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## Migration-regime liberalization and social security : political-economy effect

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# Migration-Regime Liberalization and Social Security: Political-Economy Effect

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JEL Code: F22, H55, J11, P16.

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# Migration-Regime Liberalization and Social Security: Political-Economy Effect

Assaf Razin\* and Edith Sand†

May 21, 2009

## Abstract

The pay-as-you-go social security system, which suffers from dwindling labor force, can benefit from immigrants with birth rates that exceed the native-born birth rates in the host country. Thus, a social security system provides effectively an incentive to liberalize migration policy. The paper examines a political- economy, inter-generational, mechanism through which the social security system influences voter attitudes in favor of more liberal immigration regime. We demonstrate that the Markov equilibrium, with social security, consists of more liberal migration policies, than the corresponding Markov equilibrium with no social security.

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## 1 Introduction

All over the world, the combination of declining birth rates, and rising life expectancy, presents major fiscal challenge to the social security system. From an economic perspective, a rise in the dependency ratio (i.e., the proportion of retirees per worker) increases the number of people drawing from the system, while it decreases the number of contributors to the system. Because Immigrants typically have higher birth rates than the native-born population of the host countries, immigration may help pay PAYG social security system. That is, the inflow of immigrants can help alleviate the current demographic imbalance by influencing the age structure of the host economy, in a way which strengthens the social security system. We therefore expect that countries with more

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comprehensive and Beveridgian-type social security systems will be more liberal in their migration policies. This paper analyzes a political-economy mechanism through which social security systems influence the degree of liberalization of the immigration policy.

Our analysis of the dynamic interactions between the political and economic decisions is conducted in an analytical framework, developed by Krussel and Rios-Rull (1996) and Krusell, Quadrini, and Rios-Rull (1997). Our paper also follows Forni (2006), who provides a neat analysis of Markov sub-game perfect equilibrium of pay-as-you-go social security system in an overlapping generations model with capital accumulation.

Earlier literature on the political economics of immigration includes Benhabib and Jovanovitch (1996), Scholten and Thum (1996), and Ortega (2005). The present paper draws heavily on Sand and Razin (2007), who analyze a political-economy equilibrium model, in which both migration and taxes interact, focusing on the intergenerational aspects of the social security and migration regimes.

The paper is organized as follows. Section 2 develops the analytical framework. Section 3 presents the political economy equilibrium. In Section 4 we characterize the equilibrium with a social security system. In section 5 we characterize the equilibrium with no social security system. Section 6 concludes.

## 2 Analytical Framework

The economy is populated by overlapping generations of representative individuals, who live for two periods. The tax-transfer system is a "pay as you go", where in every period the government levies a flat tax on the wage income of the young generation and pays social security benefits paid to the old generation. The representative individual makes labour-leisure and saving-consumption decisions, and pays social security taxes, in the first period of her life. The individual retires in the second period. The retired individual receives interest income from private savings (made in the first period, when she was young), and social security benefits. Migrants enter the economy when young, and gain the right to vote only in the next period, when old. They have the same preferences as those of the native born, except from having a higher birth rate. We assume that  $n > 0$  is the native-born birth rate, and  $m(> n)$  is the birth rate of migrants. On arrival, migrants are fully integrated into the social security system. That is, they pay the social security tax when young, and receive the social security benefits when old. Offspring of immigrants are like native born

in all respects (in particular, they have the same birth rate as the offspring of the native born). As is standard in such Diamond type overlapping generations model, the aggregate savings of the current young population generates next period aggregate capital. The latter is used as a factor of production, along with the labour input in the next period. The production function exhibits constant returns to scale. Both the wage rate and the rate of interest, are endogenously determined along the equilibrium path.

