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Working Paper

Rationalizing Formula Apportionment

CESifo Working Paper, No. 11234

Provided in Cooperation with:

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Becker, Johannes (2024) : Rationalizing Formula Apportionment, CESifo Working Paper, No. 11234, CESifo GmbH, Munich

This Version is available at:

<https://hdl.handle.net/10419/302719>

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Impressum:

CESifo Working Papers

ISSN 2364-1428 (electronic version)

Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH

The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute

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Editor: Clemens Fuest

<https://www.cesifo.org/en/wp>

An electronic version of the paper may be downloaded

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Rationalizing Formula Apportionment

Abstract

Many federal tax systems employ formula apportionment to allocate the taxable profits of large businesses to the federal subunits, where the subunits' specific tax rates are then applied. The formulas—such as the one recently proposed by the EU Commission and the one agreed upon by the OECD/G20 Inclusive Framework—are remarkably similar, yet their underlying rationale, particularly the normative criteria from which they are derived, remains largely unexplored. To address this gap, I adopt an axiomatic approach and derive a generalized system of formula apportionment based on three criteria: fixed coverage, positive responsiveness, and external independence. I demonstrate that any formula apportionment system meeting these three criteria will suffer from the same distortion, unless all local tax rates are identical. The generalized system encompasses existing real-world systems, as seen in federations like the US, Canada, and Germany, as special cases, while offering a degree of flexibility that holds the (thus far unrealized) potential for surplus-increasing reforms.

JEL-Codes: H250, H710, F230.

Keywords: corporate taxation, business taxation, federalism, formula apportionment.

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July 9, 2024

I thank Reuven Avi-Yonah, Kristoffer Berg, James Hines jr., Rick Krever, Marco Runkel, Dirk Schindler, Esteban Muñoz Sobrado, Paul Steger and participants at the 2022 MATAX conference, the 2023 Oxford University Centre for Business Taxation Symposium as well as the 2024 ZEW Public Finance Conference for helpful comments. The usual disclaimer applies.

1 Introduction

In October 2021, the OECD/G20 Inclusive Framework agreed on a “two-pillar solution to address the tax challenges arising from the digitalization of the economy” (OECD 2021). Pillar One of the agreement involves reallocating taxing rights concerning the profits of multinational firms and incorporates an element of *formula apportionment*, meaning that part of the profit is allocated in proportion to the multinational firm’s sales in each country.¹ In 2023, the EU Commission launched the BEFIT² proposal which includes a renewal of its 2011 plan to introduce formula apportionment as a method for allocating multinational firm profits within the EU.³

Formula apportionment is a method of allocating the tax base across jurisdictions when a firm operates in multiple jurisdictions and at least two jurisdictions claim the right to tax the firm’s income “at source”.⁴ In federations like the US, Canada, and Germany, where federal subunits (i.e., US states, Canadian provinces, and German municipalities) have the right to tax business income, total business income within the federation is consolidated and then allocated to the subunits based on a formula. Each subunit then applies its own tax rate to its share of the total tax base. The formula is typically based on factors such as tangible capital (or property), payroll (sometimes number of employees), and/or sales. In contrast, the predominant international method for allocating multinational firms’ income across countries is to tax each of the firm’s affiliates separately based on the arm’s length principle (*separate accounting*). Growing discontent with the international tax system, particularly concerning evidence of multinational firms engaging in base erosion and profit shifting, has fueled initiatives to reform the international system towards a formula-based tax base allocation. Examples include the aforementioned OECD/G20 agreement and the EU Commission’s BEFIT proposal.

The apportionment formulas used in real-world federations and proposed in the EU Commission’s plans, as well as in the Two-Pillar Reform Agreement, share an intriguingly similar structure. Formulas that rely on a single indicator

¹The Pillar One agreement mandates that very large firms use formula apportionment for 25 percent of their ‘residual’ profit, which is the profit exceeding a specified return.

²BEFIT is an acronym for “Business in Europe: Framework for Income Taxation”.

³The original 2011 proposal by the Commission for a Common Consolidated Corporate Tax Base (CCCTB), which included formula apportionment for large multinational firms in the EU, failed to gain sufficient political support.

⁴To be precise, the firm’s level of activity typically must surpass a certain minimum threshold, establishing “nexus” to the jurisdiction in question.

typically follow the form: “allocate x percent of the tax base where x percent of the indicator is.” Formulas using multiple indicators and allocating $b \cdot 100$ percent according to a specific indicator follow the form: “allocate bx percent of the tax base where x percent of the indicator is.” While the specific indicators and their weights may vary, the overall shape of the formula remains remarkably consistent across different systems.

In this paper, I aim to rationalize the general shape of apportionment formulas. I argue that, while existing formulas may have been introduced in an ad-hoc manner, their consistent shape and simultaneous adoption in different parts of the world suggest they appeal to common sense principles. This paper seeks to make these implicit normative criteria explicit, exploring the foundations from which the formulas can be derived. So far, theoretical studies on formula apportionment have justified their analysis by stating, “because it exists in real-world tax systems.” This paper provides a more robust motivation: formula apportionment is the only system of profit allocation that adheres to certain principles, which are introduced and detailed within the study.

I make three key contributions. First, I identify the normative criteria that can be used to derive a generalized formula that encompasses most, if not all, existing formulas observed in real-world tax systems. Notably, for single-indicator formulas (e.g., sales-based), the observed formula (which assigns the tax base in proportion to sales) is the only one that satisfies these criteria. Second, I demonstrate that formulas adhering to these criteria imply two distinct biases in firm behavior, one of which is unavoidable. Third, for formulas using two or more indicators, I show that the generalized formula offers greater flexibility, potentially improving the efficiency of the tax system under certain circumstances, though only to a limited extent.

