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A Real-Business-Cycle model with an informal sector:

Lessons for Bulgaria (1999-2018)

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October 18, 2023

Abstract

We introduce an informal sector into a real-business-cycle setup augmented with a detailed government sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2016). We investigate the quantitative importance of the presence of a grey economy for the cyclical fluctuations in Bulgaria. We find that incorporating an informal sector improves the model fit against data, as compared to the standard RBC model, and thus this sector

is an important ingredient that needs to be considered by researchers interested in

business cycle-, or public finance issues.

**Keywords:** business cycles, informal sector, Bulgaria

JEL Classification Codes: E24, E32

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## 1 Introduction and Motivation

Informal economic activity is a phenomenon that has been detected/observed worldwide, as reported in Schneider and Enste (2013). The grey economy is particularly problematic in developing and emerging countries where the size of informality is much larger relative to the official output. In contrast, the size of the informal sector is relatively small in the developed world, and thus the topic is largely ignored in the literature that focuses on Western countries.

In the developed world, however, the situation is drastically different, as the informal sector is responsible for a substantial share of total employment and income of an economy. As pointed above, most of the models in the modern macroeconomic literature are not directly applicable as - beside many other features - they ignore the existence of that sector. As a result, when researchers use an off-the shelf model developed originally for the US, it would provide a misleading picture about the size of economic fluctuations, and the cyclical movement of labor market variables in particular. More specifically, the grey economy could be serving an important role in the economy, and acting as a safety valve in the economy; the presence of an informal market allows for alternative employment opportunities and income generation, which are particularly in demand during bad times. Therefore, the presence of an informal sector may have important quantitative effects on aggregate allocations over the business cycle, as it introduces a second labor market, which interacts with the official one. With more alternatives for work available, both hours and wages are expected to become more responsive, when it comes to adjuctments in behavior in the face of the shocks hitting the economy. Therefore, a good business-cycle model, calibrated for a developing economy, requires that an informal sector be included in the theoretical framework in order for the setup to capture the observed fluctuations in economic variables better.

We take these conjectures seriously, and incorporate an informal sector in an otherwise standard real-business-cycle (RBC) model with a detailed government sector. We calibrate the model for Bulgaria in the period 1999-2016, as Bulgaria provides a good testing case for the theory. Bulgaria is an EU member state, but as a former transition economy, it is still developing. In addition, Bulgaria is the country with the largest degree of informality in the

We then proceed to quantitatively evaluate the effect of informal sector on business cycle fluctuations, and the implications for public finances. To the best of our knowledge, this is the first study on the issue using modern macroeconomic modelling techniques, and thus an important contribution to the field. We find that incorporating an informal sector improves the model fit against data, as compared to the standard RBC model. In addition, the model predicts that informal activity is countercyclical, as in good times people move out of informality. Therefore, the informal sector is an important ingredient that needs to be considered by researchers interested in business cycle-, or public finance issues.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

# 2 Model Description

There is a representative households which derives utility out of consumption and leisure. The time available to households can be spent working in official sector, the shadow economy, or enjoyed as leisure. The government taxes consumption spending, and levies a common proportional ("flat") tax on all income, in order to finance purchases of government consumption goods, and government transfers. On the production side, there is a representative firm, which hires labor and capital to produce a homogenous final good, which could be used for consumption, investment, or government purchases.

<sup>&</sup>lt;sup>1</sup>Still, the Bulgarian economy underwent a number of significant changes in the last 20 years (1998- 2018), during which period the informal economy size shrank from 37% of official GDP to 31%.

#### 2.1 Households

There is a representative household, which maximizes its expected utility function

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - h_t^o - h_t^u) \right\}$$
 (2.1)

where  $E_0$  denotes household's expectations as of period 0,  $c_t$  denotes household's private consumption in period t,  $h_t^o$  and  $h_t^u$  are hours worked in period t in the official and unofficial sector, respectively,  $0 < \beta < 1$  is the discount factor,  $0 < \gamma < 1$  is the relative weight that the household attaches to leisure.<sup>2</sup>

