Gollier, Christian

Working Paper
Expected net present value, expected net future value, and the Ramsey rule

CESifo Working Paper, No. 2643

Provided in Cooperation with:
Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Gollier, Christian (2009) : Expected net present value, expected net future value, and the Ramsey rule, CESifo Working Paper, No. 2643

This Version is available at:
http://hdl.handle.net/10419/30504

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Abstract

Weitzman (1998) showed that when future interest rates are uncertain, using the expected net present value implies a term structure of discount rates that is decreasing to the smallest possible interest rate. On the contrary, using the expected net future value criterion implies an increasing term structure of discount rates up to the largest possible interest rate. We reconcile the two approaches by introducing risk aversion and risk-neutral probabilities. We show that if the aggregate consumption path is optimized, the two criteria are equivalent. Moreover, they are also equivalent to the Ramsey rule extended to uncertainty.

JEL Code: D61.

Keywords: discount rate, asset price, Ramsey rule, cost-benefit analysis.

Christian Gollier

Toulouse School of Economics
University of Toulouse I
Place Anatole France
31042 Toulouse cedex
France

gollier@cict.fr

March 14, 2009

This paper benefited from the financial support of the Chair "Sustainable finance and responsible investment" at TSE.
1 Introduction

Consider a simple investment that generates a sure payoff $Z$ at date $t$ per euro invested at date 0. If $\rho$ is the continuously compounded interest rate during the period, it is optimal to undertake the project if its Net Present Value

$$NPV = -1 + Ze^{-\rho t}$$

is positive. This NPV rule is sustained by a simple arbitrage argument: implementing the project and borrowing $Ze^{-\rho t}$ at date 0 until date $t$ would generate a sure payoff $NPV$ today, with no other net payoff along the lifetime of the project. One could alternatively consider another arbitrage strategy, in which the investor borrows one euro at date 0 to finance the project. In that case, only one net payoff is generated. It takes place at date $t$ and is equal to the Net Future Value

$$NFV = -e^{\rho t} + Z.$$ Implementing the project is optimal if $NFV$ is positive. Of course, because $NFV = e^{\rho t} NPV$, the two decision criteria are equivalent.

Suppose now that the interest rate $\tilde{\rho}$ that will prevail between dates 0 and $t$ is uncertain. It is unknown at the time the investment decision must be made, but the uncertainty $\tilde{\rho}$ is fully resolved at date $t = 0$. Because the investment opportunity cost is uncertain, it is likely to affect the optimal decision. Weitzman (1998, 2001) assumes that the optimal decision criterion in that context is to invest if the expected NPV is positive. Obviously, this is equivalent to using a discount rate equaling

$$r_p(t) = -\frac{1}{t} \ln Ee^{-\tilde{\rho} t}. \quad (1)$$

It is easy to check that $r_p(t)$ is less than $E\tilde{\rho}$, and that it tends to its lowest possible rate for large $t$. The discount rate $r_p$ is sustained by a strategy in which the project is implemented and in which the investor borrows at date 0 a random amount $Ze^{-\tilde{\rho} t}$ at interest rate $\tilde{\rho}$ until $t$. All the risk is thus borne at date 0.

As initially observed by Pazner and Razin (1975), and then by Gollier (2004), Hepburn and Groom (2007) and Buchholz and Schumacher (2008), one could use the alternative strategy to invest if and only if the expected
NFV is positive. This is equivalent to using a discount rate equaling

\[ r_f(t) = \frac{1}{t} \ln Ee^{\bar{\rho} t}. \]  

(2)

It is easy to check that \( r_f(t) \) is larger than \( E\bar{\rho} \), and that it tends to its largest possible rate when \( t \) tends to infinity. The discount rate \( r_f \) is sustained by an arbitrage strategy in which the investment is implemented, the initial investment cost being financed by a loan at rate \( \bar{\rho} \) between 0 and \( t \). All the investment risk is thus borne at the future date \( t \) in this case.