To be specific, I consider the class of apportionment systems characterized by three key principles: (1) fixed coverage, (2) positive responsiveness, and (3) external independence. *Fixed coverage* refers to the requirement that the fraction of the total tax base allocated according to the formula remains constant. This includes the scenario where the entire tax base is allocated based on the formula, known as full coverage. *Positive responsiveness* means that an increase in the firm’s activity in a given region—whether through increased input use, higher sales, or a greater concentration of firm functions—weakly increases the tax base fraction allocated to that region. Conversely, an increase in activity outside the region weakly decreases the tax base fraction allocated to the region. *External independence* stipulates that the tax base fraction assigned to a par-

ticular location is unaffected by changes in the distribution of activity outside that location. For example, if tangible assets are moved from one location to another (both of which are outside of location i), the tax base fraction allocated to i remains unchanged.

I derive a general formula for tax base apportionment that satisfies these three properties. This formula encompasses existing real-world formulas as special cases. I demonstrate that such systems generally face two types of biases. First, with fixed coverage and strict positive responsiveness, there is an inefficient dispersion of effective local tax burdens, implying that a re-allocation of activity from low-taxed to high-taxed locations within the firm increases total surplus (i.e. economic pre-tax profit). I show that this *dispersion bias* is an unavoidable consequence of these properties. Second, depending on the firm-specific geographical allocation of activity, the aggregate activity level within the firm may be inefficiently low (or high) due to the incentives provided by the apportionment formula. That is, an increase (or a decrease) in activity in all locations of the firm would increase surplus. I demonstrate that, under certain conditions, a general form of the formula (derived from the three properties above) has the potential to mitigate the *distortion of the aggregate activity level*. This corrective effect is not achievable with the simplified formulas typically used in real-world tax systems.

Who invented formula apportionment as a method of profit allocation? Krever and Mellor (2020) report that formula apportionment emerged independently at the end of the 19th Century in several locations, including the German State of Baden, France, Australia, and various US states.

“The intuitive solution adopted almost universally for the earliest income and predecessor taxes when an enterprise operated across borders was to allocate a share of the profits to each jurisdiction that contributed to the total profits by reference to formulas incorporating the factors that were presumed to contribute to the generation of those profits.” (Krever and Mellor 2020, p. 9)

In the US, individual States unilaterally applied this “intuitive solution” to specific industrial sectors such as railways and telegraphs. For example, the length of railway tracks in a state served as an indicator for determining the railway company’s nexus and tax liability within that State. As Krever and Mellor (2020) report, these early formulas were initially used to calculate the

property taxes owed by out-of-State businesses to the State. Later, this formula-based approach was extended to levy the "capital tax," which required assigning a share of the distributed profit to each State.

Over time, formula apportionment became the preferred method for profit allocation within federations such as the US and Australia (between states), Canada (between provinces), Germany (between municipalities), and others. In contrast, the international system for allocating profit across countries did not adopt formula apportionment but instead established a system based on separate accounting (grounded in the arm's length principle and regulated by double tax treaties, see Weiner 1999).

The coexistence of two distinct systems of tax base allocation has fueled ongoing debates about potential reforms, particularly in the international tax system.⁵ Each of the two systems has its flaws. Tax base allocation based on formulas is likely to distort the firm's decision where to invest and to hire labor, see McLure (1981) as well as Gordon and Wilson (1986).⁶ In contrast, the current international system of business taxation is prone to base erosion and profit shifting, as a large literature⁷ starting with Hines and Rice (1994) shows, and may have revenue implications in violation of inter-nation equity.⁸ Moreover, the growing awareness of tax competition between the EU member led to a number of studies that considered the incentives for (non-cooperative) tax policy under competition under the two alternative systems, separate accounting and formula apportionment,⁹ with some studies focussing on the choice of the formula weights.¹⁰

⁵The literature on this topic is large, see Hellerstein and McLure (2004), Weiner (2005), Avi-Yonah and Clausing (2008), Altshuler and Grubert (2009), Bettendorf et al. (2010), Clausing (2016) for more recent contributions.

⁶Empirical studies offering evidence for behavioral distortions include Goolsbee and Maydew (2000) measuring the employment effect of payroll-based formula, Klassen and Shackelford (1998) as well as Gupta and Mills (2002) focussing on strategic choice of sales locations, Riedel (2010) analyzing employment decision by German firms, as well as Buettner et al. (2011) looking at strategic consolidation under formula apportionment. Mardan and Stimmelmayer (2018) analyze strategic loss offsets under a system with common tax bases and formula apportionment. Hines (2010) as well as Becker and Runkel (2013) focus on distortions of ownership patterns.

⁷See Dharmapala (2014) and Riedel (2018) for literature reviews.

⁸In the early 1970s, Musgrave and Musgrave (1972) pointed out major disadvantages of the separate accounting for international profit allocation specifically for developing countries and promoted unitary taxation with formula apportionment as an alternative. These arguments have been adopted nowadays by tax justice activists.

⁹See e.g. Pethig and Wagener (2003), Eggert and Schjelderup (2003), Nielsen et al. (2003, 2010), Riedel and Runkel (2010), Gresik (2010) and Becker and Fuest (2010). Mintz and Smart (2004) show that using formula apportionment reduces the tax base elasticity in Canadian provinces, as compared to a system with separate accounting.

