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Restricted Dominant Unanimity and Social Discounting

Bach Dong-Xuan* and Xiangyu Qu[†]

Abstract

This paper addresses the intricate challenge of establishing social discount rates across far-reaching generations, particularly in the face of divergent social viewpoints. We introduce several principles related to *Dominant Unanimity*, which enable nondictatorial social discounting, and we characterize different ranges of social discount factors based on individual one-period discount factors. Our findings suggest that societies adhering to these principles exhibit varying degrees of patience and different ranges of social discounting. This approach provides theoretical pathways to enhance the consideration of distant future welfare, particularly in the context of long-term economic activities and policies.

1 INTRODUCTION

The discourse surrounding social discounting remains fiercely debated, highlighted prominently by contrasting viewpoints from economists (Weitzman (2001); Drupp et al. (2018)). Stern (2007), for instance, advocates for a near-zero social discount rate, emphasizing the imperative for immediate action against climate change. This stance posits that the welfare of future generations should weigh heavily in present decisions. However, the paternalistic nature of this method disregards the individual opinions and impedes practical

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implementation in democracies (Marglin (1963); Feldstein (1964)). In contrast, Nordhaus (2007) proposes the adoption of market rates as a more pragmatic and balanced approach. The deficiency of parameters, particularly for far-reaching market, hinders the use of conventional methodologies in determining the social rate. The divergence in opinions and the absence of a convergent consensus complicate the establishment of a unanimous methodology. This discord defies easy resolution through conventional principles, as highlighted by Zuber (2011) and Jackson and Yariv (2015), who discover that discount rates founded on unanimity principle either flout time consistency or become subservient to paternalistic decision rule. This ongoing discourse illustrates the pressing need to reconcile divergent views, striving for a methodological framework that effectively harmonizes various perspectives to formulate an impactful social discount rate.

This paper aims to confront the challenge of determining social discount rates across far-reaching generations. Specifically, it delves into the quandary of establishing a novel and rational principle when social individuals hold divergent perspectives on the discount rate. The objective is to devise a non-paternalistic approach that ensures both time consistency and effective safeguarding of the welfare of far-reaching distant generations, facilitating environmental preservation and sustainable development. The typical method widely applied in the literature is to weaken the classical unanimity principle to find possibility results. In this paper, we aim to understand to what extent relaxing the unanimity principle allows a society to achieve discounting in a non-dictatorial manner. Additionally, we expect social discounting to respect *individualism*, meaning that the range of discount rates should be bounded by the individual discount factors.

The principle of *dominant unanimity* asserts that if one consumption stream is superior to another time-wisely and is universally preferred by all individuals, then society should adopt this ranking. This unanimity might be the most persuasive principle, implicitly assumed as a behavioral principle in most frameworks without being formally established as one. Therefore, to achieve meaningful social discounting, this paper considers several variations of dominance consistency and describes the boundaries of social discounting accordingly.

Moreover, most public interventions impose costs on one generation (today) for the benefit of others (in the future), so society often faces the challenge of ranking two consumption streams where one stream *almost* dominates, as will be explained later. Motivated by this observation, it seems natural to restrict unanimity to pairs of consumption streams that exhibit an almost-dominance property. First, we consider the *almost*

dominance relation, where one consumption stream almost dominates another if it yields higher utility for all but at most one generation. However, we find that this dominance relation has moral flaws. In this relation, the superiority of the first consumption stream over the latter may result from sacrificing the future to benefit the present. Therefore, we propose the *almost weak dominance* relation. If one consumption stream almost weakly dominates another, there exists one generation before which the consumption of both streams is identical, and after which the former stream dominates the latter. The almost weak dominance relation ensures that the dominant consumption stream does not benefit the present at the expense of future consumption. We believe that the almost weak dominance relation is morally superior to the almost dominance relation. Consequently, we propose three different unanimity principles based on the comparison of consumption streams under the almost weak dominance relation. These three dominance unanimity principles characterize different margins of social discounting, thereby axiomatizing the different possible ranges of social discounting.

This paper reaches a significant conclusion within our established framework, highlighting several novel principles related to dominant unanimity. The key finding suggests that adopting different principles leads to a social discount factor with varying choice domains. In simpler terms, the principle a society adopts reflects its attitudes toward patience and its concern for the well-being of future generations. Consequently, the paper's conclusions offer theoretical pathways for societies to more effectively consider the welfare of generations far into the future.

We further explore the possibility of deviating from the unanimity principle by examining the *unilateral* principle for pairs of streams with an almost weakly dominant relation. Our findings suggest that this principle indicates the social discount factor should go beyond individual ones. This result aligns with the claims made by Caplin and Leahy (2004); Farhi and Werning (2007); Feng and Ke (2018) in different contexts. However, our study does not take into account the preferences of future generations, making this principle less compelling. Consequently, the result could be viewed as a *negative* outcome against such social discounting within the framework of the current generation alone. Additionally, we discuss how our results might be extended to situations where individuals differ in both instantaneous utility and discount factors.

The theoretical framework is outlined in the following section. Our primary result will be presented and discussed in Sections 3 and 4. We discuss some related issues in Section 5 and conclude in the final section.

2 The Model

Let X be a set of *outcomes*, formally a convex and compact subset of a vector space. Let $\mathcal{L} = \Delta(X)$ be a consumption space, namely a set of lotteries over outcomes X. Let $\mathcal{T} = \{0, 1, 2, ...\}$ be the set of generations or time periods, 0 represents the current generation. A *consumption stream* is denoted by bold letters, *e.g.* $\mathbf{x} = (x_0, x_1, ...) \in \mathcal{L}^T$. We use *a*, *b*, and *c* to denote generic elements of \mathcal{L} . For every $t \in \mathcal{T}$, $b_t \mathbf{a}$ denotes the *binary* consumption stream \mathbf{x} that is defined by $x_t = b$ and $x_s = a$ for all $s \neq t$. Note that while we assume an infinite time horizon, our results remain valid within the context of a finite time horizon.

Society is a set of individuals $\mathcal{I} = \{1, ..., n\}$. Individual $i \in \mathcal{I}$ has preferences $\geq_i \subset \mathcal{L}^T \times \mathcal{L}^T$. Similarly, society's preferences are denoted by $\geq_0 \subset \mathcal{L}^T \times \mathcal{L}^T$. We assume that individual preferences \geq_i are represented by a *discounted utility*.