The utility of the representative young individual is logarithmic:

$$U^y(\tau_t, s_t, b_{t+1}) = \text{Log}(wl_t(1 - \tau_t) - s_t - \frac{l_t^{\Psi+1}}{\Psi + 1}) + \beta \text{Log}(b_{t+1} + (1 + r)s_t) \quad (1)$$

and the utility function of the representative old individual is given by:

$$U^o(s_{t-1}, b_t) = b_t + (1 + r)s_{t-1} \quad (2)$$

where  $\tau_t$  is period  $t$  tax rate,  $s_t$  is period  $t$  individual saving,  $b_{t+1}$  is period  $t+1$  social security benefits,  $l_t$  is period  $t$  individual labor supply,  $w_t$  is period  $t$  wage rate, and  $r_t$  is period  $t$  interest rate.

The production function is of a Cobb-Douglas form, which is assumed to use both labour and capital as its factors of production:

$$Y_t = N_t^{1-a} K_t^\alpha, \quad (3)$$

where  $K_t$  is the aggregate amount of capital,  $\gamma_t$  is the ratio of migrants to the young native born population, and  $N_t = (1 + \gamma_t)l_t$  is period  $t$  aggregate labor supply (native born and migrants).

The wage rate and interest rate are determined competitively by the marginal productivity conditions (for simplicity, capital is assumed to depreciate completely at the end of the period):

$$w_t = (1 - a)(1 + \gamma_t)^{-a} l_t^{-a} k_t^\alpha \quad (4)$$

$$r_t = \alpha(1 + \gamma_t)^{1-a} l_t^{1-a} k_t^{\alpha-1} - 1, \quad (5)$$

where  $k_t$  is capital per (native-born) worker. The balanced government budget constraint is derived as in the previous section:

$$b_{t+1} = \frac{\tau_{t+1} w_{t+1} l_{t+1} [(1 + n) + \gamma_t (1 + m)] (1 + \gamma_{t+1})}{(1 + \gamma_t)} \quad (6)$$

The saving-consumption decisions of young individuals are made by maximizing their utility while taking the prices and policy choices as given, and the labour-leisure decision is given as in the previous section:

$$s_t = \frac{1}{1+\beta} \left( \beta \frac{\Psi}{\Psi+1} w_t l_t (1-\tau_t) - \frac{b_{t+1}}{1+r_{t+1}} \right) \quad (7)$$

$$l_t^\Psi = w_t (1-\tau_t) \quad (8)$$

The market clearing condition requires that net domestic savings generate net domestic investment:

$$s_t = k_{t+1} \left( \frac{1+n+\gamma_t(1+m)}{(1+\gamma_t)} \right) \quad (9)$$

Solving equations (20) and (21) for  $b_{t+1}$  and substituting  $b_{t+1}$  in equations (15), we can write the indirect utility function of the young as follows.

$$\begin{aligned} V^y(w_t, \tau_t, r_{t+1}, \tau_{t+1}) = & \text{Log} \left( \frac{1}{1+\beta} \frac{\Psi}{\Psi+1} w_t l_t (1-\tau_t) (1+\beta f(\tau_{t+1})) \right) \\ & + \beta \text{Log} \left( \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} w_t l_t (1-\tau_t) (1+\beta f(\tau_{t+1})) (1+r_{t+1}) \right) \end{aligned} \quad (10a)$$

where  $f(\tau_{t+1}) = \frac{\frac{1-\alpha}{\alpha} \frac{1}{1+\beta} \tau_{t+1}}{1 + \frac{1-\alpha}{\alpha} \frac{1}{1+\beta} \tau_{t+1}}$ ,  
such that,

$$k_{t+1} = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t) w_t l_t (1-\tau_t) (1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)} \quad (11)$$

$$l_t^\Psi = w_t (1-\tau_t) \quad (12)$$

$$l_{t+1}^\Psi = w_{t+1} (1-\tau_{t+1}), \quad (13)$$

Now, substituting  $b_t$  from equation (20) and  $k_t$  from equation (23), and using equation (16), the indirect utility function of the old is:

$$\begin{aligned} V^o(\gamma_{t-1}, k_t, w_t, r_t, \tau_t) = & \frac{\tau_t w_t l_t [(1+n)+\gamma_{t-1}(1+m)](1+\gamma_t)}{(1+\gamma_{t-1})} + \\ & (1+r_t) k_t \left( \frac{1+n+\gamma_{t-1}(1+m)}{(1+\gamma_{t-1})} \right) \end{aligned} \quad (14)$$

such that,

$$l_t^\Psi = w_t (1-\tau_t), \quad (15)$$

As expected, the old individual favours a maximizing-revenue level of the social security tax rate ( the "Laffer Point"),  $\tau^* = \frac{\Psi}{\Psi+1}$ , and the largest immigration quota,  $\gamma = 1$ .