¹⁰Eichner and Runkel (2008) argue in favor of local sales as a formula factor. Runkel and

An obvious candidate for adopting a (US style) system with formula apportionment is the EU (or its predecessor, respectively), as observers like McLure (1989) and Daly and Weiner (1993) have noted early on. In 2004, the EU Commission proposed for a Common Consolidated Corporate Tax Base (CCCTB) with formula apportionment.¹¹ So far, the proposal failed to gain the necessary political support among EU Member States, but it was nevertheless renewed in the 2023 BEFIT proposal. Recently, the OECD/G20 Inclusive Framework agreed on a fundamental reform of the international system that introduces formula apportionment of profits according to sales (Navarro 2021), however only for large multinational firms and only with regard to part of their profit.

To the best of my knowledge, all of the studies and literature mentioned above share the common characteristic of treating the general formula as a given, accepting its structure as an exogenously imposed policy constraint.¹² With the formula's shape taken as given, existing studies vary the formula's weights to analyze its effects on behavior and the resulting efficiency costs. However, these studies have not addressed or questioned the general form of the formula itself. This paper sets out to fill this gap.

The remainder of the paper is organized as follows. The following section provides the model analysis. Section 3 concludes.

2 Model analysis

Consider a federation with I regions and a multiregional firm, which is active in all I regions. Each region levies a linear profit tax at source with a rate of t_i with $i = 1, \dots, I$. The total tax base is the firm's consolidated pre-tax profit π .¹³ Region i 's share in the total tax base is denoted as b_i and the vector of regional tax base fractions is $\mathbf{b} = (b_1, \dots, b_I)$, which can be understood as the federation's apportionment system. A b_i can be a function of the firm's

Schjelderup (2011) discuss the choice between capital, labor and sales and show that the formula should use capital even if it is perfectly mobile across borders. A somewhat US centered part of this literature considers competitive choices of US states of formula weight reflecting the fact that, in the US, States can choose their own formula weights (implying that income may be double-taxed and double-non-taxed), see e.g. Shackelford and Slemrod (1998), Anand and Sansing (2000), Edmiston (2002), Omer and Shelley (2004).

¹¹See e.g. Fuest et al. (2007), Devereux and Loretz (2008), Cobham and Loretz (2014), de Mooij et al. (2021) for impact assessments and tax revenue estimations.

¹²In some rare cases, alternative shapes are discussed, e.g. Gordon and Wilson (1986), without reference to deeper-level principles, though. I will discuss two of these alternatives below.

¹³I ignore any differences between economic profit and accounting profit.

activities (both inside and outside of region i) or simply a scalar. For instance, in a federation with two regions, the apportionment system $\mathbf{b} = (\frac{1}{3}, \frac{2}{3})$ implies that one third of the tax base is allocated to region 1 and the remainder to region 2.

Note that, so far, I did not impose any restrictions on b_i . Specifically, it may be that the sum $\sum b_i$ is smaller or larger than one (e.g. it is possible that only part of the tax base is allocated to the regions and the remainder is taxed by the federal government). Let $\tilde{T} = T + T^F$ denote the total effective (average) tax rate on firm profits, which is the weighted sum of the effective tax on allocated profit T , equal to

$$T = \sum b_i t_i \quad (1)$$

and the tax on non-allocated profit $T^F = (1 - \sum b_i) t^F$, possibly levied by the federal government, with t^F denoting the associated tax rate.

The firm's consolidated pre-tax profit π depends on a range of choices made by the firm. For instance, the firm chooses a level of tangible assets and employment in each region (*input choice*), it decides how much of the total product to sell in a given region (*sales shares*) and where to locate the marketing department, the risk management etc. (*firm functions*). I summarize these choices under the label *activities* \mathbf{x} that have an impact on total profit, i.e. $\pi = \pi(\mathbf{x})$.¹⁴ Let $a = 1, \dots, A$ be the index for the activities and $\mathbf{x}_i = (x_{i1}, \dots, x_{iA})$ the vector of the A activities in region i . Then, $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_I)$ denotes the vector of activities in regions $i = 1, \dots, I$. Although some of the activity considered here imply discrete choices (e.g. the marketing department being located either in i or in j , but not split-up and located in both), I will treat all activities as continuous variables.¹⁵

Activities are called bound, if their level is naturally limited. For instance, a firm cannot sell more than 100 percent of its product; i.e. when the firm chooses the fraction of total sales for a given region i , an increase of this share requires a reduction in some other region by the same amount. Similarly, some firm functions (e.g. headquarters services) may be considered a constant for a given level of output. An example for an unbound activity is employment. An increase in employment in one location does not necessitate a reduction in employment somewhere else (although it may affect the return to employment

¹⁴In the Appendix, I present a slightly more specific model with explicit expressions for inputs, sales shares and functions.

¹⁵Adapting the model to account for discrete choices is straightforward, but involves some notational effort. In any way, it does not alter the results in important ways.

in other locations). Any activity that can be measured and aggregated across regions can, in principle, be used for purpose of allocation of the tax base.

With this in mind, the optimization problem of the firm is given by

$$\max_{\mathbf{x}} \pi(\mathbf{x}) (1 - \tilde{T}) \quad \text{s.t.} \quad \sum x_{ia} = \bar{x}_a \text{ for all } a \in \bar{A} \quad (2)$$

where \bar{A} is the set of bound activities. The first order conditions for the x_{ia} are

$$\pi_{x_{ia}} (1 - \tilde{T}) - \pi \frac{d\tilde{T}}{dx_{ia}} + \lambda^{x_a} \leq 0 \quad \forall i, a \quad (3)$$

where λ^{x_a} is the Lagrangian parameter with $\lambda^{x_a} \neq 0$ if $a \in \bar{A}$ and $\lambda^{x_a} = 0$ if $a \notin \bar{A}$. Moreover, $\frac{d\tilde{T}}{dx_{ia}} = \frac{dT}{dx_{ia}} + \frac{dT^F}{dx_{ia}}$ with $\frac{dT}{dx_{ia}} = \sum_j \frac{db_j}{dx_{ia}} t_j$. The first order condition holds with a strict less-than sign, if no activity a in region i is the optimal choice. It holds with a strict greater-than sign, if all of activity a is optimally located in i , which may happen with bound activities. An inner solution emerges, if the first order condition holds with a strict equality sign. The following discussion refers to a setting with inner solutions. Moreover, for simplicity, I will assume that $T^F = 0$ and, thus, $\tilde{T} = T$.

