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Portfolio Optimization at Damascus Securities Exchange: A Fractal Analysis Approach

Kinda Dooba^{1*} and Sulaiman Mouselli²

Abstract: This paper adopts the fractal analysis approach, specifically a Hurst exponent index in portfolio optimization at the Damascus Securities Exchange (DSE). We construct three portfolios based on the Hurst index from stocks listed on the DSE during the period between 2019 and 2021 and find that these portfolios outperform the market portfolio in terms of returns, Sharpe ratio, Treynor ratio, and alpha. In addition, selecting stocks with high Hurst coefficients further enhances the performance of the portfolio. Importantly, even in out-of-sample tests, the three fractal portfolios continue to outperform the market portfolio. Furthermore, we find that fractal portfolios outperform portfolios formed using the momentum and size strategies demonstrating the superiority of the fractal analysis approach. We conclude that the incorporation of fractal analysis into the portfolio optimization problem allows the creation of a more efficient portfolio. Hence, we recommend that investors consider the Hurst exponent index in their portfolio optimization for better investment decisions.

Subjects: Risk Management; Economics; Finance

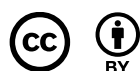
Keywords: market efficiency; Portfolio optimization; fractal analysis; Hurst exponent; Long-Memory (LM) effect; Damascus Securities Exchange (DSE)

1. Introduction

At the core of financial decision-making theory, portfolio optimization plays a pivotal role. Markowitz (1952) developed the mean-variance analysis, a work that became the center of modern portfolio theory (MPT), in which the optimal portfolio choice is presented as the solution to a simple-constrained optimization problem (Chaweewanchon & Chaysiri, 2022; Wu et al., 2021). Many influential portfolio selection models emerged after Markowitz's classic

work, such as the capital asset pricing model (CAPM) (Lintner, 1965; Mossin, 1966; Sharpe, 1964), and the option pricing model (Black & Scholes, 1973).

The efficient market hypothesis (EMH) implies that movements in stock returns are random events independent from historical values and follow the random walk hypothesis. The rationale is that prices contain all publicly available information and if patterns do exist, rational agents (arbitrageurs) would exploit them, and thereby they are quickly eliminated (Blackledge & Lamphiere, 2022; Fama, 1970). Furthermore, the EMH posits that the stock returns exhibit characteristics of linearity, continuity, static nature, and independence. These attributes allow for the estimation and management of investment risks effectively (Liu et al., 2022).



The irrationality of investors, market friction, and incomplete arbitrage represent violations of the EMH (Daniel & Titman, 1999). In empirical terms, many financial anomalies are evident in financial markets such as long-range dependency (Giacalone & Panarello, 2022; Lo, 1991; Urquhart & Hudson, 2013), higher peak and fat tail (Enders, 1995; Fama, 1965; Wu et al., 2021), volatility clustering (Bae et al., 2020; Peters, 2015), and multifractal (Mandelbrot, 1971; Mandelbrot et al., 1997; Zhang & Fang, 2021). This challenges the validity of both the normal distribution assumption and the random walk assumption in describing financial returns (Sun et al., 2007).

Financial markets are unstable, complex, and dynamic systems with nonlinear behavior (Tebyaniyan et al., 2020; Wu et al., 2021). Financial time series appear to exhibit fractal properties, with their patterns becoming increasingly complex when enlargement. These patterns repeat themselves, showing a qualitative similarity to the overall structure of the market (Anderson & Noss, 2013; Blackledge & Lamphiere, 2022). Also, there is a certain long memory in prices as opposed to the memory-free according to the EMH (Chen et al., 2019). That is, prices may rise or fall substantially to a certain degree (Deboeck, 1994; Giacalone & Panarello, 2022; Mandelbrot, 1971). Consequently, the assumptions of classical finance are insufficient in explaining the behavior and characteristics observed in financial markets (Anderson & Noss, 2013; Mandelbrot et al., 1997; Peters, 1991). To address this, Peters (1991) proposed the fractal market hypothesis (FMH) as an alternative to the EMH.

According to the FMH, financial market time series are primarily influenced by the long memory (LM) Effect. That is, the evolution of a time series in the future is influenced by past events (Giacalone & Panarello, 2022; Peters, 1994).

The FMH mainly focuses on two aspects: studying the fractal characteristics of the stock market and building various models to try to predict market trends. In addition, it seeks to introduce new perspectives and approaches for enhancing portfolio optimization according to several alternative criteria. These approaches include fractal theory, chaos theory, artificial neural network, artificial intelligence theory, and so on (Deboeck, 1994).

The Damascus Securities Exchange (DSE) is a relatively new stock exchange in the Middle East and North African (MENA) region, as it started trading in 2009. The efficiency of this market has been tested by most traditional parametric and non-parametric tests, most of which have rejected the weak form of the EMH. Abbas (2014) rejected the random walk hypothesis at the DSE using the daily returns of the index DWX between 2009 and 2014. Likewise, Mouselli and Al-Samman (2016) found evidence in favor of the month-of-the-year effect in the DSE between 2010 and 2015. Mahmoud and Wardeh (2018) also provided evidence in favor of the momentum effect in the DSE market during the period (2010–2016) suggesting that the weak form of the EMH does not hold at the DSE. On the contrary, Ismaiel (2017) found that the weak form of the EMH holds at the DSE during the period between 2010 and 2017, using tests that account for endogenously capturing structural breaks in the time series.

