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Can Growth Stabilize Debt? A Fiscal Theory Perspective

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June 13, 2024

Abstract

This paper studies price stability and debt sustainability when the real rate exceeds trend growth (r > g) in a New Keynesian model with endogenous technology growth through R&D. Endogenous growth constitutes a self-financing mechanism for deficits which backs debt and attenuates fiscal inflation. A dynamic r - g stability criterion characterizes the set of feasible monetary-fiscal frameworks. If surpluses do not adjust to stabilize debt, the central bank must permit r - g to fall with inflation. Monetary policy which follows the Taylor principle can be consistent with a unique stable equilibrium under active fiscal policy as growth endogenously creates fiscal capacity and policy space.

JEL Classification: E31, E52, E62, E24, 042

Keywords: Public Debt, Inflation, Monetary-Fiscal Interactions, Fiscal Theory of the Price Level, Endogenous Growth.

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1. Introduction

The long-run trend is a central determinant of the debt-to-GDP ratio, which is essential for price stability and fiscal sustainability considerations. However, standard results concerning monetary-fiscal interactions are grounded in models that abstract from the determinants of the long-term aggregate output path. These modeling paradigms support a particular view of the "appropriate" monetary-fiscal framework, characterized as follows: Under a debt-stabilizing ("passive") fiscal policy, price stability requires a monetary policy that satisfies the "Taylor Principle" by raising nominal interest rates more than one-for-one with inflation. Without a passive fiscal surplus policy backing the public debt ("active" fiscal policy), monetary policy must violate the Taylor Principle and therefore let the real interest rate, r, fall with a persistent rise in inflation (Leeper (1991)).

This paper argues that the assumption of a constant, exogenous long-term output path shuts off an additional channel affecting fiscal sustainability and price stability: technological innovation. Drawing from the well-established result in the endogenous growth literature that research and development (R&D) and technology growth constitute key drivers of long-run growth, we re-evaluate fundamental questions concerning monetary-fiscal interactions in a New Keynesian model with endogenous long-run trend dynamics determined by technological innovation through R&D. We show that accounting for general equilibrium effects on the long-run trend margin and the presence of forward-looking innovators alters the conditions for price stability and debt sustainability and the interaction between monetary and fiscal policy. These results follow from a central mechanism: changes in aggregate demand affect entrepreneurs' incentives to innovate, which generates endogenous adjustment in R&D investment, technology growth and an endogenous long-run path of aggregate output.

Fiscal capacity in our framework is endogenous. Under active fiscal policy, Ricardian equivalence fails and an increase in public debt generates an expansion in demand, which raises R&D and thus trend growth. Since government revenues scale with GDP on the economy's balanced growth path (BGP), this permanent output expansion endogenously backs the debt by creating new tax revenues and fiscal capacity. Therefore, endogenous growth substitutes for fiscal inflation by creating fiscal space: the inflation response to a fiscal expansion in an active fiscal regime is dampened and may even be deflationary, depending on the country-specific supply-side characteristics, in particular in the innovation sector. Moreover, the effect of fiscal policy on inflation is further attenuated for long maturity structures of public debt and when the Phillips curve is flat.

We further show that under an endogenous long-run growth trend, a monetary policy which satisfies the Taylor Principle is not sufficient for local determinacy when fiscal policy is passive. Instead, monetary policy should adhere to the growthaugmented Taylor principle (GrTP) which formalizes a more stringent requirement with respect to the trend output growth rate (g): the central bank should raise r - gin response to a persistent rise in inflation, and not just r as suggested by the Taylor Principle. The feedback between expected inflation and the payoff from technological innovation through R&D warrants this especially hawkish monetary policy stance. Consequently, the discrepancy between the Taylor Principle and GrTP depends on the responsiveness of technology to aggregate demand.

On the other hand, violating the GrTP is sufficient for stability under a debtdestabilizing ("active") fiscal policy. This result can be formulated in terms of a *dynamic* r - g criterion: fiscally-unbacked fiscal expansions can be financed through a combination of inflation and changes in the long-run output trend, so long as the monetary authority lets r - g fall dynamically with a persistent rise in inflation. A monetary policy that violates the GrTP rules out changes in the government's debt-service costs that offset the debt-stabilizing effect of inflation and technological innovation. The latter fiscal-financing margin is absent under exogenous growth (gis constant), and as a result, violation of the Taylor Principle is a strict requirement for fiscal sustainability in standard macro models. Crucially, since the GrTP is a stronger condition than the Taylor Principle, a model with endogenous growth can admit a unique stable equilibrium under active fiscal policy and a monetary policy which follows the Taylor principle.

Our mechanism operates in a tractable representative agent model in which r > gholds and which nests the standard three-equation New Keynesian model. The model is populated by a representative household that consumes final output, holds government bonds and is the owner of the firms. The household supplies two types of labor: unskilled labor, for goods production, and skilled labor, for technological innovation through R&D. There are two layers of production. A unit mass of final good firms produces differentiated final output goods under monopolistic competition using intermediate inputs. Their price setting is subject to a Calvo pricing friction. Intermediate goods are imperfect substitutes in final goods production and intermediate good firms produce using unskilled labor under monopolistic competition. The aggregate technology stock is a function of the number of intermediate good varieties which expands endogenously through R&D. Entrepreneurs in the R&D sector create new intermediate goods varieties using skilled labor and obtain the payoffs from a new intermediate good production line. The monetary authority sets nominal interest rates through a Taylor rule. The fiscal authority issues public debt and sets taxes via a fiscal surplus rule. These ingredients combine the monetary policy framework of Woodford (2003) with the endogenous growth model of Romer (1990), resulting in a simple framework ideally suited for the study of fundamental price stability and fiscal sustainability issues. We describe in what follows how this paper contributes to the previous literature.

Previous literature

Monetary-fiscal interaction and the FTPL. The fiscal theory of the price level (FTPL) literature, which follows Leeper (1991), Sims (1994), Woodford (1995), among others, emphasizes that the joint behavior of monetary and fiscal policy determines prices in a manner that ensures fiscal backing for the public debt, as described above. Leeper and Leith (2016) and Cochrane (2023) provide comprehensive reviews of this literature. This paper contributes to this literature by studying monetaryfiscal interaction under endogenous technology growth which, as Cochrane (2023) also observed, has not been previously analyzed in the FTPL literature. We show that the typical assumption of an exogenous technology stock has non-trivial implications for monetary-fiscal interaction as it omits the long-run trend as an additional adjustment margin. Endogenous movements in technology growth require monetary policy to respond more hawkishly under passive fiscal policy, but endogenous growth also relaxes the conditions for debt sustainability under active fiscal policy compared with standard results. Crucially, an anti-inflationary monetary policy which follows the Taylor principle can be consistent with a unique stable equilibrium under active fiscal policy. Our model features a self-financing channel in which endogenous trend growth can create fiscal space and reduce the need for fiscal inflation.

Conditions for Stability. A strand of the literature shows how conditions for macroeconomic stability depend on the features of the economy not included in standard New Keynesian models, such as heterogenous agents (Kaplan et al. (2023),

Aguiar et al. (2023)) the presence of partially unfunded debt (Bianchi et al. (2023)), bubble terms (Brunnermeier et al. (2022)), deviations from rational expectations (Eusepi and Preston (2012), Eusepi and Preston (2018)), the possibility of regime change (Ascari et al. (2020), Cho and Moreno (2021)), and the presence of sovereign risk premia or convenience yields (Bonam and Lukkezen (2019), Bonam (2020)).

DeLong and Summers (2012) show by means of a stylized, reduced-form arithmetic that fiscal stimulus can be self-financing in a depressed economy with high unemployment and a binding zero lower bound (ZLB) constraint on the nominal interest rate, if fiscal multipliers are sufficiently large, and hysteresis, i.e. permanent, crisisinduced output loss, is possible.¹ Angeletos et al. (2023) establish micro-foundations for self-financing fiscal deficits, which result from an expansion of the tax base from a boom caused by a breakdown of Ricardian equivalence under finite lifespans. In contrast, our self-financing mechanism operates through endogenous growth in the context of the fiscal theory of the price level: fiscally-unbacked fiscal deficits raise demand through a breakdown in Ricardian equivalence, which boosts R&D, and hence the long-term output path. Trend adjustment substitutes for fiscal inflation and can permit the central bank to adhere to the Taylor principle even under an active fiscal regime.

Relation to r < g. A growing literature focuses on episodes in which the real interest rate ranges below the long-term growth rate of the economy, and derives models which can generate r - g < 0 (e.g., see Blanchard (2019), Mehrotra and Sergeyev (2021), Bassetto and Cui (2018), Mian et al. (2022), Reis (2022), Galí (2021)). As in this literature, we emphasize the importance of r and g for debt sustainability. However, our methodology, underlying mechanisms, and outcomes diverge significantly. First, we work with a representative agent model in which r > g, both dynamically and at the BGP. Second, we link transitory changes in r - garound its BGP level to both price stability and fiscal sustainability, and we isolate assumptions about the joint conduct of monetary and fiscal policy that give rise to stabilizing dynamics of r - g in equilibrium. Finally, our results highlight that growth does not unambiguously relax the conditions for macroeconomic stability. In fact, a more responsive interest rate rule is necessary for price stability under a passive fiscal policy. Similarly, monetary policy cannot respond too aggressively to inflation

¹Hysteresis is introduced by means of an ad-hoc parameter which governs the degree of the crisis-induced output loss.

under active fiscal policy (the growth-augmented Taylor Principle must be violated in this case).

Endogenous growth. We model long-run trend dynamics following the endogenous growth literature, which identifies investment in R&D and technology growth as key drivers of long-run growth (Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1999)).² This paper is closely linked to a growing literature which incorporates endogenous growth through technology-enhancing investment in New Keynesian DSGE models. However, our paper is the first to introduce public debt and study monetary-fiscal interaction under endogenous growth. Our approach to introducing endogenous growth via expanding varieties through R&D as in Romer (1990) into a simple New Keynesian environment (Woodford (2003), Galí (2015)) is closely related to Queralto (2022) who studies optimal monetary policy in a model without public debt. Garga and Singh (2021) study optimal monetary policy in a tractable New Keynesian DSGE model with R&D and Schumpeterian growth. Key results from estimated, medium-scale DSGE models (Smets and Wouters (2007), Christiano et al. (2005)) with endogenous growth through investment in innovation include the persistent output effects of demand-side shocks (Moran and Queralto (2018), Anzoategui et al. (2019), Bianchi et al. (2019), Ikeda and Kurozumi (2019), Elfsbacka Schmöller and Spitzer (2021)).

The effect of monetary policy shocks on R&D and TFP is shown by time series and firm-level evidence (Moran and Queralto (2018), Jordà et al. (2020), Ma and Zimmermann (2023)). Ilzetzki (2023) shows that exogenous aggregate demand increases raise technology growth.³ As to fiscal policy, Antolin-Diaz and Surico (2022) and Cloyne et al. (2022) show the persistent effects of, respectively, government spending shocks and corporate tax cuts via investment in innovation. Elfsbacka Schmöller (2022) studies fiscal stimulus targeted to innovation investment. Using Keynesian growth models, i.e. growth models with Keynesian elements, Benigno and Fornaro (2018) study stagnation traps at the ZLB and Fornaro and Wolf (2020) examine supply-side disruptions as observed in the pandemic.

This paper is structured as follows. Section 2 presents the model. We study growth and debt sustainability in section 3. Section 4 studies monetary-fiscal inter-

 $^{^{2}}$ As typical in the standard New Keynesian model, we abstract from population growth to allow for direct comparability with the exogenous technology benchmark framework.

³Further, Furlanetto et al. (2021) estimate the persistent effects of demand shocks.

actions with endogenous trend growth. Section 5 shows additional results regarding the role of the demand-growth link, debt maturity, and the Phillips curve. Section 6 concludes.

2. Model

We proceed to describe our theoretical framework, which is a tractable New Keynesian model with nominal public debt, and endogenous technology growth through R&D. As in Queralto (2022), our model generalizes the standard 3-equation New Keynesian system by allowing for endogenous growth through expanding varieties in the spirit of Romer (1990), and novel to this paper, public debt. A representative household consumes final consumption goods, holds government bonds and owns the firms in the economy. The household supplies two types of labor: unskilled labor used in the production of intermediate goods and skilled labor which serves as input in the R&D sector. There are two types of firms, corresponding to two layers of production in the economy. A unit mass of final good firms produces differentiated final consumption goods under monopolistic competition with intermediate goods as the only input. These firms are assumed to face a Calvo pricing friction. Because the intermediate input varieties are imperfect substitutes in final goods production, they are produced under monopolistic competition, by firms that use unskilled labor as the sole input. Aggregate technology and thus TFP growth is driven by the range of intermediate good varieties, denoted as A_t , which is endogenously determined through research and development. R&D entrepreneurs create new intermediate goods varieties using skilled labor and receive the payoffs from a newly created intermediate good production variety. In this environment, monetary policy sets nominal interest rates to target inflation and output via a Taylor rule. A fiscal authority issues public debt, makes fiscal expenditures, and sets lump-sum taxes through a fiscal surplus rule.

Endogenous trend growth: The aggregate technology stock, A_t , is subject to endogenous growth. Technologies, i.e. intermediate good varieties, can become obsolete at rate $1 - \phi$ and V_t denotes new technologies created through R&D in period t and available for production in t + 1. Thus, the technology stock is governed by the process

$$A_{t+1} = \phi A_t + V_t$$

which states that the time t + 1 technology stock equals to the surviving technologies from the previous period, ϕA_t , and technological innovations, V_t . The long-run trend or total factor productivity equals to $A_t^{\frac{1}{\vartheta-1}}$, where ϑ is the elasticity of substitution in intermediates. In this section, we will show that output, Y_t , follows the trend $A_t^{\frac{1}{\vartheta-1}}$. As a result, detrended output is given by $y_t := Y_t / A_t^{\frac{1}{\vartheta-1}}$, and trend growth, g_y , is a function of technology growth: $g_{y,t} = (g_{A,t})^{\frac{1}{\vartheta-1}}$, where $g_{A,t} := \frac{A_{t+1}}{A_t}$. We describe innovation and growth in detail in section 2.3.