Definition 1. $U : \mathcal{L}^{\mathcal{T}} \to \mathbb{R}$ is a *discounted utility* (DU) if there exist a nontrivial affine *instantaneous* utility function $u : \mathcal{L} \to \mathbb{R}$ and a decreasing *discount function* $d : \mathcal{T} \to (0, 1]$, with d(0) = 1 and $\sup_{t \in \mathcal{T}} \frac{d(t+1)}{d(t)} < 1$, such that, for all $\mathbf{x} \in \mathcal{L}^{\mathcal{T}}$,

$$U(\boldsymbol{x}) = \sum_{t=0}^{\infty} d(t)u(x_t)$$

DU is *regular* if there is $s \in \mathcal{T}$ such that $\frac{d(s+1)}{d(s)} = \sup_{t \in \mathcal{T}} \frac{d(t+1)}{d(t)}$.

Note that when \mathcal{T} is finite, the regularity condition is automatically satisfied. The *one-period discount factor* $\frac{d(t+1)}{d(t)}$ is strictly less than 1 reflects that each individual discounts every future generation. We remark that the requirement that the sequence of one-period discount factors $\left\{\frac{d(t+1)}{d(t)}\right\}_{t\in\mathcal{T}}$ is bounded away from 1 is stronger than the assumption that the sum of all weights is bounded above, *i.e.* $\sum_{t=0}^{\infty} d(t) < \infty$. This latter condition ensures that the discounted utility of a consumption stream \mathbf{x} with bounded utility does not take the value of infinity. We use (d_i, u_i) to represent individual preferences \gtrsim_i .

We assume that social preferences \geq_0 are represented by an *exponential discounted* utility (EDU) of Koopmans (1960), which is a DU whose discount function $d_0(t) = \delta_0^t$ is exponential for $0 < \delta_0 < 1$. We use (δ_0, u_0) to represent social preferences \geq_0 .

We assume that both individuals and society have identical preferences over constant consumption streams, *i.e.* $u_i = u$ for all $i \in \mathcal{I} \cup \{0\}$ (this assumption of homogeneous instantaneous utility will be relaxed in Section 5.2). Therefore, individual preferences \gtrsim_i can be represented by a pair (d_i, u) and social preferences \gtrsim_0 can be represented by (δ_0, u) .

We define *unanimous relation* $\succeq_{\mathcal{I}}$ as usual: $\mathbf{x} \succeq_{\mathcal{I}} \mathbf{y}$ if and only if $\mathbf{x} \succeq_i \mathbf{y}$ for all $i \in \mathcal{I}$.

3 UNANIMITY AND IMPOSSIBILITY

The literature widely acknowledges the challenge of deriving an aggregate discount factor for a society with heterogeneous individual impatience rates. However, for theoretical relevance, the assumptions regarding social and individual preferences must be reasonable, and the formation of social preferences should have a normative basis. To this end, this paper adopts an axiomatization approach for preference aggregation to form social preferences.

On one hand, we assume that social preferences are represented by an EDU function, which inherently follows the property of time consistency. The EDU model, as introduced by Ramsey (1928), Samuelson (1937) and Koopmans (1960), is recognized as the canonical model for a representative agent. Despite warnings from Marglin (1963) and Feldstein (1964) about the challenges of deriving a social EDU by aggregating heterogeneous individuals, this form remains widely used in policy evaluation due to its simplicity and elegance. Moreover, this assumption is crucial as it ensures stable and credible decisions over time, thereby enhancing policy effectiveness and fostering a stable economic environment.

On the other hand, we assume that individual preferences are represented by a DU function, rather than an EDU function, as supported by previous studies such as Zuber (2011) and Jackson and Yariv (2015). This assumption not only extends the domain of individual preferences we consider but also aligns more closely with empirical evidence. Since Thaler (1981), it has been a well-established finding that individuals become time-inconsistent as the delay increases. Frederick, Loewenstein and O'Donoghue (2002), along with more recent work by Cohen et al. (2020), demonstrate that individual discount factors are generally not stationary and vary significantly across different studies and estimations.

It is evident that considering DU individuals is more practical and descriptively valid. However, this approach comes with certain trade-offs. In our context, the unanimity principle is incompatible with non-dictatorial aggregation rule.

Unanimity: For any $x, y \in \mathcal{L}^T$, if $x \gtrsim_{\mathcal{I}} y$, then $x \gtrsim_0 y$.

Consider a simple situation in which each individual *i* has an EDU utility function with the representation (δ_i, u) . We say that the social planner is *non-dictatorial* if there

does not exist any $i \in \mathcal{I}$ such that $\delta_0 = \delta_i$. It is well known that a conflict arises between a non-dictatorial EDU society and Unanimity.

Proposition 1. Suppose $\delta_i \neq \delta_j$ for all $i, j \in \mathcal{I}$. Unanimity is satisfied iff there exists some $i \in \mathcal{I}$ such that $\delta_0 = \delta_i$.

This result has been formally established by Zuber (2011) and Jackson and Yariv (2015), which indicates that if an EDU society is required to be unanimous, the social discount factor must equal the discount factor of some individual.

4 Dominance, Unanimity and Social Discounting

A common approach to addressing the aforementioned impossibility result is to explore alternative principles that relax Unanimity. In this section, we will introduce several principles that are less stringent than Unanimity. We will then examine the potential implications for social discounting under these alternative principles.

4.1 Restricted Dominant Unanimity

The limitations of the Unanimity principle have been extensively discussed in the literature (Chambers, Echenique and Miller (2023); Billot and Qu (2021*a*); Hayashi and Lombardi (2021)). If Unanimity is not deemed the most compelling principle for a society to accept normatively, what alternative principle would be? If society only applies the Unanimity principle to pairs of consumption streams where one stream dominates the other period-wise, it might be one of the most compelling principles to accept. This principle can be formally stated as follows:

Dominant Unanimity: For any $\mathbf{x}, \mathbf{y} \in \mathcal{L}^{\mathcal{T}}$ with $u(x_t) \ge u(y_t)$ for all $t \in \mathcal{T}$, if $\mathbf{x} \succeq_{\mathcal{I}} \mathbf{y}$, then $\mathbf{x} \succeq_{0} \mathbf{y}$.

In our framework, similar to other literature, if every generation derives higher utility from x than from y, then all individuals and society will undoubtedly prefer x over y due to the homogeneity of instantaneous utilities and the satisfaction of the dominance property by the discounted utility form. Therefore, Dominant Unanimity is a very weak and compelling principle that is already implicitly assumed in the framework.

In order to explore meaningful possibility result, it is essential to consider various ways to strengthen Dominant Unanimity. One potential variation involves applying the Unanimity principle to pairs of *nearly* dominant consumption streams. We consider two possible concepts related to almost dominance.