The main objective of this study is to develop a portfolio optimization approach for investors at the DSE. This approach utilizes the estimation of the Hurst exponent to identify the long memory property in stock returns, which is then combined with the Markowitz (mean-variance) model to form optimal portfolios.

This study has three main contributions. First, this study proposes a novel approach for portfolio formation that combines the Hurst exponent estimation with the Markowitz (mean-variance) model. The Hurst exponent measures the long memory of a time series, indicating the persistence of trends. In portfolio construction, this is valuable information as it helps identify stocks that exhibit strong trends and are likely to continue performing well. Hence, if high H-index stocks performed well in the past, they are expected to continue this performance in the future. Second, our method includes a stock selection process that ensures the trend-reinforcing behavior of stock

inputs, in which stocks with higher Hurst exponents are selected for optimal portfolio construction. Third, this study contributes to the empirical literature on portfolio optimization at the DSE by examining the FMH by investigating the existence of the long memory property and utilizing fractal analysis in portfolio construction.

Our findings reveal strong evidence of a fractal structure with a long memory effect at the DSE, consistent with (Giacalone & Panarello, 2022; Tebyaniyan et al., 2020). Furthermore, portfolios constructed using fractal analysis outperform the market portfolio, corresponding to the findings of Wu et al. (2021) and Chun et al. (2020), and also outperform portfolios formed using the momentum and size strategies, providing supporting evidence for the effectiveness of portfolios that rely on fractal statistical measures in their creation.

The rest of this paper is structured as follows: Section 2 reviews the literature on the LM processes in capital markets. Section 3 illustrates our research methodology. Section 4 describes the sample and data collection, and Section 5 presents the empirical results. Section 6 concludes and provides recommendations.

2. Literature review

The analysis of the long-memory property in financial returns is one of the attractive topics in finance (Mensi et al., 2019). It presents a challenge to the EMH, as it suggests that random shocks can have long-term effects on future returns (Blackledge & Lamphiere, 2022). From a statistical perspective, the long memory process is characterized by the persistence of the autocorrelation function (Giacalone & Panarello, 2022). In other words, it refers to the strength of statistical dependence between lagged observations in a time series, and how the lagged autocorrelation functions decrease slower than exponential decay in more persistent time series (Ding et al., 2021; Peters et al., 2021). This concept is important in understanding market behavior and trends and it can also be the basis of portfolio construction.

The empirical evidence regarding the presence of the LM effect in stock returns is indeed mixed. Several studies have provided evidence supporting the existence of the LM effect, while others have found no such evidence. For example, Greene and Fielitz (1977) found evidence of persistence in most stock returns listed on the NYSE (Peters, 1992). also found evidence in favor of the LM in the S&P 500 Index. Similar conclusions were reached in many developed and emerging markets such as the German Equity Market (Sun et al., 2007), London Stock Exchange (Lillo et al., 2004), Indian Stock Market (Mishra et al., 2011), and several other stock markets (Taiwanese, Japanese, South Korean and German) (Henry, 2002). Contrary to previous studies (Lo, 1991), found no evidence of the LM in U.S. stock returns (Bhattacharya et al., 2018) also provided empirical evidence against the presence of the LM in 10 indices of certain emerging and developed markets. Similarly (Mensi et al., 2019), failed to find any evidence of the LM in European stock markets. The mixed empirical evidence suggests that the presence of the LM in stock returns may depend on factors related to the dataset and market (Saha et al., 2020)

Despite the existence of little evidence of the LM in the MENA markets, there is evidence of the long-memory effect in many Arabic stock markets. For example (Aloui & Hamida, 2014), examined the existence of long memory, structural breaks, asymmetry, and fat-tails in Gulf Cooperation Council (GCC) markets for the period between 2003 and 2013. They found that only two markets, namely Saudi Arabia and Oman exhibit the LM effect. Hoyfi (2021) also found similar results in the Tunisian stock market (Boubaker et al., 2022). Also found evidence in favor of the LM effect in the GCC equity markets over the recent period including the global financial crisis of 2007-2009 using the ARFIMA-Hyperbolic Asymmetric Power ARCH modeling process. Assaf (2016) investigated the LM effect in a group of the MENA equity markets using the Hurst exponent. The results suggested that there was evidence of weakening in the LM effect. Similar results were found in the MENA stock markets regarding returns volatility using FIABARCH models (Boubaker & Sghaier, 2015). In

addition, Al-Hajieh (2017) provided strong evidence of the long memory of returns volatility for 12 Arabic stock markets. However, no such tests were performed on the DSE.

3. Research methodology

The long-memory property in time series can be analyzed through the Hurst exponent index (H) proposed by the English hydrologist Hurst (1951) (Bui & Ślepaczuk, 2022). It is considered suitable for distinguishing a non-random time series from a random one, regardless of its distribution type. This tool is suitable for a phenomenon that appears to be random but may have a regular pattern.

The rescaled range estimation (R/S) is commonly used to calculate the Hurst exponent and the fractal properties of the time series (Sánchez Granero et al., 2008). Mandelbrot and Wallis (1969) introduced this analysis based on the earlier work of Hurst (1951). The R/S analysis is a robust nonlinear method as it allows for comparison across different time intervals by rescaling the data. This rescaling process helps capture the fractal properties of the time series. In addition, this analysis is superior to the autocorrelation and variance analysis (Sánchez Granero et al., 2008).