The household starts with an initial stock of physical capital  $k_0 > 0$ , and has to decide how much to add to it in the form of new investment. The law of motion for physical capital is

$$k_{t+1} = i_t + (1 - \delta)k_t \tag{2.2}$$

and  $0 < \delta < 1$  is the depreciation rate. Next, the real interest rate is  $r_t$ , hence the before-tax capital income of the household in period t equals  $r_t k_t$ . In addition to capital income, the household can generate labor income. Hours supplied to the representative firm are rewarded at the hourly wage rate of  $w_t^o$ , and the imputed wage in the unofficial sector is  $w_t^u$ , so total labor income equals  $w_t^o h_t^o + w_t^u h_t^u$ . Note that the government is only able to tax labor income earned in the official sector. Lastly, the household owns the firm in the economy and has a legal claim on all the firm's profit,  $\pi_t$ .

Next, the household's problem can be now simplified to

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - h_t^o - h_t^u) \right\}$$
 (2.3)

s.t.

$$(1+\tau^c)c_t + k_{t+1} - (1-\delta)k_t = (1-\tau^y)[r_t k_t + w_t^o h_t^o + \pi_t] + w_t^u h_t^u + g_t^t$$
(2.4)

<sup>&</sup>lt;sup>2</sup>This utility function is equivalent to a specification with a separable term containing government consumption, e.g. Baxter and King (1993). Since in this paper we focus on the exogenous (observed) policies, and the household takes government spending as given, the presence of such a term is irrelevant. For the sake of brevity, we skip this term in the utility representation above.

where where  $\tau^c$  is the tax on consumption,  $\tau^y$  is the proportional income tax rate (0 <  $\tau_t^c, \tau^y$  < 1), levied on both labor and capital income, and  $g_t^t$  denotes government transfers. The household takes the two tax rates  $\{\tau_t^c, \tau^y\}_{t=0}^{\infty}$ , government spending categories,  $\{g_t^c, g_t^t\}_{t=0}^{\infty}$ , profit  $\{\pi_t\}_{t=0}^{\infty}$ , the realized technology process  $\{A_t\}_{t=0}^{\infty}$ , prices  $\{w_t^o, w_t^u, r_t\}_{t=0}^{\infty}$ , and chooses  $\{c_t, h_t^o, h_t^u, k_{t+1}\}_{t=0}^{\infty}$  to maximize its utility subject to the budget constraint.<sup>3</sup>

The first-order optimality conditions as as follows:

$$c_t : \frac{1}{c_t} = \lambda_t [1 + \tau^c] \tag{2.5}$$

$$h_t^o : \frac{\gamma}{1 - h_t^o - h_t^u} = \lambda_t (1 - \tau^y) w_t^o$$
 (2.6)

$$h_t^u : \frac{\gamma}{1 - h_t^o - h_t^u} = \lambda_t w_t^u \tag{2.7}$$

$$k_{t+1} : \lambda_t = \beta E_t \lambda_{t+1} [1 + (1 - \tau^y) r_{t+1} - \delta]$$
 (2.8)

$$TVC : \lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0$$
 (2.9)

where  $\lambda_t$  is the Lagrangean multiplier attached to household's budget constraint in period t. The interpretation of the first-order conditions above is as follows: the first one states that for each household, the marginal utility of consumption equals the marginal utility of wealth, corrected for the consumption tax rat. The second and third equations state that when choosing labor supply optimally, at the margin, each hour spent by the household working for the firm or in the unofficial sector should balance the benefit from doing so in terms of additional income generates, and the cost measured in terms of lower utility of leisure. The forth equation is the so-called "Euler condition," which describes how the household chooses to allocate physical capital over time. The last condition is called the "transversality condition" (TVC): it states that at the end of the horizon, the value of physical capital should be zero.

## 2.2 Firm problem

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is Cobb-Douglas and uses

<sup>&</sup>lt;sup>3</sup>Note that by choosing  $k_{t+1}$  the household is implicitly setting investment  $i_t$  optimally.

both physical capital,  $k_t$ , and labor hours,  $h_t^o$ , to maximize static profit

$$\Pi_t = A_t k_t^{\alpha} (h_t^o)^{1-\alpha} - r_t k_t - w_t^o h_t^o, \tag{2.10}$$

where  $A_t$  denotes the level of technology in period t. Since the firm rents the capital from households, the problem of the firm is a sequence of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, i.e.:

$$k_t : \alpha \frac{y_t^o}{k_t} = r_t, \tag{2.11}$$

$$h_t^o: (1-\alpha)\frac{y_t^o}{h_t^o} = w_t^o.$$
 (2.12)

In equilibrium, given that the inputs of production are paid their marginal products,  $\pi_t = 0$ ,  $\forall t$ .