Clearly, using the ENFV and ENPV approaches yield opposite shapes of the term structure of discount rates, except in the special case of certainty. This shows that, in general, the choice of the discount rate cannot be disentangled from how the investment is financed, and from how the risky payoff of the project is allocated through time. Hepburn and Groom (2007) generalized the analysis above by showing that one can consider other evaluation dates than 0 (for NPV) or \( t \) (for NFV), each choice yielding a different term structure. They show that the decision criterion is highly sensitive to the arbitrary evaluation date, noticing that "a fuller analysis is required", and concluding that "in the murky waters of intergenerational policy, any theoretical advance providing a ray or two of light is to be welcomed". They suggested that the solution of the puzzle should be find in the intergenerational conflicts that prevail when discount rates are not constant.

In this paper, we propose to introduce risk aversion into the picture to solve the puzzle. We make explicit the investor's objective function, which is assumed to be the standard Discounted Expected Utility criterion. Using exactly the same arbitrage strategies than the one presented above, we show how to generalize the discount rates based on the NPV and NFV rules to the case of risk aversion. In fact, we show that the expectation operators appearing in equations (1) and (2) must be based in that general case on risk-neutral probabilities, which are proportional to the marginal utility of consumption at the evaluation date. This is in line with the standard consumption-based methodology in finance to estimate risk premia.

Introducing risk aversion alone does not solve the puzzle. Another step must be made, in which the consumption path is optimized. Under the condition that the investor optimizes her consumption between 0 and \( t \) contingent to the observed interest rate, we show that the two approaches lead to exactly the same term structure of discount rates, which is decreasing and tends to the lowest possible interest rate. Moreover, we show that the two equivalent
approaches are then also perfectly compatible with the well-known Ramsey rule, which has been used recently to provide alternative justifications to the decreasing term structure (Gollier (2002, 2007), Weitzman (2007)).

Our approach is mostly non-technical, contrary to Gollier (2008). This companion paper also shows that the decreasing nature of the term structure obtained in this framework depends heavily upon the assumption that shocks on the interest rate are permanent. If they are purely transitory, the term structure of discount rates should be flat. This paper is also related to a recent paper by Buchholz and Schumacher (2008), who also recognize the necessity to introduce risk aversion into the analysis. They propose an interesting criterion in which investing at the discount rate \( r_{bs}(t) \) is defined in such a way that it yields the same expected utility than investing at the uncertain rate of return of capital: \( u(\exp r_{bs}(t)t) = E u(\exp \tilde{\rho} t) \). They conclude that the discount rate \( r_{bs} \) is decreasing or increasing with the time horizon \( t \) depending upon the intensity of risk aversion. In particular, a relative risk aversion less than unity yields an increasing term structure. Our approach differs much from Buchholz and Schumacher’s one mostly because we use the more standard marginalist approach to asset pricing.

2 Three discount rates

Consider a representative investor with an increasing and concave von Neumann-Morgenstern utility function \( u \). Her intertemporal welfare function is written as

\[
W = \sum_{t=0} e^{-\delta t} E u(c_t),
\]

where \( \delta \) is the investor’s rate of pure preference for the present. Consumption at date \( t \), which is denoted \( c_t \), is a random. Because the psychological discount factor is exponential, this discounted expected utility model generates time-consistent behaviors. Finally, let \( \rho \) denote the rate of return of capital per period. It is a random variable \( \tilde{\rho} \) at the time of the investment decision prior to \( t = 0 \), but the uncertainty is fully revealed at date 0. Using a standard arbitrage argument, this implies that the socially efficient discount rate after \( \tilde{\rho} \) is realized is \( \rho \). We hereafter determine three different ways to characterize the socially efficient discount rate prior to the realization of \( \tilde{\rho} \).

Consider an investment that generates with certainty \( Z = e^{rt} \) euros at date \( t \) per euro invested at date 0, where \( r \) is the sure internal rate of return.
of the project. If $r$ is large enough, the project is socially desirable, i.e., it would increase $W$. We define the discount rate associated to maturity $t$ as the critical $r$ such that investing a small amount $\varepsilon$ in the project would have no effect on welfare $W$. However, measuring the impact of the implementation of the investment project on $W$ requires determining first how the benefit of the project is transformed into changes in consumption at different dates. We explore three different consumption strategies, which leads to three potentially different discount rates.