If $\frac{d\tilde{T}}{dx_{ia}} = 0$, the firm's activity choice is unbiased, i.e. the optimum condition is $\pi_{x_{ia}} = 0$ for an unbound activity, and $\pi_{x_{ia'}} = \pi_{x_{ia''}}$ for any bound activities a', a'' . In contrast, if the tax payment \tilde{T} responds to a marginal increase in activity in i , there is a behavioral bias due to the tax system. The magnitude and the sign of this bias depends on the apportionment system chosen by the federation.

I will now limit the set of apportionment systems to those with specific properties. The following two properties help characterizing the formula effects on firm behavior.

Properties 1 (i) Fixed coverage: A fixed fraction $\bar{b} \in (0, 1]$ of total profit π is allocated to the regions:

$$\sum_i b_i = \bar{b} \quad (4)$$

with $\bar{b} = 1$ implying full coverage.

(ii) Positive responsiveness:

$$\frac{\partial b_i}{\partial x_{ia}} \geq 0 \quad \text{and} \quad \frac{\partial b_i}{\partial x_{-ia}} \leq 0 \quad (5)$$

with the stronger form (strictly positive responsiveness) requiring $\frac{\partial b_i}{\partial x_{ia}} > 0$ and $\frac{\partial b_i}{\partial x_{-ia}} < 0$.

With fixed coverage, the total fraction of tax base allocated to the regions does not respond to an increase in activity in some region i , i.e. $\sum \frac{db_j}{dx_{ia}} = 0$ and, thus, $\frac{db_i}{dx_{ia}} = -\sum_{j \neq i} \frac{db_j}{dx_{ia}}$. Using this, I can write the above first order condition (3) as follows

$$\pi_{x_{ia}} + \frac{\lambda^{x_a}}{1-T} = \bar{b}(t_i - \tilde{t}_{-i}) \frac{db_i}{dx_{ia}} \frac{\pi}{1-T} \quad (6)$$

where $\tilde{t}_{-i} = \sum_{j \neq i} \sigma_{ij} t_j$ is a weighted average of the tax rates outside of i with $\sum_{j \neq i} \sigma_{ij} = 1$ and $\sigma_{ij} = -\frac{db_j}{dx_{ia}} / \frac{db_i}{dx_{ia}}$. The above equation (6) is the equivalent to eq. (4) in Gordon and Wilson (1986).

In what follows, I will use the term *distorted* or *biased*, if an activity is higher or lower than its Pareto efficient level *for a given level of all other activities*. In order to make statements on Pareto efficiency, I assume that $\pi(\mathbf{x})$ is the total surplus.¹⁶

Proposition 1 *With fixed coverage and strictly positive responsiveness, the choice of activity x_{ia} is biased downwards if $t_i > \tilde{t}_{-i}$ and upwards if $t_i < \tilde{t}_{-i}$.*

Proof. Consider a small increase of an unbound activity a_i . The effect on total surplus is $\pi_{x_{ia}} dx_{ia}$ or $T_{a_i} \frac{\pi}{1-T} dx_{ia}$, which is equal to the right hand side of the first order condition in eq. (6). With strictly positive responsiveness and $t_i > \tilde{t}_{-i}$, the effect is unambiguously positive. Now, consider a small increase of a bound activity a_i and an associated decrease of a_j with $j \neq i$. Furthermore, let $t_i > \tilde{t}_{-i}$ and $t_j < \tilde{t}_{-j}$. With $dx_{ia} = -dx_{ja}$, the effect on total surplus is $(T_{a_i} - T_{a_j}) \frac{\pi}{1-T} dx_{ia}$ or $\left((t_i - \tilde{t}_{-i}) \frac{db_i}{dx_{ia}} - (t_j - \tilde{t}_{-j}) \frac{db_j}{dx_{ja}} \right) \frac{\bar{b}\pi}{1-T}$ which is positive, again. Thus, eq. (6) implies that activity a in region i is distorted downwards if $t_i > \tilde{t}_{-i}$ and vice versa. ■

The above Proposition generalizes a notion that occurs repeatedly in the literature (e.g. Gordon and Wilson 1986): Formula apportionment has an efficiency cost. The firm has an incentive to shift activity to locations with below 'average' (i.e. below \tilde{t}_{-i}) tax rates. This incentive is closely linked to the formula itself, and the shape and the weights in the formula affect this incentive. However, the above Proposition tells us that, independent of its specific shape

¹⁶This is the case if one assumes that input remuneration just covers the opportunity cost of input providers and that there are no other externalities.

(which I have not specified yet), any formula characterized by fixed coverage and strictly positive responsiveness causes a behavioral distortion that cannot be avoided (unless all local tax rates are identical).

It is helpful to differentiate two aspects of the tax distortion. First, efficient production requires that the tax-inclusive marginal cost of activity (e.g. the cost of capital) is equal in all regions. However, with different tax rates across regions, strictly positive responsiveness implies *dispersion* of marginal tax cost (the right hand side of eq. (6)), captured by the term $(t_i - \bar{t}_{-i})$ in the above formula. Second, for unbound activities, $\lambda^{x_a} = 0$, the *aggregate level of activity* may be too low (or too high). Note that, if the tax rates are the same across locations, the dispersion of tax rates is zero and, following eq. (6), the tax on the marginal unit of activity is zero in each region. In this case, the aggregate activity is at its efficient level as well. This implies that, in the absence of a dispersion distortion, there cannot be a distortion of the aggregate activity level. However, in the presence of dispersion, the aggregate activity level may be too high or too low – in the following sense: An increase in the marginal tax on activity in each region (i.e. a uniform increase of the right hand side of eq. 6 for all i), increases or decreases total pre-tax profit.