## 2.1 Political-Economic Equilibrium

The Markov Perfect equilibrium is defined as follows.

**Definition 1** *A Markov perfect political equilibrium is a vector of policy decision rules,  $\Pi = (T, G)$ , and private decision rule,  $S$ , where  $T : [0, 1] \rightarrow [0, 1]$ , is the tax policy rule,  $\tau_t = T(k_t)$ , and  $G : [0, 1] \rightarrow [0, 1]$ , is the immigration policy rule,  $\gamma_t = G(k_t)$ , and  $S(k_t)$  is the saving decision rule so that  $k_{t+1} = S(\pi_t, k_t)$ , such that the following functional equations hold:*

$$\begin{aligned} (1) \quad & \hat{\Pi}(k_t) = \arg \max_{\pi_t} V^i(\gamma_{t-1}, \pi_t, \pi_{t+1}) \\ & \text{subject to } \pi_{t+1} = \Pi(\gamma_t, S(\pi_t, k_t)). \\ (2) \quad & S(k_t) = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t)w_t l_t (1-\tau_t)(1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)}, \\ & \text{with } \tau_{t+1} = T(S(k_t)). \end{aligned}$$

(3) *A fixed-point condition requiring that given the next period policy outcome (the vector of policy decision rules-  $\Pi(k_{t+1})$ ), the maximization of the indirect utility of the current decisive voter, subject to the law of motion of the capital stock, will reproduce the same law of motion,  $\hat{\Pi}(k_t) = \Pi(k_t)$ , (as in condition (1)).*

This means that in equilibrium, policy variables have to maximize the decisive voter's indirect utility function, while taking into account the law of motion of capital, and expect that next period decision rules depend on the state variables, i.e. the current period migration quota and next period capital per (native-born) worker.

## 3 Equilibrium With Social Security

The Markov sub-game Perfect equilibrium is as follows.

**Proposition 2** *The equilibrium is given by the policy rules, and the saving rate are:*

$$T(k_t) = \begin{cases} \tau(k_t) & \text{if } k_t \in [k(\bar{\tau}), k(0)] \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

$$G(k_t) = \begin{cases} 1 & \text{if } k_t \in [k(\bar{\tau}), k(0)] \\ \gamma^* & \text{otherwise} \end{cases} \quad (17)$$

$$S(\tau_t, \gamma_t, k_t, \tau_{t+1}) = \begin{cases} S(\tau(k_t), 1, k_t, \tau(k_{t+1})) & \text{if } k_t \in [k(\bar{\tau}), k(0)] \\ S(0, \gamma^*, k_t, 0) & \text{otherwise} \end{cases} \quad (18)$$

where



$$\gamma^* = \frac{\beta(1-\alpha)\Psi(m-n) - \alpha(1+\Psi)(1+n)(1 + \frac{\beta\alpha(1+\Psi)}{\alpha+\Psi})}{\alpha(1+\Psi)(1+m)(1 + \frac{\beta\alpha(1+\Psi)}{\alpha+\Psi})},$$

$$S(\tau_t, \gamma_t, k_t, \tau_{t+1}) = \frac{\beta}{1+\beta} \frac{\Psi}{1+\Psi} \frac{1+\gamma_t}{1+n+(1+m)\gamma_t} \frac{(1-\alpha)(1+\gamma_t)^{-\alpha} (k_t)^\alpha (1-\tau_t)^{\frac{1+\Psi}{\Psi+\alpha}}}{1 + \frac{(1-\alpha)(1-\tau_{t+1})}{\alpha+\Psi}},$$

and

$$k(\tau) = ((1 + \frac{(1-\alpha)}{\alpha}\tau)^{1+\beta} (1-\tau)^{\frac{\beta(1-\alpha)}{\alpha+\Psi}} \frac{1}{c})^{-(1 + \frac{(1+\Psi)\alpha\beta}{\Psi+\alpha})}, \text{ for } \tau = \bar{\tau} \text{ and } \tau = 0,$$

where  $c$  is a constant of integration (see Appendix).

