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## A Theory of Digital Ecosystems

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# A Theory of Digital Ecosystems

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# A Theory of Digital Ecosystems

## Abstract

We develop a theory of digital ecosystems built on the premise that a multi-market firm can steer users it has in one market toward its products in other markets. Due to this “cross-market leverage,” a leader in an “access-point” market (where users begin their online journeys) derives a high value from offering services in connected markets (where users continue their journeys), and can thus make profitable takeovers. Indeed, because the firm has the outside option of acquiring, and steering users toward, its target’s competitor, it can take over the target at a discount. In contrast, other firms have no or smaller incentives for takeovers, explaining why ecosystems grow out of market leaders at access points. Conversely, cross-market leverage also implies that once an ecosystem has grown, it has an increased value of controlling access points, so it may go to great lengths to dominate these markets. Our theory suggests that ecosystems have mixed implications for consumer welfare. Under plausible assumptions, a to-be ecosystem takes over market leaders, and this consolidation of good services across markets benefits consumers in the short run. But an ecosystem’s takeovers and dominance of access points lower incentives for entry and innovation, and lower the efficiency of access-point markets with superior alternatives. Hence, the long-run welfare implications of ecosystems are often negative.

Keywords: digital ecosystems, takeover, contestability, entry, envelopment, default effects, steering.

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# 1 Introduction

Digital ecosystems — firms that offer a wide range of online services — have become so dominant that they are familiar to any consumer with internet access. In this paper, we formulate a novel theory of these firms by connecting two observations that are central in policy reports as well as academic research investigating them. First, ecosystems aim to keep consumers through setting defaults, manipulating attention, or employing other means of steering. Second, ecosystems tend to develop in a major part through takeovers.

Based on the first fact, we make the core assumption that an ecosystem can use a sale in or data from one market to steer consumers toward its product in another market. For instance, Google Search preferentially lists videos from Google-owned Youtube and connections from Google Flights, surely to increase traffic to the latter services (Schechner et al., 2020).<sup>1</sup> Due to this “cross-market leverage,” a leader in an “access-point” market (where users begin their online journeys) has a high value from taking over firms that offer services in connected markets. Indeed, because the acquirer has the threatening outside option of taking over and then steering users toward a target’s competitor, it can obtain the target at a discount. We show that other firms do not have similar incentives for takeovers, explaining the second observation, and adding the prediction that ecosystems grow out of firms that control access points. Conversely, cross-market leverage also means that once an ecosystem has grown, it can have an extremely high value for attracting consumers at access points. It is therefore willing to spend tremendous resources to steer consumers toward itself in these markets, for instance by buying the default position there or developing its product to be the first mover.

Our analysis indicates that ecosystems have mixed implications for consumer welfare. Focusing on services that are socially valuable, we argue that the short-run implications are often positive, while the long-run implications are often negative. Under plausible as-

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<sup>1</sup> Or, as Mark Zuckerberg put it in a leaked email, “the more cross-linking [between services], the better” (<https://t1p.de/x3w8o>, accessed July 19, 2024).

sumptions, a to-be ecosystem takes over market leaders, and this consolidation of good services across markets benefits consumers. But an ecosystem harms existing firms through takeovers, lowering incentives for entry and innovation and thus often harming consumers. Furthermore, an ecosystem may stifle superior rivals at a new access point, or even newly superior rivals at an old access point, again potentially harming consumers.

We introduce our model in Section 2. Consumers use two services,  $a$  followed by  $b$ , that are offered by single-market firms and possibly (two-market) ecosystems. In each market, consumers are steered toward one “default” firm, which obtains an increase in its market share at the expense of other firms. In our basic model, the default at the access point  $a$  is allocated exogenously. Default determination in market  $b$ , in turn, captures cross-market leverage: if the consumer bought from an ecosystem in market  $a$ , then the ecosystem becomes the consumer’s default in market  $b$ . Otherwise, the consumer’s default in market  $b$  is selected randomly. For predictions regarding the takeover targets of to-be ecosystems and the short-run welfare effects of ecosystem growth, we also impose quality-steering complementarity, whereby firms offering a better service benefit more from the default position.

We discuss several possible microfoundations for the default firm’s benefit, such as pre-selecting the service in an app or link, including it in the consideration set of consumers with limited attention, using data from previous interactions to manipulate the consumer’s attention, or setting one’s product as the consumer’s reference point. While different in the underlying psychology, the microfoundations have some common qualitative implications for a firm’s valuation of the default position. Indeed, in Appendix A we identify plausible conditions under which the microfoundations also satisfy quality-steering complementarity.

In Section 3, we study the emergence of ecosystems from an economy with only single-market firms. Suppose that a single firm  $G$  in market  $a$  can make sequential take-it-or-leave-it acquisition offers to firms in market  $b$  until one accepts or all reject. Because of cross-market leverage,  $G$  can direct its users in market  $a$  toward the service of an acquired target, so it can make more profits than the target can by itself. But in addition to this valuation incentive

for growing,  $G$  also has a strategic advantage in the process. Namely, if a target  $t$  rejected  $G$ 's offer, then  $G$  would take over a competitor  $t'$  of  $t$ , and direct its users to  $t'$ . Because this would divert users away from  $t$ , it diminishes the position of  $t$ . As a result,  $G$  can take over any preferred target  $t$  at a discount.

To sharpen our understanding of takeover incentives, we study several alternative games. First, taking the perspective that the most profitable takeovers are the most likely to occur, we compare the profits of all cross-market acquisitions. We establish that the takeover in which the leader in market  $a$  acquires the best firm in market  $b$  is the most profitable. In particular, this forward integration is more profitable than backward integration, where the leader in market  $b$  acquires the leader in market  $a$ . The reason is that a firm in market  $b$  cannot steer users away from a firm in market  $a$ , so it cannot negotiate a takeover discount. Second, we show in an example with multiple equally valuable targets in market  $b$  that the takeover discount is not an artifact of a monopsonistic acquirer. Intuitively, with multiple acquirers the equilibrium price for takeovers is determined by the last target's value, which is low because some consumers are steered toward already acquired firms.

Taken as a whole, the above logic yields several conclusions. First and most importantly, it implies that market leaders at access points grow into digital ecosystems, and do so primarily by taking over the best available firms. To mention two of numerous examples, Google added both YouTube and Google Flights to its services through takeovers. Second, if appropriate targets are unavailable, a dominant firm at the access point has an incentive to develop products for connected markets itself. Because of cross-market leverage, it still benefits more from the new service than other firms. For example, Microsoft (a leader in operating systems) spent tremendous resources to develop its own browser and search engine. Third, cross-market leverage explains why it is increasingly tech firms rather than investors specializing in tech that make takeovers in digital markets (Eisfeld, 2024).

In Section 4, we return to the beginning of the consumer's journey, the access-point market  $a$ . We endogenize the determination of the default firm by assuming that an outside

seller auctions off this position to firms in the market. This corresponds to a situation where an original equipment manufacturer (OEM) like Apple or Samsung sets the default search engine for a fee. Other negotiation mechanisms, for instance bilateral bargaining, yield similar qualitative insights. Alternatively, the race to offer a new access-point product first may often be seen as an all-pay auction whose winner obtains the default position through prominence, again yielding similar insights.

As a benchmark, we show that with single-market firms, the best firm wins the auction, solidifying its leader position. Combining this with our previous results, we conclude that as a start, the best firm grows into an ecosystem. Once an ecosystem has developed and benefits from cross-market leverage, however, it may outbid superior rivals at the access point. Its payment for the default position can be especially high when there is a second ecosystem, and both aim to prevent the other from drawing consumers into its web of services. A striking example consistent with this prediction is the \$18-20 billion annual payment that Google makes for being the default search engine on Apple devices — which constitutes 36% of its search advertising revenue from these devices.<sup>2</sup>

In Section 5, we build on the above analysis to identify the welfare effects of both ecosystem growth and possible policies. Reflecting the core mechanism of our paper, we focus on interventions that disrupt the ability of ecosystems to steer consumers. These interventions include “leverage policies” aimed at weakening cross-market leverage, and “access-point policies” designed to regulate the sale of defaults. For instance, Articles 6(3) and 6(4) of the European Union’s Digital Market Act, which mandate easy changeability of default settings on operating systems, virtual assistants, web browsers, and app stores, can be seen

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<sup>2</sup> See, for example, <https://t1p.de/r9drd> (accessed on June 11, 2024). Google also pays \$400-\$450 million per year for the default position on Firefox (see <https://t1p.de/1gvpq> accessed on June 11, 2024). That capturing the consumer’s attention at access points is a key part of Google’s strategy is clear from Google’s internal communications as well. For instance: “There’s tremendous power in the default OS access points but it’s pay to play [...]. There is no substitute for the default access points: we should continue to explore broad default access across all OS (including newer and emerging access points)” (see, e.g., <https://www.justice.gov/d9/2023-09/416682.pdf>).



as access-point policies. Similarly, Articles 6(5) and 6(6), which prohibit access-point firms from favoring their own products through rankings or impediments to switching, can be seen as leverage policies.

On the one hand, the development of a digital ecosystem tends to increase short-run welfare by merging the best services, improving defaults and thereby consumer choices. This prediction is consistent with the observation that many consumers appear to love (steering by) ecosystems.<sup>3</sup> On the other hand, the long-run welfare effects of an ecosystem are more nuanced and often negative. First, potential inefficiencies arise as an ecosystem is willing to outbid a better rival at a new access point or a newly superior rival at an old access point. In such a case, access-point policies raise welfare in these critical markets. Second, our theory validates recent regulatory concerns about the stifling effects of ecosystems on innovation and entry (e.g., Crémer et al., 2019, Scott Morton et al., 2019, Furman et al., 2019). Because an ecosystem often outbids better rivals at the access point and cheaply acquires leaders in other markets, it lowers market “contestability” — the entry capacity of non-dominant firms — throughout the economy. Accordingly, both access-point and leverage policies can increase welfare by increasing contestability.

We conclude in Section 6 by speculating why ecosystems are less common in offline settings. While analogues do exist (e.g., tropical hotels with many services), in general offline firms appear to have less steering ability than their online counterparts.

**Related Literature** Our paper belongs to the quickly growing theoretical literature on ecosystems and the digital economy. No previous paper, however, analyzes our main question, the role of cross-market leverage and steering in the emergence of digital ecosystems. Indeed, although there are informal accounts (e.g., Eisenmann et al., 2011, Condorelli and Padilla, 2020), the only formal theories applicable to ecosystem growth are based on consumer-side (Chen and Rey, 2023) or producer-side (Condorelli and Padilla, 2023) com-

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<sup>3</sup> See, for example, <https://t1p.de/htpki> (accessed June 17, 2024).

plementarities. More closely related, Chen and Rey assume exogenously that consumers obtain a convenience or bundling benefit from buying multiple services from the same firm, which motivates conglomerate mergers. Our theory also features a consumer benefit from ecosystems, but this arises endogenously due to the nature of the mergers taking place. More generally, our microfoundation leads to different main predictions. First, while our theory predicts that takeovers will be initiated by firms at (or closer to) consumers’ access points, complementarity-based theories make no prediction in this regard. Second, while our theory predicts that ecosystems will go to great lengths to secure access-point defaults (like Google does with its astronomical payment to Apple), previous models do not make such a prediction. If multi-market firms had an inherent complementarity advantage, it is not obvious why they should spend a lot to steer consumers in their direction.

In addition, a small literature explores the implications of default effects or inertia in single-market models of the digital economy (Chen and Schwartz, 2023, Ostrovsky, 2023, Hovenkamp, 2023, Decarolis et al., 2022, Denicolò and Polo, 2024), without analyzing ramifications for ecosystems. Most closely related, Chen and Schwartz (2023) and Hovenkamp (2023) develop theories based on what we refer to as a switching-cost model of default effects. In line with our benchmark Proposition 4, they show that usually better firms have a greater willingness to pay for the default position.<sup>4</sup>

Our assumption that there is a default firm that consumers are more likely to choose can — consistent with our terminology — be seen as incorporating a generalized default effect into our model. A vast literature in behavioral economics documents that defaults matter (e.g., Madrian and Shea, 2001, Choi et al., 2004, Johnson and Goldstein, 2003, 2004, Jones, 2012, Chetty et al., 2014, Blumenstock et al., 2018, Jachimowicz et al., 2019, Brown and Previtero, 2020, Brot-Goldberg et al., 2023) and academics, policymakers, as well as firms

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<sup>4</sup> Recent theoretical research also investigates aspects of the digital economy, including pricing (e.g., Jeon et al., 2023), within-market steering (e.g., Teh and Wright, 2022, Hidir and Vellodi, 2020, Heidhues et al., 2023b), entry for buyout (e.g., Bryan and Hovenkamp, 2020), and innovation (e.g., Madsen and Vellodi, forthcoming), that are completely different from our main focus.

recognize that defaults are equally if not more important in digital settings.<sup>5</sup> Furthermore, it is universally recognized that firms have some control over consumers’ defaults, giving rise to the steering ability that is central to our model.

## 2 Basics: Firm Payoffs with Steering

We begin by specifying a model of demand when consumers are susceptible to default effects and steering. Because our main interest will be on firm behavior rather than the default effects per se, we specify firm payoffs in reduced form. In Appendix A, we provide micro-foundations for our assumptions based on the main mechanisms for default effects described in the literature. These mechanisms presume that consumers choose between products imperfectly or in a context-dependent way. First, consistent with a strict notion of defaults, a consumer may receive the default option unless she actively avoids it. Then, the default matters because searching for or switching to something else is costly. Second, in a generalization of default effects, the default option may attract a consumer’s attention. Then, the default matters because it is more likely to enter the consumer’s consideration set, become her reference point, or be taken by her as a recommendation.

### 2.1 Setup

**Basic Assumptions** There are two markets,  $a$  and  $b$ , served by firms 1 through  $n$ . Market  $a$  is an access point, such as a browser, search engine, map, or operating system, or in the future potentially a service on a wearable or self-driving car. Market  $b$  is a follow-on service. We denote by  $\mathcal{N}^s \subseteq \{1, \dots, n\}$  the set of firms offering service  $s \in \{a, b\}$ , define  $n^s := |\mathcal{N}^s|$ ,

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<sup>5</sup> Among academics, see for instance Johnson et al. (2002), Altmann et al. (2019), Fletcher and Vasas (2023), and Fletcher et al. (2023). Among policy reports, see Scott Morton et al. (2019) and Furman et al. (2019). Among firms, extensive Google memos document that default settings “can be a powerful strategic weapon”; a view that is shared by its competitors, as is evident from the testimony by Neeva, DuckDuckGo, and Microsoft managers (see <https://t1p.de/sco1p>). Accessed July 8, 2024.

and assume  $n^s \geq 2$ . We call a firm serving one market a single-market firm, and a firm serving both markets an ecosystem. Prices are fixed at zero, and a firm's profit equals its total demand across the two markets.<sup>6</sup> We normalize the size of the potential market  $a$  to 1.

In each market  $s$ , consumers are steered toward the product of one firm, which we call the default firm. Firm  $i$  obtains exogenously given demand  $q_{ij}^s \geq 0$  in market  $s$  when firm  $j$  is the default in market  $s$ . Throughout the paper, we maintain:

**Assumption 0** (Steering). For all  $i$  and  $j \neq i$ , we have  $q_{ii}^s > q_{ij}^s$ .

Assumption 0 can be thought of as the definition of steering or default effects in our context: that having the default position is beneficial for a firm. All of our microfoundations in Appendix A imply choice behavior satisfying this assumption.

For now, we assume that the default in market  $a$  is assigned exogenously, and endogenize this selection in Section 4. At the same time, firms' probabilities of becoming the default in market  $b$  depend in the following way on the outcome in market  $a$ . First, any ecosystem is allocated a probability equal to its market- $a$  demand. Second, the probability that remains is divided equally between all firms in market  $b$  (including ecosystems). These assumptions capture the cross-market leverage that results if consumers use service  $b$  after service  $a$ . If a consumer used an ecosystem in market  $a$ , then the ecosystem sets itself as the default in market  $b$  for that consumer. If, however, the consumer used a single-market firm in market  $a$ , then no firm in market  $b$  is systematically favored.

**Quality-Steering Complementarity** For some of our results, we impose more structure on the effect of steering on demand. Such structure is not necessary for our main positive results, that ecosystems grow from leaders at access points through takeovers, and that they have a high value from controlling access points. The additional structure is necessary,

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<sup>6</sup> This is consistent with the common situation in digital markets, where a firm benefits not from direct sales to consumers, but from collecting data and/or showing ads.

however, for our predictions regarding the takeover targets of to-be ecosystems and the short-run welfare effects of ecosystem growth.

We define firm  $i$  as “better” or “higher-quality” than firm  $j$  in market  $s$  if

$$\sum_{\ell \in \mathcal{N}^s} q_{i\ell}^s > \sum_{\ell \in \mathcal{N}^s} q_{j\ell}^s,$$

i.e.,  $i$ ’s demand when the default is assigned randomly and with equal probability is strictly higher than  $j$ ’s. This definition reflects another common property of all of our microfoundations: if no firm is favored on average, then demand indicates quality. In that case, even a boundedly rational consumer is more likely to choose a better option.

**Assumption 1** (Quality-Steering Complementarity). Let  $i, i', j, j' \in \mathcal{N}^s$ .

- I. If  $j$  is better than  $j'$ , then  $q_{ij'}^s \geq q_{ij}^s$  for all  $i \neq j, j'$ .
- II. If  $i$  is better than  $i'$ , then  $q_{ii}^s - q_{ij}^s > q_{i'i'}^s - q_{i'j}^s$  for all  $j \neq i, i'$ .
- III. If  $i$  is better than  $j$ , then  $q_{ii}^s - q_{ij}^s > q_{jj}^s - q_{ji}^s$ .

Broadly, Assumption 1 imposes that better firms benefit more from holding the default position than worse firms. Specifically, a better default attracts more consumers from all options, lowering any competitor’s demand (Part I), gaining more than a worse firm from replacing a given rival as the default (Part II), and benefiting more from replacing a worse firm as the default than vice versa (Part III).

In Appendix A, we identify plausible conditions under which our microfoundations lead to consumer behavior satisfying Assumption 1. Depending on the microfoundation, this is either always the case, or if the default effect is sufficiently weak. Intuitively, a better firm often benefits more from drawing a boundedly rational consumer’s attention, as it induces the consumer to take into account the firm’s quality in her choice.

**Discussion of Setup** For transparency and tractability, our model makes some unrealistic simplifying assumptions. First, while we have posited just one access point and one follow-on

market, actual online journeys are varied and often more complex. For example, there may be multiple possible access points, multiple possible follow-on markets after a given access point, and further follow-on markets after the first ones. In addition, consumers may use the same services in different orders. These alternatives add an obvious and realistic qualification to our results: that ecosystems will consist not just of two, but of many connected services. Nevertheless, our basic points continue to hold if there are a few central services that a disproportionate number of consumers start with.

Second, it seems plausible that even beyond sales in market  $a$ , a prominent firm in market  $b$  is systematically favored. For example, consumers may use the leading search engine simply because its name has become a synonym for search. Our results continue to hold as long as ecosystems can steer consumers across markets.

Third, our model ignores two considerations that are commonly considered central for the dominance of large digital firms (see Calvano and Polo, 2021, for a review): network effects and economies of scope in data. Network effects — that more users make the service better — apply almost exclusively within market, while all of our results are driven by cross-market effects. Economies of scope in data — that combining consumer data from different markets may increase conversions for advertisers (e.g. Ullrich et al., 2024) — are, we show, unnecessary for digital ecosystems to emerge. To go further, the data view does not explain our main results regarding the direction of ecosystem growth or the tremendous effort ecosystems expend to secure access points. Still, our findings are not only consistent with, but typically reinforced by these effects, which increase the value of large firms.

## 2.2 Preliminaries: Default Advantage and Default Externality

As a useful step for our analysis, we decompose steering effects. We define firm  $i$ 's *default advantage* as its gain in demand when it rather than a randomly chosen firm is the default:

$$\alpha_i^s := q_{ii}^s - \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} q_{i\ell}^s.$$

Relatedly, we define firm  $j$ 's *default externality* on firm  $i$  as the change in firm  $i$ 's demand when firm  $j$  rather than a randomly chosen firm is the default:

$$\eta_{ij}^s := q_{ij}^s - \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} q_{i\ell}^s.$$

Using Assumption 0, it is easy to establish the following.

