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# Equity and Efficiency When Needs Differ

# Abstract

There is no consensus on how to measure social welfare and inequality when households have different needs. As we show, a dilemma emerges between holding households responsible for their needs or compensating them. This dilemma is of first-order importance for social welfare, but generally plays a minor role in the measurement of inequality. To address this impasse, we introduce partial compensation. Our axiomatic characterizations reveal novel families of welfare criteria and, with the extension to multidimensional commodity spaces, provide ready-to-use criteria for the analysis of redistributive policies.

JEL-Codes: D300, D630, I310.

Keywords: household needs, transfer principles, equity-efficiency trade-off, welfare criteria.

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# 1 Introduction

Different households need different levels of income to achieve the same level of wellbeing. In this paper, we re-examine the measurement of social welfare and inequality when the needs of households differ. Our analysis clarifies the role of efficiency and equity considerations, reveals a dilemma between responsibility and compensation, proposes to resolve this dilemma with partial compensation, and shows how these ethical choices translate into welfare criteria.

The importance of heterogeneous needs cannot be stressed enough. These differences are ubiquitous and central to a proper evaluation of redistributive policies. Arrow (1973, p.254) brings the example of a haemophiliac who needs about \$4,000 worth per annum of coagulant therapy to arrive at a state of security from bleeding at all comparable to that of the normal person. Sen (1982, p.366-367) concludes that need differences of which hard cases are just extreme examples are pervasive, and they deserve a more central place in a theory of justice. In this paper, we take up this challenge regardless of whether needs arise due to household size, composition, characteristics of its members such as age, health, disability, etc.

To illustrate the difficulties addressed here, suppose that a couple needs 50% more income than a single person to be equally well off. Then, with a household income of \$60,000, each member of the couple enjoys the same *equivalent consumption* as a single household with an income of \$40,000.<sup>1</sup> Would social welfare increase if the single person had one dollar less and the couple had one dollar more?

<sup>&</sup>lt;sup>1</sup>We use equivalent consumption as the measure of the value of consumption of one individual in terms of a single household. In the example, the equivalence scale is 1.5; thus, the equivalent consumption each member of the couple is obtained by dividing the household income by the equivalence scale. Note that, while we refer to equivalence scales (Pollak & Wales, 1979; Blundell & Lewbel, 1991), switching to indifferences scales (as recently suggested by Browning, Chiappori, & Lewbel, 2013; Chiappori, 2016) does not affect our results: in this paper, we address the measurement of social welfare and inequality when households can be readily compared by their equivalent consumptions.

The above transfer has two effects: an inequality effect and a "consumption efficiency" effect. On the one hand, the transfer increases the inequality of equivalent consumption between households and thus reduces social welfare. On the other hand, the transfer provides a consumption efficiency gain: since couples are more efficient at transforming household income into equivalent consumption, the total equivalent consumption of households increases. The ethical stance on consumption efficiency is central to the dilemma between responsibility and compensation we identify.

One option is to trade off the inequality and consumption efficiency effects. This is achieved by the *individual-centered utilitarian criterion*, which measures welfare as the sum of a concave transformation of individuals' equivalent consumption. This approach is also consistent with the standard practice of measuring inequality in terms of individuals' equivalent consumption. However, for any finite level of inequality aversion, some income transfer from the single to the equally well-off couple increases social welfare: the consumption efficiency effect dominates the inequality effect and justifies a regressive transfer from a worse-off to a better-off household (Glewwe, 1991).<sup>2</sup>

An alternative is to prioritize income transfers to worse-off households. Thus, welfare cannot increase when money is transferred from a single to a couple with the same equivalent income. The result is that inequality can only be justified when income transfers are costly. As our characterization shows, this is achieved by the *household-centered utilitarian criterion*, which measures welfare as the sum of an appropriately weighted concave transformation of households' equivalent consumption.

In synthesis, the ethical dilemma is whether households should be held re-

<sup>&</sup>lt;sup>2</sup>As Glewwe (1991, p.213) clarifies, this occurs because large households are more efficient at using a given amount of money to raise the welfare levels than smaller households, so that in fact the total (equivalence scale adjusted) income in society changes when money income is shifted from a small household to a larger one.

sponsible or compensated for their different needs.<sup>3</sup> When households are held responsible, equality of equivalent consumption is generally suboptimal (except at the limit of infinite inequality aversion): the consumption efficiency effect obtained by transferring income to a more efficient household dominates the increase in inequality and leads to a greater social welfare.

In contrast, if households are to be compensated for their needs, society is willing to forgo some consumption efficiency. All households are considered entitled to achieve the same equivalent consumption, and the government prioritizes income transfers to households with lower equivalent consumption. Jorgenson and Slesnick (1984, p. 379) argue along these lines: they impose that a transfer of total expenditures from a rich household to a poor household that does not reverse their relative positions in the distribution of total expenditure must increase the level of social welfare and show that well-beings need to be weighted by their equivalence scale (see also Bruno & Habib, 1976; Slesnick, 1994; Ebert, 1997). Intuitively, these weights sterilize the efficiency effect of different household equivalence scales and avoid the regressive income transfers of the individual-centered criteria.

Our first contribution is to clarify the ethical dilemma between compensation and responsibility for needs. The existence of this dilemma is not new in the literature. In the context of income distributions with income-independent equivalence scales, the household and individual-centered criteria where axiomatically characterized by Ebert (1997).<sup>4</sup> We generalize the charactzerization to any type

<sup>&</sup>lt;sup>3</sup>The tension between compensation and responsibility has been mostly highlighted in the multidimensional context of skill-heterogeneity, where individuals have different preferences over consumption and leisure (see Fleurbaey & Maniquet, 2006, 2011), as well as in the literature on inequality of opportunities, where individuals ought to be compensated for circumstances and held accountable for effort variables (see Roemer & Trannoy, 2016).

<sup>&</sup>lt;sup>4</sup>Ebert (1995) adopts a more flexible transfer principle that bounds the weight on households with different needs and highlights the relationship between inequality and welfare. Fleurbaey, Hagneré, and Trannoy (2003) discusses robust welfare comparisons when varying the needs of households. Contributing to the debate about the correct welfare weight of households, Trannoy (2003) clarifies that weighting households differently does not imply their unfair treatment. Finally, Ebert (2010) proposes similar criteria for the measurement of poverty.

of household, to income-dependent equivalence scales, and to multidimensional settings. Moreover, we also uncover the relationship between social welfare and inequality measurement and explain a puzzling result. The attitudes towards responsibility/compensation for needs are of first order importance for social welfare, but have a negligible relevance for inequality. More precisely, we prove that the level of measured inequality is the same when the within-type equivalent-consumption inequality and the mean level of equivalent consumptions are equal across types. This insight is confirmed by our empirical exercise with Norwegian data and in line with earlier literature (see, among others, Coulter, Cowell, & Jenkins, 1992; Jenkins & Cowell, 1994).

Our next contribution is to address the ethical dilemma between compensation and responsibility by introducing and axiomatically characteriting criteria that allow for partial compensation of differences in needs. These criteria bridge the gap between the individual-centered and household-centered criteria. Our result builds on a novel hybrid transfer axiom, according to which the transfer from a better to a worse off household occurs partly in terms of equivalent consumption and partly in terms of income. The balance between these two types of transfers determines the degree of compensation for differences in needs. A higher degree of compensation implies a higher relative priority for households with greater needs.

Finally, the extension of our criteria to multi-commodity settings with heterogeneous preferences contributes to the recent literature on multidimensional social welfare and inequality measures. This setting accommodates the realistic cases where commodities are assigned differently across household members and prepares our results for applications. For example, one can adopt our criteria to study the jointness of income taxation (Kleven, Kreiner, & Saez, 2009; Bierbrauer, Boyer, Peichl, & Weishaar, 2023; Golosov & Krasikov, 2023), the design and targeting of the earned income tax credit (Hansen, 2021), or the provision of public goods and services (Aaberge, Bhuller, Langørgen, & Mogstad, 2010).

The rest of the paper is organized as follows. In Section 2, we present the framework and the different fairness views. In Section 3, we study the simpler case of homothetic social welfare functions and the corresponding measures of inequality, and clarify their properties. In Section 4, we extend the results to non-homothetic criteria and to multi-commodity settings with heterogeneous needs and preferences. Section 5 concludes. All proofs are collected in the appendix.

# 2 Equity as transfer principles

#### 2.1 Model

An economy consists of a finite number of households H with  $|H| \ge 3$ , where each household h consists of  $n_h > 0$  individuals. Let H and I denote the set of households and individuals respectively.