The following analysis will yield that a generalized formula may mitigate the *aggregate level* distortion, but not the *dispersion* distortion. Before, I illustrate both distortions with an example.

Example 1

Consider a simple apportionment formula based on assets k_i and employment l_i (both of which are unbound activities). Region i 's share of the tax base is given by

$$b_i = \bar{z} \frac{k_i}{K} + (1 - \bar{z}) \frac{l_i}{L} \quad \text{with} \quad (7)$$

where $\bar{z} \in [0, 1]$ is a fixed weight, $K = \sum_i k_i$ and $L = \sum_i l_i$.

After tax profit is $\pi(\mathbf{k}, \mathbf{l})(1 - T) - \tau K$ where τ is set by the federal government. The latter's sole purpose is to analyze the aggregate level distortion. The first order condition with respect to a regional capital stock k_i can be expressed as

$$\pi_{k_i} = (t_i - \bar{t}^K) \frac{\bar{z}}{K} \frac{\pi}{1 - T} + \frac{\tau}{1 - T} \quad (8)$$

where $\bar{t}^K = \sum_j t_j \frac{k_j}{K}$ denotes the capital-weighted average tax rate.

The distortion due to *dispersion* is obvious. The difference between the

marginal tax cost levels of the two locations with the highest and lowest local tax rates, t^{\max} and t^{\min} , is $(t^{\max} - t^{\min}) \frac{\bar{z}}{K} \frac{\pi}{1-T}$. The distortion of the *aggregate level* of capital inputs can be illustrated by considering an increase of τ starting from $\tau = 0$. For simplicity, assume that production is separable in labor and capital. Then, with $\sum_i \frac{dk_i}{d\tau} = \frac{dK}{d\tau}$, an increase in τ has the following effect on pretax profit π :

$$\sum_i \pi_{k_i} \frac{dk_i}{d\tau} d\tau = \frac{\bar{z}}{K} \frac{\pi}{1-T} d\tau \sum_i \left[(t_i - \bar{t}^K) \frac{dk_i}{d\tau} \right] + \frac{\tau}{1-T} \frac{dK}{d\tau} d\tau \quad (9)$$

Starting at $\tau = 0$, the effect of a small increase in τ on pretax profit is

$$\frac{\bar{z}}{K} \frac{\pi}{1-T} (\bar{t}^\Delta - \bar{t}^K) \frac{dK}{d\tau} d\tau \quad (10)$$

where $\bar{t}^\Delta = \sum_i t_i \frac{dk_i/d\tau}{dK/d\tau}$ is an average tax rate weighted by the effects of τ on local capital stocks.

For the sake of the argument, consider the case in which the $dk_i/d\tau$ are equal for all i (which may be the case with quadratic production functions). Then, \bar{t}^Δ equals the average tax rate \bar{t} and the above eq. (10) can be interpreted as follows. If the k_i are larger in low-tax regions, the average tax rate is larger than the capital-weighted one, $\bar{t} > \bar{t}^K$. In this case, an increase in τ decreases the firm's pretax profit since $dK/d\tau < 0$, implying that the aggregate level of capital inputs is too low. Similarly, if the firm's capital stocks are evenly distributed across regions and the $dk_i/d\tau$ are larger (i.e. less negative) for low-tax regions, the expression on the right hand side of eq. 9 is negative, which – again – means that the aggregate level of capital inputs is too low.

These admittedly stylized examples show that there may be an aggregate level distortion that needs to be distinguished from the dispersion distortion.

A generalized apportionment formula

I will now further restrict the set of apportionment systems to derive a generalized apportionment formula. I will restrict my attention to apportionment formulas characterized by fixed coverage, positive responsiveness and a third property:

Properties 2 (iii) External independence: With constant total activity, b_i does not depend on in which specific region $j \neq i$ activity x_j is located.

Let \mathbf{x}_{-i} denote the vector of all \mathbf{x}_j outside of region i , i.e. $j \neq i$. That is, $\mathbf{x} = (\mathbf{x}_i, \mathbf{x}_{-i})$. Then, for a given \mathbf{x}_i and any two \mathbf{x}_{-i} and \mathbf{x}'_{-i} , external independence implies

$$b_i(\mathbf{x}_i, \mathbf{x}_{-i}) = b_i(\mathbf{x}_i, \mathbf{x}'_{-i}) \text{ if the } \mathbf{X}_{-i} \text{ remain constant} \quad (11)$$

with $\mathbf{X}_{-i} = (X_{-i1}, \dots, X_{-iA})$ and $X_{-ia} = \sum_{j \neq i} x_{ja}$, with the latter denoting the aggregate activity a outside of i , i.e. $X_a - x_{ia}$.

External independence implies that, if some activity outside of i is shifted to another place outside of i , the tax base fraction allocated to i does not change. For instance, in an EU apportionment system, if a production unit moves from Portugal to Spain, the tax base fraction allocated to Germany stays the same.