First, we explain why the equilibrium tax function  $\tau(k)$  is decreasing in  $k$ , in the range  $k \in [k(\bar{\tau}), k(0)]$ . (The equilibrium tax function  $\tau(k)$  is increasing in the range  $k \in [k(\bar{\tau}), k(1)]$ , but this is irrelevant because at this range  $\tau = 0$ .) There are two conflicting forces at work when the next period tax rate rises. On one hand, an increase in period  $t+1$  tax rate raises period  $t+1$  tax revenues and thereby social security benefits. This reduces the incentive to save in period  $t$ . On the other hand, an increase in period  $t+1$  tax rate reduces the labor supply in period  $t+1$ . This effect as by itself reduces period  $t+1$  tax revenues and thereby social security benefits. Thus social security benefits tend to fall. Because also the rate of return on capital also falls, the incentives to save in period  $t$  rise. Because savings in period  $t$  are equal to the capital stock in period  $t+1$ , the latter is subject to negative and positive effects from the increase in period  $t+1$  tax rates. If  $k \in [k(\bar{\tau}), k(0)]$ , the negative effect exceeds the positive effect, and the rise in period  $t+1$  tax rate reduces the capital per (native-born) worker in period  $t+1$ . That is, The equilibrium tax rate,  $\tau(k_t)$ , is a decreasing function in  $k_t$  for  $k \in [k(\bar{\tau}), k(0)]$ . If, however,  $k \in [k(\bar{\tau}), k(1)]$ , the increase in period  $t$  tax rate has diminished effect on tax revenue, and can also turn negative for a  $\bar{\tau}$  beyond the Laffer point. The second positive effect of the increase in period  $t+1$  tax rate on the capital per (native-born) worker in period  $t+1$  dominates the first negative effect. But beyond the range  $k \in [k(\bar{\tau}), k(0)]$  the equilibrium tax rate is equal to zero, and the function  $\tau(k)$  becomes irrelevant.

We now turn to the migration quota. Consider the equation

$$\gamma^* = \frac{\beta(1-\alpha)\Psi(m-n) - \alpha(1+\Psi)(1+n)(1 + \frac{\beta\alpha(1+\Psi)}{\alpha+\Psi})}{\alpha(1+\Psi)(1+m)(1 + \frac{\beta\alpha(1+\Psi)}{\alpha+\Psi})}.$$

The positive term in the numerator on the right hand side captures the beneficial effect of immigrants offspring in strengthening the social security system. The negative term in the numerator on the right hand side captures the wage depressing effect of immigrants. If the numerator is negative, the quota is set equal to zero. This is the case, for example, if  $m=n$ . In this case the beneficial effect to period  $t$  migration, which arises from an increase in the period  $t+1$  share of the young working force in the total population vanishes completely. Because  $n > 0$ , the decisive voter is always the young. there is neither economic

or political-economy reason to let migrants in. If, however,  $m > n$  there is a beneficial economic effect to bring in migrants. For a sufficiently large gap between  $m$  and  $n$ , the young decisive voter in period  $t$ , anticipating an increase in social security benefits in period  $t+1$ , will admit immigrants. In this case  $\gamma^* > 0$ . There is a positive effect of the aging of the native born on the migration quota, captured by reducing  $n$ , and a positive effect of the increase in the gap between the immigrant birth rate and the native-born birth rates on the equilibrium migration quota, captured by  $m-n$ .

