

Arango-Castillo, Lenin; Martínez-Ramírez, Francisco J.; Orraca, María José

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Lenin Arango-Castillo
Banco de México

Francisco J. Martínez-Ramírez
Banco de México

María José Orraca
Banco de México

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Univariate Measures of Persistence: A Comparative Analysis

Lenin Arango-Castillo[†]
Banco de México

Francisco J. Martínez-Ramírez[‡]
Banco de México

María José Orraca[§]
Banco de México

Abstract: Persistence is the speed with which a time series returns to its mean after a shock. Although several measures of persistence have been proposed in the literature, when they are empirically applied, the different measures indicate incompatible messages, as they differ both in the level and the implied evolution of persistence. One plausible reason why persistence estimators may differ is the presence of data particularities such as trends, cycles, measurement errors, additive and temporary change outliers, and structural changes. To gauge the usefulness and robustness of different measures of persistence, we compare them in a univariate time series framework using Monte Carlo simulations. We consider nonparametric, semiparametric, and parametric time-domain and frequency-domain persistence estimators and investigate their performance under different anomalies found in practice. Our results indicate that the nonparametric method is, on average, less affected by the different types of time series anomalies analyzed in this work.

Keywords: Persistence, Monte-Carlo simulations, time series

JEL Classification: C15, C53, C22

Resumen: La persistencia es la velocidad con la cual una serie de tiempo regresa a su media luego de un choque. Aunque se han propuesto varias medidas de persistencia en la literatura, cuando se aplican empíricamente las diferentes medidas arrojan mensajes incompatibles, pues difieren tanto en el nivel como en la evolución implícita de la persistencia. Una razón plausible por la cual los estimadores pueden diferir es la presencia de particularidades en los datos, como tendencias, ciclos, errores de medición, observaciones atípicas y cambios estructurales. Para comprender la utilidad y robustez de distintas medidas de persistencia, las comparamos en un marco de series de tiempo univariadas utilizando simulaciones de Monte Carlo. Consideramos estimadores de persistencia no paramétricos, semi paramétricos y paramétricos e investigamos su desempeño bajo distintas particularidades. Nuestros resultados indican que el método no paramétrico es, en promedio, menos afectado por las diferentes particularidades en series de tiempo analizadas en este trabajo.

Palabras Clave: Persistencia, series de tiempo, simulaciones de Monte Carlo

[†] Dirección General de Investigación Económica. Email: larangoc@banxico.org.mx

[‡] Dirección General de Investigación Económica. Email: franciscomr@banxico.org.mx

[§] Dirección General de Investigación Económica. Email: maria.orraca@banxico.org.mx

1 Introduction

The term persistence refers to the extent to which future values of a particular variable are related to past observations of the same variable.¹ From an econometric point of view, the concept of persistence is closely related to the order of integration of a variable (Paya et al. 2007), and it is defined as the speed of a univariate time series to return to its long-run level after a shock (Andrews and Chen 1994; Willis 2003; Marques 2004; Pivetta and Reis 2007).² Different factors could be associated with the persistence of a time series, including the duration (or persistence) of the shock that is affecting the process, if the shock happens in isolation or together with another shock so that non-linearities are induced, and the possibility that internal propagation mechanisms translate serially uncorrelated shocks into highly correlated time series.

Understanding the dynamics of persistence is a crucial issue with significant policy implications in different disciplines since it can play a central role in determining how policy responds to shocks over time (Canarella and Miller 2016). Therefore, to enhance policymaking, it is desirable to have a good measure of persistence that allows for an accurate monitoring of the evolution of the persistence of the time series of interest and a more profound understanding of how shocks affect their underlying stochastic properties. In practice, although there are several estimators of persistence, the estimated persistence may differ substantially depending on the estimator used.

To motivate this claim, table 1 shows three measures of persistence applied to the U.S. monthly-headline inflation rate in four different periods (see the description of these estimators

¹ This is a property of interest for policymakers in different disciplines. For example, relevant questions include “How long is inflation going to remain high after a shock?”, “How long will temperatures stay elevated?”, “Have precipitations permanently decreased in some regions?”, “How fast will rivers return to their normal levels after heavy rain?”

² This should be understood as reduced-form persistence in the economics literature (Fuhrer 2010).

in section 3.2).³ It shows two ways of modeling inflation: short- and long-memory models.⁴ The table illustrates the inconsistency between the messages obtained from three different measures of persistence commonly used in literature. Said inconsistency is not only present in terms of the level of persistence but also in its evolution. As an example of the former, take the period 1950M1-2023M9: the nonparametric measure indicates a medium persistence. In contrast, the parametric and the long-memory semiparametric measures indicate high persistence. The inconsistency does not seem to be a consequence of the atypical evolution of inflation during the pandemic. Suppose we exclude the latter period and focus on three different subperiods. In that case, we observe that the long-memory estimator suggests that persistence was lower between 2010-2019 compared to 2000-2009, while short-term memory models suggest that persistence increased over the period. Moreover, said increase is just marginal according to the nonparametric estimator, while it turned from anti-persistent to persistent according to the parametric one. This provides an illustrative example applied to a macroeconomic time series, but equivalent discrepancies could be observed in different cases that interest other disciplines.

The discrepancy between the estimators of persistence shown in table 1 is not new in the literature, as Dias and Marques (2010) noticed. For instance, the studies of Andrews and Chen (1994) and Pivetta and Reis (2007) have found significant differences between the estimates of the cumulative impulse response (CIR), the half-life, and the largest-root autoregressive estimators of inflation persistence.⁵ According to them, for example, if the persistence of a series is estimated with the sum of autoregressive coefficients ($\hat{\rho}$) and the data is generated by an AR(p) model with $p > 1$ and a linear time trend, the least-squares estimators of the process can exhibit substantial biases, which can be downward and large (Andrews and Chen 1994).

³ We use an inflation example due to the recent importance that this variable has taken worldwide. However, the same can be said by applying the estimators to variables regarding other sciences (see section 2 for a discussion).

⁴ Short-memory models have mainly been used to derive estimators of persistence, especially in economics. Long-memory models prove helpful in many fields of science as a parsimonious way of modeling highly persistent processes.

⁵ While in table 1 we did not use the estimators CIR, the half-life, and the largest-root autoregressive estimators, all of them are related to the parametric estimator ($\hat{\rho}$) also described in section 3.

Table 1: Estimators of Persistence Applied to the U.S. Headline Inflation for Different Periods¹

| Period | Short-memory Models | | Long-memory Model |
|----------------|----------------------------------|-----------------------------|---|
| | Nonparametric ($\hat{\gamma}$) | Parametric ($\hat{\rho}$) | Semiparametric (\hat{d}_{GPH}) |
| 1950M1-2023M9 | 0.712 (0.016) | 0.851 (0.039) | 0.410 (0.150) |
| 1990M1-1999M12 | 0.658 (0.045) | 0.311 (0.168) | 0.408 (0.150) |
| 2000M1-2009M12 | 0.592 (0.044) | -0.074 (0.214) | 0.204 (0.150) |
| 2010M1-2019M12 | 0.675 (0.045) | 0.291 (0.106) | 0.161 (0.150) |

¹ The estimators are described in section 3.2. Standard errors are in parenthesis. The colors indicate low (green), medium (orange), and high (red) inflation persistence for the nonparametric and parametric estimators.

Further, Dias and Marques (2010) notice that if the data is generated with an $AR(p)$ model with $p > 1$, the estimated persistence may diverge depending on the persistence measure used. The difference between estimates stems from the fact that the shape of the impulse-response function of the process varies with a specific combination of the autoregressive coefficients. Thus, if two processes have the same ρ , but the sum of elements (coefficients) are different between the two $AR(p)$ models, the CIR, the largest-root, and the half-life autoregressive estimators of persistence may yield different levels of persistence.⁶

One plausible explanation for the discrepancies in the messages obtained regarding persistence through different estimators is that some require specific time series properties that seldom hold in real data. Moreover, real-time series data may have some particularities (hereafter called contaminations). By contamination, we refer to time series displaying certain “particularities or anomalies”, including measurement errors, outliers, structural changes, seasonal and cyclical patterns, or unit roots. We use the term “contamination” to indicate that we do not observe the time series of interest. We observe the addition of the data of interest plus contamination.⁷

⁶ See table 2 in Dias and Marques (2010).

⁷ The term contamination is used in the literature in the same sense (see Haldrup and Nielsen 2007; McCloskey and Perron 2013; Hou and Perron 2014, for example).

These anomalies could be affecting the accuracy of persistence measurements. However, it is unclear whether precision losses are similar or homogeneous across all measures of persistence or if they are responsible for the discrepancies in the messages obtained. Thus, the researcher may not know which estimator is an appropriate indicator of the “true” persistence of the series of interest. Therefore, the goal of the paper is to understand how different anomalies alter the measurements of persistence and to study if one of the indicators provides a more accurate or robust measurement of persistence on average, despite the possible contamination of the series that is sometimes inevitable when working with real data. This could justify focusing on certain estimators when the different measures indicate incompatible messages. To that end, this paper presents a statistical exercise to compare the performance of different estimators of persistence that are usually found in the literature under contaminated and uncontaminated data. We contribute to the literature by providing researchers with some elements to choose the most appropriate measure of persistence for the specific question they wish to address.

Two words of caution are in order. First, we focus on univariate time-series frameworks and abstract from discussing whether the research question requires a more sophisticated framework.⁸ Second, our starting point is the assumption that researchers are generally interested in the intrinsic properties of the time series they study, so they want to abstract from the impact that the anomalies may have in the time series of interest. This may implicitly assume that these anomalies do not reflect the intrinsic evolution of the latter, or that the evolution of interest is that of the uncontaminated time series. Nonetheless, whether the anomalies are of interest is ultimately up to the researcher and the particular question at hand.⁹

⁸ For example, a structural or multivariate time series framework may be in order. This could be related to the earlier discussion of understanding the behavior of the time series, where the nuances of how shocks affect its behavior may depend on other variables that could also be observable to the researcher and could be used to nurture the analysis.

⁹ As an illustrative example, consider inflation persistence. In this case, there is a question of whether persistence should be calculated over the monthly changes of seasonally adjusted price indices (so that it is estimated for the inflation level) or if it should be calculated for the detrended time series (so that persistence is estimated for the inflation gap). From a purely statistical point of view, the latter would seem appropriate since the researcher is interested in forecasting the stochastic variation of the time series, which would require removing the trend that may be thought of as a deterministic component. One could think of, for example, the long-term convergence of inflation to target when inflation targeting is adopted as a deterministic trend. However, from an economic point of view, removing the trend is not appealing because, intuitively, the existence of a trend in the inflation series indicates persistence, which is precisely what is being studied. Therefore, removing

Although this study focuses on how time-series' contamination may affect the measure of persistence in a univariate time-series framework, we encourage the discussion of whether the anomalies that we highlight are part of the intrinsic process of the time series, that could be of interest for the researcher or relevant for policymaking, or if they are merely contamination that may blur the underlying behavior that reflects the expected evolution of the time series. By stressing the importance of this discussion, we hope to contribute to the literature on the topic by encouraging a more profound understanding of the concept of persistence and drawing attention to the nuances in interpreting this property.

To analyze the differences among estimators of persistence and gauge the robustness of the different measures, in this paper, we perform a statistical exercise that uses Monte Carlo simulations to compare them in univariate time series. Concretely, we consider nonparametric, semiparametric, parametric time-domain and frequency-domain persistence measures and investigate their performance when data is affected by contamination commonly found in practice.

Overall, our results confirm that persistence measures are sensitive to contamination. In particular, if the data-generating process is an autoregressive of order one, AR(1), with parameter ρ and no contamination, the autoregressive estimators, both the sum of autoregressive coefficient ($\hat{\rho}$) and the autoregressive coefficient of order one ($\hat{\rho}_1$), along with the nonparametric

these trends modifies the definition of persistence. Moreover, the mere characterization of trends as deterministic components is questionable, as these trends may reflect stochastic properties of the time series that are desirable to understand. In this view, it would be preferable to focus on the persistence of the level of inflation. However, if a shock induces a positive trend to the inflation series (albeit this trend is temporary), the estimated persistence may be close to 1, which, from a strictly time series perspective, would indicate that the series will not return fast to its mean level. However, in a case such as this one, the policymaker may know (or expect) that the trend only reflects a shock that has temporarily deviated inflation from its mean but is expected to revert as the shock disappears and the economy adjusts. From this perspective, removing the linear trend may be helpful in evaluating the behavior of inflation without considering the structural changes of the time series that are expected to be temporary. Although, to our knowledge, this discussion has not drawn sufficient attention in the literature, some previous work has delved into related discussions. For example, Marques (2004) emphasizes that the mean of inflation should be seen as exogenous to the model and allowed to vary over time. Other previous studies had already touched on this subject, and while they took the mean of inflation as exogenous, they allowed for structural breaks of that level. See, for example, Burdekin and Siklos (1999), Bleaney (2000), Levin and Piger (2004), Marques (2004), and O'Reilly and Whelan (2005). However, a lot of the literature has not focused on the justification of removing or not the trend but has just estimated persistence either on inflation level (Benati 2008; Pivetta and Reis 2007; Gambetti et al. 2008), or inflation gap (Cogley and Sargent 2001, 2005; Cogley and Sbordone 2008), without acknowledging what interpretation should be given to the measures reported.

estimator ($\hat{\gamma}$), perform the best at any true value of ρ .¹⁰ The integer order of integration tests, the Augmented Dickey-Fuller (ADF), and the Kwiatkowski et al. (1992) (KPSS $_{\mu}$), perform as expected and improve their performance with sample size. The long-memory estimators are the worst performers, as they suggest that the data is generated by a long-memory process when this is not the case: the data is generated by an AR(1) (short memory) model.¹¹ In the presence of contaminations, table 2, which summarizes the performance of each measure under different anomalies, indicates that the ADF test is the most effective measure of persistence among all. The bias in the rejection probability of the ADF test is zero for almost all the contaminations studied in this paper. This tells us that even if the data-generating process (DGP) is contaminated, the ADF test correctly identifies that the persistence is not infinite. However, a drawback regarding this measure is that it does not yield an objective, scalar measure of persistence. Thus, it is useless for most applications (see studies in the next section). The nonparametric estimator, $\hat{\gamma}$, according to table 2, is the second best measure of persistence, measured by a lower deviation from the uncontaminated case in most anomalies analyzed in this paper. Thus, the nonparametric estimator is more robust to contaminations than even the most commonly used autoregressive measures, $\hat{\rho}$ and $\hat{\rho}_1$.¹²

When the DGP is a fractionally integrated process with parameter d , FI(d), and the process is not contaminated, only the long-memory persistence measures perform correctly, as they are the only ones that consider the long-memory property of the data. The nonparametric estimator, $\hat{\gamma}$, increases with the value of d , while the autoregressive measures, in contrast to the AR(1) case, fail to estimate the true persistence of the process, especially when the DGP is nonstationary, i.e., the true $d \geq 0.5$. Surprisingly, in the presence of contaminations, the long-memory estimators do not perform well in most situations. If the data belongs to the stationary

¹⁰ See section 3.2 for details on the estimation of all parameters used.

¹¹ This is to be expected as said indicators are built for long-memory processes.

¹² The nonparametric and the long-memory estimators were not designed for the autoregressive process, and we do not know the true value of γ or the long-memory parameter (d) for the AR(1) processes. Therefore, we evaluate their performance using the difference between the estimated value under contamination versus the estimated value under no contamination.

region, the ADF performs as expected and rejects the null of infinite persistence; however, as before, it does not yield a scalar measure of persistence and might not be completely useful. Thus, in the stationarity region, the γ estimator of persistence deviates less from the uncontaminated case, being the most robust estimator. In contrast, when the DGP belongs to the nonstationary mean-reverting region, the autoregressive estimators, $\hat{\rho}$ and $\hat{\rho}_1$, deviate less from the uncontaminated case. However, this is not because they are correctly estimating the persistence, it is because they were made to measure persistence for stationary short-memory processes, so long-memory nonstationary processes lead said estimators to measure persistence as almost infinite and the bias under contamination is small compared to the uncontaminated case. The tests for the integer order of integration perform badly as well: the null hypothesis rejection probability decreases (increases) for the ADF (KPSS $_{\mu}$) test, the greater the true d is, leading to erroneous conclusions of infinite persistence. Regarding the long-memory and nonparametric estimators of persistence, their performance depends on the type of contamination and the true d , being the γ estimator the one deviating less most of the times from the uncontaminated case. Therefore, if the DGP is FI(d) and $d \geq 0.5$, the nonparametric estimator $\hat{\gamma}$ or the long-memory estimators are the most robust.

Table 2: Summary of the Performance of Persistence Estimators or Test¹

| Estimator or test | Performance under contamination | | Discussion and recommendation | Ranks among the best for data with | Reference | |
|----------------------------|---------------------------------|--|---|---|---|-------------------------|
| | Short-memory process | Long-memory process | | | | |
| Autoregressive | $\hat{\rho}_1$ | It performs well on average. Even in contaminated processes, it is not among the worst estimators, and the differences with the estimation without contamination are small. | The differences with the estimation without contamination are small. However, it indicates short memory or infinite persistence. | Very good for short memory process. | A trend and structural changes. | |
| | $\hat{\rho}$ | It underperforms relative to $\hat{\rho}_1$ (except in the case of small measurement errors). | The differences with the estimation without contamination are small. However, it indicates short memory or infinite persistence. | Choose $\hat{\rho}_1$ over $\hat{\rho}$. | Additive outliers and structural changes. | |
| | \widehat{CIR} | It performs poorly under contamination, except under temporary change outliers that do not affect it severely. | High errors under contamination. | Sensitive to contamination. We do not recommend using it if we suspect contamination that cannot be removed. | Never ranks among the best. | Andrews and Chen (1994) |
| | \widehat{LAR} | It performs poorly under contamination except under temporary change outliers that do not affect it severely. | High errors under contamination. | Sensitive to contamination. We do not recommend using it if we suspect contamination that cannot be removed. | Never ranks among the best. | |
| | \widehat{HLC} | Does not perform well under contamination with low values of true autoregressive parameters. A higher true AR(1) parameter improves performance for all contamination processes. | High errors under contamination. | It could be a good alternative if the true process is known to be a short memory process with high autoregressive parameters. | A trend and structural changes. However, only for short-memory processes with a high ρ . | |
| Non-parametric | $\hat{\gamma}$ | Very good performance across types of contamination and level of true autoregressive parameters. | Very good performance across types of contamination and d parameters. | Seems to be the most robust scalar measure of persistence: it performs well across contaminated short- and long-memory processes. We recommend choosing this measure if we wish to have a scalar measure if DGP is unknown, contamination is suspected, and its type is unknown or not easy to remove. | All but temporary change outliers. Marques (2004) | |
| Order of integration tests | ADF test | It ranks as the best in many cases (across contamination types and AR parameter values). | It ranks as the best in many cases (across contamination types and d values). | It is a robust estimator that may be used under different DGPs if only an integral order of integration test is desired. The drawback is that it is not a scalar, so it may not be desirable because it does not indicate the persistence level. | All but structural changes. Kwiatkowski et al. (1992) | |
| | KPSS test | It performs well across contaminated processes and AR parameter values. It is among the best estimators in the presence of cycles or small measurement errors. | It performs relatively well across contaminated processes and levels of d . For higher levels of d , it ranks among the best estimators in the presence of cycles, small measurement errors, and additive outliers. | It is a robust estimator that may be used under different DGPs if only an integral order of integration test is desired. The drawback is that it is not a scalar, so it may not be desirable because it does not indicate the persistence level. Choose ADF over KPSS, although using both can test for structural changes. | A cycle and measurement errors. | |
| Long-memory | \hat{d}_{GPH} | It does not work for short-memory processes. | It performs well under cycle contaminations. It is the most robust to a wider range of contaminations, although it does not beat the unit root tests. However, it is perhaps an adequate scalar measure. | We strongly recommend its use if the process is long-memory. | A cycle and additive outliers. Geweke and Porter-Hudak (1983) | |
| | \hat{d}_{HP} | It does not work for short-memory processes. | With a trend, under low levels of d , has a good performance. | We recommend its use if the process is long-memory. | A trend and a structural change. Hou and Perron (2014) | |
| | \hat{d}_{LW} | It does not work for short-memory processes. | It is robust to structural changes. | We recommend its use if the process is long-memory. | A structural change and additive outliers, but for low levels of d . Robinson (1995) | |

¹ The estimators ranking displayed in column 5 is a summary of tables 7, 12, B.1, and C.1. See the description and the references of these estimators in section 3.2.

In an effort to facilitate the interpretation of the results presented and provide researchers and policymakers with a tool that allows to apply our findings in practice, table 2 summarizes, for each measure of persistence analyzed, its performance under the different anomalies studied when the underlying time series is a short or long-memory process. The last column of the table provides our recommendation based on the statistical exercise presented in this work. The purpose of this table is to provide guidelines, based on our results, on the estimator that would be more appropriate for measuring persistence in the presence of different contamination processes. Naturally, this table will be more useful if the researcher has more information about the underlying time series, including the type of anomalies that it displays and its DGP. Unfortunately, having this knowledge is not always easy in practice, so we recommend using this table at the discretion of the researcher, and just as a guideline of what estimator could be more robust to the kind of contamination that the series may display.

The rest of the paper is organized as follows. In section 2 we summarize the existing literature on inflation persistence. In section 3 we describe our methodology and show how the autoregressive and long-memory processes are contaminated. Besides, we describe the measures of persistence that will be compared. Section 4 presents results for short-memory autoregressive processes, and section 5 for the case of long-memory processes. Section 6 discusses our results and concludes.

2 Literature

In a time series context, the term persistence indicates the extent to which future values of a particular variable are related to past observations of the same variable. In other words, the persistence of a univariate time series refers to its tendency to converge slowly towards its long-run level (Paya et al. 2007). In this sense, this property of time series is of great interest to some branches of science, such as physics, hydrology, climatology, geopolitics, finance, and economics.

Some studies have focused on getting a better understanding of the persistence property, either through comparing the effect of temporal aggregation on different measures of persistence (Paya et al. 2007); the implications of noisy data (Haldrup and Nielsen 2007); the robustness of some measures of short-term memory to levels shifts processes and to deterministic time trends (McCloskey and Perron 2013); or the link between outliers, nonlinearities, and the degree of persistence in some data generating processes (Ahmad and Donayre 2016).

Some researchers have instead studied the characteristics of the models used to measure persistence (Hendry 1980; Hendry and Mizon 2000; Clements and Hendry 2001). As the concept of persistence refers to the extent to which future values of a series are related to past observations, it is straightforward to link the persistence of a univariate time series with its forecasting ability. Thus, Castle (2006) reviews, examines, and proposes practical solutions regarding model specification and nonstationarity in a forecasting context, which can, in principle, be applied to measure persistence. Along these lines, in a model specification framework, for instance, Bontemps and Mizon (2003) and Hendry and Krolzig (2005) argue that using information criteria is not enough to select a model, especially as it does not ensure congruence and, consequently, a misspecified model could be selected. Therefore, using only information criteria to fit a model to measure persistence could lead to erroneous conclusions about the in-sample characteristics of the series, provided that the model does not match the data-generating process.

The very interesting mathematical properties of the persistence of a univariate time series presented in the studies above reflect the importance of understanding each of the existing techniques to measure this property and their flaws, especially for its wide applications in different branches of science. For instance, Akgul et al. (2011) use the Hurst exponent (long-memory parameter) to measure network traffic flow, which is fundamental in diagnosing computer networks and potential anomalies. Further, Gil-Alana (2021) estimates a linear time trend in the global mean sea level data, assuming that the errors in the regression model

might be fractionally integrated. He found that estimating the persistence of sea level data by imposing the strong assumption that errors are either $I(0)$ or $I(1)$, leads to a downward bias in the estimated persistence. This study is consistent with other works such as Ercan et al. (2013), which discusses various autoregressive techniques to forecast sea levels in the Caspian Sea and the region of Peninsular Malaysia.

Similar to the previous examples, more hydrology papers measuring persistence for different time series have been published. An excellent example of this is Habib (2020), who uses detrended fluctuation analysis to estimate the long-term memory of 17,359 hydrology series. He concluded, as Gil-Alana (2021) and Ercan et al. (2013), that persistence is an important characteristic to consider when forecasting or explaining the properties of hydrology and climatology series. In a more operational sense, Chaves and Lorena (2019) use the Hurst exponent and the autocorrelation function to analyze the presence of persistent processes in the Descoberto reservoir (Brazil) inflows to assess its operational reliability after an important failure in 2017. In a study regarding the chemical properties of the Arctic Ocean, Abbott et al. (2015) measures the persistence of carbon nutrients exported to the aquatic ecosystems by the collapsing of upland permafrost across the North Slope of Alaska.

Measures of persistence have also been used to estimate temperature readings and their anomalies. For instance, Triacca et al. (2014) use the cumulative impulse response (CIR) to measure the persistence of global temperature, while Gil-Alana et al. (2022), using fractional differentiation, investigates the properties of the temperature and precipitation anomalies in the U.S. Both studies find the presence of persistence in the analyzed time series. Measures such as the coefficient of determination (R^2), reduction of error (RE), and coefficient of error (CE) have been used for paleo climatic research, which uses indirect estimates of past climate for climate reconstruction Macias-Fauria et al. (2012). A seasonal $I(d)$ was proposed by Gil-Alana (2017) to model de El Niño Southern Oscillation in order to understand its persistence, besides using the classic $I(0)$ versus $I(1)$ approach. Similarly, Yaya et al. (2020) measure the

persistence of low air quality in California, U.S., using fractionally integrated models; while Deo and Hurvich (1998), applying the same type of models as Yaya et al. (2020), measure the persistence of globally rising temperatures.

In a different spectrum of science, the persistence property has also found its applications in finance, for instance, for calculating risk management measures such as the value-at-risk (VaR) via long-memory GARCH models (Aloui and Mabrouk 2010). In a different study, Bariviera et al. (2017) uses the Hurst exponent to test the presence of long memory in bitcoin return series. Also, the sum of autoregressive coefficients has been applied to measure the persistence of bank's profit coefficients and to know its determinants by regressing the estimated persistence measure against some macroeconomic covariates (Goddard et al. 2011). Unit root tests have also been proposed, for instance, Yarovaya et al. (2022) use them to test for persistence in the return series of four board classes of financial assets: equity indexes, precious metals, 10-year benchmark bonds, and cryptocurrencies. Deo and Hurvich (1998) consider estimating a linear trend for a time series in the presence of additive long-memory noise with memory parameter and apply it to the S&P 500 return data.

