open question, since technology adoption among firms will affect each other, that is, the efforts of one firm to reduce output fluctuations will affect the efforts of other firms through the impact on prices. The First Welfare Theorem no longer holds automatically due to the firms' enlarged action space. Whether competitive market equilibrium can achieve the social optimum is the main research question studied in this paper.

To answer this question, we construct a competitive market model with the above production characteristics to examine how the interaction between firms determines the scale of technology adoption. While market prices are determined by supply and demand, as in a standard competitive market model, firms's efforts to adopt technology are determined in the following way: each firm's effort choice depends on price, which depends on the total effort of the industry; but each firm, whose output is too small compared with the total output of the industry, ignores the effect its own effort choice has on price.[1] Hence the equilibrium concept we define, though consisting of more variables than the standard competitive equilibrium, is competitive in sprit. And we find that the First Welfare Theorem no longer holds in general, and depending on the characteristics of the consumer demand function, the scale of technology adoption in the competitive market may be greater or less than the social optimal scale. Hence governments need to determine the optimal intervention policy according to the demand characteristics.

We end the introduction with a brief review of the literature. The existence of uncertainty is well acknowledged in economics, and it has been integrated into general equilibrium models (e.g., Borch (1962) and Arrow and Neave (1978)) and partial equilibrium models (e.g., Joskow and Tirole (2006) and Joskow and Tirole (2007)). It is well-known that when market price can be contingent on the realized state, competitive equilibrium achieves social optimum. However, to our best knowledge, the problem of

[1]The equilibrium determination of efforts has some flavor of Nash equilibrium in large games, see Schmeidler (1973) and Khan and Sun (2002).

mitigating uncertainty in competitive market has rarely been addressed in the literature of economics.

On the other hand, there are many studies in the literature of operation that focused on preventing risks caused by external factors such as natural disasters through supply chain risk management (SCRM) (e.g. Singhal et al. (2011) and Ritchie and Brindley (2007)) or mitigating uncertainty in the production process through production planning (e.g., Bertrand and Rutten (1999), Graves (2011)). For the production activities studied in this paper, which feature persistent and large output fluctuations, the role of SCRM and production planning are both limited. Furthermore, the aforementioned literature has mainly centered on mitigating uncertainty for individual firms, whereas this paper discusses how the scale of firms' technology adoption to reduce uncertainty interact with each other, and whether competitive markets necessitate policy intervention.

2 Model Setting

On the supply side, the model consists of two industries, one with deterministic output, and the other with stochastic fluctuating outputs as well as technology that can mitigate fluctuations. And they are referred to as deterministic industry and stochastic industry, respectively. The demand side is comprised of representative consumers who own capital that can be invested in the two industries and who consume products from both industries.

We first describe the production of each industry. For the deterministic industry, one unit of capital can produce one unit of product. For the stochastic industry, the output rate per unit of capital is $i \in [0, 1]$, where i is a uniform random variable whose realization only depends on exogenous factors like weather or temperature. And we further assume the realization of output rate is identical for each unit of capital. That is, when the capital stock of all firms in the stochastic industry is K, the total output level is iK.

The variable cost of production in each industry is assumed to be zero. Each industry has a large number of firms, and they are all price takers.

Firms in the stochastic industry can adopt some kind of technology that reduces output fluctuations.[2] Let $\theta \in [0, 1]$ be the measure of technology adoption scale. Fixed θ, the output rate per unit of capital becomes $j = (1-\theta)i + \frac{\theta}{2}$. Note that $j \in [\frac{\theta}{2}, 1-\frac{\theta}{2}]$, and $E[j] = (1-\theta)E[i] + \frac{\theta}{2} = E[i]$, $Var[j] = (1-\theta)^2 Var[i]$. When θ is larger, the range of j is narrower and the variance is smaller. Let $h(\theta)$ be the amount of capital that is needed to achieve technology adoption scale θ for each unit of capacity investment, where $h(0) = h'(0) = 0$, $h'(\theta) > 0$ for $\theta > 0$, $\lim_{\theta \to 1} h'(\theta) = \infty$, and $h''(\theta) > 0$.