The simplest method consists in assuming that the investor adapts consumption to the investment payoffs when they are generated, i.e., without using credit markets. This means that implementing the project reduces consumption at date $0$ by $\varepsilon$, and increases it by $\varepsilon e^{rt}$ at date $t$. This reduces felicity at date $0$ by $\varepsilon E u'(\tilde{c}_0)$, and it increases felicity at date $t$ by $\varepsilon e^{rt} E u'(\tilde{c}_t)$. Taking account of impatience implies that investing in the project has no effect on welfare at the margin if $E u'(\tilde{c}_0) = e^{-\delta t} e^{rt} E u'(\tilde{c}_t)$. This defines the "Ramsey" discount rate $r_r(t)$ as

$$r_r(t) = \delta - \frac{1}{t} \ln \frac{E u'(\tilde{c}_t)}{E u'(\tilde{c}_0)}. \quad (3)$$

For example, assume that relative risk aversion is constant, i.e., $u'(c) = c^{-\gamma}$, and that the growth rate of consumption is certain, i.e. $c_t = c_0 e^{gt}$. In that particular case, equation (3) implies that $r_r(t) = \delta + \gamma g$, which is the well-known Ramsey rule: the discount rate net of the rate of impatience equals the product of growth rate of consumption by the index of relative risk aversion.

Suppose alternatively that, in addition to investing $\varepsilon$ into the project, the investor borrows $\varepsilon e^{rt} e^{-\rho t}$ at rate $\rho$ from date $0$ to date $t$. Because the reimbursement $\varepsilon e^{rt}$ at date $t$ is exactly equal to the payoff of the project, this means that all cash flows of the project are transferred to date $0$, with a net increase in consumption equaling the net present value $\varepsilon(-1 + e^{(r-\rho)t})$. Prior to the resolution of the uncertainty on $\tilde{\rho}$, the effect on welfare equals $\varepsilon E(-1 + e^{(r-\rho)t}) u'(\tilde{c}_0)$. This is equal to zero if $r = r_p(t)$, with

$$r_p(t) = -\frac{1}{t} \ln \frac{E u'(\tilde{c}_0)e^{-\tilde{\rho}t}}{E u'(\tilde{c}_0)}. \quad (4)$$

This is the "NPV" discount rate under risk aversion. The ratio of in the right-hand side of this equality is a weighted expectation of $e^{-\tilde{\rho}t}$. Because the overall risk of the strategy is allocated to consumption at date $0$, the
weights are proportional to the marginal utility at that date. In the finance literature, this distortion of objective probabilities is referred to as "risk-neutral probabilities". Notice that the NPV discount rate characterized by equation (4) boils down to the Weitzman discount rate (1) when the representative agent is risk neutral, or when consumption \( \bar{c}_0 \) at date 0 is independent of the interest rate \( \bar{\rho} \). Introducing risk aversion does not change the shape of the term structure, which is decreasing and tends to the smallest possible interest rate for large maturities.

A third approach consists in converting all cash flows into consumption at date \( t \) rather than at date 0. This leads to the "NFV" discount rate. Suppose that the agent both invests \( \varepsilon \) in the project and finance the initial cost by borrowing \( \varepsilon \) at rate \( \rho \) from date 0 to date \( t \). The single payoff then occurs at date \( t \), and is equal to the net future value \( \varepsilon (e^{rt} - e^{\rho t}) \) of the project. The effect of this strategy on the investor’s welfare prior to the resolution of \( \bar{\rho} \) is \( \varepsilon E(e^{rt} - e^{\rho t})u'(\bar{c}_t) \). Equalizing this expression to zero defines the NFV discount rate \( r_f(t) \):

\[
r_f(t) = \frac{1}{t} \ln \frac{E u'(\bar{c}_t)e^{\rho t}}{E u'(\bar{c}_t)}.
\]

This NFV discount rate generalizes the discount rate (2) to the case of risk aversion. The risk-neutral probability distribution weights the objective probabilities to the marginal utility of consumption at date \( t \). Without any information about the statistical relationship between \( \bar{c}_t \) and \( \bar{\rho} \), one cannot characterize the term structure of the NFV discount rate, except in the case of risk neutrality.

