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Working Paper

Inefficiencies in coupled electricity markets with different time granularity: Exploring the zigzag pattern

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Inefficiencies in coupled electricity markets with different time granularity: Exploring the zigzag pattern

HEMF Working Paper No. 06/2024

by

Christopher Jahns

August 31, 2024

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Abstract

In European intraday electricity markets, a systematic zigzag pattern can be observed, characterized by alternating maxima and minima at the shift of hourly products. This price formation contradicts the fundamental understanding, that prices to are a monotonously increasing function of residual demand and correspondingly would evolve smoothly across hours. This study investigates the phenomenon of restricted participation as the primary cause of the zigzag pattern. Restricted participation occurs when market participants maintain constant output through sub-hourly products. A notable instance is the lack of sub-hourly cross-border trading where foreign market participants are restricted from engaging in trading sub-hourly products. This is closely linked to is the differing time granularities observed in electricity trading across European countries. Three research questions guide this investigation: (1) Is restricted participation the main cause of non-smooth intraday prices? (2) How do technical restrictions on cross-border trading contribute? (3) What role do ramping and start-up costs play? Utilizing regression models and openly available data from Germany, the research confirms that restricted participation is the primary cause of the zigzag pattern. Furthermore, an analytical model of restricted participation in cross-border trading is developed, along with empirical parameterization. Based on this model it is estimated that lifting all technical restrictions on trading sub-hourly products would reduce systematic non-smoothness in intraday prices by only about 27%. These findings are unexpected, suggesting that despite economic incentives, a significant number of domestic power plants do not adjust their output based on intraday price signals. Ramping and start-up costs appear to have little influence.

Keywords: Intraday electricity market; Econometric modeling; Market coupling

JEL-Classification: C10; C54; Q41

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1 Introduction

After the liberalization of European markets, electricity trading was initially limited to the national level. However, cross-border trading between different bidding zones was gradually introduced over the years, with the goal of enhancing overall welfare (Weber, 2023). Within this context, inefficiencies in the market design and the market coupling mechanism have been repeatedly debated, and improvements have been made to increase overall social welfare and the security of supply. Enhancements notably include flow-based market coupling (e.g., Felten et al., 2021) and intraday market coupling (e.g., Hirsch and Ziel, 2024).

Knaut and Paschmann (2017, 2019) discuss inefficiencies in the European market coupling mechanism considering the zigzag price pattern in European wholesale electricity markets. This price pattern is characterized by alternating maxima and minima in sub-hourly intraday prices (as shown in Figure 1). Knaut and Paschmann (2017) hypothesize that the origin of the pattern lies in the lack of sub-hourly cross-border trading. They demonstrate that the zigzag pattern can be replicated in a bottom-up model of the European market, where sub-hourly products cannot be traded across borders. Additionally, the authors show that ramping costs can enhance this effect. Knaut and Paschmann (2019) generalize this effect under the term 'restricted participation', wherein some market participants can adjust their output on an hourly basis but are required to maintain a constant output level across sub-hourly products. They argue that the absence of sub-hourly cross-border trading represents a specific instance of restricted participation, as foreign market participants are restricted to trading at the hourly average level. Based on an analytical model they find evidence for restricted participation in Germany. Furthermore, they estimate welfare losses attributable to restricted participation to be approximately 108 million euros in 2015. To the best of the author's knowledge, apart from the latter publication, there are currently no other publications addressing the fundamental causes of this prominent pattern. This reveals a research gap: while the overall impact of restricted participation on the volatility of intraday prices has been analyzed, the role of cross-border trading has not been empirically assessed.

The paper correspondingly addresses three research questions: 1) Is the non-smoothness (zigzag pattern) in intraday prices mainly caused by restricted participation? 2) To what extent does the restricted sub-hourly cross-border trading, as a specific instance of restricted participation, contribute to the non-smoothness (zigzag pattern) in intraday prices? 3) Do ramping and start-up costs, alongside with restricted participation, reinforce the zigzag pattern?

The first research question addressed by this publication is hence whether restricted participation in intraday markets is the primary cause of the observed zigzag pattern in intraday prices. The second research question examines the extent to which the restricted sub-hourly cross-border trading, as a specific instance of restricted participation, contributes to the zigzag pattern. Finally, the third research question explores how ramping and start-up costs may amplify this effect.

By addressing these research questions, valuable insights are expected for market participants, policymakers, and researchers. This research is crucial as it may offer a basis for assessing and mitigating this phenomenon, potentially enhancing market efficiencies. For researchers and market participants, understanding whether cross-border trading truly drives the zigzag pattern is valuable, as such a cost-based mechanism can be replicated in bottom-up models and used to simulate intraday prices. For market participants, especially those operating storage and thermal power plants, the zigzag pattern can result in additional profits. Additionally, understanding the cause of the pattern can help form expectations for market development.

To address the research questions, regression models are employed using openly available data. The empirical investigation utilizes an innovative estimation procedure. Based on the assumption that, in a scenario without restricted participation, intraday prices and residual demand should exhibit smooth¹ time series properties, counterfactual price and residual demand time series can be estimated and directly compared to the observed values. This approach allows for isolating and analyzing the specific impact of the absence of restricted participation and cross-border trading. Additionally, an analytical model is developed based on Knaut and Paschmann (2019), incorporating both restricted participation and cross-border trading. This model is used to estimate the effects of removing all technical restrictions on trading sub-hourly products across borders on the zigzag pattern.

The structure of this paper is as follows: Section 2 provides background information, including an overview of the European market design, a characterization of the zigzag pattern, and a literature review on price formation in intraday markets with a specific focus on the zigzag pattern. Section 3 outlines the analytical model proposed by Knaut and Paschmann (2019), which is used to derive insights into the price differences between scenarios with and without restricted participation. Additionally, Section 3.4 examines the effects of ramping and start-up costs on intraday prices, while Section 3.5 analyzes the impact of cross-border trading. Section 4 details the hypotheses derived from the the research questions and the general approach for addressing these questions, incorporating insights from Section 3. Section 5.1 summarizes the data and preprocessing methods employed. Section 5 describes the empirical strategy, including the smoothing technique used to create a counterfactual intraday price time series without restricted participation and the parameterization of the model extensions discussed in Sections 3.4 and 3.5. Section 6 presents the findings, and Section 7 summarizes the variations and robustness checks. Finally, Section 8 offers a summary and discussion of the main findings.

¹ Here, 'smoothness' refers to a reduction in abrupt changes and irregular fluctuations over time, as opposed to the mathematical definition of a smooth function. A detailed discussion on the definition of smoothness for time series can be found in Froeb and Koyak (1994).

2 The zigzag pattern in European intraday prices

This section provides a foundational overview of the zigzag pattern observed in European intraday electricity prices. First, a brief description of the European market design and market coupling mechanisms is presented. Following this, the zigzag pattern itself is outlined, highlighting its characterization as a non-smooth price pattern, with the zigzag pattern being an extreme case of this non-smoothness. Lastly, a summary of the literature on this pattern is provided.

2.1 European electricity market auctions, intraday markets and market coupling mechanism

The European Day-Ahead auction, which primarily handles hourly products, allows bid submissions until 12:00 PM on the day preceding the delivery date. The market coupling mechanism for the Day-Ahead auction extends to enable cross-border trading, involving a majority of European countries (ENTSO-E, 2023b) through the Single Day-Ahead Coupling (SDAC).

In the intraday market in Germany, the intraday auction bid submission is open until 3:00 PM, allowing trading of quarter-hourly products. Subsequently, continuous intraday trading begins, providing market participants with an opportunity to trade any remaining quantities after the intraday auction (IDA). This is particularly relevant due to Germany's requirement for balanced schedules on a quarter-hourly basis. A comprehensive summary of the market design is available in Viehmann (2017). Additionally, Hirsch and Ziel (2024) offer a recent summary focused on intraday markets.

As a market reform, a multiple intraday auction with an implicit market coupling mechanism was introduced on June 13th, 2024 (Nemo committee, 2024). Three intraday auctions were introduced, where the results of the first intraday auction (IDA1) are published at 15:00 the day before

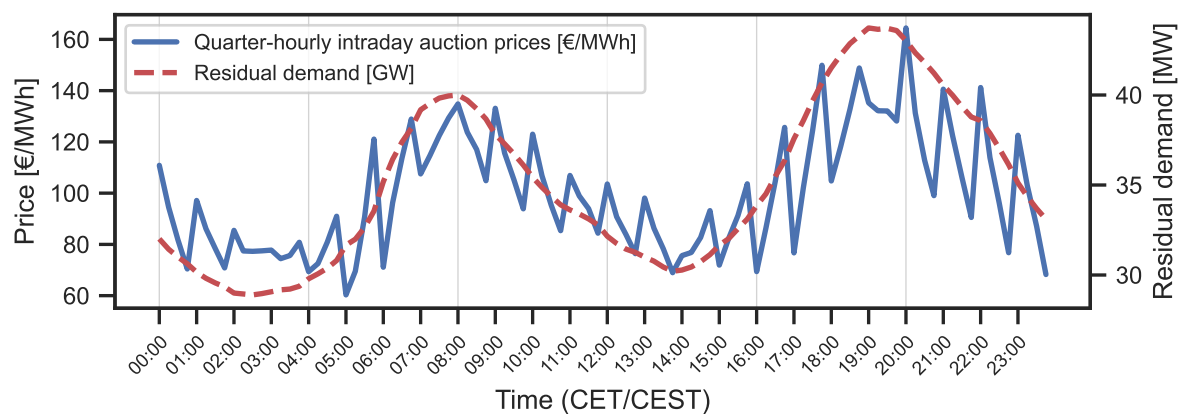


Figure 1: Average intraday auction prices and average residual demand in Germany (2019-2023)

delivery, the results of the second auction (IDA2) are published at 22:00 the day before delivery, and the results of the third auction (IDA3) are published at 10:00 on the delivery day. Within this framework, cross-border capacities in most European countries are allocated using sub-hourly products, although some restrictions remain due to differing time granularities across countries. In Europe, there is a requirement for balancing at various time intervals, such as 15 minutes, 30 minutes, or 60 minutes, as indicated by Ocker and Jaenisch (2020). Specifically, 30-minute products are traded in France, Germany, the United Kingdom, Luxembourg, and Switzerland, while 15-minute products are traded in Austria, Belgium, Germany, Hungary, Luxembourg, the Netherlands, Slovenia, and Switzerland (IRENA, 2019).

As the analysis within the publication focuses on the period before the market reform, the following describes the market design prior to the change. Until June 13th, 2024, the intraday auction (IDA) in Germany did not incorporate a market coupling mechanism. As a result, trading within the intraday auction was confined to the German/Luxembourg bidding zone (Kath, 2019, p. 4). Notably, since June 2018, a market coupling mechanism called the Single Intraday Coupling (SIDC) has been operational within the continuous intraday market (ENTSO-E, 2023a). Consequently, although cross-border trading was already established, Belgium remained the only country where quarter-hourly products were traded with Germany (ENTSO-E, 2023c). More details on the implementation of the intraday cross-border mechanism (SIDC) are available in Kath (2019).

2.2 The zigzag anomalie: A non-smooth price pattern

The following describes the zigzag pattern in sub-hourly products in wholesale electricity markets and explains why this pattern is in apparent contradiction to the basic principles of price formation in these markets. The pattern in Germany is depicted in Figure 1. The prices are characterized by alternating local minima and maxima. Notably, during periods of increasing prices, the maximum often occurs in the last quarter of the hourly product followed by a price minimum in the first quarter of the following hour. During periods of decreasing prices, it is the other way around. This zigzag pattern is not confined to a specific market segment; it manifests in both the intraday auction (IDA) and the continuous intraday market (IDC). For example, in Hungary (HUPX, 2024) and Slovenia (BSP SouthPool, 2024), similar patterns can be observed in quarter-hourly intraday prices.