Definition 2. A consumption stream \mathbf{x} almost dominates \mathbf{y} if there is $t \in \mathcal{T}$ such that $u(x_s) \ge u(y_s)$ for all $s \ne t$. Furthermore, \mathbf{x} almost weakly dominates \mathbf{y} if there is $t \in \mathcal{T}$ such that $u(x_s) = u(y_s)$ if s < t and $u(x_s) \ge u(y_s)$ if s > t.

The consumption stream x is said to almost dominate y means that x yields higher utility than y for all but one generation. On the other hand, we say that x almost weakly dominates y if x almost dominates y and the utility derived from both streams is identical for every generation prior to the sacrificed generation.¹ In other words, x almost dominates y, but the domination only occurs starting from a certain generation onward.

If, in Definition 2, $u(x_t) \ge u(y_t)$, then **x** actually dominates **y**. The more interesting case occurs when $u(x_t) < u(y_t)$. In this case, unlike standard dominance, both almost dominance and almost weak dominance require the sacrifice of one generation. Intuitively, **y** can be seen as the current state of society. If a public intervention is implemented, it results in a stream of consumption **x**. The pair of consumption streams (x, y) represents a public intervention that is against generation t, as she would be worse off if the intervention is implemented.

We propose two principles in which unanimity only applies to pairs of consumption streams with an almost (weakly) dominant relation.

Almost Dominant Unanimity (A-DU): For any $x, y \in \mathcal{L}^T$ where x almost dominates y, if $x \gtrsim_{\mathcal{I}} y$, then $x \gtrsim_0 y$.

A-DU requires that unanimity applies only to consumption streams where the first stream almost dominates the second. This condition necessitates not only agreement among all individuals on preferences but also that, with the exception of one generation, the consumption values from the first stream exceed those of the second. This is in contrast to the classical unanimity principle, which only considers agreement among individuals and does not take into account the extent of sacrifice across generations. Therefore, A-DU is less stringent and more compelling in its requirement for acceptance compared to classical unanimity principles.

¹It is important to note that *almost weakly dominant* relation is, in fact, stronger than *almost dominant* relation. The term "weakly" is used to emphasize that the restriction of unanimity to almost weakly dominant relation is less stringent than that applied to almost dominant relation. We hope that it will not cause any confusion for the reader.

A-DU permits unanimity to be achieved at the cost of sacrificing *at most* a single generation, a principle that appears reasonable at first glance. However, it is important to consider the two potential beneficiaries of such a sacrifice: either the preceding generation or the succeeding one. In other words, the sacrifice of a generation could either benefit the preceding generation or enhance the welfare of succeeding generations. We believe it is morally unjustifiable to prioritize the current generation's welfare at the expense of future generations, a mindset that contributes to today's environmental crisis. The following example formally illustrates our argument:

Example 1 (**Revisiting Zuber-Jackson-Yariv's result**). In this example, we propose a deeper understanding of the Zuber-Jackson-Yariv impossibility result. We say that \mathbf{x} almost strictly dominates \mathbf{y} if there is $t \in \mathcal{T}$ such that $u(x_t) < u(y_t)$ and $u(x_s) > u(y_s)$ for all $s \neq t$. Our claim is that in cases where both individuals and society exhibit EDU preferences and society is non-dictatorial, it is always possible to identify two consumption streams \mathbf{x} and \mathbf{y} , where \mathbf{x} almost strictly dominates \mathbf{y} , such that all individuals prefer \mathbf{x} while society prefers \mathbf{y} .

Formally, consider a non-dictatorial EDU society represented by $\{(u, \delta_i)\}_{i=0}^n$. Let $t \ge 1$, $\mathbf{y} \in \mathcal{L}^T$, $\alpha > 0$, and $\mathbf{z} \in \mathcal{L}^T$ such that $u(z_t) < u(y_t)$ and $u(z_s) > u(y_s) + \alpha$ for all $s \ne t$.² We aim to identify a consumption stream \mathbf{x} , which almost strictly dominates \mathbf{y} , satisfying $\mathbf{x} >_{\mathcal{I}} \mathbf{y}$ and $\mathbf{y} >_0 \mathbf{x}$. We define a real sequence $\boldsymbol{\theta} = (\theta_0, \theta_1, \theta_2, ...) \in \ell_\infty$ by

$$\theta_{s} = \begin{cases} 0 & \text{if } 0 \leq s \leq t-2, \\ \delta_{0}^{2} & \text{if } s = t-1, \\ \delta_{0}^{2} - 2\delta_{0} & \text{if } s = t, \\ (1-\delta_{0})^{2} & \text{if } s \geq t+1. \end{cases}$$

A simple computation yields that, for all $i \in \mathcal{I} \cup \{0\}$,

$$\sum_{s=0}^{\infty} \delta_i^s \theta_s = \frac{\delta_i^{t-1} (\delta_i - \delta_0)^2}{1 - \delta_i}$$

Thus, we get $\sum_{s=0}^{\infty} \delta_0^s \theta_s = 0$. In addition, since the social planner is non-dictatorial, $\delta_0 \neq \delta_i$ for all $i \in \mathcal{I}$, which implies $\sum_{s=0}^{\infty} \delta_i^s \theta_s > 0$ for all $i \in \mathcal{I}$. Now, define the real sequence

²Such a pair of consumption streams and a scalar α always exists due to the nontriviality and linearity of *u*.

 $\boldsymbol{\eta} \in \ell_{\infty}$ by

$$\eta_{s} = \begin{cases} \beta & \text{if } 0 \le s \le t - 2, \\ \delta_{0}^{2} & \text{if } s = t - 1, \\ \delta_{0}^{2} - 2\delta_{0} - \gamma & \text{if } s = t, \\ (1 - \delta_{0})^{2} & \text{if } s \ge t + 1. \end{cases}$$

for some $\beta, \gamma > 0$. It is easy to show that there are $\beta, \gamma > 0$ such that $\sum_{s=0}^{\infty} \delta_i^s \eta_s > 0 > \sum_{s=0}^{\infty} \delta_0^s \eta_s$ for all $i \in \mathcal{I}$.

Clearly, $\eta_t < 0$ and $\eta_s > 0$ for all $s \neq t$. Since $u(z_t) < u(y_t)$ and $u(z_s) > u(y_s) + \alpha$ for all $s \neq t$, there exists $\lambda > 0$ such that $u(y_s) + \lambda \eta_s$ is between $u(y_s)$ and $u(z_s)$ for all $s \in \mathcal{T}$. Because u is affine, for all $s \in \mathcal{T}$, there is $x_s \in \mathcal{L}$ such that $u(x_s) = u(y_s) + \lambda \eta_s$. Now if we define $\mathbf{x} = (x_0, x_1, x_2, ...)$, it is evident that \mathbf{x} almost strictly dominates \mathbf{y} ; and we have $\mathbf{x} >_{\mathcal{I}} \mathbf{y}$ and $\mathbf{y} >_0 \mathbf{x}$.

