

A Service of



Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Choi, Jay Pil; Gerlach, Heiko

Working Paper Patent Exhaustion and Licensing in the Supply Chain

CESifo Working Paper, No. 11313

Provided in Cooperation with: Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Choi, Jay Pil; Gerlach, Heiko (2024) : Patent Exhaustion and Licensing in the Supply Chain, CESifo Working Paper, No. 11313, CESifo GmbH, Munich

This Version is available at: https://hdl.handle.net/10419/305555

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



WWW.ECONSTOR.EU



Patent Exhaustion and Licensing in the Supply Chain

Jay Pil Choi, Heiko Gerlach



Impressum:

CESifo Working Papers ISSN 2364-1428 (electronic version) Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute Poschingerstr. 5, 81679 Munich, Germany Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de Editor: Clemens Fuest https://www.cesifo.org/en/wp An electronic version of the paper may be downloaded • from the SSRN website: www.SSRN.com

- from the RePEc website: <u>www.RePEc.org</u>
- from the CESifo website: <u>https://www.cesifo.org/en/wp</u>

Patent Exhaustion and Licensing in the Supply Chain

Abstract

This paper analyzes private and social incentives to levy an ad valorem licensing fee in a supply chain governed by the legal principle of patent exhaustion. With perfect competition at the upstream and downstream stage, the choice of the licensing segment is irrelevant for the patent holder and consumers. When exactly one segment of the value chain is monopolistic while the other one is competitive, the patent holder prefers licensing at the monopolistic stage leading to an alignment of private and social incentives. With imperfect competition at both stages, excessive downstream licensing can arise. We demonstrate that charging licensing fees at both stages of the supply chain ("double-dipping") can be profitable for the patent holder and beneficial for consumers. We discuss the implications of this result for the application of the patent exhaustion principle.

JEL-Codes: D430, L410, L440.

Keywords: patent licensing, supply chain, first sale doctrine, patent exhaustion, double-dipping.

Jay Pil Choi Department of Economics Michigan State University 220A Marshall-Adams Hall USA – East Lansing, MI 48824-1038 choijay@msu.edu Heiko Gerlach School of Economics University of Queensland Brisbane / Australia h.gerlach@uq.edu.au

August 2024

We would like to thank In-Koo Cho, Kyungmin Kim, Volker Nocke, Nicolas Schutz, Satoru Takahashi, Joel Watson as well as participants in seminars and workshops for useful comments and discussions.

1 Introduction

This paper studies the privately and socially optimal level of patent licensing in multi-tiered vertical supply chains. The legal principle of "patent exhaustion", also known as the "first sale doctrine" in copyright law, limits the control that a patent holder can exert over a patented product once it has been sold.¹ Following the sale of a patented item, the patent holder may forfeit the right to restrict or control its subsequent use, resale, or further distribution. The underlying rationale behind patent exhaustion is to prevent the perpetual control of a specific product by the patentee, promoting competition and innovation by facilitating the unrestricted flow of goods in the market post-initial sale. This legal principle serves to deter the practice of "double-dipping," where a patent holder seeks repeated licensing fees for the same patented technology.² In Quanta Computer et al vs LG Electronics (2008), the US Supreme Court recently re-affirmed the principle of patent exhaustion and extended its applicability to various facets of patented technologies, including method-based patents.³

In the context of supply chains, patent exhaustion signifies that once a patentee grants a license to an upstream firm, the patent holder relinquishes the right to enforce the patent against producers in subsequent layers of the value chain. In contrast, when engaging in licensing agreements with downstream firms in practical scenarios, patent holders often include contractual provisions granting licensees so-called "have made" rights. This allows the

¹This principle was first expressed in the US Supreme Court case Adam v. Burke, 84 U.S. (17 Wall.) 453 (1873). Massachusetts undertaker Burke purchased patented coffin lids from patentee and manufacturer Lockhart & Seelye. Burke then used the lids in area outside of Boston where Adams, the plaintiff in this case, was the assignee of the patent. Adams sued Burke. The Supreme Court argued in favor of Adams stating:

That is to say the patentee or his assignee having in the act of sale received all the royalty or consideration which he claims for the use of his invention in that particular machine or instrument, it is open to the use of the purchaser without further restriction on account of the monopoly of the patentees.

²The scope and application of patent exhaustion can vary, and legal interpretations may differ based on jurisdiction and specific circumstances.

³In this case, Intel obtained a license from LG Electronics for patents covering methods used in the manufacturing of microprocessors. Quanta Computers purchased microprocessors from Intel for computer assembly but did not independently license the relevant patents from LG Electronics. In response, LG Electronics filed a lawsuit against Quanta for patent infringement. The U.S. Supreme Court's decision in favor of Qantas re-affirmed the principle of patent exhaustion and extended its scope to method patents.

licensed end-product manufacturer to exercise the right to have components produced, which, if made by upstream third-party manufacturers, might infringe upon the patent holder's rights. This indirect insulation from patent infringement raises a myriad of questions. A fundamental query is what factors drive both private and social incentives to license a particular technology at different levels of the value chain. This aspect of patent licensing has been at the core of various antitrust and patent litigation cases and is a contentious policy issue. For example, consider the high-profile case DAIMLER V NOKIA (C-182/21) referred to the Court of Justice of the European Union (CJEU). Nokia holds standard essential patents on 3G/4G wireless communication technology. They offered car maker Daimler license agreements including "have made" rights for its suppliers such as Continental, Huawei, Bosch and others. At the same time, Nokia refused to license the upstream firms directly. Continental brought a complaint to the European antitrust agency, DG Competition, for abuse of dominant position by Nokia. Daimler and its suppliers contended that supplier licensing is standard practice in the automobile industry. Nokia argued that efficiency mandates downstream licensing. The case was settled in May 2021 before the start of proceedings which precluded the CJEU from rendering a decision statement on the appropriate level of patent licensing in supply chains.

To examine the implications of the licensing stage choice, we develop a simple model of a supply chain with two-stage production involving a component (incorporating a patented product or method) and the final product. Patent exhaustion introduces two primary options for the patent holder: licensing the component at the upstream stage or licensing the end product at the downstream stage. These represent distinct approaches to intellectual property licensing within supply chains. With per-unit licensing royalties, the choice of licensing level becomes inconsequential, as the patent holder generates identical licensing revenue regardless of whether upstream or downstream producers are licensed. Instead, we consider ad-valorem licensing fees which are used in all of the above-mentioned examples. There is also empirical evidence supporting this approach, demonstrating that royalties based on revenues are prevalent. For instance, Bousquet et al. (1998) examined a sample of 278 contracts and found that 225 included royalties, but only nine of them were paid per unit sold. Similarly, Hegde (2014) finds that in all 505 sampled license contracts from the pharmaceutical and biomedical industry, the licenses specify a royalty, expressed as a percentage of annual gross sales revenues. With revenue-based licensing fees, licensing at the downstream level seems attractive due to the higher values associated with end products compared to component products.⁴ However, royalty payment results from two components: a royalty rate and a royalty base to which the rate is applied. The importance of considering both royalty base and rate was, for example, noted in Lucent Technologies v. Gateway (543 F.3d 710, Fed. Cir. 2008) by the Court of Appeals for the Federal Circuit: "There is nothing inherently wrong with using the market value of the entire product, ..., so long as the multiplier accounts for the proportion of the base represented by the infringing component or feature".

Our analysis commences with a benchmark scenario where both upstream and downstream stages are characterized by perfect competition. In this scenario, we establish a strong neutrality result such that the choice of the licensing stage is irrelevant, and the patent holder can implement the full integration outcome. Upstream licensing is able to extract a higher royalty rate. Downstream licensing benefits from a higher royalty base. With perfect competition, these two effects cancel each other out. We then consider scenarios where one stage is monopolistic while the other remains competitive. In such cases, the patent holder prefers to license at the stage where market power resides, aligning perfectly with the incentives of a social planner.

We extend the analysis to encompass cases of imperfect competition with market power at both stages, utilizing a competition parameter approach in a more general framework. This approach encompasses all possible market structures, including oligopolistic competition. In this instance, the choice of licensing stage becomes contingent on the relative production costs and the distribution of relative market power associated with each stage of production. Licensing tends to occur at the layer with relatively lower production cost and relatively less competition. Our findings also reveal a potential misalignment between private and social incentives due to the patent holder's excessive choice of downstream licensing. This misalignment is more likely

⁴A senior executive from Ericsson, a company issuing licenses for wireless technology patents, emphasized this advantage, stating that "...one big advantage with this strategy is also that the royalty income will be higher since we calculate the royalty on a more expensive product" (Ericsson Inc. v. D-Link Systems, Case No. 6:10-CV-473, E.D. Tex., trial testimony of Christina Petersson, June 4, 2013, pm session).

to occur if production cost and the degree of competition are similar across the two layers of the supply chain.

Finally, our analysis introduces a novel perspective on the concept of "double-dipping," a scenario where a patent holder seeks compensation at multiple stages of the supply chain for a single patented product or process. This often involves pursuing royalties or licensing fees both from component manufacturers and device manufacturers selling to end-users. As discussed, courts and regulatory bodies may be wary of permitting double-dipping, as it may be viewed as an unfair or excessive monetization of a single invention. Our analysis uncovers a potential positive role for double-dipping. This arises as a mechanism to mitigate overall price distortions in supply chains stemming from the successive exercise of market power at each stage. We find that double-dipping can be profitable for the patent holder and lead to lower prices for consumers. As such, this paper offers a new perspective on the legal principle of patent exhaustion and sheds light on the potential benefits of a nuanced approach to licensing strategies in multi-tiered supply chains.

Despite the prevalence of litigation concerning the licensing stage and the associated policy discussions, economic research with formal modeling is relatively scant. As in our paper, Layne-Farrar et al. (2014) derive a royalty allocation neutrality result, demonstrating that how royalty rates are divided along the production chain has no substantial impact on social welfare in the case of per-unit royalty rates, assuming Nash bargaining between upstream and downstream firms. The assumption of Nash bargaining implies that there is no money left on the table, allowing firms to internalize any externalities and collectively behave as if they were integrated. In contrast, our analysis considers a scenario with *ad-valorem* royalty rates to better capture the contentious issues in recent cases and the potential for double marginalization. This departure from per-unit royalty rates introduces complexities and contributes to a more comprehensive understanding of the economic implications associated with the choice of licensing stage.

Llobet and Padilla (2016) compare the total value of the product and the value of the component as the base for a royalty in licensing contracts. They show that these two royalty bases are equivalent to using ad valorem and perunit royalties. However, they consider a much simpler setting in which the upstream firm is the patent holder who controls both the component price and royalty rate. We consider a more complex situation with an independent intellectual property (IP) holder who determines the stage in the production process where patent licensing takes place, and their equivalence results do not hold. Neven and Llobet (2022) develop a model of patent licensing in a vertical chain with investments. They show that licensors prefer to license their patents downstream, whereas upstream and downstream producers prefer upstream licensing. When the licensor chooses the stage of licensing, the static deadweight loss is lower than in the case where the producers choose. However, this comes at the cost of discouraging investment and innovation by upstream and downstream producers.

We can also make a connection between our paper and the public finance literature that investigates a Leviathan government that maximizes tax revenue. This is because the IP holder's licensing revenue maximization is formally equivalent to the government's tax revenue maximization problem. Gaudin and White (2014), for instance, compare unit and ad valorem tax regimes under the assumption that the government seeks to maximize tax revenue and derive conditions under which the ad-valorem tax regime welfare-dominates the unit tax regime. However, their focus is on the comparison of two different tax instruments. They do not consider a vertical supply chain and the issue of the optimal level of taxation does not arise.⁵ As in this paper, Peitz and Reisinger (2014) consider a vertical supply chain and study taxation. However, in line with the previous literature in public finance, their focus is on the comparison of specific and ad valorem taxes. They show that tax revenues should be raised only through ad valorem taxes.

In terms of methodology, both Johnson (2017) and our paper use a competition (or conduct) parameters approach to analyze vertical relationships under conditions of bilateral imperfect competition. However, Johnson's work does not involve licensing and focuses on the comparison between two common business practices in online markets: the agency model and the wholesale model.