Hence, even during periods where residual demand is monotonically increasing, large price drops can be observed at the shift of hourly products. This zigzag pattern is not limited to average prices but appears persistent in the original time series as well. More broadly, given the smooth nature of residual demand over time, wholesale prices, as a transformation of residual demand by a monotonically increasing and approximately smooth function, should also exhibit smoothness. Thus, the observation that sub-hourly prices are non-smooth can be categorized under a broader irregularity, namely, non-smoothness in electricity prices. This irregularity may not always manifest

as alternating local maxima and minima but can also present as abrupt changes in price development. In the subsequent analysis, we will investigate the non-smoothness in wholesale electricity prices, where the zigzag formation represents an extreme instance of this irregularity.

2.3 Literature overview: Price formation on intraday markets and the zigzag pattern

In the following, the relevant literature on price formation in intraday electricity markets and the zigzag pattern is explored. A comprehensive review of the current literature on European intraday electricity markets is provided by Birkeland and Alskaif (2024). It's important to note that the literature directly addressing this pattern is scarce. Only two publications, specifically Knaut and Paschmann (2017) and Knaut and Paschmann (2019), discuss possible fundamental effects.

In the realm of intraday markets, Pape et al. (2016) introduced a comprehensive pricing model. The observed increase in volatility in intraday markets, according to this model, is attributed to a shortened intraday supply stack. This reduction in the supply stack arises primarily from the limited ramping capabilities of inflexible power plants as the time of delivery approaches. The shortened supply curve represents a rotation of the original supply curve around the point that corresponds to market expectations, highlighting the additional costs incurred from deviations from the expected power plant schedules. Consequently, adjustments to forecast updates and events occurring after the Day-Ahead auction are considered to have a more pronounced impact on prices compared to the full supply stack in the Day-Ahead auction. This dynamic contributes to the heightened volatility observed in intraday prices.

Märkle-Huß et al. (2018) specifically analyzed intraday prices to examine the impact of introducing the intraday auction in Germany, addressing the causal effect of shorter trading intervals on electricity prices and trading volumes. For electricity prices, they reported a decrease in price levels following the launch of the 15-minute auctions. Regarding trading quantity, Märkle-Huß et al. (2018, p. 371) reported findings of a zigzag pattern. They conducted expert interviews with electricity retailers on the origin of this pattern. They report that the reason is that average quantities are traded on the Day-Ahead market and that the remainder is traded in the intraday market, leading to a zigzag pattern in trading quantity for quarter-hourly products.

The zigzag pattern in prices has been thoroughly examined in a study by Kiesel and Paraschiv (2017). In their research, they specifically investigated intraday prices using regression analysis. Their focus is on understanding how forecast errors affect 15-minute intraday prices, with a specific emphasis on the last price of continuous intraday trading. Additionally, Kiesel and Paraschiv (2017) proposed that this observed zigzag pattern could be attributed to intertemporal restrictions within fossil fuel power plants. Ramping costs associated with these plants might outweigh fuel costs in terms of short-run marginal costs, thus limiting the ability of power plant operators to respond to fluctuations in demand. Building on this work, Kremer et al. (2021) reevaluated the

analysis conducted by Kiesel and Paraschiv (2017) and improved the model by incorporating the slope of the merit order. Additionally, they investigated the effect of neighboring contracts on the price dynamics of a given contract and the influence of the intraday auction price on the intraday continuous price. In a descriptive analysis, Kremer et al. (2021) reported that trading volume is higher in the first and last quarters of the hour, supporting the hypothesis of trading residual quantities on the intraday market. Their research highlights that the slope of the merit order curve significantly affects the impact of forecast updates on intraday prices. Regarding the origin of the zigzag pattern in electricity prices, Kremer et al. (2021, p. 5) proposed an explanation based on trading pressure experienced by solar energy generators. After trading average quantities on the Day-Ahead auction, the renewable electricity supplier experiences buy and sell pressure for quarter-hourly products when trading residual quantities in the intraday auction. This pressure varies depending on the incline of the solar ramp and, according to Kremer et al. (2021), this is in line with the zigzag price formation.

A broader perspective on the zigzag pattern is offered by Braun and Brunner (2018). This study identified limitations in existing explanations for this phenomenon. Notably, it challenges the notion that the pattern is caused by the sequential market clearing of the Day-Ahead and intraday market. An example illustrating this point is provided by the Energy Exchange Austria (EXAA), where both the quarter-hourly and hourly Day-Ahead auctions have been cleared together since 2014, yet the market still exhibits the zigzag pattern (Braun and Brunner, 2018). Furthermore, Braun and Brunner (2018) question the idea that a higher variance following information updates provides a satisfactory explanation. They notably point out that despite the relatively small difference in information between the Day-Ahead and intraday auctions, in contrast to the Intraday Continuous (IDC) market, the zigzag pattern persists.

Knaut and Paschmann (2017) as well as Knaut and Paschmann (2019) hypothesized that the zigzag pattern may be sourced by restricted participation. This term refers to situations where certain market participants can adjust their output on an hourly basis but are required to maintain a constant output level across sub-hourly products. Four potential drivers of restricted participation are: i) inertia as a state of not knowing, ii) costs of market entry, iii) inflexibility of power plants, and iv) a lack of cross-border market coupling. The authors argue that the costs of market entry should not be as relevant, illustrating with an example that the potential revenues outweigh the participation costs. Furthermore, they demonstrate the effect of the lack of sub-hourly cross-border trading in a bottom-up model (Knaut and Paschmann, 2017). For the demonstration, a linear optimization model called 'DIMENSION' is used, which incorporates ramping constraints and start-up costs, and considers Germany and its neighboring countries. Without cross-border trading for quarter-hourly products, the authors show that the zigzag pattern can be reproduced. In a variation, they show how technical constraints, such as ramping constraints, enhance the effect of the absence of cross-border trading for quarter-hours. Knaut and Paschmann (2019) further examined the effect of restricted participation using an analytical model, where they separate market participants into 'restricted' and 'unrestricted' categories (see Section 3.1). Based on this model, the authors

concluded that the main predictor of intraday price volatility with restricted participation is the difference between the hourly average demand and the sub-hourly demand. Using this analytical framework, they defined a regression model that implicitly estimates the slope of the supply curve for unrestricted market participants as well as the aggregated supply curve. Furthermore, the authors show that the simulated volatility from the regression model closely matches the observed volatility in intraday prices. Hence, they concluded that by considering restricted participation, most of the volatility in intraday markets, including the zigzag price formation, can be explained. Additionally, they estimate a significant welfare loss due to restricted participation of 108 million euros for 2015. Based on the bottom-up model from Knaut and Paschmann (2017), they concluded that the lack of sub-hourly trading is the primary driver of restricted participation but do not further empirically validate this assumption.

Given the trading volumes observed in the intraday auction, it appears quite plausible that market participants choose to trade average quantities on the Day-Ahead market and subsequently engage in transactions for the remaining quarter-hourly quantities on the intraday market (Braun and Brunner, 2018, p. 262).² On this basis, Kremer (2021, p. 5) hypothesized that trading pressure for providers of renewable energies, in combination with trading residual quantities, is the main mechanism resulting in the observed zigzag pattern. These explanations seem straightforward but fall short for several reasons. Firstly, following the model of Pape et al. (2016), it is essential to note that residual demand, and not renewable generation alone, is the main driver of price formation. Both the demand side and the renewable side experience the same trading pressure to balance at a quarter-hourly level. Simultaneously, intertemporal constraints may induce trading pressure on operators of thermal power plants as well.

More generally, a steeper supply curve in intraday markets is not a sufficient explanation for the zigzag pattern. According to the model from Pape et al. (2016), an increased supply curve should rotate around a center representing the market expectations before the shortening. Consequently, intraday market prices should experience higher volatility around the Day-Ahead expectation, with observed prices randomly fluctuating above or below the price expectation. The systematic non-smoothness, where prices are consistently above or below the smooth price formation depending on the quarter-hour and the inclination of the residual demand, requires additional assumptions: that market participants are precluded from realizing the optimal power plant schedule in quarter-hourly periods, that the intraday market is not fully anticipated by market participants, or the presence of market power.

Hence, Knaut and Paschmann (2017) as well as Knaut and Paschmann (2019) provide the only fundamental explanation so far for the origin of the zigzag pattern, supported by empirical evidence. However, a research gap remains, which this paper aims to address. While Knaut and Paschmann (2019) conducted an empirical analysis on restricted participation, they primarily investigated the effect on increased volatility in intraday prices and did not quantify the extent to

²However, it should be noted that Narajewski and Ziel (2022) as well as others, including Weber (2005) and Pinson et al. (2007), have clarified that hedging according to the expected volume is not always optimal.

which the zigzag pattern can be explained by restricted participation. Additionally, the authors did not differentiate between the effects of cross-border trading and other institutional factors. This distinction is crucial, as the lack of cross-border trading itself aligns with bottom-up modeling and is inherently different from the explanation of institutional influences. Furthermore, despite the observed effect of ramping costs in the bottom-up model from Knaut and Paschmann (2017), the regression model in Knaut and Paschmann (2019) does not account for ramping events.

3 Restricted participation as the main cause of the zigzag pattern

The study by Knaut and Paschmann (2019) analytically demonstrates how restricted participation in intraday markets can generally lead to a zigzag pattern in prices. This section provides a summary of their analytical model. Next, additional propositions are derived to specifically address the non-smoothness observed in intraday prices. The effects of market power, as well as ramping and start-up costs, are then discussed. Further extensions incorporate cross-border trading into the model. These additions and extensions are utilized to derive testable hypotheses Section 4, while the empirical strategy, including model parameterization, is described in Section 5.

3.1 Analytical model restricted participation

In the following, the analytical model from Knaut and Paschmann (2019) is summarized. In the context of the electricity market, we consider a common product, electricity, which is supplied by both restricted and unrestricted suppliers. This common product is split into n different sub-products in the second market, each identified by $\tau \in \{1, 2, \dots, n\}$, where τ represents different time intervals. The quantity of electricity demand in each time interval τ is denoted by D_τ as consumers may demand different quantities in different intervals. For unrestricted suppliers, q_τ^u represents the quantity supplied in time interval τ , and $C_u(q_\tau^u)$ is the total cost of producing q_τ^u units of electricity, reflecting the linearly increasing marginal costs of generation. The unrestricted suppliers adjust their production levels to meet the demand in both markets. On the other hand, q^r is the fixed quantity supplied by restricted suppliers across all time intervals, and $C_r(q^r)$ is the total cost of producing q^r units of electricity, which also include linearly increasing marginal costs.

The objective function is then:

$$\min \sum_{\tau} [C_u(q_\tau^u) + C_r(q^r)], \quad (1)$$

subject to the constraint that the total demand is met:

$$D_\tau = q_\tau^u + q^r \quad \forall \tau. \quad (2)$$

The marginal cost functions for both suppliers are defined as follows:

$$C'_r(q^r) = a_0^r + a_1^r q^r \quad (3)$$

and

$$C'_u(q^u) = a_0^u + a_1^u q^u. \quad (4)$$

Thereby, a_0^u and a_0^r are the offsets³, a_1^r is the gradient of the restricted inverse supply curve, and a_1^u is the gradient of the inverse unrestricted supply curve.