In a steady state equilibrium the levels of the pair  $k$  and  $\tau$  can be solved from the following (non linear) equations:

$$k = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} ((1-\alpha)2^{-\alpha}k^\alpha(1-\tau))^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau)) \quad (19)$$

$$k^{-1-\frac{\alpha\beta(1+\Psi)}{\Psi+\alpha}} = \left(1 + \frac{1-\alpha}{\alpha}\tau(k)\right)^{1+\beta} (1-\tau)^{\frac{(1-\alpha)\beta}{\Psi+\alpha}} \quad (20)$$

## 4 Equilibrium With No Social Security

In order to emphasize the role of the social security system in the model, we now consider a similar model, but without any transfer payments from the young to the old.

The equilibrium migration policy rule, and the saving rate are:

$$G(k_t) = \gamma^* \quad (21)$$

$$S(\gamma^*) = \frac{\beta}{1+\beta} \frac{\Psi}{1+\Psi} \frac{1+\gamma^*}{1+n+(1+m)\gamma^*} \frac{(1-\alpha)(1+\gamma^*)^{-\alpha}(k_t)^\alpha}{1+\frac{(1-\alpha)}{\alpha+\Psi}} \quad (22)$$

Comparing migration policies with and without a social security system, we can verify that in the former migration policies are either the same or more liberal than in the latter regime. The reason that the social security system creates an incentive to bring in migrants is that the migrants' positive effect on social security tax revenues.

## 5 Conclusion

The pay-as-you-go social security system, which in recent time suffers from dwindling labor force, can benefit from immigrants with birth rates that exceed the native-born birth rates. Thus, a social security system provides effectively an incentive, through the political economy mechanism, to liberalize migration policy. The paper examines a political- economic, inter-generational, mechanism through which the social security system affects voter attitudes in favor of more liberal immigration regime. We demonstrate that the Markov equilibrium with social security consists of more liberal migration policies than the corresponding Markov equilibrium with no social security.

Related empirical work (e.g., Cohen and Razin(2009) demonstrates a positive effect of the generosity of the welfare state on the skill composition of migration, due to the fiscal benefits that are afforded by skilled migrants.

## 6 References

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## 7 Appendix

**Proof.** The proof of the proposition is as follows. Because  $n > 0$ , the majority resides with the young voters. Thus, the policy decisions concerning the tax rate and migration quotas maximizes the young indirect utility function. (We follow the proof of Forni (2004) to derive the tax policy decision rule.) The policy decision rules are derived by using, as a constraint, the first derivative with respect to the policy variables of the logarithm of the capital accumulation equation. The policy decision rules are:

$$\left(1 + \frac{1-\alpha}{\alpha}\tau_t(k_t)\right)^{1+\beta} (1 - \tau_t(k_t))^{\frac{\beta(1-\alpha)}{\Psi+\alpha}} = k_t^{-x} c \quad (23)$$

$$\gamma_t = 1 \quad (24)$$

where  $x = 1 + \frac{(1+\Psi)\alpha\beta}{\Psi+\alpha}$ , and  $c$  is a positive constant of integration. The policy decision rule of the immigration quotas is at its maximal value, and the policy decision rule of the tax rate is implicitly given in equation (23). Define the following function:  $k(\tau) = \left( \left(1 + \frac{1-\alpha}{\alpha}\tau\right)^{1+\beta} (1 - \tau)^{\frac{\beta(1-\alpha)}{\alpha+\Psi}} \frac{1}{c} \right)^{-\frac{1}{x}}$ . Thus we can rewrite the policy decision rule of the tax rate as:  $k(\tau_t) = k_t$ . The function  $k(\tau)$  is decreasing in  $\tau$ , for  $\tau \in [0, \bar{\tau}]$ , where  $\bar{\tau} = \frac{\Psi(1+\beta)+\alpha}{\Psi(1+\beta)+\alpha+\beta}$ , and increasing in  $\tau$ , for  $\tau \in [\bar{\tau}, 1]$ . Thus, according to equation (23), for every value of capital per (native-born) worker,  $k_t$ , there are two solutions for  $\tau(k_t)$  in the range  $[0, 1]$ . The solution which satisfies the equilibrium conditions, which is denoted by  $\tau(k_t)$ , is decreasing in  $k_t$  for  $k_t \in [k(\bar{\tau}), k(0)]$ .