The Hurst exponent has been widely applied in analyzing various financial assets, including stocks and indices (Assaf, 2016; Matos et al., 2008; Tebyaniyan et al., 2020), commodities (Alvarez-Ramirez et al., 2008; Tiwari et al., 2021), cryptocurrencies (Kristoufek & Vosvrda, 2019), and currencies (Shahzad et al., 2018). It helps in assessing the presence of the LM effect, which has many implications for market efficiency and return predictability. The R/S Hurst method is particularly suitable for studying returns at the DSE because it has been greatly affected by many crises, starting from the Syrian crisis in 2011, and the recent COVID-19 pandemic. This method is invaluable for understanding the long-term impact of these events on the exchange's performance.

In line with (Peters, 1994; Sánchez Granero et al., 2008), the procedure for calculating the Hurst exponent using the R/S analysis method is as follows:

We divide the return time series, which has a length of N , into d sub-series ($Z_{i,m}$) of length n , so that $d * n = N$ where n is an integral divisor of N .

Then, we calculate the mean (E_m) and the standard deviation (S_m) for each sub-series ($Z_{i,m}$) where $m = 1, 2, 3, \dots, d$.

Next, we calculate the demeaned return of the sub-series ($Z_{i,m}$) by subtracting the sub-sample mean:

$$X_{i,m} = Z_{i,m} - E_m \text{ for } i = 1, 2, \dots, n \quad (1)$$

After that, we create a cumulative time series by summing up the demeaned sub-series:

$$Y_{i,m} = \sum_{j=1}^i X_{j,m} \text{ for } i = 1, 2, \dots, n \quad (2)$$

We calculate the range for each sub-series:

$$R_m = \max(Y_{1,m}, \dots, Y_{n,m}) - \min(Y_{1,m}, \dots, Y_{n,m}). \quad (3)$$

For each sup-series, we obtain the rescaled range (R_m/S_m) by dividing the range (R_m) by the standard deviation (S_m) that corresponds to it. We then calculate the mean value $(R/S)_n$ of the rescaled range for all sub-series of length n :

$$(R/S)_n = (1/d) \left(\sum_{m=1}^d R_m/S_m \right) \quad (4)$$

Taking into account that the R/S statistic follows $(R/S)_n \sim cn^H$, we can estimate the index- H by running a simple linear regression:

$$\log(R/S)_n = H \log(n) + \log c \quad (5)$$

For a small value of time series n , there is a deviation from the 0.5 slope. Annis and Lloyd (1976) introduced the theoretical value of the R/S statistics to be as follows (Weron, 2002):

$$E[(R/S)_n] = \begin{cases} \frac{\Gamma(\frac{n-1}{2})}{\sqrt{\pi} \Gamma(\frac{n}{2})} \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}}, & \text{for } n \leq 340 \\ \frac{1}{\sqrt{n}} \sum_{i=1}^{n-1} \sqrt{\frac{n-i}{i}}, & \text{for } n > 340 \end{cases} \quad (6)$$

Where Γ is the Euler Gamma Function.

Finally, the Hurst exponent is calculated as $0.5 + H_n$ where H_n is the slope obtained by running a simple linear regression of $\log n$ as independent variable on $\log[(R/S)_n] - \log E(R/S)_n$ as dependent variable.

Hence, we have the following cases:

$0 \leq H < 0.5$: The process exhibits fractional Brownian motion and confirms the FMH. The time series is characterized by anti-persistent, meaning that trends revert to the mean.

$H = 0.5$: The process is standard Brownian motion. This confirms the EMH and suggests that the time series is random.

$0.5 < H \leq 1$: The process exhibits fractional Brownian motion and confirms the FMH. The time series has a long memory meaning that it is characterized by persistence. The higher Hurst coefficient values indicate a greater level of persistence in the time series.

In this study, we will use the Hurst exponent coefficient as a filter to select stocks with a long memory characteristic for inclusion in the portfolio. By doing so, we aim to construct an optimized portfolio that outperforms the benchmark portfolio. Moreover, we will demonstrate the benefits of selecting stocks with high Hurst coefficients in comparison to stocks with Hurst values close to 0.5. The following steps will be performed:

- (1) Find the Hurst exponent of each stock over the period between 7/1/2019 and 30/12/2021.
- (2) Filter the stocks that have a Hurst exponent greater than 0.5 and a positive daily average return.
- (3) Rank the filtered stocks based on their Hurst exponent values and divide them into two groups: the top 50% and the bottom 50%.
- (4) Construct three equally weighted portfolios: The first portfolio includes all stocks and is named as the "All stocks" portfolio. The second portfolio, referred to as the "High H-Value" portfolio, includes stocks with the highest Hurst exponent value, which make up the top 50%. The third portfolio, referred to as the "Low H-Value" portfolio, includes stocks with the lowest Hurst exponent value, which make up the bottom 50%.
- (5) Compute the mean, variance, Sharpe ratio, Treynor ratio, alpha, and the Hurst exponent coefficient for each portfolio's annual returns, and compare its performances with the market portfolio, for the same period.