The example above motivates us to propose a principle that is weaker than A-DU to prevent such immoral forms of sacrifice:

Almost Weakly Dominant Unanimity (AW-DU): For any $x, y \in \mathcal{L}^T$ where x almost weakly dominates y, if $x \gtrsim_{\mathcal{I}} y$, then $x \gtrsim_0 y$.

AW-DU requires that society follows unanimity only with respect to pairs of consumption streams where the first stream almost weakly dominates the second, making it weaker than A-DU. Notably, AW-DU permits unanimity to be achieved through the sacrifice of a single generation. However, this sacrifice is limited to succeeding generations, not preceding ones. Consequently, the consumption values prior to this specific sacrificed generation remain identical between the two streams. This principle effectively eliminates the possibility of benefiting current generations at the expense of future ones. In this regard, we believe that AW-DU is normatively sound compared to A-DU.

Before delving into the properties of AW-DU, we would like to discuss some variations of this concept. Specifically, we propose a weaker version of AW-DU by restricting the almost weak dominance relation to binary consumption streams that differ only in two consecutive periods. Simplifying the comparison to binary streams can facilitate decision-making for individuals in practical scenarios. This approach allows us to relate our principle to the concept of "dated reward," thereby enhancing its empirical relevance.

Binary Almost Weakly Dominant Unanimity (BAW-DU): For any $a, b \in \mathcal{L}$ with u(b) > u(a) and any $c \in \mathcal{L}$, if there is $t \in \mathcal{T}$ such that $b_{t+1}a \gtrsim_{\mathcal{I}} c_t a$, then $b_{t+1}a \gtrsim_0 c_t a$.

If u(b) > u(a), then the consumption stream $b_{t+1}a$ almost weakly dominates c_ta . In this pair of consumption streams, only generation t may experience a sacrifice, while only generation t+1 may benefit from it. All consumption before time t and after time t+1 remains identical across the two streams. Therefore, the principle of BAW-DU asserts that if every individual prefers consumption b at period t+1 over consumption c at period t, then society should also accept these preferences. It is evident that BAW-DU is a weaker principle than AW-DU. Furthermore, apart from the binary almost weak dominance relationship between the compared pairs of streams, the cost-benefit generations must be consecutive. This restriction is particularly relevant in experimental settings.

It is important to note that when society accepts the unanimity condition $b_{t+1}a \gtrsim_0 c_t a$, this preference is stationary. This means that shifting from period t to any other period s should not alter social preferences. This invariance stems from social EDU preferences. However, in our context, we assume that individuals possess DU preferences rather than EDU preferences. Consequently, it is entirely possible for an individual to prefer $b_{t+1}a$ over $c_t a$, yet prefer $c_s a$ over $b_{s+1}a$ for some $s \neq t$. This raises a question: when social preferences are stationary, is it necessary for individual preferences to be unanimous at *some* identical times? To address this question, we propose two alternative principles below.

Binary Pointed Almost Weakly Dominant Unanimity (BAW⁻-**DU**): For any $a, b \in \mathcal{L}$ with u(b) > u(a) and any $c \in \mathcal{L}$, if for any $i \in \mathcal{I}$, there is $t_i \in \mathcal{T}$ such that $b_{t_i+1}a \gtrsim_i c_{t_i}a$, then $b_1a \gtrsim_0 c_0a$.

This axiom seeks to identify the conditions under which it is compelling for a society to accept the preference that a one-period delayed consumption of b is preferred to c, regardless of when c is consumed. BAW⁻-DU encapsulates this idea by asserting that if for every individual there exists some period in which consuming c is less preferred than the one-period delayed consumption of b, then society should also prefer the one-period delayed b over c. This concept of time-varying unanimity is grounded in the assumption of DU individual preferences. When individuals exhibit EDU, the BAW⁻-DU and BAW-DU axioms actually align.

One way to understand the rationale behind BAW⁻-DU is as follows: society recognizes the time-inconsistency of individual preferences and may paternalistically adjust these preferences to be time-consistent. Whenever society observes an individual preferring $b_{t+1}a$ over c_ta at some period t, it regards this preference as an ethical one and paternalistically assumes that such an individual's anchored moral preferences should favor $b_{s+1}a$ over c_sa for all periods s. In this manner, it becomes compelling for a society to accept the BAW⁻-DU principle. Moreover, it is worth mentioning that BAW⁻-DU implies BAW-DU. Evidently, BAW-DU is a special situation in BAW⁻-DU whenever there is $t \in T$ such that $t_i = t$ for all $i \in I$.

Another approach to addressing the time considerations in BAW-DU is to require that a society accepts a preference for one-period delayed consumption of *b* over *c*, provided that each individual consistently holds this preference across all periods. In this scenario, the society fully respects individual preferences without treating time inconsistency as a potential issue. We formally state this principle below.

Binary Strong Almost Weakly Dominant Unanimity (BAW⁺-**DU):** For any $a, b \in \mathcal{L}$ with u(b) > u(a) and any $c \in \mathcal{L}$, if $b_{t+1}a \gtrsim_{\mathcal{I}} c_t a$ for all $t \in \mathcal{T}$, then $b_1a \gtrsim_0 c_0 a$.

BAW⁺-DU requires that if all individuals prefer binary stream $b_{t+1}a$ to c_ta at every time period t, then society has to accept this preference. Clearly, BAW⁺-DU implies both BAW-DU and BAW⁻-DU.

4.2 Social Discounting

In the above section, we have introduced several axioms related to dominant unanimity. In this section, we will examine the corresponding properties of these axioms. We first discuss how the AW-DU would characterize the domain of social discounting.

Proposition 2. The following statements hold:

- 1. If AW-DU is satisfied, then $\delta_0 \ge \sup_t \min_i \frac{d_i(t+1)}{d_i(t)}$.
- 2. If $\delta_0 \ge \min_i \sup_t \frac{d_i(t+1)}{d_i(t)}$, then AW-DU is satisfied.

The result establishes two thresholds for the social discount factor. The necessary condition for AW-DU to hold is that the social discount factor must be greater than the minimum one-period discount factors across all individuals in any given time period. However, this condition is not sufficient for AW-DU to be satisfied.³ To ensure AW-DU, the social discount factor must exceed the minimum value of the supremum one-period discount factors across all individuals over all time periods. These two thresholds will be

³In general, we do not have $\sup_t \min_i \frac{d_i(t+1)}{d_i(t)} = \min_i \sup_t \frac{d_i(t+1)}{d_i(t)}$. In fact, the former is less than or equal to the latter.

discussed later when we present complete characterizations of the lower bound for the social discount factor.