The persistence of different macroeconomic series has also been of great interest in economics. Within this context, scholars have extensively delved into the discussion of inflation persistence. Fuhrer (2010) contributes by providing an extensive discussion regarding the different measures of persistence used in the economics literature for inflation. Davig and Doh (2014) use a Markov-switching time series model using Bayesian methods to measure inflation persistence within different monetary policy regimes in the United States: pre- and post-Volcker disinflation. In a nonparametric fashion, Marques (2004) measures inflation persistence in the euro area and the United States by exploring the close relationship between persistence and mean reversion. Canarella and Miller (2016) detect structural breaks in inflation persistence series for the U.S. and countries that target their inflation levels. Alogoskoufis and Smith (1991) explore the relationship between the persistence of price inflation and exchange-rate

regimes. Utilizing Bayesian methods, Cogley et al. (2010) analyze post-World War II U.S. inflation gap persistence. Furthermore, others have researched the persistence of other indicators such as purchasing power parity (PPP) and GDP. For instance, Murray and Papell (2002) demonstrates that univariate methods offer limited insights into the magnitude of the half-lives of PPP deviations. Cheung and Lai (2000) explore heterogeneity in the persistence of deviations from parity. Conversely, other authors such as Estrada et al. (2015), Gil-Alana et al. (2023), and Hess and Iwata (1997) focus on the persistence of shocks in GDP, while the studies of Capistrán and Ramos-Francia (2009); Noriega and Ramos-Francia (2009); Noriega et al. (2013) are examples of the estimation of inflation persistence for the case of Mexico. Despite the large number of measures assessing the persistence of time series,¹³ the significance of this property in the context of different disciplines, and the apparent variations in the evolution of persistence that are suggested by different measurements, there is, to the best of the authors' knowledge, a notable absence of studies addressing the question of which estimate provides a more appropriate measure. The objective of this paper is to contribute to filling in this gap.

3 Methodology

3.1 The Design of Autoregressive or Long-memory Processes with Contamination

We consider two types of processes: short-memory (autoregressive of order one) and long-memory processes. An *autoregressive model* of order p , abbreviated $AR(p)$, is of the form

$$X_t = \alpha + \rho_1 X_{t-1} + \rho_2 X_{t-2} + \dots + \rho_p X_{t-p} + w_t, \quad (1)$$

where X_t is weakly stationary,¹⁴ α is a constant, $\rho_1, \rho_2, \dots, \rho_p$ are nonzero constants, and w_t is a white noise series with mean zero and variance σ_w^2 , $WN(0, \sigma_w^2)$. The univariate *long-memory*

¹³ See, for example, Burdekin and Siklos (1999), Bleaney (2000), Pivetta and Reis (2007).

¹⁴ See Hamilton (1994, 45) for the definition of weakly stationary.

processes under consideration are *fractionally integrated process* given by

$$\begin{aligned} (1-L)^d Y_t &= \varepsilon_t, & t = 1, 2, \dots, \\ Y_t &= 0, & t \leq 0, \end{aligned} \tag{2}$$

where ε_t is a $WN(0, \sigma_\varepsilon^2)$. We assume that X_t or Y_t are not observable due to data contamination.¹⁵ Instead, we observe a series Z_t defined as the addition of X_t or Y_t plus v_t , with v_t being the error term contaminating the process X_t or the process Y_t .

For the contamination we consider the following noise mechanisms:

1. Linear trend. In this case, $v_t = \beta t$, $t = 1, 2, \dots, T$ and β is a nonzero constant. This type of contamination has been normally addressed in the literature of persistence, for example, in Tsay (1984); Deo and Hurvich (1998); Gil-Alana (2003); Craigmile et al. (2004); Gil-Alana (2005); McCloskey and Perron (2013); Gil-Alana et al. (2020, 2022). We present results for different values of β .
2. Cycles. In this case, $v_t = A \sin(2\pi f t)$, $t = 1, 2, \dots, T$, where A is the amplitude and f the frequency. Akgul et al. (2011) and Yaya et al. (2020) study time series persistence with the presence of cycles or seasonalities in long- and short-memory models, respectively. We present results for different amplitudes, A , and different frequencies, f .
3. A contamination mechanism, used in Haldrup and Nielsen (2007), given by

$$v_t = \frac{\theta}{(1-\alpha L)} \delta_t + \eta_t, \tag{3}$$

where $\eta_t \sim iid(0, \sigma_\eta^2)$ is a measurement error and δ_t is a Bernoulli variable which can take either of the values +1 or -1 with probability $p/2$. Otherwise, the value of δ_t equals

¹⁵ According to Schennach (2016), the statistical analysis of error-contaminated data has a long history, dating back at least to the early days of econometrics, and it remains a fairly active field.

zero. Both η_t and δ_t are independent of w_t and ε_t . It is assumed that $\alpha \leq 1$, with the lag-operator L .

- (a) Simple measurement errors. Set $\theta = 0$, therefore, $v_t = \eta_t$. This type of error has been largely studied in the literature. For short-memory models, see, for example, Ashley and Vaughan (1986); Staudenmayer and Buonaccorsi (2005). For long-memory models, see, for example, Haldrup and Nielsen (2007); Perron and Qu (2010); McCloskey and Perron (2013). In this case, the variable that we do not observe has a well-defined, quantitative meaning (e.g., annual income), but our recorded measure of it may contain an error (e.g., reported income). We present results for different signal-to-noise ratios (σ_η^2/σ_w^2 for short-memory processes or $\sigma_\eta^2/\sigma_\varepsilon^2$ for long-memory processes).
- (b) Additive outliers (AO). Set $\alpha = 0$, therefore, $v_t = \theta\delta_t$. According to Dias and Marques (2010), this type of contamination corresponds to shocks that affect observations in isolation due to some nonrepetitive events (one-off events that cause a spike in the series), which may occur as a result of special events, for example, changes in VAT rates or union strikes. Haldrup and Nielsen (2007) study the effects of AO in estimators of the long-memory parameter used to measure persistence. We present results for different values of θ .
- (c) Temporary change outliers (TC). Set a nonzero α and $|\alpha| < 1$ and $v_t = \theta\delta_t/(1-\alpha L)$. It is not a one-time effect like the additive outlier; the effect of a temporary change outliers will persist, but its effect will die out. The effect of temporary change outliers was introduced and analyzed in Fox (1972); Ih Chang and Chen (1988); Chen and Liu (1993b,a) for short-memory models and in Haldrup and Nielsen (2007) for long-memory models. We present results for different values of θ .
- (d) Structural level shifts (SC). Set $\alpha = 1$, therefore, the outliers have a permanent effect, and the v_t term is the sum of all past outlier shocks to the process, $v_t =$

$\theta\delta_t/(1-L)$. In economics, one particular example of a structural change is a reallocation of labor and other resources across the economy and a structural level shift is a particular case of structural change (Tsay 1988). Tsay (1988) analyzed the effect of structural level shifts on SARIMA models and Haldrup and Nielsen (2007); McCloskey and Perron (2013) on long-memory models.

The contamination mechanism might not be visible using an eyeball test. In figures 1 and A.2, we illustrate the effect that contamination has in a single time series realization for autoregressive of order one with parameter $\rho_1 = 0.5$ and long-memory processes with differencing parameter $d = 0.25$. Figure 1 shows that small measurement errors, additive outliers, and temporary change outliers (their effects are shown in the gray lines in panels c, d and f) have a small effect on the shape of the time series (blue lines). However, a deterministic trend, a cycle, and a structural change (whose effects are displayed in the gray lines in panels a, b, and e) have a visual larger effect on the shape of the uncontaminated time series (blue line). In this paper, we investigate which contamination causes larger effects on the estimators of persistence.

3.2 Measures of Persistence

We compare different estimators of persistence, including tests for unit root and asymptotic weak stationarity, autoregressive measures of persistence, estimators of the long-memory parameter, and a nonparametric measure of persistence. The specific estimators used are detailed below:

Unit root tests and I(0) test. If the time series contains a unit root, its persistence is unquestionably large (infinite), and its variance is unbounded. A series with a unit root has

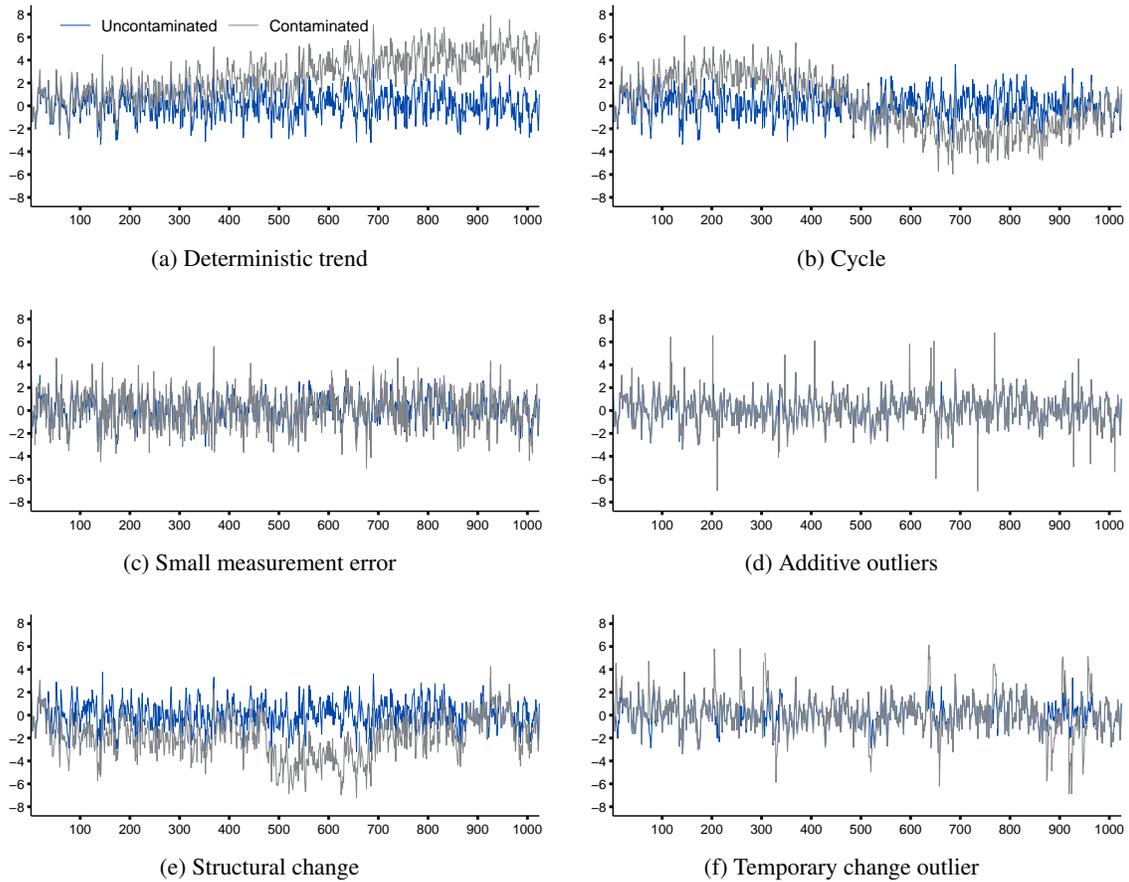


Figure 1: Autoregressive Realization Contamination¹

¹ Each time series has 1,024 observations. For panel (a), the slope of the deterministic trend used is 0.005. For panel (b), the amplitude and the frequency of the cycle are three and 0.001, respectively, while for panel (c), the signal-to-noise ratio of the small measurement error takes the value of one. For panel (d), the outlier size used is five. For panel (e), the probability of a structural change is 0.035, while for panel (f), the persistence of the temporary change takes the value of 0.8.

infinite “memory,” in the sense that a shock in period t influences all periods $t + k$, $k > 0$, which means that any shock to a series with a unit root persists forever. In the literature, Gadea and Mayoral (2006) uses several tests: the Augmented Dickey-Fuller test (Dickey and Fuller 1979, 1981), the Phillips Perron test (Phillips and Perron 1988), and the MZ-GLS test proposed by Ng and Perron (2001) to test for unit root, $I(1)$. They also use the KPSS of Kwiatkowski et al. (1992) to test for asymptotic weak stationarity, $I(0)$. They applied these tests to quarterly inflation rates in 21 OCDE economies, and

they found that for most of the countries, both the I(0) and the I(1) hypotheses are rejected. This result has been interpreted in the literature as an indicator of a behavior midway between the I(0) and the I(1) formulations. Fuhrer (2010) uses the Augmented Dickey-Fuller (ADF) test and the Phillips Perron (PP) test for unit root. He applies the tests to three different series of inflation computed using the Consumer Price Index (CPI), Price Consumer Expenditure Index (PCE), and the Gross Domestic Product (GDP) from the United States. He concludes that the results of the tests are ambiguous: one rejects the null hypothesis of unit root for some series of inflation and for some periods. Lovcha and Perez-Laborda (2018) use the ADF test for I(1) and the KPSS to test for I(0). Both tests are used with the specifications of (i) drift stationary and (ii) trend stationary with drift. They applied the test to United States' PCE quarterly inflation, and they found that the KPSS and ADF tests contradict each other for the pre-Volker period (1954Q3 to 1979Q2), which is often regarded as a symptom of long memory in the data. In this paper we evaluate the performance of the ADF and KPSS tests.

Autoregressive measures. This method assumes that the inflation process, $\{X_t\}$, $t \in \mathbb{Z}$, follows a weakly-stationary autoregressive process of order p , $\text{AR}(p)$, which we write as (1).

The model (1) may be re-parameterized as

$$\Delta X_t = \alpha + \sum_{j=1}^p \delta_j \Delta X_{t-j} + (\rho - 1)X_{t-1} + w_t, \quad (4)$$

where

$$\rho = \sum_{j=1}^p \rho_j, \quad (5)$$

$$\delta_j = - \sum_{i=1+j}^p \rho_i .$$

In the context of model (1), persistence can be defined as the speed at which inflation converges to equilibrium after a shock in the disturbance term: if a shock raises inflation today by 1%, how long does it take for the effect of the shock to die off? The concept of persistence is therefore intimately linked to the impulse response function (IRF) of the AR(p) process. However, the impulse response function is not a useful measure of persistence, as it is an infinite-length vector. Therefore, to overcome this difficulty, several scalar statistics have been proposed in the literature to summarize the information contained in the IRF and serve as a measure of inflation persistence. These include the “sum of the autoregressive coefficients in (5),” the “cumulative impulse response function,” (CIR), the “largest autoregressive root” (LAR), (Stock 1991, 2001), and the “half-life” cycle (HLC). The half-life cycle is the number of periods for which the effect of a unit shock remains above 1/2. For $|\rho| < 1$, a large value of ρ indicates higher persistence (Pivetta and Reis 2007). In this context, X_t is said to be (highly) persistent if, following a shock to the error term, X_t converges slowly to its mean. In other words, persistence and mean reversion are inversely related: high persistence implies low mean reversion and vice versa (Fuhrer 2010).

The cumulative impulse response measure of persistence based on an estimator of ρ is given by

$$\text{CIR} = \frac{1}{1 - \rho} . \tag{6}$$

Note that a large value of ρ implies that the cumulative impact of the shock will be large. Andrews and Chen (1994) argued that the CIR is a good way of summarizing the information contained in the IRF and, thus, a good scalar measure of persistence.

Long-memory parameter. The integration parameter d (Granger 1966; Hosking 1981) has also been used as a measure of persistence (see Hassler and Wolters (1995); Hsu (2005); Zagaglia (2009), for example). The LRD parameter measures the intensity of dependence; higher (lower) values of d indicate higher (lower) dependence. It is desirable to test for stationarity before estimating any LRD parameter. Some work has used the estimated d parameter to test for short-memory ($d = 0$) or unit-root time series ($d = 1$) (see, for example Kim and Phillips (2006) and Phillips (2007)). The long-memory parameter is estimated using three different semiparametric methods: GPH (Geweke and Porter-Hudak 1983), local Whittle (LW) (Robinson 1995), and HP (Hou and Perron 2014). They are semiparametric because they do not make explicit assumptions on the behavior of the autocovariances at short lags or on the spectral density apart from the origin.

These methods are based on the behavior of the spectral density at the origin. Under the assumption that the process $\{Y_t\}$, $t \in \mathbb{Z}$, is stationary, the model near the origin is given by

$$S_X(f) \sim S_*(f)|f|^{-2d} \quad (7)$$

as $f \rightarrow 0$, where $-1/2 < d < 1/2$, and $S_*(f)$ is a function with $S_*(0) = b_f > 0$ (Beran 2013). The model is semiparametric because it treats the short-memory component, $S_*(\cdot)$, nonparametrically (Beran 2013). Among these methods, we use the one proposed by Geweke and Porter-Hudak (1983) (GPH). Taking logarithms on both sides of (7) and evaluating the spectral density function at the Fourier frequency, $f_j = j/N$, we have that

$$\log S_Y(f_j) \approx \log b_f - 2d \log |f_j| + \log \left[\frac{S_*(f_j)}{b_f} \right]. \quad (8)$$

Noting that the logarithm of the periodogram, $\hat{S}_Y^{(p)}(\cdot)$, may be written as

$$\log \hat{S}_Y^{(p)}(f_j) = \log \left[\frac{\hat{S}_Y^{(p)}(f_j)}{S_Y(f_j)} \right] + \log S_Y(f_j), \quad (9)$$

and combining (8) and (9), we have

$$\begin{aligned} \log \hat{S}_Y^{(p)}(f_j) &\approx \log b_f - 2d \log |f_j| + \log \left[\frac{S_*(f_j)}{b_f} \right] + \log \left[\frac{\hat{S}_Y^{(p)}(f_j)}{S_Y(f_j)} \right] \\ &= (\log b_f - C) - 2d \log |f_j| + \log \left[\frac{S_*(f_j)}{b_f} \right] + u_j, \end{aligned}$$

where $u_j = \log \left[\frac{\hat{S}_Y^{(p)}(f_j)}{S_Y(f_j)} \right] + C$ and $C = 0.577216\dots$ is the Euler's constant (Geweke and Porter-Hudak 1983; Hurvich et al. 1998). One can estimate H using the linear regression

$$y_j = \alpha_0 + \alpha_1 \log |f_j| + e_j, \quad (10)$$

$j = 1, \dots, m$, where $y_j = \log \hat{S}_Y^{(p)}(f_j)$, $\alpha_0 = \log b_f - C$, $\alpha_1 = -2d$, and $e_j = \log (S_*(f_j)/b_f) + u_j$. The number m is known as the *bandwidth parameter* (Beran 2013, 442). Geweke and Porter-Hudak (1983) propose the estimator of d given by $\hat{d}_{\text{GPH}} = -\alpha_1/2$ with the bandwidth parameter chosen such that as $m \rightarrow \infty$, $m/T \rightarrow 0$ (Beran 2013, 442). We show in results for $m = \lfloor T^{0.65} \rfloor$. Results from other bandwidth exponents are available from the authors.

We also use the local Whittle estimator, denoted by LW. This estimator only requires that we specify the parametric form of the spectral density around the origin (Künsch 1987). This essentially involves Whittle's estimation of the "model" (7) over frequencies $f_j = j/T$, $j = 1, \dots, m$, where m plays a similar role as in the GPH estimator satisfying, as $T \rightarrow \infty$, $1/m + m/T \rightarrow 0$ (see assumption A4 in Robinson

1995). The local Whittle estimator is denoted by \hat{H}_{LW} and Robinson (1995) shows that $\sqrt{m}(\hat{H}_{\text{LW}} - H) \xrightarrow{d} N(0, 1/4)$.

Finally, we also test the modified local Whittle estimator by Hou and Perron in Hou and Perron (2014), denoted by HP. This estimator has good properties under local contaminations, including processes whose spectral density functions dominate at low frequencies: (i) random level shifts, (ii) deterministic level shifts, and (iii) deterministic trends. The data-generating process is given by

$$Z_t = C + X_t + U_t, \quad (11)$$

where C is a constant, $\{X_t\}$, $t \in \mathbb{Z}$, is a process with memory parameter $H \in [1/2, 1)$, and $\{U_t\}$, $t \in \mathbb{Z}$, is a low-frequency contamination process.

Nonparametric estimator of persistence. Marques (2004) introduced a nonparametric measure of persistence, denoted as γ . The measure, γ , defines persistence as the unconditional probability of a stationary stochastic process $\{X_t\}$, $t \in \mathbb{Z}$, not crossing its mean, μ , in period t . By noticing that $\{X_t\}$ does not cross the mean in period t if and only if $(X_t - \mu)(X_{t-1} - \mu) > 0$, γ is defined as

$$\gamma = \Pr \{ [(X_t - \mu) > 0 \wedge (X_{t-1} - \mu) > 0] \vee [(X_t - \mu) < 0 \wedge (X_{t-1} - \mu) < 0] \}. \quad (12)$$

If a time series converges slowly to its equilibrium level (the mean) after a shock, then such a series, by definition, must exhibit a low level of mean reversion and must cross its mean infrequently. In this case, γ simply measures how infrequently a given stationary process crosses its mean. This estimator requires stationarity and is defined independently of the specific underlying data-generating process. Moreover,

γ has a broader scope than ρ because the latter requires the DGP to follow a pure autoregressive process.

Marques (2004) proposed to estimate γ by

$$\hat{\gamma} = 1 - \frac{n}{T}, \quad (13)$$

where n stands for the number of times the series crosses the mean during the time interval with $T + 1$ observations. Note that γ , by definition, and $\hat{\gamma}$ by construction, are always between zero and one. Values of $\hat{\gamma}$ close to 0.5 signal the absence of any significance persistence, values above 0.5 signal significant persistence, and values below 0.5 signal negative long-run autocorrelation. Being nonparametric, this measure has the advantage of robustness against model misspecifications and outliers in the data, as will become evident below.

4 Analyzing the Performance of Persistence Measures under Contamination for Autoregressive (Short-memory) Processes

In this section, we study the performance of some of the most popular measures of time-series persistence. These measures are (i) the autoregressive measures of persistence ($\hat{\rho}_1$, $\hat{\rho}$, $\widehat{\text{CIR}}$, $\widehat{\text{LR}}$, and $\widehat{\text{HLC}}$); (ii) the tests of the integer order of integration, I(1) or I(0); (iii) the nonparametric estimator ($\hat{\gamma}$); and (iv) three estimators of long-memory (\hat{d}_{GPH} , \hat{d}_{LW} , \hat{d}_{HF}). All the estimators were described in section 3.2.

Before starting with the different types of contamination, we illustrate, using simulations, the performance of these persistence estimators on uncontaminated simulated autoregressive processes of order one (short-memory models). We simulate $n_{\text{sim}} = 1,000$ autoregressive process of size $T = 1,024$; 2,048, and 4,096 with autoregressive parameters of $\rho = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$, and 0.95. We report the mean, bias, and root-

mean-square error (RMSE) for the autoregressive methods, the mean and standard deviation for the nonparametric estimator, and the long-memory-parameter estimators and the corresponding rejection probabilities for the integer order of integration tests. The results are presented in table 3 and can be summarized as follows:

Autoregressive estimators. Table 3 shows the simulation results for five persistence estimators based on autoregressive processes. For $T = 1024$, the estimators $\hat{\rho}_1$ and $\hat{\rho}$ are the best overall regarding bias and RMSE. The cumulative impulse response estimator, $\widehat{\text{CIR}}$, performs well for lower values of ρ , and its bias and RMSE increase with ρ . The largest root of the autoregressive process and the half-life cycle estimators, $\widehat{\text{LR}}$, and $\widehat{\text{HLC}}$, perform well for higher ρ values, and their bias and RMSE decrease with ρ . Table 3 also shows that the performance of these estimators improves with the sample size. If the DGP is an autoregressive process, using the estimators $\hat{\rho}_1$ and ρ is recommended.