As observed by Hepburn and Groom (2007), one could define an infinite number of such discount rates by choosing arbitrarily the evaluation date. The translation of this observation into the framework of this paper is that the socially efficient discount rate depends upon the way the project is financed. This demonstrates that, as suggested by Hepburn and Groom (2007), both Weitzman (1998) and Gollier (2004) are right. Whether one should use a NPV or a NFV approach depends upon who will benefit from the investment.
3 Unifying field: Optimization of the consumption strategy

The conclusion of the previous section is problematic. Standard cost-benefit analysis states that the way one evaluates an investment project should not depend upon the way the project is financed. There should be only one term structure of discount rates. This property is one of the cornerstones of public finance. In this section, we show that the three discount rates are in fact all equal if we recognize that economic agents optimize their consumption path.

From date 0 on, the investor knows the rate of return on capital $\rho$. Reducing consumption by $\varepsilon$ at date $t_1$ allows the agent to increase consumption by $\varepsilon e^{(\rho-\delta)(t_2-t_1)} u'(c_1)$ at date $t_2$. Therefore, the optimal consumption path conditional to $\rho$ must be such that $u'(c_0) = e^{(\rho-\delta)t} u'(c_t)$, otherwise transferring consumption from one date to the other would increase intertemporal welfare $W$. This implies in particular that conditional to $\rho$, we must have that

$$u'(c_0) = e^{(\rho-\delta)t} u'(c_t).$$

Observe that it implies that optimal consumption plans in general depend upon $\rho$.

This optimality condition implies that $r_r(t) = r_p(t) = r_f(t)$, and more generally that the socially efficient discount rate is independent upon the evaluation date, or upon the way (marginal) projects are financed. Indeed, using condition (6), we have that

$$r_r(t) = \delta - \frac{1}{t} \ln \frac{E u'(\tilde{c}_1)}{E u'(\tilde{c}_0)} = -\frac{1}{t} \ln \frac{E u'(\tilde{c}_0) e^{-\tilde{\rho}t}}{E u'(\tilde{c}_0)} = r_p(t),$$

and

$$r_f(t) = \frac{1}{t} \ln \frac{E u'(\tilde{c}_t) e^{\tilde{\rho}t}}{E u'(\tilde{c}_t)} = -\frac{1}{t} \ln \frac{E u'(\tilde{c}_0) e^{-\tilde{\rho}t}}{E u'(\tilde{c}_0)} = r_p(t).$$

Thus, we have reconciled the various approaches that have recently been used to discuss whether the discount rate should be decreasing with the time horizon. Under this framework in which the shock on interest rates is permanent, the socially efficient discount rate is decreasing and tends to the smallest possible interest rate.
4 Conclusion

We have shown that the choice of the discount rate for marginal projects depends in general upon how the benefits and costs of the project are allocated through time. This explains the differences between the approaches examined respectively by Ramsey (1928), Weitzman (1998) and Gollier (2004). The Ramsey rule is obtained by assuming that the cash flows of the investment are consumed at the time at which they occur. In the Weitzman’s NPV approach, it is assumed that all costs and benefits are consumed at date 0, whereas they are all consumed at the terminal date in the Gollier’s NFV approach. Of course, the risk associated to the various financing strategies must be taken into account given the risk aversion of the representative agent. This must be done by using risk-neutral probability distributions that vary with the evaluation date.

Our main contribution is to show that these different approaches to discounting are completely equivalent once we recognize that consumers react optimally to changes in the interest rate. Ramsey, Weitzman and Gollier’s approaches lead to the same term structure of discount rates when consumption paths are optimal. Otherwise, the way the benefits of the project are allocated into changes in consumption will not be neutral when evaluating the project.

