

A Service of

ZBU

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Honkapohja, Seppo; McClung, Nigel

## Working Paper Average inflation targeting in Calvo model with imperfect knowledge and learning

Bank of Finland Research Discussion Papers, No. 13/2024

**Provided in Cooperation with:** Bank of Finland, Helsinki

*Suggested Citation:* Honkapohja, Seppo; McClung, Nigel (2024) : Average inflation targeting in Calvo model with imperfect knowledge and learning, Bank of Finland Research Discussion Papers, No. 13/2024, Bank of Finland, Helsinki, https://nbn-resolving.de/urn:nbn:fi-fe2024111894883

This Version is available at: https://hdl.handle.net/10419/306369

#### Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

#### Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



# WWW.ECONSTOR.EU

Bank of Finland Research Discussion Papers 13 • 2024

Seppo Honkapohja – Nigel McClung

Average inflation targeting in Calvo model with imperfect knowledge and learning



Bank of Finland Research Bank of Finland Research Discussion Papers Editor-in-Chief Esa Jokivuolle

Bank of Finland Research Discussion Papers 13/2024 18 November 2024

Seppo Honkapohja – Nigel McClung: Average inflation targeting in Calvo model with imperfect knowledge and learning

ISSN 1456-6184, online

Bank of Finland Research Unit

PO Box 160 FIN-00101 Helsinki

Phone: +358 9 1831

Email: research@bof.fi Website: <u>www.suomenpankki.fi/en/research/research-unit/</u>

The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the Bank of Finland.

# Average Inflation Targeting in Calvo Model with Imperfect Knowledge and Learning

Seppo Honkapohja Aalto University School of Business Nigel McClung Bank of Finland

November 17, 2024

#### Abstract

Properties of average inflation targeting under imperfect knowledge and learning have been studied only for the Rotemberg NK model, where price stickiness arises from adjustment costs in price setting. This note fills the gap by studying average inflation targeting in the NK model with Calvo price stickiness. It is shown that in this setting the two models have the same basic properties, unless the central bank aggressively stabilizes output instead of average inflation.

### 1 Introduction

The US Federal Reserve Board reformed its monetary policy strategy in August 2020 by changing the inflation targeting framework to average inflation targeting (AIT) to have better response to developments leading to the zero lower bound (ZLB) for policy interest rates. The reformed monetary policy adopted an opaque AIT strategy. (Powell (2020)) clarified at the 2020 Jackson Hole Symposium that the Federal Reserve would not be tying their hands "to a particular mathematical formula that defines the average" and that their "approach could be viewed as a flexible form of average inflation targeting". The AIT regime is currently maintained.

The research literature about AIT mostly relies on the rational expectations (RE) hypothesis despite the opacity of the Fed framework.<sup>1</sup> Honkapohja and McClung (2024) introduce the realistic assumption of imperfect knowledge about the interest rate policy of an average inflation targeting central bank. If agents do not know the RE equilibrium, they must engage in learning about the dynamics of the variables that must be forecasted as part of agents' intertemporal decision-making. A fundamental issue is whether the learning dynamics converge or fail to converge to the inflation target RE equilibrium.<sup>2</sup>

Honkapohja and McClung (2024) develop their analysis using the assumption that there are adjustment costs in price setting due to Rotemberg (1982). The Rotemberg model is one standard New Keynesian (NK) model with price stickiness. Opaque AIT can result in non-robust performance in the sense that learning behavior does necessarily not converge to an intertemporal equilibrium even under standard assumptions about the interest rate rule.

 $<sup>^{1}</sup>$ See Honkapohja and McClung (2024) for discussion and references of the literature.

 $<sup>^{2}</sup>$ For general discussions of adaptive learning, see e.g. Evans and Honkapohja (2001), Evans and Honkapohja (2009) and Evans and McGough (2020).