**Lemma 1** (Default Advantage and Externalities).

- I. For any firm  $i \in \mathcal{N}^s$ , we have  $\alpha_i^s > 0$ .*
- II. For any firm  $i \in \mathcal{N}^s$ , there exists another firm  $j \neq i$  such that  $\eta_{ij}^s < 0$ .*

Part I restates the premise that the default position is valuable. Part II says that any firm has a rival that imposes a negative default externality on it. Intuitively, if firm  $i$  benefited from each competitor's default position, then the default position would be undesirable, contradicting Assumption 0.

### 3 The Emergence of Ecosystems

In this section, we show that cross-market leverage leads to the emergence of ecosystems through — primarily — takeovers. Since for now the default in market  $a$  is exogenously given, we suppress it in the notation for a firm's market- $a$  demand.

#### 3.1 Takeovers in the Shadow of Cross-Market Leverage

To start, suppose that all firms are single-market firms. Now one firm in the access-point market  $a$ , firm  $G$ , can make sequential take-it-or-leave-it offers to the firms in market  $b$  until one accepts or all reject. We look for subgame-perfect equilibrium outcomes.

As a point of comparison, notice that a target  $t$ 's “standalone value,” i.e., its demand in

the absence of a takeover, is

$$V_t^b := \frac{1}{n^b} \sum_{\ell \in \mathcal{N}^b} q_{t\ell}^b.$$

Without cross-market leverage, this is how much  $G$  would value  $t$ , and also how much  $t$  would be willing to sell for. Hence, in that case the profit from a takeover would be zero.

**Proposition 1** (Takeover Discounts and Profits).

*I. In any subgame-perfect equilibrium of the takeover game, there are  $t_1, t_2 \in \mathcal{N}^b$  such that (i)  $G$  takes over  $t_1$  at a price  $V_{t_1}^b - q_G^a |\eta_{t_1 t_2}^b|$ , thereby making a profit of  $q_G^a (\alpha_{t_1}^b + |\eta_{t_1 t_2}^b|)$ ; and (ii)  $t_1$  and  $t_2$  solve  $\max_{t'_1, t'_2 \in \mathcal{N}^b} \alpha_{t'_1}^b - \eta_{t'_1 t'_2}^b$ .*

*II. Under Assumption 1,  $t_1$  and  $t_2$  are firms with the best and second-best service  $b$ .*

The proposition states that  $G$  makes a profitable takeover. Its profit is due to two considerations that arise from cross-market leverage. First, straightforwardly, if  $G$  acquires a target  $t_1$ , then it can use its cross-market leverage to raise  $t_1$ 's demand. Hence,  $G$  values  $t_1$  above  $t_1$ 's standalone value. Second, less obviously,  $G$  obtains a “takeover discount” — it can buy  $t_1$  below  $t_1$ 's standalone value. Intuitively, in equilibrium  $t_1$  knows that if it did not sell to  $G$ , then another target  $t_2$  would. Cross-market leverage means that this would raise  $t_2$ 's demand and lower  $t_1$ 's demand, creating a threat that weakens  $t_1$ 's bargaining position. Furthermore, in equilibrium the firms  $t_1$  and  $t_2$  maximize  $G$ 's takeover profit.<sup>7</sup>

In general,  $t_1$  and  $t_2$  could be any two firms in market  $b$ , but under Assumption 1, they are the best firms in the market. Then,  $G$  buys the best firm because this firm benefits the most from the default position, and hence is the most profitable firm to steer consumers to. In addition, the second-best firm is the best threat because (by Part I of Assumption 1) making it the default attracts the most consumers away from the best firm.

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<sup>7</sup> Our proof establishes the last result by noting that  $G$  can first make low-ball offers to all firms but  $t_1$  and  $t_2$ , and once these offers are rejected, approach  $t_1$ . Then,  $t_1$  knows that if it rejected the offer,  $G$  would take over  $t_2$ . This logic of course relies on the assumption that  $G$  cannot approach firms that previously rejected it. If  $G$  had less commitment power regarding which firms it will approach after getting rejected by  $t_1$ , its profit could be lower, but the qualitative logic of Proposition 1 would still hold; indeed, often there is also a subgame-perfect equilibrium in which  $G$  approaches its preferred target first.



Having analyzed takeovers by an exogenously given firm, we now explore which firm is most likely to initiate a takeover. To do so, we calculate the expected profit each firm can earn in a takeover game of the type above, where it makes sequential take-it-or-leave-it offers to the firms in the other market. Because a takeover is costly to prepare and execute, the firm with the most to gain is the most likely acquirer. Similarly, if a law firm or consultancy organizes the acquisition and reaps part of the profits, it also has an interest in bringing about the most profitable takeover.

**Proposition 2** (Profits from Forward vs. Backward Integration).

*I. Consider any firm  $i$  in market  $b$ . The highest profit it can earn is through taking over the leader  $\ell$  in market  $a$ . It can do so at a price  $V_\ell^a$ , making an additional profit of  $q_\ell^a \alpha_i^b$ .*

*II. The leader  $\ell$  in market  $a$  can earn strictly more from a takeover than  $q_\ell^a \alpha_i^b$ , and it can earn weakly more than any other firm in market  $a$ .*

Part I identifies the profits from “backward integration” — when a firm in market  $b$  expands into market  $a$ . Cross-market leverage allows the firm to direct some of the target’s consumers to itself, so it makes a profit from such a takeover. In doing so, however, the firm does not obtain a takeover discount. Intuitively, taking over the target’s competitor does not decrease the target’s demand, so it is not threatening. Accordingly, Part II says that the profit from backward integration is lower than that from “forward integration” — when a firm in market  $a$  expands into market  $b$ . Overall, the firm with the greatest incentive to make a takeover is the leader in market  $a$ .

Propositions 1 and 2 imply that market leaders at access points, like Google or Microsoft, are prone to growing into ecosystems through profitable mergers. Both specific examples and general observations are consistent with our story. For instance, Google took over Youtube and added it to its services, Google took over Where 2 Technologies and developed it into Google Maps, Google took over ITA Software to create Google Flights, and Microsoft took

over Hotmail and turned it into Outlook.<sup>8</sup> More generally, despite the growing importance of acquisitions by financial firms elsewhere (e.g., Vild and Zeisberger, 2014), strategic mergers are common in digital markets (Eisfeld, 2024).<sup>9</sup> Furthermore, consistent with our takeover discount, digital firms “tend to acquire companies at lower prices” than financial investors (Eisfeld, 2024, p. 57) — who have no threatening outside option.

The logic of our model also allows us to make further observations. First, suppose that for exogenous reasons firms in market  $b$  are not willing to sell, but the leader  $G$  in market  $a$  can internally develop service  $b$  at cost  $c_{\text{int}}$ , while an outsider can develop the same-quality service at cost  $c_{\text{ext}}$ . For simplicity, we impose that the outsider makes the development decision first, and if it does not develop, then  $G$  can do so. In this situation, the outsider develops the service if its standalone value exceeds  $c_{\text{ext}}$ . Otherwise it does not, in which case  $G$  develops the service if the same standalone value plus  $G$ ’s default advantage exceeds  $c_{\text{int}}$ . Hence,  $G$  may develop the service even if  $c_{\text{int}} > c_{\text{ext}}$ . In this sense,  $G$  has a larger incentive than other firms to develop new services in adjacent markets. Without cross-market leverage, in contrast,  $G$  would never be the one developing the service if  $c_{\text{int}} > c_{\text{ext}}$ . Consistent with our prediction, for instance, Microsoft spent tremendous resources to develop and improve its browser (first Internet Explorer, now Edge), Google developed its email service Gmail in house, and Uber developed its food delivery service UberEats.<sup>10</sup>

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<sup>8</sup> See, for example, <https://t1p.de/lsvbk> (YouTube), <https://t1p.de/86x05> (Google Maps), <https://t1p.de/aia1s> (Google Flights), and <https://t1p.de/w3dhr> (Outlook). Other examples include Google taking over Upstartle and 2Web Technologies to create Google Docs and Google Sheets (<https://t1p.de/med6m>); Apple buying the MP3 player SoundJam MP to create iTunes (<https://t1p.de/vfuyv>); Microsoft extending its range of services by taking over Skype and LinkedIn (<https://t1p.de/1dddj> and <https://t1p.de/g9bwj>); Amazon acquiring Audible, LoveFilm (now Amazon Prime Video), and Twitch (<https://t1p.de/52vaf>, <https://t1p.de/7cpmu>, and <https://t1p.de/4p5ue>); Meta taking over WhatsApp and Instagram (<https://t1p.de/g98rr>). All links accessed on June 6, 2024.

<sup>9</sup> The observation that digital ecosystems grow from access points into adjacent markets through takeovers is not restricted to western societies. The two dominant ecosystems in China, Alibaba and Tencent, grew out of Alibaba’s dominant marketplace (Taobao) and Tencent’s dominant social media and messaging service (WeChat) into ecosystems that offer a broader range of services (Prüfer et al., 2024).

<sup>10</sup> See <https://t1p.de/l6y2x> and <https://t1p.de/a5egg> (Microsoft), (<https://t1p.de/86x05>) (Google), and <https://t1p.de/2u84f> (Uber). Notably, also Apple and Google developed their own web browsers (<https://t1p.de/pqiz6> and <https://t1p.de/69unz>). All links accessed June 6, 2024.

Second, notwithstanding its incentive to self-develop service  $b$  if necessary, firm  $G$  often prefers to acquire the service through a takeover instead. In particular, suppose that  $G$  could hire a team and self-develop the service, or let the team start a company offering the service and take over the company. Firm  $G$  prefers the takeover if  $c_{\text{int}}$  is above the takeover price. Hence,  $G$  may use a takeover even if  $c_{\text{int}}$  is well below the profits it can make with the service. Again, without cross-market leverage this would never be the case. Note, however, that even if firm  $G$  makes a takeover in equilibrium, its option to self-develop the product remains relevant. Much like taking over a competitor, this can serve as a threatening outside option that lowers the preferred target's takeover price.<sup>11</sup>

Third, while we simplify things by assuming two markets, the logic of our results applies equally or even more strongly to the kinds of more complex situations we have mentioned in Section 2.1. As one relevant extension, there may be multiple markets that consumers want to use immediately after the access point. Then, the iterated application of Propositions 1 and 2 implies that the leader at the access point takes over firms in each of the other markets. Alternatively, there may be further services that consumers want to use after market  $b$ . In particular, consider a situation in which the consumer uses another service  $c$  after service  $b$ , and for simplicity suppose that market  $c$  emerges after markets  $a$  and  $b$  have been established. Applying Propositions 1 and 2 first to markets  $a/b$  says that the leader in market  $a$  takes over a firm in market  $b$ . Now applying the same logic to markets  $b/c$  says that if the new ecosystem is the leader in market  $b$ , it will take over a firm in market  $c$ . Indeed, the ecosystem will be the leader in market  $b$  if cross-market leverage is sufficiently strong, or if it was created through the takeover of a sufficiently good firm in market  $b$ . Under Assumption 1, in particular, the ecosystem merges the best firms in markets  $a$  and  $b$  (Proposition 1, Part II), so it will always be the leader in market  $b$ . In that case, it expands further by taking over the leader in market  $c$ .

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<sup>11</sup> Relatedly, Katz (1987) argues that large downstream buyers can more credibly threaten to enter an input market in the presence of scale economies, and hence get offered a lower input price even absent entry.

Fourth, consider the possibility that (for exogenous reasons) forward integration is not feasible, but backward integration is. This may be the case, for instance, when there are already ecosystems in place, and a new access point with small single-market firms emerges. For practical reasons, such as financing or know-how, it seems implausible that a small startup would take over an ecosystem like Google or Meta. Then, since backward integration is profitable (Proposition 2, Part I), it takes place instead. Consistent with this prediction, Google took over Fitbit and Meta acquired Oculus VR.<sup>12</sup>

Note that in our analysis, we have abstracted from the option of employing between-firm contracts instead of making takeovers. Our motivation is similar to that of many previous researchers studying vertical relations: that designing and enforcing contracts — especially in highly dynamic digital markets — can be difficult, rendering takeovers the superior option. If steering is based on choice architecture, for example, firms would have to agree every time the service in market  $a$  adapts its customer interface. Similarly, a firm improving its service in market  $b$  would face a hold-up problem unless the steering price is credibly fixed. Furthermore, in the plausible scenario with further markets following market  $b$ , a contracting solution would have to balance the interests of three or more parties.

### 3.2 Multiple Acquirers and Takeover Prices

Above, we have assumed that a single firm can make takeover bids. We now show through an example that the presence of multiple potential acquirers — which one would hope introduces competition for takeovers — does not necessarily protect targets from low takeover prices.

Consistent with Proposition 2, we focus on forward integration. Market- $a$  firms in  $\mathcal{A} \subseteq \mathcal{N}^a$  can make takeovers, and they differ in their market- $a$  demands. All market- $b$  firms are potential takeover targets, and they offer an equally good service with standalone value  $V^b$  and default externality  $\eta^b$ . We denote by  $\bar{\mathcal{A}}(x)$  the  $x$  firms in  $\mathcal{A}$  with the highest market- $a$

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<sup>12</sup> See <https://t1p.de/o9wui> (Google) and <https://t1p.de/p8kvb> (Meta), both accessed June 19, 2024.

demands, and set  $\bar{\mathcal{A}}(x) = \mathcal{A}$  for  $x > |\mathcal{A}|$ . In every round of the game, one of the remaining targets is offered for sale, and each potential acquirer that has not yet made a takeover can submit a takeover offer. The target can accept any offer. The game ends when all potential targets have been up for sale.

We characterize subgame-perfect equilibrium outcomes satisfying a recursive variant of the convention that takeover offers are cautious, i.e., not above the profits from the takeover. Formally, if a subgame following current offers has a unique cautious subgame-perfect equilibrium outcome, we require firms to use iteratively weakly undominated strategies in the reduced game in which these subgames are replaced by their equilibrium payoffs.<sup>13</sup>

**Proposition 3** (Multiple Acquirers). *In any equilibrium with cautious offers, firms in  $\bar{\mathcal{A}}(n^b)$ , but no other firms, make a takeover. The equilibrium takeover price is  $f^* < V^b$  satisfying*

$$f^* = \begin{cases} V^b - |\eta^b| \sum_{i \in \bar{\mathcal{A}}(n^b-1)} q_i^a & \text{if } n^b \geq |\mathcal{A}|, \\ V^b - |\eta^b| \sum_{i \in \bar{\mathcal{A}}(n^b-1)} q_i^a + \alpha^b \max_{j \in \mathcal{A} \setminus \bar{\mathcal{A}}(n^b-1)} q_j^a & \text{if } n^b < |\mathcal{A}|. \end{cases} \quad (1)$$

Proposition 3 says that even with multiple acquirers, the acquirers can always take over their targets at a price below the targets' standalone value. This means that the possibility of takeovers always harms targets. Intuitively, other acquirers not only introduce competition for an acquiring firm, but also affect the value of targets. Specifically, on the equilibrium path, the last target taken over is less valuable to an acquirer, because the ecosystems created in preceding takeovers will steer consumers away. Furthermore, in equilibrium the price of all takeovers is determined by this marginal acquisition, and hence is low.

The above logic implies that with an abundance of targets ( $n^b > |\mathcal{A}|$ ), targets are actually made worse off with an increase in the number of acquirers. In that case, other acquirers lower a target's value without inducing competition. Even if there are more acquirers than targets ( $n^b < |\mathcal{A}|$ ), the decrease in value due to other takeovers outweighs the price-increasing

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<sup>13</sup> Requiring *iteratively* undominated strategies rules out that takeover bids are driven by the fear that a low-demand firm in  $a$  bids above its possible additional profits.

effect of competition. For an intuition, notice that non-acquirers have lower demand than acquirers. With identical targets, this means that the marginal non-acquirer has fewer consumers to steer, so that the premium it is willing to pay for a takeover is lower than the decrease in value other acquisitions have caused.

## 4 Default Setting at the Access Point

Having studied the emergence of ecosystems, we now investigate the implications of ecosystems for steering incentives at the access point. Our main motivation is the fact that digital firms often use large payments or other methods to influence default settings for basic services on digital devices.<sup>14</sup> But a firm may also attempt to steer at the access point in less direct ways. It may, for example, spend extra resources to develop a new access-point service, e.g., one on self-driving cars or smart wearables, and thus reap a first-mover advantage.

Formally, we assume that the default position in market  $a$  is sold through a sealed-bid second-price auction. We look for cautious Nash equilibria, in which all firms use a weakly undominated strategy.<sup>15</sup> For stating our results, we define  $\Delta_{ij}^a := q_{ii}^a - q_{ij}^a \in (0, 1)$ , which is a single-market firm  $i$ 's willingness to pay to replace  $j$  as the default in market  $a$ .

### 4.1 Benchmark: Single-Market Firms

As a benchmark, we suppose that all firms are single-market firms.

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<sup>14</sup> Chris Barton (Google): “We need to incentivize carriers to ship Google by using the same approach we at Google have used for many years: ‘We will pay you revenue share in return for exclusive default placement.’ This contract is an exchange.” (internal email, April 2011, <https://www.justice.gov/d9/2023-09/416302.pdf>). Going further, as Heidhues et al. (2023a) explain, Google enforces its default position on Android devices through a nexus of usage contracts for the operating system (Google’s Android) and “must-have” apps (e.g., Google Play Store or Google Maps).

<sup>15</sup> It is well-known that the sealed-bid second-price auction has a continuum of equilibria in weakly dominated strategies both with independent private values as well as known values (Blume and Heidhues, 2004).

**Proposition 4** (Superior Defaults). *If Assumption 1 holds, then a firm  $i \in \mathcal{N}^a$  that offers the best service  $a$  wins the auction for the default position, and it pays at most  $\max_{j \in \mathcal{N}^a \setminus \{i\}} \Delta_{ji}^a$ .*

At least under Assumption 1, a top-quality firm becomes the default. Such a firm attracts more consumers through winning the default position than do other firms, so it has the highest value for the position. Chen and Schwartz (2023) and Hovenkamp (2023) establish similar results in related settings.

## 4.2 Ecosystems and Access Points

We turn to our main interest, how ecosystems bid for access points. For simplicity, we suppose that there are two firms in market  $a$ ,  $G$  and  $M$ , of which  $G$  is an ecosystem, and  $M$  may be a single-market firm or an ecosystem.

**Proposition 5** (Potential for Inferior Defaults).

*I. Suppose that  $M$  is a single-market firm. If  $\Delta_{GM}^a(1 + \alpha_G^b) > \Delta_{MG}^a$ , then  $G$  wins the auction for the default position with probability 1, and pays  $\Delta_{MG}^a$ .*

*II. Suppose that  $M$  is an ecosystem, and let  $\eta_{GM}^b, \eta_{MG}^b < 0$ . If*

$$\Delta_{GM}^a(1 + \alpha_G^b + \eta_{MG}^b) > \Delta_{MG}^a(1 + \alpha_M^b + \eta_{GM}^b),$$

*then  $G$  wins the auction for the default position with probability 1, and pays*

$$\Delta_{MG}^a(1 + \alpha_M^b) + \Delta_{GM}^a|\eta_{MG}^b|.$$

By standard arguments,  $G$  wins the auction if it has a higher valuation for the default position than  $M$  does; and if  $G$  wins, it pays  $M$ 's valuation. To understand the proposition, therefore, we discuss  $G$ 's valuation;  $M$ 's valuation is either  $\Delta_{MG}^a$  like in the benchmark case, or analogous to  $G$ 's valuation.

If  $G$  rather than  $M$  becomes the default in market  $a$ , its market- $a$  demand rises by  $\Delta_{GM}^a$ . Because of cross-market leverage, this increases its profit in market  $b$  by  $\Delta_{GM}^a \alpha_G^b$ , where  $\alpha_G^b$

is its default advantage in market  $b$ . Hence, if  $M$  is a single-market firm (Part I),  $G$ 's total valuation for the default position in market  $a$  is  $(1 + \alpha_G^b)\Delta_{GM}^a$ . Relative to a single-market firm, therefore,  $G$ 's valuation is increased by the “default multiplier”  $1 + \alpha_G^b$ . In an extension of our model in which  $G$  offers services in many markets, it earns extra profits in all those markets too. We can capture such a situation in reduced form by assuming that  $G$ 's default advantage  $\alpha_G^b$  is large, leading to a large default multiplier.<sup>16</sup>

If  $M$  is another ecosystem (Part II), then  $G$ 's valuation for the default position in market  $a$  is even higher. In that case, if  $M$  wins the auction and thereby raises its market- $a$  demand, its cross-market leverage hurts  $G$  in market  $b$ . Preventing this provides an extra motive for  $G$  to snatch the default position in market  $a$  from  $M$ .