The rest of the paper is organized as follows. In section 2, we set up

⁵There is a large literature analyzing the relative merits of ad-valorem vs. unit tax in terms of welfare comparison. See Suits and Musgrave for a monopoly setting and Delipalla and Keen (1992) and Anderson, et al. (2001) for oligopolistic and imperfectly competitive market settings.

a model of patent licensing in the supply chain. In section 3, we consider the benchmark case of perfect competition at both stages of production and establish a strong neutrality result. Sections 4 and 5, respectively, consider upstream and downstream monopoly while the other segment is competitive. Section 6 considers a competition parameter approach that encompasses imperfect competition at both stages. Section 7 considers an extension with probabilistic intellectual property rights. The last section follows with concluding remarks. All missing proofs are in Appendix A. Appendix B provides an analysis of the same framework with per-unit licensing rates and establishes neutrality results.

2 Model of Licensing in a Supply Chain

We consider a vertical supply chain with upstream and downstream firms. One unit of input from an upstream firm is needed to produce one unit of final output at the downstream stage. Downstream demand by end consumers is given by Q(p), where p is the downstream market price with Q'(p) < 0. Throughout the paper, we use as measure of the sensitivity of demand

$$\lambda(p) \equiv -\frac{Q(p)}{Q'(p)} > 0,$$

which is the inverse semi-price elasticity of demand. If demand is derived from a distribution function reflecting consumer heterogeneity, then the measure $\lambda(p)$ is the Mills' ratio of this distribution.⁶

Let $p^m(k)$ denote the monopoly price with a constant marginal cost k, which solves (p-k)Q'(p) + Q(p) = 0 or

$$p^m(k) - k = \lambda(p^m(k)).$$

Thus, $\lambda(p^m(k))$ represents the mark-up (i.e., price over marginal cost) of the monopolist. We assume that both $\lambda(p)$ and $\lambda(p)(2 - \lambda'(p))$ have slopes strictly less than one to ensure the quasi-concavity of profit functions below.

The set-up involves a patent holder (PH) who owns a technology essential for the production of the product. We assume that the technology is imple-

⁶See Johnson (2017) for a discussion of the properties of $\lambda(p)$ in relation to the logcurvature of demand and the cost pass-through rate of a monopolist.

mented first at the upstream level. The upstream firms, with a marginal cost denoted c, supply the technology and charge a linear wholesale price (w) to the downstream firms. The downstream firms have a marginal cost (d) in addition to the wholesale price.

To address the discussions regarding the appropriate stage for patent licensing, we consider two scenarios. First, considering the current legal landscape shaped by the first-sale doctrine, we assume that licensing can only occur at a single stage within the supply chain. If licensing takes place at the upstream stage, the principle of patent exhaustion provides protection to downstream producers who purchase the input supplied by the licensed upstream firm. In contrast, if licensing occurs at the downstream stage, the licensed downstream firm may be able to secure "have-made rights" from the patent holder, which can shield its unlicensed suppliers from infringement. Second, in order to discuss the efficiency of the first-sale doctrine in this setup, we allow the patent holder to charge license fees to both layers of the supply chain. We refer to this scenario as "double-dipping" licensing.

Throughout the main part of the paper, we focus on the implementation of ad-valorem licensing fees $r_u \in [0,1]$ and $r_d \in [0,1]$ for upstream and downstream firms, respectively.⁷ This fee is applied to the revenue of the firm obtaining the license. Hence, the royalty base price for upstream firms is the wholesale price while for downstream firms it is the final consumer price. The analysis looks at the patent holder's preference regarding the licensing stage. We investigate how the preference changes with vertical market structures and whether the patent holder's preference aligns with that of a social planner.

3 Perfect Competition Benchmark

As a benchmark consider an industry with perfectly competitive upstream and downstream segments in the supply chain. We investigate market outcomes with upstream and downstream licensing, respectively, and first establish a strong neutrality result that shows that patent holder and consumers are indifferent with respect to the choice of the licensing stage.

⁷In Appendix B we solve the general competition parameter model introduced in Section 6 for per-unit royalty rates and show that the choice of the licensing stage is inconsequential both for the patent holder and consumers.

3.1 Downstream Licensing

Suppose the patent holder sets an ad-valorem license fee of r_d for all downstream firms. Let w_d and p_d denote the upstream wholesale input and the downstream retail device price with downstream licensing, respectively. For a retail given price p_d , the downstream sector makes profits of

$$[p_d(1-r_d) - w_d - d]Q(p_d) = (1-r_d)[p_d - \frac{w_d + d}{1-r_d}]Q(p_d)$$

where $(w_d + d)(1 - r_d)^{-1}$ can be interpreted as the "perceived marginal cost" facing the downstream sector in the presence of downstream licensing. Hence, in a perfectly competitive downstream equilibrium, the retail price is given by

$$p_d = \frac{w_d + d}{1 - r_d}.\tag{1}$$

Moreover, perfectly competitive upstream firms set their price equal to their marginal cost, which yields $w_d = c$. Now consider the patent holder's choice of the ad-valorem royalty rate. Even though the patent holder chooses r_d with licensing at the downstream level, it is more convenient to view that the patent holder indirectly chooses p_d with r_d equilibrating according to (1). Let $r_d(p_d)$ be the royalty rate that implements a downstream price p_d . The patent holder's license revenues (\pounds) as a function of the implemented retail price are then

$$\pounds_d(p_d) = r_d(p_d) p_d Q(p_d) = (1 - \frac{c+d}{p_d}) p_d Q(p_d).$$

The optimal downstream price p_d^* thus satisfies

$$\frac{\partial \pounds_d(p_d)}{\partial p_d} = p_d Q(p_d) \frac{\partial r_d}{\partial p_d} + r_d(p_d) Q(p_d) + r_d(p_d) p_d Q'(p_d) = 0.$$
(2)

Increasing the retail price has three effects on licensing revenues. A marginal increase in p_d (i) raises the licensing rate r_d , (ii) increases the royalty base, and (iii) reduces consumer demand. The second and third terms in (2) are the marginal revenue of a price increase. Since the rate effect in the first term is strictly positive, the marginal revenue has to be negative at the optimal price. The patent holder thus trades off a higher royalty rate to extract rents against the loss of revenues from the downstream sector. Substituting $r_d(p_d)$

into (2) and re-arranging the first-order condition yields $p_d^* - \lambda(p_d^*) = c + d$, that is, $p_d^* = p^m(c + d)$. The patent holder implements the optimal retail price of a fully integrated firm owning intellectual property (IP) and both upstream and downstream production facilities. Furthermore, check that the optimal royalty rate with downstream licensing is

$$r_d^* = 1 - \frac{c+d}{p_d^*} = \frac{\lambda(p_d^*)}{c+d+\lambda(p_d^*)}$$

and overall licensing revenues are

$$\pounds_d(p_d^*) = \frac{\lambda(p_d^*)}{c+d+\lambda(p_d^*)} [c+d+\lambda(p_d^*)]Q(p_d^*) = \lambda(p_d^*)Q(p_d^*).$$

3.2 Upstream Licensing

Similarly, suppose the patent holder sets an ad-valorem license fee of r_u for all upstream firms. Let w_u and p_u denote the upstream wholesale and the downstream retail price with upstream licensing, respectively. With perfect competition and upstream licensing, downstream firms set their price equal to their effective marginal input cost, that is, $p_u = d + w_u$. With a retail price p_u and a wholesale price $w_u = p_u - d$, the upstream sector makes profits of

$$[(p_u - d)(1 - r_u) - c]Q(p_u)$$

and the price in a perfectly competitive equilibrium is given by

$$p_u = d + \frac{c}{1 - r_u} \tag{3}$$

where the RHS is the perceived marginal cost of upstream firms with upstream licensing.

Now consider the patent holder's pricing decision. Let $r_u(p_u)$ be the licensing rate for a given p_u from (3) and let $w_u(p_u) = p_u - d$. We can then write the patent holder's licensing revenues as a function of the retail price as

$$\pounds_u(p_u) = r_u(p_u)w_u(p_u)Q(p_u) = (1 - \frac{c}{p_u - d})(p_u - d)Q(p_u).$$

It is easy to verify that maximizing license revenues with respect to the retail price yields $p_u^* - \lambda(p_u^*) = c + d$ and $p_d^* = p_u^* = p^m(c+d)$. The corresponding royalty rate with upstream licensing is

$$r_u^* = 1 - \frac{c}{p_u^* - d} = \frac{\lambda(p_u^*)}{c + \lambda(p_u^*)} > r_d^*.$$

Moreover, the patent holder's licensing revenues are

$$\pounds_{u}(p_{u}^{*}) = \frac{\lambda(p_{u}^{*})}{c + \lambda(p_{u}^{*})} [c + \lambda(p_{u}^{*})]Q(p_{u}^{*}) = \lambda(p_{u}^{*})Q(p_{u}^{*}) = \pounds_{d}(p_{d}^{*}).$$

We immediately get the following strong neutrality result with perfect competition at both stages of the vertical supply chain.

Proposition 1. Consider perfectly competitive upstream and downstream industries. The patent holder is indifferent between upstream and downstream licensing. The level of licensing does not affect consumer surplus.

With perfectly competitive upstream and downstream segments, the patent holder is able to set licensing conditions that extract the profits that a monopolist owning all production facilities and IP would earn. This conclusion holds regardless of where licensing takes place in the supply chain. While licensing revenues and downstream market outcomes are the same, the licensing terms depend on the layer where licensing is taking place.

When licensing occurs downstream, the patent holder can apply an ad valorem fee to a higher price base, since the retail price is typically higher than the wholesale price. The relationship in this case is $p > w_u(p) =$ p-d. This is the royalty base effect, which makes downstream licensing more advantageous in terms of the fee base. However, there is a counterbalancing factor: upstream licensing allows the patent holder to extract a higher perunit revenue share compared to downstream licensing, that is, $r_u(p) > r_d(p)$. This is the rate extraction effect, which favors upstream licensing. In a supply chain, the downstream firm's cost reflects the input price. Thus, when an ad valorem royalty rate is applied at the downstream level, the perceived aggregate cost of the supply chain is higher.

More specifically, under upstream licensing with a royalty rate of r, only the upstream cost c is adjusted by a factor of 1/(1-r), leading to a perceived aggregate cost of c/(1-r) + d. In contrast, under downstream licensing, both the upstream and downstream costs (c + d) are adjusted by a factor of 1/(1-r), resulting in a perceived aggregate cost of (c + d)/(1-r) > c/(1-r) + d. This makes the retail price under downstream licensing more sensitive to changes in the royalty rate, which induces a lower royalty rate compared to upstream licensing.

With perfect competition at both layers of the supply chain, these two effects - royalty base and rate extraction - balance each other out. Ultimately, the patent holder sets royalty fees in such a way that the downstream price and licensing revenues remain the same regardless of whether licensing is upstream or downstream. With downstream licensing, the royalty rate is lower but is applied to a larger price base, while upstream licensing involves a higher royalty rate but a smaller price base.

We can further extent this equivalence of upstream and downstream licensing. In the proof of Proposition 1 in Appendix A, we show the irrelevance of the patent exhaustion principle with perfectly competitive markets at both stages. The same market outcome obtains when we allow the patent holder to "double dip" by charging ad-valorem fees at both stages.⁸

4 Upstream Monopoly

We now consider alternative market structures where one stage of the supply chain is monopolistic while the other remains perfectly competitive. We first analyze the market structure with upstream monopoly and perfectly competitive downstream. We again consider first downstream licensing, then upstream licensing and compare.