The Calvo model, developed in Calvo (1983), is probably the most widely used NK framework for modelling price stickiness.<sup>3</sup> This raises the question of whether, under AIT policy, the properties of Calvo and Rotemberg models under learning are similar. It is shown that the dynamics of learning with AIT monetary policy are indeed very similar. However, in cases where the central bank aggressively stabilizes output instead of average inflation, stability concerns are mitigated under the Calvo pricing friction but not under Rotemberg.

#### 2 The Calvo model with Imperfect Knowledge

We employ a standard New Keynesian model with Calvo price stickiness and imperfect knowledge, see Preston (2005) and Preston (2008). The two behavioral equations, linearized at the target steady state, are:<sup>4</sup>

(i) the aggregate demand curve

$$\hat{y}_t = -\sigma \hat{i}_t + \hat{E}_t \sum_{i=0}^{\infty} \beta^i [(1-\beta)\hat{y}_{t+1+i} - \sigma(\beta \hat{i}_{t+1+i} - \hat{\pi}_{t+1+i})] + r_t^n,$$

where  $r_t^n$  is a demand shock. (ii) the Phillips curve

$$\hat{\pi}_t = \kappa \hat{y}_t + \hat{E}_t \sum_{i=0}^{\infty} (\alpha \beta)^i [\kappa \alpha \beta \hat{y}_{t+1+i} + (1-\alpha) \beta \hat{\pi}_{t+i+1}].$$

Here hat denotes the proportional deviation of the variable from its value at the target steady state. Thus, for example  $\hat{y}_t = (y_t - y^*)/y^*$  is aggregate demand,  $\hat{\pi}_t$  is inflation and  $\hat{\imath}_t$  is nominal interest rate for the target.  $\beta$  is the subjective discount rate,  $\sigma$  is the intertemporal elasticity of substitution in utility function,  $\kappa$  is a parameter indicating degree of price stickiness and  $\alpha$  is the fraction of firms which cannot change the price in a period. Note that  $\kappa \to \infty$  if  $\alpha \to 0$ .

In the model there is also

(iii) the linearized AIT interest rate rule<sup>5</sup>

$$\hat{i}_t = \bar{\psi}_p \sum_{k=0}^{L-1} \hat{\pi}_{t-k} + \bar{\psi}_y \hat{y}_t.$$

The denominators  $\pi^*$  and  $y^*$  are incorporated into  $\psi_p$  and  $\psi_y$ , so  $\bar{\psi}_p = \tilde{\psi}_p/\pi^*$  and  $\bar{\psi}_y = \psi_y/y^*$ . ( $\pi^*$  and  $y^*$  denote the inflation and output levels at the target steady state.)

<sup>&</sup>lt;sup>3</sup>For example, see the treatises by Gali (2008) and Woodford (2003).

<sup>&</sup>lt;sup>4</sup>See Preston (2005), equations (18) and (19).

<sup>&</sup>lt;sup>5</sup>Note that  $\hat{i}_t = (i_t - i^*)/i^* = \hat{R}_t(1 + i^*)/i^*$  as R = 1 + i. So  $\tilde{\psi}_p = \psi_p(i^*/(1 + i^*))$ , where  $\psi_p$  is the policy rule parameter in the undeviated model.

Let  $x_t = (\hat{y}_t, \hat{\pi}_t, \hat{\imath}_t)^T$ . The economic system can be written as

$$\begin{pmatrix} 1 & 0 & \sigma \\ -\kappa & 1 & 0 \\ -\bar{\psi}_y & -\bar{\psi}_p & 1 \end{pmatrix} x_t = \sum_{k=1}^{L-1} \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \bar{\psi}_p & 0 \end{pmatrix} x_{t-k} \right] + \begin{pmatrix} r_t^n \\ 0 \\ 0 \end{pmatrix} + \qquad (1)$$
$$+ \sum_{i=0}^{\infty} \beta^i \begin{pmatrix} (1-\beta) & \sigma & -\sigma\beta \\ 0 & 0 & 0 \\ 0 \end{pmatrix} \hat{E}_t x_{t+1+i}$$
$$+ \sum_{i=0}^{\infty} (\alpha\beta)^i \begin{pmatrix} 0 & 0 & 0 \\ \kappa\alpha\beta & (1-\alpha)\beta & 0 \\ 0 & 0 & 0 \end{pmatrix} \hat{E}_t x_{t+1+i}.$$