Finally, we hold each portfolio for different holding periods, also referred to as investment or buy-and-hold periods, ranging from 3 January 2022 to 22 December 2022. These (out of sample) periods spanned 3, 6, 9, and 12 months. Subsequently, we calculate the mean, variance, Sharpe ratio, Treynor ratio, and alpha for each portfolio's returns.

4. Data sample and descriptive statistics

The dataset consists of all stocks listed on the DSE and the overall index (DWX) as a benchmark (market portfolio). The full data was compiled from the DSE official website over the period from 7 January 2019 until 22 December 2022, which makes a total of 894 trading days. The selected sample period was deliberately chosen to avoid the impact of the Syrian crisis on stock performance which started in 2011. To construct investable portfolios based on the Hurst exponent index, it is necessary to include only liquid stocks. Therefore, the time period was selected to maximize the number of stocks with at least 25 percent of trading days during the sampling period. This criterion ensures that a sufficient sample size is available for analysis. As a result, our sample consisted of only 19 out of 27 listed stocks. To overcome the non-stationary problem arising from using raw daily prices data, the daily returns were computed as the natural logarithm of the difference in daily closing prices after taking into account both stock and cash dividends.

The descriptive statistics for daily returns are summarized in Table 1. All stocks (except BSO) demonstrate positive but close to zero mean returns which are considered minor when compared to standard deviation. ATI and AVOC stocks have the highest daily mean returns of 0.0034 and 0.0027 respectively, whereas BSO has the lowest daily mean returns (−0.0001). SIIB shows the highest standard deviation of 0.0206, whereas BOJS has the lowest standard deviation of 0.0066. The overall positive average daily returns reflect the recovery period at the DSE where the exchange is bouncing back after a long period of decline due to the Syrian crisis. The distribution of returns departs markedly from normality given the observed skewness and excess kurtosis statistics. Consequently, based on the Jarque-Bera test, the null hypothesis of normality for the daily returns is rejected at the 1% significance level. Furthermore, the stock returns are skewed to the right suggesting a greater probability of large increases in returns than falls. All stock returns show high levels of kurtosis indicating that these distributions have thicker tails than the normal distribution (UIC has the highest skewness value of 6.5049 and the highest level of kurtosis of 60.0286). The descriptive statistics of the DWX show that it achieves daily returns of 0.0016 at a risk level of 0.0069, which is considered a relatively low return with a high level of risk. A Kurtosis statistic of 4.9559 points out that returns are leptokurtic distributed, and indicate higher peaks than expected from a normal distribution. Jarque-Bera statistics and their corresponding p-values suggest that the hypothesis of the normal distribution of DWX returns can be rejected. Hence, based on the descriptive analysis, it can be concluded that the daily returns of the 19 selected stocks and the market index are not well approximated by the normal distribution, and there is strong ground for rejecting the random walk characteristic. Additionally, the utilization of the Dickey-Fuller unit root test within the period of 2019–2021 reveals that the time series of returns for both individual stocks and the market index (DWX) exhibit stability, signifying their predictability.

Table 1 also presents the Ljung-Box test statistics ($Q(20)$) for the returns. The test reveals that most stock returns, with the exception of SIIB, exhibit significant autocorrelations. This indicates that there is a relationship between past and current stock returns for the majority of the stocks in the sample. Additionally, when examining the rate at which the autocorrelation function of stock return time series decays over time, it is observed that SYTEL, SKIC, AVOC, and ATI exhibit slower decay up to lag 20. This suggests that these stocks are influenced by their past returns for a longer period. On the other hand, FSBS, CHB, and BJOS show faster decay at the initial lags, indicating that they quickly forget their history. The market index also displays slower decay up to lag 12. These results may suggest the presence of varying degrees of long memory in the listed stocks at the DSE.

5. Empirical results and discussion

We initially calculate the Hurst exponent coefficient for the daily returns of 19 stocks and the benchmark portfolio (DWX). Table 2 illustrates that all stock returns exhibit a Hurst coefficient

Table 1. Descriptive statistics of daily returns of the examined stocks and the benchmark portfolio (DWX)

Stock	Mean	Std. Dev	Skewness	Kurtosis	Jarque-Bera	ADF	ACF (q(20))
ARBS	0.0008	0.0099	4.0510	28.0496	19407.48***	-15.1867***	263.38***
ATI	0.0034	0.0188	0.7778	5.3786	226.1695***	-8.58321***	582.57***
AVOC	0.0027	0.0175	1.6293	6.1422	573.7642***	-9.01407***	582.57***
BASY	0.0004	0.0088	3.0050	23.7681	13088.13***	-15.7569***	207.20***
BBS	0.0010	0.0082	6.0810	49.9936	65976.76***	-19.0446***	119.20***
BBSF	0.0012	0.0126	1.9823	9.5972	1658.747***	-16.7593***	293.10***
BBSY	0.0022	0.0189	0.6612	3.8934	71.30960***	-17.0393***	92.028***
BOJS	0.0006	0.0066	5.6892	43.1379	48734.50***	-14.9739***	42.174**
BSO	(-0.0001)	0.0130	1.2759	8.4845	1024.571***	-13.439***	129.02***
CHB	0.0014	0.0188	0.7306	4.7249	143.0956***	-20.2465***	58.056***
FSBS	0.0009	0.0141	1.5410	7.1464	747.3649***	-20.6303***	54.171***
IBTF	0.0014	0.0155	1.3950	6.5102	562.9608***	-12.5394***	248.48***
QNBS	0.0020	0.0189	1.1457	4.6591	224.0813***	-18.6301***	88.643***
SGB	0.0022	0.0167	1.6897	6.3328	630.7880***	-7.53122***	527.18***
SHRQ	0.0011	0.0116	2.7226	15.2687	5044.806***	-10.673***	146.66***
SIIB	0.0005	0.0206	1.0445	9.4239	1277.647***	-24.7453***	22.253
SKIC	0.0013	0.0075	5.5981	33.8278	30119.89***	-10.088***	646.32***
SYTEL	0.0015	0.0121	2.5817	17.5403	6666.311***	-5.50962***	1042.6***
UIC	0.0015	0.0097	6.5049	60.0286	95802.4***	-25.1734***	42.395**
DWX	0.0016	0.0069	1.4187	7.9559	896.78***	-10.1032***	381.36***