Two economically important implications follow. First, even under Assumption 1,  $G$  may win the default position in market  $a$  despite offering the worse service there. Recall that under Assumption 1,  $G$  having the worse service  $a$  is equivalent to  $\Delta_{GM}^a < \Delta_{MG}^a$ . Hence,  $G$  may win the auction against a better single-market firm if it has a large default multiplier;

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<sup>16</sup> More concretely, consider the extensions discussed in Section 3.1. Suppose that, in addition to the access point, there are markets  $b_1, \dots, b_S$ , and the multi-market firm  $G$  is active in all of these markets. First, suppose that each consumer wants to use exactly one service  $b_s$  upon leaving the access point, but different consumers demand different services. For example, some consumers search for vacation spots (market  $a$ ) and book a flight afterwards (market  $b_1$ ) while others search for yesterday's game (market  $a$ ) to watch its highlights (market  $b_2$ ). Let  $w_i \in (0, 1)$  be the share of consumers interested in service  $b_i$ . Firm  $G$ 's willingness to pay for the default position in market  $a$  is then

$$\left(1 + \sum_{s=1}^S w_s \alpha_G^{b_s}\right) \Delta_{GM}^a.$$

Second, suppose that upon leaving the access-point market all consumers want to use all other services in a (for simplicity) fixed and identical sequence. For example, a consumer may search for an interview about yesterday's game (market  $a$ ) that she translates to her mother tongue (market  $b_1$ ) and then emails the translation to a friend (market  $b_3$ ). In this case,  $G$ 's willingness to pay for the default in market  $a$  is

$$\left(1 + \sum_{s=1}^S \prod_{m=1}^s \alpha_G^{b_m}\right) \Delta_{GM}^a.$$

Either way, firm  $G$ 's willingness to pay for the default strictly increases in the number of services it offers.



and it may win against a better ecosystem if it offers a better product in market  $b$ ,<sup>17</sup> or — considering the extension to many markets discussed above — is active in more markets. As we discuss in Section 5, this result has ramifications for welfare.

Second,  $G$  may make an extremely large payment for the default position. For a single-market firm  $M$ , this is the case if  $M$  is very good. For an ecosystem  $M$ , the same is the case also if cross-market steering is very effective. This result explains Google’s astronomical payments for the default position in the market for search (where it competes, for instance, against Microsoft).<sup>18</sup> The result also explains why Google is spending tremendous resources to develop a self-driving car, which is considered a major new access point in the making.<sup>19</sup>

Proposition 5 analyzes what happens if  $G$  competes either against a single-market firm or against an ecosystem. An interesting additional issue arises if  $G$  competes against both types of firms at the same time, and the single-market firm offers the best service  $a$ . Then,  $G$ ’s willingness to bid more when competing against another ecosystem can generate multiple equilibria. In one equilibrium, the ecosystems bid high in an effort to prevent the other ecosystem from winning. Then, an ecosystem wins and pays a lot. In the other equilibrium, the single-market firm wins, and the ecosystems bid less aggressively exactly because they are not worried about the other ecosystem winning. Then, the winner pays a lower amount.

The above logic implies that despite having no direct interest in either market, an OEM may want to exclude good single-market competitors from bidding for the default position at the access point.<sup>20</sup> By doing so, the OEM guarantees that the ecosystems compete against each other, ensuring itself the profits from the high-price equilibrium above.

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<sup>17</sup> Under Assumption 1 (Part III),  $q_{GG}^b - q_{GM}^b = \alpha_G^b - \eta_{GM}^b > \alpha_M^b - \eta_{MG}^b = q_{MM}^b - q_{MG}^b$  if and only if  $G$  offers a better product than  $M$ . Hence,  $G$  needs to offer a better service than  $M$  for  $\alpha_G^b + \eta_{MG}^b > \alpha_M^b + \eta_{GM}^b$ .

<sup>18</sup> See, for example, <https://t1p.de/r9drd> (accessed June 7, 2024).

<sup>19</sup> See, for example, <https://t1p.de/uo5sm>. Other examples include Google developing — partly, through takeovers — a smart TV platform that “amalgamates various streaming services, live TV, and a plethora of apps, all accessible through a single, user-friendly interface” (<https://t1p.de/1aetd> and <https://t1p.de/khbht>). All links accessed July 3, 2024.

<sup>20</sup> See Ostrovsky (2023) for how an auction designer who does have a vested interest in the access-point market can subtly manipulate the auction to generate more desirable outcomes.

## 5 Welfare and Policy Implications

Following high-profile reports in the US (Scott Morton et al., 2019), UK (Furman et al., 2019), and EU (Cr  mer et al., 2019), policymakers have proposed a variety of laws aiming to curb the power of “big tech.” For example, Articles 6(5) and 6(6) of the European Union’s Digital Market Act (DMA) prohibit ecosystems from giving their own complementary services preferential treatment, and Articles 6(3) and 6(4) limit the use of default settings on operating systems, virtual assistants, web browsers, and app stores.<sup>21</sup> Indeed, EU consumers now often see a choice screen when first using a browser on a new mobile device. There has also been discussion of either breaking up big-tech firms or limiting their ability to engage in further takeovers.<sup>22</sup>

Motivated by these initiatives, we identify the welfare implications of ecosystems, and ask whether policy can improve outcomes. For the former question, we compare consumer welfare when an ecosystem has developed through a merger to consumer welfare in the pre-merger situation. For the latter question, we focus on interventions that interfere with steering by ecosystems, considering “leverage policies” — weakening cross-market leverage — as well as “access-point policies” — disallowing default setting at the access point based on a payment. Articles 6(5) and 6(6) of the DMA mentioned above, as well as breaking up ecosystems or preventing further takeovers, can be seen as leverage policies. Articles 6(3) and 6(4) can be seen as access-point policies.

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<sup>21</sup> The steering of consumers is also restricted through various other EU regulations, including the EU’s Digital Service Act (e.g., Art. 25), Data Act, AI Act, as well as the Unfair Commercial Practices Directive (for an overview see Busch and Fletcher, 2024). These aim to protect consumers from online choice architecture — sometimes referred to as “dark patterns” — that misleads or otherwise induces the consumer to act in the firm’s rather than her own interest. Similarly, China’s drafted guidelines would restrict dominant ecosystems’ ability to give its own services preferential treatment (Pr  ufer et al., 2024).

<sup>22</sup> The proposal to break up big tech companies has been, for example, a central part of Senator Warren’s campaign for US President (e.g., <https://t1p.de/rtm8k>), and the US Justice Department currently considers the option to break up Google’s ecosystem after a recent court ruling found that Google monopolized the search market (e.g., <https://t1p.de/bl112>). Scott Morton et al. (2019) argue for scrutinizing big tech acquisitions more heavily. On this topic, see also Cabaral et al. (2021). Both links accessed August 27, 2024.

**Assumptions** We restrict attention to valuable services (e.g., search, maps, or the operating system of a self-driving car) that are optimal for a person to use. Hence, our analysis does not apply to services (e.g., social networks) that may be overconsumed.<sup>23</sup> For the latter products, regulating the harm directly appears to be a natural policy prescription.

We also assume that consumers benefit on average from a better default. Excluding models in which consumers overestimate the cost of switching away from the default, all of our microfoundations satisfy this assumption (see Appendix C). If the assumption is not satisfied, then forcing choice through a bad default, choice screen, or other method can raise welfare. Finally, we impose quality-steering complementarity (Assumption 1), and for simplicity, we assume that there is a single ecosystem  $G$ .

**Implications** The short-run welfare effects of a digital ecosystem are positive. Propositions 1 and 2 imply that the best firm in the access-point market  $a$  takes over the best firm in market  $b$ ; and the logic of Propositions 4 and 5 implies that the resulting ecosystem obtains the default position in market  $a$ . As a result, the probability that the best firm becomes the default increases in each market, benefiting consumers. Accordingly, both leverage policies and access-point policies lower short-run welfare.

The long-run welfare effects of a digital ecosystem, however, are more complex and often negative. For starters, Proposition 5 implies potential inefficiencies at a new access point, or when another firm becomes better than the ecosystem at an existing access point. In that situation,  $G$  (but not a single-market firm) may be willing to outbid others despite having a worse product. Hence, both leverage and access-point policies can improve welfare in the access-point market  $a$ . If  $G$  still has the best product in market  $b$ , however, such policies do come at the cost of lowering welfare there. This is because both policies lower the probability that  $G$  becomes the default in market  $b$ .

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<sup>23</sup> See Allcott et al. (2020) for empirical evidence that the average user spends too much time on Facebook, and Bursztyn et al. (2024) for experimental evidence suggesting that a large share of TikTok and Instagram users would be better off if these platforms did not exist.

More importantly, an ecosystem generates several barriers to the contestability of markets — the ability of non-dominant firms to make sufficient profits to overcome barriers to entry.<sup>24</sup> To study this question, we assume (in line with Propositions 1 and 2) that  $G$  offers the best existing service in both markets, thus imposing a negative default externality on its rivals, and consider a single-market potential entrant  $e$  that has a better product than does  $G$ . The message that ecosystems reduce contestability, and policies can help, emerges from investigating several environments.

First, we study contestability in the access-point market  $a$ . By the logic of Propositions 4 and 5,  $G$  is the highest bidder in the default auction absent entry, and it bids higher than if it was a single-market firm. Hence, ecosystem growth reduces contestability. Conversely, the logic of Proposition 5 implies that leverage policies lower  $G$ 's bid in the default auction, so they increase contestability. In the extreme case in which the policy eliminates all cross-market leverage, it ensures that  $e$  wins the auction and lowers the amount  $e$  pays conditional on winning.<sup>25</sup> More subtly, access-point policies also raise contestability unless  $e$  has a much better product than  $G$ . For concreteness, suppose that under the policy the default in market  $a$  is assigned randomly. Then, even if  $e$  wins upon entering, it benefits from the auction only if its default advantage  $\alpha_e^a$  compensates for the required payment. This can happen only if  $e$ 's product is so much better than  $G$ 's that  $\alpha_e^a/\alpha_G^a$  exceeds  $G$ 's default multiplier  $1 + \alpha_G^b$ .

Second, we study contestability in the follow-on market  $b$ . Because an ecosystem is more likely than its constituent firm to become the default in market  $b$ , it reduces  $e$ 's demand conditional on entry, so it lowers contestability. Mitigating this effect, both access-point and leverage policies lower the probability that  $G$  becomes the default in market  $b$ , so both types of policy raise  $e$ 's profit conditional on entry and thus raise contestability. In the extension of our model where additional markets are connected to market  $b$ , further implications arise.

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<sup>24</sup> Contestability is defined in Recital 32 of the DMA as “[...] ability of undertakings to effectively overcome barriers to entry and expansion and challenge the gatekeeper on the merits of their products and services.”

<sup>25</sup> Absent the policy,  $G$  wins the auction against a superior entrant if  $\Delta_{Ge}^a(1 + \alpha_G^b) > \Delta_{eG}^a$ . And if the entrant wins the auction absent the policy, it pays  $\Delta_{Ge}^a(1 + \alpha_G^b)$ , while with the policy it pays  $\Delta_{Ge}^a$ .

Since  $G$  derives a default advantage in market  $b$ , both types of policy lower the probability that  $G$  becomes the default in a connected market. As a result, both types of policy encourage entry also in connected markets. This effect is reinforced if  $e$  indeed enters market  $b$ , taking further market share from  $G$  and thus creating a “double dividend” to encouraging entry.

Finally, we discuss contestability in markets in which the ecosystem is not yet active, again distinguishing (new) access-point markets from follow-on markets. Consider an entrant  $e$  that decides whether or not to enter a market before  $G$  can take over one of the then existing firms. As we have discussed in Section 3.1, an ecosystem has a strong incentive to expand through backward integration into a new access point, so an entrant in such a market may be able to sell expensively. Leverage policies, therefore, initially lower contestability in new access-point markets. By contrast, Proposition 1 implies that conditional on entry, the ecosystem  $G$  takes over the entrant into a follow-on market at a discount that is increasing in  $G$ ’s cross-market leverage. Hence, leverage policies raise contestability in follow-on markets.

## 6 Conclusion

Given our results, the obvious question arises: would the same forces not generate ecosystems in the offline world, just like they do in the online world? While we do not have a fully precise answer, several observations may be helpful for answering the question. For starters, in some settings offline ecosystems do emerge. For example, tropical hotels often feature an array of services, including airport transfers and a selection of water sports and tours, that keep travelers from looking for other options. Nevertheless, such arrangements do seem less dominant in offline settings. Unlike tropical hotels, for example, hotels in Paris or London are usually not part of an ecosystem. A simple explanation may be that offline sellers are less able to steer consumers than online firms. Even if a hotel in Paris recommends a particular restaurant to a traveler, she may be tempted by the many other restaurants she encounters. Since takeovers are costly, they may make sense only if steering by the acquirer

can substantially increase a target’s market share, and this is simply not the case in most offline settings. Tropical hotels are arguably in a unique position to steer because many travelers are reluctant to venture outside due to their lack of knowledge about the culture or environment. We have, however, not evaluated this hypothesis systematically, and doing so is an important topic for future research.

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## A Behavioral Foundations of Default Effects

We provide a series of microfoundations for a firm’s benefit of having the default position. All proofs for the claims below can be found in Appendix C. We consider a single representative consumer who values the service  $s \in \{a, b\}$  offered by a firm  $i \in \mathcal{N}^s$  at  $v_i^s$ , which is drawn from a distribution with continuous cumulative distribution function (CDF)  $F_i^s$ . (Alternatively, we can think of a mass of ex-ante homogeneous consumers.) The consumer’s valuations are drawn independently across firms and markets. Within market  $s$ , the value distributions can be ranked in terms of (weak) first-order-stochastic dominance (FOSD) and have common support. These rankings correspond to the “better than” quality rankings that we have introduced in the main text. Throughout, we assume that the consumer chooses a service  $s$  based on within-market considerations only. All results generalize, however, to a forward-looking consumer who understands, and takes into account, the steering by ecosystems.

**Technically Unsavvy Consumer** Consider a consumer who is assigned a default product and does not know that this default can be changed or, alternatively, believes that doing so is prohibitively costly. If the value of the default product is above a critical threshold  $\underline{v} \in \mathbb{R}$ , the consumer uses the default service. Otherwise, she refrains from using any service in market  $s$  (i.e., she selects an outside-option activity). This model of a “technically unsavvy consumer” satisfies Assumption 0 if and only if  $F_i^s(\underline{v}) < 1$  for all  $i \in \mathcal{N}^s$ , and it satisfies Assumption 1 if and only if  $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$  whenever  $F_i^s$  first-order stochastically dominates  $F_j^s$ . Moreover, for valuable services, consumer welfare is higher with a better default.

**Switching Costs** In a less extreme version of the above model, the consumer incurs physical switching costs of  $\gamma > 0$  when switching away from the default product. To simplify notation, we assume that the consumer uses one of the services (i.e., there is no outside option). This model generally satisfies Assumption 0. To also establish Assumption 1, we strengthen our assumptions on the distributions of values: all distributions have full support on  $\mathbb{R}$ , and can be ranked in terms of their reversed hazard rates.<sup>26</sup> Then, if  $n^s \geq 3$  and switching costs are small enough, this model also satisfies Assumption 1.<sup>27</sup> Moreover, under the same distributional assumptions, consumer welfare is higher with a better default.

**Sequential Search** A consumer observes the value (but not the identity) of the default for free and can then search for other products at an increasing cost. For simplicity, search costs increase sufficiently fast for the consumer to search at most once. A searching consumer draws one of the non-default products at random. The model implies Assumption 0, and it satisfies Assumption 1 if and only if  $n^s \geq 3$  and  $F_i^s(v) \neq F_j^s(v)$  for small enough  $v$  whenever  $F_i^s$  first-order stochastically dominates  $F_j^s$ . Consumer welfare is higher with a better default.

**Consideration Sets** The consumer considers only  $k^s \in \{1, \dots, n^s - 1\}$  offers in market  $s$ . The consumer’s “consideration set” includes the default product with certainty, and any non-default product with probability  $(k^s - 1)/(n^s - 1)$ . From her consideration set, the consumer selects the product with the highest realized value  $v_i^s$  (as in Lleras et al., 2017). This model can be thought of as a simultaneous search model in which the consumer (perhaps optimally) chooses the size of her consideration set before her attention is drawn to the default product. For example, the consumer may consider products based on her “mood,” and the default firm is the only one being able to predict the her mood (e.g., using data from past interactions) and tailor its product accordingly. The model implies Assumption 0, and it satisfies Assumption 1 if and only if  $n^s \geq 3$  and  $k^s \geq 2$ . Consumer welfare is higher with a better default.

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<sup>26</sup> We provide a formal definition of “dominance in reversed hazard rates” in Appendix C.2.

<sup>27</sup> The literal cost of switching to another option is small; in particular, in the digital context we consider.

**Loss Aversion** Suppose that the default product acts as a reference point, and the consumer dislikes falling short of this reference point. Product  $i$  is defined by a pair  $(v_i^s, d_i^s)$  with  $v_i^s \sim G^s$  and  $d_i^s \in \mathbb{R}_{>0}$ . We think of  $d_i^s$  as an app-design dimension affecting the consumer’s utility in an additively separable way (as in most applications of Kőszegi and Rabin, 2006). Let  $F_i^s(u) = G^s(u - d_i^s)$  be the distribution of  $v_i^s + d_i^s$ . We assume that, if the consumer uses the product of firm  $j$  instead of the default product offered by firm  $i$ , she experiences a loss of  $\lambda d_i$  (with  $\lambda > 1$ ) because she cannot enjoy firm  $i$ ’s proprietary design features, and a gain of  $d_j$  from using firm  $j$ ’s product instead. In addition, the consumer has some reference point in the service dimension, and we think of the distribution  $G^s$  as being “gain-loss adjusted.” The model satisfies Assumption 0. To establish also Assumption 1, we again assume that any  $F_i^s$  and  $F_j^s$  can be ranked in terms of their reversed hazard rates. Then, Assumption 1 holds as long as the consumer is not too loss averse. But, when gain-loss utility is not welfare relevant, which seems plausible for the digital goods we have in mind, a better default does not necessarily improve consumer welfare (see also Goldin and Reck, 2022).

**Other Models of As-If Switching Costs** Our loss-aversion based model of default effects is mathematically equivalent to a model of product-specific switching costs. Similarly, other mechanism give rise to “as if” switching costs, with similar positive and normative implications.<sup>28</sup> The consumer may, for example, (mis)interpret the default status of a product as a recommendation, introducing non-physical switching costs. Or a consumer may want to procrastinate on making a choice, which again introduces non-physical switching costs.

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<sup>28</sup> Non-physical switching costs are arguably higher than the physical costs of switching, which makes the restriction to small switching costs — necessary for some of our results — stronger. At the same time, the additional structure implied by these models can help with satisfying Assumption 1.

## B Proofs

*Proof of Lemma 1. Part I.* By Assumption 0, we immediately have

$$\alpha_i^s = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s} (q_{ii}^s - q_{i\ell}^s) = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \underbrace{(q_{ii}^s - q_{i\ell}^s)}_{>0} > 0.$$

Part II. Let  $i \in \mathcal{N}^s$ , and to derive a contradiction, suppose  $\eta_{ij}^s \geq 0$  for all  $j \neq i$ . Then,

$$\alpha_i^s = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} (q_{ii}^s - q_{i\ell}^s) = \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} (\alpha_i^s - \eta_{i\ell}^s) \leq \frac{1}{n^s} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \alpha_i^s = \frac{n^s - 1}{n^s} \alpha_i^s < \alpha_i^s,$$

where the strict inequality follows from  $\alpha_i > 0$ ; a contradiction.  $\square$

*Proof of Proposition 1. Part I.* We solve the game backwards. If  $G$  takes over  $t$ , it earns additional gross profits of  $V_t^b + q_G^a \alpha_t^b$ , which by Lemma 1 (Part I), is greater than  $t$ 's standalone value  $V_t^b$ . Standard ultimatum-bargaining game analysis, hence, implies that in any subgame in which there is a single target  $t$  left,  $G$  must offer  $V_t^b$  and  $t$  always accepts that offer.