Each household  $h \in H$  is assigned a level of (total) **income**  $y_h > 0$ . A strictly increasing function  $e_h : \mathbb{R}_+ \to \mathbb{R}_+$ , named **equivalence function**, transforms household income  $y_h$  into (per capita) **equivalent consumption** of its members. This function generalizes standard equivalence scales to non-linear transformations.<sup>5</sup> An **income distribution** is a vector  $y = (y_1, \ldots, y_H)$ . Let  $Y \subset \mathbb{R}^H_+$  denote the set of all income distributions.

Social preferences  $\succeq$  are a complete, transitive, and continuous binary relation over *Y*. Thus, social preferences are represented by a social welfare function  $W: Y \to \mathbb{R}_+$ . We restrict our attention to social preferences that are monotonic and separable.<sup>6</sup> These axioms imply that the social welfare function can be writ-

<sup>&</sup>lt;sup>5</sup>The literature has addressed the question of how to identify equivalence scales. See for example, Pollak and Wales (1979), Ebert and Moyes (2003), and Chiappori (2016). Here, we assume away the identification of equivalence scales and, rather, discuss how to accommodate any equivalent function in the measurement of social welfare and inequality.

<sup>&</sup>lt;sup>6</sup>Formally, *monotonicity* requires that for each pair  $y, y' \in Y$ , if y > y' then  $y \succ y'$ . Separability requires that for each pair  $y, y' \in Y$  and each  $h \in H$ ,  $(y_h, y_{-h}) \succeq (y_h, y'_{-h})$  if and only if  $(y'_h, y_{-h}) \succeq (y'_h, y'_{-h})$ .

ten as a sum over household of some measure of their well-being. Formally, there exist real-valued continuous and strictly increasing functions  $(f_1, \ldots, f_H)$  such that  $W = \sum f_h(y_h)$ .

For clarity of exposition, we introduce our main results on a restricted domain and relax these restrictions in Section 4. First, we assume the equivalence functions are income-linear, as commonly adopted in the literature; formally,  $e_h(y_h) = \beta_h y_h$  for some  $\beta_h \in \mathbb{R}_{++}$  for each  $h \in H$ . Second, we impose that social preferences are homothetic, ensuring that inequality aversion is captured by a single parameter; formally for each  $y, y' \in Y$  and each  $\alpha > 0, y \succeq y'$  if and only if  $\alpha y \succeq \alpha y'$ . In Section 4, we also extend our analysis to multidimensional settings with heterogeneous and non-homothetic household preferences.

#### 2.2 The Pigou-Dalton transfer principle

A central tenet of distributive justice is the Pigou-Dalton transfer principle. Arthur Cecil Pigou (1912) and Hugh Dalton (1920) identified a simple thought experiment that reduces inequality and improves social welfare: a non-leaky transfer of income from a richer to a poorer individual. Abiding by this principle, an income distribution (10, 20) has more inequality and less social welfare than an income distribution (12, 18).<sup>7</sup>

**Pigou-Dalton transfer.** For each pair  $y, y' \in Y$ , each pair  $h, k \in H$ , and each  $\varepsilon > 0$ ,  $y \succeq y'$  whenever, ceteris paribus,

$$y'_h + \varepsilon = y_h \le y_k = y'_k - \varepsilon$$

As clarified in the literature, the *Pigou-Dalton transfer* entails controversial implications when applied to households with different composition and needs (see Coulter et al. (1992) and references therein). Assume the household *h* with income

<sup>&</sup>lt;sup>7</sup>Our convention for vector inequalities is as follows: For each  $y, y' \in Y$ ,  $y \ge y'$  means that  $y_h \ge y'_h$  for each  $h \in H$ . Similarly, y > y', means that  $y_h \ge y'_h$  for each  $h \in H$ , with strict inequality for at least one;  $y \gg y'$  means that  $y_h > y'_h$  for each  $h \in H$ .

10 is in perfect health, while the household *k* with income 20 has a health condition that, for simplicity, requires goods that are exactly twice as expensive. Since the *Pigou-Dalton transfer* disregards household needs, it would approve an income transfer from the (income-) richer household to the (income-) poorer household, even if the latter was better-off in terms of equivalent consumption.

This implication is illustrated graphically in Fig. 1. The incomes of households are represented in quadrant 3, where the equal-income locus  $y_h = y_k$  is highlighted. The *Pigou-Dalton transfer* splits this area in two equal parts: if  $y_h > y_k$ , a (small) transfer from household h to household k increases welfare; conversely, when  $y_h < y_k$ , a (small) transfer from household k to household h increases welfare. The latter case is illustrated with distributions y and y'.

The corresponding effect on equivalent consumption is represented in quadrant 1, while the relationship between incomes and equivalent consumption, that is, the functions  $e_h$  and  $e_k$ , are represented in quadrants 2 and 4, respectively. Following the example,  $e_h(y_h) = y_h = 2 \cdot e_k(y_k)$ . Using these transformations, we can see that the transfer of income from k to h corresponds to a specific transfer of equivalent consumption from k to h. This highlights that this is a transfer from a worse-off to a better-off household. More generally, the area of Pigou-Dalton transfers from worse-off to better-off households is the shaded area between the locus of income distributions where  $y_h = y_k$  and the one where  $e_h = e_k$ .

#### **2.3** Fair transfers

Our proposal avoids the unfair implications of the *Pigou-Dalton transfer principle*. The key idea is to identify the donor and receiver of transfers by comparing the equivalent consumptions of households. A progressive transfer is a transfer from a better to a worse off household. The issue is how to define these transfers in light of household differences in needs. Building on Ebert (1997), we consider two options.



Figure 1: Pigou-Dalton transfer

Our first proposal is to impose a transfer of *income* from better-off households to worse-off households.

**Fair income transfer.** For each pair  $y, y' \in Y$ , each pair  $h, k \in H$ , and each  $\varepsilon > 0$ ,  $y \succeq y'$  whenever, ceteris paribus,

$$e_h(y'_h + \varepsilon) = e_h(y_h) \le e_k(y_k) = e_k(y'_k - \varepsilon).$$

Importantly, the transfer  $\varepsilon$  is in terms of household incomes, while the comparison of households is made in terms of households' equivalent consumptions.

Our second proposal is to impose a transfer of *equivalent consumption* from better-off households to worse-off households. Since equivalent consumption is the level achieved by each member of a household, the transfer is balanced by weighting the amounts donated and received by the respective households' sizes.

**Fair (equivalent) consumption transfer.** For each pair  $y, y' \in Y$ , each pair  $h, k \in H$ , and each  $\varepsilon > 0, y \succeq y'$  whenever, ceteris paribus,

$$e_{h}\left(y_{h}'\right) + \frac{\varepsilon}{n_{h}} = e_{h}\left(y_{h}\right) \le e_{k}\left(y_{k}\right) = e_{k}\left(y_{k}'\right) - \frac{\varepsilon}{n_{k}}$$

In contrast to *fair income transfer*, the transfer  $\varepsilon$  is in terms of households' equivalent consumptions. Similarly to *fair income transfer*, the comparison of households is established in terms of their equivalent consumptions. For the example above, households *h* and *k* both consisted of a single individual, that is,  $n_h = n_k = 1$ .

The key difference between these transfers is whether society is entitled to exploit the efficiency gains embedded in households' consumption technology. *Fair income transfer* implies that these efficiency gains should not be exploited. If two households have the same equivalent consumption, a transfer of income does not increase social welfare independently of how differently efficient these households are in transforming income into equivalent consumption.

The opposite viewpoint is expressed by *fair consumption transfer*. If two households have the same equivalent consumption, a transfer of (population-weighted) equivalent consumption does not increase social welfare, independently of the income needed to achieve such consumption transfers. Thus, if a household is more efficient at transforming income into equivalent consumption, it is welfare improving (with the other axioms) to transfer income to more efficient households.<sup>8</sup>

Interestingly, these two axioms are not inconsistent with each other. In fact, the "maximin" social welfare function that maximizes the equivalent consumption of the worst-off household satisfies both axioms. However, these axioms cannot be jointly satisfied by monotonic and continuous social preferences, which are those considered here.

<sup>&</sup>lt;sup>8</sup>The knife-edge case is in the absence of returns to scale and differences in needs/disabilities. This emerges when  $e_h(y_h) = y_h/n_h$  for each household  $h \in H$ . Then, the two axioms are equivalent (and coincide with the *Pigou-Dalton transfer* if all households consist of a single individual).

## **3** Welfare criteria and inequality measures

We next introduce the social welfare functions and inequality measures and discuss how these relate to our transfer axioms. We also illustrate our results with simple examples.

