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Article

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Safety Stock Optimization under Lead Time Ambiguity

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A B S T R A C T

Purpose: The purpose of this study is to investigate how ambiguity faced by the risk-neutral manager affects the firm's optimal level of safety stock.
Design/methodology/approach: This study adopts the traditional model of Arrow et al. (1951) and employ a manager who has multiple prior beliefs about the probability distribution of the lead time.
Findings: This study finds that facing lead time ambiguity, the manager becomes more conservative when choosing the optimal stock level to hold and the amount of safety stock.
Research limitations/implications: The future research would consider a risk-averse manager who could be compensated or punished as a result of stock management. Then the future research would examine the interactive effects of the manager's risk aversion and lead time ambiguity on the optimal safety stock level.
Originality/value: This would be the first study that investigates the effects of ambiguity on the level of safety stock.

Keywords: ambiguity, safety stock, stochastic lead time

I. Introduction

During the COVID-19 pandemic, there are significant challenges for global supply chains. The closures of country borders and shut downs of factories and workplaces slow or even stop the flow of raw materials and products. For instance, the widespread and prevalent supply chain issues in semiconductor arise shortage of chips used by companies in other industries such as automakers and home appliance companies. Thus, global supply chain becomes more unpredictable and managers in firms sometimes make decisions about supply chain management based on significantly imprecise information.

To model the extremely uncertain circumstances of global supply chain, this study allows for ambiguity faced by inventory managers. Ambiguity means multiple prior beliefs about a probability distribution. In traditional models dealing with inventory policy such as Arrow et al. (1951), Veinott and Wagner (1965), and Weiss (1980), it is assumed that managers have the exact information about related probability distributions.[1)] Indeed, recent studies about stochastic inventory model also do not care about ambiguity. Inderfurth and Vogelgesang (2013) suggest stochastic inventory model under periodic review and Kim et al. (2015) try to

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1) See also Inderfurth (1991) and Song (1994).

optimize total logistic cost for perishable product in a multi-period newsvendor model. However, these studies develop models with certain probability distributions. To the best of our knowledge, there is not study which allows for ambiguity faced by managers in charge of inventory management.

The purpose of this study is to investigate how ambiguity faced by the risk-neutral manager affects the firm's optimal level of safety stock. To do this, the study adopts the traditional model of Arrow et al. (1951) and employ a manager who has multiple prior beliefs about probability distribution of the lead time. Our one-period model assumes stochastic demand and lead time, but for simplicity, this study only considers ambiguity for the lead time. To minimize total cost, the managers chooses the level of stock to hold after reordering and optimal amount of safety stock. To analyze the behavior of the manager under ambiguity evidenced by Ellsberg (1991) paradox, this study adopts maxmin expected utility (MEU) model (Giboa and Schmeidler, 1989), in which the manager makes inventory choice in extremely conservative way. According to the experimetnal study of Ahn et al. (2014), the MEU model explains the behavior of managers with ambiguity aversion.[2)]

This study finds that facing lead time ambiguity, the manager becomes more conservative when choosing the optimal stock level to hold and the amount of safety stock. In the presence of ambiguity, the manager increases the level of stock after reordering to minimize the cost. Furthermore, as the degree of ambiguity increases, the manager holds more stocks in a warehouse. The effects on the optimal safety stock depends on the type of ambiguity. The manager prepares more safety stock under ambiguity about the variance of lead time than without it, and moreover, an increase in degree of lead time variance ambiguity leads to a higher level of the safety stock. On the other hand, the amount of safety stock is not affected by ambiguity about the mean of lead time. Irrespective of the presence of ambiguity about the mean of the lead time, the manager chooses the same level of safety stock. This study also finds that the optimal service level is not affected by ambiguity. As long as the cost structure remains unchanged, the manager chooses the equivalent service level to minimize the cost when ambiguity is absent and when it is present.

Our result conforms to economic models in which market participatns have multipe beliefs about probability distirbution. In the presence of ambiguity, the ambiguity-averse inventory manager takes worst probability distiribution, which makes the manager choose more conservative level of safety stock. This behavior of the manager is evidenced by Ellsberg (1991) paradox and the maxmin expected utility axiomized in Giboa and Schmeidler (1989). Economic models assuming ambiguity faced by market participants reveal simialr results. In Ozsoylev and Werner (2011) and Illeditsch (2011), market participants under ambiguity have extremely conservative order schedule in the way that they refuse to take positions in intermediate price regions.

