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Expected Net Present Value, Expected Net Future Value, and the Ramsey Rule

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Abstract

Weitzman (1998) showed that when future interest rates are uncertain, using the expected net present value implies a term structure of discount rates that is decreasing to the smallest possible interest rate. On the contrary, using the expected net future value criterion implies an increasing term structure of discount rates up to the largest possible interest rate. We reconcile the two approaches by introducing risk aversion and risk-neutral probabilities. We show that if the aggregate consumption path is optimized, the two criteria are equivalent. Moreover, they are also equivalent to the Ramsey rule extended to uncertainty.

JEL Code: D61.

Keywords: discount rate, asset price, Ramsey rule, cost-benefit analysis.

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1 Introduction

Consider a simple investment that generates a sure payoff $Z$ at date $t$ per euro invested at date 0. If $\rho$ is the continuously compounded interest rate during the period, it is optimal to undertake the project if its Net Present Value

$$NPV = -1 + Ze^{-\rho t}$$

is positive. This NPV rule is sustained by a simple arbitrage argument: implementing the project and borrowing $Ze^{-\rho t}$ at date 0 until date $t$ would generate a sure payoff $NPV$ today, with no other net payoff along the lifetime of the project. One could alternatively consider another arbitrage strategy, in which the investor borrows one euro at date 0 to finance the project. In that case, only one net payoff is generated. It takes place at date $t$ and is equal to the Net Future Value

$$NFV = -e^{\rho t} + Z.$$

Implementing the project is optimal if $NFV$ is positive. Of course, because $NFV = e^{\rho t}NPV$, the two decision criteria are equivalent.

Suppose now that the interest rate $\tilde{\rho}$ that will prevail between dates 0 and $t$ is uncertain. It is unknown at the time the investment decision must be made, but the uncertainty $\tilde{\rho}$ is fully resolved at date $t = 0$. Because the investment opportunity cost is uncertain, it is likely to affect the optimal decision. Weitzman (1998, 2001) assumes that the optimal decision criterion in that context is to invest if the expected NPV is positive. Obviously, this is equivalent to using a discount rate equaling

$$r_p(t) = -\frac{1}{t} \ln E e^{-\tilde{\rho} t}. \quad (1)$$

It is easy to check that $r_p(t)$ is less than $E\tilde{\rho}$, and that it tends to its lowest possible rate for large $t$. The discount rate $r_p$ is sustained by a strategy in which the project is implemented and in which the investor borrows at date 0 a random amount $Ze^{-\tilde{\rho} t}$ at interest rate $\tilde{\rho}$ until $t$. All the risk is thus borne at date 0.

As initially observed by Pazner and Razin (1975), and then by Gollier (2004), Hepburn and Groom (2007) and Buchholz and Schumacher (2008), one could use the alternative strategy to invest if and only if the expected
NFV is positive. This is equivalent to using a discount rate equaling

$$r_f(t) = \frac{1}{t} \ln E e^{\tilde{\rho} t}.$$  

(2)

It is easy to check that $r_f(t)$ is larger than $E \tilde{\rho}$, and that it tends to its largest possible rate when $t$ tends to infinity. The discount rate $r_f$ is sustained by an arbitrage strategy in which the investment is implemented, the initial investment cost being financed by a loan at rate $\tilde{\rho}$ between 0 and $t$. All the investment risk is thus borne at the future date $t$ in this case.

Clearly, using the ENFV and ENPV approaches yield opposite shapes of the term structure of discount rates, except in the special case of certainty. This shows that, in general, the choice of the discount rate cannot be disentangled from how the investment is financed, and from how the risky payoff of the project is allocated through time. Hepburn and Groom (2007) generalized the analysis above by showing that one can consider other evaluation dates than 0 (for NPV) or $t$ (for NFV), each choice yielding a different term structure. They show that the decision criterion is highly sensitive to the arbitrary evaluation date, noticing that "a fuller analysis is required", and concluding that "in the murky waters of intergenerational policy, any theoretical advance providing a ray or two of light is to be welcomed". They suggested that the solution of the puzzle should be find in the intergenerational conflicts that prevail when discount rates are not constant.