By combining both functions, the aggregate supply function is expressed as:

$$C'(q) = a_0 + \frac{a_1^r a_1^u}{a_1^r + a_1^u} q \quad (5)$$

$$= a_0 + a_1 q. \quad (6)$$

where a_1 is the gradient and a_0 is the offset of the aggregated inverse supply curve. Proof, along with the equation for a_0 , is given in the Appendix A.

Additional, the authors show that the average price is:

$$\bar{p} = a_0 + a_1 \bar{D} \quad (7)$$

Proof is given in the Appendix B.

Knaut and Paschmann (2019) show that the price in τ is as follows:

$$p_\tau = a_0 + a_1 \bar{D} + (D_\tau - \bar{D}) a_1^u \quad (8)$$

$$= \bar{p}_\tau + (D_\tau - \bar{D}) a_1^u, \quad (9)$$

Proof is given in the Appendix C.

where \bar{p}_τ represents the average price and \bar{D} is the average demand for the full hour. This demonstrates that the volatility in the intraday market is affected by the residual demand deviation and the intraday demand, as well as the gradient a_1^u from the unrestricted participants.

3.2 Additional propositions: Difference between full participation and restricted participation

Building on the publication by Knaut and Paschmann (2019), the following section introduces additional propositions. As we are interested in the effect of restricted participation on the zigzag

³Here the analytical model has been adapted to account for different offsets for restricted and unrestricted suppliers.

pattern, and not on intraday volatility in general, we aim to isolate the impact of restricted participation by comparing it to a case with full participation, where q_τ^r can vary for every τ . The price p_τ^* in a case with full participation can therefore be expressed using Equation 5 (aggregated inverse supply curve):

$$p_\tau^* = a_0 + a_1 D_\tau. \quad (10)$$

Proposition 1 *The difference between the prices with full participation (p_τ^*) and with restricted participation (p_τ) depends on the difference between the average demand and the demand in each time period.*

$$\begin{aligned} p_\tau - p_\tau^* &= (a_0 + a_1 \bar{D} + (D_\tau - \bar{D})a_1^u) - (a_0 + a_1 D_\tau) \\ &= (a_1^u - a_1)(D_\tau - \bar{D}). \end{aligned} \quad (11)$$

The proof is in Appendix D.

With the additional assumption that the prices in the case ‘full participation’ are smooth i.e., that the non-smoothness can be attributed to restricted participation, we can conclude that the emerging zigzag pattern is determined by the ‘residual demand deviation’ ($D_\tau - \bar{D}$) and the difference ($a_1^u - a_1$) between the slope of the inverse supply curve of the unrestricted market participants and the slope of the aggregated inverse supply curve.

Proposition 2: *The average of prices in a case with restricted participation is equal to the average of prices with full participation.*

$$\frac{1}{n} \sum_{\tau} p_\tau = \frac{1}{n} \sum_{\tau} p_\tau^*. \quad (12)$$

The difference between the prices with and without restricted participation is further illustrated in Figure 2 for the example of two time intervals. The demand in the first half of the hour is D_1 , in the second half it is D_2 , and the average demand for the entire hour is \bar{D} . Therefore, the hourly price \bar{p}_t is determined by the intersection of the hourly demand and the aggregated inverse supply curve.

With full participation, the price p_t^* for the second half of the hour would be determined by the intersection of the aggregated inverse supply curve and the demand D_2 . However, with restricted participation, deviations from the average demand are more costly because only unrestricted market participants are available. Thus, the price with restricted participation is determined by the

steeper inverse supply curve that represents the ramping up and down potential for the sub-hourly periods.

It is important to note that, unlike in the model of Pape et al. (2016), the steeper inverse supply curve in this model is not rotated around the intersection between demand D_2 and the aggregated inverse supply curve, since the restricted participants cannot adjust their output on a sub-hourly basis.

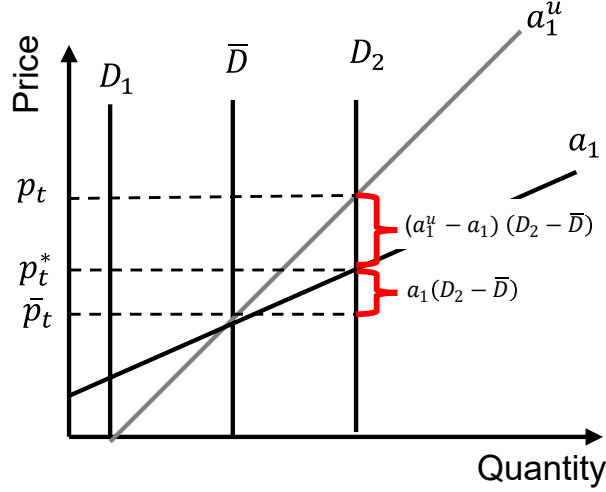


Figure 2: Supply curve restricted and unrestricted market participants

3.3 Market power

It is important to note that an unrestricted supplier with market power can strategically manipulate prices to maximize profits. Given the assumption of inflexible demand, the unrestricted supplier can adjust market prices without affecting overall consumption. By increasing prices during periods of higher production and decreasing prices during periods of lower production, the supplier can exploit market conditions to extract additional profits. This strategy is possible because profit is realized as a weighted mean, allowing the unweighted mean price to remain unchanged. As long as the average price over an hour stays constant, there is no competition from the restricted supplier.

3.4 Ramping and start-up costs

Cycling operations in power plants, encompassing activities like startup, shutdown, ramp-up, and ramp-down, accelerate plant degradation and reduce efficiency, thereby increasing operational costs (Troy et al., 2010). Building upon findings by Knaut and Paschmann (2017) that highlight

how ramping and startup costs amplify the impact of restricted participation, this section analytically examines these costs to refine the intended empirical analysis.

Figure 3 provides a concise example illustrating the hourly production impact of restricted market participants. Each hour is subdivided into half-hours labeled t_1 to t_4 , where t_1 and t_2 pertain to Hour 1, and t_3 and t_4 to Hour 2. The residual demand, depicted by bold step-wise functions, exhibits a monotonous trend. The production of restricted participants q^r is shown as a step-wise function with thin lines. Additionally, the dashed lines represent the half-hourly generation of unrestricted participants q^u . Consequently, the jumps in the generation of restricted suppliers may induce both upward and downward ramping events in the schedule of unrestricted suppliers.

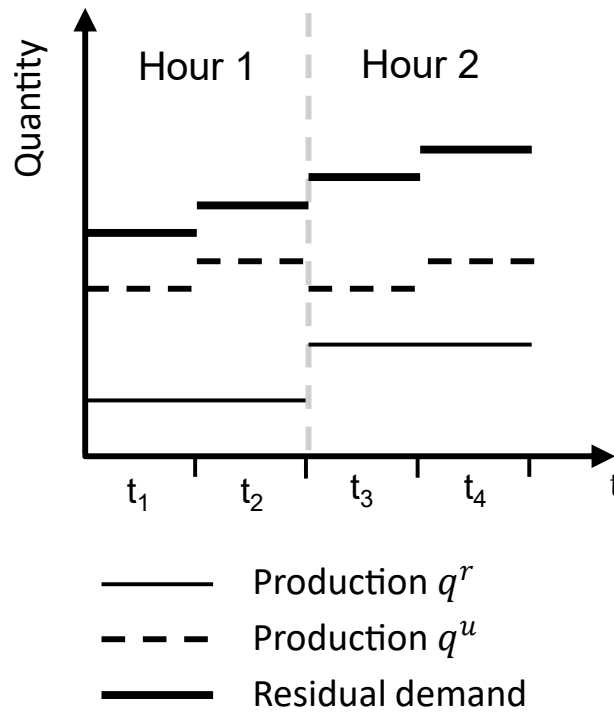


Figure 3: Depiction of the discontinuity in residual demand adjusted by the hourly cross-border trading (net residual demand)

Based on the example depicted in Figure 3, the effect of ramping costs on prices is evaluated analytically. Consequently, periods t_1 to t_4 are associated with residual demand D_t , with restricted suppliers producing q_{t_1,t_2}^r for Hour 1 and q_{t_3,t_4}^r for Hour 2. Now we assume that changes in production by unrestricted suppliers q_t^u induce costs represented by a convex function of power generation changes from one step to the next, denoted as $f(q_t^u - q_{t-1}^u)$. As the generation of the unrestricted market participants is equal to the demand subtracted by the generation of restricted suppliers, the additional costs attributable to ramping for unrestricted suppliers C_{ramp} can be formulated as follows:

$$\begin{aligned}
C_{ramp} = & f((D_{t_2} - q_{t_1,t_2}^r) - (D_{t_1} - q_{t_1,t_2}^r)) \\
& + f((D_{t_3} - q_{t_3,t_4}^r) - (D_{t_2} - q_{t_1,t_2}^r)) \\
& + f((D_{t_4} - q_{t_3,t_4}^r) - (D_{t_3} - q_{t_3,t_4}^r)).
\end{aligned} \tag{13}$$

This can be rewritten as,

$$\begin{aligned}
C_{ramp} = & f(D_{t_2} - D_{t_1}) \\
& + f(D_{t_3} - D_{t_2} - (q_{t_3,t_4}^r - q_{t_1,t_2}^r)) \\
& + f(D_{t_4} - D_{t_3}).
\end{aligned} \tag{14}$$

In a competitive environment, the corresponding price mark-up at time t_2 and t_3 equals the change in marginal costs induced by a change in demand.

$$\frac{\partial C_{ramp}}{\partial D_{t_2}} = f'(D_{t_2} - D_{t_1}) - f'(D_{t_3} - D_{t_2} - (q_{t_3,t_4}^r - q_{t_1,t_2}^r)) \tag{15}$$

and

$$\frac{\partial C_{ramp}}{\partial D_{t_3}} = f'(D_{t_4} - D_{t_3}) + f'(D_{t_3} - D_{t_2} - (q_{t_3,t_4}^r - q_{t_1,t_2}^r)). \tag{16}$$

And the derivative of the price with respect to the change of the production by restricted suppliers ($q_{t_3,t_4}^r - q_{t_1,t_2}^r$) from 'Hour 1' to 'Hour 2':

$$\frac{\partial C_{ramp}}{\partial D_{t_2} \partial (q_{t_3,t_4}^r - q_{t_1,t_2}^r)} = f''(D_{t_3} - D_{t_2} - (q_{t_3,t_4}^r - q_{t_1,t_2}^r)) > 0 \tag{17}$$

and for t_3

$$\frac{\partial C_{ramp}}{\partial D_{t_3} \partial (q_{t_3,t_4}^r - q_{t_1,t_2}^r)} = -f''(D_{t_3} - D_{t_2} - (q_{t_3,t_4}^r - q_{t_1,t_2}^r)) < 0. \tag{18}$$

This results in a price markup at t_2 and a markdown at t_3 if the production of restricted suppliers increases, and vice versa. During periods of increasing demand, it is plausible that the production of restricted suppliers will also increase. This implies that the average increase in unrestricted

production is dampened. Yet at the same time the expected zigzag pattern is reinforced by the price mark-ups before and after the jump in restricted supply.

3.5 Extending the model to account for cross-border trading

The analytical model as outlined in Section 3.1 can be adapted to incorporate cross-border trading. Consequently, four groups of suppliers are introduced, as summarized in Table 1. These groups are categorized based on origin (domestic or foreign) and level of participation (restricted or unrestricted). Restricted participation refers to the willingness to trade quarter-hourly products. For foreign suppliers, technical restrictions on cross-border trading may still be present.