The rest of the paper is organized as follows. In Section 2, this paper introduces a model of safety stock with ambiguity. The optimal level of safety stock is derived in Section 3. In Section 4, our results are numerically examined. Concluding remarks are given in Section 5.

A. Related Literature

In general, inventory control deals with trade-off relationship between meeting customer demand and the profit of of firms. Since Harris (1913) proposes the first inventory model and Wilson (1934) develops the formula of economic order quantity (EOQ), a vast amount of literature tries to construct models with more realistic assumptions, in which lead time or demands cannot be expressed as deterministic functions. Classical models such as Galliher et al. (1959) and Hadley and Whitin (1962) introduce stochastic demand in their models and characterize

2) We would consider smooth ambiguity model of Klibanoff et al. (2005), which allows for changes in the degree of ambiguity averse. In this study, however, we use MEU model to capture the behavior of managers who make extremely conservative choices to avoid the shortage of stock.

the optimal inventory level. Further, Eppen and Martin (1988) derive optimal level of safety stock when both demand and lead time are random variables.

Recently, traditional stochastic inventory models are extended to dynamic models. Farahvash and Altiok (2011) develop multi-period stochastic model under reverse auction. Zhang et al. (2009) propose stochastic programming models for single and multi-period when the demand has probability distribution. They do now show closed-form solution, but employ numerical methods to explain the optimal stock level to minimize expected loss. In the model of Song and Wang (2017), fixed order cost and uniform random yield are assumed. They find the lower and upper bound structure of the optimal order level. On the other hand, Qiu et al. (2017) employ robust optimization approaches to solve multi-period inventory management problem when the distribution of demand is not certain. Two-dimensional multi-period stochastic inventory model with random demand is developed in M'Hallah et al. (2020). In their model, there are two items to be controlled and each item has the sequence of independent identical distribution of demand.

Implications of ambiguity faced by decision makers have been extensively studied in economics and financial literature. Gilboa and Schmeidler (1989) axiomize the maxmin expected utility in which decision makers maximiae their utility under the worst-case probability distribution. Ozsoylev and Werner (2011) and Mele and Sangiorgi (2015) characterize financial markets where a fraction of rational traders has ambiguous beliefs for the future value of assets. Epstein and Schneider (2008) and Illeditsch (2011) find that, in the presence of ambiguity, the value of asset becomes highly unpredictable since the volatility of asset value may diverge to infinity.

II. The Model

There is a firm which manufactures a single product. Our model assumes that the product is replenished in continuous units and it does not deteriorate and has an infinite life time. Our model allows for stochastic demand and lead time. The demand $\tilde{d}$ during the one period is normally distributed with mean μ_d and variance σ_d^2. Lead time $\tilde{l}$ has normal distribution with mean μ_l and variance σ_l^2. Assuming the amount of demand is not affected by the time period, our model obtains the distribution of lead time demand x whose mean and variance are given by

$$\mu = \mu_d \mu_l,$$
$$\sigma^2 = \sigma_d^2 \mu_l + \sigma_l^2 \mu_d^2,$$

respectively.

The model considers three types of costs: per unit production cost c, per unit effective holding cost c_H, and penalty cost for shortage c_p. Let y be the level of stock after reordering. Let $f(x)$ denote the probability density function of x:

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2} \right).$$

Then the total expected cost function is given by

$$C(y) = cy + \int_0^y c_H (y-x) f(x) dx + \int_y^\infty c_p (x-y) f(x) dx.$$

The first term on the left side is the production cost and the second and third ones are, respectively, the total expected holding cost and expected shortage cost.

The manager of the firm does not observe the realization of the demand $\tilde{d}$ and the lead time $\tilde{l}$ when making decision about the level of safety stock. Further, the manager has ambiguous belief about the distribution of $\tilde{l}$. That is, the manager does not have the exact information about the probability distribution of $\tilde{l}$ and takes into account multiple probability distributions. This model assumes that the manager's

belief set P consists of normal distributions with mean and variance lying on interval $\left[\underline{\mu},\overline{\mu}\right]$ and $\left[\underline{\sigma}^2,\overline{\sigma}^2\right]$, respectively. As the size of interval $\left[\underline{\mu},\overline{\mu}\right]$ ($\left[\underline{\sigma}^2,\overline{\sigma}^2\right]$) increases, the manager becomes more uncertain about the true μ (σ^2, respectively). Thus, this study measures the degree of ambiguity by the size of each interval.