Note: the sample spans the period from 7 January 2019 to 30 December 2021. **, *** are significance at 5 and 1% levels, respectively.

above 0.50, indicating a fractal structure, with different degrees of long memory. Notably, SYTEL, AVOC, and ATI exhibit the highest H-values (0.76, 0.75, 0.73, respectively) signifying trend-reinforcing behavior. Conversely, SIIB, CHB, and FSBS have the lowest H-values (0.54, 0.54, 0.53, respectively), with values close to 0.50 indicating weaker trends. The market index also displays long memory with an H-value of 0.68 implying fractal and non-random structure, which confirms the hypothesis of a fractal market and rejects the hypothesis that the DSE is efficient in the weak form. This result is consistent with the findings of (Abbas, 2014; Mahmoud & Wardeh, 2018; Mouselli & Al-Samman, 2016) indicating that the DSE is not weak-form efficient, and with the findings of (Al-Hajieh, 2017; Assaf, 2016; Boubaker et al., 2022) suggesting the existence of the long memory in Middle Eastern markets. Based on these findings, we anticipate that we can leverage stocks that exhibit long memory and trending behavior to construct a robust portfolio that can outperform the benchmark portfolio. Furthermore, we expect the portfolio with high H-value stocks to outperform other portfolios.

Based on steps (2,3, and 4) explained in section 3, and considering the outcomes of the Hurst estimation presented in Table 2, three portfolios are constructed, with 18 stocks being included in the first portfolio (All stocks), 9 stocks in the second portfolio (High H-value), and 9 stocks in the third portfolio (Low H-value). BSO is excluded from the portfolios due to its negative returns during the ranking period. Table 3 summarizes the descriptive statistics of the daily returns of the three constructed portfolios.

The High H-value portfolio in Table 3 achieves the highest daily returns of 0.0018 and the lowest standard deviation of 0.0060. The return distributions for the three portfolios are skewed to the right, indicating a greater likelihood of large increases in returns than decreases. Additionally, the kurtosis values for these distributions are high, indicating thicker tails than those of the normal distribution. Specifically, the (All stocks) portfolio has the highest skewness value of 2.18, whereas the High H-value portfolio has the highest level of kurtosis (10.40). The test of Jarque-Bera confirms that the return distributions of the three portfolios deviate from normality, indicating a higher probability of observing extremely positive returns than a normal distribution.

Table 3 also reveals that the formed portfolios have a Hurst coefficient higher than 0.50, implying a fractal structure with long memory characteristics and predictive ability regarding the continuity of portfolios' superiority in the future. The (High H-value) portfolio has an H-value of 0.76 compared to H-values for the (All stocks) portfolio, (Low H-value) portfolio, and the benchmark portfolio of 0.68, 0.65, and 0.68 respectively indicating that they are trend-reinforcing portfolios.

To examine the effectiveness of the portfolio selection strategy under the combination of the Hurst exponent analysis and Markowitz (mean-variance) model, we calculate the annual returns,

Table 2. H -index estimation for daily returns of the examined stocks

STOCKS	HURST	STOCKS	HURST
SYTEL	0.76	BBSF	0.60
AVOC	0.75	ARBS	0.60
ATI	0.73	BOJS	0.59
IBTF	0.67	BBSY	0.57
UIC	0.66	BASY	0.56
SHRQ	0.66	BSO	0.56
SKIC	0.65	SIIB	0.54
SGB	0.63	CHB	0.54
BBS	0.63	FSBS	0.53
QNBS	0.61	DWX	0.68

Note: the sample spans the period from 7 January 2019 to 30 December 2021.

Table 3. Descriptive statistics of the daily returns of the three formed portfolios

Portfolio	Mean	Std. Dev	Skewness	Kurtosis	Jarque-Bera	p-value	Hurst
All stocks	0.0014	0.0061	2.18	9.89	1863.03	0.000	0.68
High H- Value	0.0018	0.0060	2.17	10.40	2054.25	0.000	0.76
Low H- Value	0.0011	0.0084	1.62	7.01	741.15	0.000	0.65

Note: the sample spans the period from 7 January 2019 to 30 December 2021.