The external independence property can be interpreted as a pragmatic approach to save on complexity and information cost. Although plausible, this property may be violated under certain circumstances. For instance, a formula designed to account for the firm's presence in tax havens would not have this property. Another example is provided in Gordon and Wilson (1986) who propose a system of allocation based on the firm's capital (or property) where a region's tax base is (in the notation used above): $b_i = \frac{k_i/(1-t_i)}{\sum_j (k_j/(1-t_j))}$, i.e. a higher tax ceteris paribus implies a higher tax base share. Obviously, such a system satisfies the properties fixed coverage and strictly positive responsiveness. However, it does not have the property external independence. If capital is shifted from a low-tax to a high-tax region outside (both outside of region i), the denominator increases and region i 's tax base share is reduced.¹⁷

I may now characterize the class of apportionment systems that satisfy the properties (i), (ii) and (iii). For this purpose, the following three Lemmas are useful.

Lemma 1 *Due to external independence,*

$$b_i(\mathbf{x}) = b_i(\mathbf{x}_i, \mathbf{X}_{-i}) \quad (12)$$

Proof. Note first that the above is trivially true if $I = 2$. For $I > 2$, external independence implies that region i 's tax base share b_i is, for a given level of

¹⁷Hines (1990) proposes a sophisticated (and, in terms of data requirements, demanding) system of formula apportionment, based on the notion of hypothetical average local profit. While having favorable efficiency properties, such a system does not satisfy the positive responsiveness property.

locational activity \mathbf{x}_i , is the same for any two \mathbf{x}_{-i} and \mathbf{x}'_{-i} as long as the total activity level \mathbf{X}_{-i} remains constant, see eq. (11). This includes the \mathbf{x}_{-i} in which all activity outside of i takes place in one single region. If the latter is region j , region i 's share is $b_i(0, \dots, \mathbf{x}_i, \dots, \mathbf{x}_j, \dots, 0)$ with $\mathbf{x}_j = \mathbf{X}_{-i}$. Thus, \mathbf{x}_i and \mathbf{X}_{-i} is sufficient information to determine $b_i(\cdot)$. ■

Lemma 2 $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X}_{-i})}{\partial x_{ia}} - \frac{\partial b_i(\mathbf{x}_i, \mathbf{X}_{-i})}{\partial X_{-ia}}$ is the same for all i .

Proof. Fixed coverage implies $\sum_i b_i = \bar{b}$ and, thus, $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X}_{-i})}{\partial x_{ia}} + \sum_{j \neq i} \frac{\partial b_j(\mathbf{x}_j, \mathbf{X}_{-j})}{\partial x_{ia}} = 0$. Due to external independence (and following the proof of Lemma 1), $\frac{\partial b_j(\mathbf{x}_j, \mathbf{X}_{-j})}{\partial x_{ia}}$ is, for a given j , equal for all i . Thus, $\frac{\partial b_j(\mathbf{x}_j, \mathbf{X}_{-j})}{\partial x_{ia}} = \frac{\partial b_j(\mathbf{x}_j, \mathbf{X}_{-j})}{\partial X_{-ja}}$ for $i \neq j$. It thus follows that $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X}_{-i})}{\partial x_{ia}} - \frac{\partial b_i(\mathbf{x}_i, \mathbf{X}_{-i})}{\partial X_{-ia}} = - \sum_j \frac{\partial b_j(\mathbf{x}_j, \mathbf{X}_{-j})}{\partial X_{-ja}}$ where the right hand side is equal for all i . ■

The above Lemma implies that an increase of x_{ia} with an associated decrease of X_{-ia} of the same size (i.e. with a constant overall level of activity, X_a) has an effect on b_i that is the same for all i . In order to use this property, it is convenient to redefine the tax base fraction $b_i(\cdot)$ as a function of the regional activities in i and the total activity levels, i.e. redefine $b_i(\mathbf{x}_i, \mathbf{X}_{-i})$ as $b_i(\mathbf{x}_i, \mathbf{X})$. While $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X}_{-i})}{\partial x_{ia}}$ holds the level of X_{-ia} constant and allows for an adjustment in X_a , $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X})}{\partial x_{ia}}$ holds X_a constant and allows for an adjustment in X_{-ia} ; to be specific, $dx_{ia} = -dX_{-ia}$. Formally speaking, $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X})}{\partial x_{ia}} = \frac{\partial b_i(\mathbf{x}_i, \mathbf{X}_{-i})}{\partial x_{ia}} - \frac{\partial b_i(\mathbf{x}_i, \mathbf{X}_{-i})}{\partial X_{-ia}}$. With Lemma 2, I can derive the following statement.

Lemma 3 $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X})}{\partial x_{ia}}$ is the same for all i .

Proof. See above. ■

The consequence from the above Lemma is that $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X})}{\partial x_{ia}}$ cannot specifically depend on x_{ia} or any other region-specific activity. This means that, for a given X_{-ia} , the tax base share b_i is linear in x_{ia} .

The following Proposition describes the general form of allocation systems with properties (i) through (iii).

Proposition 2 A system of tax base allocation system with fixed coverage, positive responsiveness and external independence has the following general form:

$$b_i = z_i^0(\cdot) + \sum_a z_i^a(\cdot) \cdot \frac{x_{ia}}{X_a} \quad (13)$$

where $\sum_i z_i^0(.) + \sum_a z^a(.) = \bar{b}$. The weights $z_i^0(.)$ and $z^a(.)$ may be functions of the firm's characteristics, e.g. its aggregate activity levels X_a and X_{a+1} , but only $z_i^0(.)$ may depend on the region's identity or its characteristics (e.g. its GDP, population size).