Let the mean and variance under probability distribution $\pi\in P$ of the lead time be denoted by μ_l^π and $\left(\sigma_l^\pi\right)^2$, respectively. Then the mean and variance of the lead time demand x when the manager takes $\pi\in P$ as a lead time probability distribution can be written by

$$\mu^\pi = \mu_d\mu_l^\pi,$$
$$\left(\sigma^\pi\right)^2 = \sigma_d^2\mu_l^\pi + \left(\sigma_l^\pi\right)^2\mu_d^2,$$

respectively. Thus, probability function of lead time demand x when $\pi\in P$ is picked by the manager is given by

$$f^\pi(x) = \frac{1}{\sqrt{2\pi}\,\sigma^\pi}\exp\left(-\frac{(x-\mu^\pi)^2}{2(\sigma^\pi)^2}\right).$$

Let

$$\underline{\mu}\equiv\mu_d\underline{\mu}_l,\quad \overline{\mu}\equiv\mu_d\mu_l$$

and for each $\mu_l^\pi\in\left[\underline{\mu}_l,\overline{\mu}_l\right]$

$$\underline{\sigma}^2\equiv\sigma_d^2\mu_l^\pi+\underline{\sigma}_l^2\mu_d^2,\quad \overline{\sigma}^2\equiv\sigma_d^2\mu_l^\pi+\overline{\sigma}_l^2\mu_d^2.$$

Obviously, μ^π ($(\sigma^\pi)^2$) is lying on the interval $\left[\underline{\mu},\overline{\mu}\right]$ ($\left[\underline{\sigma}^2,\overline{\sigma}^2\right]$) and it is minimized at $\mu^\pi=\underline{\mu}$ $\left((\sigma^\pi)^2=\underline{\sigma}^2\right)$ and is maximized at $\mu^\pi=\overline{\mu}$ ($(\sigma^\pi)^2=\overline{\sigma}^2$, respectively).

Equipped with MEU preferences of Gilboa and Schmeidler (1989), the ambiguous manager solves cost minimization problem under the worst-case probability distribution. Facing ambiguity, the manager solves

$$\min_y\max_{\pi\in P}C(y)=\min_y\max_{\pi\in P}[cy+\int_0^y C_h(y-x)f^\pi(x)dx+\int_y^\infty C_p(x-y)f^\pi(x)dx].$$

The manager chooses the optimal safety stock level through a two-step process: (i) the manager takes the worst case probability distribution which maximizes the cost (ii) the manager chooses the level y of the stock after reordering to minimize the cost.

To find the worst-case probability distribution, partially differentiate $C(y)$ with μ_π and σ_π^2. Let

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt.$$

Then,

$$\frac{\partial C(y)}{\partial\mu_\pi}=\frac{1}{2}\left[\frac{c_h}{\sigma_\pi}\sqrt{\frac{2}{\pi}}\exp\left(-\frac{\mu_\pi^2}{2\sigma_\pi^2}\right)y+c_p\left(1-erf\left(\frac{y-\mu_\pi}{\sqrt{2\sigma_\pi}}\right)\right)-c_h\left(erf\left(\frac{y-\mu_\pi}{\sqrt{2}\,\sigma_\pi}\right)+erf\left(\frac{\mu_\pi}{\sqrt{2}\,\sigma_\pi}\right)\right)\right] \tag{2.1}$$

and

$$\frac{\partial C(y)}{\partial\sigma_\pi^2}=\frac{1}{2\sqrt{2\pi}\,\sigma_\pi^3}\exp\left(-\frac{y^2-2y\mu_\pi+2\mu_\pi^2}{2\sigma_\pi^2}\right)\left[(c_h+c_p)\sigma_\pi^2\exp\left(\frac{\mu^2}{2\sigma_\pi^2}\right)-c_h\left(y\mu_\pi+\sigma_\pi^2\right)\exp\left(\frac{(y-\mu_\pi)^2}{2\sigma_\pi^2}\right)\right]. \tag{2.2}$$

Now the following proposition is obtained.