Now consider any subgame in which a target  $t'$  receives an offer and there is one remaining target  $t \neq t'$  left thereafter. By subgame perfection,  $t'$  accepts the offer only if it is greater than  $V_{t'}^b + q_G^a \eta_{t't}^b$ , and whenever it accepts the offer in equilibrium, by standard arguments, the offer cannot be strictly greater than  $V_{t'}^b + q_G^a \eta_{t't}^b$ . When an offer of  $V_{t'}^b + q_G^a \eta_{t't}^b$  is accepted,  $G$  earns net profits of  $q_G^a (\alpha_{t'}^b - \eta_{t't}^b)$  while otherwise it earns  $q_G^a \alpha_t^b$ . Hence,  $G$  earns at most  $q_G^a \max\{\alpha_{t'}^b - \eta_{t't}^b, \alpha_t^b\}$ . Similarly, if firm  $G$  instead approached  $t$  first, it would earn at most  $q_G^a \max\{\alpha_t^b - \eta_{tt'}^b, \alpha_{t'}^b\}$ . Therefore, in any subgame with just two targets left,  $G$  earns at most

$$\max_{t_1, t_2 \in \mathcal{N}^b} q_G^a (\alpha_{t_1}^b - \min\{0, \eta_{t_1 t_2}^b\}) = \max_{t_1, t_2 \in \mathcal{N}^b} q_G^a (\alpha_{t_1}^b - \eta_{t_1 t_2}^b) =: \pi, \quad (2)$$

where the equality follows because by Part II of Lemma 1, for every  $t_1$  there exists some  $t'_2$  such that  $\eta_{t_1 t'_2}^b < 0$ . Because we have a finite number of targets, the maximum in (2) exists.

Next, we observe that in any subgame with two or more remaining targets in which  $t$  accepts  $G$ 's offer,  $G$  must offer at least  $V_t^b + q_G^a \min_{t' \in \mathcal{N}^b \setminus \{t\}} \min\{0, \eta_{tt'}^b\}$ ; for otherwise the

target is better off rejecting the offer no matter what happens in later subgames. This, however, implies that  $G$ 's profits from the takeover in any subgame are at most

$$\max_{t' \in \mathcal{N}^b} q_G^a (\alpha_t^b - \min \{0, \eta_{tt'}^b\}) \leq \max_{t_1, t_2 \in \mathcal{N}^b} q_G^a (\alpha_{t_1}^b - \min \{0, \eta_{t_1 t_2}^b\}) = \pi.$$

Finally, let  $t'_1$  and  $t'_2$  be targets for which the maximum in (2) is attained. By standard arguments, in any subgame with only  $t'_1$  and  $t'_2$  left,  $G$ 's offer must be accepted by the target it first approaches. By approaching all targets other than  $t'_1$  and  $t'_2$  beforehand and offering them zero, which must be rejected,  $G$  can induce the subgame in which only  $t'_1$  and  $t'_2$  remain. In the subgame-perfect equilibrium of this subgame,  $G$  earns  $\pi$  from the takeover. Since  $G$  cannot earn more than  $\pi$  in any other subgame, offering zero to all other targets first and playing a subgame-perfect equilibrium strategy when only  $t'_1$  and  $t'_2$  remain is part of a subgame-perfect equilibrium strategy for  $G$ . Since doing so is always a possible deviation, furthermore,  $G$  must earn  $\pi$  from the takeover in any subgame-perfect equilibrium. Existence of a subgame perfect equilibrium follows from standard arguments. Since  $G$  must offer any target  $t$  at least  $V_t^b + q_G^a \min_{t' \in \mathcal{N}^b \setminus \{t\}} \eta_{tt'}^b$ , for  $G$  to earn  $\pi$ , it must take over a target  $t_1$  from a pair  $t_1$  and  $t_2$  that attains the maximum in (2) at the price specified in the proposition.

Part II. We show that *under Assumption 1*,  $(t_1, t_2) \in \{\arg \max_{t'_1, t'_2 \in \mathcal{N}^b} \alpha_{t'_1}^b - \eta_{t'_1 t'_2}^b\}$  implies  $t_1$  and  $t_2$  offer the best and second-best service  $b$ , respectively. For any  $t_1$  and  $t_2$ , we have

$$\alpha_{t_1}^b - \eta_{t_1 t_2}^b = q_{t_1 t_1}^b - q_{t_1 t_2}^b.$$

By Part I of Assumption 1, for any  $t_1$  the right-hand side is larger for a better  $t_2$ . Fixing  $t_2$ , by Part II of Assumption 1, the right-hand side is greater for a better  $t_1$ . Hence,  $t_1$  and  $t_2$  must belong to the firms offering the two best services. Focusing on the best two firms, by Part III of Assumption 1, the right-hand side is maximized if  $t_1$  is one of the best firms.  $\square$

*Proof of Proposition 2. Part I.* Suppose firm  $i \in \mathcal{N}^b$  plays the takeover game, and wants to take over a firm  $j \in \mathcal{N}^a$ . Because firm  $i$  can not direct consumers into market  $a$ , it has to

pay  $j$ 's standalone value for  $j$  to accept the offer. Upon taking over firm  $j$ , firm  $i$  can direct  $j$ 's consumers in market  $a$  to its own product in market  $b$ , generating a default advantage  $\alpha_i^b$ . Hence, firm  $i$  earns additional profits of  $q_j^a \alpha_i^b > 0$  from taking over firm  $j$ . By definition, these profits are highest when taking over the market leader  $\ell$  because  $q_\ell^a \geq q_k^a$  for all  $k \in \mathcal{N}^a$ .

Part II. By Proposition 1, the market leader  $\ell$  in market  $a$  earns additional profits of

$$q_\ell^a (\alpha_i^b + |\eta_{ii^*}^b|) > q_\ell^a \alpha_i^b$$

from taking over firm  $i$  in market  $b$  (where  $i^*$  is  $i$ 's strongest rival). By definition, the market leader  $\ell$  generates more profits from taking over  $i$  than any other firm  $j \in \mathcal{N}^a$  does.  $\square$

*Sketch of the Proof of Proposition 3.* We only sketch the proof here, and provide the full argument in Appendix D. Conceptually, the proof proceeds by backward induction in the usual way. A crucial, but tedious, step is to show the existence of a cautious equilibrium in which acquirers use iteratively weakly undominated strategies in the reduced game where future subgames with a unique cautious equilibrium outcome are replaced by that outcome.

We start by considering subgames with more potential acquirers than targets. We first show that with only a single target left, there exists a cautious equilibrium in which the potential acquirer with the greatest market- $a$  demand takes over the target at the price specified in the proposition. We then argue that this is the outcome in any cautious equilibrium with a single target left. Next, using backward induction, we move on to subgames with  $k \geq 2$  remaining targets and at least  $k + 1$  remaining acquirers. We begin by establishing the following necessary conditions for a cautious equilibrium: (i) no firm with a market- $a$  demand below the  $k$ th highest among the remaining acquirers makes a takeover in equilibrium; (ii) the takeover price must be the one specified in the proposition; and (iii) the firm with the  $k$ th highest market- $a$  demand does not take over a target other than the last one. Finally, we construct a cautious equilibrium that implements the outcome specified in the proposition.

Having solved the subgames with more targets than acquirers allows us to proceed by backward induction for the case in which there are currently as many targets as potential

acquirers. The proof proceeds similarly as above. Finally, we turn to the case in which there are currently less acquirers than targets, again proceeding by backward induction.  $\square$

*Proof of Proposition 4.* Let firm 1 offer the best service  $a$ . We first observe that for every firm  $j \neq 1$ , bidding strictly above  $\Delta_{j1}^a$  is weakly dominated. By Part I of Assumption 1,

$$\Delta_{jj'}^a = q_{jj}^a - q_{jj'}^a \leq q_{jj}^a - q_{j1}^a = \Delta_{j1}^a.$$

To see that bidding  $b > \Delta_{j1}^a$  is weakly dominated by bidding  $\Delta_{j1}^a$ , we consider three cases. First, if the highest rival bid is strictly above  $b$  or below  $\Delta_{j1}^a$ , both bids obtain the same payoff. Second, if the highest rival bid lies in  $(\Delta_{j1}^a, b]$ , bidding  $\Delta_{j1}^a$  yields a strictly higher payoff since  $j$  strictly prefers to lose the auction at any price strictly above  $\Delta_{j1}^a$ , no matter who wins. Third, if the highest bid is  $\Delta_{j1}^a$ , bidding  $\Delta_{j1}^a$  yields a strictly higher payoff if the highest rival bid is made by a firm other than 1 and the same payoff when the highest rival bid is made by firm 1. This also establishes that if firm 1 wins, it pays at most  $\max_{j \neq 1} \Delta_{j1}^a$ .

Now suppose that, for the sake of a contradiction, a firm  $j \neq 1$  that does not offer the best service in market  $a$ , wins with positive probability. Hence, there exists some interval of bids  $(\underline{b}, \bar{b}]$  in which firm  $j$  bids with positive probability and conditional on doing so wins the auction. Since bidding above  $\Delta_{j1}^a$  is weakly dominated for firm  $j$ , we must have

$$\bar{b} \leq \Delta_{j1}^a = q_{jj}^a - q_{j1}^a < q_{11}^a - q_{1j}^a = \Delta_{1j}^a, \quad (3)$$

where the strict inequality follows from Part III of Assumption 1. Because firm  $j$  wins with positive probability either firm 1 bids below  $\bar{b}$  with positive probability or both firms have a mass point at  $\bar{b}$  in their bid functions. We argue that in either case, firm 1 can profitably move the probability mass in its bid function from weakly below  $\bar{b}$  to  $\Delta_{1j}^a$ . If firm 1 would have won the auction with its original bid, this does not affect the allocation and hence does not affect firm 1's payoff. Similarly, if firm 1 still loses when bidding  $\Delta_{1j}^a$  the allocation is unaffected. If, however, firm 1 wins, it must replace some other bidder  $j'$  (not necessarily



equal to  $j$ ). Because also  $j'$  must bid below  $\Delta_{j'1}^a$ , the winning bid firm 1 replaces satisfies

$$b' \leq \Delta_{j'1}^a < \Delta_{1j'}^a. \quad (4)$$

Conditional on replacing the winning bid  $b'$ , firm 1 thus gains at least  $\Delta_{1j'}^a - \Delta_{j'1}^a > 0$ . Finally, because prior to the deviation winning bids fall in the interval  $(\underline{b}, \bar{b}]$  with positive probability, the latter happens with positive probability. Hence, firm 1 has a strictly profitable deviation; a contradiction. Hence, a firm offering the best service  $a$  wins with probability 1.  $\square$

*Proof of Proposition 5.* It is well-known that with two bidders, it is weakly dominant for each to bid its valuation. Hence, to prove the claim, we simply calculate these valuations.

Part I. Suppose  $M$  is a single-market firm. Hence,  $M$ 's willingness to pay to replace  $G$  as the default is  $\Delta_{MG}^a$ . If  $G$  replaces  $M$  as the default in market  $a$ , it earns additional profits of  $\Delta_{GM}^a$  in that market, and it increases the probability of being the default in market  $b$  by  $\Delta_{GM}^a$ . Hence,  $G$  values the default position in market  $a$  at  $\Delta_{GM}^a(1 + \alpha_G^b)$ . The claim follows.

Part II. Suppose  $M$  is an ecosystem. We start by deriving  $G$ 's willingness to pay to replace  $M$  as the default in  $a$ . When firm  $G$  is the default in market  $a$ , it makes profits of

$$q_{GG}^a + q_{GG}^a q_{GG}^b + q_{MG}^a q_{GM}^b + (1 - q_{GG}^a - q_{MG}^a) \frac{1}{n^b} \sum_{\ell \in \mathcal{N}^b} q_{G\ell}^b = q_{GG}^a + q_{GG}^a \alpha_G^b + q_{MG}^a \eta_{GM}^b + \frac{1}{n^b} \sum_{\ell \in \mathcal{N}^b} q_{G\ell}^b.$$

When firm  $M$  is the default in market  $a$ , firm  $G$  earns

$$q_{GM}^a + q_{GM}^a q_{GG}^b + q_{MM}^a q_{GM}^b + (1 - q_{GM}^a - q_{MM}^a) \frac{1}{n^b} \sum_{\ell \in \mathcal{N}^b} q_{G\ell}^b = q_{GM}^a + q_{GM}^a \alpha_G^b + q_{MM}^a \eta_{GM}^b + \frac{1}{n^b} \sum_{\ell \in \mathcal{N}^b} q_{G\ell}^b.$$

Using  $q_{GG}^a - q_{GM}^a = \Delta_{GM}^a$  and  $-(q_{MM}^a - q_{MG}^a) = -\Delta_{MG}^a$ , we obtain

$$\text{WTP}_{GM} = \Delta_{GM}^a(1 + \alpha_G^b) - \Delta_{MG}^a \eta_{GM}^b = \Delta_{GM}^a(1 + \alpha_G^b) + \Delta_{MG}^a |\eta_{GM}^b|,$$

where the last equality follows from the fact that  $\eta_{GM}^b < 0$ . Similarly, because  $\eta_{MG}^b < 0$ ,

$$\text{WTP}_{MG} = \Delta_{MG}^a(1 + \alpha_M^b) + \Delta_{GM}^a |\eta_{MG}^b|.$$

The claims follows by observing that

$$\text{WTP}_{GM} > \text{WTP}_{MG} \quad \text{if and only if} \quad \Delta_{GM}^a(1 + \alpha_G^b + \eta_{MG}^b) > \Delta_{MG}^a(1 + \alpha_M^b + \eta_{GM}^b). \quad \square$$

# Online Appendix

## C Proofs of the Microfoundations

### C.1 Technically Unsavvy Consumer

Consider market  $s$ , and let  $n^s \geq 2$ . The consumer is assigned a default service  $i$ , which she uses if its value  $v_i$  is above a  $\underline{v} \in \mathbb{R}$ . Otherwise, the consumer does not use service  $s$ .

**Lemma 2** (Technically Unsavvy Consumer).

- I. Assumption 0 holds if and only if  $F_i^s(\underline{v}) < 1$  for all  $i$ .*
- II. Assumption 1 holds if and only if  $F_i^s(\underline{v}) < 1$  and  $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$  for all  $i$  and all  $j \neq i$ .*

*Proof.* Part I. The consumer never uses a non-default product. Hence, for any  $i$  and  $j \neq i$ ,

$$q_{ii}^s - q_{ij}^s = 1 - F_i^s(\underline{v}).$$

Hence, Assumption 0 holds if and only if  $F_i^s(\underline{v}) < 1$ . This proves Part I.

Part II. Part I of Assumption 1 holds because for any  $i$  and  $j \neq i$ ,  $q_{ii}^s = 1 - F_i^s(\underline{v}) \geq 0$  and  $q_{ij}^s = 0$ . Next, we observe that for any  $i, i'$ , and  $j \neq i, i'$ ,

$$(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) = F_{i'}^s(\underline{v}) - F_i^s(\underline{v}) = (q_{ii}^s - q_{ii'}^s) - (q_{i'i'}^s - q_{i'i}^s)$$

which is positive if and only if  $F_{i'}^s(\underline{v}) > F_i^s(\underline{v})$ . Because any two distribution  $F_i^s$  and  $F_{i'}^s$  can be ranked in terms of FOSD, Parts III and IV of Assumption 1 thus hold if and only if  $F_i^s(\underline{v}) \neq F_{i'}^s(\underline{v})$  for all  $i$  and  $i'$  with  $i \neq i'$ . This completes the proof of Part II.  $\square$

**Consumer Welfare** Suppose  $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$  for all  $i$  and all  $j \neq i$ , and in line with Section 5, let  $\underline{v} \geq 0$ ; that is, the service is valuable in that the consumer “buys reasonably”

(Heidhues et al., 2023b). If  $i$  is the default, consumer welfare is  $\int_{\underline{v}}^{\infty} v \, dF_i^s(v)$ . Since all  $F_\ell^s$  have the same support, we have

$$\int_{\underline{v}}^{\infty} v \, dF_i^s(v) - \int_{\underline{v}}^{\infty} v \, dF_j^s(v) = \underline{v}(F_j^s(\underline{v}) - F_i^s(\underline{v})) + \int_{\underline{v}}^{\infty} F_j^s(v) - F_i^s(v) \, dv.$$

Because  $F_i^s(\underline{v}) \neq F_j^s(\underline{v})$  and  $\underline{v} \geq 0$ , the above is positive if  $F_i^s$  first-order stochastically dominates  $F_j^s$ . Hence, for valuable services consumer welfare is higher with a better default.

## C.2 Switching Costs

Consider market  $s$ , and let  $n^s \geq 2$ . We study a more general model than the one described in Appendix A in that we allow for product-specific switching costs. Specifically, we assume that the consumer observes the value of all products, but incurs a switching cost  $\gamma_i > 0$  when  $i$ 's product is the default and she chooses the product of  $j \neq i$ . Moreover, we assume that, if  $F_i^s$  first-order stochastically dominates  $F_j^s$ , then  $\gamma_i \geq \gamma_j$ . The consumer uses one service.

**Lemma 3** (Switching Costs: Steering). *Assumption 0 holds.*

*Proof.* When firm  $i$ 's product is the default, it is chosen if and only if  $v_i + \gamma_i \geq \max_{\ell \in \mathcal{N}^s \setminus \{i\}} v_\ell$ .

This happens with probability

$$q_{ii}^s = \int \prod_{\ell \in \mathcal{N}^s \setminus \{i\}} F_\ell^s(v + \gamma_i) \, dF_i^s(v).$$

When firm  $j$ 's product is the default, the consumer buys the product of firm  $i$  with probability

$$q_{ij}^s = \int F_j^s(v - \gamma_j) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v).$$

We obtain

$$\begin{aligned} q_{ii}^s - q_{ij}^s &= \int F_j^s(v + \gamma_i) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v + \gamma_i) \, dF_i^s(v) - \int F_j^s(v - \gamma_j) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v) \\ &> \int \left( F_j^s(v + \gamma_i) - F_j^s(v - \gamma_j) \right) \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v) \, dF_i^s(v), \end{aligned}$$

which is positive because  $\gamma_i, \gamma_j > 0$ . □

To also establish Assumption 1, we impose more structure. First, we assume that all value distributions have full support on  $\mathbb{R}$ , which implies that  $F_i^s$  admits a density  $f_i^s$ .<sup>29</sup> Second, we assume that any two distributions can be ranked in terms of their reversed hazard rates, which is stronger than a ranking in terms of first-order stochastic dominance. Formally,  $F_i^s$  dominates  $F_j^s$  in the reversed hazard rate if  $f_i^s(v)/F_i^s(v) > f_j^s(v)/F_j^s(v)$  for all  $v \in \mathbb{R}$ .

**Lemma 4** (Switching Costs: Quality-Steering Complementarity). *Suppose that  $F_i^s$  dominates  $F_j^s$  in the reversed hazard rate for all  $j \in \mathcal{N}^s$ . Then, for any  $n^s \geq 3$  and any such  $\{F_\ell^s\}_{\ell \in \mathcal{N}^s}$ , there exists some  $\bar{\gamma} > 0$ , such that for any  $\gamma_i < \bar{\gamma}$ , Assumption 1 holds.*

*Proof.* First, consider  $j$  and  $j'$  with  $F_j^s$  dominating  $F_{j'}^s$  in the reversed hazard rate. Then,

$$\begin{aligned} q_{ij'}^s - q_{ij}^s &= \int \left( F_{j'}^s(v - \gamma_{j'}) F_j^s(v) - F_j^s(v - \gamma_j) F_{j'}^s(v) \right) \prod_{\ell \in \mathcal{N}^s \setminus \{i, j, j'\}} F_\ell^s(v) dF_i^s(v) \\ &\geq \int \left( F_{j'}^s(v - \gamma_j) F_j^s(v) - F_j^s(v - \gamma_j) F_{j'}^s(v) \right) \prod_{\ell \in \mathcal{N}^s \setminus \{i, j, j'\}} F_\ell^s(v) dF_i^s(v), \end{aligned}$$

where the inequality follows from  $\gamma_j \geq \gamma_{j'}$  and  $F_j^s$  being increasing. Moreover, because  $F_j^s$  dominates  $F_{j'}^s$  in the reversed hazard rate, we have

$$\frac{\partial}{\partial v} \frac{F_j^s(v)}{F_{j'}^s(v)} \propto f_j^s(v) F_{j'}^s(v) - f_{j'}^s(v) F_j^s(v) > 0.$$

This implies, in turn, that

$$\frac{F_j^s(v)}{F_{j'}^s(v)} > \frac{F_j^s(v - \gamma_j)}{F_{j'}^s(v - \gamma_j)} \quad \text{or, equivalently,} \quad F_{j'}^s(v - \gamma_j) F_j^s(v) - F_j^s(v - \gamma_j) F_{j'}^s(v) > 0. \quad (5)$$

Hence,  $q_{ij'}^s > q_{ij}^s$ , which means that Part I of Assumption 1 holds.

Second, consider  $i$  and  $j$  with  $F_i^s$  dominating  $F_j^s$  in the reversed hazard rate. To verify

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<sup>29</sup> If  $v_i^s$  has full support on  $\mathbb{R}$ , then  $F_i^s$  is strictly increasing and thus almost everywhere differentiable.