2.1 Monetary policy

We assume that the central bank targets inflation π_t and output Y_t using a simple Taylor rule for the risk-free gross nominal interest rate R_t :

$$R_t = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_t}$$

We adopt the usual convention of a zero inflation target $(\pi^* = 1)$, while Y_t/Y_t^* denotes the gap between output and targeted output. In this paper, we abstract from the zero lower bound on interest rates, and hence we do not consider the global equilibrium multiplity issues studied in Benhabib et al. (2001b). Instead, we focus on local determinacy, i.e., the existence and uniqueness of bounded equilibrium near the inflation target steady state. It is well-known that the responsiveness of the interest rate to inflation and output plays a key role in local determinacy analyses in standard New Keynesian environments, as summarized by the Taylor Principle.

Definition. Monetary policy adheres to (violates) the **Taylor principle** if $\phi_{\pi} - 1 + \frac{(1-\beta)}{\kappa}\phi_y > 0$ ($\phi_{\pi} - 1 + \frac{(1-\beta)}{\kappa}\phi_y < 0$).

Intuitively, monetary policy adheres to the Taylor principle if it raises interest rates more than one-for-one in response to a persistent increase in inflation. A monetary policy which follows the Taylor principle is typically described as "active" in the literature and "passive" otherwise.⁴

 $^{^{4}\}mathrm{Section}$ 4.1 revisits standard results regarding the Taylor principle and existence and uniqueness of equilibrium.

2.2 Fiscal policy

The fiscal authority faces the following intertemporal budget constraint:

$$B_t P_t^m + T_t P_t = B_{t-1}(1 + \rho P_t^m) + G_t P_t$$

where P_t denotes the price level and T_t real lump-sum taxes. B_t is the nominal bond which has a geometrically decaying coupon payment structure, i.e. one unit of the bond portfolio purchased at time t at price P_t^m pays one unit of nominal income in t + 1, ρ units in t + 2, ρ^2 in t + 3, and so on. We assume that government purchases are a fraction, $\tilde{g} > 0$, of total output in a balanced growth path (BGP) equilibrium, which implies that detrended government on the balanced path is equal to $\bar{g} := \tilde{g}\bar{y} > 0$. Government expenditures are also subject to exogenous shocks and follow the process:

$$\frac{g_t}{\bar{g}} = \left(\frac{g_{t-1}}{\bar{g}}\right)^{\rho_g} \epsilon_t^G,$$

where $g_t := G_t/(A_t^{(\vartheta-1)^{-1}})$ and ϵ_t^G is i.i.d. mean one and $0 \le \rho_g < 1.^5$ The associated market clearing condition is then given by

$$y_t = g_t + c_t,$$

where $c_t := C_t/(A_t^{(\vartheta-1)^{-1}})$. Like government expenditures, both taxes, T, and debt, B, scale with GDP on the BGP. In other words, debt and taxes follow the trend $A_t^{(\vartheta-1)^{-1}}$, which implies a steady state in the debt-to-GDP and primary surplus-to-GDP ratios. We may therefore express the budget constraint in terms of detrended variables:

$$b_t P_t^m + \tilde{T}_t = b_{t-1} \frac{(1 + \rho P_t^m)}{\pi_t g_{y,t-1}} + g_t$$

 $^{^5 \}rm We$ abstract from public R&D spending and our results may therefore understate the importance of endogenous growth. In practice, public R&D spending may be increasing in government spending and public debt. Hence, public R&D would have direct effects on R&D, in addition to indirect effects on private R&D.

where $b_t = \frac{B_t}{P_t A_t^{\frac{1}{\vartheta-1}}}, \tilde{T}_t = \frac{T_t}{A_t^{\frac{1}{\vartheta-1}}}, g_{y,t} = \left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{\vartheta-1}}$. Finally, we assume a simple feedback rule for the tax rule, as in Leeper (1991):⁶

$$\frac{\tilde{T}_t}{\bar{\tilde{T}}} = \left(\frac{b_{t-1}}{\bar{b}}\right)^{\frac{\gamma \bar{P}^m \bar{b}}{\bar{\tilde{T}}}} \tag{1}$$

where \bar{b} is detrended debt on the BGP. The rule (1) summarizes whether the fiscal authority is committed to resolving fiscal imbalances using real fiscal surpluses. Sufficiently high values of the parameter γ describe a debt-stabilizing fiscal policy, which raises surpluses when debt is above its BGP level. In what follows, we distinguish between "active" and "passive" fiscal policy according to the following definition:

Definition. Fiscal policy is said to be **passive** (active) if $|\beta^{-1} - \gamma| < 1$ ($|\beta^{-1} - \gamma| > 1$).

As we discuss below, passive fiscal policy ensures real fiscal adjustments that bring debt back to its long-run steady state level following any sequences of shocks, inflation, growth or interest rates. Active fiscal policy fails to guarantee debt-stabilizing adjustments in the fiscal surplus.

2.3 Growth

Trend growth is modeled endogenously in general equilibrium. A continuum of measure one of innovators engage in research and development to generate new intermediate inputs. A successful innovator obtains the patent for the new innovation. Skilled labor serves as the R&D input. A newly created technology in t becomes available in production in period t + 1. Innovation results in the generation of new technologies, i.e. an expansion of intermediate good inputs in the spirit of Romer (1990). As described in the beginning of section 2, the technology stock evolves as $A_{t+1} = \phi A_t + V_t$.

Research and development:

An innovator which creates a new intermediate good variety obtains the patent and the real profits, Π_t , from this respective production line. The payoff from a new

⁶The log-linearized version of (1) implies that the log deviation of the real detrended fiscal surplus from the steady state BGP is proportional to the log deviation of the real detrended public debt from the steady state BGP. Note that we calibrate steady state \tilde{T} (\tilde{T}) to target a strictly positive debt-to-GDP ratio dy > 0: $\bar{b} = dy * \bar{y} > 0$.

technological innovation, J_t , thus equals the expected present value of profits

$$J_t = E_t \left\{ \sum_{k=1}^{\infty} \phi^{k-1} \beta^k \frac{U_{C,t+k}}{U_{C,t}} \Pi_{t+k} \right\} = E_t \left\{ \beta \frac{U_{C,t+1}}{U_{C,t}} \left(\Pi_{t+1} + \phi J_{t+1} \right) \right\}.$$
 (2)

One unit of skilled labor allocated to R&D creates Φ_t new technologies. The R&D production function of an individual innovator j then takes the form

$$V_t(j) = \Phi_t L_t^s(j)$$

where $L_t^s(j)$ denotes skilled labor and $V_t(j)$ newly created technologies by innovator j, respectively. The R&D production technology, Φ_t , is characterized by

$$\Phi_t = \varsigma A_t (L_t)^\eta$$

where ς denotes R&D efficiency. R&D production technology depends on the aggregate time t technology stock, A_t , and thus the existing technological knowledge stock facilitates R&D and future innovation. The learning-by-doing (Romer (1986), Arrow (1962)) term, $(L_t)^{\eta}$, permits for direct complementarities between production and innovation for $\eta > 0$.

Innovators' problem: Innovator j chooses skilled labor, $L_t^s(j)$, to maximize the expected payoff from R&D (equ. 2) subject to the costs of R&D: $\frac{W_{s,t}}{P_t}L_t^s(j)$. The optimality condition for R&D is given by

$$\Phi_t J_t = \frac{W_{s,t}}{P_t}.$$

Given symmetry, the aggregate R&D input is $L_t^s(j) = L_t^s$ and $V_t = \int_0^1 V_t(j) dj$. Aggregate new technologies at time t obtain as

$$V_t = \Phi_t L_t^s. \tag{3}$$

From (3), the law-of-motion of the aggregate technology stock can be expressed as

$$A_{t+1} = \phi A_t + \Phi_t L_t^s. \tag{4}$$

Expanding varieties and TFP dynamics: Equation (4) introduces endoge-

nous growth through expanding varieties (Romer (1990)). Quantitative, estimated medium-scale DSGE models that introduce endogenous growth à la Romer (1990)⁷ also incorporate technological adoption to account for lagged diffusion of new technologies. As the focus of this paper is on analytical results, we focus exclusively on R&D for tractability.⁸ As shown by Comin and Gertler (2006), technological adoption increases the response of technology growth to short-run shocks, and hence our simple framework may understate the effects of endogenous growth. As in Queralto (2022), we allow for the possibility of learning-by-doing (lbd) in the innovation process to compensate for the absence of the adoption margin and to increase realism in TFP dynamics in response to demand shocks by introducing direct complementary between production and innovation for $\eta > 0.^9$ $\eta = 0$ recovers the standard growth process for expanding varieties in intermediate goods through R&D as in Romer (1990). We emphasize that lbd is not necessary for results, which obtain also for the case: $\eta = 0$.

2.4 Production and price setting

Intermediate goods firms operate in a monopolistically competitive environment using unskilled labor as input. Final good firms produce using intermediate goods as inputs and are subject to nominal pricing frictions.

2.4.1 Final goods producer

There is a continuum of measure one of monopolistically competitive final good firms which produce differentiated output $Y_t(i)$. The final good composite Y_t is the CES aggregate of differentiated final good varieties:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$$

⁷See Moran and Queralto (2018), Anzoategui et al. (2019), Ikeda and Kurozumi (2019), Elfsbacka Schmöller and Spitzer (2021), Cloyne et al. (2022) for estimated medium-scale DSGE models with growth through expanding varieties.

⁸Other papers which study endogenous technology growth in New Keynesian DGSE models with a focus on analytical results also focus on R&D as the main driver of technology growth (Queralto (2022), Garga and Singh (2021)).

⁹IIzetzki (2023) provides empirical evidence that demand shocks raise TFP more strongly under relatively tighter capacity as the latter spurs learning-by-doing and innovation in response to the demand increase. Including lbd in the TFP process accounts for this mechanism, as above-trend production employment raises TFP growth and facilitates innovation. Queralto (2022) sets η to match the TFP response following a monetary policy shock consistent with empirical estimates.

Final good producers are subject to Calvo price rigidities. A final good firm produces according to the production function

$$Y_t(i) = \left[\int_0^{A_t} X_t(i,j)^{\frac{\vartheta-1}{\vartheta}} dj\right]^{\frac{\vartheta}{\vartheta-1}}.$$

 $Y_t(i)$ denotes final output i, $X_t(i, j)$ the amount of intermediate good input j used in the production of final output i, and ϑ the intermediate good elasticity of substitution $(\vartheta > 1)$. $P_{x,t}(j)$ is the price of intermediate good variety j, the demand for which can be derived as

$$X_t(i,j) = \left(\frac{P_{x,t}(j)}{P_{x,t}}\right)^{-\vartheta} Y_t(i)$$

where $P_{x,t}$ describes the price index of intermediates:

$$P_{x,t} = \left[\int_0^{A_t} P_{x,t}(j)^{1-\vartheta} dj\right]^{\frac{1}{1-\vartheta}}$$

Real marginal costs, MC_t , of production are identical for all final goods firms and can be derived as

$$MC_t = \frac{P_{x,t}}{P_t}.$$

A firm which resets its price in t sets the optimal price P_t^* and the related optimality condition can be stated as:

$$E_t \left\{ \sum_{j=0}^{\infty} U_{C,t+j} \beta^j \theta^j \left(\frac{P_t^*}{P_{t+j}} - \frac{\epsilon}{\epsilon - 1} M C_{t+j} \right) Y_{t,t+j} \right\} = 0$$

where $U_{C,t+j} = (C_{t+j})^{-1}$ and $Y_{t,t+j}$ is the demand for a goods variety in period t+jassuming the price for the goods variety was last reset in period t. The final goods price index P_t obtains as

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}.$$

2.4.2 Intermediate goods producer

Intermediate goods firms produce intermediate output $X_t(j)$ $(j \in [0, A_t])$ and are produced using unskilled labor L_t by the production technology

$$X_t(j) = L_t(j),$$

where $X_t(j)$ denotes intermediate good output of variety j. Dividends in real terms in time t, $\Pi_t(j)$, follow from the maximization problem

$$\Pi_t(j) = \max_{P_{x,t}(j), X_t(j)} \left\{ \left(\frac{P_{x,t}(j)}{P_t} - \frac{W_t}{P_t} \right) X_t(j) \right\}$$

subject to

$$X_t(j) = \left(\frac{P_{x,t}(j)}{P_{x,t}}\right)^{-\vartheta} \int_0^1 Y_t(i,j) di$$

which uses market clearing for intermediate good j $(X_t(j) = \int_0^1 X_t(i,j)di)$. The conventional pricing condition follows: $P_{x,t}(j) = \frac{\vartheta}{\vartheta - 1} W_t$. Using the pricing index for intermediates we derive real marginal costs as

$$MC_t = \frac{\vartheta}{\vartheta - 1} \frac{1}{A_t^{\frac{1}{\vartheta - 1}}} \frac{W_t}{P_t}.$$

Firm profits from intermediate good production lines obtain as

$$\Pi_t = \frac{\nu_t}{\vartheta} \frac{MC_t}{A_t} Y_t$$

with price dispersion $\nu_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di$.¹⁰

2.5 Households

Households maximize utility

$$E_t \sum_{k=0}^{\infty} \beta^s \left(log(C_{t+k}) - \frac{(L_{t+k})^{1+\varphi}}{1+\varphi} - \chi \frac{(L_{t+k}^s)^{1+\varphi}}{1+\varphi} \right)$$

¹⁰Some of the optimality conditions are expressed in terms of final good output Y_t . We describe aggregation in more detail in section 2.6.

where C_t denotes an index of final good consumption, L_t denotes labor supplied to intermediate good producers and L_t^s denotes skilled labor supplied to the R&D sector. The household budget constraint can be stated as

$$P_t^m B_t + R_t^{-1} B_t^s + \int_0^1 P_t(i) C_t(i) di = W_t L_t + W_{s,t} L_t^s + B_{t-1} (1 + \rho P_t^m) + B_{t-1}^s - P_t T_t + P_t D_t$$

where B_t denotes government bonds, R_t the risk-free gross nominal interest rate, W_t and $W_{s,t}$ the wage from skilled and unskilled labor respectively, T_t real lumpsum taxes, D_t dividends, and B_t^s short-term debt which is in net-zero supply. The following conditions follow from the household optimization problem:

$$1 = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} R_t \right\}$$
(5)

$$(L_t)^{\varphi}C_t = \frac{W_t}{P_t} \tag{6}$$

$$\chi(L_t^s)^{\varphi}C_t = \frac{W_{s,t}}{P_t} \tag{7}$$

$$P_t^m = E_t \left\{ R_t^{-1} \left(1 + \rho P_{t+1}^m \right) \right\}$$
(8)

where equation (8) is the no-arbitrage condition implied by the first-order conditions for short-term bond holdings and holdings of the government bond portfolio.