Table 1: Overview of different groups of suppliers in the analytical model

Group name	Symbol for slope parameter	
	inverse supply curve $p = f(q)$	supply curve $q = f(p)$
Domestic unrestricted suppliers	a_1^{DUS}	b_1^{DUS}
Domestic restricted suppliers	a_1^{DRS}	b_1^{DRS}
Foreign unrestricted* suppliers	a_1^{FUS}	b_1^{FUS}
Foreign restricted suppliers	a_1^{FRS}	b_1^{FRS}
Suppliers that can trade domestic quarter-hourly products	a_1^u	b_1^u
All groups combined	a_1	b_1

*Restricted participation refers to the willingness to trade quarter-hourly products. For foreign suppliers technical restrictions for cross-border trading may still be present.

Contrary to the derivation of propositions in Section 3.1, where the analytical model is based on inverse supply curves, direct supply curves are introduced to simplify the aggregation of individual curves. The slope parameter of the supply curves can be aggregated by summing the individual parameters. The slope parameter of the inverse supply curve can then be obtained by taking the inverse. This approach simplifies the aggregation process, as the sum of the parameters from both curves directly equals the parameter of the aggregated curve.

The supply curves for the four groups can be expressed as follows:

$$q^{DUS} = b_0^{DUS} + b_1^{DUS}p \quad (19)$$

$$q^{DRS} = b_0^{DRS} + b_1^{DRS}p \quad (20)$$

$$q^{FUS} = b_0^{FUS} + b_1^{FUS}p \quad (21)$$

$$q^{FRS} = b_0^{FRS} + b_1^{FRS} p \quad (22)$$

Hence, the parameter a_1 can then be calculated as:

$$a_1 = \frac{1}{b_1^{DUS} + b_1^{DRS} + b_1^{FUS} + b_1^{FRS}}. \quad (23)$$

4 Research approach and hypotheses

In the following sections, the research approach to assess the research questions introduced in the introduction is outlined along with a summary for the motivation for the individual questions.

Hypothesis 1: *The non-smoothness (zigzag pattern) in intraday prices is mainly caused by restricted participation.*

The first hypothesis addresses the first research question and is motivated by the empirical analysis conducted by Knaut and Paschmann (2019), as summarized in Section 2.3 and Section 3.1. The authors demonstrate that the model specification, which incorporates the analytical description of restricted participation, explains a significant proportion of intraday price volatility. While this volatility is closely related to the non-smoothness in intraday prices, the direct effect on the non-smoothness remains to be shown.

The general approach to address this research question involves using Proposition 1 (Section 3.2). This proposition connects the observed spot price p_τ with a counterfactual price p_τ^* that assumes no restricted participation. The counterfactual price is estimated by leveraging the fact that the average hourly price level (Proposition 2) remains unaffected by restricted participation, while the prices are expected to exhibit smoothness. After estimating $p_\tau - p_\tau^*$ (see Section 5), Proposition 1 is applied to a regression model to assess the amount of explained variance and the significance of the parameters.

Hypothesis 2: *The restricted sub-hourly cross-border trading, as a specific instance of restricted participation, contributes to the non-smoothness (zigzag pattern) in intraday prices yet does not fully explain it.*

This hypothesis takes up the second research question and is inspired by the analyses of Knaut and Paschmann (2019), who hypothesized that the main cause of restricted participation is the limitation on trading sub-hourly products across borders. This hypothesis is based on the analysis in Knaut and Paschmann (2017), which replicated a similar zigzag pattern using a bottom-up optimization model with constraints on sub-hourly cross-border trading and demonstrated the

monetary benefits of participating in the quarter-hourly market for a representative thermal power plant. However, this hypothesis requires empirical verification.

The empirical assessment strategy is grounded in Proposition 1 (see Section 3.2). According to this proposition, the magnitude of the systematic non-smoothness in intraday prices is determined by the difference between the slope parameter of unrestricted suppliers and the aggregated slope parameter of the inverse supply curve, denoted as $a_1^u - a_1$.

The primary approach involves estimating the slope parameter a_1^u for a scenario where only domestic suppliers are involved in quarter-hourly trading. This estimate is then compared to a counterfactual scenario where foreign suppliers also participate in domestic quarter-hourly trading through cross-border transactions. This comparison is facilitated by utilizing the distinct supplier groups outlined in Section 3.5.

For domestic suppliers trading quarter-hourly products, where only domestic unrestricted suppliers are active, the slope parameter is given by:

$$a_1^u = \frac{1}{b_1^{DUS}}. \quad (24)$$

In a counterfactual scenario where both domestic and foreign suppliers participate in trading quarter-hourly products, the slope parameter becomes:

$$a_1^{u'} = \frac{1}{b_1^{DUS} + b_1^{FUS}}. \quad (25)$$

To determine the reduction in systematic non-smoothness in intraday prices resulting from the inclusion of cross-border quarter-hourly trading, we calculate the following factor:

$$1 - \frac{a_1^{u'} - a_1}{a_1^u - a_1}. \quad (26)$$

The estimation strategy is described in Section 5.

Hypothesis 3: *Ramping and start-up costs, alongside restricted participation, reinforce the zigzag pattern.*

To empirically test this hypothesis, referring to the third research question and stipulated in Section 3.4, the analysis focuses on price markups associated with changes in production by restricted suppliers. Specifically, this involves examining the price markup at the end of the previous hour and the markdown at the beginning of the next hour resulting from these production changes.

A regression model is specified to evaluate the significance of variations in production by restricted suppliers. Net imports are used as a proxy for this production, and the model assesses whether fluctuations in net imports add explanatory power. Particular attention is given to how these effects vary depending on the index of the sub-hourly product within the hour under consideration.

5 Empirical strategy

This section on empirical strategy is divided into two main parts. The first part provides a broad overview of the general empirical approach, including key aspects of the constructed time series. The second part focuses on the specific empirical strategies used to test each hypothesis. This includes a discussion of the smoothing technique applied to create a counterfactual intraday price time series, without any restrictions regarding participation, and the parameterization methods for model extensions related to cross-border trading and ramping and start-up costs, as detailed in Section 3.4 and Section 3.5. These strategies build on the general research approach outlined in Section 4.

5.1 Data

All data is openly available from the ENTSO-E Transparency Platform. The dataset covers the period from January 2019 to December 2023. The data is provided in quarter-hourly resolution. A summary of the data sources is presented in Table 2. The method for outlier detection and replacement of missings is described in Appendix E. ENTSO-E distinguishes between the Day-Ahead and intraday net import, with the Day-Ahead value being utilized for the regression analysis. To ensure that the net import genuinely reflects the expectations of market participants for our analysis, the mean difference between the Day-Ahead and intraday net import was examined. The close proximity of the average value to zero reinforces the assumption that the Day-Ahead net import is indeed a reasonable approximation of the expected values. The analysis is performed with data from Germany. Germany is chosen because the intraday auction for quarter-hourly products has been in place since 2014 in contrast to most other European countries where only continuous intraday trading was implemented during the analyzed period. By utilizing prices from the intraday auction, we can rule out other influences affecting the quarter-hourly prices as the intraday auction takes place shortly after the regular auction for hourly products.

5.2 General empirical strategy

The overview of the constructed time series is available in Table 3. While most variables are self-explanatory, some require additional clarification. The smoothened intraday auction price \tilde{P}_{IDA}

Table 2: Data sources (ENTSO-E)

Data	Symbol	ENTSO-E name
Total load	L	Load Forecast Day-Ahead in GW [6.1.b]
Solar/Wind infeed	S / W	Day-Ahead forecast Day-Ahead (18:00) [14.1.D]
Day-Ahead prices	P_{DA}	Day-Ahead Prices [12.1.D]
Intraday auction prices	P_{IDA}	Day-Ahead Prices [12.1.D]
Import and export	IM / EX	Day-Ahead scheduled commercial exchanges [12.1.F]
Generation per type	G	Aggregated Generation per Type [16.1.B&C]

is calculated using the smoothing method described in Section 5.3. Hence, they act as estimators for the counterfactual time series expected without restricted participation.

Table 3: Key constructed timeseries

Symbol	Description	Calculation	Unit
NI	Net import	$IM - EX$	[GW]
ΔNI	Change in net import	$NI_{t+2} - NI_{t-2}$	[GW]
P_{IDA}	Intraday auction prices	-	[€/MWh]
\bar{P}_h	Hourly average intraday price	$\frac{1}{4} \sum_{q=0}^3 P_{IDA,h,q}$	[€/MWh]
$P_{h,q}^{dev}$	Intraday price deviation	$P_{IDA,h,q} - \bar{P}_h$	[€/MWh]
\tilde{P}_{IDA}	Smoothened intraday auction prices	-	[€/MWh]
R	Residual demand	$L - S - W$	[GW]
\bar{R}_h	Hourly average residual demand	$\frac{1}{4} \sum_{q=0}^3 R_{h,q}$	[GW]
R^D	Domestic residual demand	$R - NI$	[GW]
$R_{h,q}^{dev}$	Residual demand deviation	$R_{h,q} - \bar{R}_h$	[GW]
Z_{IDA}	Deviation from smooth intraday prices	$P_{IDA} - \tilde{P}_{IDA}$	[€/MWh]

Indices definition: h: Hour, q= Index quarter-hour (0-3), t: Timestep = 4*(h-1)+q+1

5.3 Research question 1: Relevance of restricted participation

Hypothesis 1: *The non-smoothness (zigzag pattern) in intraday prices is mainly caused by restricted participation.*

The first hypothesis tested by examining whether the derived Proposition 1 (Equation 11) can be validated in a regression model using real data. The price p_{τ}^* , which represents the price without restrictions, is not directly observable. However, under the assumption that prices would form a smooth time series without restricted participation, and with Proposition 2 suggesting that the hourly average price remains the same with restricted participation as it would with full participation, a smoothed intraday price \tilde{P}_{IDA} can serve as an estimator for p_{τ}^* . Consequently, the deviation from smooth intra day prices Z_{IDA} can be used to approximate the difference between the price with full participation and the price with restricted participation.

To create counterfactual time series of the quarter-hourly prices for a market without restricted participation (see Section 4), two key properties are utilized. First, the hourly average remains consistent regardless of restricted participation (Section 3.2). Second, without restricted participation, the time series should be smooth (Section 2.2). The smoothing technique used to separate the time series into smooth and non-smooth components is described in the following.

The particular challenge is to retain peaks and price spikes that are unrelated to the intra-hourly variation. For instance, employing a rolling window average would be inappropriate. This becomes evident when considering a convex trajectory over multiple hours, which would be smoothened even if it is unrelated to the shift of hourly products. Additionally, the objective is to ensure that the price levels i.e., the average hourly prices, remain unchanged.

Therefore, cubic splines are employed. They consist of a piecewise polynomial of the third order, and the shifts between two pieces are called knots. The cubic spline is constructed to be continuous and smooth at the knots by imposing they have the same first and second derivative on both sides of the knots (Durrleman and Simon, 1989). The steps for the applied smoothing approach can be summarized as follows.

Step 1: Fitting integrated time series Let $P(t)$ be the original discrete intraday price time series, and let's denote the cumulative time series as $\Psi(t)$, obtained by cumulatively summing up the original prices:

$$\Psi(t) = \sum_{i=1}^t P(i) \quad (27)$$

Step 2: Placing knots at the shift of hourly products Assuming there are n hourly products, $n - 1$ knots are placed in the integrated time series at the points corresponding to the shifts of the hourly products. The knot values denoted as K_i , where i represents the index of the knots:

$$K_i = \Psi(t_i) \quad (28)$$

where t_i is the time corresponding to the end of the i th hourly product.