Consumers own a fix amount of capital $\bar{K}$. And the utility function for the representative consumer is $U(q, x)$, where q is the products of the industry with stochastic output, and x is the products of the industry with deterministic output. We further assume that the utility function is separable, that is $U(q, x) = S(q) + W(x)$, where $S'(q) > 0$, $S''(q) < 0$, $W'(x) > 0$, $W''(x) \leq 0$, $\lim_{q \to 0} S'(q) > W'(\bar{K})$ and $\lim_{x \to 0} W'(x) > S'(\bar{K})$. Note that consumers prefer lower output volatility. That is, $S(E[\tilde{q}]) > E[S(\tilde{q})]$, for any $\tilde{q}$ being a random consumption amount of products from the stochastic industry.

We will first look at the socially optimal technology adoption scale. Then we will define and characterize the long-term market equilibrium. Finally, we will compare the market equilibrium results with the socially optimal results.

3 Social optimum

Let K be the capital investment level and θ be the technology adoption scale in the stochastic industry. The social optimal problem is the following.

[2] In general, technology adoption will affect both output volatility and output level. Here we only consider the impact on volatility. Some technologies can be approximated as only affecting volatility though, such as power storage technologies.

$$\max_{\theta,K} E_j[S(jK)] + W(\bar{K} - (1+h(\theta)) \cdot K).$$

Since i is uniformly distributed in $[0,1]$ and $j = (1-\theta)i + \frac{\theta}{2}$, we have that j is uniformly distributed in $[\frac{\theta}{2}, 1-\frac{\theta}{2}]$. And we can hence write the problem as

$$\max_{\theta,K} \int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} \frac{1}{1-\theta} S(jK)\mathrm{d}j + W(\bar{K} - (1+h(\theta)) \cdot K).$$

Since $\lim_{q\to 0} S'(q) > W'(\bar{K})$ and $\lim_{x\to 0} W'(x) > S'(\bar{K})$, in the optimal solution, $0 < K < \bar{K}$. Since $\lim_{\theta\to 1} h'(\theta) = \infty$, in the optimal solution, $\theta < 1$. Suppose in the optimal solution, $\theta > 0$. The the optimal solution satisfies the following first order conditions:

For K,

$$E_j[S'(jK) \cdot j] = W'(\bar{K} - (1+h(\theta)) \cdot K)(1+h(\theta)). \tag{1}$$

For θ,

$$\frac{1}{(1-\theta)^2} \int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} S(jK)\mathrm{d}j - \frac{1}{2(1-\theta)}[S(\frac{\theta}{2}K) + S((1-\frac{\theta}{2})K)] = W'(\bar{K} - (1+h(\theta)) \cdot K) \cdot h'(\theta) \cdot K. \tag{2}$$

Equation (1) indicates that the marginal benefits of capital investment in the two industries are equal. Equation (2) indicates that the marginal benefit from increasing the scale of technology application θ equals the marginal cost. Note that the marginal benefit from increasing θ consists of two parts: one is the loss of consumer utility at the two endpoints of $\frac{\theta}{2}$ and $1-\frac{\theta}{2}$, and the other is the increase in consumer utility over the entire interval $(\frac{\theta}{2}, 1-\frac{\theta}{2})$, since the probability density at each position in the interval has increased.

Since $S'' < 0$, we have that

$$\frac{\int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} S(jK)\mathrm{d}j}{1-\theta} > \frac{S(\frac{\theta}{2}K) + S((1-\frac{\theta}{2})K)}{2} \qquad \forall \theta \in [0,1),$$

that is, the average value of $S(jK)$ over the entire interval $[\frac{\theta}{2}K, 1-\frac{\theta}{2}K]$ is greater than the average value of $S(jK)$ at the two end points. Therefore, the marginal benefit from increasing θ is greater than zero for any $\theta \in [0,1)$. Since $h'(0)=0$, the optimal scale of technology adoption is indeed greater than zero.

4 Competitive equilibrium

In this section, we consider the long-term equilibrium of the competitive market. Like the standard competitive market model, the output of each firm in each industry is very small relative to the total output of the industry, so it is a price taker. The market equilibrium prices of products of both industries are determined by the supply of firms and the demand of consumers. The market has free entry, so in the long-term equilibrium, the profits of all firms in both industries are zero.