The model can be written in the form

$$x_{t} = M_{1} \sum_{i=0}^{\infty} \beta^{i} \hat{E}_{t} x_{t+1+i} + M_{2} \sum_{i=0}^{\infty} (\alpha \beta)^{i} \hat{E}_{t} x_{t+1+i} +$$

$$\sum_{k=1}^{L-1} [N_{k} x_{t-k}] + (r_{t+i}^{n} \ 0 \ 0 )^{T},$$
(2)

where T denotes matrix transpose.  $M_1$ ,  $M_2$  and  $N_k$  (k = 1, ..., L - 1) are  $3 \times 3$  matrices

$$M_{1} = S \begin{pmatrix} (1-\beta) & \sigma & -\sigma\beta \\ \kappa (1-\beta) & \kappa\sigma & -\kappa\sigma\beta \\ (\bar{\psi}_{y} + \kappa\bar{\psi}_{p}) (1-\beta) & \sigma(\bar{\psi}_{y} + \kappa\bar{\psi}_{p}) & -\sigma\beta(\bar{\psi}_{y} + \kappa\bar{\psi}_{p}) \end{pmatrix},$$
$$M_{2} = S \begin{pmatrix} -\alpha\kappa\sigma\beta\bar{\psi}_{p} & -\sigma\beta\bar{\psi}_{p} (1-\alpha) & 0 \\ \alpha\kappa\beta(\sigma\bar{\psi}_{y} + 1) & \beta(\sigma\bar{\psi}_{y} + 1) (1-\alpha) & 0 \\ \alpha\kappa\beta\bar{\psi}_{p} & \beta\bar{\psi}_{p} (1-\alpha) & 0 \end{pmatrix}$$

and

$$N_k = S \begin{pmatrix} 0 & -\sigma \bar{\psi}_p & 0\\ 0 & -\kappa \sigma \bar{\psi}_p & 0\\ 0 & \bar{\psi}_p & 0 \end{pmatrix}, \text{ where } S = (\sigma \bar{\psi}_y + \kappa \sigma \bar{\psi}_p + 1)^{-1}$$

Assume for simplicity that  $r_{t+i}^n$  is an exogenous and stationary *iid* process:  $r_t^n = \varepsilon_{s,t}$ .

It is useful to stack the system (2) into a first order form. We write

$$X_{t} = \hat{M}_{1} \sum_{i=0}^{\infty} \beta^{i} \hat{E}_{t} X_{t+1+i} + \hat{M}_{2} \sum_{i=0}^{\infty} (\alpha \beta)^{i} \hat{E}_{t} X_{t+1+i} + \hat{N} X_{t-1} + (r_{t+i}^{n} \ 0 \ \dots \ 0)^{T}.$$
(3)

Here  $\hat{M}_i$  are zero matrices of  $3 \times 3$  blocks, except that the (1, 1) block given by with  $M_i$  for i = 1, 2, and  $\hat{N}$  is blockwise  $(L - 1) \times (L - 1)$  and has blocks  $N_i = N, i = 1, ..., L - 1$  in its first row and the remaining blocks form  $(L - 1) \times (L - 1)$  identity matrix.