Part III of Assumption 1, we define  $G^s(v) := \prod_{\ell \in \mathcal{N}^s \setminus \{i,j\}} F_\ell^s(v)$  and observe

$$\begin{aligned}
q_{ii}^s - q_{ij}^s &= \int F_j^s(v + \gamma_i) G^s(v + \gamma_i) dF_i^s(v) - \int F_j^s(v - \gamma_j) G^s(v) dF_i^s(v) \\
&= 1 - \int F_i^s(u - \gamma_i) G^s(u) dF_j^s(u) - \int F_j^s(u) F_i^s(u - \gamma_i) dG^s(u) \\
&\quad - \left( 1 - \int F_i^s(u + \gamma_j) G^s(u + \gamma_j) dF_j^s(u) - \int F_j^s(u - \gamma_j) F_i^s(u) dG^s(u) \right) \\
&= q_{jj}^s - q_{ji}^s + \int \left( F_j^s(u - \gamma_j) F_i^s(u) - F_j^s(u) F_i^s(u - \gamma_i) \right) dG^s(u) \\
&\geq q_{jj}^s - q_{ji}^s + \int \left( F_j^s(u - \gamma_j) F_i^s(u) - F_j^s(u) F_i^s(u - \gamma_j) \right) dG^s(u),
\end{aligned}$$

where the inequality follows from  $\gamma_i \geq \gamma_j$  and  $F_i^s$  being increasing. Hence, by (5), it follows that  $q_{ii}^s - q_{ij}^s > q_{jj}^s - q_{ji}^s$  and thus Part III of Assumption 1.

Third, consider  $i$  and  $i'$  with  $F_i^s$  dominating  $F_{i'}^s$  in the reversed hazard rate, and denote as  $\tilde{G}^s(v) := \prod_{\ell \in \mathcal{N}^s \setminus \{i,i',j\}} F_\ell^s(v)$ . Then, since  $F_i^s(v)$  and  $F_{i'}^s(v)$  admit a density by assumption,

$$\begin{aligned}
(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) &= \int F_j^s(v + \gamma_i) \tilde{G}^s(v + \gamma_i) F_{i'}^s(v + \gamma_i) f_i^s(v) dv \\
&\quad - \int F_j^s(v + \gamma_{i'}) \tilde{G}^s(v + \gamma_{i'}) F_i^s(v + \gamma_{i'}) f_{i'}^s(v) dv \\
&\quad - \int F_j^s(v - \gamma_j) \tilde{G}^s(v) \left[ F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] dv \\
&\geq \int F_j^s(v + \gamma_i) \tilde{G}^s(v + \gamma_i) \left[ F_{i'}^s(v + \gamma_i) f_i^s(v) - F_i^s(v + \gamma_i) f_{i'}^s(v) \right] dv \\
&\quad - \int F_j^s(v - \gamma_j) \tilde{G}^s(v) \left[ F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] dv,
\end{aligned}$$

where inequality follows from  $\gamma_i \geq \gamma_{i'}$ , and  $F_j^s$ ,  $F_{i'}^s$  and  $\tilde{G}^s$  being increasing.

We distinguish two cases. First, let  $\gamma_j = \gamma_i + \epsilon$  for  $\epsilon > 0$ . Then, at  $\gamma_i = 0$ , the above is

$$\int \tilde{G}^s(v) \left[ F_j^s(v) - F_j^s(v - \epsilon) \right] \left[ F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] dv > 0,$$

where the inequality follows from  $\epsilon > 0$  and  $F_j^s$  having full support, and  $F_i^s$  and  $F_{i'}^s$  satisfying the reverse hazard rate. By continuity, there exists some  $\bar{\gamma}_{ii'j} > 0$ , so that for any  $\gamma_i < \bar{\gamma}_{ii'j}$ , we have  $q_{ii}^s - q_{ij}^s > q_{i'i'}^s - q_{i'j}^s$ . Note that the constraint is getting tighter for smaller  $\epsilon$ .

Second, let  $\gamma_j = \gamma_i - \epsilon$  for some  $\epsilon \in [0, \gamma_i]$ . Hence, if  $\gamma_i = 0$ , then also  $\gamma_j = 0$ . It follows that  $q_{ii}^s - q_{ij}^s = q_{i'i'}^s - q_{i'j}^s$  if  $\gamma_i = 0$ . We now take, for a fixed  $\epsilon$ , the partial derivative of the lower bound on  $(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s)$  derived above with respect to  $\gamma_i$ . Clearly, both integrands are bounded from above and from below. Hence, by Lebesgue's dominated convergence theorem, we can exchange differentiation and integration. This implies that this partial derivative is given by

$$\begin{aligned} \int \left[ F_{i'}^s(v + \gamma_i) f_i^s(v) - F_i^s(v + \gamma_i) f_{i'}^s(v) \right] \frac{\partial}{\partial \gamma_i} \left( F_j^s(v + \gamma_i) \tilde{G}^s(v + \gamma_i) \right) dv \\ + \int \left[ f_{i'}^s(v + \gamma_i) f_i^s(v) - f_i^s(v + \gamma_i) f_{i'}^s(v) \right] F_j^s(v + \gamma_i) \tilde{G}^s(v + \gamma_i) dv \\ + \int f_j^s(v - \gamma_j) \tilde{G}^s(v) \left[ F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] dv. \end{aligned}$$

Evaluating this partial derivative at  $\gamma_i = 0$ , we obtain

$$\int \left[ F_{i'}^s(v) f_i^s(v) - F_i^s(v) f_{i'}^s(v) \right] \left( 2f_j^s(v) \tilde{G}^s(v) + F_j^s(v) \tilde{g}(v) \right) dv,$$

which is strictly positive by the reversed hazard rate condition. Since  $q_{ii}^s - q_{ij}^s = q_{i'i'}^s - q_{i'j}^s$  at  $\gamma_i = 0$ , this implies that again, there exists some  $\bar{\gamma}_{ii'j} > 0$ , such that for any  $\gamma_i < \bar{\gamma}_{ii'j}$ ,  $q_{ii}^s - q_{ij}^s > q_{i'i'}^s - q_{i'j}^s$ . Here, the constraint gets tighter as  $\epsilon$  gets bigger. Noticing that there are only finitely many combinations of  $i$ ,  $i'$ , and  $j$ , we conclude that  $\bar{\gamma} := \min_{i, i', j \in \mathcal{N}^s} \bar{\gamma}_{ii'j} > 0$ . Hence, if  $\gamma_i < \bar{\gamma}$  for all  $i$ , also Part II of Assumption 1 holds. This completes the proof.  $\square$

**Consumer Welfare** Suppose switching costs are symmetric:  $\gamma_\ell \equiv \gamma$  for all  $\ell \in \mathcal{N}^s$ . Denote as  $\nu_i^s$  the value (net of switching costs) of the product *chosen* by the consumer when product  $i$  is the default. The value  $\nu_i^s$  is drawn from the cumulative distribution function

$$\mathbb{P}[\nu_i^s \leq v] = F_i^s(v) \prod_{\ell \in \mathcal{N}^s \setminus \{i\}} F_\ell^s(v + \gamma).$$

Notice that

$$\mathbb{P}[\nu_i^s \leq v] - \mathbb{P}[\nu_j^s \leq v] = \left[ F_i^s(v) F_j^s(v + \gamma) - F_i^s(v + \gamma) F_j^s(v) \right] \prod_{\ell \in \mathcal{N}^s \setminus \{i, j\}} F_\ell^s(v + \gamma).$$

Hence, if  $F_i^s$  dominates  $F_j^s$  in the reversed hazard rate, then  $\nu_i^s$  first-order stochastically dominates  $\nu_j^s$ . As a result, consumer welfare is higher with a better default.

**Large Switching Costs** To see that the restriction to small enough switching costs has bite, let  $n^s = 3$ ,  $\gamma_i = \gamma$ , and assume that  $v_i^s$  is drawn from a normal distribution with a mean  $\mu_i$  and a variance of 1. As illustrated in Figure 1 for the case of  $\mu_1 = \frac{1}{2}$ ,  $\mu_2 = 0$ , and  $\mu_3 = -\frac{1}{2}$ , Part II of Assumption 1 does *not* hold in general. In this particular example, the upper bound on the switching cost  $\gamma$  for Assumption 1 to hold is given by  $\bar{\gamma} \approx 2.15$ .

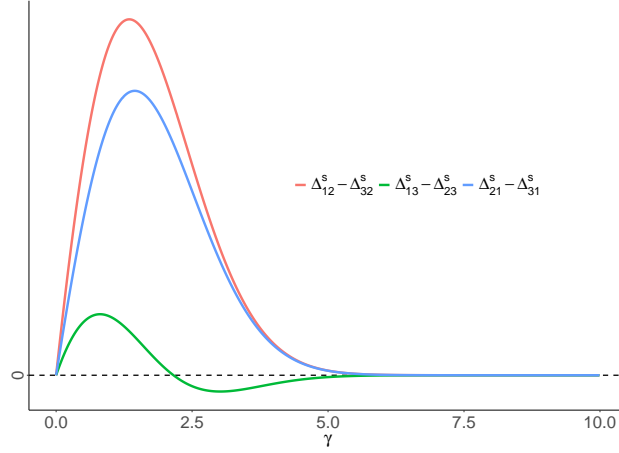


Figure 1: *Violation of Assumption 1 (Part II) with normal values. Note:  $\Delta_{ij}^s = q_{ii}^s - q_{ij}^s$ .*

### C.3 Sequential Search

Consider market  $s$ , and let  $n^s \geq 2$ . The consumer observes the value, but not the identity of the default product for free and can then search at a cost. The first search costs  $c_1 > 0$ , the second search costs  $c_2 \geq c_1$ , and the  $k$ th search costs  $c_k \geq c_{k-1}$ . For simplicity, we set  $c_2 = \infty$ , thereby effectively assuming that the consumer searches at most once. If the consumer searches for another product, she randomly draws one of the non-default products.

Suppose that firm  $i$ 's product is the default. The consumer searches if and only if the realized value  $v_i^s$  lies below some cutoff  $\hat{v}_i^s \in \mathbb{R}$ . Moreover, because the consumer does not learn the identity of the default firm, she does not learn anything about the quality of the non-default firms and thus about the value of search. Formally,  $\hat{v}_i^s = \hat{v}^s$ . Then:

**Lemma 5** (Sequential Search).

*I. Assumption 0 holds.*

*II. Assumption 1 holds if and only if  $n^s \geq 3$  and for all  $i, i' \in \mathcal{N}^s$ , there exists some  $v \leq \hat{v}^s$ , such that  $F_{i'}^s(v) \neq F_i^s(v)$ .*

*Proof. Preliminaries.* First, if firm  $i$ 's product is the default in  $s$ , its profit in market  $s$  is

$$\begin{aligned}
q_{ii}^s &= \mathbb{P}[v_i^s > \hat{v}^s] + \mathbb{P}[v_i^s \leq \hat{v}^s] \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \mathbb{P}[v_i^s > v_\ell^s | v_i^s \leq \hat{v}^s] \\
&= \mathbb{P}[v_i^s > \hat{v}^s] + \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \left\{ \mathbb{P}[v_i^s > v_\ell^s] - \mathbb{P}[v_i^s > \hat{v}^s] \mathbb{P}[v_i^s > v_\ell^s | v_i^s > \hat{v}^s] \right\} \\
&= \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \left\{ \mathbb{P}[v_i^s > v_\ell^s] + \mathbb{P}[v_i^s > \hat{v}^s] \left( 1 - \mathbb{P}[v_i^s > v_\ell^s | v_i^s > \hat{v}^s] \right) \right\} \\
&= \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \underbrace{\left\{ \mathbb{P}[v_i^s > v_\ell^s] + \mathbb{P}[v_i^s > \hat{v}^s] \mathbb{P}[v_i^s \leq v_\ell^s | v_i^s > \hat{v}^s] \right\}}_{=: \mathcal{A}_{i\ell}},
\end{aligned} \tag{6}$$

where the second equality holds by the law of total probability.

Second, suppose firm  $j$ 's product is the default. Firm  $i$  makes a sale only if the consumer searches *and* firm  $i$  is the random firm that she finds. Firm  $i$ 's profit in market  $s$  then is

$$\begin{aligned}
q_{ij}^s &= \frac{1}{n^s - 1} \mathbb{P}[v_j^s \leq \hat{v}^s] \mathbb{P}[v_i^s > v_j^s | v_j^s \leq \hat{v}^s] \\
&= \frac{1}{n^s - 1} \underbrace{\left\{ \mathbb{P}[v_i^s > v_j^s] - \mathbb{P}[v_j^s > \hat{v}^s] \mathbb{P}[v_i^s > v_j^s | v_j^s > \hat{v}^s] \right\}}_{=: \mathcal{B}_{ij}},
\end{aligned} \tag{7}$$

where the second equality again follows from the law of total probability.



Third, suppose that the default is determined at random. Using Eq. (6) and (7), we conclude that firm  $i$ 's profit in market  $s$  is then given by

$$\frac{1}{n^s(n^s - 1)} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \left\{ \mathcal{A}_{i\ell} + \mathcal{B}_{i\ell} \right\}.$$

Fourth, we express the probabilities above in terms of the underlying CDFs. We observe

$$\mathbb{P}[v_i^s > v_\ell^s] = \int_{-\infty}^{\infty} F_\ell^s(v) dF_i^s(v),$$

and

$$\mathbb{P}[v_i^s > \hat{v}^s] \mathbb{P}[v_i^s \leq v_\ell^s | v_i^s > \hat{v}^s] = \mathbb{P}[v_i^s \leq v_\ell^s, v_i^s > \hat{v}^s] = 1 - F_i^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_\ell^s(v) dF_i^s(v).$$

This implies that

$$\begin{aligned} \mathcal{A}_{i\ell} &= \mathbb{P}[v_i^s > v_\ell^s] + \mathbb{P}[v_i^s > \hat{v}^s] \mathbb{P}[v_i^s \leq v_\ell^s | v_i^s > \hat{v}^s] \\ &= \int_{-\infty}^{\hat{v}^s} F_\ell^s(v) dF_i^s(v) + 1 - F_i^s(\hat{v}^s) \\ &= 1 - F_i^s(\hat{v}^s) + F_\ell^s(\hat{v}^s) F_i^s(\hat{v}^s) - \int_{-\infty}^{\hat{v}^s} F_i^s(v) dF_\ell^s(v), \end{aligned} \tag{8}$$

where the last equality follows from integration by parts, as well as

$$\begin{aligned} \mathcal{B}_{ij} &= \mathbb{P}[v_i^s > v_j^s] - \mathbb{P}[v_j^s > \hat{v}^s] \mathbb{P}[v_i^s > v_j^s | v_j^s > \hat{v}^s] \\ &= \int_{-\infty}^{\infty} F_j^s(v) dF_i^s(v) - \left( 1 - F_j^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_i^s(v) dF_j^s(v) \right) \\ &= 1 - \int_{-\infty}^{\infty} F_i^s(v) dF_j^s(v) - 1 + F_j^s(\hat{v}^s) + \int_{\hat{v}^s}^{\infty} F_i^s(v) dF_j^s(v) \\ &= F_j^s(\hat{v}^s) - \int_{-\infty}^{\hat{v}^s} F_i^s(v) dF_j^s(v), \end{aligned} \tag{9}$$

where the second equality again follows from integration by parts.

Finally, we define

$$\begin{aligned}
\mathcal{C}_{ij} &:= \mathcal{A}_{ij} - \mathcal{B}_{ij} \\
&= 1 - F_i^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_j^s(v) dF_i^s(v) + 1 - F_j^s(\hat{v}^s) - \int_{\hat{v}^s}^{\infty} F_i^s(v) dF_j^s(v) \\
&= 2 - F_i^s(\hat{v}^s) - F_j^s(\hat{v}^s) - (1 - F_j^s(\hat{v}^s)F_i^s(\hat{v}^s)) + \int_{\hat{v}^s}^{\infty} F_i^s(v) dF_j^s(v) - \int_{\hat{v}^s}^{\infty} F_i^s(v) dF_j^s(v) \\
&= 1 - F_i^s(\hat{v}^s)(1 - F_j^s(\hat{v}^s)) - F_j^s(\hat{v}^s),
\end{aligned} \tag{10}$$

where the second equality follows from integration by parts. This also means that  $\mathcal{C}_{ij} = \mathcal{C}_{ji}$ .

Part I. Subtracting (7) from (6) gives

$$q_{ii}^s - q_{ij}^s = \frac{1}{n^s - 1} \left( \mathcal{C}_{ij} + \sum_{\ell \in \mathcal{N}^s \setminus \{i, j\}} \mathcal{A}_{i\ell} \right).$$

Because all  $F_\ell^s$  have common support, we have  $\mathcal{A}_{i\ell}, \mathcal{C}_{ij} > 0$ . Hence, Assumption 0 holds.

Part II. Suppose  $F_i^s$  first-order stochastically dominates  $F_j^s$ . Since  $\mathcal{C}_{ij} = \mathcal{C}_{ji}$ , we have

$$(q_{ii}^s - q_{ij}^s) - (q_{jj}^s - q_{ji}^s) = \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i, j\}} \{\mathcal{A}_{i\ell} - \mathcal{A}_{j\ell}\}.$$

Using Eq. (8), we conclude

$$\mathcal{A}_{i\ell} - \mathcal{A}_{j\ell} = (F_j^s(\hat{v}^s) - F_i^s(\hat{v}^s))(1 - F_\ell^s(\hat{v}^s)) + \int_{-\infty}^{\hat{v}^s} (F_j^s(v) - F_i^s(v)) dF_\ell^s(v) \geq 0,$$

holding with a strict inequality if and only if  $F_{i'}^s(v) \neq F_i^s(v)$  for some  $v \leq \hat{v}^s$ . Moreover, if  $n^s = 2$ ,  $q_{ii}^s - q_{ij}^s = q_{jj}^s - q_{ji}^s$ . Combining both, we conclude that Part III of Assumption 1 holds if and only if  $n^s \geq 3$  and for all  $i, i' \in \mathcal{N}^s$ , there exists some  $v \leq \hat{v}^s$ , so that  $F_{i'}^s(v) \neq F_i^s(v)$ .

Next, consider three firms  $i, i'$ , and  $j \neq i, i'$ . We have

$$(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) = \frac{1}{n^s - 1} \left( \mathcal{C}_{ij} - \mathcal{C}_{i'j} + \mathcal{A}_{ii'} - \mathcal{A}_{i'i} + \sum_{\ell \in \mathcal{N}^s \setminus \{i, i', j\}} \{\mathcal{A}_{i\ell} - \mathcal{A}_{i'\ell}\} \right).$$

To verify Part II of Assumption 1, let  $F_i^s$  first-order stochastically dominate  $F_{i'}^s$ . By (10),

$$\mathcal{C}_{ij} - \mathcal{C}_{i'j} = (F_{i'}^s(\hat{v}^s) - F_i^s(\hat{v}^s))(1 - F_j^s(\hat{v}^s)) \geq 0,$$

holding with a strict inequality if and only if  $F_{i'}^s(\hat{v}^s) > F_i^s(\hat{v}^s)$ . Moreover, by (8), we obtain

$$\mathcal{A}_{ii'} - \mathcal{A}_{i'i} = F_{i'}^s(\hat{v}^s) - F_i^s(\hat{v}^s) + \int_{-\infty}^{\hat{v}^s} F_{i'}^s(v) dF_i^s(v) - \int_{-\infty}^{\hat{v}^s} F_i^s(v) dF_{i'}^s(v) \geq 0,$$

holding with a strict inequality if and only if  $F_{i'}^s(v) \neq F_i^s(v)$  for some  $v \leq \hat{v}^s$ . Finally, by the same argument as above, we have  $\mathcal{A}_{i\ell} \geq \mathcal{A}_{i'\ell}$ , holding with a strict inequality if and only if  $F_{i'}^s(v) \neq F_i^s(v)$  for some  $v \leq \hat{v}^s$ . Hence, Part II of Assumption 1 holds if and only if for all  $i, i' \in \mathcal{N}^s$ , there exists some  $v \leq \hat{v}^s$ , so that  $F_{i'}^s(v) \neq F_i^s(v)$ .