#### 3.1 Individual- and Household-Centered Utilitarianism

Social preferences are **Household-Centered (HC) utilitarian** if these can be expressed as the sum of an inequality averse transformation of each household's equivalent consumption, weighted by an index of household efficiency. Formally, it can be represented by the social welfare function  $W_{HC} : Y \to \mathbb{R}$ , defined by setting, for each  $y \in Y$ ,

$$W_{HC}(y) = \sum_{h \in H} \beta_h^{-1} \frac{(e_h(y_h))^{1-\gamma}}{1-\gamma}$$

where  $\gamma > 0$  captures the degree of inequality aversion. The weight  $\beta_h^{-1}$  is a measure of each household's efficiency. This weight ensures that social welfare increases the most, when income is transferred to a household with lower equivalent consumption.<sup>9</sup> In fact, the social value of assigning a marginal increase of income to household *h* is

$$\frac{\partial W_{HC}(y)}{\partial y_h} = (e_h(y_h))^{-\gamma}.$$

Since marginal welfare change is decreasing with respect to the equivalent consumption,  $W_{HC}$  prioritizes income transfers to the worse-off households.

Social preferences are **Individual-Centered (IC) utilitarian** if these can be expressed as the sum of an inequality averse transformation of each individual's equivalent consumption. Formally, it can be represented by the social welfare

<sup>&</sup>lt;sup>9</sup>The unweighted criterion would emerge when imposing the Pigou-Dalton transfer axiom.

function  $W_{IC}: Y \to \mathbb{R}$ , defined by setting, for each  $y \in Y$ ,

$$W_{IC}(y) = \sum_{h \in H} n_h \frac{(e_h(y_h))^{1-\gamma}}{1-\gamma}$$

where  $\gamma > 0$  captures the degree of inequality aversion. The individual-centered utilitarian criterion prioritizes transfers of equivalent consumption to the worse-off individuals.

The following results characterize these criteria in terms of the above-introduced transfer axioms.

**Proposition 1** With linear equivalent functions, social preferences  $\succeq$  are monotonic, separable, homothetic, and satisfy fair income transfer if and only if these are household-centered utilitarian.

**Proposition 2** With linear equivalent functions, social preferences  $\succeq$  are monotonic, separable, homothetic, and satisfy fair consumption transfer if and only if these are individual-centered utilitarian.

#### Illustration.

To demonstrate how the two criteria differ, we compare their implied marginal welfare weights. The marginal welfare weight of household h is defined as the partial derivative of the social welfare function with respect to household h's income. It measures the priority attributed by the criterion to each household. Suppose the appropriate equivalence scale is the square root of household size, implying that the equivalence function is  $e_h(y_h) = y_h/\sqrt{n_h}$ . This implies the marginal welfare weights are given by,

$$g_h^{IC} \equiv \frac{\partial W_{IC}}{\partial y_h} = n_h \beta_h e_h^{-\gamma} = \sqrt{n_h} e_h^{-\gamma}, \qquad g_h^{HC} \equiv \frac{\partial W_{HC}}{\partial y_h} = e_h^{-\gamma}$$

Hence, for the individual-centered criterion, marginal welfare weights are increasing in the number of household members at a given equivalent consumption, whereas for the household-centered criterion, marginal welfare weights are equal when equivalent consumption levels are the same.

Since the derivatives depend on the inequality aversion, we consider  $\gamma = 1, 2$  separately for both criteria. For each criterion, we calculate the average marginal welfare weight for households of different sizes. For ease of comparison, the marginal welfare weights are normalized by the marginal welfare weight of singles. Formally,

$$\bar{g}_n^k \equiv \frac{\mathbb{E}[g_h^k \mid n_h = n]}{\mathbb{E}[g_h^k \mid n_h = 1]} \text{ for each } k \in \{IC, HC\}.$$

Thus, if the average marginal welfare weight of couples is one-half, giving an additional dollar to a randomly drawn couple is worth half of giving an additional dollar to a randomly drawn single.

To illustrate the differences between the two criteria, we consider an empirical example using the distribution of household after-tax incomes in Norway in 2018. Details about the data can be found in Appendix B.

Figure 2 plots our two criteria' average marginal welfare weights against household size for two different levels of inequality aversion. The blue line (circles) plots the welfare weight for the individual-centered criterion, while the orange one (diamonds) plots them for the household-centered. Finally, the grey (triangle) line plots the group's average equivalent consumption relative to the average equivalent consumption for singles. In terms of equivalent consumption, bigger households are, on average, better off than singles, with three-person households having the highest average.

From panel (a), we see that the planner with an individual-centered criterion is approximately indifferent between giving an additional dollar to couples or singles, although couples, on average, have 45 percent higher equivalent consumption than singles. Moreover, giving an additional dollar to a threeperson household is more valuable to this planner than to the worse-off singles. The household-centered criterion, on the other hand, recommends redistributing from, on average, better-off to worse-off households or from larger to smaller households. The average marginal welfare weights more or less mirror the average equivalent consumption.



Figure 2: Marginal welfare weights.

*Notes:* The figure plots average marginal welfare weights against household size for the two different criteria. The average marginal welfare weights are normalized by the average for singles. The blue line is the individual-centered welfare weights, while the orange line is the household-centered welfare weights. Lastly, the gray line is the average equivalent consumption relative to the average for singles. The underlying distribution is the Norwegian distribution of household incomes in 2018.

With a larger inequality aversion ( $\gamma = 2$ ), plotted in panel (b), also the individualcentered criterion recommends redistributing from 2,3 and 4-person households to singles. This is because, with more inequality aversion, the inequality effect dominates the consumption efficiency effect. However, for 5- and 6-person households, the consumption-efficiency effect still dominates, implying that the criterion favors transfers to bigger households.

#### 3.2 Inequality measures

The individual-centered utilitarian criterion is consistent with the common practice of measuring inequality in terms of the individuals' equivalent consumptions.<sup>10</sup> Welfare can be written in terms of the ordinally equivalent function

$$\tilde{W}_{IC} = \mu_y \cdot \theta_{IC} \cdot (1 - I_{IC}),$$

where  $I_{IC}$  is the Atkinson measure of inequality in terms of equivalent consumption; and  $\theta_{IC} \equiv \mu_e/\mu_y$  is a measure of allocative efficiency given by the ratio between mean per capita equivalent consumption,  $\mu_e$ , and mean (per capita) income,  $\mu_y$ . Formally, the measure of inequality is

$$I_{IC} = 1 - \frac{1}{\mu_e} \left( \frac{1}{N} \sum_{h \in H} n_h(e_h(y_h))^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

This decomposition clarifies that mean income and inequality in equivalent consumption do not provide enough information to assess social welfare in a way that is consistent with the individual-centered utilitarian criterion. In fact, at fixed mean income and fixed level of inequality, when income is redistributed to more effective households,  $\theta_{IC}$  goes up, and social welfare increases.

A similar formula emerges for the household-centered utilitarian criterion. Let the efficiency-weighted mean equivalent consumption be  $\mu_e^\beta \equiv B^{-1} \sum_{h \in H} \beta_h^{-1} e_h(y_h)$ , where  $B \equiv \sum_{h \in H} \beta_h^{-1}$ . Then, this criterion can be written as

$$\tilde{W}_{HC} = \mu_y \cdot \theta_{HC} \cdot (1 - I_{HC}),$$

where  $I_{HC}$  is the Atkinson measure of inequality of efficiency-weighted equivalent consumption; and  $\theta_{HC} \equiv \mu_e^\beta / \mu_y$  is a measure of allocative efficiency given by

<sup>&</sup>lt;sup>10</sup>Here, we generalize Atkinson (1970), which showed there is a simple relationship between measures of welfare and measures of inequality. To account for the rich households heterogeneity considered here, Atkinson's formula needs to be generalized.

the ratio between the efficiency-weighted mean equivalent consumption and the mean income. Formally, the measure of inequality is

$$I_{HC} = 1 - \frac{1}{\mu_e^{\beta}} \left( \frac{1}{B} \sum_{h \in H} \beta_h^{-1} (e_h(y_h))^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

This decomposition clarifies that the household-centered utilitarian criterion is consistent with a measure of inequality that weights households by their efficiency rather than the household size.

Clearly, the measures of inequality  $I_{IC}$  and  $I_{HC}$  are different. However, our empirical exploration suggests these do not significantly differ in the case of Norway.

As we show next, this result can be explained by the "similar" distribution of equivalent consumption within each type of household. Let a type  $t \in T$  be the set of households with the same equivalence function and the same household size.

**Proposition 3** If the within-type inequalities in equivalent consumption and the withintype mean equivalent consumptions are the same, the individual-centered and the householdcentered measures of inequality are the same. Formally, if, for each  $t, t' \in T$ ,  $\mu_e(y_t) = \mu_e(y_{t'})$  and  $I_{IC}(y_t) = I_{IC}(y_{t'})$ , then  $I_{HC} = I_{IC}$ .