The expectations  $\hat{E}_t x_{t+1+i}$  are determined by an adaptive learning process. Agents have a forecasting model, called perceived law of motion (PLM). If there is transparency about the functional form of the AIT rule, the PLM is

$$x_t = A + \sum_{k=1}^{L-1} C_k x_{t-k} + \varepsilon_{s,t}, \qquad (4)$$

where  $\varepsilon_t$  is a disturbance term. Write the PLM in first order form

$$\begin{pmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-(L-1)} \end{pmatrix} = \begin{pmatrix} A \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} C_1 & 0 & \cdots & C_{L-1} \\ I & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & I \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-L} \end{pmatrix} + \begin{pmatrix} \varepsilon_{s,t} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$
(5)

and, with new notation, the system (5) is written as

$$X_t = \Delta + \Theta X_{t-1} + \Xi_t$$

where  $\Delta = (A^T, 0..., 0)^T$  and  $\Xi_t = (\varepsilon_{s,t}^T, 0, ..., 0)^T$ . We get

$$\hat{E}_{t}(X_{t+1+i}) = (\sum_{k=0}^{i+1} \Theta^{k})\Delta + \Theta^{i+2}X_{t-1} = (I - \Theta)^{-1}(I - \Theta^{i+2})\Delta + \Theta^{i+2}X_{t-1}$$

Here  $\Delta$  is 3L dimensional vector and  $\Theta$  is  $3L \times 3L$  matrix. The summation terms in (3) are

$$F_{\beta} = \sum_{i=0}^{\infty} \beta^{i} \hat{E}_{t} X_{t+1+i} = \sum_{i=0}^{\infty} \beta^{i} \hat{E}_{t} [(I - \Theta)^{-1} (I - \Theta^{i+2}) \Delta + \Theta^{i+2} X_{t-1}]$$
  
=  $(I - \Theta)^{-1} (I(1 - \beta)^{-1} - (I - \beta\Theta)^{-1}\Theta^{2}) \Delta + (1 - \beta\Theta)^{-1}\Theta^{2} X_{t-1}$ 

and

$$F_{\alpha} = \sum_{i=0}^{\infty} (\alpha \beta)^{i} \hat{E}_{t} X_{t+1+i} = (I - \Theta)^{-1} \left( I (1 - \alpha \beta)^{-1} - (I - \alpha \beta \Theta)^{-1} \Theta^{2} \right) \Delta + (1 - \alpha \beta \Theta)^{-1} \Theta^{2} X_{t-1}.$$

This system will be used further in Proposition 2 below.

## **3** Rational Expectations Equilibrium

A policymaker would not select a policy which does not, in turn, select a unique equilibrium. Consequently, we begin by assessing conditions for the uniqueness of rational expectations equilibrium (REE). In the case of flexible prices, a version of the Taylor Principle is necessary and sufficient for existence of a unique equilibrium.<sup>6</sup>

<sup>&</sup>lt;sup>6</sup>The Rotemberg and Calvo formulations imply identical log-linearized equilibrium conditions under rational expectations and the conditions for determinacy discussed in this section apply to either model. We note that the condition  $L\bar{\psi}_p > 1$  is only generically necessary and sufficient condition, as a finite number of non-generic values of  $\bar{\psi}_p$  are omitted in the proof of Proposition 1. For comparison to earlier work, this analysis studies a log-linear approximation of the model around the inflation target steady state, whereas Honkapohja and McClung (2024) study a system that is linearized (first-order Taylor series approximation) around the target steady state, and therefore the relevant determinacy condition differs slightly between the two frameworks.

**Proposition 1** Consider AIT policy and suppose fully flexible prices. A unique bounded REE exists if and only if  $L\bar{\psi}_p > 1$ .

The proof of the proposition is in the appendix. For the case of sticky prices, numerical analysis suggests the following condition for determinacy:

Assumption 1.  $L\bar{\psi}_p > 1 - (1 - \beta)(\kappa)^{-1}\bar{\psi}_y$ .

In what follows, we maintain Assumption 1.

#### 4 E-stability of REE

Under Assumption 1, a unique REE exists and the law-of-motion for endogenous variables can be written in the minimum state variable (MSV) form:

$$x_t = \sum_{k=1}^{L-1} \Omega_k x_{t-1} + \Gamma r_t^n.$$

Agents can, in principle, learn the REE law-of-motion, and therefore learn to forecast rationally, by estimating the transparency PLM (4) recursively (i.e., if  $A \to 0$  and  $C_k \to \Omega_k$ asymptotically). We remark that the MSV solution is E-stable for standard calibrations of the model, just as the MSV solution of the corresponding Rotemberg model studied in Honkapohja and McClung (2024) is E-stable (provided that L is not too large).