### 2.3 Unofficial sector

The unofficial sector will be modelled as a "home production" technology, which is available to everyone, and uses only labor. This is to reflect the fact that the shadow economy is less capital intensive than the official economy. Unofficial output is produced as follows:

$$y_t^u = B_t(h_t^u)^{1-\alpha}, (2.13)$$

where, due to the lack of data, we will assume the same labor intensity parameter as in the formal economy.

Lastly, with free entry the unofficial wage rate is

$$w_t^u = B_t(h_t^u)^{-\alpha}. (2.14)$$

#### 2.4 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption, in order to finance spending on wasteful government purchases, and government transfers. The government budget constraint is as follows:

$$g_t^c + g_t^t = \tau^c c_t + \tau^y [w_t^o h_t^o + r_t k_t]$$
 (2.15)

Income tax rate and government consumption-to-output ratio would be chosen to match the average share in data. Finally, government transfers would be determined residually in each period so that the government budget is always balanced.

### 2.5 Dynamic Competitive Equilibrium (DCE)

For a given process followed by technology  $\{A_t\}_{t=0}^{\infty}$  tax schedules  $\{\tau^c, \tau^y\}_{t=0}^{\infty}$ , and initial capital stock  $\{k_0\}$ , the decentralized dynamic competitive equilibrium is a list of sequences  $\{c_t, i_t, k_t, h_t^o, h_t^u\}_{t=0}^{\infty}$  for the household, a sequence of government purchases and transfers  $\{g_t^c, g_t^t\}_{t=0}^{\infty}$ , and input prices  $\{w_t^o, w_t^u, r_t\}_{t=0}^{\infty}$  such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) there are no profits in the unofficial sector; (iv) government budget is balanced in each period; (v) all markets clear.

## 3 Data and Model Calibration

To characterize business cycle fluctuations with an endogenous depreciation rate in Bulgaria. we will focus on the period following the introduction of the currency board (1999-2018). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2020), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2020). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, as in Vasilev (2017a), the discount factor,  $\beta = 0.982$ , is set to match the steady-state capital-to-output ratio in Bulgaria, k/y = 13.964, in the steady-state Euler equation. The labor share parameter,  $1 - \alpha = 0.571$ , is obtained as in Vasilev (2017e), and equals the average value of labor income in aggregate output over the period 1999-2016. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Next, the average income tax rate was set to  $\tau^y = 0.1$ . This is the average effective tax rate on income between 1999-2007, when Bulgaria used progressive income taxation, and equal to the proportional income tax rate introduced as of 2008. Similarly, the average tax rate on consumption is set to its value over the period,  $\tau^c = 0.2$ .

Next, the relative weight attached to the utility out of leisure in the household's utility function,  $\gamma$ , is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. The split between official and unofficial hours follows Vasilev (2017d), who found that a third of total hours are spent in the unofficial sector. Next, the steady-state depreciation rate of physical capital in Bulgaria,  $\delta = 0.013$ , was taken from Vasilev (2016a). It was estimated as the average quarterly depreciation rate over the period 1999-2014. Finally, the processes followed by TFP processes and energy prices, are estimated from the detrended series by running an AR(1) regression and saving the residuals. Table 1 below summarizes the values of all model parameters used in the paper.

$T_0$ $h$ $h$	¬ 1•	$M_{\rm C}$	AA1	Par	$\operatorname{ameters}$
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Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
$\alpha$	0.429	Capital Share	Data average
$1 - \alpha$	0.571	Labor Share	Calibrated
$\gamma$	0.873	Relative weight attached to leisure	Calibrated
$\delta$	0.013	Depreciation rate on physical capital	Data average
$ au^y$	0.100	Average tax rate on income	Data average
$ au^c$	0.200	VAT/consumption tax rate	Data average
$ ho_a$	0.701	AR(1) persistence coefficient, TFP process	Estimated
$\sigma_a$	0.044	st. error, TFP process	Estimated