I(1) and I(0) tests. Table 4 shows the rejection probabilities for the ADF and KPSS tests for different values of ρ . For $T = 1,024$, the ADF test rejects the null hypothesis of unit root almost for all values of ρ except for $\rho = 0.95$. However, it rejects the null of the unit root for bigger samples for all values of ρ . This test only indicates that the persistence is not infinite and it doesn't provide any scalar measure of persistence. We expect this result because the simulated AR processes are weakly stationary and for higher values of ρ and small samples, the test might perform poorly. For the KPSS, we show results for the test assuming the process is I(0) and trend-I(0), named KPSS_μ and KPSS_τ , respectively. We expect the former to perform better for the simulated AR processes. The KPSS tests reject the null hypothesis of I(0) more often than the nominal significance value of 5% for higher values of ρ . However, its performance improves with the sample size. If the DGP is an autoregressive process, using the

Table 3: Autoregressive Measures of Persistence for Autoregressive Processes of Order One with Parameter ρ^1

| ρ | Stat | $T = 1,024$ | | | | | $T = 2,048$ | | | | | $T = 4,096$ | | | | |
|--------|------|----------------|--------------|-----------------------|----------------|-----------------|----------------|--------------|-----------------------|----------------|-----------------|----------------|--------------|-----------------------|----------------|-----------------|
| | | $\hat{\rho}_1$ | $\hat{\rho}$ | $\widehat{CI\bar{R}}$ | \widehat{LR} | \widehat{HLC} | $\hat{\rho}_1$ | $\hat{\rho}$ | $\widehat{CI\bar{R}}$ | \widehat{LR} | \widehat{HLC} | $\hat{\rho}_1$ | $\hat{\rho}$ | $\widehat{CI\bar{R}}$ | \widehat{LR} | \widehat{HLC} |
| 0.10 | Mean | 0.103 | 0.097 | 1.115 | 10.501 | -7.620 | 0.101 | 0.098 | 1.113 | 10.354 | -7.518 | 0.100 | 0.097 | 1.111 | 10.296 | -7.478 |
| | Bias | 0.003 | -0.003 | 0.004 | 0.501 | -0.347 | 0.001 | -0.002 | 0.002 | 0.354 | -0.245 | 0.000 | -0.003 | 0.000 | 0.296 | -0.205 |
| | RMSE | 0.027 | 0.062 | 0.035 | 2.993 | 2.076 | 0.022 | 0.044 | 0.027 | 2.414 | 1.674 | 0.016 | 0.032 | 0.020 | 1.822 | 1.264 |
| 0.20 | Mean | 0.200 | 0.194 | 1.251 | 5.135 | -3.895 | 0.200 | 0.198 | 1.250 | 5.069 | -3.850 | 0.200 | 0.197 | 1.250 | 5.042 | -3.831 |
| | Bias | 0.000 | -0.006 | 0.001 | 0.135 | -0.094 | 0.000 | -0.002 | 0.000 | 0.069 | -0.048 | 0.000 | -0.003 | 0.000 | 0.042 | -0.029 |
| | RMSE | 0.030 | 0.061 | 0.048 | 0.882 | 0.613 | 0.022 | 0.041 | 0.034 | 0.574 | 0.399 | 0.015 | 0.029 | 0.024 | 0.400 | 0.278 |
| 0.30 | Mean | 0.298 | 0.296 | 1.428 | 3.384 | -2.677 | 0.300 | 0.297 | 1.429 | 3.354 | -2.656 | 0.300 | 0.299 | 1.428 | 3.347 | -2.652 |
| | Bias | -0.002 | -0.004 | -0.001 | 0.050 | -0.035 | 0.000 | -0.003 | 0.000 | 0.021 | -0.015 | 0.000 | -0.001 | 0.000 | 0.014 | -0.010 |
| | RMSE | 0.029 | 0.053 | 0.058 | 0.345 | 0.240 | 0.021 | 0.036 | 0.043 | 0.241 | 0.168 | 0.015 | 0.027 | 0.031 | 0.172 | 0.120 |
| 0.40 | Mean | 0.397 | 0.392 | 1.661 | 2.533 | -2.083 | 0.399 | 0.396 | 1.666 | 2.513 | -2.069 | 0.399 | 0.398 | 1.665 | 2.509 | -2.066 |
| | Bias | -0.003 | -0.008 | -0.005 | 0.033 | -0.023 | -0.001 | -0.004 | -0.001 | 0.013 | -0.009 | -0.001 | -0.002 | -0.002 | 0.009 | -0.006 |
| | RMSE | 0.028 | 0.051 | 0.077 | 0.185 | 0.130 | 0.021 | 0.035 | 0.057 | 0.132 | 0.092 | 0.014 | 0.023 | 0.039 | 0.089 | 0.062 |
| 0.50 | Mean | 0.498 | 0.494 | 1.998 | 2.013 | -1.719 | 0.499 | 0.497 | 1.998 | 2.008 | -1.715 | 0.500 | 0.499 | 2.000 | 2.003 | -1.712 |
| | Bias | -0.002 | -0.006 | -0.002 | 0.013 | -0.009 | -0.001 | -0.003 | -0.002 | 0.008 | -0.006 | 0.000 | -0.001 | 0.000 | 0.003 | -0.002 |
| | RMSE | 0.027 | 0.046 | 0.106 | 0.110 | 0.077 | 0.020 | 0.031 | 0.080 | 0.081 | 0.057 | 0.014 | 0.020 | 0.056 | 0.056 | 0.040 |
| 0.60 | Mean | 0.595 | 0.592 | 2.481 | 1.683 | -1.486 | 0.599 | 0.598 | 2.501 | 1.670 | -1.477 | 0.599 | 0.599 | 2.496 | 1.670 | -1.477 |
| | Bias | -0.005 | -0.008 | -0.019 | 0.016 | -0.011 | -0.001 | -0.002 | 0.001 | 0.003 | -0.002 | -0.001 | -0.001 | -0.004 | 0.003 | -0.002 |
| | RMSE | 0.026 | 0.037 | 0.156 | 0.074 | 0.052 | 0.018 | 0.025 | 0.112 | 0.050 | 0.035 | 0.012 | 0.017 | 0.076 | 0.034 | 0.024 |
| 0.70 | Mean | 0.698 | 0.696 | 3.326 | 1.435 | -1.311 | 0.698 | 0.698 | 3.326 | 1.432 | -1.309 | 0.699 | 0.699 | 3.331 | 1.430 | -1.307 |
| | Bias | -0.002 | -0.004 | -0.007 | 0.006 | -0.004 | -0.002 | -0.002 | -0.008 | 0.004 | -0.003 | -0.001 | -0.001 | -0.003 | 0.002 | -0.001 |
| | RMSE | 0.022 | 0.029 | 0.243 | 0.047 | 0.033 | 0.016 | 0.020 | 0.177 | 0.033 | 0.024 | 0.011 | 0.015 | 0.120 | 0.022 | 0.016 |
| 0.80 | Mean | 0.796 | 0.795 | 4.951 | 1.256 | -1.184 | 0.799 | 0.798 | 4.992 | 1.252 | -1.181 | 0.799 | 0.799 | 4.978 | 1.252 | -1.181 |
| | Bias | -0.004 | -0.005 | -0.049 | 0.006 | -0.005 | -0.001 | -0.002 | -0.008 | 0.002 | -0.002 | -0.001 | -0.001 | -0.022 | 0.002 | -0.002 |
| | RMSE | 0.019 | 0.023 | 0.449 | 0.030 | 0.021 | 0.013 | 0.016 | 0.326 | 0.021 | 0.015 | 0.010 | 0.011 | 0.245 | 0.016 | 0.011 |
| 0.90 | Mean | 0.896 | 0.895 | 9.774 | 1.117 | -1.084 | 0.898 | 0.898 | 9.878 | 1.114 | -1.082 | 0.899 | 0.899 | 9.970 | 1.112 | -1.081 |
| | Bias | -0.004 | -0.005 | -0.226 | 0.006 | -0.004 | -0.002 | -0.002 | -0.122 | 0.003 | -0.002 | -0.001 | -0.001 | -0.030 | 0.001 | -0.001 |
| | RMSE | 0.015 | 0.016 | 1.349 | 0.019 | 0.013 | 0.010 | 0.011 | 0.964 | 0.012 | 0.009 | 0.007 | 0.008 | 0.685 | 0.009 | 0.006 |
| 0.95 | Mean | 0.946 | 0.946 | 19.326 | 1.057 | -1.041 | 0.948 | 0.948 | 19.591 | 1.055 | -1.040 | 0.949 | 0.949 | 19.842 | 1.054 | -1.039 |
| | Bias | -0.004 | -0.004 | -0.674 | 0.004 | -0.003 | -0.002 | -0.002 | -0.409 | 0.002 | -0.002 | -0.001 | -0.001 | -0.158 | 0.001 | -0.001 |
| | RMSE | 0.011 | 0.011 | 3.867 | 0.012 | 0.009 | 0.007 | 0.008 | 2.659 | 0.008 | 0.006 | 0.005 | 0.005 | 1.915 | 0.006 | 0.004 |

¹ Each statistic is estimated based on 1,000 realizations.

ADF test helps the researcher to discard infinite persistence. Rejecting the null of $I(0)$ indicates that the order of integration is nonzero and one should explore non integer integration orders (Lee and Schmidt 1996), in particular, in small samples.

Nonparametric estimator. Table 5 shows the results for the nonparametric estimator. First, notice that for all the values of ρ , we report the mean estimated value and its standard deviation over 1,000 simulations. We were not able to derive the true value of γ . However, because all the simulated AR processes are persistent, we expected the estimated γ to be between 0.5 and 1: lower values of ρ imply lower values γ and higher values of ρ imply higher values of γ . The results of this table show that $\hat{\gamma}$ increases with ρ , which means that the estimator performs in the right direction and

Table 4: Tests of I(1) and I(0) for Autoregressive Processes with Parameter ρ . Rejection Probabilities for Nominal Size of 5% ¹

| ρ | $T = 1,024$ | | | $T = 2,048$ | | | $T = 4,096$ | | |
|--------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|
| | ADF | KPSS $_{\mu}$ | KPSS $_{\tau}$ | ADF | KPSS $_{\mu}$ | KPSS $_{\tau}$ | ADF | KPSS $_{\mu}$ | KPSS $_{\tau}$ |
| 0.10 | 1.000 | 0.045 | 0.048 | 1.000 | 0.053 | 0.044 | 1.000 | 0.035 | 0.037 |
| 0.20 | 1.000 | 0.054 | 0.060 | 1.000 | 0.040 | 0.049 | 1.000 | 0.048 | 0.040 |
| 0.30 | 1.000 | 0.058 | 0.062 | 1.000 | 0.053 | 0.049 | 1.000 | 0.057 | 0.058 |
| 0.40 | 1.000 | 0.043 | 0.050 | 1.000 | 0.047 | 0.055 | 1.000 | 0.050 | 0.064 |
| 0.50 | 1.000 | 0.045 | 0.072 | 1.000 | 0.056 | 0.065 | 1.000 | 0.044 | 0.051 |
| 0.60 | 1.000 | 0.068 | 0.073 | 1.000 | 0.045 | 0.073 | 1.000 | 0.056 | 0.056 |
| 0.70 | 1.000 | 0.073 | 0.080 | 1.000 | 0.069 | 0.071 | 1.000 | 0.061 | 0.077 |
| 0.80 | 1.000 | 0.073 | 0.098 | 1.000 | 0.072 | 0.087 | 1.000 | 0.060 | 0.088 |
| 0.90 | 1.000 | 0.145 | 0.169 | 1.000 | 0.126 | 0.172 | 1.000 | 0.113 | 0.123 |
| 0.95 | 0.996 | 0.264 | 0.419 | 1.000 | 0.253 | 0.348 | 1.000 | 0.201 | 0.319 |

¹ The rejections probabilities were computed based on 1,000 realizations.

also performs similarly for all sample sizes. The improvement of higher sample sizes is only in terms of its variability. We also expected this estimator to be lower than one because other processes are mean-reverting and more persistent than the simulated AR in this exercise. We point out that the results in the table are close to the true values of γ reported in (Dias and Marques 2010, Table 1).¹⁶

Long-memory estimators. Table 6 shows three estimators of the long-memory parameter.

We expect the estimators to perform badly because the DGPs under consideration are short-memory. Similar to the case for the nonparametric estimator, we do not know the true long-memory parameter, d , but we expected the estimators of d to be close to zero for any ρ value, provided that the DGP has short-memory and the estimators are made to detect long-memory in the data. Table 6 shows that for $T = 1,024$, the estimated values of d increase with ρ . The estimated values of d also fall out of the stationarity region for values of $\rho \geq 0.9$, that is, $\hat{d} > 0.5$ and this result does not improve with the sample size. Hence, for the simulated AR processes in consideration, this class of estimators is providing the wrong information suggesting the presence of long-term persistence and nonstationarities for large values of ρ . In conclusion, the long-memory

¹⁶ We did not find in Dias and Marques (2010) any guide to compute these true values of γ for autoregressive processes.

estimators are not recommended to be used as a measure of persistence if the DGP is an autoregressive process with relatively high values of ρ .

Table 5: Nonparametric Estimator of Persistence for Autoregressive Processes of Order One with Parameter ρ^1

| ρ | Statistic | $T = 1,024$ | $T = 2,048$ | $T = 4,096$ |
|--------|-----------|-------------|-------------|-------------|
| 0.1 | Mean | 0.534 | 0.533 | 0.532 |
| | SD | 0.015 | 0.011 | 0.008 |
| 0.2 | Mean | 0.565 | 0.564 | 0.564 |
| | SD | 0.016 | 0.012 | 0.008 |
| 0.3 | Mean | 0.598 | 0.598 | 0.597 |
| | SD | 0.016 | 0.011 | 0.008 |
| 0.4 | Mean | 0.631 | 0.631 | 0.631 |
| | SD | 0.016 | 0.011 | 0.008 |
| 0.5 | Mean | 0.667 | 0.667 | 0.667 |
| | SD | 0.016 | 0.011 | 0.008 |
| 0.6 | Mean | 0.704 | 0.705 | 0.705 |
| | SD | 0.016 | 0.011 | 0.008 |
| 0.7 | Mean | 0.747 | 0.747 | 0.747 |
| | SD | 0.017 | 0.011 | 0.008 |
| 0.8 | Mean | 0.795 | 0.796 | 0.795 |
| | SD | 0.016 | 0.011 | 0.008 |
| 0.9 | Mean | 0.857 | 0.856 | 0.856 |
| | SD | 0.016 | 0.012 | 0.008 |
| 0.95 | Mean | 0.899 | 0.899 | 0.899 |
| | SD | 0.016 | 0.012 | 0.008 |

¹ The estimator and its standard deviation were computed using 1,000 realizations.

4.1 Contaminating Short-memory Processes

In this section, we investigate the effect of the contamination processes described in section 3.1, namely: i) linear trend (Trend), ii) cycles (Cycle), iii) small measurement errors (SME), iv) additive outlier (AO), v) structural change (SC), and vi) temporary change outliers (TCO). For comparison, we include the results for the uncontaminated processes in the column named AR(1). Table 7 shows the results for autoregressive processes for different values of ρ contaminated with the six mechanisms with a given set of parameters (specified in the corresponding columns). Results for other values of ρ are in appendix B, table B.1. We

Table 6: Long-Memory Estimators of Persistence for Autoregressive Processes of Order One with Parameter ρ^1

| ρ | Statistic | $T = 1,024$ | | | $T = 2,048$ | | | $T = 4,096$ | | |
|--------|-----------|------------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|
| | | \hat{d}_{GPH} | \hat{d}_{HP} | \hat{d}_{LW} | \hat{d}_{GPH} | \hat{d}_{HP} | \hat{d}_{LW} | \hat{d}_{GPH} | \hat{d}_{HP} | \hat{d}_{LW} |
| 0.1 | Mean | 0.009 | -0.004 | -0.005 | 0.008 | -0.003 | -0.004 | 0.002 | -0.004 | -0.005 |
| | SD | 0.061 | 0.053 | 0.048 | 0.048 | 0.041 | 0.038 | 0.034 | 0.029 | 0.028 |
| 0.2 | Mean | 0.024 | 0.013 | 0.011 | 0.013 | 0.006 | 0.004 | 0.008 | 0.004 | 0.002 |
| | SD | 0.060 | 0.051 | 0.047 | 0.048 | 0.040 | 0.038 | 0.036 | 0.029 | 0.028 |
| 0.3 | Mean | 0.040 | 0.031 | 0.026 | 0.028 | 0.021 | 0.017 | 0.017 | 0.012 | 0.011 |
| | SD | 0.061 | 0.053 | 0.048 | 0.046 | 0.040 | 0.038 | 0.036 | 0.028 | 0.027 |
| 0.4 | Mean | 0.065 | 0.060 | 0.052 | 0.044 | 0.041 | 0.036 | 0.033 | 0.029 | 0.026 |
| | SD | 0.062 | 0.051 | 0.048 | 0.046 | 0.038 | 0.037 | 0.035 | 0.029 | 0.028 |
| 0.5 | Mean | 0.104 | 0.104 | 0.093 | 0.076 | 0.074 | 0.067 | 0.053 | 0.052 | 0.048 |
| | SD | 0.062 | 0.052 | 0.050 | 0.048 | 0.040 | 0.039 | 0.037 | 0.029 | 0.028 |
| 0.6 | Mean | 0.165 | 0.168 | 0.155 | 0.124 | 0.127 | 0.118 | 0.087 | 0.090 | 0.085 |
| | SD | 0.061 | 0.051 | 0.050 | 0.047 | 0.037 | 0.037 | 0.038 | 0.030 | 0.029 |
| 0.7 | Mean | 0.255 | 0.272 | 0.257 | 0.194 | 0.207 | 0.197 | 0.149 | 0.156 | 0.150 |
| | SD | 0.061 | 0.053 | 0.052 | 0.047 | 0.039 | 0.038 | 0.035 | 0.030 | 0.030 |
| 0.8 | Mean | 0.386 | 0.426 | 0.408 | 0.318 | 0.352 | 0.340 | 0.258 | 0.283 | 0.275 |
| | SD | 0.062 | 0.052 | 0.052 | 0.047 | 0.041 | 0.041 | 0.036 | 0.032 | 0.032 |
| 0.9 | Mean | 0.600 | 0.666 | 0.649 | 0.533 | 0.602 | 0.589 | 0.470 | 0.536 | 0.527 |
| | SD | 0.063 | 0.059 | 0.058 | 0.048 | 0.044 | 0.044 | 0.036 | 0.035 | 0.035 |
| 0.95 | Mean | 0.770 | 0.820 | 0.805 | 0.713 | 0.783 | 0.771 | 0.657 | 0.737 | 0.729 |
| | SD | 0.059 | 0.054 | 0.054 | 0.046 | 0.043 | 0.043 | 0.035 | 0.034 | 0.034 |

¹ The estimator and its standard deviation were computed using 1,000 realizations.

select the set of parameters in table 7 considering that the contamination processes are not detected using an eyeball test. The results are confirmed using different sets of parameters, as shown in appendix B. The results for each contamination mechanism are presented in terms of absolute-value differences between the mean estimator (or rejection probability) under contamination and the mean estimator (or mean rejection probability) under noncontamination: values closer to zero indicate better performance. The results are presented for a sample size of $T = 4,096$, and the means or rejections probabilities were computed using 1,000 simulations.

Trend. Table 7 and table B.1 show the results for a trend value of $\beta = 0.005$ and ten values of ρ . For all values of ρ , the ADF test correctly rejects the null hypotheses of the unit root, and it is therefore considered a robust test under a linear trend. The estimator $\hat{\rho}_1$

is among the three best estimators for all values of ρ , and it is also a robust estimator under the presence of trends. Finally, we find the nonparametric estimator among the three best estimators; however, this estimator does not work correctly for large values of ρ . Figure B.1 shows the effect of a linear trend, under different slopes, for four estimators of persistence: $\hat{\rho}_1$, ρ , γ , and \hat{d}_{GPH} . For the model-specific methods, panels (a) and (b), the figure shows the results for the estimators $\hat{\rho}_1$ and $\hat{\rho}$ as a function of the true value of ρ and the slope values between 0.001 to 0.1. These results indicate that the model-specific estimators are affected, in terms of RMSE, only if the true value of ρ is below 0.7, this suggests that very low slope values of the trend can yield high estimated persistence, when it is not the case. Higher slopes also have a stronger effect, that is, the greater the slope, the greater the estimated persistence. Furthermore, we show the effect of linear trends on two other estimators: the nonparametric (panel (c)) and the long-memory (panel (d)) estimators. Trends also have a detrimental effect on these estimators: higher trend values imply higher bias of these estimators. In the tables 7 and B.1, we show detailed results with a trend value of 0.005 because small trends have enough effect on all the estimators, even though trends are possibly visible and easy to remove. This is an important finding because low-persistence time series contaminated by trends cause the estimators to indicate higher persistence; in other words, a linear trend fools the estimators. However, our results indicate that if the researcher ignores the presence of a linear trend for low slope values, one can (i) discard infinite persistence using the ADF test, (ii) if one knows that the DGP is an AR(1) process, using $\hat{\rho}_1$ is a good strategy, and finally, (iii) the nonparametric estimator is a good scalar estimator of persistence that does not require knowing the DGP process.

Cycle. Table 7 shows results for cycle contamination such that $v_t = \sin[2\pi(0.01)t]$, $t = 1, \dots, T$, that is, a cycle with unitary amplitude and a frequency of 0.01. Under

this type of contamination, the ADF test correctly rejects the null hypothesis of the unit root, and it is also robust to cycles for all values of ρ . As for the ADF test, the nonparametric estimator, $\hat{\gamma}$, is among the three best estimators for all values of ρ . Finally, the KPSS test is among the best in seven out of 10 cases. Results for different amplitudes and frequencies are shown in figure B.2. Figure B.2 shows the effect of cycles in four estimators: $\hat{\rho}_1$, $\hat{\rho}$, $\hat{\gamma}$, and \hat{d}_{GPH} . Panels (a) and (b) show the effect on the model-specific estimators. For the estimator $\hat{\rho}_1$, when the values of ρ are small, at lower and higher frequencies, the RMSE is higher than for larger values of ρ . It also shows that for frequencies around 0.20, the estimator is almost unaffected regardless of the amplitudes. For higher values of ρ , the cycles have a small effect on the estimator $\hat{\rho}_1$. For the estimator $\hat{\rho}$, panel (b) in figure B.2, the results indicate that this estimator only shows a detrimental effect caused by cycles when the frequency is small and is considered the best model-specific estimator. For the nonparametric estimator, $\hat{\gamma}$, we show the estimated value for different frequencies in the horizontal axis and amplitudes (blue lines). We show the estimated values for three different values of ρ , namely, 0.10, 0.50, and 0.95 (red lines). The results indicate that for lower values of ρ , the estimator shows a threshold value where below this threshold one “overestimates” the “true” value of γ and after this threshold one “underestimates” the “true” value of γ . When the persistence is high, say $\rho = 0.95$, the presence of a cycle always causes an “underestimation” the “true” value of γ . The effect of a cycle on the long-memory estimator is shown in figure B.2-(d): the cycle has the effect of underestimating the “true” value of d for small frequencies and is unaffected for larger frequencies and the threshold value is 0.166. Overall, for different amplitudes and frequencies, the model-specific method performs the best. We also point out that the results in the table indicate that the nonparametric estimator is among the best estimators when a cycle is present. Thus, we recommend using the nonparametric estimator if the researcher

suspects the presence of cycles, but these cannot be removed. Notice that cycles with small frequencies behave as a linear trend. Therefore, we recommend using the ADF test to discard infinite persistence, the use of a model-specific estimator if one knows that the process is an AR(1), and using the nonparametric estimator if one does not know the type of process or the presence of cycles.

Small measurement errors. In this case, we set $\sigma_w^2 = 1$ and $\sigma_\eta^2 = 1$, that is, we have a unitary signal-to-noise ratio. Under this contamination, the ADF test, as before, shows its robustness against contaminations, in this case against small measurement errors. The nonparametric estimator, $\hat{\gamma}$, appears among the best estimators in eight out of ten cases shown in the table. Interestingly, as the true value of ρ increases, the KPSS test of I(0) data becomes one of the best measures, especially when $\rho \geq 0.5$. Thus, when there are measurement errors, both the ADF and KPSS tests are indicative that, although the data is contaminated, the persistence is not infinite and the series will indeed return to its long-run value relatively fast. In the presence of measurement errors, the nonparametric estimator is, thus, an excellent complement to the ADF and KPSS test because it complements the test by providing an objective, scalar measure of persistence. Figure B.3 shows four estimators of persistence ($\hat{\rho}$, $\hat{\rho}_1$, $\hat{\gamma}$, and \hat{d}_{GPH}) when the measurement error takes on multiple values of the signal-to-noise ratio. The autoregressive estimators of panels (a) and (b) show that the bias of the estimators increases with the signal-to-noise. Between both estimators, $\hat{\rho}$ is the most robust, as the bias for lower values of ρ is smaller, in contrast to $\hat{\rho}_1$. The nonparametric estimator deviates from the uncontaminated case whenever the signal-to-noise increases. However, its bias does not decrease when ρ gets closer to the unit root, if compared to the autoregressive estimators. Finally, the long-memory estimator does not deviate much from the uncontaminated case; however, this happens because this estimator is not designed for short-memory processes and tends to indicate the

presence of long-memory (long-term persistence). Hence, using the ADF or KPSS test to discard infinite long-term persistence is a good strategy, and using the nonparametric estimator yields a scalar measure of persistence that is robust to small measurement errors.

Additive outliers. Table 7 shows the results for contamination by additive outliers with parameters $\theta = 1.2$ and $p = 0.025$, which refer to the outlier size and the probability of the outlier occurring, respectively. Again, the ADF test shows its robustness against additive outliers, being the persistence measure that deviates less from the uncontaminated case. The nonparametric estimator appears among the best estimators in nine out of ten cases shown in the table, strengthening the argument of it being one of the most robust estimators of persistence. Note, however, that if this contamination is present, the deviation from the uncontaminated estimate is small for all estimated measures, meaning that additive outliers may not be problematic if present in the data. Further, the results depicted in figure B.4 make the previous affirmation quite compelling: considering the four estimators of panels (a), (b), (c), and (d) and if we allow the size of the outlier, θ , to vary, the difference compared to the uncontaminated DGP will be negligible. Nonetheless, some comments are in order. First, the long-memory estimates assume a long-memory DGP and are almost unaffected according to the RMSE shown in the figure. Second, most of the times, the nonparametric estimator may be preferred, as it is completely model-free and, thus, the researcher lowers the risk of wrong persistence estimation.

Structural change. Table 7 shows results for structural-change contamination with parameters $\theta = 1$ and a probability of structural change given by $p = 0.06$. In this case, the results are quite different from the previous four contaminations. When the DGP is contaminated with a structural change, the ADF test does not perform as in previous

cases: it doesn't work for any value of ρ . The ADF test indicates that most of the tested series have infinite persistence, as the null hypothesis of unit root presence in the data cannot be rejected. The nonparametric estimator performs well for eight out of ten values of ρ . However, the model-specific estimator, $\hat{\rho}_1$, works in nine out of ten cases, and it is the less affected estimator by the structural change. Figure B.5 shows the behavior of the $\hat{\rho}$, $\hat{\rho}_1$, $\hat{\gamma}$, and \hat{d}_{GPH} estimators when the probability, p , and the size, θ , of the structural change are allowed to vary. The four panels of the figure show that structural change size matters for the estimators' performance. When $\theta = 0.5$ all the estimators show either low RMSEs or no deviation from the "true" values of γ or d . If θ increases to the unit, the RMSE increases for model-specific estimators: the estimators $\hat{\rho}_1$ and $\hat{\rho}$ show that the RMSE increases for higher structural changes probabilities. The increment on the RMSE is bigger for lower values of ρ . The estimator \hat{d}_{GPH} estimates values compatible with nonstationary time series: as in previous cases, this is not a good method to estimate the persistence since it indicates long-term persistence and the AR(1) processes under analysis are short-term persistent. This type of contamination has a highly detrimental effect on all the estimators: if it can be identified and its effect removed, we suggest using model-specific estimators such as $\hat{\rho}_1$ and the nonparametric estimator.