2.6 Equilibrium and balanced growth path

In equilibrium, $\int_0^1 Y_t(i) di = A_t^{\frac{1}{\vartheta-1}} L_t$ and final output, Y_t , is characterized by $Y_t = \nu_t^{-1} \int_0^1 Y_t(i) di$. Endogenous trend growth, $g_{y,t}$, is determined by technology growth, $g_{A,t}$, where $g_{y,t} = (g_{A,t})^{\frac{1}{\vartheta-1}}$ and $g_{A,t} = \frac{A_{t+1}}{A_t}$. At the BGP the economy is subject to constant trend growth: $g_y = (g_A)^{\frac{1}{\vartheta-1}}$. The rate of technology growth, g_A , at the BGP is endogenous and can be derived as $g_A = \phi + \varsigma(L)^{\eta} L^s$.¹¹ The BGP is characterized in terms of stationary variables

$$\{L_t, L_t^s, R_t, \pi_t, P_t, P_t^*, P_t^m, MC_t, g_{a,t}, g_{y,t}\}$$

and trend-stationary variables: $\{y_t, c_t, g_t, \frac{W_t}{P_t}, \frac{W_t}{P_t}, \frac{B_t}{P_t}, T_t\}$ with trend $A_t^{\frac{1}{\vartheta-1}}$ and $\{J_t, \Pi_t\}$ with trend $A_t^{\frac{2-\vartheta}{\vartheta-1}}$. In an equilibrium, the endogenous variables described above must satisfy the equilibrium conditions (equations (B.1)-(B.19) in Online Appendix B),

¹¹In practice, we calibrate g_A and g_y by choosing ς . See Online Appendix B for details.

given initial conditions, b_{-1} and P_{-1} , and the government spending shock $\{\epsilon_t^G\}$.

Exogenous TFP case: The model described in section 2 nests the standard 3-equation New Keynesian model with exogenous technology. We restore the exogenous technology case through the assumption of perfect substitutability of intermediate goods $(\vartheta \to \infty)$.¹²

2.7 Linearized System

We denote a log-linearized variable by \hat{z} . We assume that the central bank targets detrended output $(Y_t^* = A_t^{\frac{1}{\vartheta-1}}\bar{y})^{.13}$ The system of equilibrium conditions can be log-linearized and compactly expressed as¹⁴

$$\hat{g}_{A,t} = \eta \omega \left(\hat{y}_t - \bar{\beta} E_t \hat{y}_{t+1} \right) + \bar{\delta} E_t \hat{y}_{t+1} + \frac{\bar{\beta} \bar{\varphi}}{1 + \bar{\varphi}} E_t \hat{g}_{A,t+1}$$
(9)

$$\hat{g}_{y,t} = (\vartheta - 1)^{-1} \hat{g}_{A,t}$$
 (10)

$$\hat{y}_t = E_t \hat{y}_{t+1} - \tilde{c} (\hat{i}_t - E_t \hat{\pi}_{t+1} - \hat{g}_{y,t}) + (1 - \rho_g) \tilde{g} \hat{g}_t$$
(11)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t - \kappa \frac{g}{\varphi \tilde{c} + 1} \hat{g}_t$$
(12)

$$i_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \tag{13}$$

$$\hat{b}_t = (\beta^{-1} - \gamma)\hat{b}_{t-1} - \beta^{-1}(\hat{\pi}_t + \hat{g}_{y,t-1}) + (\frac{\rho}{g_y} - 1)\hat{P}_t^m + \frac{g}{\bar{P}^m\bar{b}}\hat{g}_t$$
(14)

$$\hat{P}_{t}^{m} = -\hat{i}_{t} + \frac{\beta\rho}{g_{y}} E_{t} \hat{P}_{t+1}^{m}$$
(15)

where $\tilde{c} := 1 - \tilde{g}, \ \bar{\phi} := \phi/g_A, \ g_y = g_A^{(\vartheta-1)^{-1}} \ge 1, \ g_A \ge 1, \ \bar{\beta} := \beta \bar{\phi}, \ \bar{\varphi} := \varphi/(1 - \bar{\phi}), \ \omega := (1 + \varphi)/(1 + \bar{\varphi}), \ \bar{\delta} := (\varphi + 1)(1 - \bar{\beta})/(1 + \bar{\varphi}).$ The endogenous determination of technology growth and trend growth (equ. 9 and equ. 10) and the presence of trend growth, $\hat{g}_{y,t}$, in the Euler equation (11) and government budget constraint (14) distinguishes the model from a textbook New Keynesian model.

We use the system (9)-(15) to conduct the stability analysis in this paper. We will focus on analytical results, supported by numerical simulations. For the latter,

¹²In this setting, an increase in the number of intermediate goods does not translate into changes in aggregate TFP and the long-run trend component. An alternative way of restoring the exogenous technology New Keynesian model is by imposing zero R&D input $(L_t^s = L^s = 0)$, thus holding technology growth constant $(A_t = \bar{A})$, which is obtained as $\chi \to \infty$. In both cases, we set $\phi = 1$ to ensure constant, positive technology stock, A_t .

¹³We could instead define the system of equations in terms of an output gap but this would not affect the main stability analysis undertaken in this paper.

¹⁴Note that $\kappa = \lambda(\varphi + \tilde{c}^{-1})$ where λ is the coefficient on marginal cost in the linearized Phillips curve.

we use the following calibration, unless stated otherwise. The discount factor β is set to 0.99, the inverse Frisch elasticity of labor supply φ to 2, and the slope of the Phillips curve κ to 0.02. We calibrate the BGP government spending to GDP ratio, \bar{g}/\bar{y} , to 0.2. The steady state growth rate is set to match 2% annual trend GDP growth $(g_y = (1.02)^{1/4})$. We calibrate the technological survival rate ϕ to 0.925 and the learning-by-doing spillover in R&D η to 1.5, as in Queralto (2022). When considering active fiscal policy, we simply assume exogenous surpluses ($\gamma = 0$). A number of papers in the literature which introduce growth via expanding varieties in intermediate goods in New Keynesian models, calibrate the elasticity parameter, ϑ , in the range $\vartheta \in [2,4]$,¹⁵ drawing on the calibration by Comin and Gertler (2006), which in turn is based on the estimates by Broda and Weinstein (2006). In what follows, we set $\vartheta = 2.5$ as the default value as this delivers $\frac{\vartheta}{\vartheta-1} = 1.67$ which is close to Comin and Gertler (2006) (1.6). We emphasize though that the evidence on this parameter in the literature is not conclusive. Broda and Weinstein (2006), among others, indicate a wide range of plausible parameter estimates across different industries and sectors, including estimates as low as $\vartheta = 1.3$. We account for this by permitting for a correspondingly wide range of ϑ values in our numerical analysis.

3. Growth and Debt Sustainability

Previous theoretical studies of monetary-fiscal interactions highlight two potential channels for the stabilization of public debt. First, through debt-stabilizing "passive" fiscal policy, i.e. by a fiscal authority which adjusts fiscal surpluses to prevent explosive debt-to-GDP dynamics. Second, in the absence of such a debt-stabilizing fiscal policy, i.e. under "active" fiscal policy, through inflation.¹⁶ We argue that the assumption of a constant, exogenous long-term output path shuts off an additional channel affecting fiscal sustainability: technological innovation. In fact, a breakdown in Ricardian equivalence under active fiscal policy means that an increase in public debt leads to higher demand, R&D and hence higher technology growth, creating an endogenous expansion in fiscal capacity (section 3.2). For a government whose revenues scale with GDP on the BGP, permanently higher output through

¹⁵E.g., see Comin and Gertler (2006), Anzoategui et al. (2019), Elfsbacka Schmöller and Spitzer (2021), Queralto (2022), among others.

¹⁶As is standard in New Keynesian studies of monetary-fiscal interactions, we abstract from the possibility of sovereign default.

an endogenous expansion of the long-run trend creates new tax revenue to back the debt. Sections 3.3 and 3.4 show that this endogenous *fiscal capacity* mitigates fiscal inflation.

3.1 Determinants of fiscal sustainability

The above-mentioned channels are directly visible in the government's intertemporal budget constraint, (14):

$$\hat{b}_t = (\beta^{-1} - \gamma)\hat{b}_{t-1} - \beta^{-1}(\hat{\pi}_t - \hat{g}_{y,t-1}) + \frac{\bar{g}}{\bar{P}^m\bar{b}}\hat{g}_t$$
(16)

where here we imposed an interest rate peg, $\hat{i}_t = 0$, for brevity (monetary policy is studied in detail in subsequent sections). This flow constraint shows the key determinants of public debt: fiscal expenditures (\hat{g}) , taxes $(\gamma \hat{b}_{t-1})$, inflation $(\hat{\pi})$, and endogenous trend growth (\hat{g}_y) . Fluctuations in fiscal expenditures, for example, cause debt to deviate from steady state $(\hat{b}_t \neq 0)$, and debt is said to be stable only if it is expected to return to steady state following any such deviation (i.e., $\lim_{j\to\infty} E_t \hat{b}_{t+j} = 0$). If $|\beta^{-1} - \gamma| < 1$, the fiscal authority is passive, i.e. they commit to keeping the public debt bounded for given inflation, nominal interest rate, shocks, and for any initial debt level. For instance, if debt is initially high, then taxes $(\gamma \hat{b}_{t-1})$ adjust to bring debt back to steady state in the future.¹⁷

Under active fiscal policy $(|\beta^{-1} - \gamma| > 1)$, the fiscal authority makes no such commitment to stabilize the public debt. In this case, inflation can adjust to revert debt to steady state. In fact, a breakdown in Ricardian equivalence under active fiscal policy explains why inflation stabilizes debt in equilibrium. For instance, suppose that the government issues debt to finance a deficit today. Households, the holders of public debt, perceive the expansion in public debt as an increase in net wealth since an active fiscal authority does not back the higher debt by promising future tax hikes. This breakdown in Ricardian equivalence raises demand and thus creates a "fiscal inflation" that reduces the real value of debt (provided that interest rates are not too responsive, as we will discuss in section 4). Under exogenous growth ($\hat{g}_{y,t-1} =$ 0), fiscal inflation is necessary for fiscal sustainability; it is the only mechanism

¹⁷For existence of unique stable equilibrium under passive fiscal policy it is in addition necessary that the central bank adheres to the Growth-augmented Taylor principle. These specific conditions are derived in section 4.

that stabilizes fiscal imbalances under active fiscal policy, as is evident from (16) if $|\beta^{-1} - \gamma| > 1$ and $\hat{g}_{y,t-1} = 0$.

Growth as an additional adjustment channel for public debt. However, in the model described in section 2, trend growth is determined endogenously in general equilibrium. Changes in trend growth, captured by $\hat{g}_{y,t-1}$ in (16), directly affect the evolution of debt. Thus, an increase in debt which is not backed by passive fiscal policy can, in principle, be financed by a mix of inflation and changes in long-run trend output growth.

3.2 Endogenous fiscal capacity

As it turns out, the same breakdown in Ricardian equivalence that delivers fiscal inflation also rationalizes why transitory debt-stabilizing changes in trend growth occur in equilibrium with active fiscal policy. If, say, public debt rises, then so does demand when Ricardian equivalence fails. Higher demand encourages investment in R&D and technology growth. This additional channel, which is absent in models assuming exogenous technology, reflects a positive feedback between demand and innovation, as summarized by equation (9). Thus, a two-way mechanism links growth and debt in the absence of a debt-stabilizing fiscal authority: 1) demand rises with debt under active fiscal policy, 2) R&D and technology growth rise with demand, expanding the long-term path of GDP (see equation (10)).

An expansion in the long-term path of GDP enhances the ability of the fiscal authority to generate new tax revenue. That is, growth creates *fiscal capacity*. Our model implicitly features a steady state in the long-run surplus/GDP ratio, and hence a permanent upward shift in the BGP generates higher revenue in the future. Thus, an increase in public debt under active fiscal policy translates to higher demand, higher growth and hence greater fiscal capacity. The presence of this endogenous fiscal capacity has consequences for inflation as well.