#### **Illustration.**

To illustrate the two different inequality measures, we plot the evolution in inequality over time using the same data as above. We choose inequality aversion  $\gamma = 1$  for both measures but note that the patterns are the same with  $\gamma = 2$ . Figure 3, Panel(a), shows that, according to both measures, there has been a slight increase in household inequality from 2006 to 2018. Moreover, it shows that the two measures yield very similar results, both in levels and trends. Proposition 3, together with Figure 3, Panel (b) suggests why this is the case. The proposition states that if within-inequality and average equivalent consumption are constant across household types, the two inequality measures will coincide. Within-inequality, for both measures, is the inequality among households of the same type, as measured by Atkinson's inequality index with the corresponding level of inequality aversion.

As panel (b) shows, there is not much variation in either inequality in or average of equivalent consumption across different household types. Inequality varies from 0.105 for singles to 0.08 among four-person households. In terms of average equivalent consumption, singles are about 50 percent worse off compared to four-person households.







Figure 3: Inequality over time and across household types.

*Notes:* Panel (a) plots the evolution of inequality in the distribution of family income for the two different measures of inequality. Panel (b) plots within-inequality for the different households of the same sizes and their average consumption.

#### 3.3 First-best optimum

We now consider first-best redistribution, where the government optimally allocates income across households at no cost of redistribution. Hence, the government maximizes welfare given that total income stays constant. Each criterion maximizes welfare by redistributing incomes such that social marginal welfare weights are equal for all households, since if not, a welfare improvement could be made by redistributing to households with higher weights.

Since  $g_h^{HC} = e_h^{-\gamma}$ , the household-centered criterion sets equivalent consumption equal for all households. Thus, at the first-best allocation for this criterion, there is no inequality in terms of equivalent consumption.

In contrast, the individual-centered criterion equalizes  $g_h^{IC} = \sqrt{n_h} e_h^{-\gamma}$ . Across same-size households, each household member is assigned the same equivalent consumption. However, across households with different sizes, equivalent consumption differs. The reason is that this criterion exploits efficiency gains due to scale. For example, when larger households require a smaller per-capita income for the same equivalent consumption, the criterion chooses to provide larger households with relatively more income to exploit the efficiency gains, up to the point where the above weights are equalized across households.

The following equation shows how equivalent consumption varies with household size, in relative terms to the equivalent consumption of singles, for the individual-centered optimum

$$\frac{e_h^{IC}(n_h)}{e_h^{IC}(1)} = (\sqrt{n_h})^{1/\gamma} \,.$$

If  $\gamma$  is small, the criterion is willing to generate more inequality at the expense of smaller households, in order to increase the total amount of equivalent consumption. As  $\gamma \to +\infty$ , the criterion ensures all households achieve the same amount of equivalent consumption.

#### Illustration.

Using the same Norwegian data as above, Figure 4a shows the relative change in equivalent consumption at the first best for the individual-centered and household-

centered criteria, differentiated by household size. The household-centered criterion would increase equivalent consumption by 30 percent for one-person households at the expense of households with two-five members, in order to equalize equivalent consumption across households. The individual-centered criterion would instead increase equivalent consumption by more than 40 percent for households with five or more members, at the expense of households with two or fewer members. The relative gains are larger for the individual-centered criterion as it exploits the efficiency gains from transferring to larger households.

Figure 4b shows the relative increase in average equivalent consumption in the first-best allocation according to the individual-centered criterion compared to the first-best according to the household-centered criterion. It illustrates that the extent to which the individual-centered criterion exploits efficiency gains by transferring more to larger households depends crucially on the level of inequality aversion. When the inequality aversion is 1, the individual-centered criterion implements an allocation with around 7 percent higher equivalent consumption than what the household-centered criterion would do. This comes at the expense of smaller households, such that in the optimum according to individual-centered utilitarianism, smaller households are worse off than under household-centered utilitarianism. (a) First-best allocations across household (b) Relative efficency gains and inequality sizes for each criterion.

aversion.



Figure 4: First-best allocations.

*Notes:* Panel (a) plots the change in equivalent consumption for each household size compared to the actual distribution for both the individual-centered and householdcentered criterion. Panel (b) plots the gain in average equivalent consumption in the individual-centered first-best allocation compared to the household-centered first-best allocation for different levels of inequality aversion.

#### **Extensions** 4

In this section, we generalize our criteria and relax our simplifying assumptions. First, we characterize a criterion with partial compensation for differences in need by an axiom that relies on a transfer partly in terms of household income and partly in terms of household equivalent consumption. Second, the equivalence function need not be a linear transformation from household income to individual equivalent consumption. This is in line with the empirical evidence that scale effects are generally non-linear (Aaberge & Melby, 1998; Koulovatianos, Schröder, & Schmidt, 2005; Biewen & Juhasz, 2017; Dudel, Garbuszus, & Schmied, 2021). Due to the non-linear equivalence functions, the assessment of income distributions cannot be scale invariant: the ranking of two income distributions might then change when comparing a multiple or fraction of these distributions. Finally, we introduce multidimensional consumption spaces and heterogeneous preferences.

#### 4.1 Partial compensation

Here, we develop criteria that generalize and bridge the gap between the individualand household-centered utilitarian criteria. The idea is that the households can be *partially* compensated for their different equivalence scales. For simplicity, we present this generalization for linear equivalence scales.

We introduce a new transfer axiom whereby transfers take place partly in terms of income and partly in terms of equivalent consumption. This hybrid transfer consists of an income transfer  $\alpha\varepsilon$  and of an equivalent consumption transfer  $(1 - \alpha)\varepsilon/n_h$ , with  $\alpha \in [0, 1]$ . The extent to which the transfer occurs in terms of income rather than equivalent consumption,  $\alpha$ , determines degree of compensation for differences in needs. If  $\alpha = 0$ , the axiom is the same as fair consumption transfer whereas when  $\alpha = 1$ , the axiom is the same as fair income transfer.

**Fair mixed transfer.** There exists  $\alpha \in [0, 1]$ , such that for each pair  $y, y' \in Y$ , each pair  $h, k \in H$ , and each  $\varepsilon > 0, y \succeq y'$ , whenever, ceteris paribus,

$$e_h\left(y_h' + \alpha\varepsilon\right) + (1 - \alpha)\frac{\varepsilon}{n_h} = e_h\left(y_h\right) \le e_k\left(y_k\right) = e_k\left(y_k' - \alpha\varepsilon\right) - (1 - \alpha)\frac{\varepsilon}{n_k}.$$

Social preferences  $\succeq$  are **Partial compensation (PC) utilitarian** if these can be expressed by the social welfare function  $W_{PC} : Y \to \mathbb{R}$ , defined by setting, for each  $y \in Y$ ,

$$W_{PC}(y) = \sum_{h \in H} \left( \alpha \beta_h + \frac{(1-\alpha)}{n_h} \right)^{-1} \frac{(e_h(y_h))^{1-\gamma}}{1-\gamma},$$

where  $\alpha$  captures the degree of compensation for differences in equivalence scales. When  $\alpha = 1$ , there is full compensation (household-centered utilitarianism), and when  $\alpha = 0$ , there is no compensation (individual-centered utilitarianism). The weight  $(\alpha \beta_h + (1 - \alpha) / n_h)^{-1}$  accounts for the extent of compensation for each household's (in)efficiency.

The following result characterizes this family of criteria.

**Proposition 4** With linear equivalent functions, social preferences  $\succeq$  are monotonic, separable, homothetic, and satisfy fair mixed transfer if and only if these are partial compensation utilitarian.

Another interpretation of  $\alpha$  is that for a fixed level of inequality aversion,  $\gamma$ , it determines the equity-efficiency trade-off between higher total equivalent consumption and a more equal distribution of equivalent consumption.

As in Section 3.1, we illustrate the criterion with the marginal welfare weights for the equivalence function  $e_h(y_h) = \beta_h y_h = y_h / \sqrt{n_h}$ . Hence,

$$g_h^{PC} \equiv \frac{\partial W_{PC}}{\partial y_h} = \left(\alpha + \frac{(1-\alpha)}{\sqrt{n_h}}\right)^{-1} e_h^{-\gamma}.$$

The way in which marginal welfare weights vary with household size at the same equivalent consumption,  $e_h = \bar{e}$ , depends on the degree of compensation,  $\alpha$ , such that

$$\frac{\partial g_h^{PC}}{\partial n_h}\bigg|_{e_h=\bar{e}} = \frac{1}{2} \left(\alpha + \frac{(1-\alpha)}{\sqrt{n_h}}\right)^{-2} \frac{(1-\alpha)}{n_h^{3/2}} \bar{e}^{-\gamma}.$$

If  $\alpha = 1$ , weights are constant across households at constant and equal equivalent consumption. Whereas the lower is  $\alpha$ , the higher is the weight on larger households with the same equivalent consumption (as  $1 > 1/\sqrt{n_h}$ ). This establishes that the degree of compensation is a key normative parameter for the extent of redistribution that policymakers will introduce across households with different needs.