Temporary change outliers. Table 7 shows the results for the size and duration of the outlier $\theta = 1$ and $\alpha = 0.50$, respectively. Again, the ADF test shows its robustness for all values of ρ , now against a temporary-change outlier contaminating the DGP. The nonparametric estimator appears among the best estimators in eight out of ten cases shown in the table, while some model-specific estimators appear among the three best estimators. Hence, the results on the table suggest that in the presence of temporary change outliers contaminating the data, the ADF test will always correctly identify that, although contaminated, the DGP is not infinitely persistent, which can be further com-

plemented by applying the nonparametric estimator, $\hat{\gamma}$, to obtain a more informative scalar measure of the DGP persistence. According to Table 7, the use of autoregressive measures is not fully recommended for this type of contamination, even though that set of estimators is among those that deviate less from the uncontaminated case, because the true value of ρ is crucial for selecting the appropriate estimator. On the one hand, in panels (a) and (b) of figure B.6, we show the performance of two autoregressive measures of persistence when the temporary change outlier can take on multiple values of θ and α . It can be appreciated that for the $\hat{\rho}_1$ ($\hat{\rho}$) estimator, only when the duration of the outlier is one (greater than 0.9), the RMSE is greater than zero, and it increases with the outlier size. On the other hand, if we look at the performance of the $\hat{\gamma}$ and \hat{d}_{GPH} estimators in panels (c) and (d) of the figure, we can notice that the deviation also increases with the size of the outlier, but a much lower value of α is needed for the measures to deviate from the uncontaminated case, being the \hat{d}_{GPH} estimator the one that deviates the most. Thus, although the autoregressive estimators (according to figure B.6) appear to be the ones with lower RMSE, the estimators are still sensitive to the true value of ρ , which can be a problem if it is unknown that this type of contamination is present. Therefore in the presence of temporary change outliers, although more sensitive to the duration of the outlier, the autoregressive measures and the $\hat{\gamma}$ estimators may be preferred, especially the latter, as it is parameter-free and its deviations from the uncontaminated case are almost similar to the $\hat{\rho}$ and $\hat{\rho}_1$ estimators.

Table 7: Estimators or Tests for Time-series Persistence for Autoregressive Processes of Order One with Parameter ρ ¹

| Parameter | Estimator or test | AR(1) | Contamination (Difference between contaminated and uncontaminated, AR(1), in absolute value) ^{2,3} | | | | | |
|---------------|----------------------------------|-------|---|--------------------------------|------------------|-------------------------------------|----------------------------------|---------------------------------------|
| | | | Trend ($\beta = 0.005$) | Cycle ($A = 1, f = 0.01$) | SME (SNR = 1) | AO ($\theta = 1.2, p = 0.025$) | SC ($\theta = 1, p = 0.06$) | TCO ($\theta = 1, \alpha = 0.5$) |
| $\rho = 0.1$ | $\hat{\rho}_1$ | 0.10 | 0.61 | 0.34 | 0.07 | 0.00 | 0.77 | 0.01 |
| | $\hat{\rho}$ | 0.10 | 0.88 | 0.55 | 0.08 | 0.00 | 0.88 | 0.02 |
| | \hat{CTR} | 1.12 | 2.42 | 0.67 | 0.08 | 0.00 | 11.10 | 0.02 |
| | \hat{LAR} | 10.50 | 9.10 | 8.22 | 688.83 | 0.48 | 9.34 | 1.20 |
| | HLC | -7.62 | 6.34 | 5.71 | 477.46 | 0.34 | 6.51 | 0.83 |
| | $\hat{\gamma}$ | 0.53 | 0.36 | 0.13 | 0.02 | 0.00 | 0.37 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.87 | 0.00 |
| | KPSS _{μ} | 0.04 | 0.96 | 0.04 | 0.00 | 0.00 | 0.92 | 0.00 |
| | d_{GPH} | 0.01 | 0.56 | 0.17 | 0.00 | 0.01 | 0.71 | 0.02 |
| | d_{HP} | 0.00 | 0.00 | 0.51 | -0.01 | 0.00 | 0.06 | 0.00 |
| d_{LW} | -0.01 | 0.53 | 0.50 | -0.01 | -0.01 | 0.70 | 0.00 | |
| $\rho = 0.3$ | $\hat{\rho}_1$ | 0.30 | 0.47 | 0.25 | 0.20 | 0.01 | 0.59 | 0.01 |
| | $\hat{\rho}$ | 0.30 | 0.68 | 0.44 | 0.19 | 0.01 | 0.68 | 0.01 |
| | \hat{CTR} | 1.43 | 2.84 | 0.79 | 0.32 | 0.02 | 13.27 | 0.01 |
| | \hat{LAR} | 3.38 | 2.08 | 1.55 | 8.73 | 0.11 | 2.25 | 0.07 |
| | HLC | -2.68 | 1.46 | 1.09 | 6.06 | 0.08 | 1.58 | 0.05 |
| | $\hat{\gamma}$ | 0.60 | 0.31 | 0.10 | 0.07 | 0.00 | 0.31 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.79 | 0.00 |
| | KPSS _{μ} | 0.06 | 0.94 | 0.06 | 0.00 | 0.00 | 0.90 | 0.00 |
| | d_{GPH} | 0.04 | 0.51 | 0.17 | 0.02 | 0.04 | 0.66 | 0.04 |
| | d_{HP} | 0.03 | 0.08 | 0.45 | 0.01 | 0.03 | 0.13 | 0.03 |
| d_{LW} | 0.03 | 0.50 | 0.44 | 0.00 | 0.03 | 0.66 | 0.03 | |
| $\rho = 0.5$ | $\hat{\rho}_1$ | 0.50 | 0.31 | 0.16 | 0.31 | 0.01 | 0.41 | 0.00 |
| | $\hat{\rho}$ | 0.49 | 0.47 | 0.28 | 0.25 | 0.01 | 0.48 | 0.00 |
| | \hat{CTR} | 2.00 | 3.28 | 0.90 | 0.77 | 0.05 | 14.88 | 0.00 |
| | \hat{LAR} | 2.01 | 0.78 | 0.48 | 3.63 | 0.05 | 0.91 | 0.00 |
| | HLC | -1.72 | 0.55 | 0.34 | 2.53 | 0.04 | 0.64 | 0.00 |
| | $\hat{\gamma}$ | 0.67 | 0.24 | 0.07 | 0.11 | 0.00 | 0.25 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 |
| | KPSS _{μ} | 0.04 | 0.96 | 0.04 | 0.00 | 0.00 | 0.92 | 0.01 |
| | d_{GPH} | 0.10 | 0.48 | 0.19 | 0.06 | 0.10 | 0.62 | 0.10 |
| | d_{HP} | 0.10 | 0.20 | 0.41 | 0.05 | 0.10 | 0.25 | 0.10 |
| d_{LW} | 0.09 | 0.47 | 0.40 | 0.05 | 0.09 | 0.62 | 0.09 | |
| $\rho = 0.8$ | $\hat{\rho}_1$ | 0.80 | 0.09 | 0.04 | 0.36 | 0.01 | 0.14 | 0.00 |
| | $\hat{\rho}$ | 0.80 | 0.13 | 0.04 | 0.19 | 0.01 | 0.17 | 0.00 |
| | \hat{CTR} | 4.95 | 3.95 | 1.05 | 3.17 | 0.24 | 17.15 | 0.09 |
| | \hat{LAR} | 1.26 | 0.13 | 0.05 | 1.05 | 0.02 | 0.19 | 0.01 |
| | HLC | -1.18 | 0.09 | 0.04 | 0.74 | 0.01 | 0.14 | 0.00 |
| | $\hat{\gamma}$ | 0.80 | 0.12 | 0.02 | 0.15 | 0.00 | 0.13 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| | KPSS _{μ} | 0.07 | 0.93 | 0.05 | 0.00 | 0.00 | 0.87 | 0.00 |
| | d_{GPH} | 0.39 | 0.56 | 0.42 | 0.30 | 0.38 | 0.66 | 0.38 |
| | d_{HP} | 0.43 | 0.53 | 0.52 | 0.32 | 0.42 | 0.60 | 0.42 |
| d_{LW} | 0.41 | 0.53 | 0.51 | 0.30 | 0.41 | 0.64 | 0.41 | |
| $\rho = 0.95$ | $\hat{\rho}_1$ | 0.95 | 0.01 | 0.00 | 0.18 | 0.00 | 0.02 | 0.00 |
| | $\hat{\rho}$ | 0.95 | 0.01 | 0.00 | 0.06 | 0.00 | 0.02 | 0.00 |
| | \hat{CTR} | 19.33 | 4.37 | 1.12 | 14.93 | 1.19 | 17.29 | 0.54 |
| | \hat{LAR} | 1.06 | 0.01 | 0.00 | 0.25 | 0.00 | 0.02 | 0.00 |
| | HLC | -1.04 | 0.01 | 0.00 | 0.18 | 0.00 | 0.02 | 0.00 |
| | $\hat{\gamma}$ | 0.90 | 0.03 | 0.00 | 0.12 | 0.00 | 0.04 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| | KPSS _{μ} | 0.26 | 0.65 | 0.04 | 0.01 | 0.00 | 0.56 | 0.00 |
| | d_{GPH} | 0.77 | 0.80 | 0.78 | 0.66 | 0.77 | 0.84 | 0.77 |
| | d_{HP} | 0.82 | 0.83 | 0.84 | 0.69 | 0.82 | 0.85 | 0.82 |
| d_{LW} | 0.81 | 0.81 | 0.82 | 0.67 | 0.80 | 0.84 | 0.80 | |

¹ Results for other values of ρ are available in the appendix B, table B.1.

² Each entry in the table, in columns 4-9, shows the difference, in absolute value, between the mean under contamination and the mean under noncontamination. The means were computed over 1,000 replications. For example, when $\rho = 0.1$, the uncontaminated mean of the $\hat{\rho}_1$ estimator yields a persistence of 0.10 (see column 3, AR(1)), and if we estimate it again, but with the trend, the mean of the $\hat{\rho}_1$ estimator is 0.71. Therefore, the difference is 0.61. Notice that for the ADF and KPSS tests, we report the mean probability of rejections and the difference between these probabilities between contaminated and noncontaminated processes.

³ In blue are the three estimators or tests with the best performance. Darker blue indicates the best performance. Lower values indicate better performance. Remember that the long-memory estimators are excluded from the race of the top three estimators because they are not designed for short-memory processes. We show them to illustrate that they tend to indicate long-term persistence mistakenly. Some observations might not be colored when their difference is zero, however, it is not because we are not considering them for the analysis but because the criteria to select the top three estimators used more than two decimals, thus being greater than the colored observations.

5 Analyzing the Performance of Persistence Measures under Contamination for Fractionally Integrated (Long-memory) Processes

In this section, we study the performance of the same measures of time-series persistence of section 4, described in section 3.2, but applied to uncontaminated and contaminated fractionally integrated processes of order d , $\text{FI}(d)$. As before, we start illustrating, using simulations, the performance of the persistence estimators on uncontaminated simulated fractionally integrated processes. We simulate $n_{sim} = 1,000$ $\text{FI}(d)$ processes of order d of size $T = 1,024; 2,048$, and $4,096$ with parameters $d = 0.1, 0.2, 0.3, 0.4, 0.45, 0.55, 0.6, 0.7, 0.8, 0.9$, and 0.95 . We remind the reader that the $\text{FI}(d)$ with $d \in [0, 0.5)$ are stationary mean-reverting processes while $\text{FI}(d)$ with $d \in (0.5, 1)$ are nonstationary mean-reverting processes. We report for the long-memory methods, mean, bias, and root-mean-square error (RMSE). We report the corresponding rejection probabilities for the integer order of integration tests. We also report the mean and standard deviation for the nonparametric and autoregressive estimators. Recall that for the nonparametric and autoregressive estimators, the true value of γ or ρ is unknown for long-memory processes. The results are the following:

Long-memory estimators. Table 8 shows the simulation results for three persistence estimators based on long-memory models and described in section 3.2. For each d we show the mean, bias, and root-mean-square error. For $T = 1024$, the \hat{d}_{GPH} is the best overall regarding bias and RMSE, especially when d belongs to the stationarity region, $d < 0.5$. According to the table, the second best estimator is \hat{d}_{LW} , whose performance improves with d , especially if it belongs to the nonstationary mean-reverting region. The \hat{d}_{HP} estimator, although not the best, also yields estimates near the true parameter and its performance improves for processes with $d \geq 0.5$. Table 8 also shows that the performance of these three estimators improves with the sample size. We recommend

using the local Whittle estimator, \hat{d}_{LW} , if the DGP is long-term persistent because its performance yields the lower RMSE among the three estimators.

I(1) and I(0) tests. Table 9 shows the rejection probabilities for the ADF and KPSS tests for different values of d . The ADF test performs well for $d \leq 0.5$ for all sample sizes: the test correctly rejects the null of the unit root. For values of $d \in (0.5, 1)$, a nonstationary FI(d), the test does not perform well; however, its performance improves for higher values of d , that is, the rejections probabilities get closer to the nominal size of 5%. The $KPSS_{\mu}$ should reject the null of $d = 0$; however, this only happens for values of higher values of d . The $KPSS_{\tau}$ test performance shows a similar pattern to the $KPSS_{\mu}$ test, but recall that the simulated processes don't have a trend.

Nonparametric estimator. Table 10 shows the results for the nonparametric estimator, $\hat{\gamma}$. For all values of d , we report the mean and its standard deviation over 1,000 simulations. We were not able to derive the true value of γ . However, because all the simulated FI processes are long-term persistent, we expected the estimated $\hat{\gamma}$ to be between 0.5 and 1: lower values of d imply lower values of γ , while higher values of d imply higher values of γ . In the table, the estimated value of $\hat{\gamma}$ increases with the value of d , meaning that the estimator behaves as expected. Further, one thing to notice is that when the process is fractionally integrated, the $\hat{\gamma}$ estimator is greater than the estimator when an autoregressive process generates the data. This can indicate the estimator's robustness due to its model-free characteristics and that long-memory processes are more persistent than short-memory processes. Moreover, the performance of $\hat{\gamma}$ is almost the same for different sample sizes.

Autoregressive estimators. Table 11 shows five autoregressive estimators of persistence. As with the case of the long-memory estimators applied to autoregressive data, we expect that these estimators perform poorly as well because the DGPs under consideration

in this section are long-memory and the autoregressive estimators are designed for short-memory time series. As for the nonparametric estimator, we do not know the true parameter, ρ . One expects that, for long-memory time series, the autoregressive processes indicate high persistence. Table 11 shows that for $T = 1,024$, the estimated values of $\hat{\rho}$ increases with d ; that is, the autoregressive measures indicate lower values of persistence for $d < 0.40$ and indicate higher persistence for higher values of persistence for $d \geq 0.40$.

Table 8: Long-Memory Measures of Persistence for Fractionally Integrated Processes with Parameter d^1

| d | Statistic | $T = 1,024$ | | | $T = 2,048$ | | | $T = 4,096$ | | |
|------|-----------|------------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|
| | | \hat{d}_{GPH} | \hat{d}_{HP} | \hat{d}_{LW} | \hat{d}_{GPH} | \hat{d}_{HP} | \hat{d}_{LW} | \hat{d}_{GPH} | \hat{d}_{HP} | \hat{d}_{LW} |
| 0.10 | Mean | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 |
| | Bias | 0.00 | -0.01 | -0.01 | -0.00 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 |
| | RMSE | 0.06 | 0.06 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 |
| 0.20 | Mean | 0.20 | 0.18 | 0.18 | 0.20 | 0.19 | 0.19 | 0.20 | 0.19 | 0.19 |
| | Bias | 0.00 | -0.02 | -0.02 | 0.00 | -0.01 | -0.01 | -0.00 | -0.01 | -0.01 |
| | RMSE | 0.06 | 0.06 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 |
| 0.30 | Mean | 0.30 | 0.28 | 0.28 | 0.30 | 0.29 | 0.29 | 0.30 | 0.29 | 0.30 |
| | Bias | 0.00 | -0.02 | -0.02 | 0.00 | -0.01 | -0.01 | 0.00 | -0.01 | -0.00 |
| | RMSE | 0.06 | 0.06 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 |
| 0.40 | Mean | 0.40 | 0.37 | 0.39 | 0.40 | 0.39 | 0.39 | 0.40 | 0.39 | 0.39 |
| | Bias | 0.00 | -0.03 | -0.01 | 0.00 | -0.01 | -0.01 | 0.00 | -0.01 | -0.01 |
| | RMSE | 0.06 | 0.07 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.03 | 0.03 |
| 0.45 | Mean | 0.45 | 0.42 | 0.43 | 0.45 | 0.43 | 0.44 | 0.45 | 0.44 | 0.44 |
| | Bias | 0.00 | -0.03 | -0.02 | 0.00 | -0.02 | -0.01 | 0.00 | -0.01 | -0.01 |
| | RMSE | 0.06 | 0.07 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.03 |
| 0.55 | Mean | 0.56 | 0.52 | 0.54 | 0.55 | 0.53 | 0.54 | 0.55 | 0.54 | 0.55 |
| | Bias | 0.01 | -0.03 | -0.01 | 0.00 | -0.02 | -0.01 | 0.00 | -0.01 | -0.00 |
| | RMSE | 0.06 | 0.08 | 0.05 | 0.05 | 0.06 | 0.04 | 0.04 | 0.04 | 0.03 |
| 0.60 | Mean | 0.61 | 0.56 | 0.59 | 0.61 | 0.58 | 0.60 | 0.61 | 0.58 | 0.60 |
| | Bias | 0.01 | -0.04 | -0.01 | 0.01 | -0.02 | -0.00 | 0.01 | -0.02 | -0.00 |
| | RMSE | 0.07 | 0.09 | 0.05 | 0.05 | 0.07 | 0.04 | 0.04 | 0.04 | 0.03 |
| 0.70 | Mean | 0.72 | 0.66 | 0.70 | 0.71 | 0.68 | 0.70 | 0.71 | 0.69 | 0.70 |
| | Bias | 0.02 | -0.04 | -0.00 | 0.01 | -0.02 | -0.00 | 0.01 | -0.01 | 0.00 |
| | RMSE | 0.07 | 0.13 | 0.05 | 0.05 | 0.08 | 0.04 | 0.04 | 0.05 | 0.03 |
| 0.80 | Mean | 0.83 | 0.78 | 0.80 | 0.82 | 0.79 | 0.81 | 0.82 | 0.81 | 0.81 |
| | Bias | 0.03 | -0.02 | 0.00 | 0.02 | -0.01 | 0.01 | 0.02 | 0.01 | 0.01 |
| | RMSE | 0.07 | 0.15 | 0.05 | 0.06 | 0.10 | 0.04 | 0.05 | 0.05 | 0.04 |
| 0.90 | Mean | 0.92 | 0.90 | 0.90 | 0.93 | 0.91 | 0.91 | 0.92 | 0.91 | 0.91 |
| | Bias | 0.02 | -0.00 | -0.00 | 0.03 | 0.01 | 0.01 | 0.02 | 0.01 | 0.01 |
| | RMSE | 0.07 | 0.11 | 0.05 | 0.06 | 0.05 | 0.04 | 0.05 | 0.05 | 0.03 |
| 0.95 | Mean | 0.96 | 0.93 | 0.94 | 0.96 | 0.94 | 0.95 | 0.96 | 0.95 | 0.95 |
| | Bias | 0.01 | -0.02 | -0.01 | 0.01 | -0.01 | -0.00 | 0.01 | 0.00 | 0.00 |
| | RMSE | 0.06 | 0.12 | 0.05 | 0.05 | 0.15 | 0.04 | 0.04 | 0.06 | 0.03 |

¹ Each statistic is estimated based on 1,000 realizations.

Table 9: Tests of I(1) and I(0) for Fractionally Integrated Processes with Parameter d . Rejection Probabilities for Nominal Size of 5% ¹

| d | $T = 1,024$ | | | $T = 2,048$ | | | $T = 4,096$ | | |
|------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|
| | ADF | KPSS $_{\mu}$ | KPSS $_{\tau}$ | ADF | KPSS $_{\mu}$ | KPSS $_{\tau}$ | ADF | KPSS $_{\mu}$ | KPSS $_{\tau}$ |
| 0.10 | 1.00 | 0.13 | 0.15 | 1.00 | 0.16 | 0.17 | 1.00 | 0.18 | 0.22 |
| 0.20 | 1.00 | 0.27 | 0.29 | 1.00 | 0.34 | 0.40 | 1.00 | 0.43 | 0.51 |
| 0.30 | 1.00 | 0.41 | 0.48 | 1.00 | 0.51 | 0.60 | 1.00 | 0.64 | 0.75 |
| 0.40 | 1.00 | 0.57 | 0.64 | 1.00 | 0.67 | 0.76 | 1.00 | 0.78 | 0.88 |
| 0.45 | 1.00 | 0.61 | 0.72 | 1.00 | 0.73 | 0.84 | 1.00 | 0.83 | 0.92 |
| 0.55 | 0.97 | 0.72 | 0.81 | 1.00 | 0.84 | 0.93 | 1.00 | 0.94 | 0.98 |
| 0.60 | 0.90 | 0.78 | 0.84 | 0.98 | 0.89 | 0.94 | 1.00 | 0.96 | 0.98 |
| 0.70 | 0.65 | 0.85 | 0.89 | 0.84 | 0.93 | 0.98 | 0.94 | 0.99 | 1.00 |
| 0.80 | 0.29 | 0.90 | 0.93 | 0.45 | 0.95 | 0.98 | 0.58 | 0.98 | 1.00 |
| 0.90 | 0.11 | 0.93 | 0.96 | 0.14 | 0.98 | 0.99 | 0.18 | 1.00 | 1.00 |
| 0.95 | 0.09 | 0.95 | 0.96 | 0.09 | 0.98 | 0.99 | 0.10 | 1.00 | 1.00 |

¹ The rejections probabilities were computed based on 1,000 realizations.

Table 10: Nonparametric Estimators of Persistence for Fractionally Integrated Processes with Parameter d ¹

| d | Statistic | $T = 1,024$ | $T = 2,048$ | $T = 4,096$ |
|------|-----------|-------------|-------------|-------------|
| 0.10 | Estimator | 0.54 | 0.54 | 0.54 |
| | SD | 0.02 | 0.01 | 0.01 |
| 0.20 | Estimator | 0.58 | 0.58 | 0.58 |
| | SD | 0.02 | 0.01 | 0.01 |
| 0.30 | Estimator | 0.64 | 0.64 | 0.64 |
| | SD | 0.02 | 0.02 | 0.01 |
| 0.40 | Estimator | 0.71 | 0.71 | 0.71 |
| | SD | 0.03 | 0.03 | 0.02 |
| 0.45 | Estimator | 0.75 | 0.75 | 0.76 |
| | SD | 0.04 | 0.03 | 0.03 |
| 0.55 | Estimator | 0.83 | 0.83 | 0.84 |
| | SD | 0.05 | 0.04 | 0.04 |
| 0.60 | Estimator | 0.86 | 0.87 | 0.88 |
| | SD | 0.05 | 0.04 | 0.04 |
| 0.70 | Estimator | 0.92 | 0.92 | 0.94 |
| | SD | 0.04 | 0.03 | 0.03 |
| 0.80 | Estimator | 0.95 | 0.96 | 0.97 |
| | SD | 0.03 | 0.02 | 0.02 |
| 0.90 | Estimator | 0.97 | 0.98 | 0.98 |
| | SD | 0.02 | 0.02 | 0.01 |
| 0.95 | Estimator | 0.98 | 0.98 | 0.99 |
| | SD | 0.02 | 0.01 | 0.01 |

¹ The estimator and its standard deviation were computed using 1,000 realizations.

Table 11: Autoregressive Estimators of Persistence for Fractionally Integrated Processes with Parameter d^1

| d | Stat | $T = 1,024$ | | | | | $T = 2,048$ | | | | | $T = 4,096$ | | | | |
|------|-----------|----------------|--------------|-----------------|----------------|-----------------|----------------|--------------|-----------------|----------------|-----------------|----------------|--------------|-----------------|----------------|-----------------|
| | | $\hat{\rho}_1$ | $\hat{\rho}$ | \widehat{CIR} | \widehat{LR} | \widehat{HLC} | $\hat{\rho}_1$ | $\hat{\rho}$ | \widehat{CIR} | \widehat{LR} | \widehat{HLC} | $\hat{\rho}_1$ | $\hat{\rho}$ | \widehat{CIR} | \widehat{LR} | \widehat{HLC} |
| 0.10 | Estimator | 0.11 | 0.20 | 1.12 | 10.64 | -7.72 | 0.11 | 0.22 | 1.12 | 9.71 | -7.07 | 0.11 | 0.25 | 1.12 | 9.33 | -6.81 |
| | SD | 0.03 | 0.09 | 0.04 | 6.86 | 4.76 | 0.02 | 0.08 | 0.03 | 2.43 | 1.69 | 0.02 | 0.07 | 0.02 | 1.55 | 1.07 |
| 0.20 | Estimator | 0.24 | 0.44 | 1.32 | 4.27 | -3.29 | 0.24 | 0.48 | 1.32 | 4.18 | -3.23 | 0.25 | 0.51 | 1.33 | 4.10 | -3.18 |
| | SD | 0.04 | 0.10 | 0.06 | 0.67 | 0.47 | 0.03 | 0.08 | 0.05 | 0.50 | 0.35 | 0.02 | 0.06 | 0.03 | 0.32 | 0.22 |
| 0.30 | Estimator | 0.39 | 0.63 | 1.65 | 2.60 | -2.13 | 0.40 | 0.68 | 1.67 | 2.52 | -2.08 | 0.41 | 0.72 | 1.70 | 2.45 | -2.03 |
| | SD | 0.04 | 0.09 | 0.12 | 0.30 | 0.21 | 0.03 | 0.07 | 0.09 | 0.21 | 0.14 | 0.02 | 0.05 | 0.07 | 0.14 | 0.10 |
| 0.40 | Estimator | 0.56 | 0.77 | 2.29 | 1.81 | -1.57 | 0.58 | 0.81 | 2.39 | 1.74 | -1.53 | 0.59 | 0.84 | 2.47 | 1.69 | -1.49 |
| | SD | 0.05 | 0.06 | 0.27 | 0.16 | 0.11 | 0.04 | 0.05 | 0.25 | 0.13 | 0.09 | 0.03 | 0.04 | 0.21 | 0.09 | 0.07 |
| 0.45 | Estimator | 0.64 | 0.82 | 2.81 | 1.58 | -1.41 | 0.66 | 0.86 | 3.00 | 1.52 | -1.37 | 0.68 | 0.89 | 3.17 | 1.47 | -1.34 |
| | SD | 0.05 | 0.06 | 0.42 | 0.13 | 0.09 | 0.04 | 0.04 | 0.43 | 0.10 | 0.07 | 0.03 | 0.03 | 0.36 | 0.07 | 0.05 |
| 0.55 | Estimator | 0.78 | 0.89 | 4.90 | 1.28 | -1.20 | 0.81 | 0.92 | 5.52 | 1.24 | -1.17 | 0.83 | 0.94 | 6.28 | 1.20 | -1.14 |
| | SD | 0.05 | 0.04 | 1.29 | 0.08 | 0.06 | 0.04 | 0.03 | 1.34 | 0.06 | 0.04 | 0.03 | 0.02 | 1.39 | 0.05 | 0.03 |
| 0.60 | Estimator | 0.84 | 0.92 | 6.92 | 1.19 | -1.14 | 0.87 | 0.94 | 8.03 | 1.16 | -1.11 | 0.89 | 0.96 | 9.72 | 1.12 | -1.09 |
| | SD | 0.05 | 0.03 | 2.35 | 0.07 | 0.05 | 0.03 | 0.02 | 2.38 | 0.05 | 0.03 | 0.03 | 0.01 | 2.85 | 0.03 | 0.02 |
| 0.70 | Estimator | 0.92 | 0.96 | 14.95 | 1.09 | -1.06 | 0.94 | 0.97 | 19.72 | 1.06 | -1.05 | 0.96 | 0.98 | 27.48 | 1.04 | -1.03 |
| | SD | 0.03 | 0.02 | 6.95 | 0.04 | 0.03 | 0.02 | 0.01 | 8.36 | 0.02 | 0.02 | 0.02 | 0.01 | 12.66 | 0.02 | 0.01 |
| 0.80 | Estimator | 0.97 | 0.98 | 39.77 | 1.04 | -1.03 | 0.98 | 0.99 | 56.32 | 1.02 | -1.02 | 0.99 | 0.99 | 92.09 | 1.01 | -1.01 |
| | SD | 0.02 | 0.01 | 25.10 | 0.02 | 0.01 | 0.01 | 0.01 | 32.23 | 0.01 | 0.01 | 0.01 | 0.00 | 64.36 | 0.01 | 0.01 |
| 0.90 | Estimator | 0.99 | 0.99 | 116.99 | 1.01 | -1.01 | 0.99 | 0.99 | 215.43 | 1.01 | -1.01 | 1.00 | 1.00 | 364.15 | 1.00 | -1.00 |
| | SD | 0.01 | 0.01 | 156.26 | 0.01 | 0.01 | 0.00 | 0.00 | 236.23 | 0.01 | 0.00 | 0.00 | 0.00 | 356.46 | 0.00 | 0.00 |
| 0.95 | Estimator | 0.99 | 0.99 | 196.10 | 1.01 | -1.01 | 1.00 | 1.00 | 290.49 | 1.00 | -1.00 | 1.00 | 1.00 | 984.51 | 1.00 | -1.00 |
| | SD | 0.01 | 0.01 | 872.74 | 0.01 | 0.00 | 0.00 | 0.00 | 2205.00 | 0.00 | 0.00 | 0.00 | 0.00 | 5962.44 | 0.00 | 0.00 |

¹ The estimator and its standard deviation were computed using 1,000 realizations.