3.3 Growth substitutes for fiscal inflation

We have discussed that under active fiscal policy, endogenous fiscal capacity through R&D and trend growth introduces an additional stabilizer of fiscal imbalances. In what follows we analyze the importance of this adjustment mechanism relative to

inflation in general equilibrium. Under an active fiscal policy, the law-of-motion for inflation in a unique bounded equilibrium¹⁸ assumes the form

$$\hat{\pi}_t = \Omega_{\pi,b} \hat{b}_{t-1} + \Omega_{\pi,g} \hat{g}_{A,t-1} + \Gamma_\pi \hat{g}_t.$$
(17)

The coefficients in (17) depend on the model parameters, and $\Omega_{\pi,b} = \partial \hat{\pi}_t / \partial \hat{b}_{t-1}$ and $\Gamma_{\pi} = \partial \hat{\pi}_t / \partial \hat{g}_t$ capture how inflation responds to changes in public debt and in fiscal expenditures, respectively. For now, we abstract from direct complementarities between production and R&D ($\eta = 0$), and focus on the commonly studied case of exogenous fiscal surpluses ($\gamma = 0$), an interest rate peg ($\phi_{\pi} = \phi_y = 0$), and linear disutility in labor ($\varphi = 0$) (more general assumptions are considered in the subsequent sections). Under these specific assumptions, the coefficients in equation (17) derive as

$$\Omega_{\pi,b} = \frac{\kappa(1-\beta\lambda_1)}{\kappa+\beta\lambda_1(1-\beta\lambda_1)\mu} > 0,$$
(18)

$$\Gamma_{\pi} = \xi \Omega_{\pi,b}, \tag{19}$$

$$\xi = \frac{\bar{g}(g_y - \beta \rho)}{\bar{z}(g_y - \beta \rho)} - \frac{2\beta \mu \rho_g \tilde{g}}{\bar{z}(g_y - \beta \rho)}, \qquad (19)$$

$$\zeta = \overline{\overline{b}(1-\beta\rho_g)} - \frac{1}{1+\beta(1-2\rho_g)+\beta(1+\widetilde{c}\mu)\sqrt{\gamma_1^2-4\gamma_0}+\widetilde{c}(\kappa+\mu(1-2\beta\rho_g))},$$

where $\mu := (1 - \bar{\beta})/(\vartheta - 1), \gamma_0 := \frac{1}{\beta(1 + \tilde{c}\mu)}, \gamma_1 := \frac{1 + \beta + \kappa \tilde{c} + \tilde{c}\mu}{\beta(1 + \tilde{c}\mu)}, \lambda_1 := 0.5(\gamma_1 - \sqrt{\gamma_1^2 - 4\gamma_0}) \in [0, 1).$ See Appendix A.1 for more details.

From (18)-(19), it is apparent that higher debt or higher government spending implies high inflation when growth is strictly exogenous (i.e., $\Omega_{\pi,b} = 1 - \beta \lambda_1 > 0$ and $\xi = \frac{\bar{g}(g_y - \beta \rho)}{\bar{b}(1 - \beta \rho_g)} > 0$ when $\vartheta = \infty$). This underscores the indispensable debtstabilizing role played by inflation in textbook models of monetary-fiscal interactions. We summarize the result in Proposition 1.

Proposition 1 Consider (9)-(15) and suppose $\varphi = \eta = \gamma = \phi_{\pi} = \phi_y = 0$. If growth is exogenous, then $\frac{\partial \hat{\pi}_t}{\partial \hat{g}_t} = \Gamma_{\pi} > 0$ and $\frac{\partial \hat{\pi}_t}{\partial \hat{b}_{t-1}} = \Omega_{\pi,b} > 0$.

The detailed proof is shown in Appendix A.2. The effect of fiscal policy on inflation is potentially quite different under growth. From (18), it is apparent that the inflation response to debt may be very small, perhaps even close to zero, when ϑ is small. Additionally, the coefficient in (19) can be smaller, perhaps even negative,

¹⁸We describe the conditions for a unique bounded equilibrium and in particular the requirements the monetary policy must satisfy in detail in section 4.

when growth is endogenous. These observations capture the notion that growth substitutes for inflation. Whereas fiscal expansions are unambiguously inflationary under exogenous growth, they generate less inflation, and may even be deflationary, in the presence of innovation through R&D, depending on the respective supply-side characteristics (see also section 5.1).



Figure 1: Dynamics: Elevated Public Debt

Note: Annual inflation, public debt, technology growth, and R&D (skilled employment) are expressed in log deviations from their steady state values (e.g., 0.01 equals 1% annual inflation rate). % Δ Cons. (Output) Level= $ln(C_t/C_t^*)$ ($ln(Y_t/Y_t^*)$) where C_t/C_t^* (Y_t/Y_t^*) is the ratio of the level of consumption (output) to the counterfactual BGP level in the absence of the shock. In all subsequent figures that illustrate macroeconomic dynamics, we adhere to this convention.

3.4 Dynamics: Debt, Growth, and Inflation

We now consider the joint dynamics of public debt, inflation and trend growth when debt is initially elevated above steady state (section 3.4.1) and following an expansionary government spending shock (section 3.4.2).

3.4.1 Stabilization of high debt levels

Elevated debt levels, the risk they pose for inflation, and the question of how to revert debt to sustainable levels, are often identified as key challenges for policymakers (e.g., see Bianchi et al. (2020), Cochrane (2022), Bianchi et al. (2023)). In the economy we now consider, outstanding government debt is assumed to be initially elevated above steady state ($\hat{b}_{t-1} > 0$ in the context of our model at time-t). Figure 1 shows the impulse responses to an initial debt stock which is 50 percent above steady state level in the model with endogenous growth (red line), relative to the model with exogenous technology (blue line). We observe two major differences in the endogenous vs. exogenous growth dynamics. First, fiscal inflation is reduced under endogenous growth, both in terms of magnitude and persistence of the surge in fiscal inflation. Second, despite the muted response of inflation, the reversal of public debt is accelerated.

Under endogenous growth, high initial debt leads to a surge in demand that reflects a breakdown in Ricardian equivalence. In turn, the higher demand raises the expected discounted value from a new innovation, which stimulates investment in R&D and thus technology growth g_{A_t} . This expansion in the long-term trend path contributes to the erosion of public debt by creating fiscal capacity which helps return debt to steady state and reduces the need for fiscal inflation.

3.4.2 Expansion in government spending

Our analysis of $\Omega_{\pi,b}$ and Γ_{π} concerned only the effect of fiscal policy on impact. Figure 2 depicts impulse responses to a persistent increase in government spending. Following an expansionary shock to government spending under active fiscal policy, the inflation response is attenuated in the model with endogenous technology (red line), compared with the exogenous technology model (blue line). This is the case because government expenditures raise demand and thus the payoff from R&D investment. The latter translates into an increase in R&D investment and thus an expansion in technology and TFP growth, inducing a permanent upward shift of the trend of aggregate output. This adjustment on the long-run margin reduces fiscal inflation.

4. Monetary-fiscal interactions with endogenous trend growth

In the previous sections, we focused on the debt-stabilizing role of growth in the case of an active fiscal authority, under the simplifying assumption of a nominal interest



Figure 2: Dynamics: Government Spending Shock

rate peg. More generally, whether fiscal policy is active or passive has implications for price stability, and the actions of the monetary authority has implications for both fiscal sustainability and price stability. In what follows, we describe how monetary and fiscal policy jointly stabilizes inflation and debt through the selection of a unique bounded equilibrium.

4.1 Stability and monetary-fiscal interactions under exogenous trend growth

We first revisit some established results related to stability and monetary-fiscal interactions in the standard framework with exogenous technology. In general, macroeconomic policy should: 1) determine the price level and 2) stabilize public debt. A vast literature established that the joint behavior of monetary and fiscal authorities affects both price stability and fiscal sustainability. The hawkishness of the central bank, as summarized by the Taylor Principle, is frequently associated with price stability. In the exogenous growth model examined by Woodford (2001) (i.e. (9)-(15) with $\vartheta \to \infty$), the Taylor Principle can be interpreted as:

$$\phi_{\pi} - 1 + \frac{(1-\beta)}{\kappa} \phi_y = \frac{\partial \hat{r}}{\partial \hat{\pi}} > 0,$$

where $\partial \hat{r} / \partial \hat{\pi}$ represents the long-run response of the real interest rate, \hat{r} , to a permanent rise in inflation. That is:¹⁹

$$\frac{\partial \hat{r}}{\partial \hat{\pi}} := \lim_{k \to \infty} \frac{\partial (\hat{i}_{t+k} - \hat{\pi}_{t+k+1})}{\partial \hat{\pi}_t} = \lim_{k \to \infty} \frac{\partial \left(\phi_\pi \hat{\pi}_{t+k} - \hat{\pi}_{t+k+1} + \phi_y \hat{y}_{t+k}\right)}{\partial \hat{\pi}_t} = \phi_\pi - 1 + \frac{(1-\beta)}{\kappa} \phi_y$$

A monetary policy that satisfies this principle may eliminate positive, self-fulfilling feedback between expected inflation, demand and income. Suppose, for example, that agents in the economy believe inflation will go up by a percentage point in the current period and all future periods. Under the Taylor Principle, equilibrium inflation does not validate the exogenous change in expectations; the implied promise by the central bank to raise the real interest rate would reduce demand, and hence equilibrium inflation through the Phillips curve. Exogenous changes in expectations do not seed multiple dynamically stable equilibria. In fact, there can be only one bounded equilibrium, and in this bounded equilibrium, inflation, output, and consequently interest rates and bond prices are in steady state ($\hat{y}_t = \hat{\pi}_t = \hat{i}_t = \hat{P}_t^m = 0$) in the absence of shocks.²⁰

However, a bounded equilibrium may not exist under the Taylor Principle. Suppose there are no shocks, and that the Taylor Principle is satisfied (such that $\hat{y}_t = \hat{\pi}_t = \hat{i}_t = \hat{P}_t^m = 0$ describes the only potential stable equilibrium). Then the government's intertemporal budget constraint, (14), becomes:

$$\hat{b}_t = (\beta^{-1} - \gamma)\hat{b}_{t-1} - \beta^{-1}\hat{\pi}_t + (\frac{\rho}{g_y} - 1)\hat{P}_t^m = (\beta^{-1} - \gamma)\hat{b}_{t-1}.$$

Clearly $|\hat{b}_t| \to \infty$ for $\hat{b}_{-1} \neq 0$ unless fiscal policy is *passive*. If debt explodes $(|\hat{b}_t| \to \infty)$, then $\hat{y}_t = \hat{\pi}_t = \hat{i}_t = \hat{P}_t^m = 0$ cannot be an equilibrium due to a breakdown in Ricardian equivalence. For example, if $\hat{b}_{-1} > 0$ and fiscal policy is active, then households understand that their bond wealth will increase without bound, and without a commensurate increase in their tax burden. Consequently, agents perceive

¹⁹See also Chapter 4 of Galí (2015) for a related interpretation of the Taylor Principle.

²⁰The adaptive learning literature provides a different logic for the stabilizing effect of active monetary policy; in a sequence of *temporary* equilibria in which expectations are formed adaptively, the Taylor Principles raises the real interest rate when expected inflation is high, thus reducing actual inflation and hence the level of inflation expectations formed in the next period. In this environment, the Taylor Principle can rationalize how a rational expectations equilibrium emerges through a process of statistical learning. This logic does not apply to our rational expectations framework. Under rational expectations, the Taylor Principle entails off-equilibrium promises that render all but one solution of the model dynamically unstable, as explained above.

the initial public debt stock $(\hat{b}_{-1} > 0)$ as net wealth, and this spurs demand and inflation in the initial period. Under the Taylor Principle, an increase in inflation begets higher real interest rates, which implies higher debt service costs and hence higher debt, which in turn implies higher demand and inflation. Therefore, adhering to the Taylor Principle under active fiscal policy may expose the economy to both hyperinflation and explosive debt. Violating the Taylor Principle breaks this explosive feedback loop between debt, inflation and real interest rates, by ensuring that real debt service costs fall when debt, and therefore inflation, rises. The fall in real interest rates helps return public debt to steady state, which neutralizes wealth effects of high public debt. Some simple analytics using the government's intertemporal budget constraint, (14), shed light on the importance of violating the Taylor Principle under active fiscal policy. For simplicity, suppose a short maturity structure ($\rho = 0$) and exogenous fiscal surpluses ($\gamma = 0$). The budget constraint, (14), becomes:

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\hat{\pi}_t + \hat{i}_t + \frac{\bar{g}}{\bar{P}^m\bar{b}}\hat{g}_t.$$

Solving the constraint forward, substituting for $\hat{r}_{t+j} = \hat{i}_{t+j} - \hat{\pi}_{t+j+1}$, taking expectations and imposing $\lim_{j\to\infty} \beta^{j+1} E_t \hat{b}_{t+j} = 0$:

$$\underbrace{\hat{b}_{t-1} - \hat{\pi}_t}_{\text{Real Value of Debt}} = \underbrace{-\sum_{j\geq 0} \beta^{j+1} \frac{\bar{g}}{\bar{P}^m \bar{b}} E_t \hat{g}_{t+j} - \sum_{j\geq 0} \beta^{j+1} E_t \hat{r}_{t+j}}_{\text{Discounted P.V. of Expected Fiscal Surpluses}} .$$
(20)

Equation (20) implies that the real value of government debt must equal the discounted present value of expected fiscal surpluses in every period.²¹ Thus, a fiscal expansion at time-t (an increase in $\{\hat{g}_{t+j}\}_{j\geq 0}$) which lowers the first term on the righthand-side of (20) must be offset by an increase in current inflation $(\hat{\pi}_t)$ or a fall in the path of expected path of future real interest rates $(\{\hat{r}_{t+j}\}_{j\geq 0})$. The Taylor Principle implies positive co-variation in \hat{r}_t and $\hat{\pi}_t$, which is not debt-stabilizing. Violating the Taylor Principle permits negative debt-stabilizing co-movements in inflation and real interests that offset the effect of a fiscal expansion on (20).