#### 4.2 Generalized criteria

Here, we generalize our criteria to address non-linear transformations from household income to individual equivalent income. Social preference  $\succeq$  are *Generalized Household-Centered (GHC) utilitarian* if these can be represented by the function  $W_{HI}: Y \to \mathbb{R}$ , defined by setting, for each  $y \in Y$ ,

$$W_{GHC} = \sum_{h \in \mathcal{H}} \int_0^{y_h} g(e_h(z)) dz$$

where g is a strictly positive, decreasing function of equivalent consumption. Here, the social value of assigning a marginal income to household h is exactly given by

$$\frac{\partial W_{GHC}}{\partial y_h} = g(e_h(y_h)).$$

Since g is decreasing, the GHC utilitarian criterion prioritizes households with lower equivalent consumption.

Social preference  $\succeq$  are *Generalized Individual-Centered (GIC) utilitarian* if these can be represented by the function  $W_{GIC} : Y \to \mathbb{R}$ , defined by setting, for each  $y \in Y$ ,

$$W_{GIC} = \sum_{h \in \mathcal{H}} n_h v(e_h(y_h))$$

where v is a strictly increasing and concave function.

**Proposition 5** Social preferences  $\succeq$  are monotonic, separable, and satisfy fair income transfer if and only if these are generalized household-centered utilitarian.

**Proposition 6** Social preferences  $\succeq$  are monotonic, separable, and satisfy fair consumption transfer if and only if these are generalized individual-centered utilitarian.

#### 4.3 Multidimensional consumption space

Next, we relax the assumption of a single dimensional consumption space. For the sake of example, we focus on the case where each member of the household is assigned a different consumption and labor supply. Household income remains one-dimensional and consists of the sum of the incomes achieved by the household members. The challenge consists in comparing households with heterogenous preferences over different bundles of consumption and labor supply.

Let  $(\mathbf{c}_h, \mathbf{l}_h)$  denote the assignment of consumption and labor of each member of household *h*, where  $\mathbf{c}_h$  and  $\mathbf{l}_h$  denotes the vector of consumption and labor suply allocations in each household, respectively. The preferences of household *h* are represented by a utility function  $u_h(\mathbf{c}_h, \mathbf{l}_h)$ .<sup>11</sup>

Furthermore, the relationship between household income and consumption is described by a household-specific domestic production function. Specifically, for each income  $y_h$  of household h, the consumption vector  $\mathbf{c}_h$  satisfies  $\sum_{i \in h} c_i = P_h(y_h)$ , where  $P_h$  is strictly increasing and continuous.<sup>12</sup>

How can households be compared? In the previous sections, the equivalence function was exogenously determined by income and was identifying the level of equivalent consumption by which households were compared. Here, household income is not a sufficient statistic for how well the household fares: available consumption might be unequally distributed and labor supply might differ. Thus, we need to derive an equivalence function that: *i*) establishes comparability across households; and *ii*) respects heterogeneous preferences.

Our suggestion combines the equally-distributed equivalent introduced in Atkinson (1970) and imposes a household-specific reference vector of labor supply  $\bar{l}_h$  (Berg & Piacquadio, 2023). The first addresses within-household consumption heterogeneity, while the second ensures the respecting of preferences while allowing some flexibility with respect to how much time each member of each type of household should spend working.

For a given bundle  $(\mathbf{c}_h, \mathbf{l}_h)$  assigned to household *h*, let the equally distributed

<sup>&</sup>lt;sup>11</sup>We assume the household utility function fairly aggregates the preferences of its members and, thus, is worth respecting. We leave to future research the debate about how to "correct" for household utilities that reflect inefficient household bargaining or where its members are unfairly treated.

<sup>&</sup>lt;sup>12</sup>The simplifying assumption of an additive domestic production function is without loss of generality. As we shall see, the shape of this function, within the standard assumptions, is qualitatively irrelevant.

level of consumption be  $ede_h(\mathbf{c}_h, \mathbf{l}_h) \equiv k$  whenever  $u_h(\{k\}, \overline{\mathbf{l}}_h) = u_h(\mathbf{c}_h, \mathbf{l}_h)$ . Next, the equivalent function is defined by setting  $e_h(y_h) \equiv ede_h(\mathbf{c}_h, \mathbf{l}_h)$ , where the equally distributed level of consumption consumption is exactly affordable for household income  $y_h$ .

The household- and individual-centered utilitarian criteria for the multidimensional settings are directly obtained by replacing the equivalence function and defining the social preferences in terms of allocations of bundles to households. An allocation is defined as  $a \equiv ((\mathbf{c}_h, \mathbf{l}_h)_{h \in \mathcal{H}})$ . Let A be the set of all allocations and  $\overline{A}$  the subset at which household members achieve the same consumption within household and labor supply is at the reference level.

**Multidimensional fair income transfer.** For each pair  $a, a' \in \overline{A}$ , each pair  $h, k \in H$ , and each  $\varepsilon > 0$ ,  $a \succeq a'$  whenever, ceteris paribus,

$$P_h^{-1}(n_h \cdot c'_h) + \varepsilon = P_h^{-1}(n_h \cdot c_h); P_k^{-1}(n_k \cdot c_k) = P_k^{-1}(n_k \cdot c'_k) - \varepsilon; c_h \le c_k,$$

where  $P_h^{-1}$  and  $P_k^{-1}$  measure the income needed by each household to afford the allocation.

Note that for any allocation  $a \in \overline{A}$  it holds that  $e_h \left( P_h^{-1}(n_h \cdot c_h) \right) = e_h (y_h) = c_h$ . Thus, the condition of the axiom is equivalent to the one-dimensional version of the transfer principle:  $e_h \left( y'_h + \varepsilon \right) = e_h \left( y_h \right) \le e_k \left( y_k \right) = e_k \left( y'_k - \varepsilon \right)$ ,

**Multidimensional fair consumption transfer.** For each pair  $a, a' \in \overline{A}$ , each pair  $h, k \in H$ , and each  $\varepsilon > 0$ ,  $a \succeq a'$  whenever, ceteris paribus,

$$c'_h + \varepsilon = c_h \le c_k = c'_k - \varepsilon.$$

Note that for any allocation  $a \in \overline{A}$  it holds that  $e_h(y_h) = ede_h(\mathbf{c}_h, \mathbf{l}_h) = c_h$ . Thus, the definition is equivalent to the one-dimensional version of the transfer principle:  $e_h(y'_h) + \frac{\varepsilon}{n_h} = e_h(y_h) \le e_k(y_k) = e_k(y'_k) - \frac{\varepsilon}{n_k}$ .

Next, we define social preferences in the multidimensional setting.

Social preference  $\succeq$  are *Multidimensional Household-Centered (MHC) utilitarian* if these can be represented by the function  $W_{MHC} : A \to \mathbb{R}$ , defined by setting, for each  $a \in A$ ,

$$W_{MHC} = \sum_{h \in \mathcal{H}} \int_{0}^{P_{h}^{-1}(n_{h}ede_{h}(\mathbf{c}_{h},\mathbf{l}_{h}))} g(e_{h}(z))dz$$

where *g* is a strictly positive, decreasing function of equivalent consumption.

Social preference  $\succeq$  are *Multidimensional Individual-Centered (MIC) utilitarian* if these can be represented by the function  $W_{MIC} : A \to \mathbb{R}$ , defined by setting, for each  $a \in A$ ,

$$W_{MIC} = \sum_{h \in \mathcal{H}} n_h v(ede_h(\mathbf{c}_h, \mathbf{l}_h))$$

where v is a strictly increasing and concave function.

The difference with respect to the one-dimensional setting is immediate. For the household-centered criterion, the income needed at the consumption equivalent substitutes the income. For the individual-centered criterion, the equallydistributed equivalent substitutes the consumption equivalent.

The characterization results extend, when Pareto efficiency substitutes monotonicity and the above multidimensional transfer principles are adopted.

**Proposition 7** Social preferences  $\succeq$  are Pareto efficient, separable, and satisfy multidimensional fair income transfer if and only if these are multidimensional householdcentered utilitarian.

**Proposition 8** Social preferences  $\succeq$  are Pareto efficient, separable, and satisfy multidimensional fair consumption transfer if and only if these are multidimensional individualcentered utilitarian.

# 5 Conclusions

This paper considers how society should prioritize households with different needs. Household size, disability, and health status pose serious challenges to existing theories of justice and call into question their use for policy evaluation. We revisit the issue, present a novel ethical dilemma between compensation and responsibility, and axiomatically characterize novel social welfare functions along this tension.

Should the government allow individuals with a food allergies to deduct the extra food expenditures from their taxes? As we show, even if the government prioritizes individuals and households who are worse off, as we assume throughout, the answer is not obvious. In fact, the willingness to prioritize the worse off must generally be confronted with the higher costs of doing so. The cost is higher because creating the same consumption gain for an allergic person requires a larger income transfer than for a non-allergic person. This leads to an ethical dilemma.