5.1 Contaminating Long-memory Processes

In this section, we investigate the effect of the contamination processes described in section 3.1. Again, we present the results for i) Trend, ii) Cycle, iii) SME, iv) AO, v) SC, and vi) TCO, and for comparison, we include the results for the uncontaminated processes in the column $FI(d)$. Table 12 shows the results for long-memory processes for different values of d contaminated with the six mechanisms with a given set of parameters. Results for other values of ρ are in appendix C, table C.1. We select the set of parameters in table 12 considering that the contaminations are not detected using an eyeball test. The results for each contamination mechanism are presented in terms of the absolute value difference between the estimator's means (or probability of test rejection) for the uncontaminated process and the estimator's means (or probability of test rejection) for the contaminated process: lower values imply better performance and higher values imply worse performance. Results for a different set of parameters are presented in appendix C, and the conclusions are similar to the ones presented in this section. The results are for a sample size of $T = 4,096$ and the means or rejection probabilities were computed using 1,000 simulations.

Trend. Table 12 shows the results for a trend value of $\beta = 0.005$. Similarly to the autoregressive case, when the data is contaminated with a deterministic trend, the ADF is the best performer; however, the same drawback as before applies: it does not yield us a scalar measure of persistence; it only tells us that the process is not infinitely persistent. Therefore, additional measures are needed if the researcher is looking for a result different from a dichotomous indicator of infinite persistence or nonpersistence. The estimator $\hat{\gamma}$ performs well in eight out of eleven cases of d displayed in the table. Something noteworthy is that although the $\hat{\rho}$, \widehat{HLC} and \widehat{LR} estimators are rated among the best three for certain values of d , this is not because the estimators seem to provide

an adequate measure of persistence, but because the uncontaminated case is already indicating an incorrect level of persistence, plausibly because the DGP of the data is long-memory, and the estimators assume short-memory properties. Therefore, when compared to the estimated persistence under the uncontaminated case, these estimators seem robust, but we know that they were inappropriate for this kind of processes. Hence, the best candidates to measure persistence under the DGP analyzed are the long-memory estimators, which do not deviate much from the estimated persistence of the uncontaminated process and are, consequently, near the true d of the process. If we let the value of the trend vary, in figure C.1 we observe that between the long-memory estimators \hat{d}_{GPH} and \hat{d}_{HP} (panels (a) and (b)) the \hat{d}_{HP} estimator is the best performer, as the RMSE is not as sensitive to the trend and the value of d as the \hat{d}_{GPH} estimator. The $\hat{\gamma}$ and $\hat{\rho}$ estimators depicted in panels (c) and (d) show that the estimators perform as expected only for high values of d , compared to the uncontaminated case, and they show a positive bias for smaller values of d . Hence, when the $\text{FI}(d)$ DGP is contaminated with a deterministic trend, the estimator $\hat{\gamma}$, although biased for increasing values of the trend slope, is the best estimator according to figure C.1. Therefore, we propose to use the ADF test if one seeks to reject infinite persistence. In terms of scalar measures of persistence, we propose using the \hat{d}_{HP} estimator if one knows that the process is an $\text{FI}(d)$, and using the nonparametric estimator if the DGP is unknown.

Cycle. Table 12 shows the estimated persistence of a long-memory process contaminated with a deterministic cycle, such that $v_t = \sin[2\pi(0.01)t]$, $t = 1, \dots, T$; that is, a cycle with unitary amplitude and a frequency of 0.01. The best performer is the nonparametric estimator, $\hat{\gamma}$: for all values of d , the nonparametric estimator deviates less from the estimator based on the uncontaminated series. The table also shows that the ADF test is the best performer in nine out of eleven cases of d . Both estimators are the best and second-best performers when $d < 0.5$. When d belongs to the mean-reverting

region, the $\hat{\rho}$, $\hat{\rho}_1$, and $\widehat{\text{HLC}}$ autoregressive estimators of persistence display misleading information regarding the true persistence of the data, as the uncontaminated case is already measured as practically infinitely persistent, which causes a low deviation when measuring contaminated data. Accordingly, if the DGP is stationary and fractionally integrated, the $\hat{\gamma}$ estimator is preferred. In figure C.2, we extend the analysis by letting the cycle take on multiple amplitude and frequency values. We can see that the \hat{d}_{GPH} estimator in panel (a) is the one that displays the best performance in terms of RMSE. If the true d of the process is low, the estimator becomes sensitive to cycles with low frequencies combined with high amplitudes. The \hat{d}_{HP} estimator performs well also, but not as good as the \hat{d}_{GPH} estimator. This estimator is also sensitive to cycles with high amplitudes and low frequencies; however, the RMSE is higher for processes with low d . The $\hat{\gamma}$ and $\hat{\rho}$ estimators depicted in panels (c) and (d) of figure C.2 show that the estimated persistence will deviate from the uncontaminated case with increasing amplitudes of the cycle. Besides, it seems that both measures will accurately estimate the persistence of the process irrespective of the amplitude of the cycle if its frequency is equal to a certain value that depends on the integration parameter, d . Therefore, for a $\text{FI}(d)$ process contaminated with a deterministic cycle, the \hat{d}_{GPH} estimator is the indisputable winner. We also should consider the nonparametric estimator, which shows a lower deviation from the uncontaminated case for higher values of d .

Small measurement errors. In table 12, we show the estimated persistence of an $\text{FI}(d)$ process contaminated with a small measurement error. To construct the error, we set $\sigma_w^2 = 1$ and $\sigma_\eta^2 = 1$ from equation (3): we have a unitary signal-to-noise ratio, SNR. As before, the ADF and KPSS tests are the best performers for all values of d . If a scalar measure of persistence is required by the researcher, the $\hat{\gamma}$ estimator follows the tests as the third-best measure of persistence, in particular, when the DGP belongs to the stationarity region. However, according to figure C.3, if the process has a $d > 0.5$, the

long-memory estimators of persistence outperform the rest, as the estimated persistence of the uncontaminated case goes in line with the true d of the process; therefore, the RMSE of the estimators under contamination is small. As before, the autoregressive parameters yield incorrect information regarding their robustness: their deviations from the uncontaminated case are small, relative to other contaminations, because the estimated persistence under no contamination is already measured as almost infinite. The robustness of the long-memory estimators to this contamination is displayed in figure C.3, where we let the signal-to-noise ratio take on different values greater, equal, and less than one. Panel (a) shows that the \hat{d}_{GPH} estimator is the most robust of all the four estimators due to the RMSE being approximately zero irrespective of the size of the SNR and the process' true value of d . The \hat{d}_{HP} estimator in panel (b) only performs right if $d < 0.5$ and the error has a low SNR. The $\hat{\gamma}$ and $\hat{\rho}$ estimators of panels (c) and (d), respectively, deviate from the uncontaminated estimate with the size of the SNR, that is, the greater the SNR, the further the estimator deviates from the uncontaminated estimator. Hence, in the presence of small measurement errors, the \hat{d}_{GPH} is the safest option to obtain a scalar, (slightly) unbiased, estimate of persistence. We also recommend the use of the nonparametric estimator if one does not know that the process is, but if it is suspected to be contaminated by SME.

Additive outliers. Table 12 shows the results for contamination by additive outliers with parameters $\theta = 1.2$ and $p = 0.025$, which refer to the outlier size and the probability of the outlier occurring, respectively. The nonparametric estimator is the best performer; it works well in ten out of eleven cases. The ADF test performs well in seven out of eleven cases. If the process is nonstationary but mean-reverting ($d > 0.5$), the autoregressive estimators are, according to the table, some of the most robust; however, as before, the uncontaminated case measures almost infinite persistence. Thus, when applying said estimators to contaminated processes, the difference between estimates

of persistence is small. Figure C.4 shows that, actually, the long-memory estimators \hat{d}_{GPH} and \hat{d}_{HP} are the best performers for any value of d and θ . The \hat{d}_{GPH} estimator in panel (a) has the lowest RMSE of both long-memory estimators, becoming the best performer among the four depicted, followed by the \hat{d}_{HP} estimator, which RMSE increases only with the value of d . The $\hat{\gamma}$ and $\hat{\rho}$ estimators of panels (c) and (d) show that both estimators do not deviate from the uncontaminated case if the true d of the process or the θ of the outlier increase. Therefore, when measuring the persistence of a long-memory process, we recommend using the \hat{d}_{GPH} and the nonparametric estimators. We do not encourage the use of the $\hat{\rho}$ estimator, as it assumes a short-memory process to perform the estimation.

Structural change. Table 12 shows the results of the estimated persistence of an $\text{FI}(d)$ process contaminated with a structural change with parameters $\theta = 1$, and a probability of structural change $p = 0.06$. The table shows that, in contrast to the previous contaminations, the ADF test is not robust to this contamination. Under structural changes, the \hat{d}_{HP} and $\hat{\gamma}$ estimators are the best performers. Although the tables show that the autoregressive measures are the best by deviating little from the uncontaminated case, it is the same as in the previous contaminations: autoregressive-based estimators deviate less because they are, already, measured as infinitely persistent. Figure C.5 shows the \hat{d}_{GPH} , \hat{d}_{HP} , $\hat{\gamma}$, and $\hat{\rho}$ estimators when the structural change can take on different sizes ($\theta = 1.0, 1.5$) and different probabilities of happening. As depicted, the $\hat{\gamma}$, and $\hat{\rho}$ estimators do not perform well under structural changes, as they estimate the processes as very persistent when, in fact, it is not, e.g., if $d = 0.45$. The performance of long-memory estimators improves when the probability of a structural change is low and the true value of d is high. Between both estimators, the \hat{d}_{HP} displays the lowest RMSE for low values of d ; besides, it is more robust to low values of θ than the \hat{d}_{GPH} estimator. Thus, in the presence of structural change in the data-generating process, all the estimators will be

biased; however, the \hat{d}_{HP} estimator yields the lowest RMSE among the long-memory estimators. We also recommend the nonparametric estimator, which performs well if one does not know the type of process or the presence of structural changes. Again, this exercise stresses that this type of contamination has a highly detrimental impact on the estimation of persistence, so, if possible, the researcher is advised to test for its presence, and if it can be identified, to remove it.

Temporary change outliers. Table 12 shows the estimated persistence of an $FI(d)$ process contaminated with a temporary change outlier with size and duration $\theta = 1$ and $\alpha = 0.50$, respectively. In the presence of a temporary change outlier, on the one hand, the ADF test and the nonparametric estimator, $\hat{\gamma}$, perform well in ten out of eleven cases. Figure C.6 shows how the \hat{d}_{GPH} , \hat{d}_{HP} , $\hat{\gamma}$, and $\hat{\rho}$ estimators perform when there is a temporary change outlier that can take on different values of duration and size, α and θ . The \hat{d}_{GPH} estimator of panel (a) shows that the estimator is biased only if the α of the outlier is one and d belongs to the stationarity region. Besides, the RMSE increases with θ , meaning that bigger outliers fool the estimator into measuring a higher level of persistence. The \hat{d}_{HP} , in panel (b), estimator performs better when the duration is one than the \hat{d}_{GPH} estimator. Further, this estimator appears to be more robust to the size of θ compared to the \hat{d}_{HP} estimator. However, irrespective of the level of α , the RMSE of the \hat{d}_{HP} estimator will be greater than zero for values of $d \in (0.5, 0.9]$. The estimators in panels (c) and (d) show the $\hat{\gamma}$, and $\hat{\rho}$ estimators. These estimators perform well if d the parameter α is small, below 0.8; otherwise, they deviate from the estimated persistence of the uncontaminated process with increasing values of d . Both estimators appear to function similarly. Therefore, if the DGP is contaminated with a temporary change outlier, according to our results the best estimators are the long-memory estimators, provided that they are robust to any value of α and θ , in contrast to the other two measures. We also recommend using the nonparametric estimator when temporary

change outliers are present and one suspects or knows that the DGP is long-memory, as its performance is also good.

Table 12: Estimators or Tests for Time-series Persistence for Fractional Integrated Processes with Parameter d^1

| Parameter | Estimator or Test | FI(d) | Contamination (Difference between contaminated and uncontaminated estimators, FI(d), in absolute value) ^{2,3} | | | | | |
|----------------|-------------------|-----------|--|--------------------------------|------------------|-------------------------------------|---------------------------------|---------------------------------------|
| | | | Trend ($\beta = 0.005$) | Cycle ($A = 1, f = 0.01$) | SME (SNR = 1) | AO ($\theta = 1.2, p = 0.025$) | SC ($\theta = 1, p = 0.6$) | TCO ($\theta = 1, \alpha = 0.5$) |
| $d = 0.1$ | $\hat{\rho}_1$ | 0.11 | 0.71 | 0.44 | 0.03 | 0.11 | 0.87 | 0.12 |
| | $\hat{\rho}$ | 0.20 | 0.98 | 0.68 | 0.05 | 0.20 | 0.97 | 0.21 |
| | \widehat{CIR} | 1.12 | 3.52 | 1.79 | 1.04 | 1.12 | 12.48 | 1.14 |
| | \widehat{LAR} | 10.64 | 1.40 | 2.28 | -40.89 | 11.39 | 1.16 | 9.11 |
| | \widehat{HLC} | -7.72 | -1.29 | -1.91 | 28.00 | -8.24 | -1.11 | -6.66 |
| | $\hat{\gamma}$ | 0.54 | 0.36 | 0.13 | 0.02 | 0.00 | 0.37 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.86 | 0.00 |
| | KPSS $_{\mu}$ | 0.13 | 0.87 | 0.13 | 0.05 | 0.00 | 0.83 | 0.00 |
| | \hat{d}_{GPH} | 0.10 | 0.47 | 0.13 | 0.06 | 0.00 | 0.62 | 0.00 |
| | \hat{d}_{HP} | 0.09 | 0.00 | 0.45 | 0.06 | 0.00 | 0.03 | 0.00 |
| \hat{d}_{LW} | 0.09 | 0.46 | 0.44 | 0.06 | 0.00 | 0.62 | 0.00 | |
| $d = 0.3$ | $\hat{\rho}_1$ | 0.39 | 0.78 | 0.59 | 0.14 | 0.38 | 0.89 | 0.39 |
| | $\hat{\rho}$ | 0.63 | 0.96 | 0.81 | 0.35 | 0.62 | 0.97 | 0.63 |
| | \widehat{CIR} | 1.65 | 4.57 | 2.44 | 1.16 | 1.62 | 14.71 | 1.65 |
| | \widehat{LAR} | 2.60 | 1.29 | 1.71 | 7.97 | 2.68 | 1.12 | 2.58 |
| | \widehat{HLC} | -2.13 | -1.21 | -1.51 | -5.86 | -2.19 | -1.09 | -2.12 |
| | $\hat{\gamma}$ | 0.64 | 0.27 | 0.08 | 0.09 | 0.00 | 0.27 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.77 | 0.00 |
| | KPSS $_{\mu}$ | 0.41 | 0.59 | 0.32 | 0.08 | 0.01 | 0.55 | 0.00 |
| | \hat{d}_{GPH} | 0.30 | 0.27 | 0.08 | 0.13 | 0.00 | 0.41 | 0.00 |
| | \hat{d}_{HP} | 0.28 | 0.00 | 0.26 | 0.14 | 0.00 | 0.02 | 0.00 |
| \hat{d}_{LW} | 0.28 | 0.26 | 0.24 | 0.12 | 0.00 | 0.41 | 0.00 | |
| $d = 0.45$ | $\hat{\rho}_1$ | 0.64 | 0.82 | 0.73 | 0.29 | 0.63 | 0.91 | 0.63 |
| | $\hat{\rho}$ | 0.82 | 0.95 | 0.87 | 0.65 | 0.82 | 0.97 | 0.82 |
| | \widehat{CIR} | 2.81 | 6.06 | 3.72 | 1.41 | 2.72 | 17.25 | 2.79 |
| | \widehat{LAR} | 1.58 | 1.22 | 1.38 | 3.61 | 1.61 | 1.10 | 1.58 |
| | \widehat{HLC} | -1.41 | -1.16 | -1.27 | -2.83 | -1.43 | -1.07 | -1.42 |
| | $\hat{\gamma}$ | 0.75 | 0.16 | 0.03 | 0.13 | 0.00 | 0.17 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 |
| | KPSS $_{\mu}$ | 0.62 | 0.38 | 0.22 | 0.05 | 0.00 | 0.32 | 0.00 |
| | \hat{d}_{GPH} | 0.45 | 0.14 | 0.05 | 0.14 | 0.00 | 0.26 | 0.01 |
| | \hat{d}_{HP} | 0.42 | 0.00 | 0.16 | 0.17 | 0.00 | 0.03 | 0.01 |
| \hat{d}_{LW} | 0.43 | 0.13 | 0.14 | 0.14 | 0.00 | 0.26 | 0.01 | |
| $d = 0.8$ | $\hat{\rho}_1$ | 0.97 | 0.97 | 0.97 | 0.85 | 0.96 | 0.97 | 0.97 |
| | $\hat{\rho}$ | 0.98 | 0.98 | 0.98 | 0.95 | 0.98 | 0.98 | 0.98 |
| | \widehat{CIR} | 39.77 | 43.81 | 40.85 | 8.35 | 37.33 | 57.84 | 38.61 |
| | \widehat{LAR} | 1.04 | 1.03 | 1.03 | 1.19 | 1.04 | 1.03 | 1.04 |
| | \widehat{HLC} | -1.03 | -1.02 | -1.02 | -1.13 | -1.03 | -1.02 | -1.03 |
| | $\hat{\gamma}$ | 0.95 | 0.00 | 0.00 | 0.06 | 0.00 | 0.01 | 0.00 |
| | ADF | 0.29 | 0.00 | 0.02 | 0.09 | 0.01 | 0.08 | 0.00 |
| | KPSS $_{\mu}$ | 0.90 | 0.03 | 0.01 | 0.00 | 0.00 | 0.04 | 0.00 |
| | \hat{d}_{GPH} | 0.83 | 0.01 | 0.01 | 0.12 | 0.00 | 0.03 | 0.01 |
| | \hat{d}_{HP} | 0.78 | 0.01 | 0.04 | 0.34 | 0.01 | 0.04 | 0.00 |
| \hat{d}_{LW} | 0.80 | 0.01 | 0.02 | 0.12 | 0.00 | 0.03 | 0.00 | |
| $d = 0.95$ | $\hat{\rho}_1$ | 0.99 | 0.99 | 0.99 | 0.96 | 0.99 | 0.99 | 0.99 |
| | $\hat{\rho}$ | 0.99 | 0.99 | 0.99 | 0.98 | 0.99 | 0.99 | 0.99 |
| | \widehat{CIR} | 196.10 | 248.66 | 508.84 | 36.11 | 197.65 | 211.98 | 60.98 |
| | \widehat{LAR} | 1.01 | 1.01 | 1.01 | 1.05 | 1.01 | 1.01 | 1.01 |
| | \widehat{HLC} | -1.01 | -1.01 | -1.01 | -1.03 | -1.01 | -1.01 | -1.01 |
| | $\hat{\gamma}$ | 0.98 | 0.00 | 0.00 | 0.03 | 0.00 | 0.00 | 0.00 |
| | ADF | 0.09 | 0.00 | 0.01 | 0.01 | 0.00 | 0.01 | 0.00 |
| | KPSS $_{\mu}$ | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 |
| | \hat{d}_{GPH} | 0.96 | 0.00 | 0.01 | 0.09 | 0.00 | 0.00 | 0.00 |
| | \hat{d}_{HP} | 0.93 | 0.00 | 0.01 | 0.21 | 0.01 | 0.02 | 0.00 |
| \hat{d}_{LW} | 0.94 | 0.00 | 0.01 | 0.09 | 0.00 | 0.00 | 0.00 | |

¹ Results for other values of d are in the appendix C in table C.1.

² Each entry in the table, in columns 4-9, shows the absolute difference between the mean under contamination and the mean under noncontamination. The means were computed over 1,000 replications. For example, when $d = 0.1$, the uncontaminated mean of the \hat{d}_{GPH} estimator yields a persistence of 0.10 (see column 3, FI(d), and if we estimate it again, but with the trend, the mean of the \hat{d}_{GPH} estimator is 0.57. Therefore, the difference is 0.47. Notice that for the ADF and KPSS tests, we report the mean probability of rejections and the difference between these probabilities between contaminated and noncontaminated processes.

³ In blue are the three estimators or tests with the best performance. Darker blue indicates the best performance. Lower values indicate better performance. Remember that autoregressive model-based estimators are excluded from the race of the top three estimators because they are not designed for long-memory processes. We show them to illustrate that they tend to indicate short-term persistence mistakenly. Some observations might not be colored when their difference is zero, however, it is not because we are not considering them for the analysis, but because the criteria to select the top three estimators used more than two decimals, thus being greater than the colored observations.

6 Discussion and Concluding Remarks

In this paper we compared the robustness of different univariate measures of persistence used in the literature when the data is contaminated with trends, cycles, measurement errors, additive and temporary change outliers, or structural changes. For the analysis, we considered autoregressive (short-memory) and fractionally integrated (long-memory) processes with different parameters ρ and d , respectively. Our results indicate that if the data-generating process is an autoregressive process of order one, AR(1), with parameter ρ and no contamination, the autoregressive estimators, $\hat{\rho}$ and $\hat{\rho}_1$, along with the nonparametric estimator, $\hat{\gamma}$, perform the best at any true value of ρ . The integer order of integration tests, ADF, and $KPSS_\mu$, perform as expected and improve their performance with the sample size. In the presence of contamination processes, the ADF test is the most effective measure of persistence, correctly identifying that persistence is not infinite in almost all cases. However, if a scalar measure is needed, the nonparametric estimator, $\hat{\gamma}$, is the best among the scalar estimators, as it deviates less from the uncontaminated case in all contamination processes analyzed but the temporary change outliers. When the DGP is a fractionally integrated process with parameter d and the process is not contaminated, only the long-memory persistence measures perform correctly, as they are the only ones that consider the long-memory property of the data. Under contamination, if the data belongs to the stationarity region, the ADF test and the $\hat{\gamma}$ estimator perform well. However, if the data belongs to the nonstationary region, the best estimators are the long-memory and the nonparametric one, depending on the contamination and the nominal value of d .