How do firms in the stochastic industry determine the scale of technology adoption in the long-term equilibrium? We assume that each firm will determine its own scale of technology adoption according to the market prices, while the market prices are taken as given. That is, any single firm ignores the impact of its own choice of technology adoption scale on market prices. Given that the output of each firm is very small relative to the total market output, this assumption is reasonable. And we only consider symmetric equilibrium, that is, all firms choose the same technology adoption scale θ at equilibrium. Specifically, θ determines the range of output rate j. The market forms different prices p_j based on different realizations of the output rate j. Each firm chooses its technology adoption scale to maximize its expected profit given the prices.

Note that since the firms' capital investment in both industries have the characteristics of constant returns to scale, we only need to consider the total capital investment level of the whole industries at equilibrium. For the deterministic industry, suppose it invests K_d units of capital, then it produces K_d units of products; since firms earn zero profit, the price of the products equals the price of the capital, which we denote by p_c. In equilibrium, consumers consume K_d units of products from the deterministic industry, and the marginal utility, $W'(K_d)$, equals the price p_c.

Consider the stochastic industry. Suppose the capital investment level of the stochastic industry is K, and the technology adoption scale of each firm is θ, and the market price of its product is p_j when the output rate is $j \in [\frac{\theta}{2}, 1 - \frac{\theta}{2}]$. The equilibrium is then characterized by the following conditions:

(i) At any output rate, firms maximize profits, consumers maximize utilities, and market supply equals demand.

Specifically, in each state j, firms maximize profits by supplying all outputs, and the total demand of the market equals the total supply, which equals jK. Consumers maximize utilities by consuming to the extent such that the marginal utility equals the market price, that is,

$$S'(jK) = p_j \qquad \forall j. \tag{3}$$

(ii) The long-term profits of the firms are zero.

Specifically, the cost of one unit capacity investment plus the corresponding technology adoption cost equals the expected marginal return, that is,

$$E_j[p_j \cdot j] = (1 + h(\theta))p_c, \tag{4}$$

where $p_c = W'(\bar{K} - (1 + h(\theta))K)$.

(iii) For each firm, given market prices, the scale θ of technology adoption maximizes its profit.

Specifically, given p_j, where $j = (1 - \theta)i + \frac{\theta}{2}$, and p_c, we have that

$$\theta = \arg \max_{\theta' \in [0,1]} E_{j'}[p_j \cdot j'] - p_c \cdot h(\theta'),$$

where $j' = (1-\theta')i + \frac{\theta'}{2}$.

Now we analyze condition (iii). The firm's objective function can be written as follows.

$$\max_{\theta' \in [0,1]} \int_{\frac{\theta'}{2}}^{1-\frac{\theta'}{2}} \frac{1}{1-\theta'} p_j \cdot j' \mathrm{d}j' - p_c \cdot h(\theta').$$

Note that $j = \frac{j'-\theta'/2}{1-\theta'}(1-\theta) + \frac{\theta}{2}$. Hence when $j' = \frac{\theta'}{2}$, $j = \frac{\theta}{2}$, and when $j' = 1 - \frac{\theta'}{2}$, $j = 1 - \frac{\theta}{2}$. Suppose the optimal solution is an interior solution, then the first-order condition is as follows.

$$\frac{1}{(1-\theta')^2} \int_{\frac{\theta'}{2}}^{1-\frac{\theta'}{2}} p_j \cdot j' \mathrm{d}j' - \frac{1}{2(1-\theta')}[p_{\frac{\theta}{2}} \frac{\theta'}{2} + p_{1-\frac{\theta}{2}}(1-\frac{\theta'}{2})] = p_c \cdot h'(\theta').$$

As in the social optimal problem, the marginal revenue of increasing θ' can be divided into two parts. One part is the loss of revenue at the two endpoints of $\frac{\theta'}{2}$ and $1-\frac{\theta'}{2}$. The other part is the increase in revenue over the entire interval $(\frac{\theta'}{2}, 1-\frac{\theta'}{2})$, since the probability density at each point in the interval has increased.

In equilibrium, $\theta' = \theta$, $p_j = S'(jK)$ and $p_c = W'(\bar{K} - (1+h(\theta))K)$. Hence we obtain

$$\frac{1}{(1-\theta)^2} \int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} S'(jK) \cdot j \mathrm{d}j - \frac{1}{2(1-\theta)}[S'(\frac{\theta}{2}K)\frac{\theta}{2} + S'((1-\frac{\theta}{2})K) \cdot (1-\frac{\theta}{2})] = W'(\bar{K} - (1+h(\theta))K) \cdot h'(\theta). \tag{5}$$

Above we consider the case where the equilibrium scale of technology adoption is greater than zero. However, depending on the parameters of the model, the equilibrium scale of technology adoption may be zero.