Step 3: Constructing a cubic spline In the next step, a cubic spline⁴ $S(t)$ is constructed that passes through each of the knots K_i . A cubic spline consists of piecewise polynomials of the third order. The spline function $S(t)$ can be represented as:

⁴For the implementation the python function 'interpolate.LSQUnivariateSpline' from the package 'scipy' has been utilized (Virtanen et al., 2020).

$$S(t) = a_i(t - t_i)^3 + b_i(t - t_i)^2 + c_i(t - t_i) + d_i \quad \text{for } t \in [t_i, t_{i+1}) \quad (29)$$

where a_i , b_i , c_i , and d_i are constants that depend on the coefficients of the cubic polynomial for each interval $[t_i, t_{i+1})$.

To ensure that the smoothed price series retains the mean for the hourly products from the original prices, increased weights are applied during the last quarter-hour. Let's denote the weights as $w(t)$, where $w(t)$ takes a higher value during the last quarter-hour and a value of 1 otherwise:

$$w(t) = \begin{cases} 1, & \text{for } t \in [t_i, t_{i+1} - \Delta t) \\ w_i, & \text{for } t \in [t_{i+1} - \Delta t, t_{i+1}) \end{cases} \quad (30)$$

where w_i represents the increased weight during the last quarter-hour of the i th hourly product. The proposed method ensures that the hourly average of the time series is maintained by appropriately adjusting the weights during the last quarter-hour.

Step 4: Taking the first difference To obtain the smoothed price series, the first difference of the cubic spline is taken, resulting in a second-order polynomial for each individual hour. Let the smoothed price series be denoted as $S'(t)$, then:

$$S'(t) = S(t) - S(t - \Delta t) \quad (31)$$

where Δt represents the time interval (e.g. quarter-hourly interval).

Due to the second-order polynomial nature of the differentiated fitted function, it is not capable of capturing the zigzag pattern with two turning points within one hour. Nevertheless, it demonstrates flexibility in smoothly adjusting to convex or concave shapes in the hourly time series. The procedure has been tested with both real and simulated data, confirming its ability to yield the expected results.

The analytical model from Knaut and Paschmann (2019) does not account for fluctuating renewable energy, so the residual demand is used as an equivalent to the demand D_τ in the analytical model. Consequently, the demand deviation $D_\tau - \bar{D}$ from the analytical model is replaced with the residual demand deviation R^{dev} . Therefore, the regression model corresponding to Proposition 1 is written:

$$\text{Model 1: } Z_{IDA} = \beta_0^{M1} + \beta_1^{M1} R^{dev} + \epsilon^{M1}. \quad (32)$$

Derived from Proposition 1, β_1^{M1} corresponds to $(a_1^{DUS} - a_1)$, which is the difference between the slope of the supply function from the domestic unrestricted suppliers and the aggregated supply function. A high proportion of explained variance and a significant regression coefficient then support the hypothesis that restricted participation is the main cause for the observed zigzag pattern.

Since Z_{IDA} includes a stochastic component, and as the motivation for this publication partly comes from the observation of the zigzag price formation in average prices (see Figure 2) it is intriguing to explore how much of the non-smoothness (zigzag pattern) in average prices it explains. To measure this, we calculate the sum of squares of the deviation of average intraday prices from average smooth intraday prices conditional on the quarter-hour of the day, $\bar{z}_{H,q}$, where H serves as an index for the hour of the day (0-23) and q as an index for the quarter-hour (0-3). Additionally, we assess the sum of squares of the residuals conditional on the quarter-hour of the day, $\bar{\epsilon}_{H,q}$, which measures non-smoothness after accounting for factors included in the regression models. The ratio of these components provides an R^2 -like measure for the non-smoothness in average price that can be explained by the regression model, denoted as:

$$R_{Aver.Prices}^2 = 1 - \frac{\sum_{H,q} (\bar{\epsilon}_{H,q})^2}{(\sum_{H,q} \bar{z}_{H,q}^{Ind})^2}. \quad (33)$$

5.4 Research question 2: Relevance of restrictions on cross-border trading

Hypothesis 2: *The restricted sub-hourly cross-border trading, as a specific instance of restricted participation, contributes to the non-smoothness (zigzag pattern) in intraday prices yet does not fully explain it.*

Based on Section 3.5, we can assess the proportion of non-smoothness attributable to restricted sub-hourly cross-border trading by examining the difference between the factors $(a_1^u - a_1)$ and $(a_1^{u'} - a_1)$. Here, the factor a_1^u is derived from the supply curve of the domestic unrestricted suppliers, characterized by the slope parameter b_1^{DUS} . In contrast, $a_1^{u'}$ is derived from the slope parameters of the supply curves of both the domestic unrestricted suppliers (b_1^{DUS}) and the foreign unrestricted suppliers (b_1^{FUS}), to depict a counterfactual scenario where foreign unrestricted suppliers can trade domestic quarter-hourly products. Therefore, to implement the research approach, it is necessary to estimate a_1 , b_1^{DUS} and b_1^{FUS} .

To estimate the parameter a_1 we employ a second regression model in the following form:

$$\text{Model 2: } \bar{P} = \beta_0^{M2} + \beta_1^{M2} \bar{R} - \beta_2^{M2} \epsilon^{M2}, \quad (34)$$

where \bar{R} is the hourly average of the residual demand and \bar{P} is the hourly average of intraday prices. Following Equation 7, β_1^{M2} is an estimate of a_1 . As β_1^{M1} from the first regression model is equal to $(a_1^u - a_1)$ and as unrestricted suppliers consist exclusively of domestic suppliers due to the technical restriction in cross-border trading, we have $a_1^u = \frac{1}{b_1^{DUS}}$.

Therefore we can estimate b_1^{DUS} as:

$$b_1^{DUS} = \frac{1}{\beta_1^{M1} + \beta_1^{M2}}. \quad (35)$$

In order to estimate the slope of the foreign unrestricted suppliers b_1^{FUS} , the first step is to estimate the combined slope of foreign suppliers $b_1^{FRS,FUS} = b_1^{FRS} + b_1^{FUS}$. Rather obviously, we estimate how much an increase of residual demand triggers an increase of net import.

$$b_1^{FRS,FUS} = \frac{\partial NI}{\partial P} = \frac{\partial NI}{\partial R} \frac{\partial R}{\partial P}. \quad (36)$$

$\frac{\partial P}{\partial R}$ is the change in price relative to change in the residual demand, which is equivalent to $\frac{1}{a_1}$. Now we have to estimate $\frac{\partial NI}{\partial R}$ with the third regression model:

$$\text{Model 3: } NI = \beta_0^{M3} + \beta_1^{M3} R + \epsilon^{M3}, \quad (37)$$

where NI are the net imports and R represents the residual demand. As β_1^{M3} is an estimator for $\frac{\partial NI}{\partial R}$, we can estimate $b_1^{FRS,FUS}$ as:

$$b_1^{FRS,FUS} = \beta_1^{M3} \frac{1}{a_1}. \quad (38)$$

To decompose $b_1^{FRS,FUS}$ and obtain b_1^{FUS} , we use the assumption that the ratio of restricted to unrestricted suppliers is the same for both foreign and domestic suppliers. Hence, the ratio of the increase in production of domestic unrestricted suppliers to the increase in one unit of aggregated suppliers can be expressed as:

$$\frac{\partial q^{DUS}}{\partial q^{DRS,DUS}} = \frac{b_1^{DUS}}{b_1^{DUS,DRS}} = \frac{b_1^{DUS}}{b_1^{DUS} + b_1^{DRS}}. \quad (39)$$

The slope parameter for the domestic restricted suppliers is given by:

$$b_1^{DRS} = \frac{1}{a_1} - b_1^{FRS,FUS} - b_1^{DUS}, \quad (40)$$

where this formula is derived by subtracting the slope parameters of the foreign suppliers and the domestic unrestricted suppliers from the overall aggregated supply curve. The remaining parameter represents the slope of the domestic restricted suppliers.

Accordingly, the assumption that the ratio of restricted to unrestricted suppliers is the same for both foreign and domestic suppliers can be written as:

$$\frac{\partial q^{DUS}}{\partial q^{DRS,DUS}} = \frac{\partial q^{FUS}}{\partial q^{FRS,FUS}}. \quad (41)$$

Analogous to Equation 39, we have:

$$\frac{b_1^{FUS}}{b_1^{FUS,FRS}} = \frac{b_1^{FUS}}{b_1^{FUS} + b_1^{FRS}}. \quad (42)$$

By inserting Equation 39 and Equation 42 into Equation 41 and isolating b_1^{FUS} we get:

$$b_1^{FUS} = b_1^{FUS,FRS} \frac{b_1^{DUS}}{b_1^{DUS} + b_1^{DRS}} \quad (43)$$

Having estimated a_1 , b_1^{DUS} , and b_1^{FUS} , the method for deriving the reduction in systematic non-smoothness resulting from lifting the technical restrictions on trading sub-hourly contracts can be applied as described in Section 4.

5.5 Research question 3: Relevance of ramping and start-up costs

Hypothesis 3: *Ramping and start-up costs, alongside restricted participation, reinforce the zigzag pattern.*

In Section 3.4 it has been derived that the price mark-up due additional costs of ramping and start-up costs is dependent on the change in net import from one hour to the next. Since the effect of net import depends on the quarter-hour (0-3), an interaction term between a variable that indicates which quarter of the current hour and the 'change in net imports' ΔNI is used. The variable 'change in net import' is used as an indicator for discontinuities caused by changes in net export and shifts in hourly products. The change in net import is shifted by half an hour because, for quarter-hours 3 and 4, the relevance of the change in net exports to the next hour is more significant than its comparison to the previous hour.

$$\text{Model 3: } Z_{IDA} = \beta_0^{M3} + \beta_1^{M3} R^{dev} + \beta_2^{M3} \Delta NI + \beta_3^{M3} \Delta NI \cdot q + \epsilon^{M3} \quad (44)$$

If these additional variables significantly enhance the explanatory power of the model, it demonstrates how ramping and start-up costs further contribute to non-smoothness in intraday prices. It should be noted, however, that we can only observe net export as one component of the production from restricted suppliers, q^r . Consequently, the effect may not be fully captured in the model if the impact of restricted sub-hourly cross-border trading provides only a small amount of explanatory power, and other forms of restricted participation outweigh this effect.

6 Results

This section starts with a broad overview of the results related to the general empirical analysis of non-smoothness in intraday prices. It covers aspects such as the non-smoothness in residual demand, the relationship between power generation and non-smoothness, and the descriptive statistics of the constructed time series. Following this, the focus shifts to a detailed discussion of the empirical methods specifically designed to address each research question. This detailed analysis builds on the foundational research approach outlined in Section 4 and the empirical strategy described in Section 5.

6.1 General empirical results

Descriptive statistics of selected time series are provided in Table 4.

Table 4: Descriptive statistics

	mean	std	min	25%	50%	75%	max
Residual demand deviation R^{dev}	0.00	0.82	-5.32	-0.40	-0.00	0.39	5.36
Residual demand R	34.76	13.00	-13.65	26.28	35.56	43.87	71.74
Change in net import ΔNI	0.00	1.47	-9.28	-0.85	-0.00	0.79	9.34
Deviation from smooth intraday prices Z_{IDA}	0.00	16.52	-115.84	-6.45	-0.13	6.19	108.87

The domestic residual demand, representing the production of restricted and unrestricted domestic suppliers, is calculated as the difference between the residual demand and net imports. Due to the non-smooth production of foreign restricted suppliers, the domestic residual demand is also expected to be non-smooth. The hypothesized non-smoothness in the domestic residual demand caused by the absence of quarter-hourly cross-border trading is observable and illustrated in Figure 4. In this figure, two time series are depicted, representing the average first difference (i.e., the change from one time step to the next) for both residual demand and domestic residual demand, conditioned on the quarter-hour of the day. Consequently, the observation is that the net imports indeed induces non-smoothness in the domestic residual demand and consequently in the schedule of dispatchable power plants, which appears as a sudden change in slope of the domestic residual demand time series.