When all individuals have EDU, these two thresholds are equal. Therefore, AW-DU is both necessary and sufficient to ensure that the social discount factor exceeds that of the least patient individual.

Corollary 1. For all $i \in I$, suppose individual i has an EDU with the representation (δ_i, u) . AW-DU is satisfied iff $\delta_0 \ge \min_i \delta_i$.

On the other hand, BAW-DU, BAW⁻-DU, and BAW⁺-DU provide complete characterizations of the lower bound for the social discount factor.

Theorem 1. *BAW-DU is satisfied* iff $\delta_0 \ge \sup_t \min_i \frac{d_i(t+1)}{d_i(t)}$.

The theorem requires that the social discount factor must be at least as large as the supremum, over time, of the minimal one-period discount factors among all individuals within each period. One possible intuition for this lower bound is as follows: the social discount factor is compared to the one-period discount factors of all individuals on a *period-by-period* basis. Specifically, society focuses on evaluating the trade-offs between any two consecutive time periods, ensuring that it is always more patient than the *least* patient individual. This approach guarantees that the social discount factor exceeds the minimal one-period discount factors across all individuals in any given time period.

Theorem 2. Assume the regularity of individual preferences. Then, BAW⁻-DU is satisfied iff $\delta_0 \ge \min_i \sup_t \frac{d_i(t+1)}{d_i(t)}$.

This result asserts that BAW⁻-DU is equivalent to the social discount factor being greater than or equal to the minimum, across all individuals, of the supremum of their one-period discount factors over all time periods. Intuitively, the social discount factor is compared to *individual patient parameters*, which are defined by the entire sequence of individuals' one-period discount factors. In this case, each individual's patient parameter is determined by their highest one-period discount factor across all time periods. To satisfy the BAW⁻-DU principle, social discounting must be at least as patient as that of the least patient individual in society.

Theorem 3. BAW⁺-DU is satisfied iff $\delta_0 \ge \min_i \inf_t \frac{d_i(t+1)}{d_i(t)}$.

The lower bound for the social discount factor in this theorem is determined by taking the infimum of the ratio $\frac{d_i(t+1)}{d_i(t)}$ over time *t* for each individual *i*, and then finding the minimum across all individuals. The interpretation of this threshold is similar to that presented in Theorem 2, but the individual patient parameters are now defined by individuals' lowest one-period discount factor across all time periods.

So far, previous results have focused on characterizing the lower bound of the social discount factor. However, one might wonder whether there are principles that could define its upper bound. In fact, this can be achieved by simply removing the requirement that u(b) > u(a) from the existing principles. Below, we formally demonstrate the principle of BAW⁺-DU by eliminating the requirement of an almost weakly dominant relation, and we characterize it in the following corollary. Similarly, other principles can be modified in this manner to characterize the upper bound of the social discount factor.

Restricted Binary Unanimity (RBU): For any $a, b, c \in \mathcal{L}$, if $b_{t+1}a \gtrsim_{\mathcal{I}} c_t a$ for all $t \in \mathcal{T}$, then $b_1a \gtrsim_0 c_0 a$.

Corollary 2. *RBU is satisfied* iff $\min_i \inf_t \frac{d_i(t+1)}{d_i(t)} \le \delta_0 \le \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$.

This result indicates that our restricted binary unanimity principle helps establish both the lower and upper bounds of the social discount factor. The principle offers society a remarkably wide range of choices, as long as these choices fall within the range of individual one-period discount factors. It is important to note that this result is flexible enough to accommodate the aggregation rules derived in numerous studies, such as those by Boylan and McKelvey (1995), Hayashi and Lombardi (2021), and Billot and Qu (2021*b*).

5 Discussions

5.1 Unilateral Dominance

The previous discussion suggests that the range of the social discount factor should be confined within the limits established by individual discount factors. This approach is generally applicable to most situations. However, Farhi and Werning (2007) argue that social discounting should encompass broader social concerns, such as equity and longterm welfare, which are often overlooked in individual discounting decisions. They contend that the social discount factor should be higher than individual discount factors, particularly when considering the inequality and social welfare of future generations. In this section, we will focus on the principles that can help characterize this result, thereby providing an axiomatic foundation for such social discounting. However, implementing such social discounting would require abandoning the principle of unanimity. We begin by presenting the relevant axiom.

Unilateral Almost Weakly Dominance (UAWD): For any $x, y \in \mathcal{L}^T$ where x almost weakly dominates y, if $x \geq_i y$ for some $i \in \mathcal{I}$, then $x \geq_0 y$.

Similar to AW-DU, UAWD considers only pairs of consumption streams where one stream almost weakly dominates the other. However, UAWD is clearly weaker than AW-DU: society will choose the almost dominant stream, which benefits the distant future, whenever at least one individual favors this option. Moreover, unlike AW-DU, UAWD does not require unanimous preferences among individuals to accept a preference. Therefore, in summary, UAWD is significantly less compelling for normative acceptance compared to the various dominant unanimity principles we proposed in the previous section.

It is not surprising that UAWD characterizes a society that is more patient than any individual

Theorem 4. UAWD is satisfied iff $1 > \delta_0 \ge \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$.

The maximum of individual's one-period discount factors represents the most substantial discounting that occurs across different periods for any individual in society. By setting the social discount factor greater than or equal to this maximum, social discounting is more patient than the *most* patient individual's long-run discounting.

This result indicates that, in general, society struggles to be more patient than any individual. This is because achieving such patient social discounting would require an individual's preference – for an almost weakly dominant consumption stream – to dominate those of the rest of society. This principle is unlikely to be widely accepted in a democratic context. Given that Theorem 4 provides a complete characterization, it can also be seen as a *negative* result. Therefore, the more normatively appealing construction of such social discounting might consider alternative frameworks, such as the one proposed by Feng and Ke (2018), which includes the preferences of future generations.

It is worth noting that the two principles we introduce above characterize the Weitzman (1998) Discounting, where the social discount factor is equal to the maximum of individual's one-period discount factors.

Corollary 3. UAWD and RBU are satisfied iff $\delta_0 = \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$.