This conclusion holds only when there is no friction on credit markets, and when investment projects are marginal. For non-marginal projects, their existence requires re-optimizing the consumption path, as claimed by Stern (2006). In the case of the climate change problem, the choice of the discount rate remains dependent upon which generations will bear the mitigation and adaptation costs.
Bibliography


Gollier, C., (2004), Maximizing the expected net future value as an alternative strategy to gamma discounting, Finance Research Letters, 1, 85-89.


Gollier, (2008), Should we discount the far-distant future at its lowest possible rate?, unpublished manuscript, Toulouse School of Economics.


CESifo Working Paper Series
for full list see www.cesifo-group.org/wp
(address: Poschingerstr. 5, 81679 Munich, Germany, office@cesifo.de)

2581 António Afonso and Christophe Rault, Bootstrap Panel Granger-Causality between Government Budget and External Deficits for the EU, March 2009

2582 Bernd Süßmuth, Malte Heyne and Wolfgang Maennig, Induced Civic Pride and Integration, March 2009

2583 Martin Peitz and Markus Reisinger, Indirect Taxation in Vertical Oligopoly, March 2009

2584 Petra M. Geraats, Trends in Monetary Policy Transparency, March 2009

2585 Johannes Abeler, Armin Falk, Lorenz Götte and David Huffman, Reference Points and Effort Provision, March 2009

2586 Wolfram F. Richter, Taxing Education in Ramsey’s Tradition, March 2009

2587 Yin-Wong Cheung, Menzie D. Chinn and Eiji Fujii, China’s Current Account and Exchange Rate, March 2009

2588 Alexander Haupt and Silke Uebelmesser, Voting on Labour-Market Integration and Education Policy when Citizens Differ in Mobility and Ability, March 2009

2589 Hans Jarle Kind, Marko Koethenbuerger and Guttorm Schjelderup, Should Utility-Reducing Media Advertising be Taxed?, March 2009

2590 Alessandro Cigno, How to Avoid a Pension Crisis: A Question of Intelligent System Design, March 2009

2591 Helmut Lütkepohl and Fang Xu, The Role of the Log Transformation in Forecasting Economic Variables, March 2009

2592 Rainald Borck, Hyun-Ju Koh and Michael Pflüger, Inefficient Lock-in and Subsidy Competition, March 2009

2593 Paolo M. Panteghini, On the Equivalence between Labor and Consumption Taxation, March 2009

2594 Bruno S. Frey, Economists in the PITS?, March 2009

2595 Natalie Chen and Dennis Novy, International Trade Integration: A Disaggregated Approach, March 2009

2596 Frédérique Bec and Christian Gollier, Term Structure and Cyclicity of Value-at-Risk: Consequences for the Solvency Capital Requirement, March 2009
2597 Carsten Eckel, International Trade and Retailing, March 2009
2598 Gianni De Nicolò and Iryna Ivaschenko, Global Liquidity, Risk Premiums and Growth Opportunities, March 2009
2599 Jay Pil Choi and Heiko Gerlach, International Antitrust Enforcement and Multi-Market Contact, March 2009
2600 Massimo Bordignon and Guido Tabellini, Moderating Political Extremism: Single Round vs Runoff Elections under Plurality Rule, April 2009
2601 Ana B. Ania and Andreas Wagener, The Open Method of Coordination (OMC) as an Evolutionary Learning Process, April 2009
2602 Simon Gächter, Daniele Nosenzo, Elke Renner and Martin Sefton, Sequential versus Simultaneous Contributions to Public Goods: Experimental Evidence, April 2009
2603 Philippe Jehiel and Andrew Lilico, Smoking Today and Stopping Tomorrow: A Limited Foresight Perspective, April 2009
2604 Andreas Knabe, Steffen Rätzel, Ronnie Schöb and Joachim Weimann, Dissatisfied with Life, but Having a Good Day: Time-Use and Well-Being of the Unemployed, April 2009
2606 Erkki Koskela and Jan König, Can Profit Sharing Lower Flexible Outsourcing? A Note, April 2009
2607 Michel Beine, Frédéric Docquier and Çağlar Özden, Diasporas, April 2009
2608 Gerd Ronning and Hans Schneeweiss, Panel Regression with Random Noise, April 2009
2609 Adam S. Booij, Bernard M.S. van Praag and Gijs van de Kuilen, A Parametric Analysis of Prospect Theory’s Functionals for the General Population, April 2009
2610 Jeffrey R. Brown, Julia Lynn Coronado and Don Fullerton, Is Social Security Part of the Social Safety Net?, April 2009
2611 Ali Bayar and Bram Smeets, Economic, Political and Institutional Determinants of Budget Deficits in the European Union, April 2009
2614 Marie-Louise Leroux, Pierre Pestieau and Gregory Ponthiere, Should we Subsidize Longevity?, April 2009