Let $T(j,K) = S'(jK) \cdot j$. Note that $T(j,K)$ is the revenue firms obtain for each unit of capital investment, and it is also consumers' marginal utility with respect to K. We obtain $T_j''(j,K) = 2KS''(jK) + jK^2 S'''(jK)$. If $S'''(jK) \leq 0$ or $S''' > 0$ smaller enough, then $T''(j,K) < 0$. And as in the social optimum problem, for any θ, the left side of Equation (5) is greater than zero. Since $h'(0) = 0$, the technology adoption scale in the competitive equilibrium is greater than zero.

However, $T''(j)$ is not necessarily negative. If $S'''(\cdot)$ is very large such that for any K greater than the socially optimal capital level with no technology adoption and any $\theta \in [0, 1)$, $T(j, K)$ has a lower average value over the interval $[\frac{\theta}{2}, 1 - \frac{\theta}{2}]$ than at the two endpoints, then the marginal benefit is negative for any $\theta \in [0, 1)$. As a result, firms will choose $\theta = 0$ at equilibrium.

5 Comparison between competitive equilibrium and social optimum

In this section we compare the competitive equilibrium and the social optimum solution.

Note that Equation (2) can be written as follows.

$$\frac{1}{(1-\theta)^2}\int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} \frac{S(jK)}{K}\mathrm{d}j - \frac{1}{2(1-\theta)}[\frac{S(\frac{\theta}{2}K)}{K} + \frac{S((1-\frac{\theta}{2})K)}{K}] = W'(\bar{K} - (1+h(\theta))K) \cdot h'(\theta). \tag{6}$$

By comparing Equation (6) and Equation (5), we can see that in the social optimal problem, the effect of increasing θ acts on consumers' average utility function $S(jK)/K$, while in the competitive equilibrium, the effect of increasing θ acts on consumers' marginal utility function $S'(jK) \cdot j$, which is also firm's revenue function per unit of capital. Therefore, in general, these the two solutions are not the same. That is, in a market where the choice of technology adoption scale is incorporated, the First Welfare Theorem no longer holds in general.

Let $V(j, K) = S(jK)/K - S'(jK) \cdot j$. We first note that only when $V(j, K)$ is special would the competitive equilibrium be socially optimal.

Proposition 1. *The competitive equilibrium is socially optimal if and only* *if* $E_j[V(j,K^*)] = \frac{V(1-\frac{\theta^*}{2},K^*)+V(\frac{\theta^*}{2},K^*)}{2}$.

The condition required for the above proposition is that the average value of $V(j,K^*)$ over the interval $[\frac{\theta^*}{2}, 1-\frac{\theta^*}{2}]$ is equal to its average value at the two endpoints. Note that $V_j''(j,K) = -KS''(jK) - jK^2S'''(jK)$. If $S''(\cdot) = 0$, then $V_j''(j,K) = 0$; that is, $V(j,K)$ is a linear function of j. Then this condition holds. That is, if consumers are risk neutral, the competitive equilibrium is socially optimal; in fact, the technology to mitigate output fluctuations would be of no use either for the social planner or for competitive firms. If $S''(\cdot) < 0$, then $V(j,K)$ is in general not a linear function of j, and this condition will not hold in general.

If we further assume that $W''(x) = 0$, then we can compare the two solutions and have the following results. Note that under the condition that $W''(x) = 0$, we can reduce our model to a partial equilibrium model that only consists of one industry and a representative consumer. This partial equilibrium model is wildly used in the literature of electricity market, see e.g. Turvey (1968) and Joskow and Tirole (2007). The difference between our model and the classical one is that we consider the technology that could reduce output fluctuations.

Proposition 2. *Suppose* $W''(x) = 0$ *and* $h''(\theta)$ *is large enough.*

(i) The capacity investment level of the stochastic industry in the competitive equilibrium is greater than or equal to the socially optimal level of capacity investment.