The key insight is twofold. First, price and debt stability under exogenous growth requires a careful coordination of monetary and fiscal policy: under an active monetary policy which adheres to the Taylor principle, fiscal policy has to be passive.

 $^{^{21}}$ See Cochrane (2023) or Cochrane (2005) for interpretations of closely-related equilibrium conditions.

Under an active fiscal authority, a passive monetary authority which violates the Taylor principle is required. Two passive authorities allow for multiple equilibria, while two active authorities lead to non-existence of dynamically stable equilibrium. Second, whether real interest rates rise or fall with inflation, as summarized by the Taylor Principle, is critically important with exogenous growth, and a central bank that aims to raise real interest rates to engineer a disinflation may inadvertently destabilize the economy without the backing of passive fiscal policy. These central results concerning stability and the monetary-fiscal framework are summarized by Theorem 1.

Theorem 1 Consider the New Keynesian model with constant technology:

- *i.* Under passive fiscal policy, the Taylor Principle is a necessary and sufficient condition for local determinacy.
- *ii.* Under active fiscal policy, violating the Taylor Principle is a necessary and sufficient condition for local determinacy.

4.2 Growth-augmented Taylor Principle

According to Theorem 1, macroeconomic stability under exogenous technology requires the correct co-movements between real interest rates and inflation. However, managing the co-movement of real interest rates and inflation is not sufficient when trend growth and hence fiscal capacity are endogenous. A comparison of the Euler equation under exogenous and endogenous growth, respectively, provides useful intuition. Consider first the standard exogenous growth Euler equation (with no shocks, for brevity):

$$\hat{y}_t = E_t \hat{y}_{t+1} - \tilde{c}(\hat{i}_t - E_t \hat{\pi}_{t+1}) = E_t \hat{y}_{t+1} - \tilde{c} \hat{r}_t.$$

It is apparent that for given expected future income, the Taylor Principle promises to contract demand in response to an expected permanent increase in inflation. The analogous equation under endogenous growth can be expressed as:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \tilde{c}(\hat{r}_t - \hat{g}_{y,t}).$$

The last equation suggests that a stabilizing response of monetary policy to infla-

tion entails adjustments in $\hat{r}_t - \hat{g}_{y,t}$, and *not* just \hat{r}_t . Importantly, the trend output growth rate, $\hat{g}_{y,t}$, is endogenous and responds positively to a permanent increase in expected future inflation:

$$\begin{aligned} \frac{\partial \hat{g}_{y}}{\partial \hat{\pi}} &:= \lim_{k \to \infty} \frac{\partial \hat{g}_{y,t+k}}{\partial \hat{\pi}_{t}} = \lim_{k \to \infty} (\vartheta - 1)^{-1} \frac{\partial \hat{g}_{A,t+k}}{\partial \hat{\pi}_{t}} \\ &= \lim_{k \to \infty} (\vartheta - 1)^{-1} \left(\eta \omega \frac{\partial \hat{y}_{t+k}}{\partial \hat{\pi}_{t}} + (\bar{\delta} - \bar{\beta} \eta \omega) \frac{\partial \hat{y}_{t+k+1}}{\partial \hat{\pi}_{t}} + \frac{\bar{\beta} \bar{\varphi}}{1 + \bar{\varphi}} \frac{\partial \hat{g}_{A,t+k+1}}{\partial \hat{\pi}_{t}} \right) \\ &= \lim_{k \to \infty} \frac{(1 - \bar{\beta}) \eta \omega + \bar{\delta}}{(\vartheta - 1) \left(1 - \frac{\bar{\beta} \bar{\varphi}}{1 + \bar{\varphi}} \right)} \frac{\partial \hat{y}_{t+k}}{\partial \hat{\pi}_{t}} = \frac{(1 - \bar{\beta}) \eta \omega + \bar{\delta}}{(\vartheta - 1) \left(1 - \frac{\bar{\beta} \bar{\varphi}}{1 + \bar{\varphi}} \right)} \left(\frac{1 - \beta}{\kappa} \right) > 0. \end{aligned}$$

The fact that expected inflation temporarily raises trend growth reflects incentives to innovate; higher expected inflation implies higher aggregate demand, which raises the demand for input varieties and therefore the return to innovation and R&D. Thus, demand raises trend growth, and in turn, higher trend growth feeds back to aggregate demand through the Euler equation. Innovation through R&D therefore strengthens the overall responsiveness of demand to expected inflation, relative to the textbook exogenous growth case. Raising $\hat{r} - \hat{g}_y$ to lean against the inflation requires an especially hawkish monetary policy:

Definition. The growth-augmented Taylor Principle (GrTP) is satisfied (violated) if:

$$\phi_{\pi} - 1 + \frac{(1-\beta)}{\kappa} \left(\phi_y - \frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{(\vartheta-1)\left(1 - \frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)} \right) = \frac{\partial(\hat{r} - \hat{g}_y)}{\partial\hat{\pi}} > 0 (<0).$$

The GrTP generalizes the Taylor Principle to the endogenous growth economy. It constitutes a stricter requirement for monetary policy, as any interest rate rule that satisfies the GrTP will also satisfy the Taylor Principle, but not vice versa. In essence, the GrTP modifies the Taylor Principle to account for the role of R&D investment and the expectations of forward-looking innovators in general equilibrium. We observe that the wedge introduced by the endogeneity of growth vis-à-vis the Taylor principle is a function of the structural parameters of the model, including from R&D, the growth block and the slope of the Phillips curve (we will examine this in greater detail in sections 5.1 and 5.3).

4.3 A Dynamic r-g Stability Criterion

The growth-augmented Taylor Principle formalizes a dynamic r-g stability criterion. Building on this insight, we now present our key results regarding monetary-fiscal interaction and macroeconomic stability under endogenous growth (Proposition 2).

Proposition 2 Consider the endogenous TFP model, and suppose η is sufficiently small as defined in Appendix A.3. Then

- *i.* Under passive fiscal policy, the growth-augmented Taylor Principle is a necessary condition for local determinacy.
- *ii.* Under active fiscal policy, violating the growth-augmented Taylor Principle is a sufficient condition for local determinacy.

Proposition 2 reveals several novel implications of endogenous growth and fiscal capacity for price stability and debt sustainability and the interaction between monetary and fiscal policy. First, growth necessitates an especially active monetary policy to lean against the amplification of expectations coming from forward-looking innovators and R&D. Note further that the violation of the GrTP is not sufficient for price stability under passive fiscal policy as in particular, a weak response of interest rates to the output gap can lead to indeterminacy, as depicted by the white region in Figure 3. In fact, multiple bounded equilibria exist in the white region depicted in Figure 3 when fiscal policy is *active* as well. Hence, the model can admit multiple stable equilibria under the GrTP–regardless of whether fiscal policy is active or passive–if monetary policy is not sufficiently responsive to output.²²

Moreover, endogenous fiscal capacity implies relatively lax conditions for monetary policy under active fiscal policy. Growth stabilizes debt-to-GDP by creating

²²In general, the complete necessary and sufficient conditions for determinacy under passive fiscal policy do not admit a form that can be interpreted analytically. When the GrTP holds but indeterminacy obtains under passive (active) fiscal policy then we have two (one) "degrees of indeterminacy" in the terminology used by Lubik and Schorfheide (2003) or Bianchi and Nicolò (2021). We note that the simple New Keynesian model with exogenous technology can admit two (one) degrees of indeterminacy under passive (active) fiscal policy and a simple Taylor rule (13) that satisfies the Taylor Principle if ϕ_y is sufficiently negative. Hence, the Taylor Principle is *not* a necessary and sufficient for determinacy in the simple New Keynesian model with standard Taylor rule (13) unless one additionally imposes some lower bound on ϕ_y , such as $\phi_y \ge 0$. See also Benhabib et al. (2001a), which identifies assumptions about money-in-utility and production that deliver analogous order two indeterminacy results under active monetary policy, or Acharya and Dogra (2020), who show that order two indeterminacy can arise in a tractable heterogeneous-agents New Keynesian model with counter-cyclical income risk even when their "income-augmented Taylor Principle" is satisfied.

fiscal capacity, which in turn generates monetary policy space by permitting a more hawkish response by the central bank when fiscal policy is active. Crucially, a unique bounded equilibrium can exist when there is both active fiscal policy and "active" monetary policy in the sense that the Taylor Principle is satisfied, as stated in Corollary 1.

Corollary 1 A unique bounded equilibrium exists under active fiscal policy and the Taylor Principle if the growth-augmented Taylor Principle is violated.



Figure 3: Uniqueness and Existence



The corollary result reflects the fact that endogenous growth substitutes for the kind of fiscal inflation that is necessary for fiscal sustainability under active fiscal policy and exogenous growth, and which the central bank tolerates by violating the Taylor Principle.²³ To see this, consider the government budget constraint (14), and assume short maturity structure ($\rho = 0$) and exogenous fiscal surpluses ($\gamma = 0$) for simplicity:

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}(\hat{\pi}_t + \hat{g}_{y,t-1}) + \hat{i}_t + \frac{\bar{g}}{\bar{P}^m\bar{b}}\hat{g}_t.$$

Solving the constraint forward, substituting for $\hat{r}_{t+j} = \hat{i}_{t+j} - \hat{\pi}_{t+j+1}$, and taking expectations and imposing $\lim_{j\to\infty} \beta^{j+1} E_t \hat{b}_{t+j} = 0$:

$$\hat{b}_{t-1} - \hat{\pi}_t - \hat{g}_{y,t-1} = -\sum_{j\geq 0} \beta^{j+1} \frac{\bar{g}}{\bar{P}^m \bar{b}} E_t \hat{g}_{t+j} - \sum_{j\geq 0} \beta^{j+1} E_t \left(\hat{r}_{t+j} - \hat{g}_{y,t+j} \right).$$
(21)

 $^{^{23}}$ See Online Appendix C.1 for the dynamic effects of high initial public debt, as in Figure 1, in a regime with simultaneous active monetary and active fiscal policy.

Equation (21) is the endogenous growth analog of (20) from section 4.1. As in the exogenous growth case, an unbacked fiscal expansion (increase in $\{\hat{g}_{t+j}\}_{j\geq 0}$) which comes at time-t lowers the discounted present value of expected fiscal surpluses (right-hand-side of (21)). To satisfy (21), inflation $(\hat{\pi}_t)$ can reduce the real value of debt, or $\{\hat{r}_{t+j}\}_{j\geq 0}$ and $\{\hat{g}_{y,t+j}\}_{j\geq 0}$ can offset the effect of fiscal policy on the discounted present value term. Notice that the expected path of growth rates $(\{g_{y,t+j}\}_{j\geq 0})$ can also adjust to satisfy (20), unlike in the exogenous growth case. Thus, changes in current and expected future trend output growth can substitute for inflation or variation in the real interest rate and provide backing for the public debt. Moreover, the second sum on the right-hand-side of (21) reveals why violations of the GrTP permit negative debt-stabilizing co-movements in inflation and $\hat{r} - \hat{g}_y$. Adhering to the GrTP, on the other hand, means that the discounted present value of expected fiscal surpluses falls in inflation, which suggests that inflation will not finance a rise in fiscal expenditures.

4.4 Relationship to r < g

To the extent that growth is endogenous, the dynamics of $\hat{r} - \hat{g}_y$ are of central importance for macroeconomic stability. If the fiscal authority commits to resolving fiscal imbalances using real fiscal revenues (passive fiscal policy) then the central bank should at least ensure that $\hat{r} - \hat{g}_y$ rises with persistent inflation. Otherwise, policymakers must let $\hat{r} - \hat{g}_y$ fall with persistent inflation in order to stabilize both inflation and debt. The *dynamics* of $\hat{r} - \hat{g}_y$ matter for stability, and the Taylor Principle is not a relevant guide to policy in the case of endogenous trend growth.

Blanchard (2019), among others, have recently examined the costs of public debt when the rate of return on sovereign debt (r) is less than the economy's growth rate, (g_y) - a case regarded as the norm in the US by Blanchard (2019). For given r, a higher g_y improves conditions for debt stability by lowering the level of primary surpluses needed to sustain a given level of public debt. Permanent fiscal deficits can even be consistent with constant debt/GDP in cases where r is always less than g_y . This discussion clearly highlights the importance of growth for questions related to the sustainability of debt.

Our insight is fundamentally different. We show that policy should orchestrate the appropriate *changes* in $\hat{r} - \hat{g}_y$ that are needed to stabilize inflation and debt in a dynamic economy. The result does not hinge on $r - g_y < 0$ in steady state. Our model is a representative agent model in which $r = \frac{g_y}{\beta}$ and thus $r > g_y$ holds in the steady state. Therefore, the key mechanism in the model does not result from the assumption of a fiscal free lunch of the kind implied by $r < g_y$. Despite the unfavorable steady state financing conditions implied by $r > g_y$, endogenous growth through R&D relaxes the conditions for fiscal sustainability under active fiscal policy relative to the analogous conditions in canonical exogenous growth model (i.e., $\partial(\hat{r} - \hat{g}_y)/\partial\hat{\pi} < 0$ versus $\partial \hat{r}/\partial \hat{\pi} < 0$).