Suppose that the patent holder licenses downstream firms with an ad valorem rate of r_d . The upstream monopolist sets a wholesale price w_d . This implies that the downstream price for end users is set according to (1). Hence, for a given p_d , the wholesale price has to satisfy $w_d = (1 - r_d)p_d - d$. We let the upstream monopolist implicitly choose p_d to maximize profits,

$$\max_{p_d} (w_d - c)Q(p_d) = [(1 - r_d)p_d - c - d]Q(p_d)$$
$$= (1 - r_d)[p_d - \frac{c + d}{1 - r_d}]Q(p_d).$$

Taking the derivative with respect to p_d yields the monopoly price $p^m(k)$

⁸We also discuss double-dipping in more detail in Section 6.3 below.

with perceived marginal cost of $k = (c+d)(1-r_d)^{-1}$ or

$$p_d - \lambda(p_d) = \frac{c+d}{1-r_d}.$$
(4)

Let $\bar{r}_d(p_d)$ be the licensing rate that satisfies condition (4), for a given p_d , with equality. As the price in (4) is higher due to the monopolistic mark-up of $\lambda(p)$, the corresponding rate $\bar{r}_d(p_d)$ is lower compared to perfect competition at both stages in the previous section. The patent holder's revenues as a function of the retail price are then

$$\mathcal{L}_{d}(p_{d}) = \bar{r}_{d}(p_{d})p_{d}Q(p_{d}) = (1 - \frac{c+d}{p_{d} - \lambda(p_{d})})p_{d}Q(p_{d})$$

$$= \frac{p_{d}}{p_{d} - \lambda(p_{d})}[p_{d} - \lambda(p_{d}) - c - d]Q(p_{d})$$
(5)

Consider upstream licensing instead. With a perfectly competitive downstream segment, the retail price is simply $p_u = w_u + d$. Writing the upstream profits as a function of the realized retail price, the monopolist solves

$$\max_{p_u} \quad (1 - r_u)[p_u - \frac{c}{1 - r_u} - d]Q(p_u)$$

which yields

$$p_u - \lambda(p_u) = \frac{c}{1 - r_u} + d. \tag{6}$$

Compare this condition with (4) under downstream licensing. At equal royalty rates $r_d = r_u = r$, downstream licensing leads - again - to a higher retail price. This is due to the rate extraction effect discussed above. Let $\bar{r}_u(p_u)$ be the licensing rate that satisfies (6), for a given p_u , with equality. The patent holder then chooses p_u to maximize its revenues

$$\mathcal{L}_{u}(p_{u}) = \bar{r}_{u}(p_{u})w_{u}(p_{u})Q(p_{u}) = (1 - \frac{c}{p_{u} - \lambda(p_{u}) - d})(p_{u} - d)Q(p_{u}) \quad (7)$$
$$= \frac{p_{u} - d}{p_{u} - \lambda(p_{u}) - d}[p_{u} - \lambda(p_{u}) - c - d]Q(p_{u}).$$

We solve the patent holder's problems with downstream and upstream licensing in the appendix and summarize the results as follows.

Proposition 2. Consider an upstream monopoly and perfect competition downstream. If the elasticity of Mills ratio, $p\lambda'(p)/\lambda(p)$, is less than one,

then $p_u^* < p_d^*$. The patent holder prefers to license at the upstream level. Private and social incentives regarding the level of licensing are aligned.

As shown in the previous section, with competitive upstream and downstream sectors, the level of licensing is irrelevant for consumer surplus and licensing revenues. In the competitive benchmark, the revenue extraction effect and the royalty base effect canceled each other out. When the upstream sector is monopolized, this neutrality results does no longer hold. Upstream licensing is optimal both for the IP owner and consumers. It is straightforward to see from the second lines in in (5) and (7) that the licensing revenues are higher with upstream licensing for any retail price that the patent holder implements. Moreover, we can show that the per-unit revenue (license rate times base) is less elastic with downstream licensing leading to higher retail prices. The intuition for these results is as follows. The royalty base effect of downstream licensing is unchanged relative to the competitive benchmark as the difference between retail and wholesale price is equal to the downstream cost (d). However, the rate extraction effect of upstream licensing becomes more pronounced with a monopolistic upstream sector. Introducing a price mark-up means that the retail price with downstream and upstream licensing is more responsive to the cost of the supply chain. Hence, the advantage of having a lower perceived marginal cost with upstream licensing in (6) relative to downstream licensing in (4) becomes more important. This tilts the patent holder towards upstream licensing with rates that implement lower retail prices.

While we have focused on single-layer licensing, our analysis is without loss of generality. Even if the patent holder is allowed to license at both layers, the patent holder will not choose such an option. We show in the Appendix that the patent holder imposes an ad-valorem royalty fee only on the monopoly sector even if "double-dipping" is allowed.

5 Downstream Monopoly

We now consider the diametrically opposed market structure where the downstream segment is monopolistic and the upstream is perfectly competitive.

Consider downstream licensing first. With a fully competitive upstream sector, the unit wholesale price is $w_d = c$ and the downstream monopolist

maximizes profits of

$$[(1 - r_d)p_d - c - d]Q(p_d) = (1 - r_d)[p_d - \frac{c + d}{1 - r_d}]Q(p_d)$$

which yields the price implicitly given by (4). This means that the licensing revenues as a function of the implemented downstream price are the same as in (5) in the previous section. Hence, the patent holder faces the same problem with downstream licensing independent of whether there is a monopolist in the upstream or downstream segment.

Remark. Suppose a patent holder is licensing the downstream segment. When only one stage of the vertical supply chain is monopolistic (whereas the other stage is perfectly competitive), the final end user prices are the same regardless of where the monopoly power resides.

Let us turn to upstream licensing. With a perfectly competitive upstream segment, the wholesale price follows from $(1 - r_u)w_u = c$. The downstream monopolist maximizes its profits,

$$[p_u-\frac{c}{1-r_u}-d]Q(p_u),$$

which yields the same price as with an upstream monopolist in (6). The royalty rate as a function of the price is $\bar{r}_u(p_u)$ and the corresponding wholesale price is given by

$$w_u(p_u) = \frac{c}{1 - \bar{r}_u(p_u)} = p_u - d - \lambda(p_u).$$

The patent holder then maximizes the following licensing revenues

$$\mathcal{L}_{u}(p_{u}) = \bar{r}_{u}(p_{u})w_{u}(p_{u})Q(p_{u}) = (1 - \frac{c}{p_{u} - d - \lambda(p_{u})})(p_{u} - d - \lambda(p_{u}))Q(p_{u})$$
$$= [p_{u} - c - d - \lambda(p_{u})]Q(p_{u}).$$
(8)

Let us compare optimal prices and licensing revenues.

Proposition 3. Consider a perfectly competitive upstream and a monopolistic downstream market. If the elasticity of Mills' ratio is less than one $(p\lambda'(p)/\lambda(p) \leq 1)$, then $p_d^* < p_u^*$. The patent holder prefers to license at the

downstream stage. Private and social incentives concerning the choice of the licensing stage are aligned.

A monopolistic downstream sector also leads to a break-down of the neutrality result from the competitive benchmark model. We observe the converse results compared to the scenario of an upstream monopoly in Section 4. Facing a competitive upstream and monopolistic downstream industry, the patent holder prefers to license the downstream monopolist. In fact, the patent holder's licensing revenues are higher with downstream licensing for any implemented retail price (compare (5) and (8)). This licensing arrangement also leads to lower prices for consumers as the per-unit revenue is less elastic with upstream licensing. Hence, private and social incentives are again aligned.

To understand the change in results relative to an upstream monopolist, consider the two effects discussed above. Compared to the previous section, the licensing rates as a function of the price are the same. The *rate extraction effect* of upstream licensing is thus unchanged. However, the mark-up of a monopolistic downstream sector drives a wedge between the upstream royalty base and the downstream royalty base. This strengthens the *royalty base* effect, reverses the equilibrium price ranking and makes downstream licensing more profitable. Additionally, like in the previous section, we find that multi-layer licensing (or "double-dipping") is dominated by licensing only to the monopolistic sector.

Overall, our analysis so far, has shown that market power in the upstream and downstream market affects the licensing rate and royalty base differently. The optimal licensing stage is at the monopolistic layer given that the other layer is perfectly competitive. In the next section we extend our framework to allow for imperfect competition at both stages of production. Note that our results in this and the previous section are distinct from an explanation that is based on the minimization of transaction costs. As the transaction costs would increase with the number of licensing contracts that need to be executed, with everything being equal, the considerations of transaction costs would imply that licensing should occur at the stage where the number of licensees is smaller, that is, the monopolistic sector.

6 Bilateral Imperfect Competition

6.1 Analytical Framework

We extend the previous analysis to a situation of imperfect competition at both layers of the supply chain. The analysis of bilateral imperfect competition generalizes our previous results and encompasses all previous cases as special ones. To analyze this general set-up, we adopt the competition parameter approach used among others by Genesove and Mullin (1998), Weyl and Fabinger (2013) and Johnson (2017). This approach does not rely on specifications of a particular model. Instead, it postulates that the inverse demand elasticity rule of the Lerner index is adjusted by a parameter $\theta \in [0, 1]$, which captures the degree of competition or collective market power of the firms in the market, that is,

$$p - k = \frac{\theta p}{\epsilon_d} = \theta \lambda(p),$$

where k is a constant marginal cost and ϵ_d is the price elasticity of demand. This modelling approach nests a wide range of forms of imperfect competition models as discussed in detail in Weyl and Fabinger (2013). We apply this parameter approach to a vertical supply chain with imperfect competition and licensing at both stages. Let $\theta_u \in [0, 1]$ and $\theta_d \in [0, 1]$ be the market power parameter for the upstream and downstream industry, respectively. A value $\theta_u = 1$ ($\theta_d = 1$) corresponds to a situation where the upstream (downstream) segment is a monopoly. By contrast, $\theta_u = 0$ ($\theta_d = 0$) implies that the upstream (downstream) industry is perfectly competitive.

To fix ideas, let us analyze price formation in the chain supply for given licensing fees (r_u, r_d) . First consider the downstream segment and let $m_d(p) = (1 - r_d)p - w - d$ be the margin of this layer when all upstream (downstream) firms charge the same price w(p). The aggregate downstream profit is $\pi_d(p) = m_d(p)Q(p)$ with a derivative of

$$\frac{\partial \pi_d(p)}{\partial p} = m'_d(p)Q(p) + m_d(p)Q'(p).$$

Implicitly define the price set by the downstream sector as

$$\frac{m_d(p)}{m'_d(p)} = p - \frac{w+d}{1-r_d} = \theta_d \lambda(p).$$

Check that the cost pass-through rate of the downstream sector is given by $(1 - \theta_d \lambda'(p))^{-1}$. Solving for w yields the wholesale price $\bar{w}(p)$ implementing retail price p,

$$\bar{w}(r_d, p) = (1 - r_d)[p - \theta_d \lambda(p)] - d.$$

This is the royalty base for upstream licensing. Let ε_w be the price elasticity of the royalty base $\bar{w}(p)$. The upstream segment's margin is then $m_u(p) = (1 - r_u)\bar{w}(r_d, p) - c$ and profits are $\pi_u(p) = m_u(p)Q(p)$. Similar to the downstream segment, we then introducing the parameter θ_u and define the price for the upstream industry by

$$\frac{m_u(p)}{m'_u(p)} = \theta_u \lambda(p)$$

Solving and re-arranging implicitly gives the retail price as

$$p - \lambda(p)\Theta(p) = \frac{c + d(1 - r_u)}{(1 - r_d)(1 - r_u)}$$
(9)

where $\Theta(p)$ is an aggregate supply chain competition index given by

$$\Theta(p) = \theta_d + \theta_u - \lambda'(p)\theta_d\theta_u.$$

This measure is symmetric and increasing in the upstream and downstream competition parameter. It takes value $\Theta(p) = 0$ when both layers are competitive and increases to $\Theta(p) = 2 - \lambda'(p)$ with a bilateral monopoly. The index can also be written as the downstream parameter θ_d plus the upstream parameter adjusted by the inverse cost pass-through rate.⁹ The RHS of (9) is the overall perceived marginal cost of the supply chain as a function of the production costs and ad valorem license fees.

⁹The interaction of the two parameters depends on whether demand is log-concave or not. With a log-concave demand $(\lambda'(p) < 0)$, the marginal effect of increased competition in one layer is stronger if the parameter in the other layer is higher. Vice versa, with log-convex demand $(\lambda'(p) > 0)$, the marginal effect is reduced, the higher the competition parameter in the other layer.