Figure 2 summarizes results based on tables 7 and B.1 for autoregressive processes and tables 12, and C.1 for long-memory processes. In particular, it shows the fraction of times where each estimator ranks among the best three, among long and short-memory processes, under different parameters and anomalies. We refer to said fraction as success rate. Recall

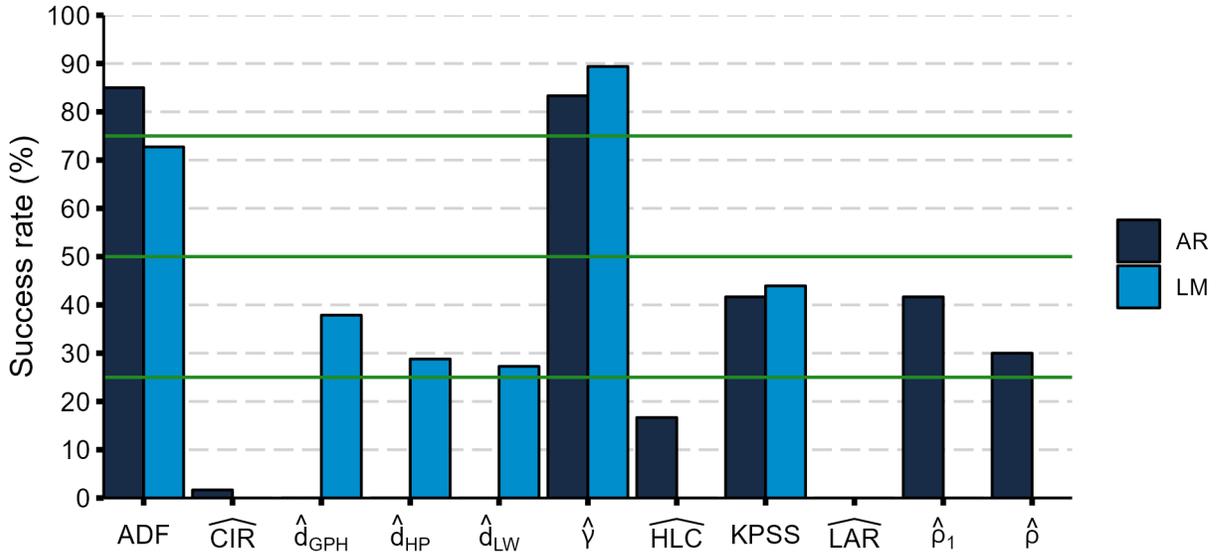


Figure 2: Estimators Performance (Success Rates in %) Under Six Types of Contamination for AR(1) and FI(d) Processes¹

¹ Results based on tables 7, 12, B.1, and C.1. The success rates are computed based on the number of times the estimators appear among the three best estimators. The green horizontal lines indicate 25%, 50%, and 75%.

that the tables were computed using a set of parameters selected such that the identification of the contamination is difficult using an eyeball test. First, note that when the DGP are autoregressive processes, the long-memory model-specific estimators (\hat{d}_{GPH} , \hat{d}_{HP} , and \hat{d}_{LW}) are excluded from the comparison as they are not meant to measure persistence for autoregressive processes. In the same way, when the DGP is a long-memory process, we eliminate the autoregressive model-specific estimators ($\hat{\rho}_1$, $\hat{\rho}$, \widehat{CIR} , \widehat{HLC} and \widehat{LAR}) from the comparison because these are not expected to measure persistence for fractionally integrated processes. Moreover, highly-persistent short-memory time series mislead the long-memory estimators by indicating the presence of long-term persistence, while long-memory time series mislead the autoregressive-based estimators by indicating short-term memory or infinite memory. Therefore, we only compare estimators designed for each DGP. The table illustrates that the order of integration tests, especially the ADF, have the highest success rates. However, as

mentioned in sections 4 and 5, these tests do not yield a scalar estimate of persistence; they only indicate whether the data has infinite persistence or not. Hence, if the researcher is interested in a scalar measure of persistence, the figure shows that the nonparametric estimator, $\hat{\gamma}$, is the best for both processes, with a success rate greater than eighty for both data-generating processes. Third, if the process is autoregressive, the estimators with the highest success rate after the $\hat{\gamma}$ estimator are the $\hat{\rho}$ and $\hat{\rho}_1$ estimators. The other three autoregressive estimators, $\widehat{\text{CIR}}$, $\widehat{\text{HLC}}$, and $\widehat{\text{LAR}}$, do not perform as well as the other two, especially the $\widehat{\text{CIR}}$ and $\widehat{\text{LAR}}$, which have two and zero percent success rate respectively. As a result, both measures should be avoided. If the DGP is a $\text{FI}(d)$, the best estimator after the $\hat{\gamma}$ is the \hat{d}_{GPH} , which has an almost 40 percent success rate. Nonetheless, notice that the \hat{d}_{HP} and \hat{d}_{LW} estimators are near the success rate of \hat{d}_{GPH} . This means that if we had only focused on one of the three long-memory estimators, its success rate would be much higher. Thus, apart from the nonparametric estimator, the long-memory estimators perform well under contaminated long-memory processes.

How should we proceed to measure the persistence of a time series? Based on the results of sections 4 and 5, the desirable thing is to know the underlying process of the time series we are interested in and if an anomaly is present or not. Therefore, we should proceed to test for trends, cycles, or structural changes. Suppose the anomalies found are not of interest to the researcher (because they do not display an intrinsic or underlying property of the time series of interest). In that case, they should be removed if possible. In the ideal case that we can identify and remove possible contamination processes, a more precise scalar, objective persistence estimate can be obtained. To do this step, we start with an eyeball test and then a statistical test to be as certain as possible regarding the series' structure. For example, if an autoregressive process is contaminated with a trend, the trend should be removed, as none of the estimators studied in this paper are constructed to consider the existence of trends.¹⁷ In this particular example, after removing the trend, either the autoregressive or the nonparametric

¹⁷ As discussed in section 1, this may run into interpretation problems.

estimators are the safest. If other types of contamination are detected and can be removed, persistence can be measured with the most appropriate estimator, depending on the underlying process.

Once we have removed the trends, cycles, or structural changes, we should test whether the data-generating process is an autoregressive process with parameter ρ or a fractionally integrated process of order d and whether the data has infinite persistence with the order of integration tests. While we consider these two steps as the most important of the process because they will dictate the set of appropriate persistence estimators available for use, it is important to keep in mind that, with the existing tests, in practice, it may be complex to identify precisely the underlying DGP, particularly for autoregressive processes with high autoregressive parameters. If the researcher can identify the DGP, and for instance, it is an AR(1), the best-performing estimators $\hat{\rho}_1$, $\hat{\rho}$, and $\hat{\gamma}$ should be the tools to consider for the analysis. If, instead, the DGP is an FI(d), the long-memory and $\hat{\gamma}$ estimators should be used. If the type of DGP cannot be easily identified, it would be preferable to use a robust estimator to both of them, which, in case the researcher wants a scalar measure of persistence, it would be $\hat{\gamma}$. The order of integration tests can be applied, according to tables 7 and 12, to almost any series, yielding robust results.

If other types of contamination are detected but cannot be removed, the researcher will have to use the tools at her disposal, and, according to the results in this paper, she should proceed as follows. If a cycle is present, the $\hat{\gamma}$ estimator is the one to choose. Although it might be biased, the results display that it has a lower deviation from the estimated persistence of the uncontaminated series among all estimators. If there is a structural change in the data, any of the model-based or $\hat{\gamma}$ estimators may be used as long as the persistence is estimated before and after the break date and not for the whole series. If there is evidence of outliers (additive or temporary change) in the data, any of the model-based estimators and the $\hat{\gamma}$ estimator can be used, as this type of contamination does not have an apparent effect on the estimators.

However, in practice, the contamination of the series may not be observable by the researcher, so obtaining a clean, uncontaminated time series will be challenging. Moreover, it can also be difficult to distinguish the DGP of the time series of interest. In these cases, the best we can do is to use the most robust measure of persistence. In light of the results shown in this paper, the nonparametric measure of persistence is the best alternative in these cases since it provides, on average, the most robust point measure of persistence in the presence of different types of contamination that are commonly found in real-time series data, and for different DGP. This provides a strong case for researchers to use this estimator as their first alternative. Finally, multiple contaminations may affect the statistical process of interest and persistence statistics. We do not have an answer to how a researcher should proceed in such a case, and we will keep this question open for future research.

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A Auxiliary and Additional Figures

Figure A.1 shows six different autocorrelation functions. On the left-hand side, the autocorrelation functions are for AR(1) processes with parameters $\rho = 0.1, 0.5,$ and 0.95 (all weakly-stationary processes). The autocorrelation functions on the right-hand side are for FI(d) processes with parameters $d = 0.1, 0.45,$ and 0.95 (two weakly stationary and one non-stationary processes). The panels on this figure show that the rate of decay for long-memory processes, FI(d) is lower (hyperbolic rate) than for autoregressive processes (exponential rate). Long-memory processes are long-term persistent processes and autoregressive processes are short-term persistent.

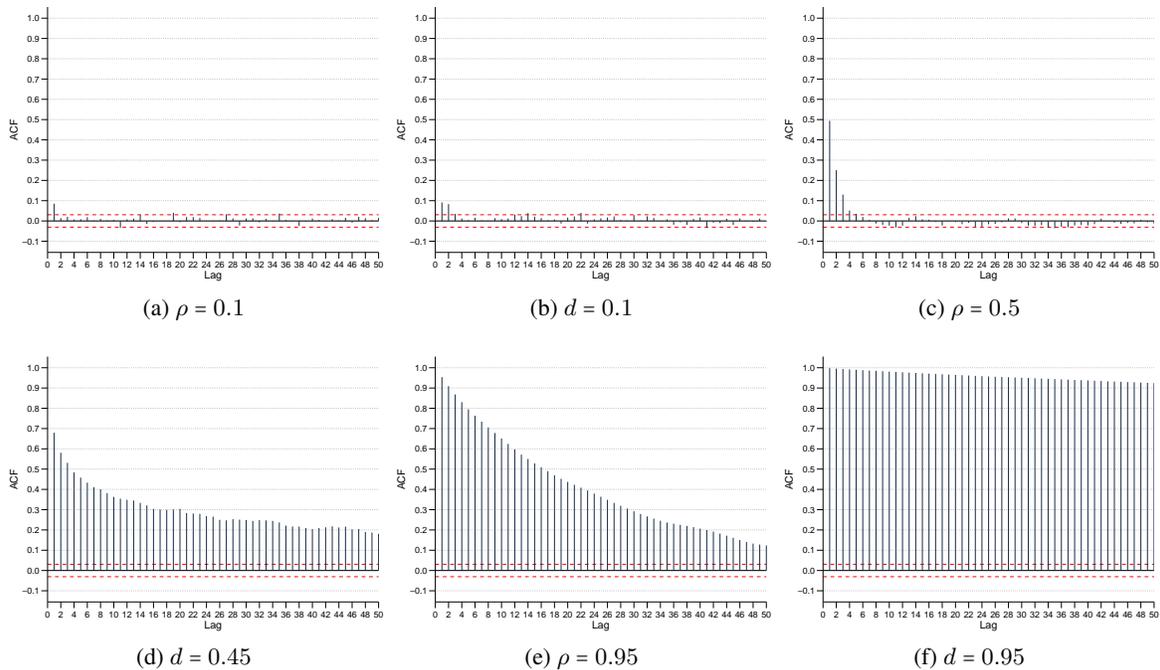


Figure A.1: Autocorrelation Function for Realizations of an AR(1) process with $\rho = 0.1, 0.5,$ and 0.95 (on the left-hand side) and an FI(d) with $d = 0.1, 0.45,$ and 0.95 (on the right-hand side).

Figure A.2 shows the effect of the six types of contaminations on an FI($d = 0.25$). The panels of this figure show that the trend, cycle, and structural change have a visible large contamination effect (gray lines) compared to other contamination mechanisms; that is, the gray line departs from the uncontaminated lines (blue lines).

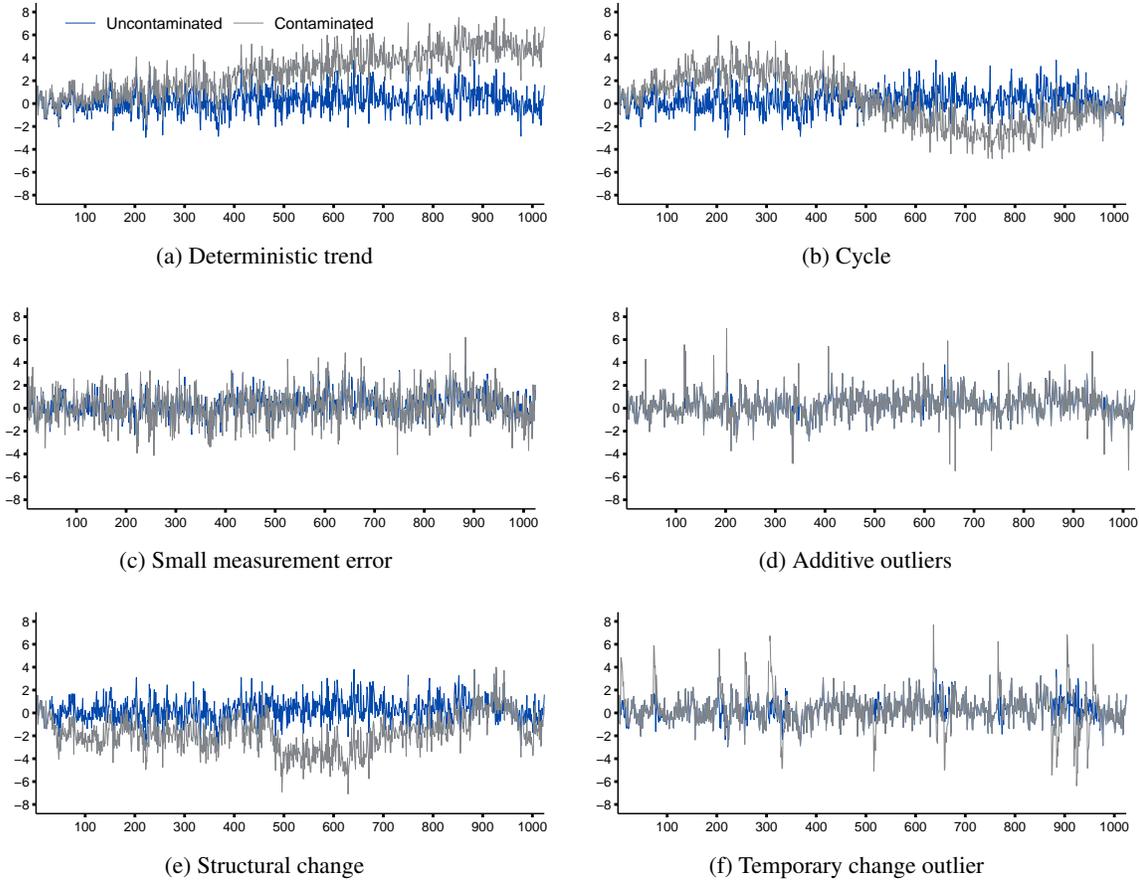


Figure A.2: Long-memory Process Realization with Six Types of Contamination¹

¹ Each time series has 1,024 observations. For panel (a), the slope of the deterministic trend used is 0.005. For panel (b), the amplitude and the frequency of the cycle are three and 0.001 respectively, while for panel (c), the signal-to-noise ratio of the small measurement error takes the value of one. For panel (d), the outlier size used is five. For panel (e), the probability of a structural change is 0.035, while for panel (f), the persistence of the temporary change takes the value of 0.8. The long-memory process is an FI($d = 0.25$).

B Additional Results for Autoregressive (Short-memory) Processes

In this appendix, we show results for the effects of the six contamination mechanisms described in section 3.1. In particular, this appendix shows results for different contamination parameters, $n_{sim}s = 1,000$, and realizations of length $T = 4,096$. We also show results only for two model-specific estimators (ρ_1 and ρ), the nonparametric estimator ($\hat{\gamma}$), and one estimator of long-term persistence (\hat{d}_{GPH}). For the model-specific estimators, we present the RMSE (panel (a) and panel (b)), and for the other two estimators, we present the mean over 1,000 estimators (panel (c) and panel (d)).

Contamination with Deterministic trend. Figure B.1 shows results for slope values between 0.001 and 0.01 with steps of 0.001 and different values of ρ . The results indicate that the model-specific estimators have a higher RMSE or higher deviations from the uncontaminated cases for lower values of ρ . The RMSE or deviations from the uncontaminated cases are small for higher values of ρ .

Contamination with cycles. Figure B.2 presents the results for amplitude values between 0.1 and 1.5 and frequency values between 0 and 0.5, each with steps of 0.1. The results indicate that, among the model-specific estimators, the RMSE is greater for the $\hat{\rho}_1$ estimator. As before, the RMSEs and deviations from the uncontaminated case are higher for lower values of ρ , and higher amplitude values for the nonparametric estimator.

Contamination with small measurement errors. Figure B.3 presents the estimated persistence when the signal-to-noise can take on values between 0.5 and 1.5, with steps of 0.2. The RMSE of the model-specific estimators increases until some point with ρ , then decreases to almost zero when ρ is high. The $\hat{\rho}$ estimator has lower RMSE for any ρ , than the $\hat{\rho}_1$ estimator. The RMSE and deviations increase with the signal-to-noise.

Contamination with additive outliers. Figure B.4 shows the results when the size of the outlier, θ , takes on values between 0 and 1.2 in 0.2 increments. The RMSE and deviations of the estimators from the uncontaminated case are negligible and robust to the value of θ . The estimators are robust to this type of contamination.

Contamination with structural change. Figure B.5 shows the estimated persistence when the outlier size can be either 0.5 or 1 and when the probability of the structural change takes on values between 0 and 0.1. The RMSE of the model-specific estimators increases with the size of the structural change but decreases with the level of $\hat{\rho}$; the estimators appear to be robust to the probability of the structural change. The deviations from the uncontaminated case are also due to the value of θ and irrespective of the probability.

Contamination with temporary change outlier. Figure B.6 shows the persistence estimations when the temporary change outlier contaminating the process takes on two values of θ , 1 and 1.5, and α values between 0 and 1 in 0.1 increments- The autoregressive estimators are only affected when $\alpha = 1$ and the RMSE decreases with the value of ρ . The $\hat{\rho}_1$ has the lower RMSEs. The nonparametric and long-memory estimators deviate from the uncontaminated case for lower α .

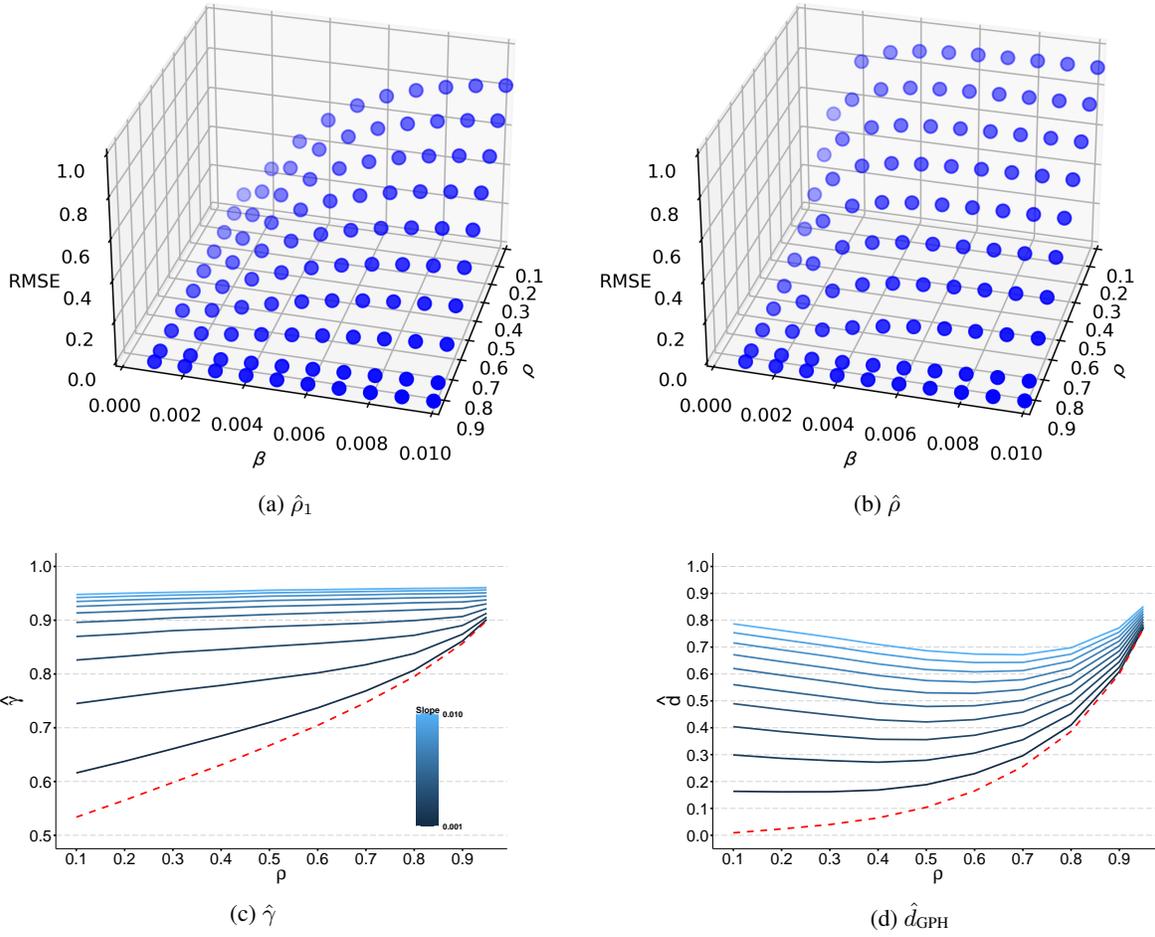


Figure B.1: Performance of $\hat{\rho}$, $\hat{\rho}_1$, $\hat{\gamma}$, and \hat{d}_{GPH} Persistence Estimators for Autoregressive Processes of Order One with Parameter ρ Contaminated with a Deterministic Trend with Different Slopes¹

¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, $\hat{\rho}_1$ and $\hat{\rho}$, and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the long-memory estimator, \hat{d}_{GPH} .

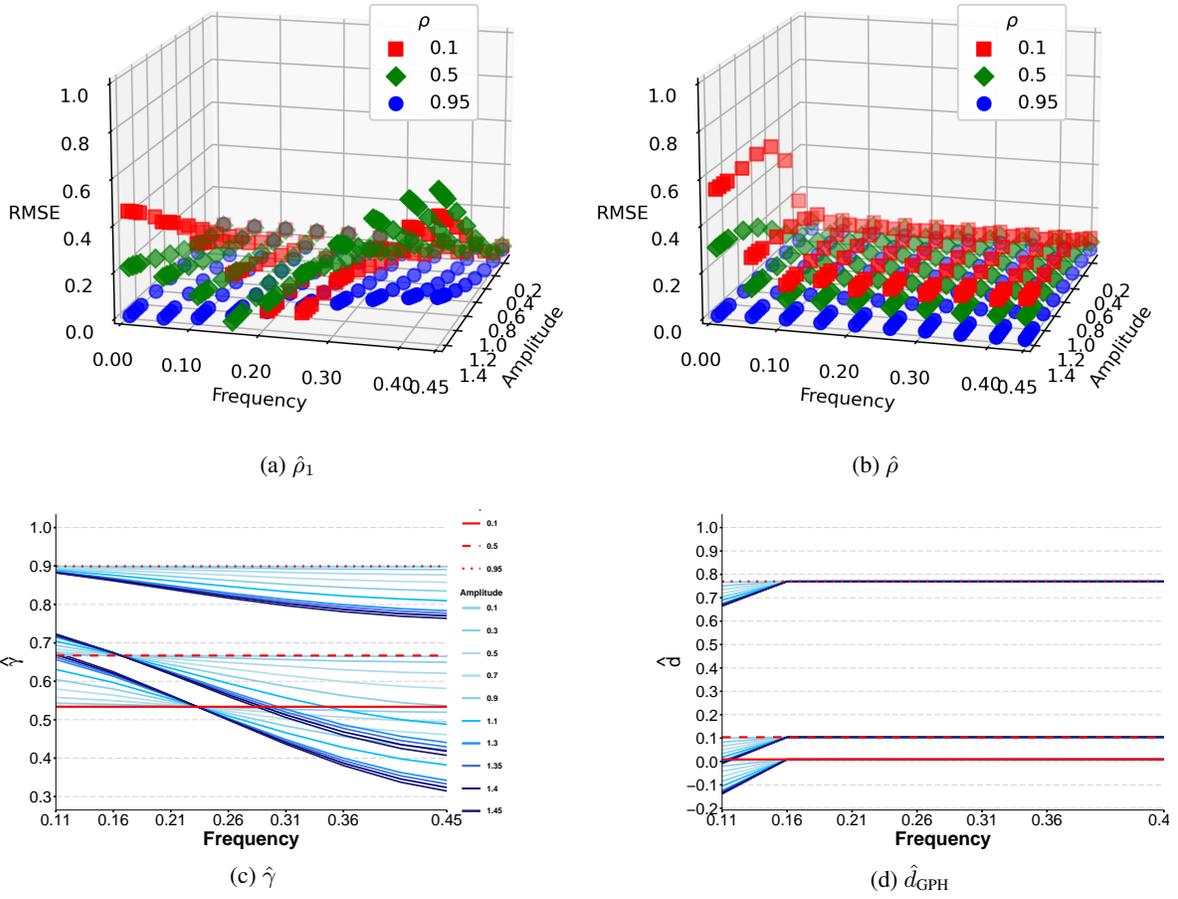
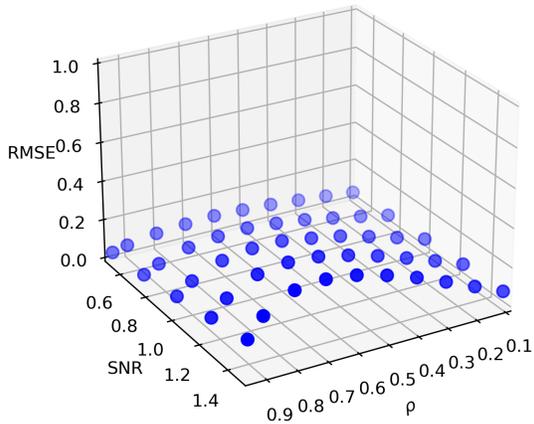
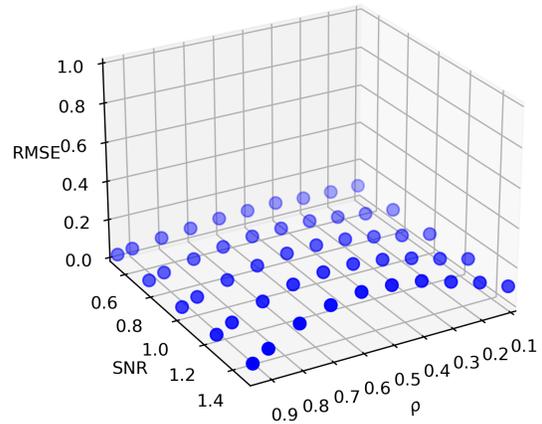


Figure B.2: Performance of $\hat{\rho}$, $\hat{\rho}_1$, $\hat{\gamma}$, and \hat{d}_{GPH} Persistence Estimators for Three Autoregressive Processes of Order One with Parameters $\rho = 0.1, 0.5$, and 0.95 Contaminated with a Deterministic Cycle with Multiple Amplitudes and Frequencies¹

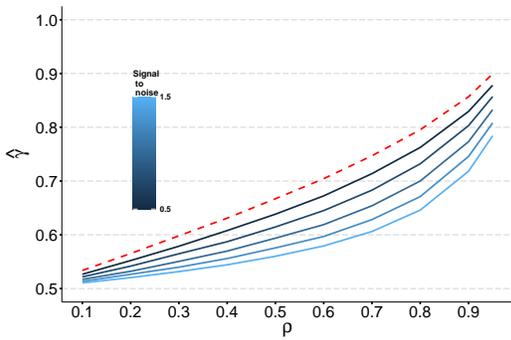
¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, $\hat{\rho}_1$ and $\hat{\rho}$, and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the long-memory estimator, \hat{d}_{GPH} .