In this paper, we propose to introduce risk aversion into the picture to solve the puzzle. We make explicit the investor’s objective function, which is assumed to be the standard Discounted Expected Utility criterion. Using exactly the same arbitrage strategies than the one presented above, we show how to generalize the discount rates based on the NPV and NFV rules to the case of risk aversion. In fact, we show that the expectation operators appearing in equations (1) and (2) must be based in that general case on risk-neutral probabilities, which are proportional to the marginal utility of consumption at the evaluation date. This is in line with the standard consumption-based methodology in finance to estimate risk premia.

Introducing risk aversion alone does not solve the puzzle. Another step must be made, in which the consumption path is optimized. Under the condition that the investor optimizes her consumption between 0 and $t$ contingent to the observed interest rate, we show that the two approaches lead to exactly the same term structure of discount rates, which is decreasing and tends to the lowest possible interest rate. Moreover, we show that the two equivalent
approaches are then also perfectly compatible with the well-known Ramsey 
rule, which has been used recently to provide alternative justifications to the 
decreasing term structure (Gollier (2002, 2007), Weitzman (2007)).

Our approach is mostly non-technical, contrary to Gollier (2008). This 
companion paper also shows that the decreasing nature of the term structure 
obtained in this framework depends heavily upon the assumption that shocks 
on the interest rate are permanent. If they are purely transitory, the term 
structure of discount rates should be flat. This paper is also related to a recent 
paper by Buchholz and Schumacher (2008), who also recognize the necessity 
to introduce risk aversion into the analysis. They propose an interesting 
criterion in which investing at the discount rate $r_{bs}(t)$ is defined in such a 
way that it yields the same expected utility than investing at the uncertain 
rate of return of capital: $u(\exp r_{bs}(t) t) = E u(\exp \tilde{\rho} t)$. They conclude that the 
discount rate $r_{bs}$ is decreasing or increasing with the time horizon $t$ depending 
upon the intensity of risk aversion. In particular, a relative risk aversion 
less than unity yields an increasing term structure. Our approach differs 
much from Buchholz and Schumacher’s one mostly because we use the more 
standard marginalist approach to asset pricing.

2 Three discount rates

Consider a representative investor with an increasing and concave von Neumann-
Morgenstern utility function $u$. Her intertemporal welfare function is written 
as

$$W = \sum_{t=0}^{\infty} e^{-\delta t} E u(\tilde{c}_t),$$

where $\delta$ is the investor’s rate of pure preference for the present. Consumption 
at date $t$, which is denoted $\tilde{c}_t$, is a random. Because the psychological 
discount factor is exponential, this discounted expected utility model generates time-consistent behaviors. Finally, let $\rho$ denote the rate of return of capital per period. It is a random variable $\tilde{\rho}$ at the time of the investment decision prior to $t = 0$, but the uncertainty is fully revealed at date 0. Using a standard arbitrage argument, this implies that the socially efficient discount rate after $\tilde{\rho}$ is realized is $\rho$. We hereafter determine three different ways to characterize the socially efficient discount rate prior to the realization of $\tilde{\rho}$.

Consider an investment that generates with certainty $Z = e^{rt}$ euros at 
date $t$ per euro invested at date 0, where $r$ is the sure internal rate of return.
of the project. If $r$ is large enough, the project is socially desirable, i.e., it would increase $W$. We define the discount rate associated to maturity $t$ as the critical $r$ such that investing a small amount $\varepsilon$ in the project would have no effect on welfare $W$. However, measuring the impact of the implementation of the investment project on $W$ requires determining first how the benefit of the project is transformed into changes in consumption at different dates. We explore three different consumption strategies, which leads to three potentially different discount rates.