The patent holder's licensing income associated with a price p is

$$\pounds(r_u, r_d, p) = r_u \bar{w}(r_d, p)Q(p) + r_d pQ(p).$$

We first consider single-layer licensing where the patent holder is only allowed to either license the upstream or the downstream sector. In subsection 6.3, we allow for "double-dipping" where the patent holder can charge a strictly positive licensing rate to both layers. With downstream licensing $(r_u = 0)$, solving condition (9) for r_d yields

$$r_d = \bar{r}_d(p) = 1 - \frac{c+d}{p - \lambda(p)\Theta(p)}.$$

Let $\varepsilon_{rd}(p)$ be the price elasticity of the royalty rate r_d . With upstream licensing $(r_d = 0)$, the royalty rate r_u for a given price p is simply

$$r_u = \bar{r}_u(p) = 1 - \frac{c}{p - d - \lambda(p)\Theta(p)}.$$

and its price elasticity is denoted by $\varepsilon_{ru}(p)$. The following relationships will be useful for the analysis.

Lemma 1. It holds that (i) $\varepsilon_w(p) \ge 1$ and (ii) $\varepsilon_{rd}(p) \ge \varepsilon_{ru}(p)$.

The upstream royalty base $\bar{w}(p)$ is more price-elastic than the downstream royalty base p. This is due to the wedge between retail and wholesale price created by the downstream cost and price mark-up θ_d . At d = 0 and $\theta_d = 0$, we have $\varepsilon_w(p) = 1$. The second point is the rate extraction effect expressed in elasticities. The downstream licensing rate is more price elastic than the upstream rate. The difference between these elasticities is larger, the higher $\bar{r}_u(p)$ is relative to $\bar{r}_d(p)$.

6.2 Single-layer licensing

We first characterize downstream prices with upstream and downstream licensing. Then we look at the patent holder's optimal licensing target and compare private and social incentives.

Suppose the patent holder only charges license fees to the downstream

sector. The patent holder implicitly chooses the retail price such that

$$p_d = \arg \max_p \ \mathcal{L}(0, \bar{r}_d(p), p) = \bar{r}_d(p)pQ(p),$$

which yields as first-order condition in terms of the price elasticities

$$\varepsilon_{rd}(p_d) + 1 = \varepsilon_d(p_d). \tag{10}$$

The patent holder implements a retail price where the price elasticity of the per-unit revenues equals the demand elasticity. As the rate elasticity $\varepsilon_{rd}(p_d)$ is strictly positive for positive licensing rates, the LHS is strictly larger than one and it has to holds that $\varepsilon_d(p_d) > 1$ at the optimal price.

Suppose the patent holder only charges license fees to the upstream sector. In this case the optimal retail price solves

$$p_u = \arg \max_p \mathcal{L}(\bar{r}_u(p), 0, p) = \bar{r}_u(p)\bar{w}(0, p)Q(p)$$

The first-order condition for revenue maximization yields

$$\varepsilon_{ru}(p_u) + \varepsilon_w(p_u) = \varepsilon_d(p_u). \tag{11}$$

With upstream licensing, the patent holder implements a price where the demand elasticity equals the sum of the elasticities of the licensing rate and the wholesale price. The more elastic the per unit-revenues are, the higher is the implemented retail price. We can now compare equilibrium retail prices with downstream and upstream licensing.

Proposition 4. There exist a unique $d^* \ge 0$ and a unique $c^*(d) \ge 0$ such that if $d \ge d^*$ and $c \le c^*(d)$, then upstream licensing leads to lower prices. Otherwise, downstream licensing leads to lower prices.

Conditions (10) and (11) provide a generalized version of the trade-off between the *rate extraction* and *royalty base* effect. A higher retail price increases the revenue share that the patent holder can extract from the licensee segment. Since $0 < \varepsilon_{ru} < \varepsilon_{rd}$, the royalty rate is more price-sensitive with downstream licensing. This puts more upward pressure on prices with downstream licensing. At the same time, it holds that $\varepsilon_w \geq 1$ and the royalty base with upstream licensing is more price elastic than with downstream licensing. The question as to which of these opposing effects is stronger depends on the parameters. The upstream cost c has a relatively stronger effect on the elasticity of per-unit revenues with upstream licensing. Hence, for low values of c, upstream licensing results in lower prices; for sufficiently high upstream cost, downstream licensing leads to lower retail prices.

We can now investigate the patent holder's incentive to license either the upstream or downstream industry. As first step, consider the difference in licensing revenues when implementing the same retail price,

$$\mathcal{L}(\bar{r}_u(p), 0, p) - \mathcal{L}(0, \bar{r}_d(p), p) = \frac{[p - c - d - \lambda(p)\Theta(p)][\Theta(p)d - \theta_d\lambda(p)(p - \lambda(p)\Theta(p))]}{(p - \lambda(p)\Theta(p))(p - d - \lambda(p)\Theta(p))}Q(p).$$

This difference has either two or three roots.¹⁰ Let p_1 and p_2 denote the respective prices at which the first and second squared bracket in the numerator equates to zero. The retail price p_1 is implemented in the absence of any licensing. At this price, the patent holder makes no profit with either licensing target as it implements a rate of 0. The patent holder optimally chooses prices above p_1 with upstream and downstream licensing. Hence, whether upstream or downstream is optimal depends on the sign of the second squared bracket. We can verify that this condition nests the results of Sections 3 to 5 as special cases. When both segments are competitive, $\theta_d = \theta_u = 0$, the second squared bracket is equal to zero and the patent holder is indifferent between upstream and downstream licensing. When only the downstream (upstream) segment is perfectly competitive, the difference is strictly positive (negative). In the case where both segments are imperfectly competitive, the difference depends on the implemented retail price. The second root p_2 is the threshold value such that if the retail price p is smaller (larger) than p_2 , the patent holder strictly prefers upstream (downstream) licensing. It follows that there are retail prices at which upstream licensing is at least as profitable as downstream licensing if and only if $p_1 \leq p_2$ or

$$\frac{c}{d} \le \frac{\theta_u}{\theta_d} [1 - \theta_d \lambda'(p)]. \tag{12}$$

If the ratio of upstream to downstream cost is relatively small compared

¹⁰A third root exists if demand has a choke price, $p_3 = \{p|Q(p) = 0\}$.

to the ratio of the upstream and downstream competition parameters, upstream licensing may be profitable for a given retail price. Condition (12) is a necessary condition for upstream licensing being preferred by the patent holder. The overall comparison then depends on the relative magnitudes of the optimal prices p_d , p_u and the threshold price p_2 . We get the following result.

Proposition 5. There exist a unique $d^{**} > 0$ and a unique $c^{**}(d)$ such that if $d \ge d^{**}$ and $c \le c^{**}(d)$, then the patent holder prefers licensing the upstream segment; otherwise, the patent holder is better off licensing the downstream industry.

This result follows from the fact that the optimal retail prices p_u and p_d increase in the cost level c while the threshold price p_2 is invariant in c. For low values of the upstream cost, the optimal retail prices are both below the threshold and upstream licensing dominates. Vice versa, for relatively high level of c, both retail prices are above the threshold price p_2 and downstream licensing is optimal. For intermediate values of c, the threshold price is between the optimal retail price with upstream and downstream licensing, $p_u < p_2 < p_d$. For these values, the patent holder prefers targeting the upstream industry if and only if $\pounds(\bar{r}_u(p_u), 0, p_u) - \pounds(0, \bar{r}_d(p_d), p_d) \ge 0$ or

$$\bar{r}_u(p_u)\bar{w}(0,p_u)[Q(p_u) - Q(p_d)] \ge [\bar{r}_d(p_d)p_d - \bar{r}_u(p_u)\bar{w}(0,p_u)]Q(p_d)$$
(13)

The lower retail price with upstream licensing allows to serve higher demand relative to downstream licensing. This has to be traded off against a higher per-unit revenue with downstream licensing (squared bracket on RHS). Low upstream costs c increase the benefit of a higher demand with upstream licensing. Condition (13) thus holds and upstream licensing is more profitable if and only if $c \leq c^{**}(d)$. For higher values of the cost level, the patent holder switches from upstream to downstream licensing leading to a discontinuous increase of the implemented retail price at the threshold $c = c^{**}(d)$.

We are now in a position to compare private and social incentives regarding the choice of the licensing target.

Proposition 6. It holds that $d^{**} > d^*$ and $c^{**}(d) < c^*(d)$ for all $d > d^{**}$. This yields the following results: (i) If $c \le c^{**}(d)$, the patent holder's choice of the upstream industry is efficient. (ii) If $c \ge c^*(d)$, the patent holder's choice of the downstream industry is efficient. (iii) Otherwise, the patent holder targets the downstream industry when the efficient licensing level is the upstream industry.

The patent holder's choice of its licensing target is socially efficient unless upstream and downstream cost levels are intermediate. For values $c^{**}(d) < c < c^*(d)$, the patent holder excessively targets the downstream industry. The reason is that for these values $p_u < p_d$ and using (13) the patent holder prefers downstream licensing if and only if

$$\frac{Q(p_u)}{Q(p_d)} < \frac{\bar{r}_d(p_d)p_d}{\bar{r}_u(p_u)\bar{w}(0,p_u)}.$$

Consumers prefer upstream licensing with lower retail prices. The patent holder also benefits from the relatively higher demand with upstream licensing. However, the patent holder also takes into account the per-unit revenues from each segment. For the given parameter values, the higher per-unit revenue with downstream licensing dominates the gains from a higher demand with upstream licensing.

FIGURE 1 illustrates the above results using a numerical, linear demand example. To account for the four main parameters of the model, panel 1.A displays a c - d diagram while panel 1.B presents the results in terms of the competition parameters. The upper panel indicates the three different regions identified in Proposition 6. In region DD the patent holder chooses downstream licensing, and the retail price is lower at this licensing level. Vice versa, in region UU the patent holder picks the socially efficient level by targeting the upstream segment. An inefficiency occurs in region DU where the downstream sector is licensed which leads to higher retail prices compared to upstream licensing. This happens when upstream and downstream cost levels are similar. If one segment has a significantly higher share of the overall cost of production, then the patent holder selects the efficient licensing layer. A similar picture emerges in the lower panel. For given cost levels, let $\theta_u^*(\theta_d)$ and $\theta_{u}^{**}(\theta_{d})$ be the values that solve $c = c^{*}(d)$ and $c = c^{**}(d)$. Licensing occurs in the segment of the chain supply with relatively more market power, that is less competition. If there is significantly more market power at one level of the supply chain relative to the other, then private and social licensing incentives are aligned. An inefficiently excessive licensing at the downstream segment

is more likely when the upstream and downstream competition parameters are similar.



[Q(p) = 1 - p, c = d = 0.2]

6.3 Double-dipping licenses

Let us now investigate multi-layer licensing and allow the patent holder to charge strictly positive rates to both layers of the supply chain. Solving the pricing condition (9) for r_d we get

$$\hat{r}_d(r_u, p) = 1 - \frac{c + d(1 - r_u)}{(1 - r_u)[p - \lambda(p)\Theta(p)]}$$

The graph of this function is illustrated in FIGURE 2 below. It is easy to check that the downstream rate is decreasing in r_u from $\bar{r}_d(p)$ at $r_u = 0$ (point A) to 0 at $r_u = \bar{r}_u(p)$ (point B). An increase in license rate to one layer requires a lower rate to the other layer to keep the price p constant. Moreover, $\hat{r}_d(r_u, p)$ is strictly concave in r_u and we know that $\bar{r}_u(p) > \bar{r}_d(p)$. This implies that the sum of upstream and downstream rates along $\hat{r}_d(r_u, p)$ is higher compared to exclusive downstream licensing (where $r_u + r_d = \bar{r}_d(p)$). Raising the retail price p increases $\hat{r}_d(r_u, p)$ and shifts the curve outwards.