One ethical perspective is to give priority to worse-off households if society bears no cost of doing so. This view holds households responsible for their needs. The government balances the trade-off between inequality in household equivalent consumption and the total amount of equivalent consumption enjoyed by society. This perspective is captured by the "individual-centered utilitarian" welfare criteria and is consistent with the standard practice of measuring inequality in terms of the equivalent consumption of individuals. However, these criteria justify regressive (non-leaky) income transfers from worse-off to better-off households.

An alternative ethical perspective is to prioritize worse-off households even if society has to bear the cost of doing so. This view holds that households ought to be compensated for their different needs. As we show, this perspective is captured by the "household-centered utilitarian" welfare criteria and the corresponding inequality measures. While these criteria avoid the regressive transfers by sterilizing the efficiency effect, they justify equality of equivalent consumption regardless of its cost to society.

We address this dilemma by introducing a compromise perspective. We axiomatically characterize a novel family of criteria that allow for partial responsibility and partial compensation for differences in needs. Further research is needed to identify how individuals and policymakers view this dilemma.

Our results shed new light on the measurement of social welfare and inequality when households have different needs. Two further generalizations prepare the criteria for application to policy evaluation. First, we extend the analysis to multi-commodity settings and heterogeneous household preferences. This extension unables the assessment of redistribution with costly income transfers, and the analysis of (joint) labor supply choices, the allocation of public services, and the provision of public goods, in particular with respect to differential effects on household members. Second, we extend our analysis to non-homothetic social preferences. This extension is necessary to capture non-homothetic household preferences naturally emerge in labor supply models (where time is bounded), but are observed quite generally. Non-linear equivalence functions allow capturing changes in household efficiency at different levels of income.

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# A Proofs

#### A.1 **Proof of Proposition 1**

Proposition 1 obtains from the characterization of Proposition 5 when adding homotheticity of social preferences. We start from the generalized household centered social welfare function  $W_{GHC}$ . This function is concave with respect to equivalent consumptions. Since the equivalence functions are linear, social welfare is also concave with respect to household incomes.

Together with continuity, monotonicity, and concavity, homotheticity implies that the social welfare function can be written as (see Katzner, 1970, Thm 2.4.4)  $W_{HC} = \sum hG_h(y_h)$  with

$$G_{h}(y_{h}) = \begin{cases} a_{h} \frac{y_{h}^{1-\gamma}}{1-\gamma} & \gamma \neq 1\\ a_{h} \ln y_{h} & \gamma = 1. \end{cases}$$

The derivative of  $G_h$  is  $G'_h = a_h y_h^{-\gamma}$ . By the functional representation of  $W_{GHC}$ ,  $G'_h = g(e_h(y_h))$ . Since  $e_h = \beta_h y_h$ ,  $a_h = \beta_h^{-\gamma}$ . Thus, the formulation of  $W_{HC}$  follows.

#### A.2 Proof of Proposition 2

Proposition 2 obtains from the characterization of Proposition 6 when adding homotheticity of social preferences.

We start from the generalized individual-centered social welfare function  $W_{GIC}$ . This function is concave with respect to equivalent consumptions. Since the equivalence functions are linear, social welfare is also concave with respect to household incomes.

Together with continuity, monotonicity, and concavity, homotheticity implies that the social welfare function can be written as (see Katzner, 1970, Thm 2.4.4)  $W_{IC} = \sum_{h} G_{h}(y_{h})$  with

$$G_{h}(y_{h}) = \begin{cases} a_{h} \frac{y_{h}^{1-\gamma}}{1-\gamma} & \gamma \neq 1\\ a_{h} \ln y_{h} & \gamma = 1. \end{cases}$$

The two social welfare functions coincide when  $G_h = n_h v(e_h(y_h))$ . Thus,  $a_h = n_h \beta^{1-\gamma}$  and the formulation of  $W_{IC}$  follows.

#### A.3 Proof of Proposition 3

Suppose that equivalence scales only vary by household type. Let t index household type and h index household. There are T types of households,  $N_t$  households of type t, and  $n_t$  members in each household of type t. Then, both inequality measures can be written as

$$I(\omega) = 1 - \frac{\left(\frac{\sum_{t=1}^{T} N_t \omega_t \mathbb{E}[e_{k(h)}^{1-\gamma}|k=t]}{\sum_{t=1}^{T} N_t \omega_t}\right)^{\frac{1}{1-\gamma}}}{\sum_{t=1}^{T} N_t \omega_t \mathbb{E}[e_{k(h)}|k=t]}}{\frac{\sum_{t=1}^{T} N_t \omega_t \mathbb{E}[e_{k(h)}|k=t]}{\sum_{t=1}^{T} N_t \omega_t}}{\frac{\left(\frac{\sum_{t=1}^{T} \omega_t \sum_{h=1}^{N_t} e_{l(h)}^{1-\gamma}}{\sum_{t=1}^{T} N_t \omega_t}\right)^{\frac{1}{1-\gamma}}}{\sum_{t=1}^{T} N_t \omega_t}}$$

with

$$\omega_t = \begin{cases} n_t & \text{if individual-centered} \\ \beta_t^{-1} & \text{if household-centered} \end{cases}$$

Therefore, if there is little variation in  $\mathbb{E}[e_{k(h)} \mid k = t]$  and  $\mathbb{E}[e_{k(h)}^{1-\gamma} \mid k = t]$  across household types, the inequality measure will be insensitive to the weights  $\omega_t$ . In the extreme case, with no variation, the weights drop out.

Let within-type *t* inequality be defined by

$$I_t(\omega) = 1 - \frac{\left(\frac{\sum_{h=1}^{N_t} \omega_t e_{t(h)}^{1-\gamma}}{\sum_{h=1}^{h} \omega_t}\right)^{\frac{1}{1-\gamma}}}{\frac{\sum_{h=1}^{N_t} \omega_h e_{t(h)}}{\sum_{h=1}^{N_t} \omega_h}} = 1 - \frac{\left(N_t^{-1} \sum_{h=1}^{N_t} e_{t(h)}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}{N_t^{-1} \sum_{h=1}^{N_t} e_{t(h)}} \equiv I_t$$

Clearly, the within-inequality does not depend on  $\omega$ .

Now suppose that for each  $t, t' \in T, I_t(\omega) = I_{t'}(\omega)$  and  $N_t^{-1} \sum_{h=1}^{N_t} e_{t(h)} = N_{t'}^{-1} \sum_{h=1}^{N_{t'}} e_{t'(h)}$ . Then,  $N_t^{-1} \sum_{h=1}^{N_t} e_{t(h)}^{1-\gamma} = N_{t'}^{-1} \sum_{h=1}^{N_{t'}} e_{t'(h)}^{1-\gamma}$ . By substituting these equalities into the expression for  $I(\omega)$ , it becomes clear that also overall inequality is independent of  $\omega$ , since  $I(\omega) = I_t$  for each t.

#### A.4 Proof of Proposition 4

( $\Leftarrow$ ). If the equivalence functions are linear and social preferences can be represented by the social welfare function  $W_{PC}$ , these are monotonic, separable, homothetic and satisfy fair mixed transfer.

*Monotonic*. The change in social welfare obtained by marginally increasing the income  $y_h$  of a household  $h \in H$  is

$$\frac{\partial W_{PC}}{\partial y_h} = \left(\alpha\beta_h + \frac{(1-\alpha)}{n_h}\right)^{-1} e_h^{-\gamma}.$$

Since this expression is positive for each  $y_h \ge 0$ , social preferences are monotonic.

Separable. The additive structure of  $W_{PC}$  immediately implies that social preferences are separable. Formally, consider a pair  $y, y' \in Y$  and a household  $h \in H$ , such that  $(y_h, y_{-h}) \succeq (y_h, y'_{-h})$ . Since social preferences are GHC utilitarian,  $W_{GHC}(y_h, y_{-h}) \ge W_{GHC}(y_h, y'_{-h})$ . By the additive structure,  $W_{PC}(y'_h, y_{-h}) \ge$  $W_{PC}(y'_h, y'_{-h})$ . Thus,  $(y'_h, y_{-h}) \succeq (y'_h, y'_{-h})$ .