However, as pointed out in the introduction, the Federal Reserve has not communicated the window length, L, and consequently there are questions about whether agents would know L and include the correct number of lags in their PLM when learning to forecast. Hence, just as in Honkapohja and McClung (2024), we are chiefly interested in the case of "opacity" in which agents exclude lags of inflation from their forecasting models due to ignorance of various aspects of the policy framework.

#### 5 The Case of Full Opacity

We consider the case of full opacity, in which private agents continue to do steady state learning as they do not introduce lagged inflation variables into their forecasting model.<sup>7</sup> Full opacity is the extreme case of insufficient lags in the PLM, because the central bank has not provided any information about the formula for computing AIT. The PLM is

$$\begin{pmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-(L-1)} \end{pmatrix} = \begin{pmatrix} A \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{pmatrix} \text{ or }$$
$$X_t = \Delta + \Xi_t.$$

Then

$$\hat{E}_t X_{t+1+i} = \hat{E}_t X_t$$
 for all  $i = 0, ..., \infty$ 

 $<sup>^7 \</sup>rm Under$  usual inflation targeting policy, the steady state learning setup above has the correct functional form in the standard NK model.

and (2) becomes

$$X_{t} = \hat{M}_{1} \sum_{i=0}^{\infty} \beta^{i} \Delta + \hat{M}_{2} \sum_{i=0}^{\infty} (\alpha \beta)^{i} \Delta + \hat{N} X_{t-1} + \hat{r}_{t}^{n}$$
  
=  $\hat{M}(\hat{E}_{t} X_{t}) + \hat{N} X_{t-1} + \hat{r}_{t}^{n},$ 

where

$$\hat{M} = (1 - \beta)^{-1} \hat{M}_1 + (1 - \alpha \beta)^{-1} \hat{M}_2.$$

Also recall

$$N_k = \hat{N} = \psi_p S \begin{pmatrix} 0 & -\sigma & 0 \\ 0 & -\kappa\sigma & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Let  $\hat{E}_t X_t = X_t^e$  and introduce steady state learning (with constant gain)

$$X_t^e = (1 - \omega) X_{t-1}^e + \omega X_{t-1}.$$
 (6)

State dynamics becomes

$$X_{t} = \hat{M}X_{t}^{e} + \hat{N}X_{t-1} + \hat{r}_{t}^{n}$$

$$(1-\omega)\hat{M}X_{t-1}^{e} + (\omega\hat{M} + \hat{N})X_{t-1} + \sum_{k=2}^{L-1} \left[\hat{N}X_{t-k}\right].$$

$$(7)$$

and note  $\hat{N}_k$  are identical, so  $\hat{N}_k = \hat{N}$  and the first and third columns of  $\hat{N}$  are zero.

**Proposition 2** Consider AIT policy and full opacity. The target REE is locally stable under learning with full opacity for  $\kappa > 0$  and  $\omega > 0$  sufficiently small.

The proof is outlined in the Appendix. We remark that the Calvo model under AIT is formally identical to the general form of the Rotemberg model, but the coefficient matrices  $\hat{M}$  and  $\hat{N}$  are different from the corresponding matrices of the Rotemberg model. When  $\alpha \to 0$  (i.e., full price flexibility) the Calvo model is identical to the Rotemberg model with zero price adjustment costs and the outcome is unstable.<sup>8</sup> This implies:

**Proposition 3** In the flexible price limit of the Calvo model the target REE is unstable under learning with full opacity.