(ii) If $V_j'' > 0$*, then the scale of technology adoption in the competitive equilibrium is greater than the socially optimal scale of technology adoption.*

(iii) If $V_j'' < 0$*, then the scale of technology adoption in the competitive equilibrium is less than the socially optimal scale of technology adoption.*[3]

[3]Suppose $j = 0$, then $V_j'' = -KS''(0) > 0$. Hence for j small enough, we have that $V_j'' > 0$. That is, the condition that $V_j'' < 0$ for all j will never hold. However, as shown by Example 1, if $V_j'' < 0$ for a large range of j, then the scale of technology adoption in the competitive equilibrium will be less than the socially optimal scale of technology adoption.

In the proof of the above proposition, we show that when $V_j'' > 0$, $T_j'' < 0$. According to the analysis in Section 4, in this case, the scale of technology adoption in the competitive equilibrium is greater than zero. According to the above proposition, we know further that the scale of technology adoption in the competitive equilibrium is greater than the socially optimal scale. When $V_j'' < 0$, the scale of technology adoption in the competitive equilibrium may be greater than zero or equal to zero, and in both cases it is less than the socially optimal scale of technology adoption.

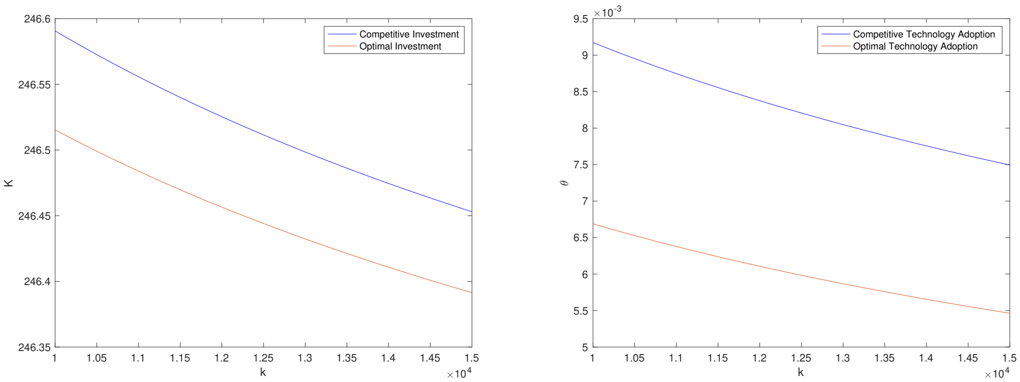
We illustrate these results using the following numerical example.

Example 1. Let $S(q) = a[b^{-1} - (q+b)^{-1}]$ and $W(x) = x$. Note that $S'(q) = a(q+b)^{-2}$, $S''(q) = -2a(q+b)^{-3} < 0$, and $V_j'' = 2a(jK+b)^{-4}(b-2jK)$. Hence $V_j'' > 0$ if and only if $b > 2jK$. Let $h(\theta) = k\theta^3$.

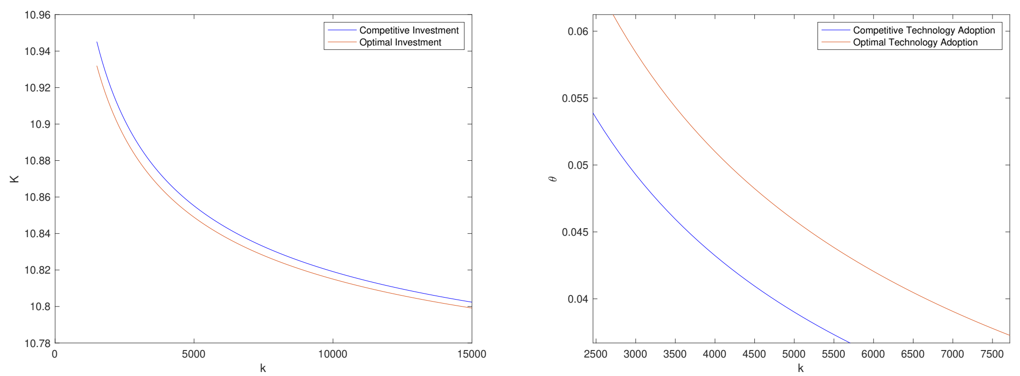
Figure 1 shows changes of the capacity investment level and technology adoption scale in the stochastic industry, as the parameter k of the technology adoption cost changes. It can be seen that in all cases, the capacity investment in the competitive equilibrium is greater than that the socially optimal investment.