Proof. Define $z_i^0(.)$ as the share of the tax base allocated to region i independent of any activity location. Let $\zeta_a(\mathbf{X}) := \frac{\partial b_i(\mathbf{x}_i, \mathbf{X})}{\partial x_{ia}}$ denote the marginal change in b_i for a constant \mathbf{X} . Due to Lemma 2, $\frac{\partial b_i(\mathbf{x}_i, \mathbf{X})}{\partial x_{ia}}$ is the same for all i , i.e. for a given \mathbf{X} , it depends neither on x_i nor on i in general. Thus, the tax base share in i can be expressed as $b_i = z_i^0(.) + \sum_a \zeta_a(\mathbf{X}) x_{ia}$ and the change in b_i in response to an increase in x_{ia} equals $\zeta_a(\mathbf{X}) + \frac{\partial \zeta_a(\mathbf{X})}{\partial X_a}$. The share of the tax base allocated according to activity a is $\zeta_a(\mathbf{X}) X_a$. Since this share is fixed in many real-world formulas (i.e. it does not depend on aggregate activity X_a), it is convenient to express $\zeta_a(\mathbf{X})$ as $\frac{z_a(\mathbf{X})}{X_a}$. It follows the above Proposition. ■

The total effective tax rate on the tax base allocated to the regions is

$$T = \sum_i b_i t_i = \bar{t}^0 + \sum_a z^a(.) \bar{t}^a \quad (14)$$

where $\bar{t}^0 = \sum_i z_i^0 t_i$ denotes the z_i^0 -weighted average tax rate (with $\bar{t}^0 = \bar{t}$ if $z_i^0(.) = z^0(.)$ for all i) and $\bar{t}^a = \sum_i t_i \frac{x_{ia}}{X_a}$ the activity- a -weighted average tax rate.

I will say that the formula satisfies the *anonymity* criterion if it is the same for all regions. So far, the formula is not anonymous since the $z_i^0(.)$ may be region-specific while $z_a(.)$ may not.

The formula above allows for the special case of $b_i = z_i^0(.)$, i.e. each region receives an individual fraction of the tax base independent of the activity located in the region, or, in a simpler (anonymous) version, $b_i = z^0(.) = \frac{\bar{b}}{I}$, i.e. the fraction is the same for all regions. In both cases, region i 's tax base satisfies the positive responsiveness property, however only in its weaker form. Specifically, the firm's activity choices do not affect the regional tax base fraction b_i . In fact, a region would be entitled to a fraction of the tax base, even if the firm were not active in this region at all. In order to avoid this kind of degenerate formula, I strengthen the positive responsiveness property by adding a nexus condition, which requires that the tax base fraction in a given region is zero, if there is no activity in this region.

Properties 3 (ii') Positive responsiveness or strictly positive responsiveness

with a nexus condition, where the latter requires

$$b_i(\mathbf{0}, \mathbf{X}) = 0 \quad (15)$$

with $\mathbf{0}$ denoting a vector of zeros, which implies $z_i^0 = 0$ for all i .

Many real-world formulas as well as the one in eq. (7) have the properties (i), (ii') and (iii). Moreover, as the following Corollary states, from properties (i), (ii') and (iii) emerges the anonymity property of the formula.

Corollary 1 *With $z_i^0 = 0$ for all i (nexus condition), the formula is fully anonymous (i.e. two regions with the same activity profile have the same tax base share).*

Moreover, the following Corollary provides a strong result for the special case in which the tax base is to be allocated based on only one variable.

Corollary 2 *Assume that the tax base is allocated based on only one activity, the quantity of which is x_i in region i and X in total, i.e. $b_i = b_i(x_i, X)$. Then, with full coverage $\bar{b} = 1$, the only formula that satisfies properties (i), (ii') and (iii) is*

$$b_i(x_i, X) = \frac{x_i}{X} \quad (16)$$

With less-than-full coverage, the formula is given by $b_i(x_i, X) = \bar{b} \frac{x_i}{X}$.

Proof. The Corollary follows directly from Prop. 2 with $z_i^0 = 0$ and $\bar{b} = 1$ implying $z^a = 1$. ■

The above Corollary shows that if the allocation of tax base uses only one factor in its formula (e.g. sales), the formula above is unique in the sense that it is the only one that satisfies the criteria (i), (ii') and (iii).

Firm behavior under generalized apportionment

In what follows, I analyze the effects of a generalized system of apportionment satisfying properties (i), (ii') and (iii) on firm behavior. In order to save on notation, I will assume that $\bar{b} = 1$ (full coverage), i.e. that the apportionment system covers the total firm profit. The findings below can easily be generalized to the case of $\bar{b} < 1$.

With a formula of the shape $b_i = \sum_a z^a(.) \cdot \frac{x_{ia}}{X_a}$, see eq. (13) with $z_i^0(.) = 0$ for all i , the first order condition with respect to some activity m can be expressed as

$$\pi_{x_{im}} + \frac{\lambda^{x_m}}{1-T} = \left(\frac{z^m}{X_m} (t_i - \bar{t}^m) + \sum_a z_{X_m}^a \bar{t}^a \right) \frac{\pi}{1-T} \quad (17)$$

where $z_{X_m}^a = \frac{\partial z^a}{\partial X_m}$ denotes the marginal response of the weight z^a to an increase in total activity level X_m . Compare the above equation to the simple formula in equation (7) with $\tau = 0$ that gives rise to the first order condition in (8), again with $\tau = 0$. The generalized formula above has an additional term reflecting that the weights z^a may respond to the choice of total activity, i.e. $z_{X_m}^a > 0$ or $z_{X_m}^a < 0$, which can be used to manipulate the firm's behavior. The example below shows how this could work.

However, it immediately becomes clear that even the general formula is unable to mitigate the dispersion bias (captured by the first term in round brackets on the right hand side of eq. 17). The new term, $\sum_a z_{X_m}^a \bar{t}^a$, is not region-specific (i.e. it does not depend on i), which makes it unsuitable to deal with the dispersion distortion.