Proposition 2.1 The following hold.

1. For a given $\sigma_\pi^2\in\left[\underline{\sigma},\overline{\sigma}\right]$, the cost-maximizing probability distribution is achieved at $\mu_\pi=\overline{\mu}$ if the shortage cost is sufficiently high such that

$$c_p\rangle 2c_h. \tag{2.3}$$

2. For a given $\mu_\pi\in\left[\underline{\mu},\overline{\mu}\right]$, the cost-maximizing probability distribution is achieved at $\sigma_\pi^2=\overline{\sigma}_\pi^2$

when the stock level y after reordering is sufficiently high such that

$$y > \mu_\pi + \sigma_\pi^2. \tag{2.4}$$

Proof: (1) Since $erf(x) < 1/2$ for $x \in (-\infty, \infty)$,

$$\frac{\partial E[C(y)]}{\partial \mu_\pi} > \frac{1}{2}\left[\frac{c_h}{\sigma_\pi}\sqrt{\frac{2}{\pi}}\exp\left(-\frac{\mu_\pi^2}{2\sigma_\pi^2}\right)y + \frac{1}{2}(c_p - 2c_h)\right]$$

by (2.1). Thus, if $c_p > 2c_h$, then $\partial E[C(y)]/\partial \mu_\pi > 0$ and $E[C(y)]$ is maximized at $\mu_\pi = \bar{\mu}$.

(2) Note that

$$\frac{\partial}{\partial \mu_\pi}\left((c_h + c_p)\sigma_\pi^2 \exp\left(\frac{\mu^2}{2\sigma_\pi^2}\right) - c_h\left(y\mu_\pi + \sigma_\pi^2\right)\right.$$
$$\left.\exp\left(\frac{(y-\mu_\pi)^2}{2\sigma_\pi^2}\right)\right) = \mu_\pi\left[(c_h + c_p)\exp\left(\frac{\mu_\pi^2}{2\sigma_\pi^2}\right)\right.$$
$$\left. + \frac{c_h\left(y^2 - y\mu - \sigma_\pi^2\right)}{\sigma_\pi^2}\exp\left(\frac{(y-\mu_\pi)^2}{2\sigma_\pi^2}\right)\right],$$

which is greater than zero if (2.4) holds. Since $\partial E[C(y)]/\partial \sigma_\pi^2 > 0$ at $\mu_\pi = 0$, it is always greater than zero. Therefore, when (2.4) holds, $E[C(y)]$ increases in σ_π^2 and is maximized at $\sigma_\pi^2 = \bar{\sigma}^2$.

Note that the inequality 2.3 holds when the firm's cost minimizing service level is sufficiently high.[3] This would be examined in the next section.

III. Optimal Safety Stock

At the beginning of this section, as a benchmark, this study reviews cost minimizing safety stock level when the manager has the exact information about the lead time $\tilde{l}$. Then, this study extends the benchmark to our model in which the manager faces ambiguity. Thus study derives the optimal safety stock level when the manager multiple prior beliefs about the lead time probability distribution. Finally, the optimal safety stock level is characterized.

A. The Manager without Ambiguity

In this subsection, let us assume that the manager resolves ambiguity and knows the exact probability distribution of the lead time. In the absence of ambiguity, set

$$\mu_l = \underline{\mu}_l = \bar{\mu}_l \quad and \quad \sigma_l^2 = \underline{\sigma}_l^2 = \bar{\sigma}_l^2.$$

Then this implies that

$$\mu = \underline{\mu} = \bar{\mu} \; and \quad \sigma^2 = \underline{\sigma}^2 = \bar{\sigma}^2.$$

The total cost function is given by

$$C^\circ(y) = cy + \int_0^y c_h(y-x)f(x)dx + \int_y^\infty c_p(x-y)f(x)dx,$$

Thus the manager solves minimization problem:

$$\min_y C^\circ = \min_y [cy + \int_0^y C_h(y-x)f(x)dx + \int_y^\infty C_p(x-y)f(x)dx].$$

Let

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt.$$

The total cost is minimized when

$$\Phi\left(\frac{y-\mu}{\sigma}\right) = \frac{c_p - c}{c_p + c_h} \equiv \alpha^\circ.$$

3) Numerically, it holds when the cost minimizing service level is higher than 0.842, which is achieved when c_p is sufficiently higher than c_h.