Moreover, a central bank that overemphasizes changes in r alone when adjusting their policy stance may inadvertently expose the economy to extraneous fluctuations. Similarly, a policymaker's preoccupation with the real interest rate alone might lead them to misjudge the risks of tight monetary policy for fiscal sustainability. We show it is imperative that policymakers take into account movements in r and short-term fluctuations in the trend growth rate, g_y , when formulating policy.

5. Additional results: demand-growth link, debt maturity, and the Phillips curve

This section extends our analysis and presents additional results concerning the role of the elasticity of substitution in intermediates (section 5.1), of the duration of public debt (section 5.2), and of the slope of the Phillips curve (section 5.3).

5.1 Demand-growth link: intermediate good elasticity of substitution

As shown in section 3, spillovers from demand fluctuations to long-run aggregate supply generate debt-stabilizing changes in fiscal capacity under an active fiscal regime. The strength of the spillover is strongly affected by the elasticity of substitution in the intermediate goods sector, which is pinned down by ϑ . In particular, ϑ influences the degree to which demand shifts translate into changes in firm profits and thus to adjustments in R&D investment, TFP and the trend GDP path. As discussed earlier, the limiting case $\vartheta \to \infty$ delivers the exogenous technology model, and consequently we interpret ϑ as a measure of the strength of the endogenous growth mechanism, and of the disparity between the exogenous and endogenous growth frameworks.



Figure 4: Growth-augmented Taylor Principle: role of ϑ

Stronger growth effects (lower ϑ) have several important implications for monetary and fiscal policy. First, the wedge between the GrTP and the standard Taylor Principle is dramatically affected by the elasticity of substitution between intermediate goods.²⁴ Figure 4 depicts the GrTP in (ϕ_{π}, ϕ_{y}) -space for values of ϑ within the range of plausible calibrations discussed in section 2, showing that an especially hawkish central bank is required for determinacy under passive fiscal policy when ϑ is fairly small. Because a low elasticity of substitution widens the disparity between the GrTP and Taylor Principle, it also expands the parameter space in which an active fiscal regime combined with a central bank that adheres to the Taylor Principle delivers a unique stable equilibrium (see Proposition 2). However, for the larger values of ϑ depicted in the figure, the wedge between the GrTP and Taylor Principle is less substantial, and unsurprisingly, the wedge disappears as $\vartheta \to \infty$.

Second, ϑ matters for the responsiveness of growth and hence fiscal capacity to changes in demand, which has implications for the amount of inflation generated by a fiscal expansion in an active fiscal regime. Figure 5 shows that the contemporaneous response of inflation to public debt (Panel (a)) and government spending (Panel (b)) is monotonically increasing in ϑ under the benchmark calibration for all other parameters. We observe that for sufficiently low ϑ , the inflation response to government expenditures turns negative, i.e. an unbacked fiscal expansion under active fiscal policy can yield deflation on impact. Online Appendix C.2 shows the dynamics for

²⁴The definition of the GrTP in 4.2 clearly shows that the wedge between the GrTP and standard Taylor principle is decreasing in ϑ .

such a calibration ($\vartheta = 1.3$). When intermediate varieties are highly imperfect substitutes in the production of final consumer goods (low ϑ), producers of intermediates have more market power and charge larger markups, which induces entrepreneurs to innovate relatively aggressively in response to a rise in demand caused by a fiscal expansion. Additionally, lower ϑ implies higher sensitivity of TFP, $A_t^{(\vartheta-1)^{-1}}$, to the innovation of new product varieties, A_t , which amplifies the response of the trend output path to innovation. The end result is that fiscal expansions generate larger booms, relatively more substantial fiscal capacity, and less inflation when ϑ is small.



Figure 5: Inflation and Fiscal Policy

Since we work with a tractable model, with the focus on analytical results, we emphasize qualitative results regarding the role ϑ . We show that if intermediate producers have fairly little market power (high ϑ) then endogenous technology may matter relatively little for the stability issues studied in this paper. However, our results also suggest the importance of accounting for endogenous technology in cases of relatively low ϑ , in line with the range of calibrations used in the literature and discussed in section 2. How one should calibrate ϑ is ultimately an empirical question which depends on country-specific supply-side characteristics. In particular, a country with relatively intense spillovers from demand to growth may experience less inflation for a given deviation of debt from its steady state level. Countries such as the US, Japan and Italy are among the many economies experiencing elevated public debt levels, but there are considerable differences in trend output growth rates, the percentage of GDP devoted to R&D, and potentially the responsiveness of technological innovation and the trend to fluctuations in demand, across these countries.²⁵ Our model suggests that quantitative predictions about the effects of high debt in different national economies should account for heterogeneity in the intensity of R&D and its role in TFP growth. Our results thus also highlight that depending on structural characteristics at the supply-side, unbacked fiscal expansions may be associated in some countries with high fiscal inflation and in others with low inflation or even deflation. We emphasize that a full quantitative assessment of these issues requires a richer model than the one we consider in this paper.

5.2 Public debt maturity: go long to keep inflation low

Under an active fiscal regime, the maturity structure of public debt affects monetary policy transmission and the dynamics of inflation, as many earlier papers have shown (e.g., Cochrane (2001), Cochrane (2018)). We show that debt maturity also matters for the sign of the response of inflation to a fiscal expansion and for the long-run effects of fiscal policy when growth is endogenous. Recall from equations (18) and (19) in section 3.3, that $\partial \hat{\pi}_t / \partial \hat{b}_{t-1} = \Omega_{\pi,b} > 0$, and $\frac{\partial \hat{\pi}_t}{\partial \hat{g}_t} = \Gamma_{\pi} > 0$ when growth is strictly exogenous ($\vartheta = \infty$). However, equation (18) suggests that $\partial \hat{\pi}_t / \partial \hat{b}_{t-1} = \Omega_{\pi,b}$ can be close to zero for very low ϑ . Moreover, (19), $\frac{\partial \hat{\pi}_t}{\partial \hat{g}_t} = \Gamma_{\pi} < 0$ if and only if $\xi < 0$. Notice that ξ is strictly decreasing in the duration of debt, ρ . Therefore, there exists a ρ^* such that $\partial \hat{\pi}_t / \partial \hat{g}_t = \Gamma_{\pi} \leq 0$ if and only if $\rho \geq \rho^*$.²⁶

Proposition 3 Consider (9)-(15) and suppose $\varphi = \eta = \gamma = \phi_{\pi} = \phi_{y} = 0$. Then $\frac{\partial \hat{\pi}_{t}}{\partial \hat{g}_{t}} = \Gamma_{\pi}$ is strictly decreasing in ρ , and $\frac{\partial \hat{\pi}_{t}}{\partial \hat{g}_{t}} = \Gamma_{\pi} \leq 0$ if and only if $\rho \geq \rho^{*}$.

See Appendix A.4 for the proof.²⁷ Intuitively, a longer debt maturity structure (higher ρ) delays the impact of a fiscal expansion on the government's finances, since lengthening the maturity structure reduces the share of debt that needs to be rolled over in a given period. Formally, higher ρ implies a smaller coefficient on \hat{g}_t in the budget constraint (14), because longer maturity raises the steady state bond price, \bar{P}^m . Since the financing needs of the government greatly impacts the amount of

²⁵See Online Appendix C.3 for the dynamic effects of high initial public debt, as in Figure 1, under different steady state values of g_y .

²⁶We note that nothing in the model implies that $\rho^* \in [0, 1]$. If $\rho^* < 0$ ($\rho^* > 1$) then any maturity structure is consistent with deflationary (inflationary) fiscal policy.

²⁷The dynamic responses to a government shock are depicted in Online Appendix C.4 for different values of ρ .

inflation generated by an unbacked fiscal expansion, we should expect lower inflation for longer maturity structures in the period the government raises its expenditures. At the same time, the direct relationship between aggregate demand, (11), innovation (9) and government spending is unaffected by the public debt maturity structure. Therefore, increasing ρ reduces the direct effect of fiscal policy on public debt, but does not change the direct effect of fiscal policy on aggregate demand. The former effect tends to be inflationary, while the latter effect is deflationary under endogenous growth. The net effect depends on debt maturity structure.

It should be noted that our results implicitly hinge on the maturity structure of bonds held by *households*, and consequently this analysis suggests that quantitative easing policies may expose the economy to greater fiscal inflation by shortening the duration of debt held by the private sector.²⁸

5.3 Slope of the Phillips curve

Additionally, the price rigidity faced by finals goods firms, captured by the slope of the Phillips curve κ , shapes the wedge between the traditional Taylor Principle and its growth-augmented counterpart. Figure 6 shows the growth-augmented Taylor Principle for respectively low (Panel (a)) and high κ (Panel (b)). In general, a flatter



Figure 6: Growth-augmented Taylor Principle and Price Rigidity

Phillips curve means that a persistent rise in inflation has larger effects on output

 $^{^{28}}$ In principle, we can study the effect of quantitative easing by adding central bank reserves to the government's consolidated budget constraint (e.g., see Maćkowiak and Schmidt (2023)).

(consistent with the Phillips curve) which implies a larger response of the longrun real interest rate to persistent inflation for given $\phi_y > 0$. Hence, price rigidity has a stabilizing effect which allows the central bank to substitute some weight on inflation-stabilization (high ϕ_{π}) for output-stabilization ($\phi_y > 0$), as captured by the traditional Taylor Principle defined above. However, because price rigidity amplifies the response of output to inflation, it also amplifies the response of trend growth, i.e., $\frac{\partial}{\partial \kappa} (\frac{\partial \hat{g}_y}{\partial \hat{\pi}}) < 0$, where $\partial \hat{g}_y / \partial \hat{\pi}$ is defined in section 4.2. This destabilizing effect of price rigidity on growth shifts the GrTP out in (ϕ_{π}, ϕ_y)-space, requiring a more hawkish monetary policy for lower values of κ , as shown in Figure 6.

6. Conclusion

This paper studies monetary-fiscal interactions under endogenous growth through R&D in an otherwise standard representative agent New Keynesian model subject to r > g. We show that the standard assumption of a constant, exogenous long-term trend path omits a channel essential for both debt sustainability and price stability: technological innovation. Movements in aggregate demand affect the R&D choice of forward-looking innovators and the long-term output path. This property makes fiscal capacity endogenous, alters the conditions for debt sustainability and price stability, and the set of feasible monetary-fiscal regimes.

Expansions in demand due to the breakdown of Ricardian equivalence under active fiscal policy raise R&D and generate a permanent output expansion which creates fiscal space. Since growth substitutes for fiscal inflation, the inflation response to a fiscal expansion is attenuated and may (for some parameterizations) even be deflationary.

Local determinacy under passive fiscal policy requires that monetary policy adheres to the growth-augmented Taylor Principle (GrTP). The latter is more stringent than the conventional Taylor Principle as it requires the central bank to raise not only r, but r - g in response to a persistent inflation increase. Under a debt-destabilizing ("active") fiscal policy, in turn, violating the GrTP suffices for stability. Unbacked fiscal expansions can be self-financing through an expansion in the long-run trend if the monetary authority permits a fall of r - g in response to a persistent inflation increase. Crucially, our model can feature a unique stable equilibrium under active fiscal policy and monetary policy which adheres to the Taylor Principle.

Taken together, these results highlight that a focus on real interest rates alone can be misguided, and policymakers should in addition factor in endogenous long-run trend adjustments. Our findings also beg additional questions about the interaction of supply side trend developments, monetary and fiscal policy. The joint analysis of optimal monetary and fiscal policy²⁹ is a particularly promising avenue for future research. Additional optimal policy issues (e.g., Debortoli et al. (2014) Debortoli et al. (2021)) and game-theoretic considerations (Bassetto (2002)) should be considered in the context of endogenous growth theory. Further, monetary-fiscal interactions and stability of equilibrium can be explored in a stochastic model with r < g, such as Galí (2021). Finally, future work should analyze the interactions of long-term supply and monetary and fiscal policy in a monetary union, along the lines of Jarociński and Maćkowiak (2018), Bassetto and Caracciolo (2021) and Maćkowiak and Schmidt (2023).

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²⁹See Benigno and Woodford (2003), Leeper and Zhou (2021), Leeper et al. (2021), and Schmitt-Grohe and Uribe (2004), among others.

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A. Derivations and Proofs

A.1 Active Fiscal Policy Solution

This appendix provides some details about the solution of the model under active fiscal policy in the special case: $\varphi = \eta = 0.3^{0}$ If fiscal policy is active and the GrTP is violated, then by Proposition 2 there exists a unique bounded equilibrium, which can be obtained by solving the government's budget constraint forward:

$$\hat{b}_{t-1} = E_t \sum_{j \ge 0} (\beta^{-1} - \gamma)^{-j-1} \left(\frac{1}{(\vartheta - 1)\beta} \hat{g}_{A,t+j-1} + \frac{1}{\beta} \hat{\pi}_{t+j} + (1 - \frac{\rho}{g_y}) \hat{P}_{t+j}^m - \frac{\bar{g}}{\bar{P}_m \bar{b}} \hat{g}_{t+j} \right). (22)$$

³⁰The same solution approach delivers an analytical solution for $\varphi \ge 0$ and $\eta \ge 0$, but the simple case is needed for Propositions 1 and 3.