Finally, for any  $j, j'$ , and  $i \neq j, j'$ , we obtain

$$q_{ij'}^s - q_{ij}^s = \frac{1}{n^s - 1} \left( \mathcal{C}_{ij} - \mathcal{C}_{ij'} + \mathcal{A}_{ij'} - \mathcal{A}_{ij} \right).$$

If  $F_j^s$  first-order stochastically dominates  $F_{j'}^s$ , then, by (10),

$$\mathcal{C}_{ij} - \mathcal{C}_{ij'} = (1 - F_i^s(\hat{v}^s))(F_{j'}^s(\hat{v}^s) - F_j^s(\hat{v}^s)) \geq 0,$$

holding with a strict inequality if and only if  $F_{j'}^s(\hat{v}^s) > F_j^s(\hat{v}^s)$ . Similarly, by (8),

$$\mathcal{A}_{ij'} - \mathcal{A}_{ij} = \int_{-\infty}^{\hat{v}^s} \{F_{j'}^s(v) - F_j^s(v)\} dF_i^s(v) \geq 0,$$

holding with a strict inequality if and only if  $F_{j'}^s(v) \neq F_j^s(v)$  for some  $v \leq \hat{v}^s$ . Hence, Part I of Assumption 1 holds if and only if  $F_{j'}^s(v) \neq F_j^s(v)$  for some  $v \leq \hat{v}^s$ .  $\square$

**Consumer Welfare** If product  $i$  is the default product, then consumer welfare equals

$$\begin{aligned} & \mathbb{P}[v_i^s \geq \hat{v}^s] \mathbb{E}[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s] + (1 - \mathbb{P}[v_i^s \geq \hat{v}^s]) \frac{1}{n^s - 1} \sum_{\ell \in \mathcal{N}^s \setminus \{i\}} \mathbb{E}[\max\{v_i^s, v_\ell^s\} - c_1 | v_i^s \leq \hat{v}^s] \\ &= \int_{\hat{v}^s}^{\infty} v dF_i^s(v) + \frac{1}{n^s - 1} \left[ \int_{-\infty}^{\infty} v - c_1 dG_{ij}^s(v) + \sum_{\ell \in \mathcal{N}^s \setminus \{i, j\}} \int_{-\infty}^{\infty} v - c_1 dG_{i\ell}^s(v) \right] \\ &= \frac{n^s - 2}{n^s - 1} \int_{\hat{v}^s}^{\infty} v dF_i^s(v) + \frac{1}{n^s - 1} \left[ \int_{\hat{v}^s}^{\infty} v dF_i^s(v) + \int_{-\infty}^{\infty} v - c_1 dG_{ij}^s(v) + \sum_{\ell \in \mathcal{N}^s \setminus \{i, j\}} \int_{-\infty}^{\infty} v - c_1 dG_{i\ell}^s(v) \right], \end{aligned}$$

where  $G_{i\ell}^s$  is the CDF of  $\max\{v_i^s, v_\ell^s\}$ . Because  $v_i^s$  and  $v_\ell^s$  are independent, we have

$$G_{i\ell}^s(v) = \mathbb{P}\left[\{\max\{v_i^s, v_\ell^s\} \leq v\} \cup \{v_i^s \leq \hat{v}^s\}\right] = \begin{cases} F_i^s(v)F_\ell^s(v) & \text{if } v < \hat{v}^s, \\ F_i^s(\hat{v}^s)F_\ell^s(v) & \text{if } v \geq \hat{v}^s, \end{cases}$$

and the corresponding density is given by

$$g_{ij}^s(v) = \begin{cases} f_i^s(v)F_j^s(v) + F_i^s(v)f_j^s(v) & \text{if } v < \hat{v}^s, \\ F_i^s(\hat{v}^s)f_j^s(v) & \text{if } v \geq \hat{v}^s. \end{cases}$$

First, we observe that, for any  $\ell \neq i, j$ ,

$$G_{i\ell}^s(v) - G_{j\ell}^s(v) = \begin{cases} (F_i^s(v) - F_j^s(v))F_\ell^s(v) & \text{if } v < \hat{v}^s, \\ (F_i^s(\hat{v}^s) - F_j^s(\hat{v}^s))F_\ell^s(v) & \text{if } v \geq \hat{v}^s. \end{cases}$$

Hence, if  $F_i^s$  first-order stochastically dominates  $F_j^s$ , then  $G_{i\ell}^s$  first-order stochastically dominates  $G_{j\ell}^s$  for all  $\ell \neq i, j$ , and as a consequence, also

$$\int_{-\infty}^{\infty} v - c_1 dG_{i\ell}^s(v) > \int_{-\infty}^{\infty} v - c_1 dG_{j\ell}^s(v) \quad \text{for all } \ell \neq i, j.$$

Second, we observe that

$$\begin{aligned} & \int_{-\infty}^{\infty} v - c_1 dG_{ij}^s(v) - \int_{-\infty}^{\infty} v - c_1 dG_{ji}^s(v) \\ &= F_i^s(\hat{v}^s) \int_{\hat{v}^s}^{\infty} v - c_1 dF_j^s(v) - F_j^s(\hat{v}^s) \int_{\hat{v}^s}^{\infty} v - c_1 dF_i^s(v) \\ &= F_i^s(\hat{v}^s)(1 - F_j^s(\hat{v}^s))\mathbb{E}[v_j^s - c_1 | v_j^s \geq \hat{v}^s] - F_j^s(\hat{v}^s)(1 - F_i^s(\hat{v}^s))\mathbb{E}[v_i^s - c_1 | v_i^s \geq \hat{v}^s], \end{aligned}$$

and thus

$$\begin{aligned} & \left( \int_{\hat{v}^s}^{\infty} v dF_i^s(v) + \int_{-\infty}^{\infty} v - c_1 dG_{ij}^s(v) \right) - \left( \int_{\hat{v}^s}^{\infty} v dF_j^s(v) + \int_{-\infty}^{\infty} v - c_1 dG_{ji}^s(v) \right) \\ &= (1 - F_i^s(\hat{v}^s))(1 - F_j^s(\hat{v}^s)) \left( \mathbb{E}[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s] - \mathbb{E}[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s] \right) + c_1 (F_j^s(\hat{v}^s) - F_i^s(\hat{v}^s)). \end{aligned}$$

Notice that

$$\mathbb{P}[v_i^s \leq v | v_i^s \geq \hat{v}^s] = \begin{cases} \frac{F_i^s(v)}{1 - F_i^s(\hat{v}^s)} & \text{if } v \geq \hat{v}^s \\ 0 & \text{otherwise.} \end{cases}$$

Hence, for any  $v \geq \hat{v}^s$ , if  $F_i^s$  first-order stochastically dominates  $F_j^s$ , then

$$\begin{aligned} & \mathbb{P}[v_i^s \leq v | v_i^s \geq \hat{v}^s] - \mathbb{P}[v_j^s \leq v | v_j^s \geq \hat{v}^s] \\ &= \frac{1}{(1 - F_i^s(\hat{v}^s))(1 - F_j^s(\hat{v}^s))} \left[ F_i^s(v) - F_j^s(v) + F_i^s(\hat{v}^s)F_j^s(v) - F_j^s(\hat{v}^s)F_i^s(v) \right] \\ &\leq \frac{1}{(1 - F_i^s(\hat{v}^s))(1 - F_j^s(\hat{v}^s))} \left[ F_i^s(v) - F_j^s(v) + F_i^s(\hat{v}^s)F_j^s(v) - F_i^s(\hat{v}^s)F_i^s(v) \right] \\ &= -\frac{F_j^s(v) - F_i^s(v)}{1 - F_j^s(\hat{v}^s)}, \end{aligned}$$

where the inequality follows from  $F_j^s(v) \geq F_i^s(v)$ . We conclude that the dominance relation is preserved when truncating the distributions of  $v_i^s$  and  $v_j^s$  at  $\hat{v}^s$ . This, in turn, implies that if  $F_i^s$  first-order stochastically dominates  $F_j^s$ , then  $\mathbb{E}[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s] \geq \mathbb{E}[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s]$ .

Combining both observations, if  $F_i^s$  first-order stochastically dominates  $F_j^s$ , then consumer welfare increases by more than

$$\frac{n^s - 2}{n^s - 1} \left[ \mathbb{P}[v_i^s \geq \hat{v}^s] \mathbb{E}[v_i^s \geq \hat{v}^s | v_i^s \geq \hat{v}^s] - \mathbb{P}[v_j^s \geq \hat{v}^s] \mathbb{E}[v_j^s \geq \hat{v}^s | v_j^s \geq \hat{v}^s] \right] > 0$$

if product  $i$  replaces  $j$  as the default. Hence, a better default improves consumer welfare.

## C.4 Consideration Sets

Consider market  $s$ , and let  $n^s \geq 2$ . The consumer considers  $k^s \in \{1, 2, \dots, n^s - 1\}$  offers, and selects one with the highest realized value  $v_i^s$  among the considered ones. The consumer considers the default product with certainty. Any non-default product enters the consideration set with probability of  $(k^s - 1)/(n^s - 1)$ ; that is, attention to non-default products is random.

**Lemma 6** (Consideration Sets).

I. *Assumption 0 holds.*

II. *Assumption 1 holds if and only if  $n^s \geq 3$  and  $k^s \geq 2$ .*

*Proof. Preliminaries.* Consider firm  $i$ , and refer to  $r_{ii}^s \in [0, 1]$  as its (average) “conversion rate” conditional on being in the consideration set and facing  $k^s - 1$  random competitors. This is exactly what we call in the main text the firm’s demand conditional on being default.

To see how the conversion rate depends on the value that firm  $i$  and its rivals offer, we denote as  $\mathcal{N}^s(x)$  a subset of  $x \geq 1$  firms that are active in market  $s$ . We introduce the CDF

$$q_{\mathcal{N}^s(x)}(v) := \prod_{\ell \in \mathcal{N}^s(x)} F_{\ell}^s(v);$$

that is, the probability that the products of all firms in the set  $\mathcal{N}^s(x)$  have a realized value less or equal to  $v$ . Using this notation, we can write the conversion rate of firm  $i$  as

$$r_{ii}^s = \sum_{\mathcal{N}^s(k^s-1): i \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v). \quad (11)$$

If  $i$  is in the consideration set, all  $n^s - 1$  rivals are equally likely to be drawn into one of the  $k^s - 1$  remaining spots. Hence, there are  $\binom{n^s-1}{k^s-1}$  different sets of competitors, each of which occurs with probability  $1/\binom{n^s-1}{k^s-1}$ . Fixing a set of competitors  $\mathcal{N}^s(k^s - 1)$ , the consumer uses  $i$ ’s service if and only if it offers the highest value; that is,  $v_i^s > v_j^s$  for all  $j \in \mathcal{N}^s(k^s - 1)$ . This happens with probability  $\int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v)$ . Because all distributions  $F_j^s$  have common support, we have  $r_{ii}^s > 0$ . By the same argument, we get  $r_{ii}^s < 1$  if and only if  $k^s \geq 2$ .

Now consider firms  $i$  and  $j$  with  $F_i^s$  first-order stochastically dominating  $F_j^s$ . Then,

$$\begin{aligned}
r_{ii}^s - r_{jj}^s &\propto \sum_{\mathcal{N}^s(k^s-1): i, j \notin \mathcal{N}^s(k^s-1)} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v) dF_j^s(v) \right\} \\
&+ \sum_{\mathcal{N}^s(k^s-1): i \notin \mathcal{N}^s(k^s-1), j \in \mathcal{N}^s(k^s-1)} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) - \sum_{\mathcal{N}^s(k^s-1): j \notin \mathcal{N}^s(k^s-1), i \in \mathcal{N}^s(k^s-1)} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_j^s(v) \\
&\geq \sum_{\mathcal{N}^s(k^s-1): i, j \notin \mathcal{N}^s(k^s-1)} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v) dF_j^s(v) \right\} \\
&+ \sum_{\mathcal{N}^s(k^s-1): i \notin \mathcal{N}^s(k^s-1), j \in \mathcal{N}^s(k^s-1)} \left\{ \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) - \int q_{\mathcal{N}^s(k^s-1)}(v) dF_j^s(v) \right\} \\
&\geq 0,
\end{aligned}$$

where the first inequality holds because  $F_j^s(v) \geq F_i^s(v)$  and the second one follows from the fact that  $q_{\mathcal{N}^s(k^s-1)}(v)$  is increasing in  $v$  together with  $F_i^s$  first-order stochastically dominating  $F_j^s$ . Moreover, whenever  $k^s \geq 2$ , the second inequality is strict, which in turn implies  $r_{ii}^s > r_{jj}^s$ .

Similarly, we refer to  $r_{ij}^s \in [0, 1]$  as the (average) conversion rate of firm  $i$  conditional on being in the consideration set and facing  $j$  as well as  $k^s - 2$  random competitors. Notice that this conversion rate is *not* the same as the demand of firm  $i$  when firm  $j$ 's product is the default. Using the same notation as above, we can write this conversion rate as

$$r_{ij}^s = \sum_{\substack{\mathcal{N}^s(k^s-1): i \notin \mathcal{N}^s(k^s-1), \\ j \in \mathcal{N}^s(k^s-1)}} \frac{1}{\binom{n^s-2}{k^s-2}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v). \quad (12)$$

This implies, in particular, that

$$r_{ij}^s - r_{ij'}^s = \sum_{\mathcal{N}^s(k^s-2): i, j, j' \notin \mathcal{N}^s(k^s-2)} \frac{1}{\binom{n^s-2}{k^s-2}} \int q_{\mathcal{N}^s(k^s-2)}(v) [F_j^s(v) - F_{j'}^s(v)] dF_i^s(v).$$

Hence, if  $F_j^s$  first-order stochastically dominates  $F_{j'}^s$  and  $k^s \leq n^s - 1$ , then  $r_{ij}^s < r_{ij'}^s$ .

Part I. For any  $i$  and  $j \neq i$ , we have

$$q_{ii}^s - q_{ij}^s = r_{ii}^s - \frac{k^1 - 1}{n^1 - 1} r_{ij}^s = \sum_{\mathcal{N}^s(k^s-1): i, j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v). \quad (13)$$

Since all  $F_\ell^s$  have the same support, the above is postive. Hence, Part Assumption 0 holds.

Part II. Suppose  $F_j^s$  first-order stochastically dominates  $F_{j'}^s$ . Then,

$$\begin{aligned}
q_{ij'}^s - q_{ij}^s &= \sum_{\mathcal{N}^s(k^s-1): i, j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) \\
&\quad - \sum_{\mathcal{N}^s(k^s-1): i, j' \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) \\
&= \frac{1}{\binom{n^s-1}{k^s-1}} \sum_{\substack{\mathcal{N}^s(k^s-1): i, j \notin \mathcal{N}^s(k^s-1), \\ j' \in \mathcal{N}^s(k^s-1)}} \int (F_{j'}^s(v) - F_j^s(v)) \prod_{\ell \in \mathcal{N}^s(k^s-1) \setminus \{j'\}} F_\ell^s(v) dF_i^s(v) > 0,
\end{aligned}$$

where the second equality follows from the definition of  $q_{\mathcal{N}^s(k^s-1)}(v)$  and the inequality holds (weakly) as  $F_j^s$  first-order stochastically dominates  $F_{j'}^s$ . The inequality is strict since all  $F_\ell^s$  have no mass points and common support. Hence, if  $k^s \geq 2$ , Part I of Assumption 1 holds.

Next, consider firms,  $i$  and  $i'$ , with  $F_i^s$  first-order stochastically dominating  $F_{i'}^s$ . Then,

$$\begin{aligned}
(q_{ii}^s - q_{ij}^s) - (q_{i'i'}^s - q_{i'j}^s) &= \sum_{\mathcal{N}^s(k^s-1): i, j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) \\
&\quad - \sum_{\mathcal{N}^s(k^s-1): i', j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_{i'}^s(v) \\
&\geq \sum_{\mathcal{N}^s(k^s-1): i, j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) \\
&\quad - \sum_{\mathcal{N}^s(k^s-1): i', j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_{i'}^s(v) \\
&= \frac{1}{\binom{n^s-1}{k^s-1}} \sum_{\substack{\mathcal{N}^s(k^s-1): i, j \notin \mathcal{N}^s(k^s-1), \\ i' \in \mathcal{N}^s(k^s-1)}} \int (F_{i'}^s(v) - F_i^s(v)) \prod_{\ell \in \mathcal{N}^s(k^s-1) \setminus \{i'\}} F_\ell^s(v) dF_i^s(v) \\
&\geq 0,
\end{aligned}$$

where the first inequality follows from  $q_{\mathcal{N}^s(k^s-1)}(v)$  being monotonically increasing in  $v$  and  $F_i^s$  first-order stochastically dominating  $F_{i'}^s$  while the second one simply follows from first-order stochastic dominance. Whenever  $k^s \geq 2$ , the inequalities are strict because the distributions have no mass points and common support. Hence, if  $k^s \geq 2$ , Part II of Assumption 1 holds.



Finally, let  $F_i^s$  first-order stochastically dominate  $F_j^s$ , and observe that

$$\begin{aligned}
(q_{ii}^s - q_{ij}^s) - (q_{jj}^s - q_{ji}^s) &= \sum_{\mathcal{N}^s(k^s-1): i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) \\
&\quad - \sum_{\mathcal{N}^s(k^s-1): i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_j^s(v) \\
&\geq \sum_{\mathcal{N}^s(k^s-1): i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) \\
&\quad - \sum_{\mathcal{N}^s(k^s-1): i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} \int q_{\mathcal{N}^s(k^s-1)}(v) dF_i^s(v) \\
&= 0,
\end{aligned}$$

where the inequality holds since  $q_{\mathcal{N}^s(k^s-1)}(v)$  is increasing and  $F_i^s$  first-order stochastically dominates  $F_j^s$ , and it is strict if  $k^s \geq 2$ . Hence, if  $k^s \geq 2$ , Part III of Assumption 1 holds.  $\square$

**Consumer Welfare** Denote as  $\nu_i^s$  the value of the product *chosen* by the consumer when product  $i$  is the default. The value  $\nu_i^s$  is drawn from the cumulative distribution function

$$\mathbb{P}[\nu_i^s \leq v] = F_i^s(v) \sum_{\mathcal{N}^s(k^s-1): i \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} q_{\mathcal{N}^s(k^s-1)}(v).$$

Notice that

$$\mathbb{P}[\nu_i^s \leq v] - \mathbb{P}[\nu_j^s \leq v] = \left( F_i^s(v) - F_j^s(v) \right) \sum_{\mathcal{N}^s(k^s-1): i,j \notin \mathcal{N}^s(k^s-1)} \frac{1}{\binom{n^s-1}{k^s-1}} q_{\mathcal{N}^s(k^s-1)}(v).$$

Hence, if  $F_i^s$  first-order stochastically dominates  $F_j^s$ , then  $\nu_i^s$  first-order stochastically dominates  $\nu_j^s$ , and consumer welfare is higher with a better default.

## C.5 Loss Aversion

Consider market  $s$ , and let  $n^s \geq 2$ . Each product  $i$  is defined by a pair  $(v_i^s, d_i^s)$  with  $v_i^s \sim G^s$  and  $d_i^s \in \mathbb{R}_{>0}$ . We think of  $d_i^s$  as an app-design dimension, which affects the consumer's

utility in an additively separable way. Let  $F_i^s(u) = G^s(u - d_i^s)$  be the distribution of  $v_i^s + d_i^s$ . We assume that, if the consumer uses the product of firm  $j$  instead of the default product  $i$ , she experiences a loss of  $\lambda d_i$  (with  $\lambda > 1$ ) because she cannot enjoy firm  $i$ 's proprietary design features, and a gain of  $d_j$  from using firm  $j$ 's product instead. In addition, she has some reference point in the service dimension, and we think of  $G^s$  as being “gain-loss adjusted.”

**Lemma 7** (Loss Aversion).

*I. Assumption 0 holds.*

*II. For any  $n^s \geq 3$  and any  $\{F_\ell^s\}_{\ell \in \mathcal{N}^s}$  that can be ranked in terms of the reversed hazard rate, there exists some  $\bar{\lambda} > 1$ , such that for any  $\lambda \in (1, \bar{\lambda})$ , Assumption 1 holds.*

*Proof.* We first show that the model is mathematically equivalent to the switching-cost model in Section C.2. Let product  $i$  be the default. Because  $v_i^s$  is gain-loss adjusted, and we assume that the reference point in this dimension is independent of a product's default status, the consumer chooses the product of firm  $i$  over that of a rival  $j$  if and only if

$$v_i > v_j + d_j - \lambda d_i \quad \text{or, equivalently,} \quad v_i + d_i > v_j + d_j - \underbrace{(\lambda - 1)d_i}_{=:\gamma_i}.$$

The claim now follows immediately from Lemmas 3 and 4. □

**Consumer Welfare** In the context of digital markets, it seems plausible to assume that gain-loss utility is not welfare relevant. Thus, from a normative perspective, our loss-aversion based model is different from the switching-cost model in Section C.2. In fact, with loss aversion, a better default does not necessarily improve consumer welfare (see also Goldin and Reck, 2022). To see this most clearly, suppose that there exists a product  $w \in \mathcal{N}^s$  with

$$\mathbb{P}[v_w + d_w + \gamma_w < v_j + d_j] \approx 1$$

for all  $j \neq w$ . Then, if  $w$  is the default, the consumer makes an active choice with almost probability 1, and thus almost certainly chooses the welfare-maximizing option. Hence, with loss-aversion based default effects it can be optimal to force choice by setting a bad default.