Note that α° is the cost minimizing service level. Then the following proposition is obtained.

Proposition 3.1 Suppose the manager has the exact information about the probability distribution of the lead time. Then the following hold.

1. The cost minimizing stock level after reordering is given by

$$y^\circ = \mu + \sigma \Phi^{-1}\left(\frac{c_p - c}{c_h + c_p}\right).$$

2. The optimal level of safety stock is

$$SS^\circ = \sigma \Phi^{-1}\left(\frac{c_p - c}{c_h + c_p}\right).$$

where $\Phi^{-1}(x)$ is the inverse function of $\Phi(x)$.

3. The cost minimizing service level is α°.

The optimal level y° of the stock after reordering increases in the mean μ_x and volatility σ of lead time demand. On the other hand, the safety stock level is not affected by μ, but it increases in lead time demand volatility. The cost minimizing service level α° consists of production cost, holding cost, and shortage cost, but is not affected by the probability distributions of demand and lead time. The manager provides a higher service level as c_p increases or c and c_h decreases regardless of the probability distributions.

B. The Manager under Ambiguity

Now this study extends the benchmark and derive optimal safety stock level under ambiguity. For simplicity, henceforth, let assume that (2.3) and (2.4) hold. As a worst-case probability distribution, the manager picks probability distribution of lead time demand x with mean $\bar{\mu}$ and variance $\bar{\sigma}^2$. Let denote probability distribution with mean $\bar{\mu}$ and variance $\bar{\sigma}^2$ by $\bar{\pi} \in P$. Then

$$\begin{aligned}\min_y \max_{\pi \in P} C(y) &= \min_y \max_{\pi \in P} [cy + \int_0^y C_h(y-x) \\ &\quad f^\pi(x)dx + \int_y^\infty C_p(x-y) f^\pi(x)dx]. \\ &= \min_y [cy + \int_0^y C_h(y-x) f^{\bar{\pi}}(x)dx \\ &\quad + \int_y^\infty C_p(x-y) f^{\bar{\pi}}(x)dx].\end{aligned}$$

where $f^{\bar{\pi}}(x)$ is the probability density function when $\pi = \bar{\pi}$. Similar to the benchmark, the total cost is minimized when

$$\Phi\left(\frac{y - \bar{\mu}}{\bar{\sigma}}\right) = \frac{c_p - c}{c_p + c_h}.$$

Then the optimal safety stock level is obtained.

Theorem 3.1 Suppose that the manager has multiple prior beliefs $\pi \in P$ about probability distribution of the lead time and inequalities (2.3) and (2.4) are satisfied. Then the following hold.

1. The cost minimizing stock level after reordering is given by

$$y^* = \bar{\mu} + \bar{\sigma} \Phi^{-1}\left(\frac{c_p - c}{c_h + c_p}\right).$$

2. The optimal level of safety stock is

$$SS^* = \bar{\sigma} \Phi^{-1}\left(\frac{c_p - c}{c_h + c_p}\right)$$

3. The cost minimizing service level is

$$\alpha^* = \alpha^\circ = \frac{c_p - c}{c_p + c_h}.$$

Theorem 3.1 shows that the manager chooses more conservative stock management policy under ambiguity than without it. Facing ambiguity, the manager allows for increased mean and variance of the lead time demand. Then, for a given service level, the manager

needs to hold a higher level of stock after reordering and prepare more safety stock compared to the benchmark. Note that the optimal safety stock is not affected by $\bar{\mu}$, but increases in $\bar{\sigma}^2$. As long as the service level remains unchanged from the benchmark (i.e., $\alpha^* = \alpha^\circ$), the effect of ambiguity on the safety stock is determined by variance ambiguity of lead time demand irrespective of mean ambiguity. On the other hand, the cost minimizing service level is not affected by ambiguity. It is because the service level is not affected by probability distribution but only depends on production cost, holding cost, and shortage cost as in the benchmark. Ambiguity effects on stock management are summarized in the next proposition.

Proposition 3.2 Suppose that the manager has multiple prior beliefs $\pi \in P$ about probability distribution of the lead time and inequalities (2.3) and (2.4) are satisfied. Then the following hold.