As in Honkapohja and McClung (2024), the fact that stability obtains with sticky prices, but not flexible prices, begs an important question: will an opaque AIT framework anchor expectations for reasonable calibrations of the learning model? In empirical models adaptive learning is usually studied using constant gain algorithms. Then the magnitude of the gain parameter becomes an important issue and a reasonable range of values for the gain is something like [0.002, 0.04]. The calibration of model parameters is as follows:  $\beta = 0.99$ ,  $\sigma = 1, \kappa = \alpha^{-1}(1-\alpha)(1-\alpha\beta)$ . We consider different values of  $\alpha$  to illustrate the importance

<sup>&</sup>lt;sup>8</sup>Proposition 2(ii) of Honkapohja and McClung (2024) shows that in the Rotemberg model the flexible price limit is not stable under full opacity. The result holds in the flexible price limit of the log-linearized Calvo framework as well.

of price rigidity for stability, as in Honkapohja and McClung (2024). Honkapohja and McClung (2024) show that the target REE in the Rotemberg model is not robustly stable as convergence takes place only with gain values below 0.01, i.e. the speed of learning is very small.

In the next table  $\omega_0$  is maximal value of the gain parameter such that learning diverges for all  $\omega > \omega_0$ . The results for the Calvo model are:<sup>9</sup>

α	0.66	0.75	0.9
$\omega_0 (IT)$	0.13618	0.15681	0.17895
$\omega_0 \ (PLT)$	0.03467	0.20753	0.04287
$\omega_0 (AIT \text{ with } L = 6)$	0.01158	0.01938	0.10152
$\omega_0 (AIT \text{ with } L = 20)$	0.00166	0.00265	0.01044
$\omega_0 (AIT \text{ with } L = 32)$	0.00073	0.00121	0.00439

Table 1: Least upper bounds  $\omega_0$  when  $\bar{\psi}_p = 1.5$ ,  $\bar{\psi}_y = 0.125$ 

Thus, in the Calvo model with AIT policy rule convergence of learning is robustly stable when there is significant price stickiness ( $\alpha$  is large) and L is not very high. However, just as in the case of Rotemberg price stickiness, there are stability concerns when prices are less rigid or L is high.<sup>10</sup>

While the stability outcomes under opacity are similar under Rotemberg and Calvo models for standard calibrations of the policy rule, it turns out that the central bank can achieve more robust stability under Calvo pricing by targeting output aggressively. Table 2 shows the robust stability results when  $\bar{\psi}_p = 1.5/L$  and  $\bar{\psi}_y = 1$ . In this case, the REE is more robustly stable for different values of L and  $\alpha$  in the Calvo model. In the Rotemberg case, however, stability concerns remain for higher values of L (details available on request).

Table 2: Least upper bounds for  $\bar{\psi}_{\pi} = 1.5/L, \ \bar{\psi}_{y} = 1$ 

α	0.66	0.75	0.9
$\omega_0 (AIT \text{ with } L = 6)$	0.04797	0.04341	0.04039
$\omega_0 (AIT \text{ with } L = 20)$	0.04797	0.04341	0.04039
$\omega_0 \ (AIT \text{ with } L = 32)$	0.04797	0.04341	0.04039

#### 6 Concluding Remarks

Various further properties of AIT policy in the Rotemberg model are studied in Honkapohja and McClung (2024). For example, the convergence results remain unchanged if agents introduce some, but insufficient number of lagged inflation variables into their PLM. However, if agents introduce more than L - 1 inflation lags to the PLM, then they can learn the value of L and the economy can converge to the MSV REE. We conjecture that these results would be similar for the Calvo model with AIT.

Honkapohja and McClung (2024) also consider variations of the AIT rule, including cases of exponentially declining weights in the AIT rule and of asymmetric AIT rules for achieving E-stability and improving performance of the economy under ZLB. The latter issues have not been considered in the linearized Calvo model. Study of the Calvo model is a very relevant open issue for situations where nonlinearities due to the ZLB prevail.

 $<sup>{}^9</sup>IT$  refers to inflation targeting, i.e. L = 1 and PLT to price level targeting.

<sup>&</sup>lt;sup>10</sup>See Table 1 in Honkapohja and McClung (2024) for analogous results under Rotemberg model.