FIGURE 2: Optimal Multi-Layer Licensing. $[Q(p) = 1 - p, c = d = 0.1, \theta_d = \theta_u = 0.4]$

Under multi-layer licensing, the patent holder solves the following maximization problem,

$$\max_{\{p,r_u,r_d\}} \quad \pounds_m(r_u,r_d,p) = [r_u\bar{w}(r_d,p) + r_dp]Q(p) \quad \text{subject to } (9).$$

Without loss of generality, let the patent holder pick r_u and p while the downstream rate is determined by $r_d = \hat{r}_d(r_u, p) \in [0, \bar{r}_d(p)]$. First, consider the optimal choice of r_u for a given price p. In an interior solution it has to hold that

$$\frac{\partial \mathcal{L}_m / \partial r_u}{\partial \mathcal{L}_m / \partial r_d} = \frac{d\hat{r}_d(r_u, p)}{dr_u}.$$
(14)

This condition equates the slope of the iso-revenue curve of \mathcal{L}_m with the

slope of $\hat{r}_d(r_u, p)$. This is shown in FIGURE 2 where the iso-revenue curve is labelled $\overline{\mathcal{I}}_m$. The tangency point *C* depicts the optimal pair (r_u, r_d) for a given retail price *p*. Using $r_d = \hat{r}_d(r_u, p)$ and simplifying (14) further allows to implicitly define the optimal royalty rate r_u for a given price *p* as

$$(1 - r_u)^2 (1 - \theta_d \lambda'(p)) = \frac{c\theta_d}{d\theta_u}.$$
(15)

This condition has at most one solution for $r_u \in (0, 1]$. The optimal upstream rate decreases in c and θ_u , and it increases in d and θ_u . An interior solution exists, if condition (12) is satisfied, that is, when prices and parameters exist such that single-layer upstream licensing is more profitable than single-layer downstream licensing. Let $c^m(d)$ be the value such that (12) holds with equality. Multi-layer licensing is then optimal if $c \leq c^m(d)$. In the special case of $\lambda''(p) = 0$, the optimal upstream rate is independent of the retail price level. This special case includes common demand functions such as, for example, the linear, exponential and constant-elasticity demand.

Now solve for the optimal retail price for a given r_u and a downstream rate $r_d = \hat{r}_d(r_u, p)$. The first-order condition for the price is $\partial \mathcal{L}_m / \partial p = 0$ or

$$[r_u(\frac{\partial \bar{w}}{\partial p} + \frac{\partial \bar{w}}{\partial r_d}\frac{\partial \hat{r}_d}{\partial p}) + \frac{\partial \hat{r}_d}{\partial p}p + \hat{r}_d]Q(p) + [r_u\bar{w} + \hat{r}_dp]Q'(p) = 0.$$

The retail price affects both downstream and upstream per-unit revenues. Note that there is a direct and indirect effect on upstream per-unit revenues. A higher retail price raises the wholesaled price but also increases the downstream rate, which, in turn, reduces the upstream royalty base. We can again express the price condition in elasticities. Let $\varepsilon_{\hat{r}d}(r_u, p)$ be the price elasticity of the downstream rate and $\varepsilon_{w,rd}(r_d, p)$ (the absolute value of) the rate elasticity of the wholesale price $\bar{w}(r_d, p)$, respectively.¹¹ Further define the share of downstream revenues in total revenues as

$$s_d = \frac{\hat{r}_d(r_u, p)p}{r_u \bar{w}(r_d, p) + \hat{r}_d(r_u, p)p}$$

$$\varepsilon_{w,rd}(r_d, p) = -\frac{\partial \bar{w}(r_d, p)}{\partial r_d} \frac{r_d}{\bar{w}(r_d, p)}$$

¹¹Let the rate elasticity of the wholesale price be defined as

and rewrite the first-order condition for the optimal retail price as follows

$$s_d[\varepsilon_{\hat{r}d}(r_u, p) + 1] + (1 - s_d)[\varepsilon_w(p) - \varepsilon_{w,rd}(r_d, p)\varepsilon_{\hat{r}d}(r_u, p)] = \varepsilon_d(p).$$
(16)

The price elasticity of the overall per-unit revenues on the LHS has to be equal to the price elasticity of demand on the RHS. The elasticities on the LHS are grouped by their effect on downstream and upstream revenues and weighted by the respective share of downstream/upstream revenues. Note that at $c = c^m(d)$, it holds that $r_u = 0$ and there are no licensing revenues from the upstream layer. Condition (16) simplifies to the price condition with exclusive downstream licensing, that is, (10). For lower values of c, the patent holder licenses to both layers and the price elasticity is a linear combination of downstream and upstream elasticity. In the Appendix we show that this leads to a lower overall price elasticity of per-unit revenues with double-dipping compared to exclusive downstream licensing. We can thus establish the following results.

Proposition 7. Suppose $\lambda''(p) = 0$. Consider a patent holder who is allowed to charge license fees from both segments of the supply chain. If condition (12) holds, then the patent holder optimally charges $r_u > 0$ and $r_d > 0$; otherwise, the patent holder only licenses the downstream segment. Allowing double-dipping strictly increases consumer surplus when the patent holder targets the downstream segment under single-layer licensing.

We find that double-dipping is profitable for the patent holder when condition (12) holds, that is, if the upstream production cost is sufficiently small relative to the downstream production cost. Otherwise, the patent holder prefers exclusive downstream licensing. Moreover, consumers are also better off with double-dipping compared to exclusive downstream licensing. This is due to the fact that multi-layer licensing makes per-unit revenues less priceelastic which leads to lower retail prices and higher demand.

These results have significant implications for the application of the patent exhaustion principle within supply chains. Suppose this principle is enforced, and the patent holder prefers to license their technology to the downstream sector - similar to Nokia's approach in the case discussed in the introduction. Allowing double-dipping in this situation, such that the patent holder charges licensing fees to both upstream and downstream sectors, could lower retail prices and enhance the overall efficiency of the supply chain. Relating back to FIGURE 1 above, double-dipping is profitable if $c \leq c^m(d)$ or $\theta_u \geq \theta_u^m(\theta_d)$. Hence, multi-layer licensing increases social welfare for parameter values in areas A and B where the patent holder targets the downstream sector under single-layer licensing.

We can further illustrate the effect of multi-layer licensing on equilibrium prices in FIGURE 3 below. The numerical calculations use a linear demand function and a fixed value of d = 0.15 to compare equilibrium prices as a function of the upstream cost c. The upper panel assumes a bilateral monopoly $(\theta_d = \theta_u = 1)$ and depicts the equilibrium prices with exclusive upstream and downstream licensing (gray lines p_u and p_d). For values $c \ge c^{**}(d)$, the patent holder switches from upstream licensing to downstream licensing. Hence, the black line is the overall equilibrium price with single-layer licensing including a discrete upward jump at $c^{**}(d)$. The optimal price with double-dipping is depicted by the red line. An interior solution exists for any $c \leq c^m(d) = 0.3$ (which is the solution to condition (12) above). For higher values of c the patent holder chooses downstream licensing. As can be seen, multi-layer licensing reduces the retail price if $c \ge c^{**}(d)$; otherwise, single-layer licensing with the downstream segment as target is socially efficient. The only difference in the lower panel is that upstream and downstream market power is lower ($\theta_d = \theta_u = 0.4$). For these values, the patent holder prefers downstream licensing for any value of c. The resulting equilibrium price with multi-layer licensing is below both the price with exclusive downstream and upstream licensing for any value where an interior solution exists.





FIGURE 3: Welfare Effect of Multi-Layer Licensing $[Q(p) = 1 - p, d = 0.15, \text{ upper panel: } \theta_d = \theta_u = 1, \text{ lower panel } \theta_d = \theta_u = 0.4]$

7 Probabilistic Patents and Litigation

In the previous sections we have analyzed the patent holder's choice of licensing stage in vertical chains with ironclad patents. In this section, we entertain the possibility of probabilistic patents which can be challenged in court. With probabilistic patents, we show that the patent holder may decide to license at the stage with *more* competition contrary to the results above. The main reason for this result is to take advantage of "free-rider" incentives in litigation at the competitive sector and insulate the patent holder from potential challenges. Let us assume that the patent has a probability of ρ that it would be upheld in court if challenged. This probability of patent validity is common knowledge and represents the strength of the patent. If there is litigation, suppose that each party incurs litigation costs of L.

To illustrate this idea, let us consider a market structure in which upstream is competitive and downstream is monopolistic. We know that with an ironclad patent the patent holder prefers to license at the downstream stage with r_d as derived in Section 5. Suppose $\pi^m(k)$ is the profit of the downstream monopolist facing constant marginal cost k. When the patent is probabilistic, the downstream firm has an incentive to litigate if

$$(1-\rho)\left[\pi^{m}(c+d) - (1-r_{d})\pi^{m}(\frac{c+d}{1-r_{d}})\right] \ge L$$
(17)

For litigation to be relevant, let us assume that this condition is always satisfied for $\rho = 0$. Then, there exists a unique $\rho^* \in (0, 1)$ such that the downstream firm has strict incentives to litigate if and only if $\rho < \rho^*$. To prevent litigation incentives, the patent holder needs to charge a limit-license rate $\hat{r}_d(\rho)$ (< r_d) such that (17) holds with equality.¹² Note that the limit-license rate $\hat{r}_d(\rho)$ is increasing in ρ for all $\rho \leq \rho^*$. With such a limit-licensing ad valorem rate, the patent holder's licensing income is given by $\hat{r}_d(\rho)\hat{p}(\rho)Q(\hat{p}(\rho))$ where the limit-licensing induced retail price is

$$\hat{p}(\rho) = p^m(\frac{c+d}{1 - \hat{r}_d(\rho)}) < p^m(\frac{c+d}{1 - r_d}),$$

which is strictly lower than the price with ironclad patents in Section 5 for any $\rho < \rho^*$.

In contrast, if licensing takes place at the perfectly competitive upstream stage, no firm has incentives to challenge the patent because of the public good nature of invalidating the patent and the competitive industry environment. As profits are competed away, there is no benefit from litigation to outweigh the litigation cost. The patent holder can thus avoid patent challenges (or costly limit licensing) by licensing the upstream segment. As the patent becomes weaker, downstream licensing becomes less profitable as the limit-license rate is reduced to mitigate litigation incentives. It can be shown that there exists a level $\hat{\rho}$ such that if $\rho < \hat{\rho}$, the patent holder will decide to license at the upstream stage. Note that the limit-licensing induced price $\hat{p}(\rho)$ is less than the equilibrium downstream price p_d with iron ad patents which, in turn, is lower than the upstream licensing price p_u from Section 5. We thus get a paradoxical result with respect to patent strength and equilibrium retail price. For low values of ρ , licensing occurs at the upstream level which is safe from patent challenges and implements high retail prices. At $\rho = \hat{\rho}$, the patent holder licenses at the downstream level and practices limit licensing. This leads to a downward jump in the retail price. While the retail price increases further as the patent becomes stronger, it never reaches

$$(1-\rho)\left[\pi^m(c+d) - (1-\hat{r}_d)\pi^m(\frac{c+d}{1-\hat{r}_d})\right] = L.$$

 $^{^{12}}$ The limit-license rate is defined by

As the LHS decreases in ρ and increases in \hat{r}_d , we have $d\hat{r}_d/d\rho > 0$.

the level that prevails for very weak patents and upstream licensing. This is illustrated in FIGURE 4 below.



FIGURE 4: Licensing with Probabilistic Patents [Q(p) = 1 - p, c = d = 0.1]

8 Concluding Remarks

In this paper, we present a framework for analyzing the incentives of patent holders in determining which stage of the supply chain to target for licensing incomes. Our findings indicate that the private and social incentives for choosing the licensing target depend on the relative distribution of market power and production costs across the vertical stages. Specifically, when both stages of the supply chain are perfectly competitive, we observe a neutrality result: the choice of licensing stage is inconsequential for both private and social incentives. In the case where one stage of the chain is monopolistic while the other is perfectly competitive, we once again find an alignment of private and social incentives. The patent holder prefers licensing at the stage where monopoly power resides, and this aligns with greater efficiency for social welfare. However, when both stages are imperfect, we identify a potential misalignment between private and social incentives. Our analysis reveals an excess incentive for downstream licensing. This divergence suggests that under certain conditions, there may be an overemphasis on licensing activities in the downstream stage, which can reduce overall economic efficiency and welfare.