1. As the mean or variance of lead time demand becomes more unpredictable, the cost minimizing stock level y^* after reordering increases.
2. As the variance of lead time demand becomes more unpredictable, the optimal safety stock SS^* increases.
3. The cost minimizing stock level is not affected by ambiguity.

IV. Numerical Implications

This study finds that, in the presence of ambiguity, the manager holds a higher level of stock after reordering and prepares more safety stock. In this section, this study numerically examines how the lead time ambiguity affects the level of optimal safety stock.

In Figure 1, the dashed line illustrates total inventory cost when the manager knows the exact information about the probability distribution of the lead time while the solid line illustrates that when the manager faced by ambiguity. This study finds that, at the minimum cost stock level, total cost is higher when ambiguity is present than otherwise. Furthermore, as the degree of ambiguity increases, the cost minimum total inventory cost increases.

Figure 2 illustrates how ambiguity affects the cost minimizing level of stock after reordering. On the left side, this study investigates the relationship between the degree of ambiguity about the lead time variance and the holding stock level and on the right side, it is observed that between the degree of ambiguity about the lead time mean and the stock level. In both cases, the cost minimizing stock level is higher under ambiguity than without it and it linearly increases as the degree of ambiguity increases.

Figure 3 shows the effects of ambiguity on the optimal safety stock level. The relationship between the degree of ambiguity about the lead time variance and the optimal safety stock level is illustrated on the left side and it is observed that between the degree of ambiguity about the lead time mean and the optimal safety stock level on the right side. The increase in the degree of ambiguity about the lead time variance raises the optimal level of safety stock while the

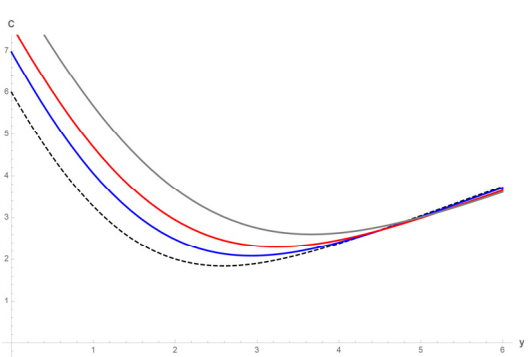

Figure 1. The effect of ambiguity on the total inventory cost at the cost minimizing stock level. The horizontal axis represents the level y of stock after reordering. The vertical axis measures total inventory cost. The dasheed line illustrates total inventory cost when ambiguity is absent and the solid line illustrate that when ambiguity is present. The common parameters are $c = 0.4$, $c_h = 0.4$, $c_p = 5$, $\mu_d = 1$, and $\sigma_d = 1$. In the dashed line, $\mu_l = \sigma_l = 1$. In the solid blue line, $\bar{\mu}_l = \bar{\sigma}_l = 1.2$. In the solid red line, $\bar{\mu}_l = \bar{\sigma}_l = 1.3$. In the solid gray line, $\bar{\mu}_l = \bar{\sigma}_l = 1.5$.

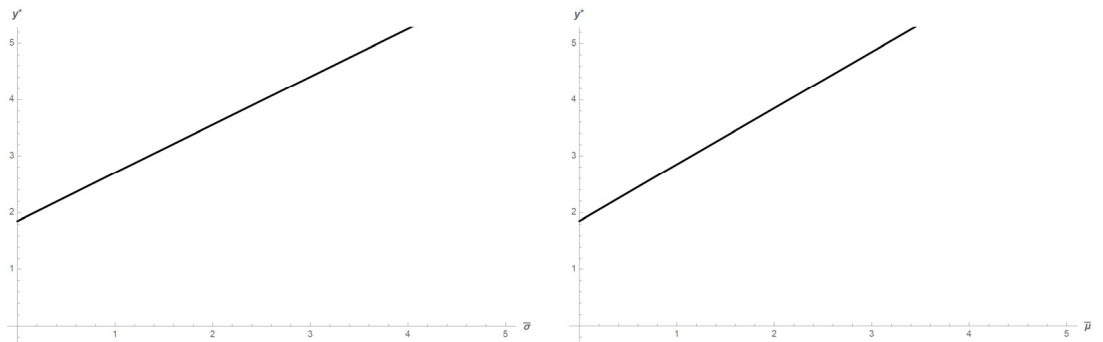

Figure 2. The effect of ambiguity on the cost minimizing stock level after reordering. The horizontal axis on the left side represents the maximum variance of the lead time and that on the right side represents the maximum mean of the lead time. The vertical axis of both plots measures the cost minimizing stock level. The common parameters are $c=0.4,\ c_h=0.4,\ c_p=5,\ \mu_d=1$. On the left side, $\mu_l=\bar{\mu}_l=1$. On the right side, $\sigma_l=\bar{\sigma}_l=1$.