2615 Ronald MacDonald, Lukas Menkhoff and Rafael R. Rebitzky, Exchange Rate Forecasters’ Performance: Evidence of Skill?, April 2009

2616 Frederick van der Ploeg and Steven Poelhekke, The Volatility Curse: Revisiting the Paradox of Plenty, April 2009

2617 Axel Dreher, Peter Nunnenkamp, Hannes Öhler and Johannes Weisser, Acting Autonomously or Mimicking the State and Peers? A Panel Tobit Analysis of Financial Dependence and Aid Allocation by Swiss NGOs, April 2009

2618 Guglielmo Maria Caporale, Roman Matousek and Chris Stewart, Rating Assignments: Lessons from International Banks, April 2009

2619 Paul Belleflamme and Martin Peitz, Asymmetric Information and Overinvestment in Quality, April 2009

2620 Thomas Dohmen, Armin Falk, David Huffman and Uwe Sunde, Are Risk Aversion and Impatience Related to Cognitive Ability?, April 2009

2621 Yin-Wong Cheung and Xingwang Qian, The Empirics of China’s Outward Direct Investment, April 2009

2622 Frédérique Bec and Christian Gollier, Assets Returns Volatility and Investment Horizon: The French Case, April 2009

2623 Ronnie Schöb and Marcel Thum, Asymmetric Information Renders Minimum Wages Less Harmful, April 2009

2624 Martin Ruf and Alfons J. Weichenrieder, The Taxation of Passive Foreign Investment – Lessons from German Experience, April 2009

2625 Yao Li, Borders and Distance in Knowledge Spillovers: Dying over Time or Dying with Age? – Evidence from Patent Citations, April 2009

2626 Jim Malley and Ulrich Woitek, Technology Shocks and Aggregate Fluctuations in an Estimated Hybrid RBC Model, April 2009

2627 Jin Cao and Gerhard Illing, Endogenous Systemic Liquidity Risk, April 2009

2628 Thiess Buettner and Bjoern Kauder, Revenue Forecasting Practices: Differences across Countries and Consequences for Forecasting Performance, April 2009


2630 Nick Johnstone and Ivan Hascic, Environmental Policy Design and the Fragmentation of International Markets for Innovation, April 2009
2631 Spiros Bougheas, Richard Kneller and Raymond Riezman, Optimal Education Policies and Comparative Advantage, April 2009

2632 Jay Pil Choi and Heiko Gerlach, Multi-Market Collusion with Demand Linkages and Antitrust Enforcement, April 2009

2633 Thor O. Thoresen, Income Mobility of Owners of Small Businesses when Boundaries between Occupations are Vague, April 2009

2634 Guido Schwerdt and Amelie C. Wuppermann, Is Traditional Teaching really all that Bad? A Within-Student Between-Subject Approach, April 2009

2635 Kurt R. Brekke, Luigi Siciliani and Odd Rune Straume, Hospital Competition and Quality with Regulated Prices, April 2009

2636 Peter Diamond, Taxes and Pensions, April 2009

2637 Shoshana Grossbard, How “Chicagoan” are Gary Becker’s Economic Models of Marriage?, May 2009

2638 Roland Strausz, Regulatory Risk under Optimal Incentive Regulation, May 2009


2640 Harald Hau and Marcel Thum, Subprime Crisis and Board (In-)Competence: Private vs. Public Banks in Germany, May 2009


2642 Balázs Õgert, Infrastructure Investment in Network Industries: The Role of Incentive Regulation and Regulatory Independence, May 2009