Case 1. In Figure 1(a), $a = 10^9$, $b = 4000$, $I = 50$, $k \in [10000, 15000]$. Note that $V_j'' > 0$. In this case, the scale of technology adoption in the competitive equilibrium is greater than the socially optimal adoption scale. As the cost of technology adoption increases, the optimal and competitive investment and technology adoption scale decrease, indicating complementarity between capacity investment and technology adoption.

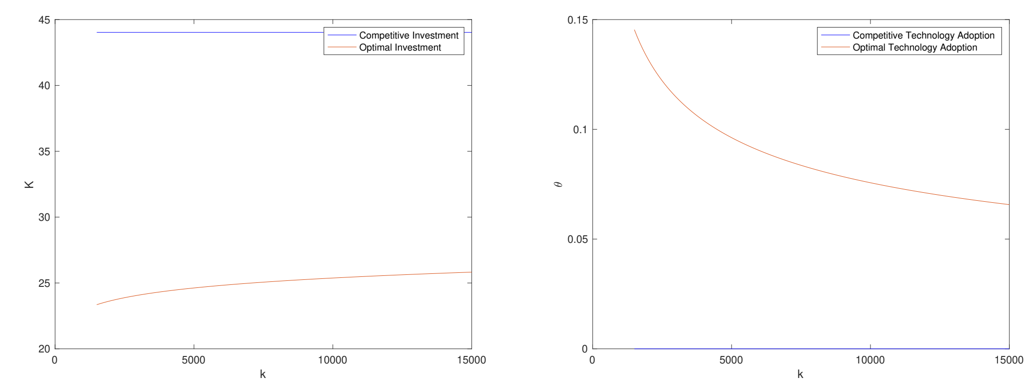
Case 2. In Figure 1(b), $a = 10^4$, $b = 4$, $I = 50$, $k \in [0, 15000]$. Hence when $j > 0.2$, $V_j'' < 0$. In this case, the scale of technology adoption in the competitive equilibrium is smaller than the socially optimal adoption scale, but greater than zero. Again as the cost of technology adoption increases, the optimal and competitive investment and technology adoption scale decrease, indicating complementarity between capacity investment and technology adoption.



(a) Case 1



(b) Case 2



(c) Case 3

Figure 1: Social optimum and competitive equilibrium

Case 3. In Figure 1(c), $a = 10^4$, $b = 0.001$, $I = 50$, $k \in [0, 15000]$. Hence when $j > 0.0001$, $V_j'' < 0$. In this case, the scale of technology adoption in the competitive equilibrium equals zero, smaller than the socially optimal adoption scale. The optimal investment increases with the increase of technology adoption costs, while the optimal scale of technology adoption decreases with the increase of technology application costs, indicating substitutability between capacity investment and technology adoption.

Since $V_j''(j, K) = -KS''(jK) - jK^2S'''(jK)$ and $S'' < 0$, we have that if $S'''(jK) \leq 0$ or $S'''(jK) > 0$ small enough, then $V_j'' > 0$. We hence obtain the following corollary.

Corollary 1. *If $W''(x) = 0$, and $S'''(jK) \leq 0$ or $S'''(jK) > 0$ small enough, then the scale of technology adoption in the competitive equilibrium is greater than the socially optimal scale of technology adoption.*

Therefore, when the inverse demand function, that is, $S'(q)$, is concave or weakly convex, the government should weaken the marginal returns to technology adoption in the competitive market to correct the tendency of over-investment in the competitive market. When $S'(q)$ shows strong convexity, the government needs to increase the marginal returns to technology adoption in the competitive market to correct the tendency of underinvestment in the competitive market.

Appendix

Let (K^*, θ^*) be the socially optimal solution, and (K^e, θ^e) be the competitive equilibrium solution.

A.1 Proof of Proposition 1

Proof. Suppose $(K^e, \theta^e) = (K^*, \theta^*)$. Then the left hand of Equation (6) is equal to the left hand of Equation (5). That is,

$$\frac{1}{1-\theta^*}\left[\frac{\int_{\frac{\theta^*}{2}}^{1-\frac{\theta^*}{2}} V(j, K^*)dj}{1-\theta^*} - \frac{V(1-\frac{\theta^*}{2}, K^*) + V(\frac{\theta^*}{2}, K^*)}{2}\right] = 0.$$

Hence $E_j[V(j, K^*)] = \frac{V(1-\frac{\theta^*}{2}, K^*)+V(\frac{\theta^*}{2}, K^*)}{2}$.