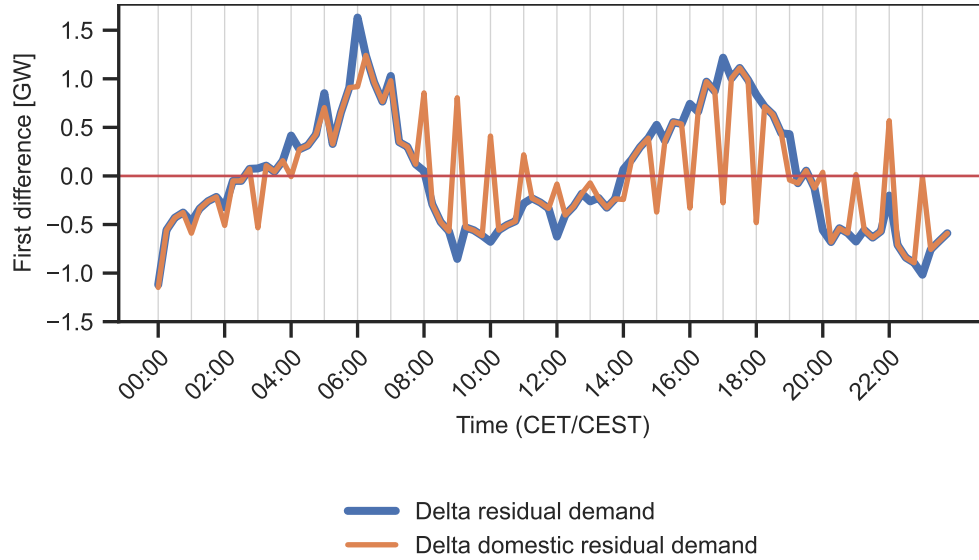


Figure 4: Illustration of the average first difference in German residual demand and domestic residual demand

To determine how power plant dispatch relates to non-smoothness in intraday prices, correlations were calculated between the deviation from smooth power plant generation, categorized by generation type (as published by ENTSO-E (2023c)), and both the deviation from smooth domestic residual demand (Z_{RD}) and the deviation from smooth intraday prices (Z_{IDA}). The correlations, along with the standard deviation, are shown in Table 5. The first observation is that substantial negative correlations are found for hydropower plant consumption, and positive correlations for hydropower production. This indicates that hydropower plants adjust their schedules in response to the economic incentives created by intraday price non-smoothness. In contrast, the positive correlations observed for biomass and gas-fired power plants suggest that their schedules are not aligned with these economic incentives, potential leading to an increase in non-smoothness in intraday prices. Therefore, this correlation could be a result of restricted participation in the market for biomass and gas-fired power plants. The standard deviations computed for the deviation from smooth generation highlight the importance at the system level, showing that hydropower plants have the most significant influence. On the other hand, although biomass power plants exhibit quite high correlation coefficients, their impact is likely to be low due to their small standard deviations.

To assess the distributional effects of non-smooth price formation, revenues for PV units in Germany have been estimated as the sum product of the intraday auction price and total infeed using both original and smoothed time series. For consumers, costs are estimated based on total consumption and intraday auction price. The results are shown in Table 6. The results show that the non-smooth prices induce higher costs for consumers and lower revenues for PV units.

Table 5: Correlation between the deviation from smooth power plant generation and the deviation from smooth domestic residual demand (Z_{RD}) as well as the deviation from smooth intraday prices (Z_{IDA})

Deviation from smooth generation	Z_{IDA}	Z_{RD}	SD
Biomass Actual Aggregated	-0.340	-0.120	0.010
Hydro Pumped Storage Actual Consumption	-0.270	-0.270	0.250
Fossil Gas Actual Aggregated	-0.120	0.020	0.080
Fossil Coal derived gas Actual Aggregated	-0.050	0.020	0.020
Fossil Hard coal Actual Aggregated	-0.040	0.030	0.060
Wind Offshore Actual Aggregated	-0.030	-0.020	0.080
Wind Onshore Actual Aggregated	-0.030	0.010	0.130
Fossil Brown coal/Lignite Actual Aggregated	-0.010	0.040	0.080
Waste Actual Aggregated	-0.000	0.000	0.010
Solar Actual Aggregated	0.000	-0.000	0.060
Nuclear Actual Aggregated	0.050	0.030	0.020
Hydro Water Reservoir Actual Aggregated	0.310	0.270	0.040
Hydro Pumped Storage Actual Aggregated	0.340	0.350	0.270

Table 6: Costs and Revenues of PV and consumers comparing cases with smooth and non-smooth prices

Year	Costs Consumers		Delta	Delta %	Revenue PV		Delta	Delta %
	Non-smooth	Smooth			Non-smooth	Smooth		
2019	18,948 M€	18,954 M€	18 M€	0.09%	1,475 M€	1,491 M€	-16 M€	-1.09%
2020	15,198 M€	15,189 M€	19 M€	0.13%	1,114 M€	1,137 M€	-24 M€	-2.15%
2021	50,386 M€	50,283 M€	123 M€	0.24%	3,548 M€	3,588 M€	-40 M€	-1.13%
2022	118,534 M€	118,205 M€	329 M€	0.28%	12,503 M€	12,648 M€	-146 M€	-1.17%
2023	45,692 M€	45,461 M€	231 M€	0.51%	4,019 M€	4,057 M€	-38 M€	-0.94%

6.2 Research question-specific results

The results for the regression models specified in Section 5 are presented in Table 7. Newey-West standard errors have been applied to all models to account for potential heteroscedasticity and autocorrelation. In Model 1, the coefficient of 'residual demand deviation', R^{dev} , is highly significant, suggesting that a deviation of 1 GW from average hourly residual demand corresponds to a 13.378 €/MWh increase in the deviation between observed and smooth intraday prices. The model explains 43% of the variation related to non-smoothness in intraday prices and 92% of the variation in average prices, supporting the hypothesis that zigzag patterns are primarily induced by restricted participation.

Model 2 and Model 3 provide additional information to estimate the supply curve parameters (see Section 4 and Section 5). Model 2 estimates the slope parameter of the aggregated supply curve a_1 with an hourly resolution. The coefficient of 2.921 is highly significant, implying that an increase of 1 GW in residual demand leads to a predicted hourly price increase of 2.92 €/MWh. The coefficient of Model 3 is also highly significant, indicating that an increase of 1 GW in residual demand results in a increase of 0.22 GW in predicted net import. These coefficients allow for the estimation of the slope parameters of the supply curves (see Sections 3.5 and 5). The estimated

Table 7: Regression results

	Z_{IDA} (1)	\bar{P}_h (2)	NI (3)	Z_{IDA} (4)
Intercept	0.000 (0.002)	-5.769 (9.548)	-9.917*** (0.474)	0.000 (0.002)
Residual demand deviation R^{dev}	13.378*** (0.347)			13.144*** (0.387)
Residual demand R			0.224*** (0.012)	
Hourly average domestic residual demand \bar{R}_h^D		2.921*** (0.407)		
Change in net import ΔNI				-0.302** (0.143)
Change in net import · Index quarter hour (0-3) $\Delta NI \cdot q$				0.179* (0.095)
Observations	184136	46034	184136	184132
R^2	0.433	0.132	0.217	0.433
Adjusted R^2	0.433	0.132	0.217	0.433
Residual Std. Error	12.377	95.884	5.467	12.375
F Statistic	1488.995***	51.429***	372.538***	526.675***
$R^2_{Aver.Prices}$	0.92	-	-	-

Note:

*p<0.1; **p<0.05; ***p<0.01

slope parameters are presented in Table 8 for all years combined, as well as for the individual years under consideration. The separate estimation for different years is discussed in Section 7.

The coefficients b_1 are slope parameters of the supply curves for the supplier groups indicated by the superscript of each parameter. They measure how an increase in electricity price affects the predicted production for each respective supplier group. Thus, the slope parameters can be compared to assess relatively how much flexibility is provided by different supplier groups. Interestingly, restricted suppliers account for a significant portion of flexibility. This distribution is evident from the fact that the increase in production from domestic unrestricted suppliers in response to a one-unit increase in electricity price is $b_1^{DUS} = 0.061$, which represents only 23% of the total combined increase in production from both domestic restricted and unrestricted suppliers, $b_1^{DUS} + b_1^{DRS} = 0.061 + 0.204 = 0.265$.

Based on the estimated supply curve parameters, the inverse supply curve parameter of the unrestricted suppliers can be considered in two cases: one where only domestic unrestricted suppliers can trade domestic quarter-hourly products (a_1^u) and another where foreign unrestricted suppliers can also trade quarter-hourly products ($a_1^{u'}$). Lifting the technical restrictions for sub-hourly cross-border trading would decrease the slope parameter of the unrestricted market participants from 16.3 to 12.64. Correspondingly, the systematic non-smoothness, represented by the difference between the slope parameter of unrestricted suppliers and that of restricted suppliers, is expected to drop by 27% (Section 4). Additionally, based on the R^2 of Model 1 indicates that 43.3% of the variance in non-smoothness is systematic and explainable, although this estimate for systematic non-smoothness is rather a lower bound since the explanatory model is rather simplified.

Table 8: Analysis supply curve slope: Results for different years

Year	b_1^{DUS}	b_1^{DRS}	b_1^{FUS}	b_1^{FRS}	a_1^u	$a_1^{u'}$	a_1	Estimated reduction in systematic non-smoothness by enabling sub-hourly cross-border trading
All	0.061	0.204	0.018	0.059	16.3	12.6	2.9	-27.3%
2019	0.120	0.624	0.034	0.177	8.3	6.5	1.0	-25.3%
2020	0.104	0.552	0.034	0.179	9.6	7.2	1.2	-27.8%
2021	0.056	0.155	0.021	0.058	17.9	13.0	3.4	-33.9%
2022	0.040	0.061	0.019	0.028	25.1	17.1	6.8	-43.6%
2023	0.048	0.192	0.019	0.077	20.8	14.9	3.0	-33.3%

Model 4 examines the effects of startup and ramping costs. While the coefficients align with expectations and are significant at least at the 10% level, the model does not lead to a significant improvement as assessed by the R^2 value. Consequently, the results provide only limited support to the hypothesis that ramping and startup cost exacerbate the price impact of restricted participation.

7 Variations and robustness checks

To ensure the validity and reliability of the findings, two additional analyses were conducted. First, the data were reexamined by analyzing each year individually to assess the robustness of the method against potential structural changes over time. Second, the impact of the market design change in June 2024 was evaluated. This change introduced intraday auctions with a market coupling mechanism for sub-hourly products and partially lifted technical restrictions on cross-border trading for these products. This allowed for a direct observation and comparison of the effects of lifting these restrictions with the results presented in Section 6.

7.1 Different years

The initial robustness check aims to determine the stability of regression results focusing on specific years. From 2019 to 2023, energy prices underwent significant fluctuations, with gas prices starting to rise from spring 2021 onwards and increasing further with the Russian attack on Ukraine in April 2022. Consequently, the Appendix F displays regression results for individual years from 2019 to 2023, covering all complete years in this dataset.

Upon closer inspection, it becomes evident that while the overall results remain robust, the magnitude of coefficients varies across the years. Notably, the R^2 is considerably lower from 2021 onwards, likely due to the inflated variance of wholesale market electricity prices. Additionally, the coefficient of residual demand deviation and the hourly average domestic residual demand increase with the escalation of energy prices. Since these coefficients depend on the slope of the inverse supply curves, their magnitude is expected to vary with overall price levels.