5.2 Heterogeneous Instantaneous Utilities

Our results can be extended to situations where individuals may differ in instantaneous utility functions. We extend only Proposition 2, as the extension of other results follows a very similar approach. Specifically, we need to assume⁴ the existence of uniform preferences over binary outcomes. That is, there exist ℓ and m in X such that $\ell >_i m$ for all $i \in \mathcal{I}$. With this assumption, we can restrict AW-DU to the domain of consumption streams where consumption belongs to $\mathcal{L}^* = \Delta(\{\ell, m\})$. We normalize individual utilities by setting $u_i(\ell) = 1$ and $u_i(m) = 0$ for all $i \in \mathcal{I}$. Consequently, we need to further revise the unilateral dominance principle and propose a constant Pareto principle as follows:

Almost Weakly Dominant Unanimity^{*} (AW-DU^{*}): For any $x, y \in \mathcal{L}^{*T}$ where x almost weakly dominates y, if $x \gtrsim_{\mathcal{I}} y$, then $x \gtrsim_{0} y$.

Constant Pareto Condition (CPC): For any $a, b \in \mathcal{L}$, if $a \succeq_{\mathcal{I}} b$, then $a \succeq_0 b$.

Together with the two principles, we can obtain a separate aggregation rule in which social utility is a weighted sum of individual ones and the social discount factor is beyond a threshold which depends on individual one-period discount factors.

Proposition 3. The following statements hold:

- 1. If AW-DU^{*} and CPC are satisfied, then there exists $\alpha_i \ge 0$ with $\sum_i \alpha_i = 1$ such that $u_0 = \sum_{i=1}^n \alpha_i u_i$, and $\delta_0 \ge \sup_t \min_i \frac{d_i(t+1)}{d_i(t)}$.
- 2. If $\delta_0 \ge \min_i \sup_t \frac{d_i(t+1)}{d_i(t)}$ and $u_0 = \sum_{i=1}^n \alpha_i u_i$ where $\alpha_i \ge 0$ and $\sum_i \alpha_i = 1$, then AW-DU^{*} and CPC are satisfied.

According to Proposition 2, it is clear that AW-DU^{*} implies the result of social discounting. As noted by De Meyer and Mongin (1995), the CPC implies that social utility is a linear aggregation of individual instantaneous utilities. Using this approach, we can effectively address aggregation problems that arise due to double heterogeneity.

6 CONCLUSION

It is well-known that even when individuals have identical instantaneous utilities, the unanimity principle and the Expected Discounted Utility (EDU) framework are not compatible. This paper addresses this issue and makes two key contributions. First, we

⁴This approach was first suggested by Qu (2017).

weaken the unanimity principle to derive a non-dictatorial method for constructing a social discount factor. Second, we explore several variations of dominant unanimity and establish different boundaries for social discounting.

The methods proposed in the literature to address the impossibility result of Zuber-Jackson-Yariv generally fall into two categories. One approach aims to ensure the time consistency of social preferences by imposing certain restrictions on the scope of the unanimity principle as this study. The other approach, to maintain the unanimity principle, relaxes the demand for time consistency of social preferences. For instance, Chambers and Echenique (2018) considers social maxmin utility, Dong-Xuan, Bich and Wigniolle (2024) examines social quasi-hyperbolic discounting, and Billot and Qu (2021*b*) explores social semi-hyperbolic discounting utility, among others. Although these two methods alleviate the impact of the impossibility theorem to some extent and can be applied in certain frameworks, the resulting social discounting remains a compromise between individual discounting and falls short of Ramsey (1928)'s ethical aspiration to equate the welfare of future generations with that of the present generation as much as possible.

APPENDIX: OMITTED PROOFS

Before presenting the proofs, we first provide two simple results that will be used repeatedly throughout.

Lemma 1. For any $a, b \in \mathcal{L}$ with u(b) > u(a), $i \in \mathcal{I}$, and $t \in \mathcal{T}$,

$$b_{t+1}\boldsymbol{a} \gtrsim_i c_t \boldsymbol{a} \iff \frac{d_i(t+1)}{d_i(t)} \ge \frac{u(c) - u(a)}{u(b) - u(a)}$$

Proof. Let $i \in \mathcal{I}$ and $t \in \mathcal{T}$. We have the following equivalence relations:

$$b_{t+1} \boldsymbol{a} \gtrsim_i c_t \boldsymbol{a}$$

$$\iff d_i(t+1)u(b) + \sum_{s \neq t+1} d_i(s)u(a) \ge d_i(t)u(c) + \sum_{s \neq t} d_i(s)u(a)$$

$$\iff d_i(t+1)(u(b) - u(a)) \ge d_i(t)(u(c) - u(a))$$

$$\iff \frac{d_i(t+1)}{d_i(t)} \ge \frac{u(c) - u(a)}{u(b) - u(a)},$$

which ends the proof of the lemma.

Lemma 2. For any $a, b \in \mathcal{L}$ with u(b) > u(a) and any $\lambda \in [0, 1]$, there exists $c \in \mathcal{L}$ such that

$$\frac{u(c) - u(a)}{u(b) - u(a)} = \lambda.$$

Proof. Let $a, b \in \mathcal{L}$ such that u(b) > u(a) and let $\lambda \in [0,1]$. Define $f : \mathcal{L} \to \mathbb{R}$ by $f(c) = \frac{u(c)-u(a)}{u(b)-u(a)}$ for all $c \in \mathcal{L}$. Since u is affine, we have

$$f((1-\lambda)a + \lambda b) = \frac{(1-\lambda)u(a) + \lambda u(b) - u(a)}{u(b) - u(a)} = \lambda$$

Defining $c = (1 - \lambda)a + \lambda b$ completes the proof.

A Proof of Proposition 2

We begin by proving the first part of the proposition. Suppose AW-DU holds. Let $a, b \in \mathcal{L}$ such that u(b) > u(a) and let $t \in \mathcal{T}$. Since $\min_i \frac{d_i(t+1)}{d_i(t)} \in (0, 1)$, by Lemma 2, there exists $c \in \mathcal{L}$ such that

$$\frac{u(c) - u(a)}{u(b) - u(a)} = \min_{i} \frac{d_i(t+1)}{d_i(t)}.$$

Clearly, $\frac{d_i(t+1)}{d_i(t)} \ge \min_i \frac{d_i(t+1)}{d_i(t)} = \frac{u(c)-u(a)}{u(b)-u(a)}$ for all $i \in \mathcal{I}$. It follows from Lemma 1 that $b_{t+1} \mathbf{a} \gtrsim_{\mathcal{I}} c_t \mathbf{a}$. Since $b_{t+1} \mathbf{a}$ almost weakly dominates $c_t \mathbf{a}$, by AW-DU, we have $b_{t+1} \mathbf{a} \gtrsim_0 c_t \mathbf{a}$, which is equivalent to $\delta_0 \ge \frac{u(c)-u(a)}{u(b)-u(a)}$. Therefore, $\delta_0 \ge \min_i \frac{d_i(t+1)}{d_i(t)}$. This equality holds for all $t \in \mathcal{T}$, which implies $\delta_0 \ge \sup_t \min_i \frac{d_i(t+1)}{d_i(t)}$.