Combining the Phillips curve (12), growth equation (9), and IS equation (11) implies a second-order difference equation in expected inflation:

$$\alpha \hat{g}_t = (E_t \hat{\pi}_{t+2} - \lambda_1 E_t \hat{\pi}_{t+1}) - \lambda_2 (E_t \hat{\pi}_{t+1} - \lambda_1 \hat{\pi}_t),$$

where $0 < \lambda_1 := 0.5(\gamma_1 - \sqrt{\gamma_1^2 - 4\gamma_0}) < 1 < \lambda_2 := 0.5(\gamma_1 + \sqrt{\gamma_1^2 - 4\gamma_0})$, and $\gamma_0 := \frac{1 + \tilde{c}\phi_y + \kappa \tilde{c}\phi_x}{\beta(1 + \tilde{c}\mu)}$, $\gamma_1 := \frac{1 + \beta + \beta \tilde{c}\phi_y + \kappa \tilde{c} + \tilde{c}\mu}{\beta(1 + \tilde{c}\mu)}$, $\alpha := \frac{\kappa \tilde{g}\tilde{c}(\mu\rho_g - \phi_y)}{\beta(1 + \tilde{c}\mu)}$, $\mu := (1 - \bar{\beta})/(\vartheta - 1)$. We solve the unstable root forward and the stable root backward obtaining:

$$E_t \hat{\pi}_{t+j} = \lambda_1^j \hat{\pi}_t - \frac{\lambda_1^j - \rho_g^j}{(\lambda_2 - \rho_g)(\lambda_1 - \rho_g)} \alpha \hat{g}_t$$

for j > 0. From the last equation, $E_t \hat{\pi}_{t+1} = \lambda_1 \hat{\pi}_t - \frac{\alpha}{(\lambda_2 - \rho_g)} \hat{g}_t$. Substituting expected inflation into the Phillips curve yields the following expression for expected output:

$$E_t \hat{y}_{t+j} = \frac{1 - \beta \lambda_1}{\kappa} E_t \hat{\pi}_{t+j} + \alpha_y \rho_g^j \hat{g}_t$$

for $j \ge 0$, where $\alpha_y = \tilde{g} + \frac{\beta \alpha}{\kappa(\lambda_2 - \rho_g)}$. Finally, the bond price is given by:

$$\hat{P}_t^m = -E_t \sum_{j\geq 0} \left(\frac{\beta\rho}{g_y}\right)^j \left(\phi_\pi \pi_{t+j} + \phi_y y_{t+j}\right).$$

Substituting the above expressions for expected inflation, output and bond price into equation (22) yields closed-form solutions for endogenous variable $z \in$ $\{\hat{\pi}, \hat{y}, \hat{i}, \hat{P}^m, \hat{b}, \hat{g}_A, \hat{g}_y\}$: $z_t = \Omega_{z,b}\hat{b}_{t-1} + \Omega_{z,g}\hat{g}_{A,t-1} + \Gamma_z\hat{g}_t$.

In the widely studied case of a passive interest rate peg ($\phi_{\pi} = \phi_y = 0$) and exogenous surplus ($\gamma = 0$), the expressions for $\Omega_{\pi,b}$ and Γ_{π} are given in (18) and (19), respectively.

A.2 Proof of Proposition 1

Consider (19) and (18). In the model with exogenous technology $(\vartheta \to \infty; \mu = (1 - \bar{\beta})/(\vartheta - 1) = 0)$, then:

$$\Gamma_{\pi} = \Omega_{\pi,b} \left(\frac{\bar{g}(g_y - \beta \rho)}{\bar{b}(1 - \beta \rho_g)} \right),$$

$$\Omega_{\pi,b} = 1 - \beta \lambda_1 > 0,$$

where $g_y = 1$, which proves the assertions.

A.3 Proof of Proposition 2

Case $\varphi > 0$. The model (9)-(15) can be expressed in matrix form as

$$\mathbf{Z}_{\mathbf{t+1}} = \bar{A}\mathbf{Z}_{\mathbf{t}} + \bar{C}\hat{g}_t$$

where $\mathbf{Z}_{t+1} = (E_t \hat{g}_{A,t+1}, \hat{g}_{A,t}, E_t \hat{\pi}_{t+1}, E_t \hat{y}_{t+1}, E_t \hat{P}_{t+1}^m, \hat{b}_t)'$ and

$$\bar{A} = \begin{pmatrix} \frac{\tilde{c}}{\vartheta-1}(\bar{\delta}-\eta\omega\bar{\beta})+1 & 0 & -\frac{\tilde{c}(\bar{\delta}-\eta\omega\bar{\beta})(\beta\phi\pi-1)}{\beta\frac{\beta\bar{\varphi}}{1-\bar{\varphi}}} & -\frac{(\bar{\delta}-\eta\omega\bar{\beta})(\beta+\beta\bar{c}\phi_y+\bar{c}\kappa)+\beta\eta\omega}{\beta\frac{\beta\bar{\varphi}}{1-\bar{\varphi}}} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 & 0 \\ -\frac{\tilde{c}}{\vartheta-1} & 0 & \tilde{c}\left(\phi_{\pi}-\frac{1}{\beta}\right) & \tilde{c}\left(\frac{\kappa}{\beta}+\phi_y\right)+1 & 0 & 0 \\ 0 & 0 & \frac{g_y\phi_{\pi}}{\beta\rho} & \frac{g_y\phi_y}{\beta\rho} & \frac{g_y}{\beta\rho} & 0 \\ 0 & -\frac{1}{\beta(\vartheta-1)} & -\frac{1}{\beta} & 0 & \frac{\rho}{g_y}-1 & \frac{1}{\beta}-\gamma \end{pmatrix} \end{pmatrix}$$

The model has a unique bounded rational expectations equilibrium (REE) if 4 roots of \bar{A} are outside the unit circle, a continuum of REE if fewer than 4 roots of \bar{A} are outside the unit circle, and no stable solution otherwise.³¹ The characteristic polynomial is:

$$P(\lambda) = \lambda(\lambda - (\beta^{-1} - \gamma))(\lambda - \frac{g_y}{\beta\rho})\tilde{P}(\lambda),$$

$$\tilde{P}(\lambda) = \lambda^3 + p_1\lambda^2 + p_2\lambda + p_3,$$

$$p_1 = -\frac{\beta(1 + C_{ga}G_{Ey}) + G_{ga}(1 + \beta + \tilde{c}(\kappa + \beta\phi_y))}{G_{ga}\beta},$$

$$p_2 = \frac{1 + C_{ga}(G_{Ey} - G_y\beta) + G_{ga} + \beta + \tilde{c}(\kappa + \beta\phi_y + G_{ga}(\phi_y + \kappa\phi_\pi))}{G_{ga}\beta},$$

$$p_3 = -\frac{1 - C_{ga}G_y + \tilde{c}(\phi_y + \kappa\phi_\pi)}{G_{ga}\beta},$$

where $G_{ga} = \frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}} \in (0,1), \ G_{Ey} = \bar{\delta} - \eta\omega\bar{\beta}, \ G_y = \omega\eta, \ C_{ga} = \frac{\tilde{c}}{\vartheta-1}$. Henceforth, we

 $^{^{31}}$ Following standard practices in the literature, we disregard boundary cases in which one or more roots lie on the unit circle throughout the proof of Proposition 2.

assume that $\eta < \bar{\eta}$:³²

$$\bar{\eta} := \min\{\frac{(\vartheta-1)(1+G_{ga}+\beta+\tilde{c}\kappa+C_{ga}\bar{\delta})}{\tilde{c}\omega(\beta+\bar{\beta})}, \frac{(\vartheta-1)(g_A-\phi+\varphi g_A(1-\beta\bar{\beta}))}{\tilde{c}(g_A-\phi)(1+\varphi)}\} \ge 0.$$

In turn, this implies that $p_2 > 0 > -1 > p_3$ and $p_1 < 0$, such that $\tilde{P}(\lambda) < 0$, $\forall \lambda \leq 0$, and $\lim_{\lambda \to -\infty} \tilde{P}(\lambda) = -\infty$, $\lim_{\lambda \to +\infty} \tilde{P}(\lambda) = +\infty$. Therefore, a necessary condition for all roots of $\tilde{P}(\lambda)$ to be outside the unit circle is: $\tilde{P}(1) < 0$. We have that $\tilde{P}(1) < 0$ if and only if:

$$\phi_{\pi} + \frac{(1-\beta)}{\kappa}\phi_{y} - 1 - \frac{(1-\beta)}{\kappa}\frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{(\vartheta-1)\left(1 - \frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)} > 0.$$

From the Phillips curve, (12), $\partial \hat{y}/\partial \hat{\pi} = (1 - \beta)\kappa^{-1}$, where $\partial \hat{y}/\partial \hat{\pi}$ denotes the longrun response of output to a permanent rise in inflation. Similarly, from the growth equation, (9), $\partial \hat{g}_A/\partial \hat{y} = \frac{(1-\bar{\beta})\eta\omega+\bar{\delta}}{\left(1-\frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)}$ where $\partial \hat{g}_A/\partial \hat{\pi}$ denotes the long run response of technology growth to a permanent rise in inflation. Therefore,

$$\phi_{\pi} + \frac{(1-\beta)}{\kappa} \phi_{y} - 1 - \frac{(1-\beta)}{\kappa} \frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{(\vartheta-1)\left(1 - \frac{\bar{\beta}\bar{\varphi}}{1+\bar{\varphi}}\right)} = \frac{\partial(\hat{i} - \hat{\pi} - (\vartheta-1)^{-1}\hat{g}_{A})}{\partial\hat{\pi}}$$
$$= \frac{\partial(\hat{r} - \hat{g}_{y})}{\partial\hat{\pi}} > 0$$

where $\partial \hat{g}_y/\partial \hat{\pi} = (\vartheta - 1)^{-1} \partial \hat{g}_A/\partial \hat{\pi}$ is the response of trend output growth to a permanent rise in inflation. Hence, if fiscal policy is passive $(|\beta^{-1} - \gamma| < 1)$ then a necessary condition for determinacy is $\frac{\partial(\hat{r}-\hat{g}_y)}{\partial \hat{\pi}} > 0$.

Now suppose that $\tilde{P}(1) > 0$, such that $\frac{\partial(\hat{r}-\hat{g}_y)}{\partial \hat{\pi}} < 0$. Then one real root, λ_1 , of $\tilde{P}(\lambda)$ is inside the unit circle and strictly positive. Let λ_2, λ_3 denote the remaining roots, and recall that any real roots cannot be negative. Then: $-\lambda_1\lambda_2\lambda_3 = p_3 < -1$. Therefore: $\lambda_2\lambda_3 = |p_3|/\lambda_1 > 1$. If the remaining roots are complex, then $\lambda_2\lambda_3 = |\lambda_2| = |\lambda_3| > 1$. If the remaining roots are real, then without loss of generality, $\lambda_2 > 1$, which implies $\lambda_3 > 1$ because $\tilde{P}(\lambda) < 0 < P(1), \forall \lambda \leq 0$, implies a maximum of one real root inside the unit circle if $\lambda_2 > 1 > \lambda_1 > 0$. We conclude that $\tilde{P}(1) > 0$ is sufficient for determinacy under active fiscal policy.

³²To our knowledge, estimates of η are not available in the literature.

Case $\varphi = 0$. If $\varphi = 0$ then the relevant characteristic polynomial becomes:

$$Q(\lambda) = \lambda(\lambda - (\beta^{-1} - \gamma))(\lambda - \frac{g_y}{\beta\rho})\tilde{Q}(\lambda),$$

$$\tilde{Q}(\lambda) = \lambda^2 + q_1\lambda + q_2,$$

$$q_1 = -\frac{(1 + C_{ga}(G_{Ey} - \beta G_y) + \beta + \tilde{c}(\kappa + \beta\phi_y))}{\beta(1 + C_{ga}G_{Ey})},$$

$$q_2 = \frac{1 - C_{ga}G_y + \tilde{c}(\phi_y + \kappa\phi_\pi)}{\beta(1 + C_{ga}G_{Ey})}.$$

The model with $\varphi = 0$ has a unique bounded REE if 3 roots of $Q(\lambda)$ are outside the unit circle, a continuum of REE if fewer than 3 roots of $Q(\lambda)$ are outside the unit circle, and no stable solution otherwise. Under passive (active) fiscal policy, determinacy therefore requires that two (only one) root(s) of $\tilde{Q}(\lambda)$ are outside the unit circle. If $\eta < \bar{\eta}$ then $\tilde{Q}(0) = q_2 > 0 > q_1$ and $Q(\lambda) > 0$ for $\lambda \leq 0$. It follows that $\tilde{Q}(1) > 0$ is a necessary condition for determinacy under passive fiscal policy which is satisfied if and only if:

$$\phi_{\pi} + \frac{(1-\beta)}{\kappa}\phi_{y} - 1 - \frac{(1-\beta)}{\kappa}\left(\frac{(1-\bar{\beta})\eta\omega + \bar{\delta}}{\vartheta - 1}\right) = \frac{\partial(\hat{r} - \hat{g}_{y})}{\partial\hat{\pi}} > 0.$$

If $\frac{\partial (r-g_y)}{\partial \hat{\pi}} < 0$ then a real root, λ_1 , is in (0, 1). Because $\tilde{Q}(1) < 0$ and $\lim_{\lambda \to +\infty} \tilde{Q}(\lambda) = +\infty$ the other root, λ_2 , is strictly greater than one. Therefore, $\frac{\partial (\hat{r}-\hat{g}_y)}{\partial \hat{\pi}} < 0$ is sufficient for determinacy under active fiscal policy.

A.4 Proof of Proposition 3

Consider (19) and (18). From (18), $\Omega_{\pi,b} > 0$ (with strict inequality) if $\vartheta > 1$ ($\mu < \infty$). Therefore, $\Gamma_{\pi} \leq 0$ if and only if $\xi \leq 0$. Clearly, ξ is strictly decreasing in ρ , and there exists a unique ρ^* such that $\xi = 0$ if and only if $\rho = \rho^*$.³³

³³Recall that $\bar{g} \neq 0$.