## D Proof of Proposition 3

Since all targets are identical, for any  $t_1, t_2 \in \mathcal{N}^b$ , we write  $\eta^b$  instead of  $\eta_{t_1 t_2}^b$ . Similarly, for any  $t \in \mathcal{N}^b$ , we write  $\alpha^b$  and  $V^b$  instead of  $\alpha_t^b$  and  $V_t^b$ . For any round of bidding, we distinguish between a *bidding subgame* in which all remaining acquirers decide what bid to submit, and an *acceptance subgame* in which the target that is currently up for sale decides whether to accept one of the bids. We will subsequently derive the equilibrium takeover price in the following three cases: (a)  $n^b < |\mathcal{A}|$ , (b)  $n^b = |\mathcal{A}|$ , and (c)  $n^b > |\mathcal{A}|$ .

Case (a). Let  $n^b < |\mathcal{A}|$ . We solve the game backwards and proceed by induction over the number of remaining targets.

**Induction hypothesis:** *Consider any bidding subgame with  $k \geq 1$  remaining targets. Denote as  $A(k) \subseteq \mathcal{A}$  the set of potential acquires in this subgame that have not yet taken over a target, and note that  $|A(k)| \geq k+1$  because there are more potential acquirers than targets (i.e.,  $n^b < |\mathcal{A}|$ ). (In slight abuse of notation, we suppress the dependence of this subgame on the prior history, which determines the identity of potential acquirers. This is without loss of generality because these identities are fixed from this point onward.) Denote by  $\ell_i \in A(k)$  the potential acquirer with the  $i$ th highest market share in market  $a$  among the firms in  $A(k)$ . Define as  $\bar{A}(k, x) := \{\ell_1, \dots, \ell_x\}$  the set of the  $x$  firms with the highest market shares in  $A(k)$ , and set  $\bar{A}(k, 0) = \emptyset$ . We hypothesize that all firms in  $\bar{A}(k, k)$  take over one of the remaining targets at a price*

$$f^*(k) := V^b - |\eta^b| \left( \sum_{i \in A \setminus A(k)} q_i^a + \sum_{i \in \bar{A}(k, k-1)} q_i^a \right) + q_{\ell_{k+1}}^a \alpha^b. \quad (14)$$

**Induction anchor:** Consider a bidding subgame with a single target left ( $k = 1$ ). We write  $f^* = f^*(1)$ . The induction hypothesis states that  $\ell_1$  takes over the last target at a price of

$$f^* = V^b - |\eta^b| \sum_{i \in A \setminus A(1)} q_i^a + q_{\ell_2}^a \alpha^b. \quad (15)$$

We verify the induction hypothesis in two steps.

1. *Step: equilibrium construction.* We construct a cautious equilibrium in which  $\ell_1$  takes over the last target at a price of  $f^*$ . Denote the target's adjusted standalone value by

$$\nu := V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus A(1)} q_i^a.$$

and the takeover revenue of a potential acquirer  $j \in A(1)$  by

$$\pi_j := \nu + q_j^a \alpha^b.$$

We argue that the target accepting any highest bid at or above  $\nu$  (breaking ties randomly),  $\ell_1$  bidding  $f^*$ ,  $\ell_2$  submitting a bid by uniformly randomizing over  $[f^* - \epsilon, f^*]$ , and any other firm  $j \neq \ell_1, \ell_2$  bidding  $\nu$  is a cautious equilibrium of this subgame for small enough  $\epsilon$ .<sup>30</sup>

We first verify that the above is a subgame perfect equilibrium. If the target rejects the highest bid, it earns  $\nu$ . Hence, it is optimal to accept any highest bid at or above  $\nu$ .

Firm  $\ell_1$  has no incentive to bid more than  $f^*$  as then it would take over the target at a strictly higher price, and it does not want to bid less than  $f^* - \epsilon$  as then it would lose and earn nothing in market  $b$  instead of  $(q_{\ell_1}^a - q_{\ell_2}^a) \alpha^b > 0$ . If firm  $\ell_1$  deviates to  $f \in (f^* - \epsilon, f^*)$ , it earns at most

$$\frac{f - (f^* - \epsilon)}{\epsilon} \left( \nu + q_{\ell_1}^a \alpha^b - f \right). \quad (16)$$

At  $f = f^*$ , the profit in (16) equals  $(q_{\ell_1}^a - q_{\ell_2}^a) \alpha^b$ ; that is, the candidate equilibrium profit.

Moreover, the derivative of (16) with respect to  $f$  is

$$\frac{1}{\epsilon} \left( \nu + q_{\ell_1}^a \alpha^b - f \right) - \left( 1 - \frac{f^* - f}{\epsilon} \right) > \frac{1}{\epsilon} \left( \nu + q_{\ell_1}^a \alpha^b - f \right) - 1 > \frac{(q_{\ell_1}^a - q_{\ell_2}^a) \alpha^b}{\epsilon} - 1 > 0,$$

which holds for all  $f \in (f^* - \epsilon, f^*)$  if  $\epsilon$  is small enough. Hence, firm  $\ell_1$  has no incentive to deviate. In fact, for  $\epsilon$  small enough bidding  $f^*$  is firm  $\ell_1$ 's strict best reply.

Finally, no firm  $j \neq \ell_1$  has an incentive to bid above  $f^*$  — a bid the target would accept — as then they would earn negative profits from the takeover, and they are indifferent between

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<sup>30</sup> The equilibrium we construct is inspired by Blume (2003).

bids (weakly) below the takeover price since independently of the size of such a bid they will lose for certain and earn zero profits in  $b$ . Hence, no firm  $j \neq \ell_1$  has an incentive to deviate.

It remains to be shown that all firms play iteratively weakly undominated strategies in the reduced bidding game in which all acceptance subgames with a unique equilibrium payoff are replaced by the corresponding equilibrium payoffs. This follows from the lemma below.

**Lemma 8** (Iteratively Weakly Undominated Strategies).

- I. Bidding  $\nu$  is iteratively weakly undominated for any potential acquirer.*
- II. For any potential acquirer  $\ell_j \neq \ell_1$ , bidding  $f \in (\nu, \pi_{\ell_j})$  is iteratively weakly undominated, and for firm  $\ell_1$ , bidding  $f \in (\nu, \pi_{\ell_2})$  is iteratively weakly undominated.*
- III. For firm  $\ell_2$ , any mixed strategy with support in  $(\nu, \pi_{\ell_2}) = (\nu, f^*)$  and a continuous CDF is iteratively weakly undominated.*
- IV. For firm  $\ell_1$ , bidding  $\pi_{\ell_2} = f^*$  is iteratively weakly undominated.*

*Proof. Preliminary.* Note that all acceptance subgames following *multiple* highest bids at or above  $\nu$  do *not* have a unique equilibrium payoff, as the equilibrium payoffs depend on whom the target selects, and are thus identical in the original and the reduced game; that is, the reduced game is still an extensive-form game. In particular, for any  $j$  making a highest bid, the target accepting  $j$ 's bid with probability 1 is an equilibrium of the acceptance subgame.

Part I. We argue that in the reduced bidding game, for any potential acquirer, it is iteratively weakly undominated to bid  $\nu$ . We do so by induction over the rounds of elimination of weakly dominated strategies. Bidding  $\nu$  is trivially undominated in the first round of elimination; for example, it is the strict best reply to every rival bidding strictly below  $\nu$ . Now suppose that  $\nu$  is weakly undominated for all potential acquirers after  $k$  rounds of elimination, and consider round  $k + 1$ . For every potential acquirer  $j$ , bidding  $\nu$  is a strict best response to all its rivals bidding  $\nu$  and the target accepting  $j$ 's offer with probability one. Hence, for every potential acquirer, bidding  $\nu$  is weakly undominated in the  $(k + 1)$ th round of elimination. This proves the claim.

Part II. Again, we proceed by induction, starting with the potential acquirer  $\ell_j \neq \ell_1$  that has the lowest demand in  $a$ . Suppose that after  $k$  rounds of elimination, bidding  $f \in (\nu, \pi_{\ell_j})$  is iteratively weakly undominated for all  $\ell_{j'}$  with  $j' \leq j$ , and note this is trivially true after zero rounds of elimination. We now argue that it then is also weakly undominated to bid  $f \in (\nu, \pi_{\ell_j})$  after  $k + 1$  rounds of elimination for all potential acquirers  $\ell_{j'}$  with  $j' \leq j$ . By Part I, it is iteratively weakly undominated to bid  $\nu$  for all acquirers, and from now on presume that all potential acquirers  $\ell_{j''}$  with  $j'' > j$  do so. For any potential acquirer  $j' \leq j$ , bidding  $f$  is a strict best response to all rivals with an index weakly below  $j$  bidding  $f$  and the target accepting  $j'$ 's offer with probability one. Hence, for any  $j'$ , bidding  $f$  is weakly undominated in the  $(k + 1)$ th round of elimination.

Part III. Suppose that firm  $\ell_1$  bids  $f' \in [\nu, \pi_{\ell_2})$  and all firms  $j \neq \ell_1, \ell_2$  bid  $\nu$ , which is weakly undominated by Parts I and II. Now consider two (potentially) mixed strategies of firm  $\ell_2$  with support in  $(\nu, \pi_{\ell_2})$ , and denote the corresponding CDFs as  $G$  and  $H$  with  $G \neq H$ . Assume that  $G$  is continuous. When bidding according to  $G$ , firm  $\ell_2$ 's additional profits in market  $b$  are

$$(1 - G(f'))(\pi_{\ell_2} - \mathbb{E}_G[f|f \geq f']) = (1 - G(f'))\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f dG(f).$$

Similarly, when bidding according to  $H$ , firm  $\ell_2$  earns additional profits of

$$(1 - H(f'))(\pi_{\ell_2} - \mathbb{E}_H[f|f \geq f']) = (1 - H(f'))\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f dH(f).$$

For the sake of a contradiction, suppose that the (potentially) mixed strategy with CDF  $H$  weakly dominates the mixed strategy with CDF  $G$ . Then, it follows from the last two equations that for any  $f' \in [\nu, \pi_{\ell_2})$ ,

$$(G(f') - H(f'))\pi_{\ell_2} - \int_{f'}^{\pi_{\ell_2}} f dH(f) + \int_{f'}^{\pi_{\ell_2}} f dG(f) \geq 0. \quad (17)$$

Integration by parts implies that (17) is equivalent to

$$-(H(f') - G(f'))(\pi_{\ell_2} - f') + \int_{f'}^{\pi_{\ell_2}} H(f) - G(f) df \geq 0. \quad (18)$$

We derive a contradiction in three steps. First, evaluating (17) at  $f' = \nu$ , we conclude

$$\mathbb{E}_H[f] \leq \mathbb{E}_G[f].$$

Hence,  $H$  cannot first-order stochastically dominate  $G$ , so that  $G(f) < H(f)$  for some  $f \in [\nu, \pi_{\ell_2})$ . Define  $\bar{f} := \sup\{\arg \sup H(f) - G(f)\}$ , and notice that  $H(\bar{f}) - G(\bar{f}) > 0$ .

Second, because  $H$  is right-continuous and  $G$  is continuous, also  $H - G$  is right-continuous. This in turn implies that the left-hand side of (18) is right-continuous. Hence, if  $\bar{f} < \pi_{\ell_2}$ ,

$$-(H(\bar{f}) - G(\bar{f}))(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(f) - G(f) df \geq 0.$$

Third, to establish a contradiction, we distinguish the following two cases:  $\bar{f} < \pi_{\ell_2}$  and  $\bar{f} = \pi_{\ell_2}$ . If  $\bar{f} < \pi_{\ell_2}$ ,  $H(f) - G(f)$  cannot be constant on  $(\bar{f}, \pi_{\ell_2})$ . Hence, there exists some  $\hat{f} \in (\bar{f}, \pi_{\ell_2})$  and some  $\zeta > 0$ , such that  $\sup_{f \in (\hat{f}, \pi_{\ell_2})} H(f) - G(f) < H(\bar{f}) - G(\bar{f}) - \zeta$ . Thus,

$$\begin{aligned} & -(H(\bar{f}) - G(\bar{f}))(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(f) - G(f) df \\ & < -(H(\bar{f}) - G(\bar{f}))(\pi_{\ell_2} - \bar{f}) + \int_{\bar{f}}^{\pi_{\ell_2}} H(\bar{f}) - G(\bar{f}) df - \int_{\hat{f}}^{\pi_{\ell_2}} \zeta df \\ & = -\zeta(\pi_{\ell_2} - \hat{f}) \\ & < 0; \end{aligned}$$

a contradiction. If  $\bar{f} = \pi_{\ell_2}$ ,  $H(f)$  jumps at  $f = \pi_{\ell_2}$ . Since  $G$  is continuous, and  $H$  is right-continuous and does not first-order stochastically dominate  $G$ , there exists  $f^s < \bar{f}$ , so that  $H(f^s) = G(f^s)$  and  $H(f) < G(f)$  for  $f \in (f^s, \pi_{\ell_2})$ . At  $f' = f^s$ , the left-hand side of (17) is

$$\underbrace{(G(f^s) - H(f^s))}_{=0} \pi_{\ell_2} - \int_{f^s}^{\pi_{\ell_2}} f dH(f) + \int_{f^s}^{\pi_{\ell_2}} f dG(f) = \mathbb{E}_G[f|f \geq f^s] - \mathbb{E}_H[f|f \geq f^s] < 0,$$

where the inequality holds as  $H$  FOSD-dominates  $G$  on  $(f^s, \pi_{\ell_2})$ ; a contradiction.

Part IV. Suppose that all of  $\ell_1$ 's rivals bid according the candidate equilibrium strategies, which are weakly undominated by Parts I and III. Then, as we have argued above, bidding  $f^*$  is firm  $\ell_1$ 's strict best reply and thus weakly undominated.  $\square$

2. *Step: unique equilibrium outcome.* We show that  $\ell_1$  takes over the last target at  $f^*$  in any cautious equilibrium. First, we argue that in any cautious equilibrium the infimum  $\underline{f}$  of the highest equilibrium takeover bid is at least  $f^*$ . Suppose not, i.e.  $\underline{f} < f^*$ . We show that then both firms  $\ell_1$  and  $\ell_2$  bid weakly above  $\underline{f}$  with probability one. If one of them does not, it must in equilibrium with positive probability submit bids below  $f^*$  that loose with probability one. This firm could then deviate and move this probability mass to a bid  $(\underline{f} + f^*)/2$  and win with positive probability thereby earning positive expected profits. Furthermore, if the distribution of the highest equilibrium takeover bid has no mass point at  $\underline{f}$ , both  $\ell_1$  and  $\ell_2$  must bid above  $\underline{f} + \epsilon'$  for some  $\epsilon' > 0$  with probability one; for otherwise, the expected profits as  $f \rightarrow \underline{f}$  from above approach zero while those of deviating and bidding  $(\underline{f} + f^*)/2$  do not; a contradiction. This, however, contradicts that  $\underline{f}$  is the infimum of the highest bid. Thus, with positive probability both  $\ell_1$  and  $\ell_2$  bid  $\underline{f}$ , and win when doing so. But since they earn strictly positive profits when winning at this price, either of them would be better off moving the probability mass from  $\underline{f}$  to  $\underline{f} + \epsilon''$  for some small enough  $\epsilon'' > 0$ . We conclude that the infimum of the highest equilibrium bid is at least  $f^*$ .

Second, we argue that the supremum of the highest equilibrium bid in a cautious equilibrium is  $f^*$ . Bidding at or above  $f^*$  is weakly dominated for all firms other than  $\ell_1$ . Hence, firms other than  $\ell_1$  must bid strictly below  $f^*$  with probability one. Thus, firm  $\ell_1$  wins with probability one when bidding  $f^*$ , and hence cannot bid strictly above  $f^*$  either. Hence, the highest equilibrium bid must be  $f^*$  in any cautious equilibrium with probability one; and it must be submitted by  $\ell_1$ , and thus  $\ell_1$  wins with probability one in any cautious equilibrium.

**Induction step:** Suppose that the induction hypothesis holds for any bidding subgame with  $k \geq 1$  remaining targets. Consider a bidding subgame with  $k + 1$  remaining targets, and note that there are at least  $k + 2$  remaining acquirers. Define  $A(k + 1)$  and  $\bar{A}(k + 1, x)$  analogously to the above.

The induction step follows in five steps: in steps one to four, we verify that the induction



hypothesis must hold in any cautious equilibrium of a bidding subgame with  $k+1$  remaining targets; in step five, we then construct a cautious equilibrium of this bidding subgame.

1. *Step: no firm  $\ell_j \in A(k+1)$  with  $j \geq k+2$  takes over the current target.* Recall that we rank acquirers in terms of their market shares in market  $a$ , with a lower index  $j$  indicating a higher market share. Let  $\ell_w$  be the firm buying target  $k+1$ , and let  $\pi_j(w)$  be firm  $j$ 's additional subgame perfect equilibrium profit in the bidding subgame with  $k$  remaining targets as a function of  $w$ .

For the sake of a contradiction, suppose firm  $\ell_j$  with  $j \geq k+2$  takes over the current target with positive probability. Then, by the induction hypothesis the remaining targets are acquired the firms in  $\bar{A}(k+1, k)$ . Hence,  $\ell_j$ 's gross profits in market  $b$  are

$$\pi_j(j) = V^b - |\eta^b| \left( \sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1, k)} q_i^a \right) + \alpha^b q_{\ell_j}^a.$$

Using the induction hypothesis, we construct a reduced game by replacing any continuation subgame following the current acceptance subgame (i.e. following the target's acceptance decision) by its unique equilibrium payoffs, and replacing all possible acceptance subgames in which the target is not indifferent (between accepting and rejecting some offer) by its unique equilibrium payoffs.

If  $\ell_j$  does not take over the current target, it will not make a takeover; thus, its profits from the takeover market will be zero. If it takes over the current target, its profits will be  $\pi_j(j)$ . In the reduced game, thus, it is weakly dominated for firm  $\ell_j$  to bid at or above  $\pi_j(j)$ . Hence, because  $\ell_j$  takes over the current target with positive probability, and because it plays a weakly undominated strategy in any cautious equilibrium, the infimum  $\underline{f}$  of the highest bid must be strictly below  $\pi_j(j)$ .

We next argue that any firm  $\ell_{j'} \in \bar{A}(k+1, k)$  makes greater profits when taking over the current target at  $\underline{f}$  than when not doing so, irrespective of which rival  $\ell_w$  would take over the current target otherwise. To see why, let  $f^* = f^*(k+1)$ , and consider three cases. First, if  $w \leq k$ , by the induction hypothesis, any firm  $\ell_{j'} \in \bar{A}(k+1, k)$  takes over a remaining

target at a price

$$f^* = V^b - |\eta^b| \left( \sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1, k)} q_i^a \right) + \alpha^b q_{\ell_{k+2}}^a \geq \pi_j(j),$$

and following the takeover in market  $b$  earns additional gross profits of

$$\pi_{j'}(w) = V^b - |\eta^b| \left( \sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1, k+1) \setminus \{\ell_{j'}\}} q_i^a \right) + \alpha^b q_{\ell_{j'}}^a.$$

This implies additional net profits of  $\alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a) > 0$ . If instead firm  $\ell_{j'}$  takes over the current target at  $\underline{f}$ , by the induction hypothesis, it earns additional profits of  $\pi_{j'}(j') - \underline{f} = \pi_{j'}(w) - \underline{f} > \pi_{j'}(w) - \pi_j(j) \geq \pi_{j'}(w) - f^* = \alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a)$ , where the first equality holds since  $\ell_w$  would take over one of the remaining targets.