Now, we show the second part of the proposition. Assume $\delta_0 \ge \min_i \sup_t \frac{d_i(t+1)}{d_i(t)}$. Let $\mathbf{x}, \mathbf{y} \in \mathcal{L}^T$ where \mathbf{x} almost weakly dominates \mathbf{y} such that $\mathbf{x} \succeq_T \mathbf{y}$. We need to prove $\mathbf{x} \succeq_0 \mathbf{y}$, which is equivalent to showing $\sum_{s=0}^{\infty} \delta_0^s u(x_s) \ge \sum_{s=0}^{\infty} \delta_0^s u(y_s)$. A simple computation shows that this is equivalent to

(1)
$$\sum_{s=t+1}^{\infty} \delta_0^{s-t} [u(x_s) - u(y_s)] \ge u(y_t) - u(x_t)$$

Let $j \in \mathcal{I}$ such that $\sup_s \frac{d_j(s+1)}{d_j(s)} = \min_i \sup_s \frac{d_i(s+1)}{d_i(s)}$. Since $\delta_0 \ge \min_i \sup_s \frac{d_i(s+1)}{d_i(s)}$, we have,

for all s > t,

$$\delta_0^{s-t} \ge \left[\sup_{t'} \frac{d_j(t'+1)}{d_j(t')}\right]^{s-t} \ge \prod_{t \le t' \le s-1} \frac{d_j(t'+1)}{d_j(t')} = \frac{d_j(s)}{d_j(t)}.$$

Because **x** almost weakly dominates **y**, $u(x_s) - u(y_s) \ge 0$ for all s > t. Thus, the inequality above implies

(2)
$$\sum_{s=t+1}^{\infty} \delta_0^{s-t} [u(x_s) - u(y_s)] \ge \sum_{s=t+1}^{\infty} \frac{d_j(s)}{d_j(t)} [u(x_s) - u(y_s)]$$

We have assumed that $x \gtrsim_i y$, which leads to the following equivalent conditions:

$$\sum_{s=0}^{\infty} d_j(s)u(x_s) \ge \sum_{s=0}^{\infty} d_j(s)u(y_s)$$
$$\iff \sum_{s=t+1}^{\infty} d_j(s)[u(x_s) - u(y_s)] \ge d_j(t)[u(y_t) - u(x_t)]$$
$$\iff \sum_{s=t+1}^{\infty} \frac{d_j(s)}{d_j(t)}[u(x_s) - u(y_s)] \ge u(y_t) - u(x_t).$$

Clearly, Equations (2) and (3) imply Equation (1). Therefore, AW-DU holds, which completes the proof.

B PROOF OF THEOREM 1

We note that, to prove the first part of Proposition 2, only BAW-DU is required. Thus, the **only-if part** of this theorem follows the same argument used in the first part of Proposition 2.

We now prove the **if part**. Assume $\delta_0 \ge \sup_t \min_i \frac{d_i(t+1)}{d_i(t)}$. Let $a, b \in \mathcal{L}$ with u(b) > u(a) and let $c \in \mathcal{L}$. If there is $t \in \mathcal{T}$ such that $b_{t+1}a \gtrsim_i c_t a$ for all $i \in \mathcal{I}$, then by Lemma 1, we have

$$\frac{d_i(t+1)}{d_i(t)} \ge \frac{u(c) - u(a)}{u(b) - u(a)}$$

for all $i \in \mathcal{I}$. Hence, $\min_i \frac{d_i(t+1)}{d_i(t)} \ge \frac{u(c)-u(a)}{u(b)-u(a)}$, which implies $\delta_0 \ge \frac{u(c)-u(a)}{u(b)-u(a)}$. Thus, $b_{t+1} \mathbf{a} \ge_0 c_t \mathbf{a}$, which ends the proof.

C Proof of Theorem 2

We first show the **if part**. Assume $\delta_0 \ge \min_i \sup_t \frac{d_i(t+1)}{d_i(t)}$. Let $a, b \in \mathcal{L}$ such that u(b) > u(a)and let $c \in \mathcal{L}$. Suppose, for any $i \in \mathcal{I}$, there is $t_i \in \mathcal{T}$ such that $b_{t_i+1} \mathbf{a} \ge_i c_{t_i} \mathbf{a}$. By Lemma 1, we get

$$\frac{d_i(t_i+1)}{d_i(t_i)} \ge \frac{u(c)-u(a)}{u(b)-u(a)}.$$

Thus, we have $\sup_t \frac{d_i(t+1)}{d_i(t)} \ge \frac{u(c)-u(a)}{u(b)-u(a)}$ for all $i \in \mathcal{I}$, which implies

$$\min_{i} \sup_{t} \frac{d_{i}(t_{i}+1)}{d_{i}(t_{i})} \ge \frac{u(c) - u(a)}{u(b) - u(a)}$$

Hence, we must have $\delta_0 \ge \frac{u(c)-u(a)}{u(b)-u(a)}$, which is equivalent to $b_1 \mathbf{a} \gtrsim_0 c_0 \mathbf{a}$

Now we demonstrate the **only-if part**. Suppose BAW⁻-DU holds. Let $a, b \in \mathcal{L}$ be such that u(b) > u(a). Note that $\min_i \sup_t \frac{d_i(t+1)}{d_i(t)} \in [0, 1)$. By Lemma 2, there is $c \in \mathcal{L}$ such that

$$\min_{i} \sup_{t} \frac{d_i(t+1)}{d_i(t)} = \frac{u(c) - u(a)}{u(b) - u(a)}.$$

Thus, $\sup_t \frac{d_i(t+1)}{d_i(t)} \ge \frac{u(c)-u(a)}{u(b)-u(a)}$ for all $i \in \mathcal{I}$. Due to the regularity of individual preferences, for each $i \in \mathcal{I}$, there exists $t_i \in \mathcal{T}$ such that

$$\frac{d_i(t_i+1)}{d_i(t_i)} \ge \frac{u(c)-u(a)}{u(b)-u(a)}.$$

That is $b_{t_i+1} \boldsymbol{a} \gtrsim_i c_{t_i} \boldsymbol{a}$ for all $i \in \mathcal{I}$ (by Lemma 1). By BAW⁻-DU, we have $b_{t_i+1} \boldsymbol{a} \gtrsim_0 c_{t_i} \boldsymbol{a}$. Hence, $\delta_0 \ge \frac{u(c)-u(a)}{u(b)-u(a)} = \min_i \sup_t \frac{d_i(t+1)}{d_i(t)}$.