The solution for the policy variables given in equations (23) and (24), will be proved to satisfy the first order conditions of the problem. The young voter's indirect utility function under the assumption that next period decisive voter is young, which sets next period policy decision rules for the tax rate and immigration quotas to be  $\tau_{t+1} = \tau(k_{t+1})$ , and  $\gamma_{t+1} = 1$  respectively, can be written

in its Lagrangian form as follows:

$$\begin{aligned}
L(k_t) = & A + (1 + \beta) \text{Log} \left( (1 - \alpha) k_t^\alpha (1 + \gamma_t)^{-\alpha} (1 - \tau_t) \right)^{\frac{1+\Psi}{\Psi+\alpha}} + \\
& (1 + \beta) \text{Log} \left[ (1 + \beta f(\tau(k_{t+1}))) + \beta \text{Log} \alpha \left( (1 - \alpha) k_{t+1}^{-\Psi} 2^\Psi (1 - \tau(k_{t+1})) \right)^{\frac{1-\alpha}{\Psi+\alpha}} \right] \\
& - \lambda_1 \left( k_{t+1} - \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t)((1-\alpha)k_t^\alpha(1+\gamma_t)^{-\alpha}(1-\tau_t))^{\frac{1+\Psi}{\Psi+\alpha}}(1-f(\tau(k_{t+1})))}{1+n+\gamma_t(1+m)} \right) \\
& - \lambda_2(\tau_t - 1) - \lambda_3(-\tau_t) - \lambda_4(\gamma_t - 1) - \lambda_5(\gamma_t)
\end{aligned} \tag{25}$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial \tau_t} = 0 = -\frac{1 + \Psi}{\Psi + \alpha} \frac{1 + \beta}{1 - \tau_t} - \lambda_1 \frac{1 + \Psi}{\Psi + \alpha} \frac{k_{t+1}}{1 - \tau_t} - \lambda_2 + \lambda_3 \tag{26}$$

$$\frac{\partial L}{\partial \gamma_t} = 0 = -\alpha \frac{1 + \Psi}{\Psi + \alpha} \frac{1 + \beta}{1 + \gamma_t} + \lambda_1 \frac{k_{t+1}}{1 + \gamma_t} \left( \frac{n - m}{1 + n + \gamma_t(1 + m)} - \alpha \frac{1 + \Psi}{\Psi + \alpha} \right) - \lambda_4 + \lambda_5 \tag{27}$$

$$\begin{aligned}
\frac{\partial L}{\partial k_{t+1}} = & 0 = \left( \frac{\beta(1 + \beta)}{1 + \beta f(\tau(k_{t+1}))} - \frac{\lambda_1 k_{t+1}}{1 - f(\tau(k_{t+1}))} \right) \frac{\partial f(\tau_{t+1})}{\partial \tau_{t+1}} \frac{\partial \tau(k_{t+1})}{\partial k_{t+1}} \\
& - \frac{\beta(1 - \alpha)}{\Psi + \alpha} \frac{1}{1 - \tau(k_{t+1})} \frac{\partial \tau(k_{t+1})}{\partial k_{t+1}} + \frac{1}{k_{t+1}} \left( -\beta \frac{\Psi(1 - \alpha)}{\Psi + \alpha} \right) - \lambda_1
\end{aligned} \tag{28}$$

$$k_{t+1} = \frac{\beta}{1 + \beta} \frac{\Psi}{\Psi + 1} \frac{(1 + \gamma_t) w_t l_t (1 - \tau_t) (1 - f(\tau(k_{t+1})))}{1 + n + \gamma_t(1 + m)} \tag{29}$$

$$\tau_t - 1 \leq 0, \lambda_2 \geq 0 \text{ and } \lambda_2(\tau_t - 1) = 0 \tag{30}$$

$$-\tau_t \leq 0, \lambda_3 \geq 0 \text{ and } \lambda_3(-\tau_t) = 0 \tag{31}$$

$$\gamma_t - 1 \leq 0, \lambda_4 \geq 0 \text{ and } \lambda_4(\gamma_t - 1) = 0 \tag{32}$$

$$-\gamma_t \leq 0, \lambda_5 \geq 0 \text{ and } \lambda_5(\gamma_t) = 0 \tag{33}$$