Second, if  $w = k + 1$ , then by the induction hypothesis any firm  $\ell_{j'} \in A(k + 1, k)$  takes over a remaining target at a price

$$f' = V^b - |\eta^b| \left( \sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1, k-1)} q_i^a + q_{\ell_{k+1}}^a \right) + \alpha^b q_{\ell_{k+2}}^a > f^*,$$

and earns additional gross profits of  $\pi_{j'}(k + 1) = \pi_{j'}(j')$ . The additional net profits are  $\pi_{j'}(j') - f'$ . If  $\ell_{j'}$  takes over the current target at  $\underline{f}$ , by the induction hypothesis, it earns additional profits of

$$\pi_{j'}(j') - \underline{f} \geq \pi_{j'}(j') - \pi_j(j) > \pi_{j'}(j') - f^* > \pi_{j'}(j') - f'.$$

Third, if  $w \geq k + 2$ , then by the induction hypothesis a firm  $\ell_{j'} \in A(k + 1, k)$  takes over a remaining target at a price

$$f'' = V^b - |\eta^b| \left( \sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1, k-1)} q_i^a + q_{\ell_w}^a \right) + \alpha^b q_{\ell_{k+1}}^a,$$

and earns additional gross profits of

$$\pi_{j'}(w) = V^b - |\eta^b| \left( \sum_{i \in \mathcal{A} \setminus A(k+1)} q_i^a + \sum_{i \in \bar{A}(k+1, k) \setminus \{\ell_{j'}\}} q_i^a + q_{\ell_w}^a \right) + \alpha^b q_{\ell_{j'}}^a.$$

and additional net profits of

$$\pi_{j'}(w) - f'' = \alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_k}^a).$$

If  $\ell_{j'}$  takes over the current target at  $\underline{f}$ , by the induction hypothesis, it earns

$$\pi_{j'}(j') - \underline{f} \geq \pi_{j'}(j') - \pi_j(j) > \pi_{j'}(j') - f^* = \alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a).$$

Hence, because

$$\alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+2}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a) > \alpha^b(q_{\ell_{j'}}^a - q_{\ell_{k+1}}^a) + |\eta^b|(q_{\ell_{j'}}^a - q_{\ell_k}^a),$$

firm  $\ell_{j'}$  strictly prefers winning at  $\underline{f}$  also in this last case.

Observe that there are at least two firms in  $\bar{A}(k+1, k)$ . Since any  $\ell_{j'} \in \bar{A}(k+1, k)$  strictly prefers winning to loosing when bidding  $\underline{f}$ , by a similar argument than in the induction anchor (2. Step), it cannot be the case that two or more firms in  $\bar{A}(k+1, k)$  have a mass point at  $\underline{f}$ . Furthermore, if no rival has a mass point at  $\underline{f}$ , as the bid of a firm  $\ell_{j'}$  approaches  $\underline{f}$  from above its profits approach those of losing for certain. In that case, however, by a similar argument than in the induction anchor, there exists an  $\epsilon' > 0$  such that firm  $\ell_{j'}$  is strictly better off moving any probability mass from below  $\underline{f} + \epsilon'$  to  $\underline{f} + \epsilon'$ . Hence, any firm in  $\bar{A}(k+1, k)$  has an incentive to bid strictly above  $\underline{f}$  with probability one. In this case, however, firm  $\ell_{j'}$  lowest bid is bounded below by  $\underline{f} + \epsilon'$ , contradicting that  $\underline{f}$  is the infimum of the highest bid. Hence, a firm in  $A(k+1, k+1)$  takes over the current target.

*2. Step: the infimum  $\underline{f}$  of the highest equilibrium bid for the current target is weakly above  $f^*$ .* Suppose otherwise. By bidding in the interval  $(\underline{f}, f^*)$ , firm  $\ell_{k+2}$  can earn positive profits. Hence, in equilibrium, firm  $\ell_{k+2}$  must win with positive probability, contradicting the first step above.

*3. Step: the supremum  $\bar{f}$  of the highest equilibrium bid for the target currently on sale is weakly below  $f^*$ .* Suppose otherwise. First, recall that bidding at or above  $f^*$  is weakly dominated for every firm  $\ell_j$  with  $j \geq k+2$ . Eliminating bids weakly above  $f^*$  for all

$j \geq k + 2$ , we now argue that it is weakly dominated to bid above  $f^*$  also for firm  $\ell_{k+1}$ . To see why, notice that if firm  $\ell_{k+1}$  bids above  $f^*$  it either wins or loses to a firm  $j \leq k$ . In either case, by the induction hypothesis, the remaining  $k + 1$  targets are bought by the firms  $\bar{A}(k + 1, k + 1)$ . Furthermore, if  $\ell_{k+1}$  loses, by the induction hypothesis, it pays  $f^*$ . Hence, when bidding  $f^*$  firm  $\ell_{k+1}$  always earns  $\alpha^b(q_{\ell_{k+1}}^a - q_{\ell_{k+2}}^a)$ , and bidding strictly above  $f^*$  is weakly dominated for firm  $\ell_{k+1}$  in the reduced bidding game.

Now consider a firm that attains the supremum; that is, a firm that bids in  $((\bar{f} + f^*)/2, \bar{f}]$  and wins with positive probability. If the firm moves the probability mass from  $((\bar{f} + f^*)/2, \bar{f}]$  to  $(\bar{f} + f^*)/2$  instead, it either wins at a lower price or loses to another firm in  $\bar{A}(k + 1, k)$  and, by the induction hypothesis, takes over another target at  $f^*$ . In either case it earns higher profits, and hence has an incentive to deviate; a contradiction.

*4. Step: firm  $\ell_{k+1}$  cannot take over the current target with positive probability.* Suppose it does. Then, it must bid  $f^*$  with positive probability, since  $\underline{f} = \bar{f} = f^*$ . By the induction hypothesis, if  $\ell_{k+1}$  takes over the current target, in the following subgame the firms in  $\bar{A}(k + 1, k)$  take over the remaining targets at  $f' > f^*$ , and hence earn strictly less than when taking over the current target at  $f^*$ . But if a firm in  $\bar{A}(k + 1, k)$  deviates and bids  $f^* + \epsilon'$  for  $\epsilon' > 0$ , it wins for certain, which is a profitable deviation for sufficiently small  $\epsilon'$ .

*5. Step: equilibrium construction.* We sketch a cautious equilibrium of the bidding subgame with  $k + 1$  remaining targets in which all firms in  $\bar{A}(k + 1, k + 1)$  take over a target at  $f^*$ . Using the induction hypothesis, following the current round of bidding a cautious equilibrium with the desired properties exist for any bidding subgame with  $k$  remaining targets. We thus specify behavior only in the current round of bidding. By an argument analogous to that in the induction anchor (1. Step and in particular, Lemma 8), the current target accepting any highest bid above

$$V^b - |\eta^b| \left( \sum_{i \in \mathcal{A} \setminus A(k)} q_i^a + \sum_{i \in \bar{A}(k, k-1)} q_i^a \right)$$

with probability one (breaking ties randomly), every firm in  $\bar{A}(k + 1, k)$  bidding  $f^*$ , firm  $\ell_{k+1}$

uniformly randomizing over  $(f^* - \epsilon, f^*)$  for small enough  $\epsilon > 0$ , and any other firm  $j$  bidding

$$V^b - |\eta^b| \left( \sum_{i \in \mathcal{A} \setminus A(k)} q_i^a + \sum_{i \in \bar{A}(k, k-1)} q_i^a \right)$$

is a weakly undominated equilibrium of the reduced game in which all subgames with a unique equilibrium outcome are replaced by the corresponding equilibrium payoffs. Hence, it is part of a cautious equilibrium of this subgame.

Finally, we argue that  $f^* < V^b$ . Let  $\underline{q} := \max_{j \in \mathcal{A} \setminus \bar{\mathcal{A}}(n^b-1)} q_j^a$ . By definition, for any  $i \in \bar{\mathcal{A}}(n^b-1)$ , we have  $q_i^a > \underline{q}$ . Thus, because all targets are symmetric, we have

$$f^* = V^b - |\eta^b| \sum_{i \in \bar{\mathcal{A}}(n^b-1)} q_i^a + \alpha^b \max_{j \in \mathcal{A} \setminus \bar{\mathcal{A}}(n^b-1)} q_j^a < V^b - \underline{q} \left( |\eta^b| (n^b - 1) - \alpha^b \right).$$

To establish  $f^* < V^b$ , it suffices to show that  $|\eta^b| (n^b - 1) = \alpha^b$ . Denote by  $\bar{Q}^b$  a target's demand when being the default and by  $\underline{Q}^b$  a target's demand when not. By Assumption 0,  $\bar{Q}^b > \underline{Q}^b$ , and by symmetry of the targets,

$$\alpha^b = \frac{n^b - 1}{n^b} (\bar{Q}^b - \underline{Q}^b) \quad \text{and} \quad \eta^b = -\frac{1}{n^b} (\bar{Q}^b - \underline{Q}^b).$$

Hence,  $(n^b - 1)|\eta^b| = \alpha^b$ .

Case (b). Let  $n^b = |\mathcal{A}|$ . We solve the game backwards. Consider first any subgame with one potential acquirer  $j$  and one target, which implies that all other firms in  $\mathcal{A}$  have taken over another target before. Since  $j$  makes a take-it-or-leave-it offer, it must offer the amount at which the target is indifferent between accepting and rejecting, and since there is a strictly positive gain of trade the target must accept in equilibrium. Hence, the takeover price is

$$V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus \{j\}} q_i^a.$$

We proceed by induction over the number of remaining targets, focusing on subgames in which there are as many targets as potential acquirers. (Notice that we can use Case (a) to deal with subgames in which there are more acquirers than targets.)

**Induction hypothesis:** Consider any subgame in which there are  $k \geq 2$  potential acquirers and  $k$  targets left. Let  $\ell_k$  be the potential acquirer with the lowest market share in market  $a$  among the  $k$  remaining potential acquirers. Then, in any cautious equilibrium of this subgame, a potential acquirer other than  $\ell_k$  takes over the current target at a price of

$$f^*(k) := V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus \{\ell_k\}} q_i^a.$$

**Induction anchor:** Consider any subgame with two potential acquirers  $\ell_1$  and  $\ell_2$  and two targets. Let  $q_{\ell_1}^a > q_{\ell_2}^a$ . We have to show that  $\ell_1$  takes over the current target at  $f^* = f^*(2)$ .

First, we observe that if the current target is not taken over, by Case (a), firm  $\ell_1$  takes over the remaining target in the next round. Anticipating this takeover, the current target's anticipated standalone value is  $f^*$ . Hence, the current target accepts any highest bid strictly above  $f^*$ , and rejects any highest bid strictly below  $f^*$ ; at  $f^*$  the current target is indifferent.

Second, we show that the infimum  $\underline{f}$  of the highest equilibrium bid is at least  $f^*$ . Suppose not. Then, by our first observation, with positive probability, the current target is not taken over. Anticipating that  $\ell_1$  would take over the last remaining target in the following subgame,  $\ell_2$  could earn net profits of  $q_{\ell_2}^a \alpha^b$  from taking over the current target at  $f^*$ . Hence, for small enough  $\epsilon' > 0$ , shifting probability mass from  $(\underline{f}, f^*)$  to  $f^* + \epsilon'$  is a profitable deviation for  $\ell_2$ ; a contradiction.

Third, we argue that in any cautious equilibrium the supremum  $\bar{f}$  of the highest equilibrium bid is at most  $f^*$ . Suppose otherwise. Because the target accepts all bids above  $f^*$ , both  $\ell_1$  and  $\ell_2$  must attain  $\bar{f}$  for otherwise the firm attaining  $\bar{f}$  would have an incentive to lower  $\bar{f}$ . To establish a contradiction, we observe that for firm  $\ell_2$  it is weakly dominated to assign positive mass to bids in  $((f^* + \bar{f})/2, \bar{f}]$ . If  $\ell_1$  submits a bid weakly below  $(f^* + \bar{f})/2$ , then firm  $\ell_2$  is strictly better off bidding  $(f^* + \bar{f})/2$  with probability one, as this way it pays a lower price, both when losing and taking over the last target at  $f^*$  or when winning at this price. If firm  $\ell_1$  submits a bid above  $(f^* + \bar{f})/2$ , firm  $\ell_2$  is (weakly) better off bidding  $(f^* + \bar{f})/2$  with

probability one and losing for certain, in which case it takes over the remaining target at  $f^*$  in the following subgame. Hence, assigning positive mass to bids in  $((f^* + \bar{f})/2, \bar{f}]$  is weakly dominated for  $\ell_2$  in the reduced bidding game; a contradiction.

Fourth, we show that firm  $\ell_1$  takes over the current target with probability 1. Note that firm  $\ell_2$  cannot take over the current target with positive probability. Suppose it does. Then, it must bid  $f^*$  with positive probability. If  $\ell_2$  takes over the current target, in the following subgame  $\ell_1$  takes over the remaining target at a price of  $f' > f^*$ , and hence earns strictly less than when taking over the current target at  $f^*$ . But if  $\ell_1$  deviates and bids  $f^* + \epsilon'$  for any  $\epsilon' > 0$ , it wins for certain, which is a profitable deviation for sufficiently small  $\epsilon' > 0$ . Similarly, the target must accept a bid of  $f^*$  by firm  $\ell_1$  with probability 1, for otherwise  $\ell_1$  could profitably deviate to a slightly higher bid.

Fifth, we construct a cautious equilibrium consistent with the induction hypothesis. Again, we specify behavior only in the current round of bidding. Since if  $\ell_2$  takes over the current target  $\ell_1$  pays a strictly higher price in the next round, similar arguments as in Case (a) imply that firm  $\ell_1$  bidding  $f^*$  and firm  $\ell_2$  uniformly randomizing over  $(f^* - \epsilon, f^*)$  is part of a cautious equilibrium of this subgame for small enough  $\epsilon > 0$ .

**Induction step:** Suppose the induction hypothesis holds for any subgame with  $k \geq 2$  remaining targets and acquirers. Consider a subgame with  $k + 1$  remaining targets and acquirers. Denote  $f^* = f^*(k + 1)$ . We follow the same five steps as in the induction anchor.

First, we observe that if the current target is not taken over, by Case (a), all firms except for  $\ell_{k+1}$  take over one of the remaining targets in the following rounds. Anticipating this, the current target's anticipated standalone value is  $f^*$ . The current target thus accepts any highest bid strictly above  $f^*$ , and rejects any highest bid strictly below  $f^*$ ; at  $f^*$  the current target is indifferent.

Second, we show that the infimum  $\underline{f}$  of the highest equilibrium bid is at least  $f^*$ . Suppose not. Then, with positive probability, the current target is not taken over. Anticipating that

it would not make a takeover otherwise, firm  $\ell_{k+1}$  could earn net profits of  $q_{\ell_{k+1}}^a \alpha^b$  from taking over the current target at  $f^*$ . Hence, for small enough  $\epsilon' > 0$  shifting probability mass from  $(\underline{f}, f^*)$  to  $f^* + \epsilon'$  is a profitable deviation for  $\ell_{k+1}$ ; a contradiction.

Third, we argue that in any cautious equilibrium the supremum  $\bar{f}$  of the highest equilibrium bid is at most  $f^*$ . Suppose not. Because the target accepts all bids above  $f^*$ , at least two firms must attain  $\bar{f}$  for otherwise the only firm attaining  $\bar{f}$  would have an incentive to lower  $\bar{f}$ . Notice that, by the same argument as in the induction anchor, assigning positive probability mass to bids above  $(f^* + \bar{f})/2$  is weakly dominated for  $\ell_{k+1}$ , so it cannot be  $\ell_{k+1}$  that attains  $\bar{f}$ . Now consider two firms  $\ell_i$  and  $\ell_j$  with  $i, j \neq k+1$  that attain  $\bar{f}$ . Both firms have an incentive to shift probability mass from  $((f^* + \bar{f})/2, \bar{f}]$  to  $(f^* + \bar{f})/2$  because they either win at a lower price or — by the induction hypothesis — take over one of the remaining targets at  $f^*$  in the following rounds; a contradiction.

Fourth, we show that the current target is taken over with probability 1 by a firm other than  $\ell_{k+1}$ . Note that  $\ell_{k+1}$  cannot take over the current target with positive probability. Suppose it does. Then, it must bid  $f^*$  with positive probability. If  $\ell_{k+1}$  takes over the current target, in the following subgame the remaining acquirers take over the remaining targets at a price of  $f' > f^*$ , and hence earn strictly less than when taking over the current one at  $f^*$ . If either of these firms deviates and bids  $f^* + \epsilon'$  for any  $\epsilon' > 0$ , it wins for certain, which is a profitable deviation for sufficiently small  $\epsilon' > 0$ ; a contradiction. Similarly, the target must accept an equilibrium bid of  $f^*$  with probability 1, for otherwise a firm other than  $\ell_{k+1}$  could profitably deviate to a slightly higher bid.

Fifth, we construct a cautious equilibrium consistent with the induction hypothesis. Again, we specify behavior only in the current round of bidding. By similar arguments as in Case (a), firm  $\ell_1, \dots, \ell_k$  bidding  $f^*$  and firm  $\ell_{k+1}$  uniformly randomizing over  $(f^* - \epsilon, f^*)$  is part of a cautious equilibrium of this subgame for small enough  $\epsilon > 0$ .

Case (c). Let  $n^b > |\mathcal{A}|$ . We solve the game backwards, and begin by solving subgames with



one more acquirer than targets. (Other relevant subgames are captured by (a) and (b).)

**Induction hypothesis:** *Consider a subgame with  $k \geq 1$  remaining potential acquirers and  $k + 1$  remaining targets. The current target is taken over at a price of*

$$f^* := V^b - |\eta^b| \sum_{i \in \mathcal{A}} q_i^a.$$

**Induction anchor:** Let  $k = 1$ , so that there are one acquirer  $\ell$  and two targets left. If the current target is not taken over, by Case (b),  $\ell$  takes over the last remaining target at a price

$$f' = V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus \{\ell\}} q_i^a.$$

The current target's profits in that case are  $f^*$ . Thus the target must reject any bid strictly less than  $f^*$ , and accept any bid strictly above  $f^*$ . Hence, firm  $\ell$  has a strict incentive to lower any bid strictly above  $f^*$ . For the sake of a contradiction, suppose the target rejects  $f^*$ . Then,  $\ell$  pays  $f' > f^*$  for the takeover, and thus has an incentive to deviate by bidding  $(f^* + f')/2$ , a bid that the current target accepts. Hence,  $\ell$  takes over the current target at  $f^*$ .

**Induction step:** Suppose that the induction hypothesis holds for  $k$  remaining acquirers and remaining  $k + 1$  targets. Now consider a subgame with  $k + 1$  remaining acquirers and  $k + 2$  targets. Let  $\ell_k$  be the remaining acquirer with the lowest market share in market  $a$ .

First, if the current target is not taken over, by Case (b), all other targets are bought at

$$f'' = V^b - |\eta^b| \sum_{i \in \mathcal{A} \setminus \{\ell_k\}} q_i^a.$$

The current target's profits in that case are  $f^*$ . Thus, it must reject any bid strictly less than  $f^*$ , and accept any bid strictly above  $f^*$ ; at  $f^*$  the current target is indifferent.

Second, we argue that the supremum  $\bar{f}$  of the highest equilibrium bid is at most  $f^*$ . Suppose not. Since the target accepts all bids above  $f^*$ , at least two firms must attain  $\bar{f}$  for otherwise the only firm attaining  $\bar{f}$  would have an incentive to lower  $\bar{f}$ . Either firm has

an incentive to shift probability mass from  $((f^* + \bar{f})/2, \bar{f}]$  to  $(f^* + \bar{f})/2$  since they either win at a lower price or by the induction hypothesis, take over a remaining target at  $f^*$  in the following rounds; a contradiction.

Third, we show that the target accepts a bid of  $f^*$ . Suppose otherwise. Then, by Case (b), all potential acquirers take over one of the remaining targets at a price  $f''$ . Hence, for small enough  $\epsilon' > 0$ , by bidding  $f^* + \epsilon'$  for the current target, each potential acquirer could make a takeover at a lower price; a contradiction.

Fourth, by an argument similar to Lemma 8, bidding  $f^*$  is weakly undominated for every potential acquirer. Hence, potential acquirers bidding  $f^*$  is part of a cautious equilibrium.

We finally argue that any acquirer takes over a target at  $f^*$  in any subgame with  $k \geq 1$  potential acquirers and  $k + x$ , for  $x \geq 2$ , targets left. First, we note that no target would sell strictly below  $f^*$  since even if all acquirers buy a rival it earns  $f^*$  upon rejecting an offer. Hence, no target can be bought at a price strictly below  $f^*$ . Second, no acquirer would bid more than  $f^*$  since such an acquirer could deviate and not bid until a subgame is reached in which there are  $k'$  acquirers and  $k' + 1$  targets left. In such a subgame, it can take over a target at  $f^*$ . Hence, in equilibrium an acquirer takes over a target at  $f^*$ .  $\square$