# 4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 below. The steady-state level of output was normalized to unity (hence the level of technology A differs from one, which is usually the normalization done in other studies), which greatly simplified the computations. Next, the model matches consumption-

to-output and government purchases ratios by construction; The investment ratios are also closely approximated, despite the closed-economy assumption and the absence of foreign trade sector. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, where  $\bar{r} = (1-\tau^y)r - \delta$  is also relatively well-captured by the model. Lastly, given the absence of debt, and the fact that transfers were chosen residually to balance the government budget constraint, the result along this dimension is understandably not so close to the average ratio in data.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
$\overline{y}$	Steady-state output	N/A	1.000
c/y	Consumption-to-output ratio	0.648	0.674
i/y	Investment-to-output ratio	0.201	0.175
k/y	Capital-to-output ratio	13.96	13.96
$g^c/y$	Government consumption-to-output ratio	0.151	0.151
$wh^o/y$	Labor income-to-output ratio	0.571	0.571
rk/y	Capital income-to-output ratio	0.429	0.429
h	Share of total time spent working	0.333	0.333
$h^o$	Hours worked in the official sector	0.280	0.280
$h^u$	Hours worked in the unofficial sector	0.083	0.083
$ar{r}$	After-tax net return on capital	0.014	0.016

# 5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total

factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

### 5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response functions (IRFs) are presented in Fig. 1 and on the next page. As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the economy, so used of output - consumption, investment, and government consumption also increase contemporaneously.

At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and capital. The representative households then respond to the incentives contained in prices and start accumulating capital, and supplies more hours worked in the official sector (while decreasing hours worked in the unofficial sector, as the two types of hours are perfect substitutes in the household's utility function). In turn, the increase in capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock. In the labor market, the official wage rate increases, and the household increases its hours worked. In turn, the increase in official hours further increases output, again indirectly. In addition, the model predicts that informal activity is countercyclical, as in good times people move out of informality.

Over time, as capital is being accumulated, its after-tax marginal product starts to decrease, which lowers the households' incentives to save. As a result, physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

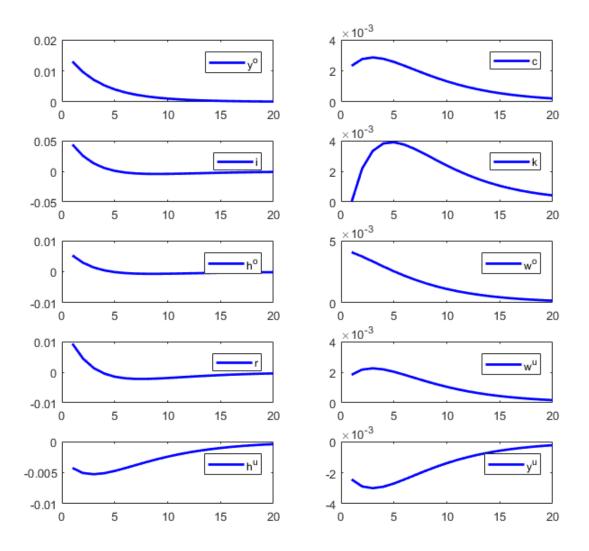


Figure 1: Impulse Responses to a 1% surprise innovation in technology

# 5.2 Simulation and moment-matching

As in Vasilev (2017b), we will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency.<sup>4</sup> To minimize the

<sup>&</sup>lt;sup>4</sup>The model-predicted 95 % confidence intervals are available upon request.

sample error, the simulated moments are averaged out over the computer-generated draws. We compare and contrast the setup to the benchmark RBC model without a grey economy. As in Vasilev (2016, 2017b, 2017c), both models match quite well the absolute volatility of output and investment. By construction, government consumption in the model varies as much as output. In addition, the predicted consumption and investment volatilies are too high. Still, both models are qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output. In addition, in the presence of a grey sector, model-generated consumption volatility is lower, and thus closer to data. This is at the expense of increased investment volatility, which is attributed to the fact that now there are two types of hours, which are perfect substitutes, and capital is a complement to official hours in the production of official output.