Table 4. The annual performance of the three constructed portfolios compared to the benchmark portfolio

Portfolio	Mean	Variance	Beta	Sharpe	Treynor	Alpha
All stocks	0.424	0.017	0.24	19.65	1.41	0.25
High H- value	0.539	0.021	0.25	20.95	1.83	0.36
Low H- value	0.308	0.027	0.23	8.12	0.96	0.14
DWX	0.441	0.104	1.00	3.36	0.35	0.00

Note: The sample spans the period from 7 January 2019 to 30 December 2021.

variance, betas, Sharpe ratio, Treynor ratio, and alpha for all portfolios over the same period. Table 4 compares the annual performance of the (All stocks), High and Low-H-value portfolios to the benchmark portfolio (DWX).

In Table 4, the (High H-value) portfolio exhibits the most efficient performance, with the highest annual returns of 0.539, the highest Sharpe ratio of 20.95, the highest Treynor ratio of 1.83, and the highest alpha of 0.36, whereas the (All stocks) portfolio has the lowest variance of 0.017. On the other hand, although the (Low H-value) portfolio has lower returns of 0.308 compared to the benchmark portfolio, it still outperforms the benchmark portfolio in terms of ratios with a Sharpe ratio of 8.12, Treynor ratio of 0.96, and alpha of 0.14. The three portfolios are considered conservative, with a beta of less than 1, but the (High H-value) portfolio is slightly more sensitive to market changes than the (Low H-value) portfolio.

One of the most interesting findings is that the (High H-value) portfolio outperforms the (All stocks) and (Low H-value) portfolios, thereby indicating a higher degree of persistence exhibited by the high Hurst value series. Hence, to further evaluate the trend-reinforcing behavior of the three formed portfolios, we use different holding (out of sample) periods spanned 3, 6, 9, and 12 months from 3 January 2022 to 22 December 2022. We report the performance of the three formed portfolios and the benchmark portfolio in the four holding periods in Table 5 below.

Table 5 shows that the three portfolios selected based on the Hurst coefficient still outperform the benchmark portfolio (DWX), indicating the effectiveness of the fractal portfolio strategy. The (High H-value) portfolio outperforms the (All stocks) portfolio, the (Low H-value) portfolio, and the benchmark Portfolio in terms of returns. It achieves the highest returns in three out of the four holding periods (3-9-12) months, with returns of 0.318, 0.539, and 1.151 respectively. Furthermore, the (High H-value) portfolio exhibits the most efficient performance across all four holding periods (3-6-9-12) months, characterized by the lowest variance (0.004, 0.013, 0.015, and 0.022 respectively), the highest Sharpe ratio (68.91, 22.90, 31.56, and 49.12 respectively), the highest Treynor ratio (18.57, 27.26, 17.97, and 9.67 respectively). It also achieves the highest alpha in three of the four holding periods (3-9-12) months (0.29, 0.47, and 1.00 respectively). Furthermore, Table 5 demonstrates that the longer the holding periods, the higher the

Table 5. The performance of the three constructed portfolios and the benchmark portfolio

Holding Period	Portfolio	Mean	Variance	Beta	Sharpe	Treynor	Alpha
3	All stocks	0.297	0.022	0.05	12.65	5.85	0.27
	High H- value	0.318	0.004	0.02	68.91	18.57	0.29
	Low H- value	0.276	0.036	0.08	7.05	3.25	0.25
	DWX	0.106	0.076	1.00	1.10	0.08	0.00
6	All stocks	0.369	0.023	0.07	14.08	4.41	0.31
	High H- value	0.348	0.013	0.01	22.90	27.26	0.30
	Low H- value	0.391	0.039	0.14	8.96	2.55	0.33
	DWX	0.196	0.085	1.00	1.79	0.15	0.00
9	All stocks	0.523	0.025	0.09	18.45	5.12	0.44
	High H- value	0.539	0.015	0.03	31.56	17.97	0.47
	Low H- value	0.507	0.041	0.15	10.64	2.90	0.41
	DWX	0.289	0.093	1.00	2.39	0.22	0.00
12	All stocks	1.149	0.026	0.14	41.35	7.55	0.98
	High H- value	1.151	0.022	0.11	49.12	9.67	1.00
	Low H- value	1.147	0.044	0.17	24.20	6.18	0.96
	DWX	0.654	0.116	1.00	4.86	0.56	0.00

Note: The sample spans the period from 3 January 2022 to 22 December 2022.

portfolio returns. This can be explained by the continuous improvement of the DSE performance due to its recovery from the war period.

It is worth noting also that the (Low H-value) portfolio underperforms both the (All stocks) portfolio and the (High H-value) portfolio in terms of returns during three of the four holding periods. This trend is also evident in the Sharpe ratio, Treynor ratio, and alpha across all holding periods, indicating that lower H values correspond to weaker trends. These results align with the research conducted by (Chun et al., 2020), which supports the effectiveness of integrating fractal correlation into the Markowitz (mean-variance) model. Moreover, the findings of Wu et al. (2021) provide further validation for the effectiveness of incorporating fractal expectation and fractal variance into the return-risk criterion.

To sum up, our results emphasize the importance of considering the fractal nature of financial time series data when managing portfolio risk, as it allows for a more accurate representation of the complex dynamics of financial markets and ultimately improves portfolio performance.