Our analysis also offers a new perspective on the legal principle of patent exhaustion and the effects of "double dipping." This practice is generally discouraged and subject to legal scrutiny due to concerns about unjust enrichment from multiple attempts to collect licensing revenues for the same patented technology. We demonstrate that the issue of double-dipping becomes inconsequential when both stages are perfectly competitive. Moreover, we find that the licensor lacks incentives to engage in double-dipping if only one stage is monopolistic. However, our analysis reveals a nuanced scenario when both stages are imperfect. In such cases, we identify situations where double-dipping can be both privately optimal and socially efficient. This is because the flexibility afforded by multi-layer licensing can help manage overall pricing distortions in the supply chain, contributing to improved social welfare. In these instances, double-dipping may not only align with the private interests of the licensor but also serve societal goals by reducing the retail price and increase output of the supply chain.

In our analysis, we have deliberately abstracted from additional considerations that could influence the choice of the appropriate licensing stage. For instance, a relevant factor not explicitly addressed is the notion that patent licensing responsibility should ideally rest with the party possessing a superior understanding of the technologies underlying the pertinent patents. Taking this perspective into account, it seems reasonable to advocate for endproduct level licensing of cellular standard essential patents, particularly for smartphone Original Equipment Manufacturers (OEMs). The rationale is that smartphone OEMs inherently possess comprehensive knowledge of cellular technologies, given that connectivity has always been integral to the functionality of mobile devices (Geradin and Katsifis, 2021). In contrast, when it comes to automobile manufacturers incorporating cellular technologies for the first time, they may lack the expertise to fully comprehend these technologies and negotiate with patent holders on an equal footing. Consequently, licensing at the component level might be deemed more appropriate in such cases. In addition, practical considerations, such as transaction costs, could play a significant role and may be contingent on the number of firms requiring licensing. In practice, it may be more efficient to choose the licensing stage where the number of firms is fewer. Although our analysis has primarily focused on pricing aspects resulting from the choice of licensing stages, we acknowledge that these practical considerations, including technological expertise and transaction costs, are crucial in real-world scenarios and merit careful consideration in future research and policy discussions.

Our analysis offers avenues for extension in several directions. Firstly, we made the assumption that patent holders have no presence in the production supply chain and did not consider vertically integrated firms. Extending our analysis to incorporate patent holders with a presence in one of the production stages would be worthwhile. This extension could involve exploring the incentives for licensing in cases where patent holders are integrated into the production chain, as, for example, in the Qualcomm case (F.T.C. v. QUAL-COMM INC. Case No. 17-CV-00220-LHK, 411 F. Supp.3d 658, 2019), where Qualcomm faced accusations regarding its "No License, No Chips" policy. In instances where licensee firms are vertically integrated, licensing at the upstream stage may give rise to strategic transfer pricing for internal transactions, introducing additional layers of complexity. Secondly, our analysis assumed the existence of ironclad patents without explicitly delving into incentives to litigate for patent invalidation, except for a brief consideration of the effects of probabilistic patents on the choice of licensing stage for the patent holder in Section 7. Recognizing the probabilistic nature of patents and the significant role that litigation plays in the enforcement of intellectual property rights can be important especially in industries with a high rate of litigation. A more comprehensive analysis considering general market power distribution across vertical stages and incorporating the strategic aspects of litigation awaits exploration in future research.

Appendix A

Proof of Proposition 1

The proof of the main results are in the text. Here we only consider doubledipping with $(r_u > 0, r_d > 0)$. With perfect competition, the upstream wholesale and downstream final prices are determined by

$$w = \frac{c}{1 - r_u}$$
 and $p = \frac{w + d}{1 - r_d}$

which yield the relationship between retail price and licensing rates,

$$r_d = 1 - \frac{c + d(1 - r_u)}{p(1 - r_u)}.$$
(18)

The patent holder's profits as a function of the implemented downstream price p can be written as

$$\begin{aligned} \mathcal{L}_{dd}(p) &= \left[r_u w + r_d p \right] Q(p) \\ &= \left[r_u \frac{c}{1 - r_u} + \left(1 - \frac{c + d(1 - r_u)}{p(1 - r_u)} \right) p \right] Q(p) \\ &= \left[\frac{r_u cp}{p(1 - r_u)} + \frac{(p - d)(1 - r_u) - c}{p(1 - r_u)} p \right] Q(p) = (p - c - d) Q(p) \end{aligned}$$

Hence, the optimal retail price and overall licensing revenues are the same as with exclusive upstream or downstream licensing. There is a continuum of (r_u, r_d) combinations satisfying (18) for $p = p^m(c+d)$ that implement the optimal price. The upstream and downstream licensing fee imposed at only one stage are special cases. The neutrality result for perfectly competitive market thus extends to multi-layer licensing. QED.

Proof of Proposition 2

1. Comparison of equilibrium prices. First we show that $p_u < p_d$. Define the following functions

$$\phi(p,d) = \frac{p-d}{p-d-\lambda(p)}, \Pi(p) = [p-\lambda(p)-c-d]Q(p).$$

The optimal price with upstream licensing satisfies

$$\phi(p,d)\Pi'(p) + \phi'(p,d)\Pi(p) = 0 \quad \text{or} \quad \Pi'(p) = -\frac{\phi'(p,d)}{\phi(p,d)}\Pi(p).$$

Check that $\partial \pounds_d(p)/\partial p \geq \partial \pounds_u(p)/\partial p$ if and only if

$$\begin{aligned} &[\phi(p,0) - \phi(p,d)]\Pi'(p) + \Pi(p)[\phi'(p,0) - \phi'(p,d)] \ge 0 \\ &- [\phi(p,0) - \phi(p,d)]\frac{\phi'(p,d)}{\phi(p,d)}\Pi(p) + \Pi(p)[\phi'(p,0) - \phi'(p,d)] \ge 0 \end{aligned}$$

or

$$\phi(p,0)\Pi(p)\left[\frac{\phi'(p,0)}{\phi(p,0)} - \frac{\phi'(p,d)}{\phi(p,d)}\right] \ge 0$$
$$\frac{d\lambda(p)\left[2p - d - \lambda(p)\right] - d(p - d)p\lambda'(p)}{p(p - d)(p - \lambda(p))(p - d - \lambda(p))} \ge 0$$

since

$$\frac{p\lambda'(p)}{\lambda(p)} < 1 < \frac{2p - d - \lambda(p)}{p - d}.$$

With quasi-concave revenues, this implies that $p_u < p_d$.

2. Patent holder's incentives. For a given downstream price p, the profit difference between upstream and downstream licensing is given by

$$\begin{aligned} \pounds_u(p) - \pounds_d(p) &= \left(1 - \frac{c}{p - d - \lambda(p)}\right)(p - d)Q(p) - \left(1 - \frac{c + d}{p - \lambda(p)}\right)pQ(p) \\ &= \frac{p - c - d - \lambda(p)}{[p - \lambda(p)][p - d - \lambda(p)]}d\lambda(p)Q(p) > 0 \end{aligned}$$

for any $p \ge p^m(c+d)$. Since $p^m(c+d) < p_u < p_d$ and revenues are quasiconcave, we get

$$\pounds_u(p_u) > \pounds_u(p_d) > \pounds_d(p_d).$$

3. Non-optimality of double-dipping. Consider the patent holder engaging in double-dipping with $(r_u, r_d) > 0$. In the presence of monopoly power at the upstream level, the upstream wholesale and downstream final prices are determined by

$$p = \frac{w+d}{1-r_d}$$

Hence, the upstream monopolist's profits as a function of the realized downstream price are

$$[(1 - r_u)w - c]Q(p) = [(1 - r_u)((1 - r_d)p - d) - c]Q(p)$$

= $(1 - r_u)(1 - r_d)[p - \frac{c}{(1 - r_u)(1 - r_d)} - \frac{d}{1 - r_d}]Q(p)$

The first-order condition for profit maximization is then simply given by

$$p - \lambda(p) = \frac{c}{(1 - r_u)(1 - r_d)} + \frac{d}{1 - r_d}.$$
(19)

Licensing revenues are given by

$$\pounds_{dd}(p) = [r_u w + r_d p] Q(p) = \{r_u[(1 - r_d)p - d] + r_d p\} Q(p)$$

Now consider an alternative licensing scheme that imposes a royalty rate only on the monopolistic upstream sector with $(r'_u, r'_d = 0)$ that induces exactly the same final price p. Then, we have the following relationship.

$$\frac{c}{1 - r'_u} + d = \frac{c}{(1 - r_u)(1 - r_d)} + \frac{d}{1 - r_d}$$

or

$$r'_{u} = 1 - \frac{c(1 - r_{u})(1 - r_{d})}{c + d(1 - r_{u})r_{d}}.$$

With this alternative licensing scheme, the licensing income is

$$\pounds_u(p) = r'_u(p-d)Q(p).$$

Thus, upstream licensing weakly dominates if $\pounds_u(p) \ge \pounds_{dd}(p)$ or

$$\frac{\left[1 - \frac{c(1 - r_u)(1 - r_d)}{c + d(1 - r_u)r_d}\right](p - d) - [r_u[(1 - r_d)p - d] + r_d p] \ge 0 \quad \text{or}}{\frac{dr_d(1 - r_u)[(1 - r_u)(p(1 - r_d) - d) - c]}{c + d(1 - r_u)r_d}} = \frac{dr_d(1 - r_u)[(1 - r_u)w - c]}{c + d(1 - r_u)r_d} \ge 0$$

which holds for any $r_d \ge 0$. QED.

Proof of Proposition 3

1. Price Comparison. Using the notation of the proof of Proposition 2, the optimal price with upstream licensing satisfies $\Pi'(p_u) = 0$ and is equal to $p_u = p^M(c+d)$. Check that at $p = p_u$ we get

$$\begin{aligned} \frac{\partial \mathcal{L}_d(p)}{\partial p} &= \phi(p,0)\Pi'(p) + \Pi(p)\phi'(p,0) \\ &= \Pi(p)\phi'(p,0) = -\Pi(p)\frac{\lambda(p) - p\lambda'(p)}{(p-\lambda(p))^2} < 0 \end{aligned}$$

since $p\lambda'(p)/\lambda(p) < 1$. With quasi-concave revenue functions, this implies that $p_d < p_u$.

2. Patent holder incentives. Verify that

$$\pounds_d(p) - \pounds_u(p) = \frac{\lambda(p)}{p - \lambda(p)} [p - \lambda(p) - c - d]Q(p) > 0$$

for any $p \ge p^m(c+d)$. Since $p^m(c+d) < p_d < p_u$ and revenues are quasiconcave, we get

$$\pounds_d(p_d) > \pounds_u(p_d) > \pounds_u(p_u).$$

3. Non-optimality of double-dipping. With double-dipping $(r_u > 0, r_d > 0)$, the downstream monopolist maximizes

$$\left[(1 - r_d)p - \frac{c}{1 - r_u} - d \right] Q(p)$$

= $(1 - r_d) \left[p - \frac{c}{(1 - r_u)(1 - r_d)} - \frac{d}{1 - r_d} \right] Q(p)$

which yields first-order condition (19) and the same revenues $\pounds_{dd}(p)$ as in the proof of the previous proposition. Again consider an alternative licensing scheme that imposes a royalty rate only on the monopolistic downstream sector with $(r'_u = 0, r'_d)$ that induces the same final price p. It then has to hold that

$$\frac{c+d}{1-r'_d} = \frac{c}{(1-r_u)(1-r_d)} + \frac{d}{1-r_d}$$

or

$$r'_{d} = 1 - \frac{(c+d)(1-r_{u})(1-r_{d})}{c+d(1-r_{u})}$$

Exclusive downstream licensing weakly dominates double-double-dipping dipping if $\pounds_d(p) = r'_d p Q(p) \ge \pounds_{dd}(p)$ or

$$\frac{\left[1 - \frac{(c+d)(1-r_u)(1-r_d)}{c+d(1-r_u)}\right]p - [r_u[(1-r_d)p - d] + r_d p] \ge 0 \quad \text{or}}{\frac{dr_u(1-r_u)[(1-r_u)(p(1-r_d) - d) - c]}{c+d(1-r_u)r_d} \ge 0}$$

which holds for any $r_u \ge 0$. QED.