Table 3:	Business	$\operatorname{Cvcl}$	e Moments

Table 5: Dusiness Cycle Moments							
	Data	Benchmark RBC Model	Model with a				
		(without a grey sector)	grey sector				
$\sigma_y$	0.05	0.05	0.05				
$\sigma_c/\sigma_y$	0.55	0.82	0.78				
$\sigma_i/\sigma_y$	1.77	2.35	2.64				
$\sigma_g/\sigma_y$	1.21	1.00	1.00				
$\sigma_h/\sigma_y$	0.63	0.28	0.30				
$\sigma_w/\sigma_y$	0.83	0.86	0.82				
corr(c, y)	0.85	0.81	0.91				
corr(i,y)	0.61	0.82	0.80				
corr(g,y)	0.31	1.00	1.00				
corr(h, y)	0.49	0.26	0.67				
corr(w, y)	-0.01	0.96	0.96				
corr(w,h)	0.33	0.36	0.45				
3							

With respect to the labor market variables, the variability of employment predicted by both model is lower than that in data, but slightly higher when an informal sector is present. Next, the variability of wages in both model is close to that in data, but the fit is better in the model with grey economy. Next, in terms of contemporaneous correlations, both model

systematically over-predicts the pro-cyclicality of the main aggregate variables - consumption, investment, and government consumption. This, however, is a common limitation of this class of models. Along the labor market dimension, the contemporaneous correlation of employment with output is too low in the benchmark model, while the setup with a grey economy overstates the correlation. With respect to wages, both models predicts a strong cyclicality, while wages in data are acyclical. This shortcoming is well-known in the literature and an artifact of the wage being equal to the labor productivity in both models.

In the next subsection, as in Vasilev (2016), we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

#### 5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 below against the averaged simulated AFCs and CCFs.<sup>5</sup>

As seen from Table 4 above, the model compares relatively well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model. The persistence of labor market variables are also relatively well-described by the model dynamics. Overall, the model with informal sector generates too much persistence in output and both employment and unemployment, and is subject to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996b), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process. In those models, e.g. Vasilev (2009), and in the current one, labor market is modelled in the Walrasian market-clearing spirit, and output and unemployment persistence is low.

<sup>&</sup>lt;sup>5</sup>Following Canova (2007), this is used as a goodness-of-fit measure.

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(u_t, u_{t-k})$	1.000	0.765	0.552	0.553
Model	$corr(u_t, u_{t-k})$	1.000	0.948	0.884	0.808
	(s.e.)	(0.000)	(0.031)	(0.058)	(0.085)
Data	$corr(n_t, n_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(n_t, n_{t-k})$	1.000	0.948	0.884	0.808
	(s.e.)	(0.000)	(0.031)	(0.058)	(0.085)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.955	0.901	0.838
	(s.e.)	(0.000)	(0.026)	(0.051)	(0.075)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.955	0.901	0.837
	(s.e.)	(0.000)	(0.027)	(0.051)	(0.075)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.958	0.907	0.849
	(s.e.)	(0.000)	(0.025)	(0.049)	(0.071)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.949	0.886	0.812
	(s.e.)	(0.000)	(0.030)	(0.057)	(0.083)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.957	0.906	0.847
	(s.e.)	(0.000)	(0.025)	(0.049)	(0.072)

Next, as seen from Table 5 below, over the business cycle, in data labor productivity leads employment. The model, however, cannot account for this fact. As in the standard RBC model a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one.

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(n_t, (y/n)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(n_t, (y/n)_{t-k})$	0.021	0.022	0.023	0.451	0.020	-0.001	-0.015
	(s.e.)	(0.322)	(0.279)	(0.229)	(0.248)	(0.194)	(0.235)	(0.270)
Data	$corr(n_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(n_t, w_{t-k})$	0.021	0.022	0.023	0.451	0.020	-0.001	-0.015
	(s.e.)	(0.322)	(0.279)	(0.229)	(0.248)	(0.194)	(0.235)	(0.270)

# 6 Conclusions

We introduce an informal sector into a real-business-cycle setup augmented with a detailed public finance side. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2018). We investigate the quantitative importance of the presence of a grey economy for the cyclical fluctuations in Bulgaria. We find that incorporating an informal sector improves the model fit against data, as compared to the standard RBC model, and thus this sector is an important ingredient that needs to be considered by researchers interested in business cycle-, or public finance issues.

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