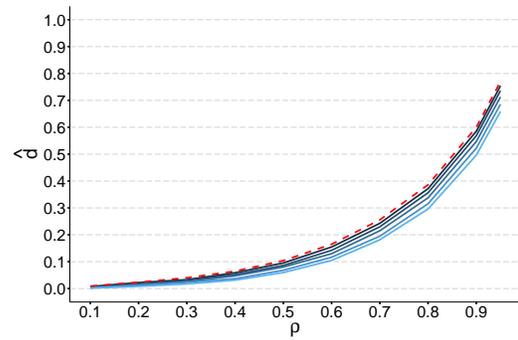
(a) $\hat{\rho}_1$



(b) $\hat{\rho}$



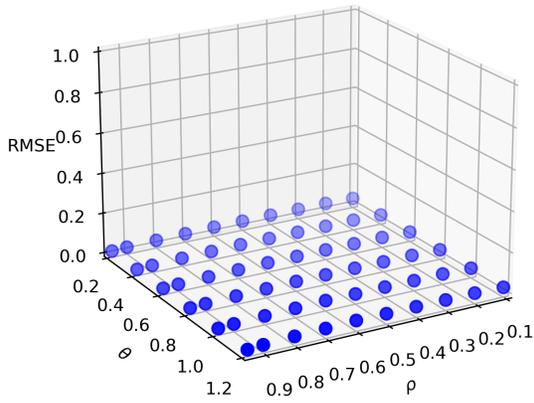
(c) $\hat{\gamma}$



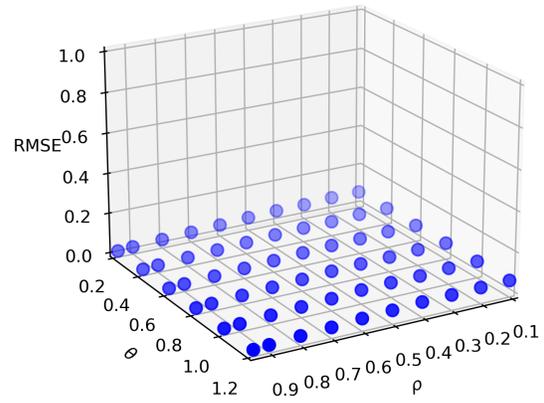
(d) \hat{d}_{GPH}

Figure B.3: Performance of $\hat{\rho}$, $\hat{\rho}_1$, $\hat{\gamma}$, and \hat{d}_{GPH} Persistence Estimators for Autoregressive Processes of Order One with Parameter ρ Contaminated with a Small Measurement Error¹

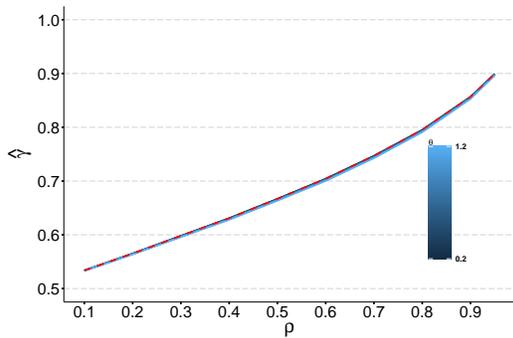
¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, $\hat{\rho}_1$ and $\hat{\rho}$, and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the long-memory estimator, \hat{d}_{GPH} .



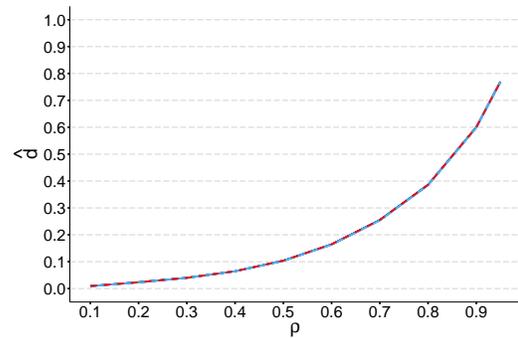
(a) $\hat{\rho}_1$



(b) $\hat{\rho}$



(c) $\hat{\gamma}$



(d) \hat{d}_{GPH}

Figure B.4: Performance of $\hat{\rho}$, $\hat{\rho}_1$, $\hat{\gamma}$, and \hat{d}_{GPH} Persistence Estimators for Autoregressive Processes of Order One with Parameter ρ Contaminated with an Additive Outlier¹

¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, $\hat{\rho}_1$ and $\hat{\rho}$, and the estimated value for the nonparametric estimator, $\hat{\gamma}$ and the long-memory estimator, \hat{d}_{GPH} .

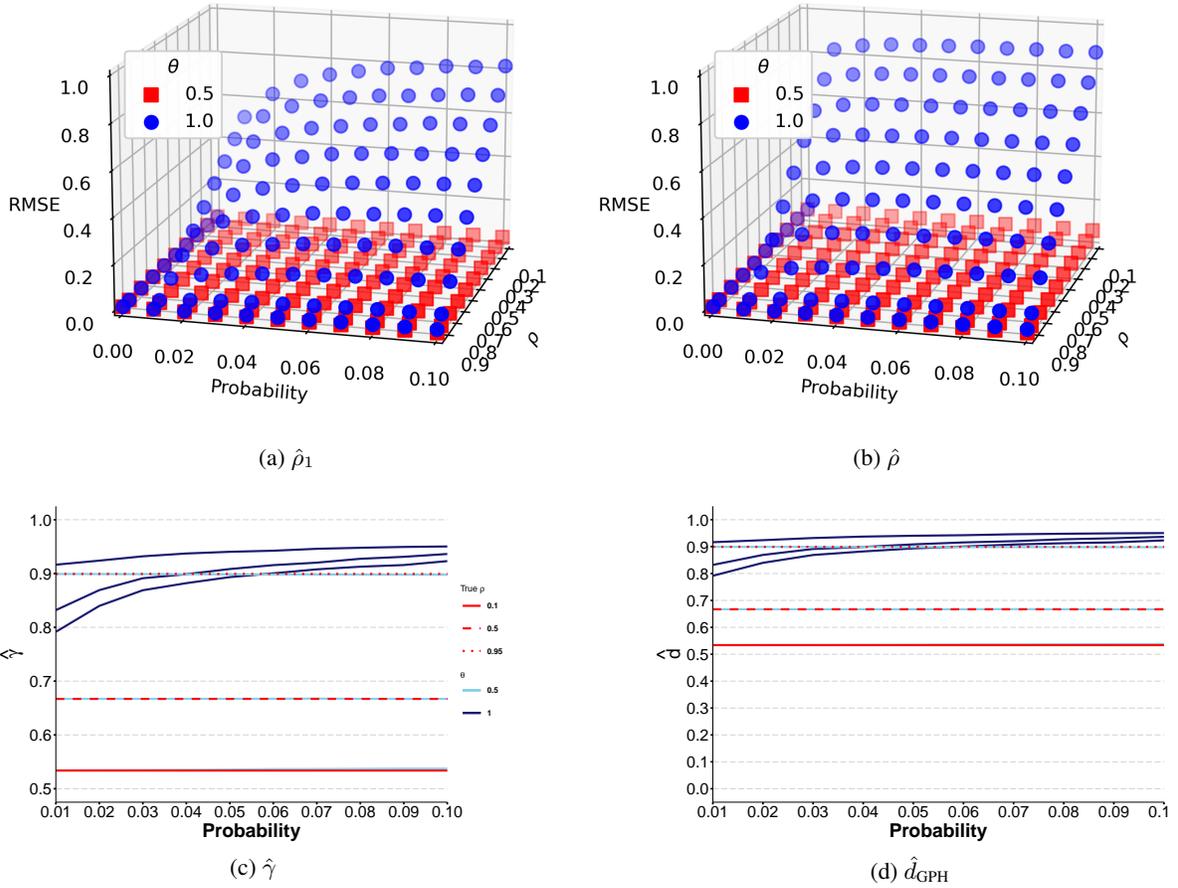


Figure B.5: Performance of $\hat{\rho}$, $\hat{\rho}_1$, $\hat{\gamma}$, and \hat{d}_{GPH} Persistence Estimators for Autoregressive Processes of Order One with Parameter ρ Contaminated with a Structural Change¹

¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, $\hat{\rho}_1$ and $\hat{\rho}$, and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the long-memory estimator, \hat{d}_{GPH} .

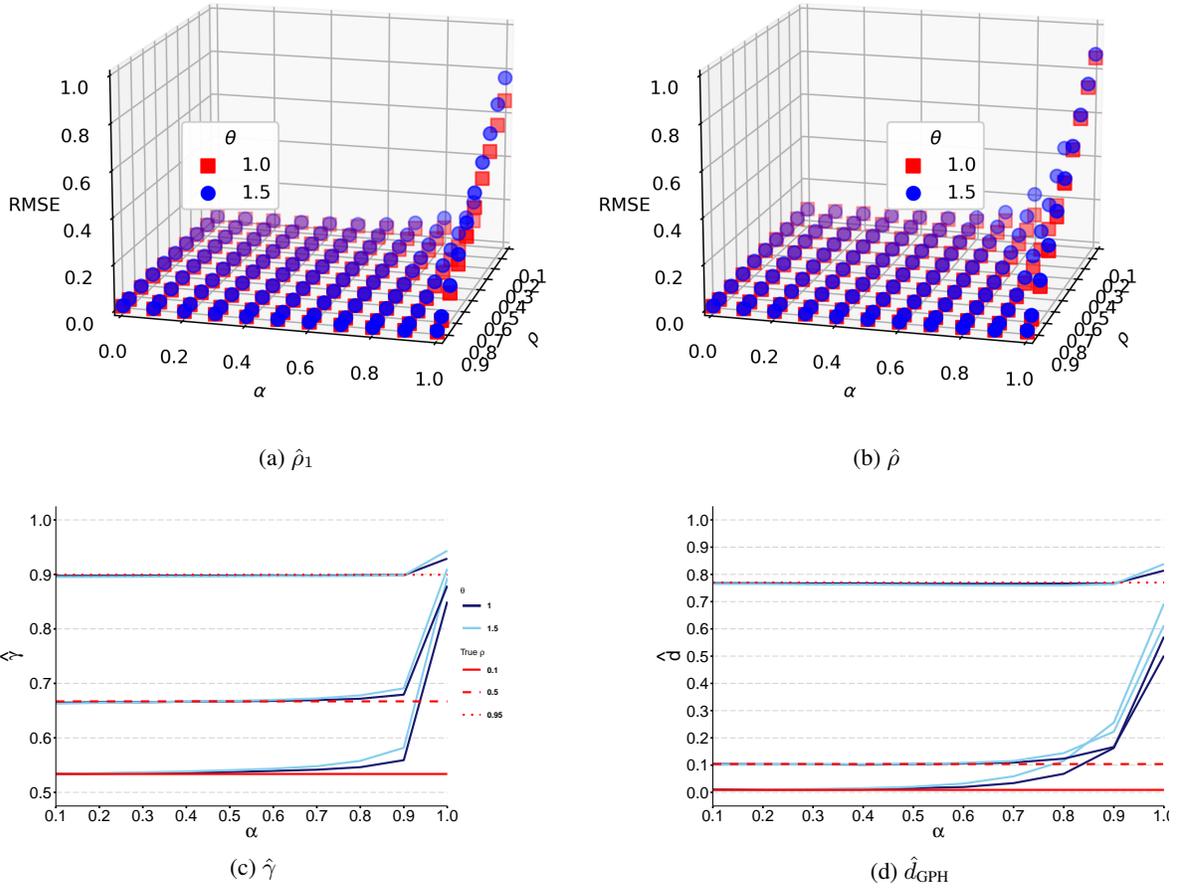


Figure B.6: Performance of $\hat{\rho}$, $\hat{\rho}_1$, $\hat{\gamma}$, and \hat{d}_{GPH} Persistence Estimators for Autoregressive Processes of Order One with Parameter ρ Contaminated with a Temporary Change Outlier¹

¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, $\hat{\rho}_1$ and $\hat{\rho}$, and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the long-memory estimator, \hat{d}_{GPH} .

Table B.1: Estimators or Tests for Time-series Persistence for Autoregressive Processes of Order One with Parameter ρ^1

| Parameter | Estimator or Test | AR(1) | Contamination (Difference between contaminated and uncontaminated estimators, AR(p), in absolute value) | | | | | |
|----------------|-------------------|-------|---|-----------------------|-----------|-------------------------------|----------------------------|--------------------------------|
| | | | Trend | Cycle | SME | AO | SC | TCO |
| | | | ($\beta = 0.005$) | ($A = 1, f = 0.01$) | (SNR = 1) | ($\theta = 1.2, p = 0.025$) | ($\theta = 1, p = 0.06$) | ($\theta = 1, \alpha = 0.5$) |
| $\rho = 0.2$ | $\hat{\rho}_1$ | 0.20 | 0.54 | 0.29 | 0.14 | 0.01 | 0.68 | 0.01 |
| | $\hat{\rho}$ | 0.19 | 0.78 | 0.50 | 0.14 | 0.00 | 0.78 | 0.01 |
| | \widehat{CIR} | 1.25 | 2.62 | 0.73 | 0.18 | 0.01 | 12.33 | 0.01 |
| | \widehat{LAR} | 5.14 | 3.79 | 3.10 | 94.48 | 0.20 | 3.99 | 0.23 |
| | \widehat{HLC} | -3.90 | 2.64 | 2.16 | 65.50 | 0.14 | 2.79 | 0.16 |
| | $\hat{\gamma}$ | 0.57 | 0.33 | 0.12 | 0.04 | 0.00 | 0.34 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.84 | 0.00 |
| | KPSS $_{\mu}$ | 0.05 | 0.95 | 0.05 | 0.00 | 0.00 | 0.91 | 0.00 |
| | \hat{d}_{GPH} | 0.02 | 0.54 | 0.17 | 0.01 | 0.02 | 0.69 | 0.03 |
| | \hat{d}_{HP} | 0.01 | 0.04 | 0.48 | -0.01 | 0.01 | 0.09 | 0.02 |
| \hat{d}_{LW} | 0.01 | 0.52 | 0.47 | -0.01 | 0.01 | 0.68 | 0.01 | |
| $\rho = 0.4$ | $\hat{\rho}_1$ | 0.40 | 0.39 | 0.20 | 0.26 | 0.01 | 0.50 | 0.00 |
| | $\hat{\rho}$ | 0.39 | 0.58 | 0.37 | 0.23 | 0.01 | 0.58 | 0.00 |
| | \widehat{CIR} | 1.66 | 3.06 | 0.85 | 0.50 | 0.03 | 13.44 | 0.01 |
| | \widehat{LAR} | 2.53 | 1.26 | 0.86 | 5.38 | 0.08 | 1.41 | 0.02 |
| | \widehat{HLC} | -2.08 | 0.89 | 0.61 | 3.74 | 0.05 | 1.00 | 0.01 |
| | $\hat{\gamma}$ | 0.63 | 0.28 | 0.09 | 0.09 | 0.00 | 0.28 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.72 | 0.00 |
| | KPSS $_{\mu}$ | 0.04 | 0.96 | 0.04 | 0.02 | 0.00 | 0.92 | 0.00 |
| | \hat{d}_{GPH} | 0.06 | 0.49 | 0.17 | 0.03 | 0.06 | 0.63 | 0.07 |
| | \hat{d}_{HP} | 0.06 | 0.13 | 0.43 | 0.02 | 0.06 | 0.17 | 0.06 |
| \hat{d}_{LW} | 0.05 | 0.48 | 0.42 | 0.02 | 0.05 | 0.63 | 0.05 | |
| $\rho = 0.6$ | $\hat{\rho}_1$ | 0.60 | 0.24 | 0.11 | 0.35 | 0.01 | 0.32 | 0.00 |
| | $\hat{\rho}$ | 0.59 | 0.37 | 0.19 | 0.26 | 0.01 | 0.38 | 0.00 |
| | \widehat{CIR} | 2.48 | 3.50 | 0.97 | 1.16 | 0.08 | 15.71 | 0.01 |
| | \widehat{LAR} | 1.68 | 0.48 | 0.27 | 2.52 | 0.04 | 0.59 | 0.01 |
| | \widehat{HLC} | -1.49 | 0.34 | 0.19 | 1.76 | 0.03 | 0.42 | 0.00 |
| | $\hat{\gamma}$ | 0.70 | 0.21 | 0.05 | 0.13 | 0.00 | 0.21 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.51 | 0.00 |
| | KPSS $_{\mu}$ | 0.07 | 0.93 | 0.07 | 0.01 | 0.00 | 0.88 | 0.00 |
| | \hat{d}_{GPH} | 0.16 | 0.48 | 0.24 | 0.10 | 0.16 | 0.61 | 0.16 |
| | \hat{d}_{HP} | 0.17 | 0.28 | 0.41 | 0.10 | 0.17 | 0.34 | 0.17 |
| \hat{d}_{LW} | 0.15 | 0.46 | 0.40 | 0.09 | 0.15 | 0.61 | 0.15 | |
| $\rho = 0.7$ | $\hat{\rho}_1$ | 0.70 | 0.16 | 0.07 | 0.38 | 0.01 | 0.23 | 0.00 |
| | $\hat{\rho}$ | 0.70 | 0.25 | 0.10 | 0.24 | 0.01 | 0.27 | 0.00 |
| | \widehat{CIR} | 3.33 | 3.69 | 1.01 | 1.85 | 0.13 | 16.40 | 0.04 |
| | \widehat{LAR} | 1.43 | 0.27 | 0.13 | 1.70 | 0.03 | 0.35 | 0.01 |
| | \widehat{HLC} | -1.31 | 0.19 | 0.09 | 1.20 | 0.02 | 0.25 | 0.01 |
| | $\hat{\gamma}$ | 0.75 | 0.17 | 0.04 | 0.14 | 0.00 | 0.18 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| | KPSS $_{\mu}$ | 0.07 | 0.93 | 0.06 | 0.00 | 0.00 | 0.86 | 0.00 |
| | \hat{d}_{GPH} | 0.26 | 0.50 | 0.31 | 0.18 | 0.25 | 0.62 | 0.25 |
| | \hat{d}_{HP} | 0.27 | 0.40 | 0.44 | 0.19 | 0.27 | 0.47 | 0.27 |
| \hat{d}_{LW} | 0.26 | 0.47 | 0.43 | 0.17 | 0.25 | 0.61 | 0.25 | |
| $\rho = 0.9$ | $\hat{\rho}_1$ | 0.90 | 0.03 | 0.01 | 0.27 | 0.01 | 0.06 | 0.00 |
| | $\hat{\rho}$ | 0.89 | 0.04 | 0.01 | 0.11 | 0.00 | 0.06 | 0.00 |
| | \widehat{CIR} | 9.77 | 4.22 | 1.14 | 7.10 | 0.56 | 18.16 | 0.24 |
| | \widehat{LAR} | 1.12 | 0.04 | 0.01 | 0.50 | 0.01 | 0.07 | 0.00 |
| | \widehat{HLC} | -1.08 | 0.03 | 0.01 | 0.35 | 0.01 | 0.05 | 0.00 |
| | $\hat{\gamma}$ | 0.86 | 0.07 | 0.01 | 0.14 | 0.00 | 0.08 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| | KPSS $_{\mu}$ | 0.14 | 0.86 | 0.04 | 0.01 | 0.00 | 0.75 | 0.01 |
| | \hat{d}_{GPH} | 0.60 | 0.69 | 0.62 | 0.50 | 0.60 | 0.76 | 0.60 |
| | \hat{d}_{HP} | 0.67 | 0.70 | 0.71 | 0.54 | 0.66 | 0.75 | 0.66 |
| \hat{d}_{LW} | 0.65 | 0.68 | 0.69 | 0.52 | 0.65 | 0.74 | 0.64 | |

¹ See notes in table 7.

C Additional Results for Fractionally Integrated (long-memory) Processes

Contamination with deterministic trend. Figure C.1 shows results for slope values between 0.001 and 0.01 with steps of 0.001 and different values of d . The results indicate that the model-specific estimators have a higher RMSE or higher deviations from the uncontaminated cases for lower values of d . The RMSE or deviations from the uncontaminated cases are small for higher values of ρ . The most robust estimator, in this case, according to the figure is the \hat{d}_{HP} estimator.

Contamination with cycles. Figure C.2 shows the results for a cycle-contaminated process with amplitudes from 0.2 to 1.5, and frequency values between 0 and 0.5. The model-specific parameters increase their RMSE when the cycle's frequency and the level of d are low. The deviations of the nonparametric and the autoregressive estimators increase with the cycle's amplitude.

Contamination with small measurement errors. Figure C.3 shows the results when the measurement error takes on multiple signal-to-noise ratio values, specifically from 0.5 to 1.5. The RMSE of the long-memory estimators increases with the signal-to-noise and the level of d ; however, the \hat{d}_{GPH} estimator displays RMSEs near zero for any values of the snr and d . The deviation from the uncontaminated case of the $\hat{\gamma}$ estimator increases with the signal-to-noise and underestimates persistence, in contrast to previous contaminations.

Contamination with additive outliers. Figure C.4 shows the results for when the size of the outlier changes and takes on values between 0.2 and 1.2. The long-memory estimators yield very low RMSEs, which only increase when d is high and the estimators are the \hat{d}_{HP} . The \hat{d}_{GPH} estimator yields very low RMSEs irrespective of the size of the outlier. The nonparametric and autoregressive estimators deviate little from the uncontaminated case; however, the latter might not provide the correct information.

Contamination with structural change. Figure C.5 shows the results when the structural change can take on different probabilities and sizes. The RMSE of the long-memory estimators is lower for high values of d and higher for increasing values of θ . The \hat{d}_{HP} estimator display the lowest RMSE for lower values of d . The deviations from the uncontaminated case of the nonparametric estimator also increase with the size of the structural change.

Contamination with temporary change outliers. Figure C.6 shows the estimated persistence when the temporary change outlier can take on different values of size, θ , and the duration, α , which lies between 0 and 1. The RMSEs of the long-memory estimators are the lowest, only increasing when the α is one or the value of d is high.

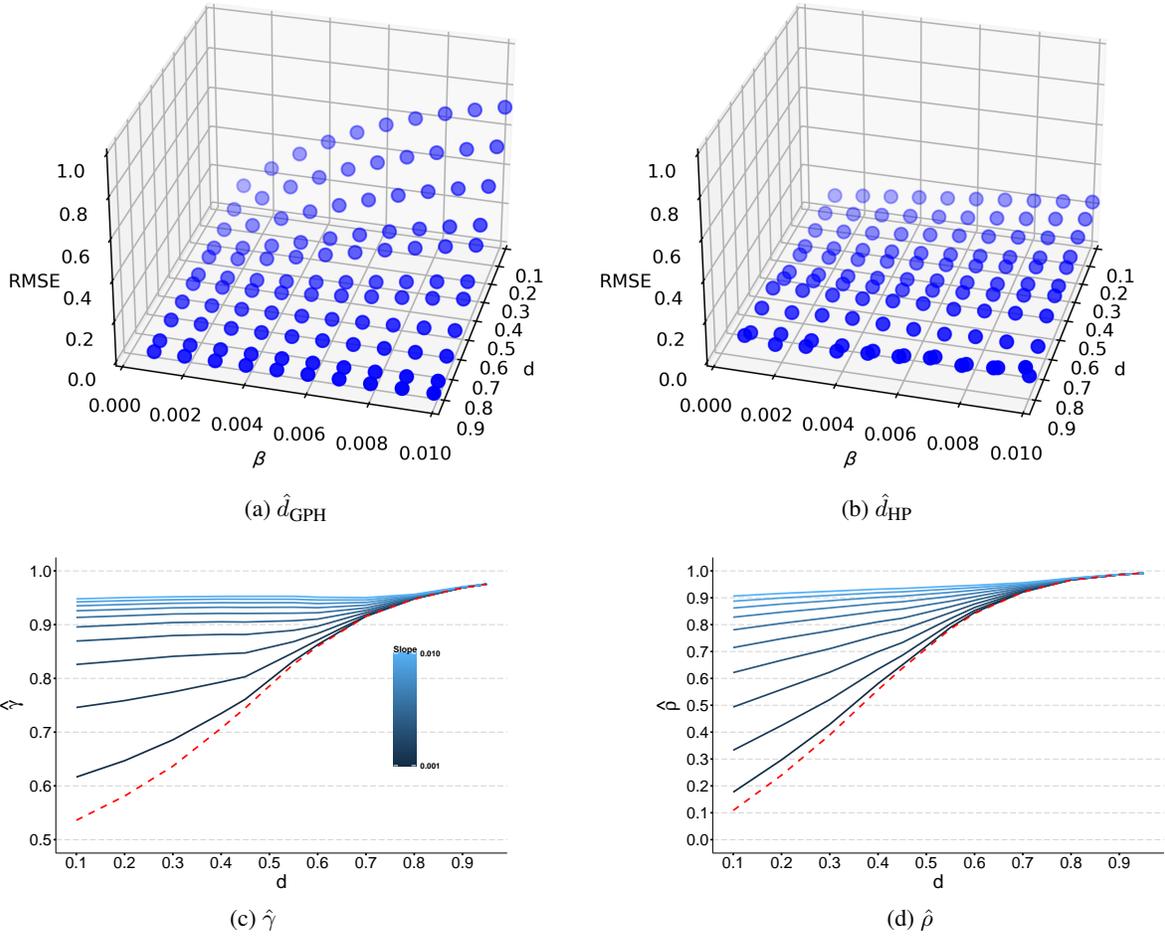


Figure C.1: Performance of \hat{d}_{GPH} , \hat{d}_{HP} , $\hat{\gamma}$, and $\hat{\rho}$ Estimators of Persistence for Fractionally Integrated Processes with Parameter d Contaminated with a Deterministic Trend¹

¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, \hat{d}_{GPH} and \hat{d}_{HP} , and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the autoregressive estimator, $\hat{\rho}$.