Suppose the condition holds. Then (K^*, θ^*) satisfy Equation (3), Equation (4), and Equation (5). That is, (K^*, θ^*) is the same as the competitive equilibrium solution. $\square$

A.2 Proof of Proposition 2

Proof. Without loss of generality, assume that $W'(x) = 1$ for any x. We first prove (i). We consider the following two cases.

Case 1: $\theta^e = 0$.

Since θ^e is the optimal choice of firms, $E_i[S'(iK^e) \cdot i] > E_j[S'(jK^e) \cdot j] - h(\theta^*)$, where $j = i(1-\theta^*) + \frac{\theta^*}{2}$. Suppose for the sake of contradiction that $K^* \geq K^e$. Since $T'_K(j, K) = S''(jK) \cdot j^2 < 0$, we have that $E_j[T(j, K^e)] > E_j[T(j, K^*)]$, that is, $E_j[S'(jK^e) \cdot j] - h(\theta^*) > E_j[S'(jK^*) \cdot j] - h(\theta^*)$. Since $E_j[S'(jK^*) \cdot j] - h(\theta^*) = 1$ (this is Equation (1)), we have that $E_j[S'(jK^e) \cdot j] - h(\theta^*) > 1$. However, $E_i[S'(iK^e) \cdot i] = 1$ (this is Equation (4)). Contradiction! Hence $K^* < K^e$.

Case 2: $\theta^e > 0$.

Let

$$F(K, \theta) := E_j[S'(jK) \cdot j] - (1 + h(\theta)),$$

and

$$G(K,\theta) := \frac{1}{(1-\theta)^2}\int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} \frac{S(jK)}{K}\mathrm{d}j - \frac{1}{2(1-\theta)}[\frac{S(\frac{\theta}{2}K)}{K} + \frac{S((1-\frac{\theta}{2})K)}{K}] - h'(\theta),$$

$$H(K,\theta) := \frac{1}{(1-\theta)^2}\int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} S'(jK)\cdot j\mathrm{d}j - \frac{1}{2(1-\theta)}[S'(\frac{\theta}{2}K)\cdot\frac{\theta}{2} + S'((1-\frac{\theta}{2})K)\cdot(1-\frac{\theta}{2})] - h'(\theta).$$

Note that $h''(\theta)$ is assumed to be large enough such that $G'_\theta < 0$ and $H'_\theta < 0$; in addition,

$$F'_K(K,\theta) = E_j[S''(jK)\cdot j^2] < 0,$$

and

$$F'_\theta(K,\theta) = H(K,\theta).$$

According to the first order conditions of the social optimal problem, we have that $F(K^*,\theta^*) = 0$ (this is Equation (1)), and $G(K^*,\theta^*) = 0$ (this is Equation (2)).

According to the conditions of competitive equilibrium, we have that $F(K^e,\theta^e) = 0$ (this is Equation (4)) and $H(K^e,\theta^e) = 0$ (this is Equation (5)).

Since $F'_\theta(K^e,\theta^e) = H(K^e,\theta^e) = 0$ and $F''_\theta(K^e,\theta^e) = H'_\theta(K,\theta) < 0$, we have that $0 = F(K^e,\theta^e) = \max_{\theta\in[0,1]} F(K^e,\theta)$. Hence $F(K^e,\theta^*) \leq 0$. Since $F'_K < 0$ and $F(K^*,\theta^*) = 0$, we have that $K^* \leq K^e$.

We now prove (ii).

Since $V''_j = -KS''(jK) - jK^2S'''(jK) > 0$, we have that $jK^2S'''(jK) < -KS''(jK) < -2KS''(jK)$. Hence $T''_j(j,K) = 2KS''(jK) + jK^2S'''(jK) < 0$. According to the analysis in Section 4, $\theta^e > 0$.

Since

$$G'_K(K,\theta) = \frac{1}{(1-\theta)^2}\int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} \frac{\partial \frac{S(jK)}{K}}{\partial K}\mathrm{d}j - \frac{1}{2(1-\theta)}[\frac{\partial \frac{S(\frac{\theta}{2}K)}{K}}{\partial K} \cdot + \frac{\partial \frac{S((1-\frac{\theta}{2})K)}{K}}{\partial K}],$$

we have that if $\frac{\partial \frac{S(jK)}{K}}{\partial K}$ is concave in j, then $G'_K(K,\theta) \geq 0$. That is, if $\frac{\partial^3 \frac{S(jK)}{K}}{\partial K \partial^2 j} \leq 0$, $G'_K(K,\theta) \geq 0$.