Proof of Lemma 1

Check that

$$\varepsilon_w = \frac{\partial \bar{w}(p)}{\partial p} \frac{p}{\bar{w}(p)} = \frac{p(1 - r_d)(1 - \theta_d \lambda'(p))}{[p - \theta_d \lambda(p)](1 - r_d) - d} \ge 1$$

if and only if $d \ge -(1-r_d)\theta_d[\lambda(p) - p\lambda'(p)]$ which holds when the elasticity of Mills ratio $p\lambda'(p)/\lambda(p)$ is less than one. For point (ii), define $\Lambda(p) = \lambda(p)\Theta(p)$ and check that

$$\varepsilon_{ru} = [1 - \bar{r}_u(p)] \frac{p(1 - \Lambda'(p))}{p - c - d - \Lambda(p)} < \varepsilon_{rd} = [1 - \bar{r}_d(p)] \frac{p(1 - \Lambda'(p))}{p - c - d - \Lambda(p)}$$

since $\bar{r}_u(p) > \bar{r}_d(p)$. QED.

Proof of Proposition 4

For notational ease, define $\Lambda(p) = \lambda(p)\Theta(p)$. The first-order condition with downstream licensing is

$$\frac{\partial \pounds(0, \bar{r}_d, p)}{\partial p} = \frac{\partial \bar{r}_d}{\partial p} pQ(p) + \bar{r}_d(p)pQ'(p) + \bar{r}_d(p)Q(p) = 0$$

$$\Leftrightarrow \varepsilon_{rd}(p) - \varepsilon_d(p) + 1 = 0$$

$$\Leftrightarrow \frac{(c+d)p[1 - \Lambda'(p)]}{[p - \Lambda(p)][p - c - d - \Lambda(p)]} - \frac{p}{\lambda(p)} + 1 = 0$$

$$\Leftrightarrow p_d - c - d - \Lambda(p_d) = \lambda(p_d)\Phi_d(p_d)$$
(20)

where

$$\Phi_d(p_d) \equiv \frac{p_d(c+d)(1-\Lambda'(p_d))}{(p_d-\lambda(p_d))(p_d-\Lambda(p_d))} > 0.$$

Similarly, the first-order condition with upstream licensing is

$$\frac{\partial \mathcal{L}(\bar{r}_u, 0, p)}{\partial p} = \frac{\partial \bar{r}_u}{\partial p} \bar{w}(p)Q(p) + \bar{r}_u(p)\bar{w}(p)Q'(p) + \frac{\partial \bar{w}}{\partial p}\bar{r}_u(p)Q(p) = 0$$

$$\Leftrightarrow \varepsilon_{ru}(p) - \varepsilon_d(p) + \varepsilon_w(p) = 0$$

$$\Leftrightarrow \frac{cp[1 - \Lambda'(p)]}{[p - d - \Lambda(p)][p - c - d - \Lambda(p)]} - \frac{p}{\lambda(p)} + \frac{p(1 - \theta_d \lambda'(p))}{p - d - \theta_d \lambda(p)} = 0$$

$$\Leftrightarrow p_u - c - d - \Lambda(p_u) = \lambda(p_u)\Phi_u(p_u)$$
(21)

where

$$\Phi_u(p_u) \equiv \frac{c[1 - \Lambda'(p_u)][p_u - \theta_d \lambda(p_u) - d]}{[p_u - d - \Lambda(p_u)][p - \theta_d \lambda(p_u) - d - \lambda(p_u)(1 - \theta_d \lambda'(p_u))]} > 0.$$

Let $p_d(c)$ and $p_u(c)$ be the solutions to (20) and (21), respectively. To ensure a revenue maximum, assume both RHS are decreasing in p for any $p - c - d - \Lambda(p) > 0$. It follows immediately from inspection that $\partial \Phi_d(p)/\partial c > 0$ and $\partial \Phi_u(p)/\partial c > 0$ which implies that $\partial p_d/\partial c > 0$ and $\partial p_u/\partial c > 0$.

Note that the lowest price $p_u(c=0)$ is given by

$$p - d - \lambda(p) - \lambda(p)\theta_d(1 - \lambda'(p)) = 0.$$

We can thus focus below on the following condition

$$\theta_d \le \frac{p - d - \lambda(p)}{\lambda(p)(1 - \lambda'(p))} \equiv \bar{\theta}_d.$$

We first show that there can be at most one intersection of $p_d(c)$ and $p_u(c)$ as a function of c. This holds if $\partial \Phi_d(p)/\partial c < \partial \Phi_u(p))/\partial c$ or

$$\frac{p}{[p-\lambda(p)][p-\Lambda(p)]} - \frac{p-\theta_d\lambda(p)-d}{[p-d-\Lambda(p)][p-d-\lambda(p)(\theta_d+1-\theta_d\lambda')]} < 0$$

$$\Leftrightarrow \quad p[p-d-\Lambda(p)][p-d-\lambda(p)(\theta_d+1-\theta_d\lambda')]$$

$$- [p-\theta_d\lambda(p)-d][p-\lambda(p)][p-\Lambda(p)] \equiv \Delta(\theta_d,\theta_u) < 0.$$

Taking the derivative of $\Delta(\theta_d, \theta_u)$ with respect to θ_u yields

$$\frac{\partial \Delta(\theta_d, \theta_u)}{\partial \theta_u} = \lambda(p)^2 (1 - \theta_d \lambda'(p)) [d + \theta(\lambda(p) - p\lambda'(p))] > 0$$

since $\lambda(p) - p\lambda'(p) > 0$. Hence, Δ takes its maximum at $\theta_u = 1$. Further, check that

$$\frac{\partial^2 \Delta(\theta_d, 1)}{(\partial \theta_d)^2} = 2\lambda(p)^2 (1 - \lambda'(p))[\lambda(p) - p\lambda'(p))] > 0$$

which implies that $\Delta(\theta_d, 1)$ is convex in θ_d . Moreover, it holds that

$$\Delta(0,1) = -d[p(p-d) - \lambda(p)^{2}] < 0$$

since the squared bracket is strictly positive for any $p - d > \lambda(p)$. Moreover,

it holds that

$$\Delta(\bar{\theta}_d, 1) = -\frac{d}{1 - \lambda'(p)} (p - \lambda(p)) [\lambda(p) - (p - d)\lambda'(p)] < 0.$$

The squared bracket is strictly positive for $\lambda'(p) < 0$. If $\lambda'(p) > 0$, then

$$(p-d)\lambda'(p) < p\lambda'(p) < \lambda(p)$$

and the squared bracket is also strictly positive. It follows that $\partial \Phi_d(p)/\partial c < \partial \Phi_u(p)/\partial c$. This implies that $\partial p_d/\partial c < \partial p_u/\partial c$. Hence, if $p_d(0) < p_u(0)$, then there is no intersection and $p_d(c) < p_u(c)$ for all c. Vice versa, if $p_d(0) > p_u(0)$, then there exists exactly one intersection $c^*(d) > 0$ such that if $c \leq c^*(d)$, then $p_u(c) \leq p_d(c)$; otherwise, if $c > c^*(d)$, then $p_u(c) > p_d(c)$.

As a last step, we characterize the condition under which $p_d(0) < p_u(0)$. Using (10) at c = 0, we know that $p_u(0) \le p_d(0)$ if and only if

$$[p_u(0) - \lambda(p_u(0))][p_u(0) - \Lambda(p_u(0))][p_u(0) - d - \Lambda(p_u(0))]$$
(22)
$$\leq dp_u(0)\lambda(p_u(0))(1 - \Lambda'(p_u(0))).$$

Let Γ denote the difference in RHS and LHS of (22). Check that

$$\frac{\partial^2 \Gamma}{(\partial d)^2} = 2[\Lambda(p) - \lambda(p)\Lambda'(p) - \theta_d \lambda(p)(1 - \lambda'(p))]$$
$$= 2\theta_u \lambda(p)[(1 - \lambda'(p))(1 - \theta_d \lambda'(p)) + \theta_d \lambda(p)\lambda''(p)] > 0$$

for any $\theta_u > 0$; from $1 - \Lambda'(p) > 0$ follows $(1 - \lambda'(p))^2 + \lambda(p)\lambda''(p) > 0$ which implies that the squared bracket is strictly positive. Hence Γ is strictly convex in *d* for any $\theta_u > 0$. As the second derivative is independent of *d*, $\Gamma(d)$ goes to positive infinity as *d* increases. Moreover, we get

$$\Gamma(d=0) = -\theta_d \lambda(p) (1-\lambda'(p)) [\lambda(p)(1-\theta_u)(1-\theta_d\lambda'(p))]^2 < 0.$$

for any $\theta_d > 0$. Hence, there exists a unique $d^* > 0$ such that if $d < d^*$, then $\Gamma(d) < 0$ and $p_d(0) < p_u(0)$. In this case, downstream licensing leads to a lower price for any c > 0. Vice versa, if $d \ge d^*$, then $\Gamma(d) \ge 0$, $p_d(0) \ge p_u(0)$ and $c^*(d) > 0$. QED.

Proof of Proposition 5 and 6

For notational ease, let $\pounds_d(p) = \pounds(0, \bar{r}_d, p)$ and $\pounds_u(p) = \pounds(\bar{r}_u, 0, p)$. Consider the difference in profits for the patent holder at the same downstream price p, that is,

$$\pounds_d(p) - \pounds_u(p) = \frac{[p - c - d - \lambda(p)\Theta(p)][\theta_d(p - \lambda(p)\Theta(p)) - \Theta(p)d]}{(p - \lambda(p)\Theta(p))(p - d - \lambda(p)\Theta(p))}Q(p).$$

This difference has either two or three roots. The first root is implicitly defined by $p_1 - \lambda(p_1)\Theta(p_1) = c + d$. This is the retail price in the absence of any licensing or implementing a rate of 0. At this price, the patent holder makes no profit with either licensing target. The second root is given by

$$p_2 - \lambda(p_2)\Theta(p_2) = \frac{\Theta(p_2)}{\theta_d}d.$$

We get that $\mathcal{L}_u(p) > \mathcal{L}_d(p)$ if $p \in [p_1, p_2]$; otherwise, the opposite profit ranking holds. This interval exists, that is, $p_1 < p_2$, if and only if

$$c < \frac{\Theta(p) - \theta_d}{\theta_d} d = \frac{\theta_u [1 - \theta_d \lambda'(p)]}{\theta_d} d$$

A third root exists if demand has a choice price, $p_3 = \{p | Q(p) = 0\}$.

The optimal licensing choice thus depends on the relative magnitude of p_u , p_d and p_2 . Note that p_u and p_d increase in c while p_2 does not depend on c. Four cases need to be considered:

1. If $\max\{p_u, p_d\} \leq p_2$, then the patent holder strictly prefers upstream licensing since

$$\pounds_u(p_u) > \pounds_u(p_d) > \pounds_d(p_d).$$

2. If $p_u < p_2 < p_d$, then upstream licensing dominates if

$$\Delta \equiv \pounds_u(p_u) - \pounds_d(p_d) \ge 0$$

It is easy to verify that $\partial \Delta / \partial c < 0$ if and only if

$$Q(p_u) > Q(p_d) \frac{p_d[p_u - d - \lambda(p)\Theta(p)]}{[p_d - \lambda(p)\Theta(p)][p_u - d - \theta\lambda(p)]}$$

This holds since $p_u < p_d$, $Q(p_u) > Q(p_d)$ and the value of the fraction is

strictly less than 1.