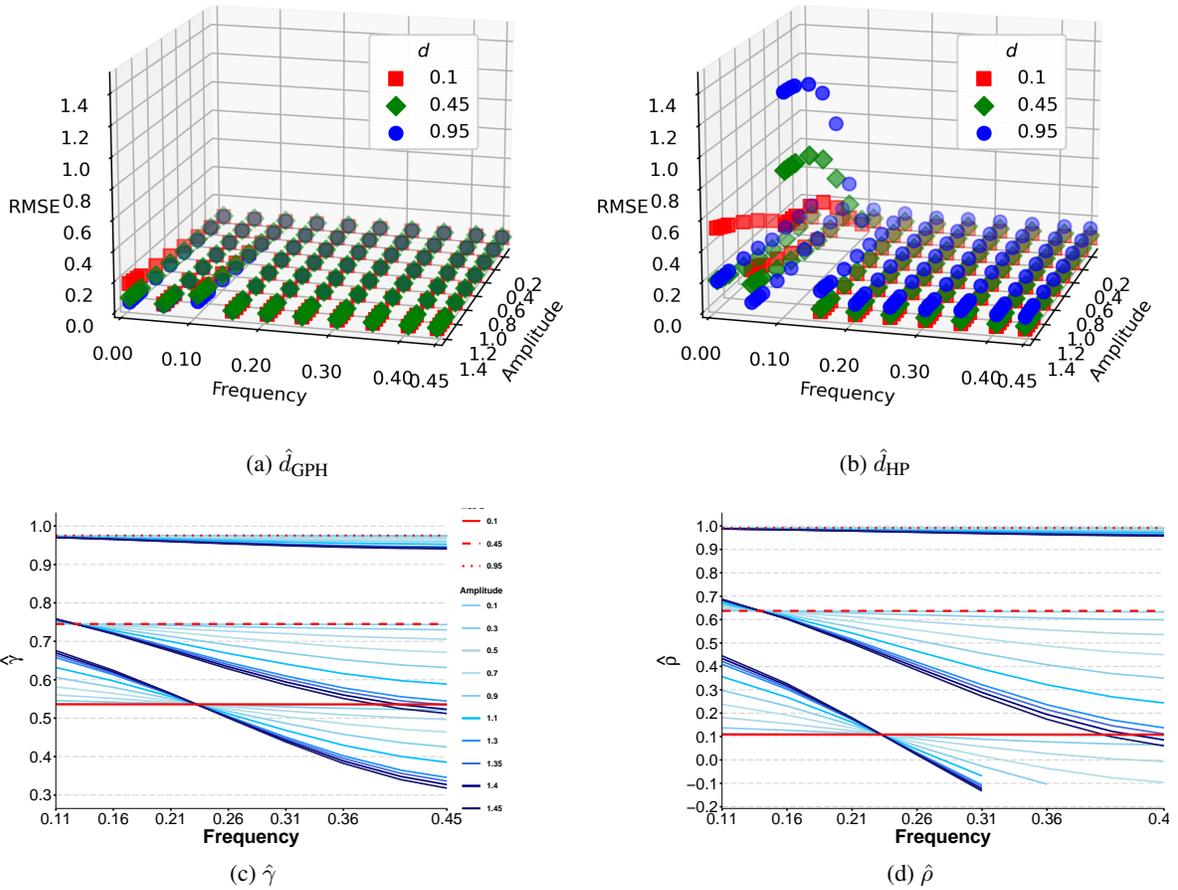
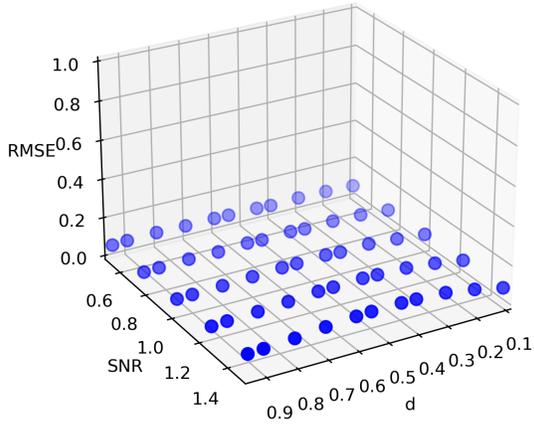
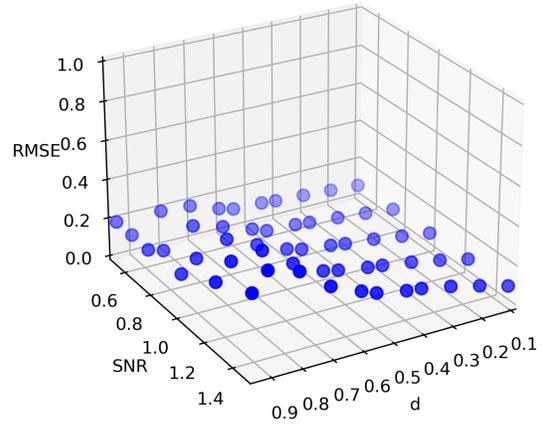


Figure C.2: Performance of \hat{d}_{GPH} , \hat{d}_{HP} , $\hat{\gamma}$, and $\hat{\rho}$ Estimators of Persistence for Fractionally Integrated Processes with Parameter d Contaminated with a Deterministic Cycle¹

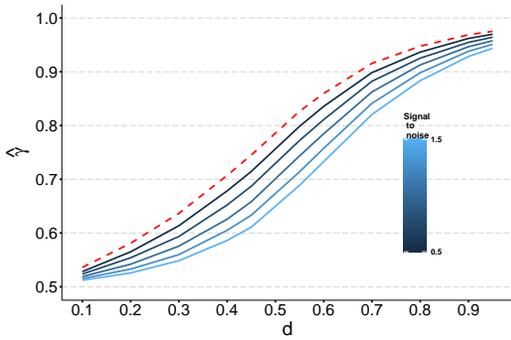
¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, \hat{d}_{GPH} and \hat{d}_{HP} , and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the autoregressive estimator, $\hat{\rho}$.



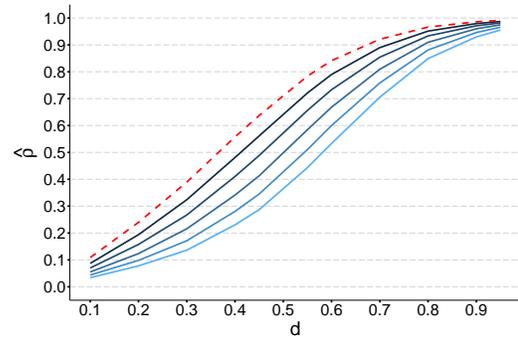
(a) \hat{d}_{GPH}



(b) \hat{d}_{HP}



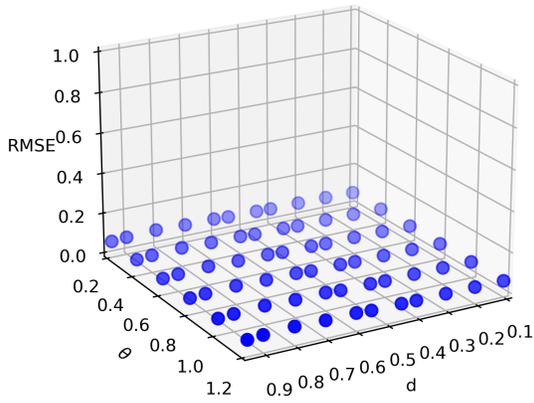
(c) $\hat{\gamma}$



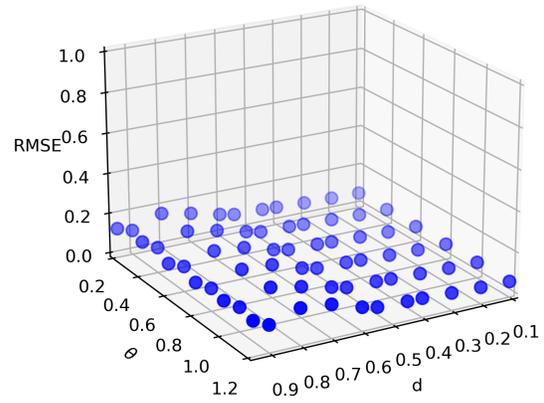
(d) $\hat{\rho}$

Figure C.3: Performance of \hat{d}_{GPH} , \hat{d}_{HP} , $\hat{\gamma}$, and $\hat{\rho}$ Estimators of Persistence for Fractionally Integrated Processes with Parameter d Contaminated with a Small Measurement Error¹

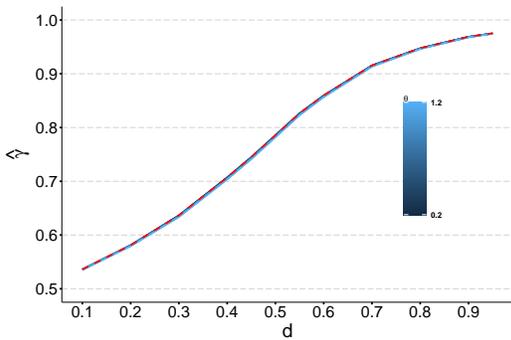
¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, \hat{d}_{GPH} and \hat{d}_{HP} , and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the autoregressive estimator, $\hat{\rho}$.



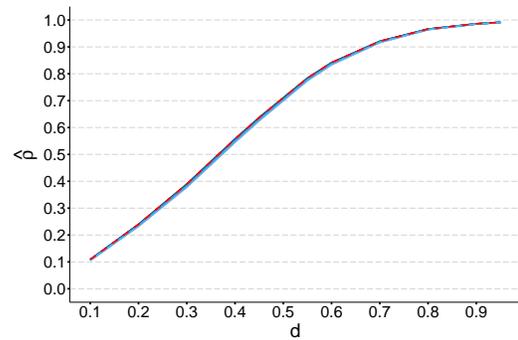
(a) \hat{d}_{GPH}



(b) \hat{d}_{HP}



(c) $\hat{\gamma}$



(d) $\hat{\rho}$

Figure C.4: Performance of \hat{d}_{GPH} , \hat{d}_{HP} , $\hat{\gamma}$, and $\hat{\rho}$ Estimators of Persistence for Fractionally Integrated Processes with Parameter d Contaminated with an Additive Outlier¹

¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, \hat{d}_{GPH} and \hat{d}_{HP} , and the estimated value for the nonparametric estimator, $\hat{\gamma}$, and the autoregressive estimator, $\hat{\rho}$.

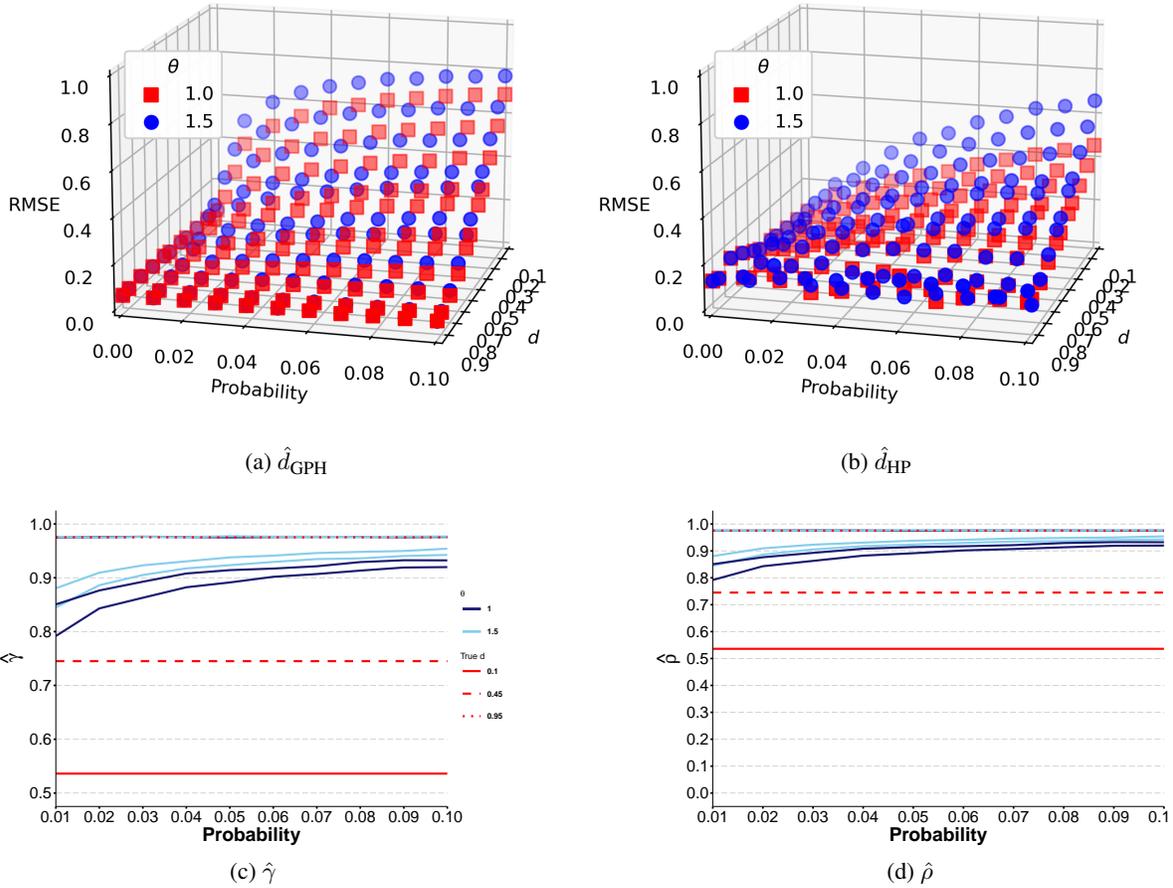
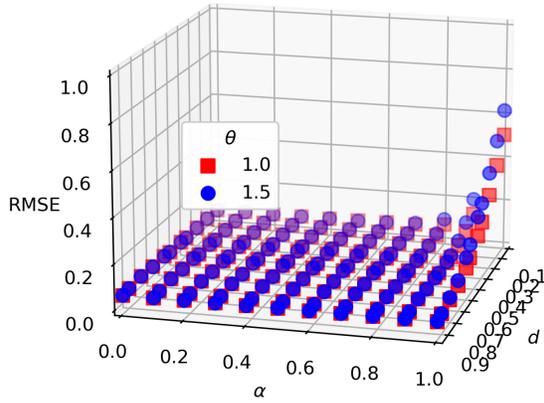
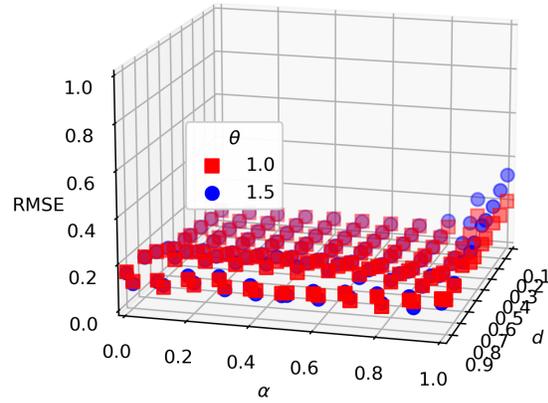


Figure C.5: Performance of \hat{d}_{GPH} , \hat{d}_{HP} , $\hat{\gamma}$, and $\hat{\rho}$ Estimators of Persistence for Fractionally Integrated Processes with Parameter d Contaminated with a Structural Change¹

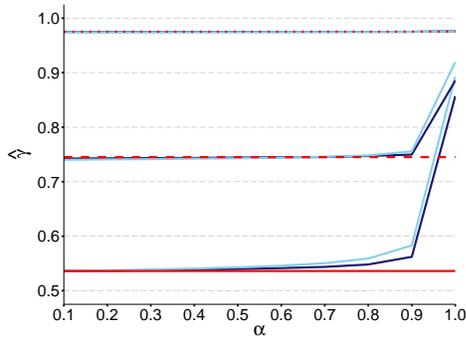
¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, \hat{d}_{GPH} and \hat{d}_{HP} , and the estimated value for the nonparametric estimator, $\hat{\gamma}$ and the autoregressive estimator, $\hat{\rho}$.



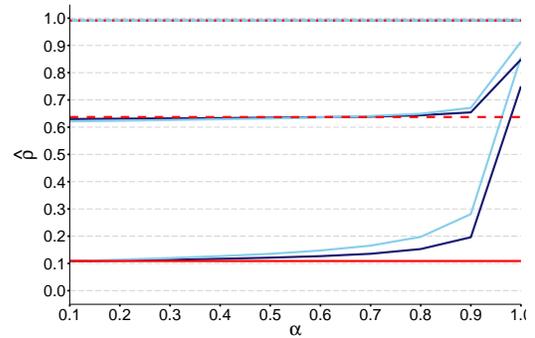
(a) \hat{d}_{GPH}



(b) \hat{d}_{HP}



(c) $\hat{\gamma}$



(d) $\hat{\rho}$

Figure C.6: Performance of \hat{d}_{GPH} , \hat{d}_{HP} , $\hat{\gamma}$, and $\hat{\rho}$ Estimators of Persistence for Fractionally Integrated Processes with Parameter d Contaminated with a Temporary Change Outlier¹

¹ Calculations based on 1,000 realizations of a time series with 4,096 observations. We show the RMSE for the model-specific estimators, \hat{d}_{GPH} and \hat{d}_{HP} , and the estimated value for the nonparametric estimator, $\hat{\gamma}$ and the autoregressive estimator, $\hat{\rho}$.

Table C.1: Estimators or Tests for Time-series Persistence for Fractional Integrated Processes with Parameter d ¹

| Parameter | Estimator or Test | FI(d) | Contamination (Difference between contaminated and uncontaminated estimators, FI(d), in absolute value) ^{1,2} | | | | | |
|------------|-------------------|-----------|--|--------------------------------|------------------|-------------------------------------|---------------------------------|---------------------------------------|
| | | | Trend ($\beta = 0.005$) | Cycle ($A = 1, f = 0.01$) | SME (SNR = 1) | AO ($\theta = 1.2, p = 0.025$) | SC ($\theta = 1, p = 0.6$) | TCO ($\theta = 1, \alpha = 0.5$) |
| $d = 0.2$ | $\hat{\rho}_1$ | 0.24 | 0.75 | 0.51 | 0.08 | 0.23 | 0.88 | 0.25 |
| | $\hat{\rho}$ | 0.44 | 0.97 | 0.75 | 0.17 | 0.43 | 0.97 | 0.44 |
| | \hat{CIR} | 1.32 | 3.99 | 2.05 | 1.09 | 1.31 | 13.38 | 1.33 |
| | \hat{LAR} | 4.27 | 1.34 | 1.97 | 23.87 | 4.41 | 1.14 | 4.13 |
| | \hat{HLC} | -3.29 | -1.24 | -1.69 | -16.89 | -3.39 | -1.10 | -3.20 |
| | $\hat{\gamma}$ | 0.58 | 0.32 | 0.11 | 0.06 | 0.00 | 0.32 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.84 | 0.00 |
| | KPSS $_{\mu}$ | 0.27 | 0.73 | 0.26 | 0.09 | 0.01 | 0.69 | 0.01 |
| | \hat{d}_{GPH} | 0.20 | 0.37 | 0.10 | 0.10 | 0.00 | 0.51 | 0.00 |
| | \hat{d}_{HP} | 0.18 | 0.00 | 0.35 | 0.10 | 0.00 | 0.02 | 0.00 |
| | \hat{d}_{LW} | 0.18 | 0.36 | 0.33 | 0.10 | 0.00 | 0.52 | 0.00 |
| $d = 0.4$ | $\hat{\rho}_1$ | 0.56 | 0.81 | 0.68 | 0.23 | 0.55 | 0.91 | 0.56 |
| | $\hat{\rho}$ | 0.77 | 0.96 | 0.85 | 0.57 | 0.77 | 0.97 | 0.77 |
| | \hat{CIR} | 2.29 | 5.53 | 3.16 | 1.30 | 2.23 | 16.60 | 2.28 |
| | \hat{LAR} | 1.81 | 1.24 | 1.48 | 4.53 | 1.85 | 1.11 | 1.81 |
| | \hat{HLC} | -1.57 | -1.17 | -1.34 | -3.47 | -1.60 | -1.08 | -1.58 |
| | $\hat{\gamma}$ | 0.71 | 0.20 | 0.05 | 0.12 | 0.00 | 0.21 | 0.00 |
| | ADF | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.71 | 0.00 |
| | KPSS $_{\mu}$ | 0.57 | 0.43 | 0.27 | 0.05 | 0.00 | 0.39 | 0.00 |
| | \hat{d}_{GPH} | 0.40 | 0.18 | 0.06 | 0.14 | 0.00 | 0.31 | 0.01 |
| | \hat{d}_{HP} | 0.37 | 0.00 | 0.19 | 0.16 | 0.00 | 0.03 | 0.01 |
| | \hat{d}_{LW} | 0.39 | 0.18 | 0.16 | 0.13 | 0.00 | 0.31 | 0.01 |
| $d = 0.55$ | $\hat{\rho}_1$ | 0.78 | 0.87 | 0.82 | 0.44 | 0.77 | 0.93 | 0.78 |
| | $\hat{\rho}$ | 0.89 | 0.95 | 0.91 | 0.79 | 0.89 | 0.97 | 0.89 |
| | \hat{CIR} | 4.90 | 8.56 | 5.87 | 1.83 | 4.68 | 21.13 | 4.83 |
| | \hat{LAR} | 1.28 | 1.16 | 1.22 | 2.32 | 1.30 | 1.08 | 1.29 |
| | \hat{HLC} | -1.20 | -1.11 | -1.16 | -1.94 | -1.21 | -1.06 | -1.20 |
| | $\hat{\gamma}$ | 0.83 | 0.08 | 0.01 | 0.14 | 0.00 | 0.10 | 0.00 |
| | ADF | 0.97 | 0.00 | 0.00 | 0.03 | 0.00 | 0.57 | 0.00 |
| | KPSS $_{\mu}$ | 0.72 | 0.26 | 0.10 | 0.02 | 0.00 | 0.21 | 0.01 |
| | \hat{d}_{GPH} | 0.56 | 0.08 | 0.04 | 0.14 | 0.00 | 0.18 | 0.01 |
| | \hat{d}_{HP} | 0.52 | 0.00 | 0.10 | 0.19 | 0.00 | 0.03 | 0.01 |
| | \hat{d}_{LW} | 0.54 | 0.07 | 0.08 | 0.14 | 0.00 | 0.18 | 0.01 |
| $d = 0.6$ | $\hat{\rho}_1$ | 0.84 | 0.89 | 0.86 | 0.53 | 0.83 | 0.94 | 0.84 |
| | $\hat{\rho}$ | 0.92 | 0.95 | 0.93 | 0.85 | 0.92 | 0.97 | 0.92 |
| | \hat{CIR} | 6.92 | 10.83 | 7.92 | 2.23 | 6.58 | 23.68 | 6.79 |
| | \hat{LAR} | 1.19 | 1.13 | 1.16 | 1.93 | 1.20 | 1.07 | 1.20 |
| | \hat{HLC} | -1.14 | -1.09 | -1.11 | -1.66 | -1.15 | -1.05 | -1.14 |
| | $\hat{\gamma}$ | 0.86 | 0.05 | 0.01 | 0.13 | 0.00 | 0.07 | 0.00 |
| | ADF | 0.90 | 0.00 | 0.00 | 0.07 | 0.00 | 0.54 | 0.00 |
| | KPSS $_{\mu}$ | 0.78 | 0.18 | 0.08 | 0.00 | 0.00 | 0.16 | 0.00 |
| | \hat{d}_{GPH} | 0.61 | 0.06 | 0.03 | 0.14 | 0.00 | 0.14 | 0.00 |
| | \hat{d}_{HP} | 0.56 | 0.01 | 0.07 | 0.21 | 0.01 | 0.03 | 0.01 |
| | \hat{d}_{LW} | 0.59 | 0.05 | 0.07 | 0.13 | 0.00 | 0.14 | 0.01 |
| $d = 0.7$ | $\hat{\rho}_1$ | 0.92 | 0.93 | 0.93 | 0.71 | 0.92 | 0.96 | 0.92 |
| | $\hat{\rho}$ | 0.96 | 0.96 | 0.96 | 0.91 | 0.95 | 0.97 | 0.95 |
| | \hat{CIR} | 14.95 | 18.78 | 16.02 | 3.77 | 14.09 | 31.50 | 14.58 |
| | \hat{LAR} | 1.09 | 1.07 | 1.08 | 1.44 | 1.09 | 1.05 | 1.09 |
| | \hat{HLC} | -1.06 | -1.05 | -1.06 | -1.31 | -1.07 | -1.03 | -1.06 |
| | $\hat{\gamma}$ | 0.92 | 0.01 | 0.00 | 0.10 | 0.00 | 0.03 | 0.00 |
| | ADF | 0.65 | 0.00 | 0.00 | 0.13 | 0.00 | 0.29 | 0.00 |
| | KPSS $_{\mu}$ | 0.86 | 0.06 | 0.03 | 0.00 | 0.00 | 0.07 | 0.00 |
| | \hat{d}_{GPH} | 0.72 | 0.02 | 0.02 | 0.13 | 0.00 | 0.07 | 0.01 |
| | \hat{d}_{HP} | 0.66 | 0.01 | 0.06 | 0.25 | 0.01 | 0.05 | 0.01 |
| | \hat{d}_{LW} | 0.70 | 0.02 | 0.04 | 0.13 | 0.00 | 0.07 | 0.00 |
| $d = 0.9$ | $\hat{\rho}_1$ | 0.99 | 0.99 | 0.99 | 0.93 | 0.99 | 0.99 | 0.99 |
| | $\hat{\rho}$ | 0.99 | 0.99 | 0.99 | 0.98 | 0.99 | 0.99 | 0.99 |
| | \hat{CIR} | 116.99 | 119.21 | 119.28 | 20.77 | 107.59 | 135.65 | 112.45 |
| | \hat{LAR} | 1.01 | 1.01 | 1.01 | 1.08 | 1.02 | 1.01 | 1.01 |
| | \hat{HLC} | -1.01 | -1.01 | -1.01 | -1.06 | -1.01 | -1.01 | -1.01 |
| | $\hat{\gamma}$ | 0.97 | 0.00 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 |
| | ADF | 0.11 | 0.00 | 0.01 | 0.03 | 0.01 | 0.01 | 0.00 |
| | KPSS $_{\mu}$ | 0.93 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 |
| | \hat{d}_{GPH} | 0.92 | 0.00 | 0.01 | 0.10 | 0.00 | 0.01 | 0.00 |
| | \hat{d}_{HP} | 0.90 | 0.00 | 0.01 | 0.32 | 0.00 | 0.00 | 0.00 |
| | \hat{d}_{LW} | 0.90 | 0.00 | 0.01 | 0.10 | 0.00 | 0.01 | 0.00 |

¹ See notes in table 12.