*Fair mixed transfer*. Consider a pair  $y, y' \in Y$ , a pair  $h, k \in H$ , and some  $\varepsilon > 0$ , such that

$$e_h\left(y_h' + \alpha\varepsilon\right) + (1 - \alpha)\frac{\varepsilon}{n_h} = e_h\left(y_h\right) \le e_k\left(y_k\right) = e_k\left(y_k' - \alpha\varepsilon\right) - (1 - \alpha)\frac{\varepsilon}{n_k}.$$

while the other households are assigned the same income at y and y'. Then,

$$W_{PC}(y) - W_{PC}(y') = \left(\alpha\beta_h + \frac{(1-\alpha)}{n_h}\right)^{-1} \frac{\left(e_h(y'_h + \alpha\varepsilon) + (1-\alpha)\frac{\varepsilon}{n_h}\right)^{1-\gamma}}{1-\gamma} \\ + \left(\alpha\beta_h + \frac{(1-\alpha)}{n_h}\right)^{-1} \frac{\left(e_h(y'_h - \alpha\varepsilon) - (1-\alpha)\frac{\varepsilon}{n_h}\right)^{1-\gamma}}{1-\gamma} \\ - \left(\alpha\beta_h + \frac{(1-\alpha)}{n_h}\right)^{-1} \frac{\left(e_h(y'_h)\right)^{1-\gamma}}{1-\gamma} \\ - \left(\alpha\beta_k + \frac{(1-\alpha)}{n_k}\right)^{-1} \frac{\left(e_k(y'_h)\right)^{1-\gamma}}{1-\gamma}$$

Differentiation with respect to  $\varepsilon$  yields

$$\frac{\partial W_{PC}}{\partial \varepsilon} = e_h(y_h)^{-\gamma} - e_k(y_k)^{-\gamma}.$$

This derivative is non-negative since  $e_h(y_h) \leq e_k(y_k)$ . Thus, the axiom holds.

*Homotheticity.* Note that for any  $\delta > 0$ , social welfare is homothetic of degree 1, that is,

$$W_{PC}(\delta y) = \sum_{h} \left( \alpha \beta_h + \frac{(1-\alpha)}{n_h} \right)^{-1} \frac{(e_h(\delta y_h))^{1-\gamma}}{1-\gamma}$$
$$= \delta^{1-\gamma} W_{PC}(y).$$

Thus, the ranking of any two alternative allocations does not change with their rescaling.

 $(\Longrightarrow)$ . If the equivalence functions are linear and social preferences are monotonic, separable, homothetic and satisfy fair mixed transfer, these can be represented by the social welfare function  $W_{PC}$ .

Monotonicity and separability, together with continuity of social preferences, imply that  $\succeq$  can be represented by  $W = \sum_{h} \tilde{G}_{h}(y_{h})$ , where, for each  $h \in H$ ,  $\tilde{G}_{h}$ is strictly increasing and continuous function. By strict monotonicity,  $\tilde{G}_{h}$  is differentiable almost everywhere. Since  $e_{h}$  is strictly increasing in  $y_{h}$ , we can define, for each h,  $G_{h}(e_{h}(y_{h})) \equiv \tilde{G}_{h}(y_{h})$ , where  $G_{h}$  inherit all the properties of  $\tilde{G}_{h}$ .

Consider a pair  $y, y' \in Y$  at which the social welfare function W is differentiable. Assume for some  $\alpha \in [0, 1]$ ,  $\varepsilon > 0$  and a pair  $h, k \in H$ 

$$e_h\left(y_h'+\alpha\varepsilon\right)+\left(1-\alpha\right)\frac{\varepsilon}{n_h}=e_h\left(y_h\right)\leq e_k\left(y_k\right)=e_k\left(y_k'-\alpha\varepsilon\right)-\left(1-\alpha\right)\frac{\varepsilon}{n_k}.$$

while  $y_j = y'_j$  for each  $j \neq h, k$ . By fair mixed transfer,  $W_{PC}(y) \geq W_{PC}(y')$ . By the

additive representation, this is equivalent to

$$\frac{G_h(e_h(y'_h + \alpha\varepsilon) + (1 - \alpha)\varepsilon/n_h) - G_h(e_h(y'_h))}{\varepsilon} + \frac{G_k(e_k(y'_k - \alpha\varepsilon) - (1 - \alpha)\varepsilon/n_k) - G_k(e_k(y'_k))}{\varepsilon} \ge 0.$$

Taking the limit for  $\varepsilon \to 0$  gives:

$$\frac{\partial G_h}{\partial e_h} \left( \beta_h \alpha + \frac{1 - \alpha}{n_h} \right) \ge \frac{\partial G_k}{\partial e_k} \left( \beta_k \alpha + \frac{1 - \alpha}{n_k} \right), \tag{1}$$

whenever  $e_h(y_h) \le e_k(y_k)$ . This demonstrates that  $G_h$  is concave with respect to  $e_h$ . Since the equivalence functions are linear, social welfare is also concave with respect to household incomes.

Together with continuity, monotonicity, and concavity, homotheticity implies that the social welfare function can be written as (see Katzner, 1970, Thm 2.4.4)  $W_{PC} = \sum_{h} \bar{G}_{h}(y_{h})$ , where

$$\bar{G}_{h}(y_{h}) = \begin{cases} a_{h} \frac{y_{h}^{1-\gamma}}{1-\gamma} & \gamma \neq 1\\ a_{h} \ln y_{h} & \gamma = 1. \end{cases}$$

This implies that  $\overline{G}(y_h) = G_h(e_h(y_h))$ . We can, thus, rewrite the expression for  $G_h$  as

$$G_h(e_h(y_h)) = a_h \frac{y_h^{1-\gamma}}{1-\gamma} = a_h \frac{(e_h(y_h)/\beta_h)^{1-\gamma}}{1-\gamma} = a_h \beta_h^{-(1-\gamma)} \frac{(e_h(y_h))^{1-\gamma}}{1-\gamma}$$

Substituting into eq. (1) yields

$$a_h \beta_h^{-(1-\gamma)} e_h(y_h)^{-\gamma} \left( \beta_h \alpha + \frac{1-\alpha}{n_h} \right) \ge a_k \beta_k^{-(1-\gamma)} e_k(y_k)^{-\gamma} \left( \beta_k \alpha + \frac{1-\alpha}{n_k} \right).$$

This equation must hold with equality whenever  $e_h = e_k$ ; that is, whenever

$$a_h = \delta \beta_h^{1-\gamma} \left( \beta_h \alpha + \frac{1-\alpha}{n_h} \right)^{-1},$$

where  $\delta > 0$  is a ranking-irrelevant constant. By substituting this expression into  $W_{PC}$ , we obtain

$$W_{PC}(y) = \sum_{h} \beta_{h}^{1-\gamma} \left(\beta_{h}\alpha + \frac{1-\alpha}{n_{h}}\right)^{-1} \frac{y_{h}^{1-\gamma}}{1-\gamma} = \sum_{h} \left(\beta_{h}\alpha + \frac{1-\alpha}{n_{h}}\right)^{-1} \frac{e_{h}(y_{h})^{1-\gamma}}{1-\gamma},$$

which completes the proof.

#### A.5 **Proof of Proposition 5**

( $\Leftarrow$ ). If social preferences can be represented by the social welfare function  $W_{GHC}$ , these are monotonic, separable, and satisfy fair income transfer.

*Monotonic*. The change in social welfare obtained by marginally increasing the income  $y_h$  of a household  $h \in H$  is  $g(e_h(y_h))$ . Since the g(z) > for each  $z \ge 0$ , social preferences are monotonic.

Separable. The additive structure of  $W_{GHC}$  immediately implies that social preferences are separable. Formally, consider a pair  $y, y' \in Y$  and a household  $h \in H$ , such that  $(y_h, y_{-h}) \succeq (y_h, y'_{-h})$ . Since social preferences are GHC utilitarian,  $W_{GHC}(y_h, y_{-h}) \ge W_{GHC}(y_h, y'_{-h})$ . By the additive structure,  $W_{GHC}(y'_h, y_{-h}) \ge$  $W_{GHC}(y'_h, y'_{-h})$ . Thus,  $(y'_h, y_{-h}) \succeq (y'_h, y'_{-h})$ .

*Fair income transfer*. Consider a pair  $y, y' \in Y$ , a pair  $h, k \in H$ , and some  $\varepsilon > 0$ , such that

$$e_{h}(y'_{h}+\varepsilon) = e_{h}(y_{h}) \le e_{k}(y_{k}) = e_{k}(y'_{k}-\varepsilon),$$

while the other households are assigned the same income at y and y'. Then,

$$W_{GHC}(y) - W_{GHC}(y') = \int_0^{y_h} g(e_h(z))dz + \int_0^{y_k} g(e_k(z))dz - \int_0^{y'_h} g(e_h(z))dz - \int_0^{y'_h} g(e_k(z))dz$$

By first order approximation and since g is positive valued and decreasing,

$$\int_0^{y_h} g(e_h(z))dz - \int_0^{y'_h} g(e_h(z))dz \ge g(e_h(y_h)) \cdot \varepsilon$$
$$\int_0^{y_k} g(e_k(z))dz - \int_0^{y'_k} g(e_k(z))dz \le g(e_k(y_k)) \cdot \varepsilon.$$

Since  $e_k(y_k) \ge e_h(y_h), W_{GHC}(y) - W_{GHC}(y') \ge 0$  and the axiom holds.