Online Appendix

B. Model Derivation Details

B.1 Stationarized equilibrium conditions

the positive the Given presence of trend growth ineconthe equilibrium conditions terms of omy, express in the we variables: $\{\mathcal{J}_t, \mathbf{\Pi}_t, y_t, c_t, g_t, w_t, w_t^s, b_t, \tilde{T}_t\}$ following stationary := $\{\frac{J_t}{A_t^{\frac{2-\vartheta}{\vartheta-1}}}, \frac{\Pi_t}{A_t^{\frac{2-\vartheta}{\vartheta-1}}}, \frac{Y_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{C_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{G_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{W_t/P_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{W_t^s/P_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{B_t/P_t}{A_t^{\frac{1}{\vartheta-1}}}, \frac{T_t}{A_t^{\frac{1}{\vartheta-1}}}\}.$ The model equilibrium conditions expressed in terms of stationary variables are given by:

$$g_{A,t} = \phi + \varsigma \left(L_t\right)^{\eta} L_t^s \tag{B.1}$$

$$\tilde{J}_t = E_t \left\{ \frac{\beta}{g_{A,t}} \frac{c_t}{c_{t+1}} \left(\tilde{\Pi}_{t+1} + \phi \tilde{J}_{t+1} \right) \right\}$$
(B.2)

$$\varsigma \left(L_t \right)^\eta \tilde{J}_t = w_t^s \tag{B.3}$$

$$g_{y,t} = (g_{A,t})^{\frac{1}{\vartheta - 1}}$$
 (B.4)

$$\tilde{\Pi}_t = \vartheta^{-1} M C_t y_t \nu_t \tag{B.5}$$

$$MC_t = \frac{\vartheta}{\vartheta - 1} w_t \tag{B.6}$$

$$E_t \left\{ \sum_{j=0}^{\infty} c_{t+j}^{-1} \beta^j \theta^j \left(\frac{P_t^*}{P_{t+j}} - \frac{\epsilon}{\epsilon - 1} M C_{t+j} \right) \left(\frac{P_t^*}{P_{t+j}} \right)^{-\epsilon} y_{t+j} \right\} = 0$$
(B.7)

$$R_t = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi} \left(\frac{y_t}{\bar{y}}\right)^{\phi_y} \tag{B.8}$$

$$b_t P_t^m + \tilde{T}_t = b_{t-1} \frac{(1+\rho P_t^m)}{\pi_t g_{y,t-1}} + g_t \tag{B.9}$$

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{b_{t-1}}{b}\right)^{\frac{\gamma P^{m}b}{\tilde{T}}} \tag{B.10}$$

$$1 = \beta E_t \left\{ \frac{c_t}{c_{t+1}} \frac{R_t}{g_{y,t}\pi_{t+1}} \right\}$$
(B.11)

$$P_t^m = E_t \left\{ R_t^{-1} \left(1 + \rho P_{t+1}^m \right) \right\}$$
(B.12)

$$(L_t)^{\varphi}c_t = w_t \tag{B.13}$$

$$\chi(L_t^s)^{\varphi}c_t = w_t^s \tag{B.14}$$

$$\frac{g_t}{\bar{g}} = \left(\frac{g_{t-1}}{\bar{g}}\right)^{\rho_g} \epsilon_t^G \tag{B.15}$$

$$y_t = \nu_t^{-1} L_t \tag{B.16}$$

$$y_t = c_t + g_t \tag{B.17}$$

$$P_{t} = \left[(1 - \theta) \left(P_{t}^{*} \right)^{1 - \epsilon} + \theta \left(P_{t-1} \right)^{1 - \epsilon} \right]^{\frac{1}{1 - \epsilon}}$$
(B.18)

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{B.19}$$

where $\nu_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di$. In an equilibrium, the endogenous variable vector:

$$\{c_t, y_t, g_t, L_t, L_t^s, g_{A,t}, g_{y,t}, \mathbf{\Pi}_t, MC_t, \mathcal{J}_t, b_t, T_t, P_t^m, R_t, P_t, P_t^*, \pi_t, w_t, w_t^s\}$$

must satisfy the equilibrium conditions (B.1)-(B.19) given the government spending shocks $\{\epsilon_t^G\}$ and initial conditions, b_{-1} , P_{-1} .

B.2 Steady state

Let \overline{Z} denote the steady state value of variable Z. Note that we define government spending policy such that $\overline{G}/\overline{Y} = \tilde{g} > 0$ and $\overline{C}/\overline{Y} = 1 - \tilde{g} = \tilde{c}$. From (B.7), the steady state marginal cost is given by

$$\bar{MC} = \frac{\epsilon - 1}{\epsilon}.$$

Combining this expression for steady state marginal cost, MC, with (B.6), (B.13) and (B.16) yields:

$$\bar{L} = \left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi + 1}}$$

and therefore, from (B.5), (B.13), and (B.16):

$$\bar{y} = \bar{L}\nu^{-1} = \nu^{-1} \left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi + 1}}$$
$$\bar{c} = \tilde{c}\bar{y} = \tilde{c}\nu^{-1} \left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi + 1}}$$
$$\bar{g} = \tilde{g}\bar{y} = \tilde{g}\nu^{-1} \left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi + 1}}$$
$$\bar{\Pi} = \frac{\bar{M}C}{\vartheta}\bar{L} = \frac{\epsilon - 1}{\vartheta\epsilon} \left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon}\right)^{\frac{1}{\varphi + 1}}$$

Given, \overline{L} , $\overline{\Pi}$, and \overline{c} from above, $\overline{\mathcal{J}}$, $g_a := \frac{A_{t+1}}{A_t}$, \overline{L}^s jointly satisfy (B.1)-(B.3) after substituting in (B.14):

$$\begin{split} \bar{\mathcal{J}} &= \frac{\beta}{g_A - \beta \phi} \bar{\Pi} \\ g_A &= \phi + \varsigma (\bar{L})^\eta \bar{L}^s \\ \chi \bar{c} (\bar{L}^s)^\varphi &= \varsigma \bar{\mathcal{J}} (\bar{L})^\eta \end{split}$$

We proceed by calibrating $g_A \geq 1$ (e.g. to match the trend output growth rate, $g_y = g_A^{\frac{1}{\vartheta-1}}$, in the economy) by solving the last three equations for $\bar{\mathcal{J}}, \bar{L}^s, \varsigma$, where ς scales the marginal product of skilled labor in R&D production.³⁴ If $\varphi > 0$:

$$\begin{split} \varsigma &= \left((g_A - \phi) \left(\frac{\nu(\epsilon - 1)(\vartheta - 1)}{\tilde{c}\epsilon\vartheta} \right)^{\frac{-\eta}{\varphi + 1}} \left(\frac{\beta \left(\frac{\nu(\epsilon - 1)(\vartheta - 1)}{\tilde{c}\epsilon\vartheta} \right)^{\frac{1 + \eta + \varphi}{\varphi + 1}}}{\chi(\vartheta - 1)(g_A - \beta\phi)} \right)^{-1/\varphi} \right)^{\frac{\varphi}{\varphi + 1}} > 0 \\ \bar{L}^s &= \varsigma^{\frac{1}{\varphi}} \left(\frac{\beta \left(\frac{\nu(\epsilon - 1)(\vartheta - 1)}{\tilde{c}\epsilon\vartheta} \right)^{\frac{1 + \eta + \varphi}{\varphi + 1}}}{\chi(\vartheta - 1)(g_A - \beta\phi)} \right)^{\frac{1}{\varphi}} \\ \bar{\mathcal{J}} &= \frac{\beta(\epsilon - 1)}{(g_A - \beta\phi)\vartheta\epsilon} \left(\frac{\nu(\vartheta - 1)(\epsilon - 1)}{\tilde{c}\vartheta\epsilon} \right)^{\frac{1}{\varphi + 1}} \end{split}$$

Otherwise, if $\varphi = 0$:

$$\varsigma = \frac{\tilde{c}\chi\epsilon\vartheta(g_A - \beta\phi)\left(\frac{\nu(\epsilon-1)(\vartheta-1)}{\tilde{c}\epsilon\vartheta}\right)^{-\eta}}{\beta\nu(\epsilon-1)} > 0$$

$$\bar{L}^s = \frac{\beta\nu(\epsilon-1)(g_A - \phi)}{\tilde{c}\chi\epsilon\vartheta(g_A - \beta\phi)}$$

$$\bar{\mathcal{J}} = \frac{\beta(\epsilon-1)}{(g_A - \beta\phi)\vartheta\epsilon}\left(\frac{\nu(\vartheta-1)(\epsilon-1)}{\tilde{c}\vartheta}\right)$$

From (B.11) and (B.12):

$$\bar{R} = r\pi^* = \beta^{-1}g_y$$
$$\bar{P}^m = \frac{\beta}{g_y - \beta\rho}$$

³⁴One alternative is that we solve these equations for $\overline{\mathcal{J}}, \overline{L}^s, \chi$ given ς . That is, we may back out a chosen g_A by adjusting the parameter χ (which affects marginal disutility of skilled labor). Another option is that we calibrate both χ and ς and solve these equations for $\overline{\mathcal{J}}, \overline{L}^s, g_A$.

where $g_y = (g_A)^{\frac{1}{\vartheta-1}}$. We calibrate $\bar{b} = dy(\bar{y})$ where dy is the debt-to-GDP ratio and solve for \tilde{T} that satisfies the government's intertemporal budget constraint (B.9):

$$\bar{\tilde{T}} = \bar{b}\left(\frac{1+\rho\bar{P}^m}{g_y} - \bar{P}^m\right) + \bar{g} = \bar{b}\left(\frac{1-\beta}{g_y - \beta\rho}\right) + \bar{g} > 0.$$

where \bar{g} , \bar{P}^m , g_y are defined in terms of deep structural parameters above. At this stage, it is possible to note that the coefficient multiplying \hat{g}_t in (14) can be expressed as $\bar{g}/(\bar{P}^m\bar{b}) = \tilde{g}(g_y - \beta\rho)/(\beta dy)$. Finally, $\bar{\nu} = \pi^* = 1$ in steady state, and \bar{w} , \bar{w}^s are given by (B.13)-(B.14) after substituting for $\bar{L}, \bar{L}^s, \bar{c}$. Therefore, we can derive the steady state for the stationarized variables: $(\bar{c}, \bar{y}, \bar{g}, \bar{L}, \bar{L}^s, g_A, g_y, \bar{\Pi}, \bar{M}C, \bar{\mathcal{J}}, \bar{b}, \bar{\tilde{T}}, \bar{P}^m, \bar{R}, \pi^*, \bar{w}, \bar{w}^s)$.

B.3 Linearization details

The model is log-linearized at the steady state described above.³⁵ Equation (9) is obtained by log-linearizing (B.1)-(B.3), (B.5)-(B.6), (B.13)-(B.14) and (B.16) and combining the resulting expressions. Equation (10) is obtained from (B.4). Equation (11) is obtained from (B.11), (B.15), and (B.17). Equation (12) is obtained from (B.6)-(B.7), (B.13), (B.15)-(B.19). Equation (13) is obtained from (B.8). Equation (14) is obtained from (B.9)-(B.10) and (B.15). Equation (15) is obtained from (B.12).

B.4 Equilibrium conditions: exogenous technology

The standard model with exogenous technology is characterized by the following, standard system of equations

$$\hat{y}_t = E_t \hat{y}_{t+1} - \tilde{c}(\hat{i}_t - E_t \hat{\pi}_{t+1}) + (1 - \rho_g) \tilde{g} \hat{g}_t$$
(B.20)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t - \kappa \frac{g}{\varphi \tilde{c} + 1} \hat{g}_t$$
(B.21)

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \tag{B.22}$$

$$\hat{b}_t = (\beta^{-1} - \gamma)\hat{b}_{t-1} - \beta^{-1}\hat{\pi}_t + (\frac{\rho}{g_y} - 1)\hat{P}_t^m + \frac{\bar{g}}{\bar{P}^m\bar{b}}\hat{g}_t$$
(B.23)

$$\hat{P}_{t}^{m} = -\hat{i}_{t} + \frac{\beta\rho}{g_{y}} E_{t} \hat{P}_{t+1}^{m}$$
(B.24)

where $g_y = 1$ under exogenous growth $(\vartheta = \infty)$.

³⁵Recall that we assume $\tilde{g} > 0$ and $\bar{b} > 0$.

C. Robustness

This section presents additional figures referenced in the main text.

C.1 Active Monetary and Active Fiscal Policy

Figures C.1 and C.2 compare the effects of high initial public debt under a regime with active monetary and active fiscal policy to the effects of the same initial debt stock when fiscal policy is active but monetary policy is passive. Figure C.1 considers a case in which growth effects are weaker ($\vartheta = 2.5$), whereas Figure C.2 considers a case with stronger growth effects ($\vartheta = 1.3$). All remaining details are the same as in Figure 1.



Figure C.1: Eroding the Public Debt: Active-Active Case ($\vartheta = 2.5$)



Figure C.2: Eroding the Public Debt: Active-Active Case ($\vartheta = 1.3$)

C.2 Role of the intertemporal elasticity of substitution

Figure C.3 shows the dynamics after a government spending shock for the case of stronger growth effects due to smaller ϑ ($\vartheta = 1.3$). All remaining details are the same as in Figure 2.



Figure C.3: Dynamics: Elevated Public Debt ($\vartheta = 1.3$)

C.3 Role of the long-run growth rate

Figure C.4 shows the dynamics under elevated debt levels for different levels of g_y on the BGP. All remaining details are the same as in Figure 1.



Figure C.4: Eroding Public Debt: Role of g_y

C.4 Fiscal Policy and the Maturity Structure of Debt

Figure C.5 shows how the average duration of public debt (determined by ρ) affects the dynamics after a government spending shock. All remaining details are the same as in Figure 2.



Figure C.5: Government Spending Shock: Role of Debt Duration (ρ)

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