The simplest method consists in assuming that the investor adapts consumption to the investment payoffs when they are generated, i.e., without using credit markets. This means that implementing the project reduces consumption at date $0$ by $\varepsilon$, and increases it by $\varepsilon e^{rt}$ at date $t$. This reduces felicity at date $0$ by $\varepsilon E[u'(\tilde{c}_0)]$, and it increases felicity at date $t$ by $\varepsilon e^{rt}E[u'(\tilde{c}_t)]$. Taking account of impatience implies that investing in the project has no effect on welfare at the margin if $E[u'(\tilde{c}_0)] = e^{-\delta t} e^{rt}E[u'(\tilde{c}_t)]$. This defines the "Ramsey" discount rate $r_r(t)$ as

$$ r_r(t) = \delta - \frac{1}{t} \ln \frac{E[u'(\tilde{c}_t)]}{E[u'(\tilde{c}_0)]}. $$

(3)

For example, assume that relative risk aversion is constant, i.e., $u'(c) = c^{-\gamma}$, and that the growth rate of consumption is certain, i.e. $c_t = c_0 e^{gt}$. In that particular case, equation (3) implies that $r_r(t) = \delta + \gamma g$, which is the well-known Ramsey rule: the discount rate net of the rate of impatience equals the product of growth rate of consumption by the index of relative risk aversion.

Suppose alternatively that, in addition to investing $\varepsilon$ into the project, the investor borrows $\varepsilon e^{rt}e^{-\rho t}$ at rate $\rho$ from date $0$ to date $t$. Because the reimbursement $\varepsilon e^{rt}$ at date $t$ is exactly equal to the payoff of the project, this means that all cash flows of the project are transferred to date $0$, with a net increase in consumption equaling the net present value $\varepsilon (-1 + e^{(r-\rho)t})$. Prior to the resolution of the uncertainty on $\tilde{\rho}$, the effect on welfare equals $\varepsilon E(-1 + e^{(r-\rho)t})u'(\tilde{c}_0)$. This is equal to zero if $r = r_p(t)$, with

$$ r_p(t) = -\frac{1}{t} \ln \frac{E[u'(\tilde{c}_0)e^{-\tilde{\rho}t}]}{E[u'(\tilde{c}_0)]}. $$

(4)

This is the "NPV" discount rate under risk aversion. The ratio of in the right-hand side of this equality is a weighted expectation of $e^{-\tilde{\rho}t}$. Because the overall risk of the strategy is allocated to consumption at date $0$, the
weights are proportional to the marginal utility at that date. In the finance literature, this distortion of objective probabilities is referred to as "risk-neutral probabilities". Notice that the NPV discount rate characterized by equation (4) boils down to the Weitzman discount rate (1) when the representative agent is risk neutral, or when consumption $c_0$ at date 0 is independent of the interest rate $\rho$. Introducing risk aversion does not change the shape of the term structure, which is decreasing and tends to the smallest possible interest rate for large maturities.

A third approach consists in converting all cash flows into consumption at date $t$ rather than at date 0. This leads to the "NFV" discount rate. Suppose that the agent both invests $\varepsilon$ in the project and finance the initial cost by borrowing $\varepsilon$ at rate $\rho$ from date 0 to date $t$. The single payoff then occurs at date $t$, and is equal to the net future value $\varepsilon(e^{rt} - e^{\rho t})$ of the project. The effect of this strategy on the investor’s welfare prior to the resolution of $\rho$ is $\varepsilon E(e^{rt} - e^{\rho t})u'(\tilde{c}_t)$. Equalizing this expression to zero defines the NFV discount rate $r_f(t)$:

$$r_f(t) = \frac{1}{t} \ln \frac{E u'(\tilde{c}_t)e^{\rho t}}{E u'(\tilde{c}_t)}.$$  \hfill (5)

This NFV discount rate generalizes the discount rate (2) to the case of risk aversion. The risk-neutral probability distribution weights the objective probabilities to the marginal utility of consumption at date $t$. Without any information about the statistical relationship between $c_t$ and $\rho$, one cannot characterize the term structure of the NFV discount rate, except in the case of risk neutrality.