5.1. Robustness tests: extended sample period

To further investigate the robustness of our fractal portfolio strategy and to enhance the accuracy of measuring the Hurst coefficient, we expand the sample period to span six years from 2017 to 2022. The period for estimating the Hurst coefficient (H) as well as forming fractal portfolios based on H is 2017–2021 while 2022 is the one-year holding period (out-of-sample period). We apply the same inclusion criteria for stock selection. That is, only liquid stocks are included. As a result, our sample consists of only 10 out of the 27 listed stocks. This implies that the size of both the High

and Low-H-value portfolios is diminished to only five stocks. Table 6 illustrates the mean, standard deviation, and Hurst coefficient for the daily return of the 10 examined stocks

Table 6 shows that all stocks demonstrate positive mean returns. ATI and BBSY stocks have the highest daily mean returns of 0.0033 and 0.0027 respectively, whereas SIIB and FSBS have the lowest daily mean returns of 0.0017. SIIB has the highest standard deviation of 0.0242, whereas ARBS has the lowest standard deviation of 0.0104. Also, Table 6 shows that when we expand the sample period all stocks remain exhibit a Hurst coefficient above 0.50, indicating a strong fractal structure at the DSE. Notably, AVOC and ATI still exhibit the highest H-values of 0.72, whereas SIIB and CHB still exhibit the lowest H-values of 0.61 and 0.59 respectively.

Following the same methodology explained in section 3, three portfolios are constructed, with 10 stocks being included in the first portfolio (All stocks), 5 stocks in the second portfolio (High H-value), and 5 stocks in the third portfolio (Low H-value). Table 7 displays the returns, risk, and Hurst coefficient for the three constructed portfolios for the period from 2017 to 2021.

It can be clearly seen from Table 7 that despite the decline in the number of stocks in the constructed portfolios, the (High H-value) portfolio and the (All stocks) portfolio outperform the benchmark portfolio. Moreover, the (High H-value) portfolio has the highest annual returns of 0.650 and the highest Hurst coefficient of 0.74, whereas the (Low H-value) portfolio has the lowest returns of 0.597 and the lowest Hurst coefficient of 0.66. These findings suggest that the three fractal portfolios serve as portfolios that reinforce trends, with the (High H-value) portfolio anticipated to demonstrate the most substantial returns during the investment duration. An evaluation of the annual performance of the three portfolios created, in addition to the benchmark portfolio, during a one-year investment period (out of sample period), is presented in the following table.

Table 8 reveals that the (High H-value) portfolio continues to exhibit superior returns throughout the investment period, displaying a robust return of 1.072, while the (Low H-value) portfolio

Table 6. Mean, standard deviation, and H-index of the examined stocks

Stocks	Mean	Std. Dev	Hurst
AVOC	0.0018	0.0148	0.72
ATI	0.0033	0.0202	0.72
IBTF	0.0020	0.0161	0.69
SGB	0.0021	0.0180	0.68
ARBS	0.0012	0.0104	0.66
FSBS	0.0017	0.0169	0.65
BBSY	0.0027	0.0192	0.63
QNBS	0.0019	0.0200	0.62
SIIB	0.0017	0.0242	0.61
CHB	0.0019	0.0194	0.59

Note: The sample spans the period from 3 January 2017 to 30 December 2021.

Table 7. Mean, variance, and H-index for the three constructed portfolios

Portfolio	Mean	Variance	Hurst
All stocks	0.623	0.028	0.72
High H- value	0.650	0.043	0.74
Low H- value	0.597	0.081	0.66
DWX	0.610	0.123	0.72

Note: The sample spans the period from 3 January 2017 to 30 December 2021.

Table 8. The performance of the three constructed portfolios and the benchmark portfolio

Portfolio	Mean	Variance	Beta	Sharpe	Treynor	Alpha
All stocks	1.046	0.044	0.23	21.79	4.15	0.83
High H- value	1.072	0.076	0.27	12.89	3.65	0.83
Low H- value	1.020	0.095	0.19	9.75	4.88	0.82
DWX	0.654	0.116	1.00	4.86	0.56	0.00

Note: The sample spans the period from 3 January 2022 to 22 December 2022.

maintains its lowest returns of 1.020 compared to the (High H-value) portfolio and the (All stocks) portfolio. These results support the robustness of the Hurst-based selection strategy.

It is worth noting that although the benchmark portfolio had a high Hurst coefficient value of 0.72 during the ranking period, it surprisingly achieved the lowest returns in the holding period. This can be attributed to the inclusion of illiquid stocks in the market index.

5.2. Robustness tests: the performance of fractal strategy in comparison to alternative investment strategies

Momentum and size-based strategies may be proposed as alternative profitable investment strategies for portfolio sorting. In this paper, we compare the performance of fractal portfolios to momentum and size-based portfolios. To achieve this objective, we use the same data sample consisting of 19 listed stocks for the period between 2019 and 2022. Moreover, all portfolios are constructed based on equal weighting and we use standard deviation as a measure of portfolio risk.