Substituting for  $\lambda_1$  from equation (28) into equations (26) and (27), we derive the following equations:

$$\frac{\partial L}{\partial \tau_t} = -\lambda_2 + \lambda_3 = 0 \tag{34}$$

$$\frac{\partial L}{\partial \gamma_t} = \frac{(1 + \beta)}{1 + \gamma_t} \left( \frac{-n + m}{1 + n + \gamma_t(1 + m)} \right) - \lambda_4 + \lambda_5 = 0 \tag{35}$$

Because  $m > n$ , from equation (35) we can derive that  $\gamma_t$  has a corner solution. The solution for the tax rate, on the other hand,  $\tau_t$ , may be bounding or not, meaning that  $\tau_t = \tau(k_t) \in [0, 1]^1$ . Substituting the solutions for the tax and openness rate into the indirect utility of the young, we obtain that the optimal solution for the openness rate is  $\gamma_t = 1$ .

<sup>1</sup>Note that the utility with  $\tau_t = 1$  is equal to minus infinity. Thus, the range for the tax rate is  $[0, 1)$ .

The optimal solutions should also satisfy the second order sufficient condition, meaning that the bordered Hessian of the Lagrangian should be negatively defined. Since the solution of the immigration quotas is a corner solution where the largest immigration quota maximizes the young voter's indirect utility function, the bordered Hessian of the Lagrangian is equal to:

$$-g_\tau \left( g_\tau \frac{\partial^2 L}{\partial^2 k_{t+1}} - g_k \frac{\partial^2 L}{\partial k_{t+1} \partial \tau_t} \right) + g_k \left( g_\tau \frac{\partial^2 L}{\partial \tau_t \partial k_{t+1}} - g_k \frac{\partial^2 L}{\partial^2 \tau_t} \right) \quad (36)$$

where  $g_\tau$  and  $g_k$  are the derivatives of the constraint of the capital per (native-born) worker from equation (29) with respect to  $\tau_t$  and  $k_{t+1}$  respectively. The bordered Hessian can be rewritten in the following way:

$$\begin{aligned} & \left( \frac{1+\Psi}{\Psi+\alpha} \right)^2 \frac{1}{(1-\tau_t)^2} \frac{2x(1+\frac{1-\alpha}{\alpha}\tau_t)(1-\tau_t)\left(\frac{1-\alpha}{\alpha}\right)}{\left( (1+\beta)\frac{1-\alpha}{\alpha}(1-\tau_t) - \frac{\beta(1-\alpha)}{\Psi+\alpha}(1+\frac{1-\alpha}{\alpha}\tau_t) \right)^2 \left( 1 + \frac{1-\alpha}{\alpha} \frac{1}{1+\beta} \tau_t \right)^2} \quad (37) \\ & \left( \begin{aligned} & x(1+\frac{1-\alpha}{\alpha}\tau_t)(1-\tau_t)\left(\frac{1-\alpha}{\alpha}\right) + \\ & \left( (1+\beta)\frac{1-\alpha}{\alpha}(1-\tau_t) - \frac{\beta(1-\alpha)}{\Psi+\alpha}(1+\frac{1-\alpha}{\alpha}\tau_t) \right) \left( 1 + \frac{1-\alpha}{\alpha} \frac{1}{1+\beta} \tau_t \right) (1+\beta) \end{aligned} \right) \end{aligned}$$

Denote by  $[\tau_1, \tau_2]$  the range of the tax rate for which the bordered Hessian of the Lagrangian is negatively defined. Since  $\tau_1$  is always negative, and tax rate is defined over the range  $\tau \in [0, 1]$ , the range of the capital optimal solution for the tax rate,  $\tau(k_t)$ , is defined in the range  $k_t \in [k(\bar{\tau}), k(0)]$ , where the function  $k(\tau)$  is decreasing in  $\tau$ . ■

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