As observed by Hepburn and Groom (2007), one could define an infinite number of such discount rates by choosing arbitrarily the evaluation date. The translation of this observation into the framework of this paper is that the socially efficient discount rate depends upon the way the project is financed. This demonstrates that, as suggested by Hepburn and Groom (2007), both Weitzman (1998) and Gollier (2004) are right. Whether one should use a NPV or a NFV approach depends upon who will benefit from the investment.
3 Unifying field: Optimization of the consumption strategy

The conclusion of the previous section is problematic. Standard cost-benefit analysis states that the way one evaluates an investment project should not depend upon the way the project is financed. There should be only one term structure of discount rates. This property is one of the cornerstones of public finance. In this section, we show that the three discount rates are in fact all equal if we recognize that economic agents optimize their consumption path.

From date 0 on, the investor knows the rate of return on capital $\rho$. Reducing consumption by $\varepsilon$ at date $t_1$ allows the agent to increase consumption by $\varepsilon e^{\rho(t_2-t_1)}$ at date $t_2$. Therefore, the optimal consumption path conditional to $\rho$ must be such that $u'(c_t)$ equals $e^{(\rho-\delta)(t_2-t_1)}u'(c_{t_2})$, otherwise transferring consumption from one date to the other would increase intertemporal welfare $W$. This implies in particular that conditional to $\rho$, we must have that

$$u'(c_0) = e^{(\rho-\delta)t}u'(c_t).$$

(6)

Observe that it implies that optimal consumption plans in general depend upon $\rho$.

This optimality condition implies that $r_r(t) = r_p(t) = r_f(t)$, and more generally that the socially efficient discount rate is independent upon the evaluation date, or upon the way (marginal) projects are financed. Indeed, using condition (6), we have that

$$r_r(t) = \delta - \frac{1}{t} \ln \frac{Eu'(c_t)}{Eu'(c_0)} = -\frac{1}{t} \ln \frac{Eu'(c_{t_0})e^{-\delta t}}{Eu'(c_0)} = r_p(t),$$

and

$$r_f(t) = \frac{1}{t} \ln \frac{Eu'(c_t)e^{\rho t}}{Eu'(c_t)} = -\frac{1}{t} \ln \frac{Eu'(c_{t_0})e^{-\delta t}}{Eu'(c_0)} = r_p(t).$$

Thus, we have reconciled the various approaches that have recently been used to discuss whether the discount rate should be decreasing with the time horizon. Under this framework in which the shock on interest rates is permanent, the socially efficient discount rate is decreasing and tends to the smallest possible interest rate.
4 Conclusion

We have shown that the choice of the discount rate for marginal projects depends in general upon how the benefits and costs of the project are allocated through time. This explains the differences between the approaches examined respectively by Ramsey (1928), Weitzman (1998) and Gollier (2004). The Ramsey rule is obtained by assuming that the cash flows of the investment are consumed at the time at which they occur. In the Weitzman’s NPV approach, it is assumed that all costs and benefits are consumed at date 0, whereas they are all consumed at the terminal date in the Gollier’s NFV approach. Of course, the risk associated to the various financing strategies must be taken into account given the risk aversion of the representative agent. This must be done by using risk-neutral probability distributions that vary with the evaluation date.

Our main contribution is to show that these different approaches to discounting are completely equivalent once we recognize that consumers react optimally to changes in the interest rate. Ramsey, Weitzman and Gollier’s approaches lead to the same term structure of discount rates when consumption paths are optimal. Otherwise, the way the benefits of the project are allocated into changes in consumption will not be neutral when evaluating the project.

This conclusion holds only when there is no friction on credit markets, and when investment projects are marginal. For non-marginal projects, their existence requires re-optimizing the consumption path, as claimed by Stern (2006). In the case of the climate change problem, the choice of the discount rate remains dependent upon which generations will bear the mitigation and adaptation costs.
Bibliography


Gollier, (2008), Should we discount the far-distant future at its lowest possible rate?, unpublished manuscript, Toulouse School of Economics.


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