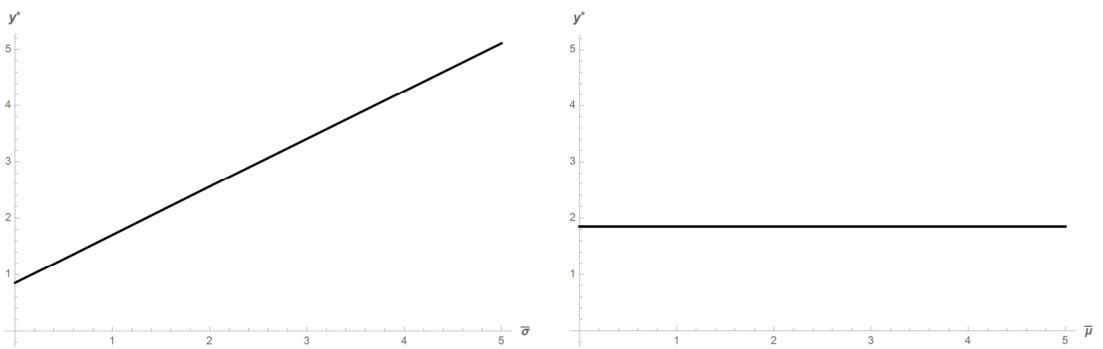

Figure 3. The effect of ambiguity on the level of optimal safety sotck. The horizontal axis on the left side represents the maximum variance of the lead time and that on the right side represents the maximum mean of the lead time. The vertical axis of both plots measures the optimal safety stock level. The common parameters are $c=0.4,\ c_h=0.4,\ c_p=5,\ \mu_d=1$. On the left side, $\bar{\mu}_l=1$. On the right side, $\bar{\sigma}_l=1$.

ambiguity about the lead time mean does not contribute to the increase in the optimal safety stock.

V. Concluding Remarks

This study investigates the effects of lead time ambiguity on the level of optimal safety stock. To do this, this study develops a model, in which there are stochastic demand and lead time and the manager has multiple prior beliefs about the distribution of lead time. This study finds that the manager under lead time ambiguity holds more stocks after reordering and prepares a higher level of safety stock compared to the case where there is no ambiguity. Further, the increasing degree of variance ambiguity of the lead time raises both the amount of the stock after reordering and the safety stock. On the other hand, the increase in mean ambiguity of lead time only increases the level of holding stock after reordering but has no influence on the optimal safety stock level.

In practice, it would be unlikely that inventory managers in firms have the exact information about the probability distribution of lead time. During the

period of Covid 19 pandemic, furthermore, the managers take into account multiple probability distributions when they determine the level of safety stock. This study shows that, facing multiple beliefs about the probability distribution of lead time, the inventory manager cares more about the loss by the shortage of stocks than holding costs. Therefore, the manager is willing to raised holding cost generated by the increase level of the safety stock.

This study employs maxmin expected utility model, in which the manager chooses extremely conservative lead time probability distribution. However, in reality, inventory managers in firms would reveal different attitude toward lead time ambiguity. This model could be extended by adopting smooth ambiguity model of Klibanoff et al. (2005). Then the research may control the degree of ambiguity averse and would observe the relationship between the manager's ambiguity preference and level of the optimal safety stock.

In the future research, one would consider a risk-averse manager. Indeed, since our model does not take into account managerial compensation, this study assumes a risk-neutral manager who makes decision only depending on overall cost. However, if the manager would be compensated or punished as a result of stock management, the manager may become risk averse when deciding safety stock level. Adopting risk-averse manager, the research would examine the interactive effects of the manager's risk aversion and lead time ambiguity on the optimal safety stock level.

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