5.2.1. Momentum strategy vs. fractal strategy

Following Jegadeesh and Titman (1993), at the end of each year, all stocks are ranked in descending order based on their past 12-month cumulative returns (ranking period). Subsequently, the stocks are categorized into three portfolios with equal weighting, where the top 30 percent of these portfolios are considered winners, whereas the bottom 30 percent are labeled losers. These portfolios are then held for a period of 12 months (holding period). The complete rebalancing approach is utilized to construct portfolios for both the winners and losers annually. After that, we compute average returns and standard deviations for winners portfolios and losers portfolios over the three years spanning from 2020 to 2022. Table 9 compares the annual performance of winners and losers to the fractal portfolios.

Table 9 shows that winners portfolios outperform losers portfolios. When compared to portfolios based on the Hurst criterion, both the winners and losers underperform fractal strategy portfolios. However, the momentum strategy (taking a long position in past winners and a short position in past losers) seems nonprofitable at the DSE, with a negative Sharpe ratio.

Table 9. The profitability of momentum strategy vs. fractal strategy in Damascus security exchange

	Portfolio	Mean	Std. Dev	Sharpe Ratio
Momentum strategy	Winners	0.593	0.464	1.35
	Losers	0.552	0.273	1.88
	Winners-Losers	0.041	0.597	-0.08
Fractal strategy	All stocks	1.149	0.847	1.25
	High H- value	1.151	1.055	1.01
	Low H- value	1.147	0.640	1.65

Note: The sample spans the period from 7 January 2019 to 22 December 2022.

Table 10. The profitability of size strategy vs. fractal strategy in Damascus security exchange

	Portfolio	Mean	Std. Dev	Sharpe Ratio
Size-based strategy	Small	0.594	0.450	1.12
	Big	0.546	0.410	1.11
	Small-Big	0.048	0.550	-0.08
Fractal strategy	All stocks	1.149	0.847	1.25
	High H- value	1.151	1.055	1.01
	Low H- value	1.147	0.640	1.65

Note: The sample spans the period from 7 January 2019 to 22 December 2022.

5.2.2. Size investment strategy vs. fractal strategy

Following Fama and French (2012), all stocks are classified according to their market capitalization in the last year (2019). Subsequently, stocks are ranked in descending order and categorized into three portfolios. The bottom 30 percent of these portfolios are considered: small portfolios, whereas the top 30 percent are considered: big portfolios. This process is carried out annually, and the small and big portfolios are held for a year. Following this, the average returns and standard deviations for both small and big portfolios over the three years (holding periods) spanning from 2020 to 2022 are computed. Table 10 compares the annual performance of small and big portfolios to the performance of the fractal portfolios.

Table 10 illustrates small portfolios outperform big portfolios by 0.048 annually. However, when compared to portfolios based on the Hurst criterion, both small and big portfolios underperform. In addition, implementing the Size strategy by taking a long position in small portfolios and a short position in big portfolios fails to generate a positive Sharpe ratio, indicating poor risk-adjusted performance.

6. Conclusions and recommendations

To enhance portfolio performance in non-linear financial markets with non-normally distributed returns and fat tails, the utilization of High-H portfolios has shown great promise. In this study, we apply the Hurst exponent coefficient to filter assets based on their long memory and construct three optimized portfolios. Our findings reveal the presence of a fractal structure with long memory at the DSE, allowing traders to make more informed investment portfolio selection decisions by incorporating fractal analysis. These results are consistent with the existing literature (Chun et al., 2020; Wu et al., 2021), providing evidence for the effectiveness of portfolios that rely on fractal statistical measures in their development. Additionally, the findings demonstrate that leveraging stocks with high Hurst coefficients can significantly enhance the performance of the fractal portfolio strategy, as higher Hurst coefficient values indicate a greater level of persistence.

Furthermore, to ensure the reliability of our findings, we expand the estimation period for the Hurst exponent to span five years, from 2017 to 2021. The results reaffirmed the superiority of fractal portfolios over the market portfolio, despite the reduced number of stocks. Moreover, portfolios with high Hurst coefficients consistently achieved the highest returns among all portfolios.

The empirical literature offers several active strategies for selecting and classifying stocks in a portfolio. In this study, we conducted a comparison between portfolios formed through fractal analysis and portfolios formed using the momentum and size strategies. However, our findings demonstrated the superiority of the fractal portfolio over both the momentum and size-based portfolios. This highlights the effectiveness of fractal analysis in achieving superior portfolio performance.

The DSE is a relatively new stock exchange, that faced many challenges due to the impact of the Syrian crisis and had a limited number of listed companies. This reality resulted in a small number of liquid stocks for portfolio construction and for the estimation of the Hurst exponent coefficient especially before 2019. In addition, this represents a challenge in evaluating the effectiveness of a contrarian strategy compared to a fractal strategy. Hence, future studies could consider extending the estimation period for the Hurst exponent beyond 2022 as well as examine the performance of contrarian strategy and other strategies once additional data become available. Moreover, future studies could explore the integration of fractal statistical measures like fractal expectation, fractal variance, fractal correlation analysis, and the Hurst exponent into alternative portfolio optimization techniques. This inclusion would enhance diversification and risk management strategies, ultimately leading to improved portfolio performance and the achievement of desired investment outcomes.

Finally, based on our empirical results we recommend that financial market participants leverage the fractal properties of the DSE to develop strategies that offer improved portfolio diversification benefits. Furthermore, investors and portfolio managers should consider stocks' long memory in their portfolio construction and utilize the Hurst exponent index in their portfolio selection.

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