 $(\Longrightarrow)$ . If social preferences are monotonic, separable, and satisfy fair income transfer, these can be represented by the social welfare function  $W_{GHC}$ .

Monotonicity and separability, together with continuity of social preferences, imply that  $\succeq$  can be represented by  $W = \sum_{h} G_{h}(y_{h})$ , where, for each  $h \in H$ ,  $G_{h}$  is strictly increasing and continuous function. By strict monotonicity,  $G_{h}$  is differentiable almost everywhere.

Consider a pair  $y, y' \in Y$  at which the social welfare function W is differentiable. Assume for some  $\varepsilon > 0$  and a pair  $h, k \in H$ 

$$e_{h}\left(y_{h}'+\varepsilon\right)=e_{h}\left(y_{h}\right)\leq e_{k}\left(y_{k}\right)=e_{k}\left(y_{k}'-\varepsilon\right),$$

while  $y_j = y'_j$  for each  $j \neq h, k$ . By *fair income transfer*,  $W(y) \geq W(y')$ . By the additive representation, this is equivalent to

$$\frac{G_h(y_h) - G_h(y_h + \varepsilon)}{\varepsilon} + \frac{G_k(y_k) - G_k(y_k - \varepsilon)}{\varepsilon} \ge 0.$$
 (2)

Taking the limit for  $\varepsilon \to 0$  gives:  $G'_h(y_h) \ge G'_k(y_k)$ , whenever  $e_h(y_h) \le e_k(y_k)$ . For each  $h \in H$ , let  $g_h$  be such that  $g_h(e_h(y_h)) = G'_h(y_h)$ . Since  $G'_h(y_h) = G'_k(y_k)$ when  $e_h(y_h) = e_k(y_k)$ , there exists  $g \equiv g_h = g_k$ . By Eq. (2), g is continuous and decreasing.

Integrating,  $G_h(y_h) = \int_0^{y_h} g(e_h(z))dz$  up to a welfare irrelevant additive constant. Substituting, the representation result holds.

#### A.6 **Proof of Proposition 6**

( $\Leftarrow$ ). If social preferences can be represented by the social welfare function  $W_{GIC}$ , these satisfy the following properties.

*Monotonic*. The change in social welfare obtained by marginally increasing the income  $y_h$  of a household  $h \in H$  is  $n_h v'(e_h(y_h))e'_h(y_h)$ . Since this derivative is positive for each  $y_h > 0$ , social preferences are monotonic.

Separable. The additive structure of  $W_{GIC}$  immediately implies that social preferences are separable. Formally, consider a pair  $y, y' \in Y$  and a household  $h \in H$ , such that  $(y_h, y_{-h}) \succeq (y_h, y'_{-h})$ . Since social preferences are GIC utilitarian,  $W_{GIC}(y_h, y_{-h}) \ge W_{GIC}(y_h, y'_{-h})$ . By the additive structure,  $W_{GIC}(y'_h, y_{-h}) \ge$  $W_{GIC}(y'_h, y'_{-h})$ . Thus,  $(y'_h, y_{-h}) \succeq (y'_h, y'_{-h})$ .

*Fair consumption transfer.* Consider a pair  $y, y' \in Y$ , a pair  $h, k \in H$ , and some  $\varepsilon > 0$ , such that

$$e_h\left(y_h'\right) + \frac{\varepsilon}{n_h} = e_h\left(y_h\right) \le e_k\left(y_k\right) = e_k\left(y_k'\right) - \frac{\varepsilon}{n_k}$$

while the other households are assigned the same income at y and y'. Then,

$$W_{GIC}(y) - W_{GIC}(y') = n_h v(e_h(y_h)) + n_k v(e_k(y_k)) - n_h v(e_h(y'_h)) - n_k v(e_k(y'_k))$$

By first order approximation and concavity of v,

$$n_h v(e_h(y_h)) - n_h v(e_h(y'_h)) \ge v'(e_h(y_h)) \cdot \varepsilon$$

$$n_k v(e_k(y_k)) - n_k v(e_k(y'_k)) \le v'(e_k(y_k)) \cdot \varepsilon.$$

Since  $e_k(y_k) \ge e_h(y_h)$ ,  $W_{GIC}(y) - W_{GIC}(y') \ge 0$  and the axiom holds.

 $(\Longrightarrow)$ . The axiom imply that social preferences can be represented by the GIC utilitarian social welfare function.

Monotonicity and separability, together with continuity of social preferences, imply that  $\succeq$  can be represented by  $W = \sum_h \tilde{f}_h(y_h)$ , where, for each  $h \in H$ ,  $\tilde{f}_h$  is strictly increasing and continuous function. For each  $h \in H$ , define the function  $f_h$  by setting  $f_h(e_h(y_h)) = \tilde{f}_h/n_h$ . Then,  $W = \sum_h n_h f_h(e_h(y_h))$ . Since  $\tilde{f}$  and  $e_h$  are strictly increasing and continuous, so is  $f_h$ .

Consider a pair  $y, y' \in Y$ , a pair  $h, k \in H$ , and some  $\varepsilon > 0$ , such that

$$e_{h}\left(y_{h}'\right) + \frac{\varepsilon}{n_{h}} = e_{h}\left(y_{h}\right) \le e_{k}\left(y_{k}\right) = e_{k}\left(y_{k}'\right) - \frac{\varepsilon}{n_{k}}$$

while the other households are assigned the same income at y and y'. Then,  $W(y) - W(y') \ge 0$  is equivalent to

$$n_h f_h(a) + n_k f_k(b) - n_h f_h(a - \varepsilon/n_h) - n_k f_k(b + \varepsilon/n_k) \ge 0,$$

where  $a \equiv e_h(y_h)$  and  $b \equiv e_k(y_k)$ .

Since this inequality holds for each pair of households, each  $\varepsilon > 0$ , and each a, b such that  $a \leq b$ , it follows that there exists a strictly increasing, continuous, and concave function v such that  $f_h = v + \theta_h$  for each  $h \in H$ , where  $\theta_h$  is an additive (welfare irrelevant) constant (the detailed steps can be found in Piacquadio, 2020). Removing these constants gives  $W_{GIC} \equiv W - \sum_h \theta_h = \sum_h n_h v(e_h(y_h))$ .

#### A.7 Proofs of Proposition 7 and 8

We show the similarity between Proposition 5 and Proposition 7 and between Proposition 6 and Proposition 8.

In Propositions 5 and 6, social preferences satisfy monotonicity with respect to household incomes. In Propositions 7 and 8, this assumption is substituted by Pareto efficiency. However, Pareto efficiency is equivalent to monotonicity with respect to households' income requirement function, defined as

$$Y_h(\mathbf{c}_h, \mathbf{l}_h) = P_h^{-1}(n_h e de_h(\mathbf{c}_h, \mathbf{l}_h)).$$

or monotonicity with respect to households' equally distributed equivalent function  $ede_h(\mathbf{c}_h, \mathbf{l}_h)$ .

Next, Proposition 5 imposes *fair income transfers*. In Proposition 7, the corresponding requirement of *multidimensional fair income transfer* is imposed on the subdomain of allocations  $\overline{A}$  where the income is replaced by the income requirement function. Note that on the subdomain  $\overline{A}$ ,  $Y_h(\mathbf{c}_h, \overline{\mathbf{l}}_h) = P_h^{-1}(n_h c_h)$ .

Similarly, Proposition 6 imposes *fair household transfers*. In Proposition 8, the corresponding requirement of *multidimensional fair consumption transfer* is imposed on the subdomain of allocations  $\bar{A}$  where the equivalent consumption is replaced by the consumption level. Note that on the subdomain  $\bar{A}$ ,  $c_h = ede_h(\mathbf{c}_h, \mathbf{\bar{l}}_h)$ .

Thus, the proofs of Propositions 5 and 6 extend when replacing household income with the income requirement function and the equivalent consumption with the equally distributed equivalent consumption.

# **B** Data

Our empirical analysis is based on administrative data from Statistics Norway. By combining annual tax records with administrative data on registered households, we observe household income and also the number of individuals that live in each household for the universe of Norwegian households. Our income measure includes wage earnings, business- and capital income, and transfers. Taxes and negative transfers are subtracted.

Households with equivalent consumption of less than one basic amount in the national insurance scheme are dropped. One basic amount was approximately 12,000 dollars in 2018. This roughly corresponds to the subsistence rates, representing the minimum financial support needed to cover essential living expenses, such as food, shelter, clothing, and healthcare. Households with reported aftertax income such that their equivalent consumption is this low are likely to live on unreported incomes in addition to the reported ones. While measurement error also matters for higher incomes, it is more problematic at the bottom of the distribution since marginal welfare weights explode as equivalent consumption approaches zero.