Overall, the observations from Section 6 are robust when examining individual years in the dataset. The estimated slope coefficients are presented in Table 8. As expected, the slope coefficients change over the year with the overall price level. Notably, in 2021 and 2022, the proportion of unrestricted suppliers relative to restricted suppliers was higher, likely due to increased incentives for participating in quarter-hourly trading amid greater price fluctuations. However, this share decreased again in 2023.

7.2 Market design change in June 2024

On June 13, 2024, intraday auctions were introduced across most European countries using a single market clearing mechanism, facilitating sub-hourly cross-border trading (Nemo committee, 2024). Despite persisting constraints due to differing time granularities among countries, many restrictions on sub-hourly cross-border production trading have been lifted with this implementation. Therefore, the difference before and after the market change can directly inform conclusions about the impact of limited sub-hourly trading on the emerging zigzag pattern. Model 1 was consequently re-estimated using data from one week before and one week after the introduction of intraday auctions. The results are presented in the Appendix G. It is evident that the coefficient of residual demand deviation decreased from 16.53 to 12.57 after the introduction, indicating a 24% reduction in the effect of restricted participation on intraday prices through the market design change which is rather similar to the estimate derived above from the full sample. As these results are consistent with our theoretical model, the remaining zigzag pattern is probably not just a transitory phenomena. Instead, a significant portion of the zigzag pattern is expected to persist in the long term, even after the removal of obstacles in sub-hourly cross-border trading.

8 Discussion and conclusion

This study addresses a significant gap in the literature by empirically investigating restricted participation as the cause of the zigzag pattern in intraday electricity prices. The zigzag pattern of alternating maxima and minima in the quarterly products of subsequent time steps is unexpected. Fundamentally, one would anticipate intraday prices to exhibit smoothness, given that residual demand is a key driver of prices and is not subject to systematic non-smooth intrahour variations. Restricted participation occurs when market participants adjust their output based on hourly prices, maintaining a constant output across sub-hourly products instead of responding to quarter-hourly price fluctuations. A special case of restricted participation is restricted sub-hourly cross-border trading. Hence, restricted participation comes along with inefficiency in cross-border trading between countries with different time granularities.

The main findings are summarized as follows. Descriptive statistics indicate that the domestic residual demand, which is the difference of the residual demand and the net import exhibits systematic non-smooth patterns. Additionally, correlation analysis shows that hydro power plants adjust their output (and intake for pumping) in response to non-smooth intraday prices to gain profit. Conversely, the schedules of biomass and gas-fired power plants seem to contradict economic incentives which indicates that these power plants are associated with a form of restricted participation. Furthermore, it was established that the non-smoothness leads to increasing costs for consumers and decreasing revenue for PV power plants.

In the first regression analysis, 43% (92% in average prices) of the variation of the non-smoothness in intraday auction prices in Germany could be explained by a single variable, namely the difference between the observed residual demand and the hourly average, which is expected to induce non-smooth prices under restricted participation. This supports the first hypothesis that the non-smoothness is indeed caused by restricted participation.

To assess the impact of technical barriers on cross-border trading, an analytical model was developed that allows to estimate ex ante the effect of removing these technical restrictions for sub-hourly cross-border trading. The empirical analysis revealed that restricted suppliers contribute a larger portion of the total flexibility compared to unrestricted suppliers. Additionally, the flexibility provided by cross-border trading is less significant compared to domestic production. The findings suggest that eliminating all technical barriers for sub-hourly cross-border trading would reduce systematic non-smoothness in intraday prices by 27.3%. Consequently, most of the restricted participation is attributed to factors other than the technical barriers to sub-hourly cross-border trading.

Additionally, the third analysis demonstrated that including variables associated with ramping and start-up costs related to the absence of sub-hourly cross-border trading does not significantly enhance the explanatory power of the model, despite the coefficients being significant and having the expected sign. This suggests that while ramping and start-up costs do have some impact on the

relationship between restricted participation and non-smoothness in intraday prices, their effect appears to be minor. Given the findings presented in this publication, this is not surprising, as the non-smoothness attributed to cross-border trading is not solely responsible for the non-smoothness in domestic residual demand and non-smoothness in domestic residual demand.

Comparing the results with Knaut and Paschmann (2019), this study confirms that restricted participation is an adequate model for describing fundamental causes behind the zigzag pattern in intraday prices. Yet, the hypothesis proposed by Knaut and Paschmann (2019), which attributes restricted participation primarily to technical barriers in sub-hourly cross-border trading is rejected by the empirical analysis in this publication. Hence, a significant portion of the non-smoothness in intraday prices is attributable to other forms of restricted participation. Primarily, the adjustment to non-smoothness is managed by hydro power plants, which do not incur additional fuel costs or start-up and ramping expenses. Regression results correspondingly indicate that ramping and start-up costs are not crucial for explaining price formation. Yet distributional effects are noteworthy, with estimated revenue losses for PV plants and increased costs for consumers which can be attributed to price non-smoothness.

The robustness of results has been confirmed by examining individual years. While the estimates appear to be within reasonable bounds, lower error bounds on in these parameters would enable the usage of a bottom-up model to support the derivation of the slopes of the net import supply curve and the domestic supply curve. A potentially critical issue is the data quality, particularly regarding day-ahead forecasts provided by ENTSO-E which are used as an estimator for market expectations. Yet, a larger variance of the forecast errors for market expectations should not impact the validity of the method. Only a structural bias in the data would be problematic. The quality of the ENTSO-E transparency platform has been debated (Hirth et al., 2018) but seems to have improved in recent years. Therefore, a large bias is not expected but cannot be ruled out. Nevertheless, this study investigated restricted participation with a focus on cross-border trading and concludes that the major part is accounted for by other forms of restricted participation from the analytical model. Although indicators of restricted participation by domestic power plants have been identified through correlation analysis, this phenomenon should be investigated further, as the evidence provided is only based on the use of aggregated generation data.

For policymakers, the results of this study provide evidence that the restricted sub-hourly trading is leads to substantial distributional effects. While hydro power plants and storage facilities benefit from these price patterns, PV power plants and consumers face disadvantages. The recent implementation of sub-hourly cross-border trading mechanisms has had a substantial effect on the non-smoothness in intraday prices yet other forms of restricted participation, both domestic and foreign, emerge as significant factors contributing to this phenomenon.

The conclusions drawn in this publication are valuable for researchers and market participants as they provide insights into the impact of structural changes on zigzag price formation, which is crucial for assessing the profitability of investment decisions, especially for flexible power plants.

The first key finding suggests that the zigzag pattern is unlikely to completely disappear despite advancements in sub-hourly cross-border trading. The estimation procedure employed enables the empirical inference of the supply curve slopes with and without cross-border trading, providing a foundation for further analysis and model development. This capability facilitates research into the effects of fundamental changes in the supply stack, input factor prices or market designs on emerging price patterns using additional bottom-up models. Moreover, these findings underscore an a so far unresolved research question: why do certain power plants not adjust their schedules in response to economic incentives from sub-hourly prices, resulting in sub-optimal market outcomes.

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A Proof: Aggregated Supply Curve

Given: We have two marginal cost functions (inverse supply curves) for two groups of suppliers⁵.

⁵Within this proof a stochastic component ϵ was added to proof the compatibility of the analytical model with the linear regression approach

- **Restricted Supplier:**

$$C'_r(q^r) = a_0^r + a_1^r q^r + \epsilon_r$$

- **Unrestricted Supplier:**

$$C'_u(q_\tau^u) = a_0^u + a_1^u q_\tau^u + \epsilon_u$$

Objective: To determine the parameters a_0 , a_1 , and ϵ of the combined marginal cost function:

$$C'(q) = a_0 + a_1 q + \epsilon$$

where $C'(q)$ is the aggregate marginal cost function.

Proof:

1. **Combine the Supply Quantities** Express the total quantity supplied, q , as the sum of the quantities supplied by each group:

$$q = q^r + q_\tau^u$$

2. **Solve the Marginal Cost Functions for q^r and q_τ^u** From the marginal cost function equations, solve for q^r and q_τ^u :

$$q^r = \frac{C'_r(q^r) - a_0^r - \epsilon_r}{a_1^r}$$

$$q_\tau^u = \frac{C'_u(q_\tau^u) - a_0^u - \epsilon_u}{a_1^u}$$

3. **Combine the Marginal Cost Functions** Substitute the expressions for q^r and q_τ^u into $q = q^r + q_\tau^u$:

$$q = \frac{C'_r(q^r) - a_0^r - \epsilon_r}{a_1^r} + \frac{C'_u(q_\tau^u) - a_0^u - \epsilon_u}{a_1^u}$$

4. **Simplify the Combined Equation** Combine the terms on the right-hand side:

$$q = \frac{(C'_r(q^r) - a_0^r - \epsilon_r)a_1^u + (C'_u(q_\tau^u) - a_0^u - \epsilon_u)a_1^r}{a_1^r a_1^u}$$

5. **Express $C'(q)$ in Terms of q** Replace $C'_r(q^r)$ and $C'_u(q_\tau^u)$ with $C'(q)$:

$$q = \frac{(C'(q) - a_0^r - \epsilon_r)a_1^u + (C'(q) - a_0^u - \epsilon_u)a_1^r}{a_1^r a_1^u}$$

Distribute and combine like terms:

$$q = \frac{C'(q)(a_1^r + a_1^u) - a_0^r a_1^u - \epsilon_r a_1^u - a_0^u a_1^r - \epsilon_u a_1^r}{a_1^r a_1^u}$$

6. **Solve for $C'(q)$** To isolate $C'(q)$, multiply both sides by $a_1^r a_1^u$ and rearrange the equation:

$$q a_1^r a_1^u = C'(q)(a_1^r + a_1^u) - a_0^r a_1^u - \epsilon_r a_1^u - a_0^u a_1^r - \epsilon_u a_1^r$$

$$C'(q)(a_1^r + a_1^u) = q a_1^r a_1^u + a_0^r a_1^u + \epsilon_r a_1^u + a_0^u a_1^r + \epsilon_u a_1^r$$

$$C'(q) = \frac{q a_1^r a_1^u + a_0^r a_1^u + \epsilon_r a_1^u + a_0^u a_1^r + \epsilon_u a_1^r}{a_1^r + a_1^u}$$

7. **Identify the Combined Parameters** To find a_0 and ϵ , observe that:

$$a_0 = \frac{a_0^r a_1^u + a_0^u a_1^r}{a_1^r + a_1^u}$$

$$\epsilon = \frac{\epsilon_r a_1^u + \epsilon_u a_1^r}{a_1^r + a_1^u}$$

8. **Determine the Combined Slope Parameter** The combined slope parameter a_1 is:

$$a_1 = \frac{a_1^r a_1^u}{a_1^r + a_1^u}$$

Conclusion Thus, the parameters for the combined marginal cost function $C'(q)$ are:

$$a_0 = \frac{a_0^r a_1^u + a_0^u a_1^r}{a_1^r + a_1^u}$$

$$a_1 = \frac{a_1^r a_1^u}{a_1^r + a_1^u}$$

$$\epsilon = \frac{\epsilon_r a_1^u + \epsilon_u a_1^r}{a_1^r + a_1^u}$$

This proves that the combined marginal cost function $C'(q)$ has the parameters a_0 , a_1 , and ϵ as given above.

B Proof: Average price

Objective: Derive the price expression $\bar{p} = a_0 + a_1 \bar{D}$.