### A Appendix

**Proof of Proposition 1**: The model under flexible prices, AIT and RE is given by:

$$E_t \pi_{t+1} = \bar{\psi}_p \sum_{k=0}^{L-1} \hat{\pi}_{t-k}$$

which is obtained by combining the Euler equation and interest rate rule. Define  $z_t := (\hat{\pi}_t, \ldots, \hat{\pi}_{t-L+1})^T$ . The system can be expressed in the first-order form:  $E_t z_{t+1} = Q_{RE} z_t$ . The model admits a unique bounded REE if and only if one eigenvalue of  $Q_{RE}$  is outside the unit circle and all the remaining eigenvalues are inside the unit circle.<sup>11</sup> The roots of  $Q_{RE}$  solve the characteristic polynomial:

$$R(\lambda) = \lambda^L - \bar{\psi}_p \sum_{k=0}^{L-1} \lambda^k$$

Define  $J(k) := 1 + k(\bar{\psi}_p)^2/((k-1)\bar{\psi}_p - 1)$  for  $k = 1, \ldots, L$ . Following the Jury stability criterion (e.g., see Honkapohja and McClung (2024)), one root of  $Q_{RE}$  is outside the unit circle and the rest are inside the unit circle if and only if J(l) < 0 for some  $l \in \{1, \ldots, L\}$  and J(m) > 0 for all  $m \in \{1, \ldots, L\}$  such that  $m \neq l$ . Clearly, this condition holds if and only if  $L\bar{\psi}_p > 1.\blacksquare^{12}$ 

**Proof of Proposition 2**: We write (6) and (7) as the first order system.

$$Z_{t} = QZ_{t-1}, \text{ where } Z_{t} = \left( \begin{array}{cccc} (X_{t}^{e})^{T} & X_{t}^{T} & X_{t-1}^{T} & X_{t-2}^{T} & \cdots & X_{t-(L-2)}^{T} \end{array} \right)^{T} \text{ and } \qquad (8)$$
$$Q = \left( \begin{array}{ccccc} (1-\omega)I_{3} & \omega I_{3} & 0 & \cdots & 0 & 0\\ (1-\omega)M & \omega M + N & N & \cdots & N & N\\ 0 & I_{3} & 0 & \cdots & 0 & 0\\ 0 & 0 & I_{3} & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & I_{3} & 0 \end{array} \right).$$

One can now follow the technique in the proof of Proposition 2(i) in Honkapohja and McClung (2024), but with the coefficient matrices M and N for Calvo model. For stability the roots of  $P(\lambda) = Det[Q - \lambda I_{3L}]$  must be inside the unit circle. It can be shown that  $P(\lambda) = \lambda^{2L-2}(1 + \omega - \lambda)\tilde{P}(\lambda)$ . Taking the limit  $\omega \to 0$ , we have

$$\tilde{P}(\lambda) = (1-\lambda)^2 \left( \lambda^{L-1} + h \sum_{k=0}^{L-2} \lambda^k \right), \text{ where } h = \kappa \sigma \bar{\psi}_p / (\sigma \bar{\psi}_y + \kappa \sigma \bar{\psi}_p + 1).$$

It is seen that  $h \in (0, 1)$ . We apply the stability criterion of Jury (1961): the roots of  $\tilde{P}(\lambda)$  are inside the unit circle if and only if

$$kh^2/(1+(k-1)h) < 1$$
, for  $k = 1, ..., L$ .

The criterion is true for all L. Thus the roots of  $P(\lambda)$  are inside the unit circle if the derivative of the non-zero eigenvalues  $\partial \lambda / \partial \omega < 0$  at  $\omega = 0$  and  $\lambda = 1$ . The latter can be established using the technique in the proof of Proposition 1(i) in Honkapohja and McClung (2024).

<sup>&</sup>lt;sup>11</sup>Following standard practice, we disregard non-generic cases in which one or more roots of  $Q_{RE}$  lie on the unit circle.