3. If $\min\{p_u, p_d\} \ge p_2$, then the patent holder strictly prefers downstream licensing as

$$\pounds_d(p_d) > \pounds_d(p_u) > \pounds_u(p_u)$$

4. The case where $p_d < p_2 < p_u$ cannot arise. To see this, note that $p_d < p_u$ implies that $\pounds_u(p) > \pounds_d(p)$ for any $p \in (p_d, p_u]$. Now suppose $p_2 > p_d$. Then it must hold that $p_2 > p_u$.

Thus, due to $\partial p_u/\partial c > 0$, $\partial p_d/\partial c > 0$ and $\partial p_2/\partial c = 0$, there exists a unique $c^{**}(d)$ such that if $c \leq c^{**}(d)$, then $\pounds_u(p_u) \geq \pounds_d(p_d)$; otherwise, $\pounds_u(p_u) < \pounds_d(p_d)$. Moreover, the threshold exists when $p_u < p_2 < p_d$ or $c < c^*(d)$. It thus has to hold that $c^{**}(d) < c^*(d)$. A strictly positive $c^{**}(d)$ requires $p_u(0) \leq p_d(0)$. From the proof of Proposition 4, we know $p_u(0) \leq p_d(0)$ if and only if $d \geq d^*$. Since $c^{**}(d) < c^*(d)$ and $c^*(d^*) = 0$, there must exist a $d^{**} > d^*$ such that $c^{**}(d^{**}) = 0$. QED.

Proof of Proposition 7

For the properties of $\hat{r}_d(r_u, p)$ in the main text, check that

$$\frac{d\hat{r}_d(r_u)}{dr_u} = -\frac{c}{(1-r_u)^2(p-\Lambda(p))} < 0, \quad \frac{d^2\hat{r}_d(r_u)}{(dr_u)^2} = -\frac{2c}{(1-r_u)^3(p-\Lambda(p))} < 0.$$

The first-order condition (15) for r_u results from

$$\frac{\partial \pounds_m}{\partial r_u} = \frac{Q(p)}{(1 - r_u)^2 [p - \Lambda(p)]} [d\Lambda(p)(1 - r_u)^2 - \theta_d \lambda(p)(c + d(1 - r_u)^2)] = 0$$

Let $r_u^m(p)$ be the solution to this condition and check that

$$r_u^m(p) = 1 - \frac{c\theta_d\lambda(p)}{\sqrt{cd\theta_d\lambda(p)[\Lambda(p) - \theta_d\lambda(p)]}} \equiv 1 - \frac{c\theta_d\lambda(p)}{V}$$

and

$$\frac{dr_u^m}{dc} = -\frac{1}{2}\sqrt{\frac{\theta_d}{cd\theta_u(1-\theta_d\lambda'(p))}} < 0.$$

Moreover, let $c^m(d)$ be the value of c such that condition (12) holds with equality. It follows that $r_u^m(p) > 0$ if and only if $c < c^m(d)$. From the proof of Proposition 5 we know that at $c = c^m(d)$, $p_1 = p_2 < p_d < p_u$. Hence, it has to hold that $c^m(d) > c^*(d)$.

Next, we show that $r_u^m(p) < \bar{r}_u(p)$ for any $\theta_d > 0$, which implies that the patent holder strictly prefers double-dipping over exclusive upstream licensing unless the downstream segment is competitive. We have

$$\bar{r}_u(p) - r_u^m(p) = \frac{p - \Lambda - c - d}{p - \Lambda - d} - (1 - \frac{c\theta_d\lambda(p)}{V})$$
$$= \frac{c[\theta_d\lambda(p)(p - \Lambda(p) - d) - V]}{V(p - \Lambda(p) - d)}.$$

The denominator is strictly positive. Using optimal prices

$$p - \Lambda(p) = \frac{c + d(1 - r_u^m(p))}{(1 - r_d)(1 - r_u^m(p))},$$

we get for the numerator

$$\theta_d \lambda(p)(p - \Lambda(p) - d) - V = \frac{r_d}{1 - r_d} c[V + d\theta_d \lambda(p)] > 0$$

for any $r_d > 0$ and $\theta_d > 0$. If $\theta_d = 0$, then $r_u^m(p) = \bar{r}_u(p)$ and double-dipping does not improve on exclusive upstream licensing as derived in section 4.

Let p_m^* and p_d^* denote the optimal price with double-dipping and downstream licensing, respectively. Next we show that $p_m^* < p_d^*$ for any $c < c^m(d)$. The revenue difference between double-dipping and downstream licensing is

$$\begin{aligned} \Delta_D &= \pounds_m(\hat{r}_d(r_u^m(p), p), r_u^m(p), p) - \pounds_m(\hat{r}_d(0, p), 0, p) \\ &= \frac{r_u^m(p)}{(1 - r_u^m(p))(p - \Lambda(p))} [d(1 - r_u^m(p)\Lambda(p)) - (c + d(1 - r_u^m(p)))\theta_d\lambda(p)]Q(p) \\ &\equiv \Psi(p)Q(p) \end{aligned}$$

Check that $\partial \Delta_D / \partial p = \Psi(p)Q'(p) + \Psi'(p)Q(p)$ and

$$\begin{aligned} \frac{\partial^2 \Delta_D}{\partial p \partial c} &= \frac{\partial \Psi(p)}{\partial c} Q'(p) + \frac{\partial \Psi'(p)}{\partial c} Q(p) \\ &= -\frac{r_u^m(p)\theta_d \lambda(p)Q'(p)}{(1 - r_u^m(p))(p - \Lambda(p))} + \frac{r_u^m(p)\theta_d Q(p)[\lambda(p)(1 - \Lambda'(p)) - (p - \Lambda(p))\lambda'(p)]}{(1 - r_u^m(p))(p - \Lambda(p))^2} \\ &= -Q'(p)\frac{r_u^m(p)\theta_d \lambda(p)[(p - \Lambda(p))(1 - \lambda'(p)) + \lambda(p)(1 - \Lambda'(p))]}{(1 - r_u^m(p))(p - \Lambda(p))^2} \ge 0 \end{aligned}$$

for any $r_u^m(p) \ge 0$. At $c = c^m(d)$, it holds that $r_u^m(p) = 0$ which implies

 $\partial p_m^*/\partial c = \partial p_d^*/\partial c$ and $p_m^* = p_d^*$. For any $c < c^m(d)$, we have $\partial p_m^*/\partial c > \partial p_d^*/\partial c > 0$ and it follows that $p_m^* < p_d^*$. QED.

Appendix B

Neutrality Result with Per-Unit Royalty Rates

We show that we have neutrality results with per-unit royalty rates regardless of market structures in vertical supply chains. Consider the competition parameter approach from Section 6. As before, let $\theta_u \in [0, 1]$ and $\theta_d \in [0, 1]$ be the parameter for the upstream and downstream industry, respectively. Suppose that the patent holder charges per-unit royalty rates of f_u and f_d to the upstream and downstream sector, respectively.

Consider the downstream segment and let $m_d(p) = p - w - d - f_d$ be the margin of this layer. The aggregate downstream profit is $m_d(p)Q(p)$ with a derivative of

$$m'_d(p)Q(p) + m_d(p)Q'(p).$$

Implicitly define the price set by the downstream sector as

$$\frac{m_d(p)}{m'_d(p)} = p - w - d - f_d = \theta_d \lambda(p).$$

Solving for w yields

$$w = \bar{w}(p) = p - \theta_d \lambda(p)) - d - f_d.$$

The upstream segment's margin is then given by $m_u(p) = \bar{w} - c - f_u$ and profits are given by $m_u(p)Q(p)$. The optimal price for the upstream industry is then defined by

$$\frac{m_u(p)}{m'_u(p)} = \frac{[p - \theta_d \lambda(p)) - d - f_d] - c - f_u}{1 - \theta_d \lambda'(p)} = \theta_u \lambda(p).$$

The optimal price induced by the upstream firm, for given license fees (f_u, f_d) , is given by

$$p - \lambda(p)\Theta(p) = c + d + (f_u + f_d)$$

where $\Theta(p)$ is the overall competition index given by

$$\Theta(p) = \theta_d + \theta_u - \lambda'(p)\theta_d\theta_u$$

Thus, the final consumer price depends only on the sum of per-unit royalty rates, $f = f_u + f_d$ and we can express this dependency as p(f). Hence, the patent holder's royalty income can be written as

$$\pounds(f_u, f_d; p) = (f_u + f_d) Q(p) = fQ(p(f))$$

and the patent holder is indifferent with respect to which layer to target. QED.

References

- Anderson, Simon P., Andre de Palma, and Brent Kreider (2001): "Tax incidence in differentiated product oligopoly," *Journal of Public Economics*, 81(2), 173–192.
- [2] Bousquet, Alain. (1998), Helmuth Cremer, Marc Ivaldi and Michel Wolkowicz: "Risk Sharing in Licensing," International Journal of Industrial Organization 16, 535–554.
- [3] Delipalla, Sofia, and Michael Keen (1992): "The comparison between ad valorem and specific taxation under imperfect competition," *Journal* of Public Economics, 49(3), 351–367.
- [4] Gaudin, Germain and Alexander White (2014): "Unit vs. Ad Valorem Taxes under Revenue Maximization" (November 26, 2014). Available at SSRN: https://ssrn.com/abstract=2440708 or http://dx.doi.org/10.2139/ssrn.2440708
- [5] Genesove, David and Wallace P. Mullin 1998: "Testing Static Oligopoly Models: Conduct and Cost in the Sugar Industry, 1890–1914," *RAND Journal of Economics*, pp. 355–77.
- [6] Geradin, Damien and Dimitrios Katsifis: "End-product- vs Component-level Licensing of Standard Essential Patents

in the Internet of Things Context" (May 18, 2021), available at SSRN: https://ssrn.com/abstract=3848532 or http://dx.doi.org/10.2139/ssrn.3848532.

- [7] Geradin, Damien and Layne-Farrar, Anne (2011): "Patent Value Apportionment Rules for Complex, Multi-Patent Products", Santa Clara Computer & High Technology Law Journal, 4(27), pp. 763-792.
- [8] Hedge, Deepak (2014): "Tacit knowledge and the structure of license contracts: evidence from the biomedical industry", Journal of Economics & Management Strategy, 23(3), Fall 2014.
- [9] Johnson, Justin P. (2017): "The Agency Model and MFN Clauses", *The Review of Economic Studies*, Vol. 84, No. 3 (300) (July 2017), pp. 1151-1185.
- [10] Layne-Farrar, Anne and Gerard Llobet and Jorge Padilla (2014), "Patent Licensing in Vertically Disaggregated Industries: The Royalty Allocation Neutrality Principle", *Communications and Strategies* 95(3), pp. 61–84.
- [11] Llobet, Gerard and Damien Neven (2022): "Investment and Patent Licensing in the Value Chain", CEPR Press Discussion Paper No. 17405.
- [12] Llobet, Gerard & Jorge Padilla (2016): "The Optimal Scope of the Royalty Base in Patent Licensing". *Journal of Law and Economics*, pp. 45-73.
- [13] Peitz, Martin, and Markus Reisinger (2014): "Indirect Taxation in Vertical Oligopoly," *Journal of Industrial Economics*, December 2014, pp. 709-755.
- [14] Shapiro, Carl and Keith Waehrer (2023), "Using and Misusing Microeconomics: Federal Trade Commission v. Qualcomm," in John E. Kwoka, Jr., Tommaso M. Valletti, and Lawrence J. White (eds.), ANTITRUST ECONOMICS AT A TIME OF UPHEAVAL: Recent Competition Policy Cases on Two Continents, Competition Policy International, pp. 294-314.

- [15] Spengler, Joseph, J. (1950): "Vertical Integration and Antitrust Policy," *Journal of Political Economy*, Vol. 58, No. 4, pp. 347-352.
- [16] Suits, Daniel B., and Richard A. Musgrave (1953): "Ad Valorem and Unit Taxes Compared," *Quarterly Journal of Economics*, 67(4), 598– 604.
- [17] Weyl, E. Glen and Michal Fabinger (2013): "Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition," *Journal of Political Economy*, Vol. 121, No. 3 (June 2013), pp. 528-583.