Proof:

Step 1: Define the Problem The objective is to minimize the total cost:

$$\min \sum_{\tau} [C_u(q_{\tau}^u) + C_r(q_{\tau}^r)],$$

subject to the constraint:

$$D_{\tau} = q_{\tau}^u + q_{\tau}^r \quad \forall \tau.$$

Step 2: Formulate the Lagrangian The Lagrangian for this problem can be written as:

$$\mathcal{L} = \sum_{\tau} [C_u(q_{\tau}^u) + C_r(q_{\tau}^r) + p_{\tau}(D_{\tau} - q_{\tau}^u - q_{\tau}^r)],$$

where λ_{τ} are the Lagrange multipliers corresponding to each constraint.

Step 3: Derive the First-Order Conditions To find the optimal quantities q_{τ}^u and q_{τ}^r , we take the derivative of the Lagrangian with respect to each variable and set it to zero.

1. For q_{τ}^u :

$$\frac{\partial \mathcal{L}}{\partial q_{\tau}^u} = C'_u(q_{\tau}^u) - p_{\tau} = 0. \quad (45)$$

Substituting $C'_u(q_{\tau}^u) = a_0^u + a_1^u q_{\tau}^u + \epsilon^u$:

$$p_{\tau} = a_0^u + a_1^u q_{\tau}^u + \epsilon^u. \quad (1) \quad (46)$$

2. For q_{τ}^r :

$$\frac{\partial \mathcal{L}}{\partial q_{\tau}^r} = C'_r(q_{\tau}^r) - \sum_{\tau} p_{\tau} = 0.$$

Substituting $C'_r(q_{\tau}^r) = a_0^r + a_1^r q_{\tau}^r + \epsilon^r$:

$$a_0^r + a_1^r q^r + \epsilon^r = \frac{1}{n} \sum_{\tau} p_{\tau} = \bar{p}. \quad (2)$$

Step 4: Derive the supply curves for average price \bar{p} This concludes that for the restricted suppliers

and for the unrestricted suppliers

- **Restricted Supplier:**

$$\bar{p} = a_0^r + a_1^r q^r + \epsilon^r$$

- **Unrestricted Supplier:**

$$\frac{1}{n} \sum_{\tau} p_{\tau} = \frac{1}{n} a_0^u + a_1^u q_{\tau}^u \quad (47)$$

$$\bar{p} = a_0^u + a_1^u \bar{q}^u + \bar{\epsilon}^u \quad (48)$$

where \bar{q}^u is the average quantity for the unrestricted suppliers. The remainder of the proof is analogously to the proof in Appendix A.

C Proof: Subhourly price

Objective: Derive the price expression:

$$p_{\tau} = a_0 + a_1 \bar{D} + (D_{\tau} - \bar{D}) a_1^u + \epsilon \quad (49)$$

$$= \bar{p}_{\tau} + (D_{\tau} - \bar{D}) a_1^u + \epsilon, \quad (50)$$

Proof: Based on the proof in Appendix B, subtracting Equation 46 from Equation 48:

$$p_{\tau} - \bar{p} = (a_0^u + a_1^u q_{\tau}^u + \epsilon^u) - (a_0^u + a_1^u \bar{q}^u + \bar{\epsilon}^u) \quad (51)$$

Simplifying we get:

$$p_{\tau} - \bar{p} = a_1^u (q_{\tau}^u - \bar{q}^u) + (\epsilon^u - \bar{\epsilon}^u) \quad (52)$$

As the difference between $D_{\tau} - \bar{D}$ can only be met with the flexibility of the unrestricted suppliers it follows that:

$$D_\tau - \bar{D} = q_\tau^u - \bar{q}^u \quad (53)$$

Inserting Equation 53 into Equation 52 and solve for p_τ :

$$p_\tau = \bar{p} + a_1^u(D_\tau - \bar{D}) + (\epsilon^u - \bar{\epsilon}^u) \quad (54)$$

D Proof: Proposition 1

Objective: Derive the price expression:

$$p_\tau - p_\tau^* = (a_1^u - a_1)(D_\tau - \bar{D}). \quad (55)$$

Proof: Without restricted participation the price p_τ^* is determined by the aggregated supply function (Equation 5):

$$p_\tau^* = a_0 + a_1 D_\tau + \epsilon. \quad (56)$$

The price p_τ is derived in the Appendix C:

$$p_\tau = a_0 + a_1 \bar{D} + \epsilon + a_1^u(D_\tau - \bar{D}) + (\epsilon^u - \bar{\epsilon}^u) \quad (57)$$

Subtracting both equations from another leads to:

$$p_\tau - p_\tau^* = a_0 + a_1 \bar{D} + \epsilon + a_1^u(D_\tau - \bar{D}) + (\epsilon^u - \bar{\epsilon}^u) - (a_0 + a_1 D_\tau + \epsilon) \quad (58)$$

Simplifying gets:

$$p_\tau - p_\tau^* = (a_1^u - a_1)(D_\tau - \bar{D}) + (\epsilon^u - \bar{\epsilon}^u) \quad (59)$$

E Outliers detection and replacement of missing values

Identifying outliers involved calculating a four-hour rolling average and the standard deviation of the difference between the observed price and this rolling average. If the absolute difference between the rolling average and the observed price exceeded three times the standard deviation, the observation was flagged as an outlier. If an outlier occurred within an hour, the entire hour was excluded from the dataset. This process was applied exclusively to the intraday auction price time series, leading to the detection and removal of 2695 outliers accounting for 1.4% of all observations. To address missing values, seasonal decomposition was employed. The non-seasonal component was then interpolated using a linear method. This interpolation process replaced a total of 2789 (94 without outlier detection) missing values in the intraday auction time series

F Regression Results: Different years

Table 1: Regression results 2019

	Z_{IDA}	\bar{P}_h	NI	Z_{IDA}
	(1)	(2)	(3)	(4)
Intercept	-0.000 (0.001)	-0.204 (1.971)	-11.686*** (0.895)	0.000 (0.001)
Residual demand deviation R^{dev}	7.289*** (0.166)			6.976*** (0.165)
Residual demand R			0.221*** (0.026)	
Hourly average domestic residual demand \bar{R}_h^D		1.047*** (0.061)		
Change in net import ΔNI				-0.367*** (0.095)
Change in net import · Index quarter hour (0-3) $\Delta NI \cdot q$				0.256*** (0.065)
Observations	35040	8760	35040	35036
R^2	0.721	0.742	0.206	0.723
Adjusted R^2	0.721	0.742	0.206	0.723
Residual Std. Error	3.476	7.281	5.123	3.462
F Statistic	1922.302***	293.461***	73.849***	642.818***
$R^2_{Aver.Prices}$	0.94	-	-	-

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 2: Regression results 2020

	Z_{IDA}	\bar{P}_h	NI	Z_{IDA}
	(1)	(2)	(3)	(4)
Intercept	0.000 (0.001)	-9.560*** (1.966)	-10.393*** (0.735)	0.000 (0.002)
Residual demand deviation R^{dev}	8.449*** (0.229)			8.132*** (0.233)
Residual demand R			0.245*** (0.026)	
Hourly average domestic residual demand \bar{R}_h^D		1.151*** (0.059)		
Change in net import ΔNI				-0.411*** (0.097)
Change in net import · Index quarter hour (0-3) $\Delta NI \cdot q$				0.279*** (0.064)
Observations	35136	8784	35136	35132
R^2	0.684	0.718	0.317	0.686
Adjusted R^2	0.684	0.718	0.317	0.686
Residual Std. Error	4.610	8.980	4.479	4.595
F Statistic	1356.921***	379.260***	90.846***	469.687***
$R^2_{Aver.Prices}$	0.92	-	-	-

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 3: Regression results 2021

	Z_{IDA}	\bar{P}_h	NI	Z_{IDA}
	(1)	(2)	(3)	(4)
Intercept	-0.000 (0.003)	-33.115* (19.279)	-12.423*** (0.439)	0.001 (0.003)
Residual demand deviation R^{dev}	14.423*** (0.819)			14.225*** (1.005)
Residual demand R			0.273*** (0.015)	
Hourly average domestic residual demand \bar{R}_h^D		3.449*** (0.837)		
Change in net import ΔNI				-0.196 (0.331)
Change in net import · Index quarter hour (0-3) $\Delta NI \cdot q$				0.147 (0.215)
Observations	35040	8760	35040	35036
R^2	0.517	0.324	0.402	0.517
Adjusted R^2	0.517	0.324	0.402	0.517
Residual Std. Error	10.635	60.853	4.082	10.634
F Statistic	309.855***	16.965***	332.185***	147.823***
$R^2_{Aver.Prices}$	0.89	-	-	-

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 4: Regression results 2022

	Z_{IDA}	\bar{P}_h	NI	Z_{IDA}
	(1)	(2)	(3)	(4)
Intercept	0.000 (0.006)	-4.864 (18.270)	-14.206*** (0.537)	-0.002 (0.006)
Residual demand deviation R^{dev}	18.306*** (0.547)			19.023*** (0.576)
Residual demand R			0.318*** (0.017)	
Hourly average domestic residual demand \bar{R}_h^D		6.758*** (0.592)		
Change in net import ΔNI				0.675** (0.291)
Change in net import · Index quarter hour (0-3) $\Delta NI \cdot q$				-0.513*** (0.198)
Observations	35040	8760	35040	35036
R^2	0.393	0.370	0.524	0.394
Adjusted R^2	0.393	0.369	0.524	0.394
Residual Std. Error	19.013	110.897	3.820	19.001
F Statistic	1121.685***	130.545***	331.350***	410.130***
$R^2_{Aver.Prices}$	0.82	-	-	-

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 5: Regression results 2023

	Z_{IDA}	\bar{P}_h	NI	Z_{IDA}
	(1)	(2)	(3)	(4)
Intercept	0.000 (0.004)	4.271 (3.295)	-7.031*** (0.970)	0.000 (0.004)
Residual demand deviation R^{dev}	17.833*** (0.337)			18.653*** (0.366)
Residual demand R			0.286*** (0.033)	
Hourly average domestic residual demand \bar{R}_h^D		2.974*** (0.110)		
Change in net import ΔNI				0.816*** (0.261)
Change in net import · Index quarter hour (0-3) $\Delta NI \cdot q$				-0.584*** (0.180)
Observations	35040	8760	35040	35036
R^2	0.532	0.803	0.283	0.534
Adjusted R^2	0.532	0.803	0.283	0.534
Residual Std. Error	14.754	19.999	6.199	14.734
F Statistic	2793.579***	725.241***	73.763***	1016.051***
$R^2_{Aver.Prices}$	0.88	-	-	-

Note:

*p<0.1; **p<0.05; ***p<0.01

G Market design change in June 2024

Table 6: Regression results for the period from 2024/06/06 to 2024/06/12

	<i>Dependent variable: Z_{IDA}</i>
	(1)
Intercept	-0.000 (0.027)
Residual demand deviation R^{dev}	16.593*** (0.948)
Observations	672
R^2	0.689
Adjusted R^2	0.688
Residual Std. Error	11.619
F Statistic	306.328***
$R^2_{Aver.Prices}$	0.82
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 7: Regression results from 2024-06-13 to 2024-06-19

	<i>Dependent variable: Z_{IDA}</i>
	(1)
Intercept	0.001 (0.030)
Residual demand deviation R^{dev}	12.571*** (1.286)
Observations	672
R^2	0.527
Adjusted R^2	0.527
Residual Std. Error	13.454
F Statistic	95.499***
$R^2_{Aver.Prices}$	0.76
Note:	*p<0.1; **p<0.05; ***p<0.01

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