<sup>&</sup>lt;sup>12</sup>We tacitly exclude the following values of  $\bar{\psi}_p$ , which form a discrete set:  $\{1, 1/2, \ldots, 1/(L-1), 1/L\}$ . Hence, Proposition 1 holds only generically for  $L\bar{\psi}_p > 1$ .

### References

- Calvo, Guillermo A., "Staggered Pricing in a Utility-Maximizing Framework," Journal of Monetary Economics, 1983, 12, 383–398.
- City, F.R.B. Kansas, Navigating the Decade Ahead: Implications for Monetary Policy, Kansas City: Federal Reserve Bank of Kansas City, 2020.
- Evans, George W. and Bruce McGough, "Adaptive Learning in Macroeconomics," Oxford Research Encyclopedia of Economics and Finance, 2020, https://doi.org/10.1093/acrefore/9780190625979.013.508.
- **\_\_\_\_** and Seppo Honkapohja, *Learning and Expectations in Macroeconomics*, Princeton, New Jersey: Princeton University Press, 2001.
- **and** \_\_\_\_\_, "Learning and Macroeconomics," *Annual Review of Economics*, 2009, 1, 421–451.
- Gali, Jordi, Monetary Policy, Inflation, and the Business Cycle, Princeton New Jersey: Princeton University Press, 2008.
- Honkapohja, Seppo and Nigel McClung, "On Robustness of Average Inflation Targeting," Working Paper 6/2021, Bank of Finland 2024.
- Jury, E.I., "A Simplified Stability Criterion for Linear Discrete Systems," Proceedings of the IRE, 1961, 49, 1493–1500.
- Powell, Jerome H., "New Economic Challenges and the Fed Monetary Policy Review,"
- **Preston, Bruce**, "Learning about Monetary Policy Rules when Long-Horizon Expectations Matter," *International Journal of Central Banking*, 2005, 1, 81–126.
- \_\_\_\_\_, "Adaptive Learning and the Use of Forecasts in Monetary Policy," Journal of Economic Dynamics and Control, 2008, 32, 3661–3681.
- Rotemberg, Julio J., "Sticky Prices in the United States," Journal of Political Economy, 1982, 90, 1187–1211.
- Woodford, Michael, Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton, NJ: Princeton University Press, 2003.

## Bank of Finland Research Discussion Papers 2024

ISSN 1456-6184, online

1/2024	Gene Ambrocio – Iftekhar Hasan – Xiang Li Global political ties and the global financial cycle
2/2024	Michaela Elfsbacka-Schmöller – Nigel McClung Price stability and debt sustainability under endogenous trend growth
3/2024	Diego Moreno – Tuomas Takalo Stress test precision and bank competition
4/2024	Eleonora Granziera – Pirkka Jalasjoki – Maritta Paloviita The bias of the ECB inflation projections: a state-dependent analysis
5/2024	Maria Eskelinen – Christopher G. Gibbs – Nigel McClung <b>Resolving new Keynesian puzzles</b>
6/2024	Markus Haavio – Joni Heikkinen – Pirkka Jalasjoki – Juha Kilponen – Maritta Paloviita – Ilona Vänni Reading between the lines - Uncovering asymmetry in the central bank loss function
7/2024	Aleksi Paavola – Ville Voutilainen Central bank collateral policy and credit pricing: evidence from Finland
8/2024	Joni Heikkinen – Kari Heimonen Media tone: the role of news and social media on heterogeneous inflation expectations
9/2024	Dominik Hecker – Hun Jang – Margarita Rubio – Fabio Verona Robust design of countercyclical capital buffer rules
10/2024	Gonçalo Faria – Fabio Verona Unlocking predictive potential: the frequency-domain approach to equity premium forecasting
11/2024	Vito Cormun – Kim Ristolainen Exchange rate narratives
12/2024	Gene Ambrocio – Markus Haavio – Nigel McClung Monetary policy announcements and sacrifice ratios
13/2024	Seppo Honkapohja – Nigel McClung Average inflation targeting in Calvo model with imperfect knowledge and learning