D Proof of Theorem 3 and Corollary 2

Since RBU implies BAW⁺-DU, without loss of generality, we prove Corollary 2 here as the proof of Theorem 3 is part of the proof of Corollary 2.

We first demonstrate **only-if part**. Assume RBU holds. Let $a, b \in \mathcal{L}$ such that u(b) > u(b)

u(a). Clearly, $\min_i \inf_t \frac{d_i(t+1)}{d_i(t)} \in [0, 1)$. By Lemma 2, there is $c \in \mathcal{L}$ such that

$$\frac{u(c)-u(a)}{u(b)-u(a)} = \min_i \inf_t \frac{d_i(t+1)}{d_i(t)}.$$

Since $\frac{d_i(t+1)}{d_i(t)} \ge \min_i \inf_t \frac{d_i(t+1)}{d_i(t)} = \frac{u(c)-u(a)}{u(b)-u(a)}$ for all $t \in \mathcal{T}$ and $i \in \mathcal{I}$, one can deduce $b_{t+1} \mathbf{a} \gtrsim_{\mathcal{I}} c_t \mathbf{a}$ for all $t \in \mathcal{T}$. From RBU, we obtain $b_1 \mathbf{a} \gtrsim_0 c_0 \mathbf{a}$, implying $\delta_0 \ge \frac{u(c)-u(a)}{u(b)-u(a)}$. Therefore, $\delta_0 \ge \min_i \inf_t \frac{d_i(t+1)}{d_i(t)}$.

Fix $a, b \in \mathcal{L}$ with u(b) < u(a). It is evident that $b_{t+1}a \gtrsim_i c_t a$ is equivalent to the following inequality:

$$\frac{d_i(t+1)}{d_i(t)} \le \frac{u(a) - u(c)}{u(a) - u(b)}.$$

For every $i \in \mathcal{I}$, $\frac{d_i(t+1)}{d_i(t)} \in (0,1)$ for all $t \in \mathcal{T}$, which implies that $\max_i \sup_t \frac{d_i(t+1)}{d_i(t)} \in (0,1]$. It follows from Lemma 2 that there exists $c \in \mathcal{L}$ such that

$$\frac{u(a)-u(c)}{u(a)-u(b)} = \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}.$$

This implies $b_{t+1} \boldsymbol{a} \gtrsim_{\mathcal{I}} c_t \boldsymbol{a}$ for all $t \in \mathcal{T}$. From RBU, we get $b_1 \boldsymbol{a} \gtrsim_0 c_0 \boldsymbol{a}$, implying $\delta_0 \leq \frac{u(a)-u(c)}{u(a)-u(b)} = \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$.

Now, we show the **if part**. Assume $\min_i \inf_t \frac{d_i(t+1)}{d_i(t)} \le \delta_0 \le \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$. Let $a, b, c \in \mathcal{L}$ such that $b_{t+1} \mathbf{a} \gtrsim_{\mathcal{I}} c_t \mathbf{a}$ for all $t \in \mathcal{T}$. If u(b) > u(a), then by Lemma 1, $\frac{d_i(t+1)}{d_i(t)} \ge \frac{u(c)-u(a)}{u(b)-u(a)}$ for all $t \in \mathcal{T}$ and $i \in \mathcal{I}$, which implies $\min_i \inf_t \frac{d_i(t+1)}{d_i(t)} \ge \frac{u(c)-u(a)}{u(b)-u(a)}$. Hence, $\delta_0 \ge \frac{u(c)-u(a)}{u(b)-u(a)}$, concluding $b_1 \mathbf{a} \gtrsim_0 c_0 \mathbf{a}$. Similarly, we can show that RBU holds when u(a) > u(b), completing the proof.

E Proof of Theorem 4

We first prove the **only-if part**. Assuming UAWD, we will prove $1 > \delta_0 \ge \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$. By Definition 1, we have $1 > \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$. It remains to show $\delta_0 \ge \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$.

Let $a, b \in \mathcal{L}$ such that u(b) > u(a), and let $i \in \mathcal{I}$ and $t \in \mathcal{T}$. Because $\frac{d_i(t+1)}{d_i(t)} \in (0,1)$, by Lemma 2, there exists $c \in \mathcal{L}$ such that $\frac{d_i(t+1)}{d_i(t)} = \frac{u(c)-u(a)}{u(b)-u(a)}$. Then it follows from Lemma 1 that $b_{t+1}a \gtrsim_i c_t a$ (indeed, $b_{t+1}a \sim_i c_t a$). Since $b_{t+1}a$ almost weakly dominates $c_t a$, from UAWD, we get $b_{t+1}a \gtrsim_0 c_t a$, which is equivalent to $\delta_0 \geq \frac{d_i(t+1)}{d_i(t)}$. Since this inequality holds true for every $i \in \mathcal{I}$ and $t \in \mathcal{T}$, we conclude $\delta_0 \geq \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$. Let us now prove the **if part**. Assume $1 > \delta_0 \ge \max_i \sup_t \frac{d_i(t+1)}{d_i(t)}$. We will show that UAWD holds. Let $\mathbf{x}, \mathbf{y} \in \mathcal{L}^T$ such that \mathbf{x} almost weakly dominates \mathbf{y} and $\mathbf{x} \succeq_i \mathbf{y}$ for some $i \in \mathcal{I}$. This is equivalent to

(4)
$$\sum_{s=t+1}^{\infty} \frac{d_i(s)}{d_i(t)} [u(x_s) - u(y_s)] \ge u(y_t) - u(x_t).$$

Since $\delta_0 \ge \max_j \sup_s \frac{d_j(s+1)}{d_j(s)} \ge \sup_s \frac{d_i(s+1)}{d_i(s)}$, we get, for all s > t,

$$\delta_0^{s-t} \ge \left[\sup_{t'} \frac{d_i(t'+1)}{d_i(t')}\right]^{s-t} \ge \prod_{t \le t' \le s-1} \frac{d_i(t'+1)}{d_i(t')} = \frac{d_i(s)}{d_i(t)}.$$

Because $u(x_s) - u(y_s) \ge 0$ for all s > t, the inequality above implies

$$\sum_{s=t+1}^{\infty} \delta_0^{s-t} [u(x_s) - u(y_s)] \ge \sum_{s=t+1}^{\infty} \frac{d_i(s)}{d_i(t)} [u(x_s) - u(y_s)],$$

which, by Equation (4), leads to

$$\sum_{s=t+1}^{\infty} \delta_0^{s-t} [u(x_s) - u(y_s)] \ge u(y_t) - u(x_t).$$

Thus, $\boldsymbol{x} \succeq_0 \boldsymbol{y}$, which implies that UAWD is satisfied.

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