Note that $\frac{\partial^3 \frac{S(jK)}{K}}{\partial K \partial^2 j} = S'''(jK)jK + S''(jK) = -\frac{V''_j}{K}$. Since $V''_j > 0$, we have that $\frac{\partial^3 \frac{S(jK)}{K}}{\partial K \partial^2 j} < 0$ and $G'_K(K,\theta) > 0$.

Note that

$$G(K,\theta) - H(K,\theta) = \frac{1}{1-\theta}[\frac{\int_{\frac{\theta}{2}}^{1-\frac{\theta}{2}} V(j,K)dj}{1-\theta} - \frac{V(1-\frac{\theta}{2},K) + V(\frac{\theta}{2},K)}{2}].$$

Since $V''_j > 0$, $G(K,\theta) < H(K,\theta)$.

Suppose for the sake of contradiction that $\theta^* \geq \theta^e$. We then have

$$0 = G(K^*,\theta^*) \leq G(K^e,\theta^*) < H(K^e,\theta^*) \leq H(K^e,\theta^e) = 0 \quad .$$

Note that the first inequality holds because $G'_K > 0$ and $K^* \leq K^e$, and the second inequality holds because $G(K,\theta) < H(K,\theta)$, and the third inequality holds because $H'_\theta(K,\theta) < 0$ and according to (i), $\theta^* \geq \theta^e$. Contradiction!

Hence $\theta^* < \theta^e$.

Now we prove (iii). If $\theta^e > 0$, then we can prove (iii) using similar arguments as in the proof for (ii). If $\theta^e = 0$,then $\theta^* > 0 = \theta^e$.

References

[1] Kenneth J. Arrow and Julius A. S. Neave. Risk Allocation and Information: Some Recent Theoretical Developments. *The Geneva Papers on Risk and Insurance*, (8):1–19, 1978.

[2] J Will M Bertrand and Werner GMM Rutten. Evaluation of three production planning procedures for the use of recipe flexibility. *European journal of operational research*, 115(1):179–194, 1999.

[3] Karl Borch. Equilibrium in a Reinsurance Market. *Econometrica*, 30(3):424–444, 1962.

[4] Stephen C Graves. Uncertainty and production planning. *Planning Production and Inventories in the Extended Enterprise: A State of the Art Handbook, Volume 1*, pages 83–101, 2011.

[5] Chrwan-Jyh Ho. Evaluating the impact of operating environments on mrp system nervousness. *The International Journal of Production Research*, 27(7):1115–1135, 1989.

[6] Paul Joskow and Jean Tirole. Retail Electricity Competition. *The RAND Journal of Economics*, 37(4):799–815, 2006.

[7] Paul Joskow and Jean Tirole. Reliability and competitive electricity markets. *The RAND Journal of Economics*, 38(1):60–84, March 2007.

[8] M. Ali Khan and Yeneng Sun. *Handbook of Game Theory with Economic Applications*, volume 3, chapter Non-cooperative games with many players, pages 1761–1808. Elsevier, January 2002.

[9] Phoebe Koundouri, Céline Nauges, and Vangelis Tzouvelekas. Technology adoption under production uncertainty: theory and application to irrigation technology. *American Journal of Agricultural Economics*, 88(3):657–670, 2006.

[10] Bob Ritchie and Clare Brindley. Supply chain risk management and performance: A guiding framework for future development. *International journal of operations & production management*, 2007.

[11] David Schmeidler. Equilibrium points of nonatomic games. *Journal of Statistical Physics*, 7(4):295–300, April 1973.

[12] Oliver Schmidt, Adam Hawkes, Ajay Gambhir, and Iain Staffell. The future cost of electrical energy storage based on experience rates. *Nature Energy*, 2(8):1–8, 2017.

[13] Piyush Singhal, Gopal Agarwal, and Murali Lal Mittal. Supply chain risk management: review, classification and future research directions. *International Journal of Business Science & Applied Management*, 6(3):15–42, 2011.

[14] Ralph Turvey. *Optimal Pricing and Investment in Electricity Supply.* George Allen and Unwin LTD, London, 1968.

□