The fact that the new term in eq. (17) is common across regions suggests, though, that the $z^a(.)$ functions could be chosen in order to affect the distortion of the aggregate level of activity. While this is true in some cases, see the example below, it turns out that the potential gains of a formula manipulation depend on the geographical structure of the firm's activity allocation. With $\sum_a z^a(.) = 1$ and, thus, $\sum_a z_{X_m}^a = 0$ follows that $\sum_a z_{X_m}^a \bar{t}^a = 0$ if the \bar{t}^a do not differ across activities. What does equal \bar{t}^a across activities mean? Recall that $\bar{t}^a = \sum_i t_i \frac{x_{ia}}{X_a}$. That is, the \bar{t}^a are the same for all a , if the $\frac{x_{ia}}{X_a}$ are the same (though this is not a necessary condition) across activities (not across locations). Thus, even if the activity within the firm is unequally distributed across locations, flexible weights are toothless if all the activities are distributed in the same way (i.e. the \bar{t}^a are the same).

The following Proposition summarizes these results.

Proposition 3 *Consider a generalized formula for apportionment of tax base of the following form: $b_i = \sum_a z^a(.) \cdot \frac{x_{ia}}{X_a}$.*

(1) The dispersion of tax biases across locations under formulas adhering to properties (i), (ii') and (iii), measured as the difference between the highest and

the lowest marginal tax cost of activity is:

$$(t^{\max} - t^{\min}) \cdot \frac{z^m}{1 - T} \cdot \frac{\pi}{X_m} \quad (18)$$

which is the same as with formulas using constant weights.

(2) The level of tax biases is the same as under formulas with constant weights if the \bar{t}^a are the same for all a .

The above Proposition is kind of a negative result and tells us that, under certain circumstances, the generalized formula does not have advantages over the simplified one. There are cases, however, in which the additional flexibility of the generalized formula may be used to increase surplus. The following example illustrates such a case.

Example 2

Consider the example 1 from above with $\pi(\mathbf{k}, \mathbf{l})$ and with $b_i = z \frac{k_i}{K} + (1 - z) \frac{l_i}{L}$ for all i where z is a function of K and L , i.e.

$$z(K, L) = \alpha_0 + \alpha_1 K + \alpha_2 L \quad (19)$$

and α_0, α_1 and α_2 are constants. Note that the above formula satisfies properties (i) with full coverage, (ii') due to $b_i = 0$ if $k_i = l_i = 0$ and (iii).

The first order conditions with respect to k_i and l_i are given by

$$\pi_{k_i} = \left[\frac{\alpha_0 + \alpha_1 K + \alpha_2 L}{K} (t_i - \bar{t}^K) + \alpha_1 (\bar{t}^K - \bar{t}^L) \right] \frac{\pi}{1 - T} \quad (20)$$

$$\pi_{l_i} = \left[\frac{1 - \alpha_0 - \alpha_1 K - \alpha_2 L}{L} (t_i - \bar{t}^L) + \alpha_2 (\bar{t}^K - \bar{t}^L) \right] \frac{\pi}{1 - T} \quad (21)$$

An increase of α_0 by $d\alpha_0$ and an associated decrease of $d\alpha_1 = -\frac{1}{K} d\alpha_0$ leaves z unaffected. However, the tax cost of capital use, π_{k_i} , is reduced by $-\frac{1}{K} (\bar{t}^K - \bar{t}^L) \frac{\pi}{1 - T} d\alpha_0$, while the marginal tax cost of labor remains unaffected. That is, without revenue loss, the firm can be incentivized to increase all capital stocks, which – as is shown in Example 1 above – is able to increase surplus under certain conditions. Similarly, an adequate variation in α_0 and α_2 may increase (or decrease, if desired) all labor inputs across firm locations.

Example 2 shows that a generalized formula could, in principle, be used to mitigate the distortion of the aggregate activity level and increase surplus. This

finding should be taken with caution, though. Since the aggregate level distortion (just like the dispersion distortion) is firm-specific, a surplus-increasing reform is possible, if the specific activity profile of the firms under consideration is known. It is, however, impossible to design a formula that improves the efficiency of the allocation independent of where (and how) the firms are active.

3 Discussion

I demonstrate that formula apportionment systems meeting two plausible criteria—*fixed coverage* and *strictly positive responsiveness*—are inherently prone to a dispersion bias due to variations in local tax rates, a bias that is unavoidable if local tax rates are not uniform. Additionally, depending on the firm’s regional activity structure, the aggregate level of activity may be distorted, resulting in either an excessive or insufficient overall activity level.

By introducing an additional criterion, *external independence*, I derive a generalized formula for the apportionment of the tax base within a federation. This formula is slightly more general than those used in real-world tax systems or proposed in existing reform plans, thus offering a framework for rationalizing these systems.

By tightening the second criterion through the addition of a nexus condition—where a region’s tax base share is zero if the firm’s activity in that region is zero—two strong results about formula apportionment are derived. First, apportionment systems that satisfy all three criteria (including the nexus condition) must be anonymous, meaning the formula is uniform across regions. Second, if profit is allocated based on a single indicator, the formula becomes fully determined: the only apportionment system meeting the three criteria equates the region’s share of taxable profit to its share of the chosen indicator.

Utilizing the full potential of the generalized formula may, in some cases, mitigate the distortion of the aggregate activity level and lead to a surplus gain. However, this is only possible under specific conditions related to the geographical allocation of activity within the firm. The dispersion bias, on the other hand, remains unavoidable. If the surplus-enhancing potential of the generalized formula is exhausted, the only effective way to minimize distortions caused by the formula is through the optimal selection of weights, as demonstrated in the works of Eichner and Runkel (2008) and Runkel and Schjelderup (2011).

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4 Appendix

A more specific model of the firm

In this Appendix, I present a slightly more explicit model of the multiregional firm, which chooses inputs, sales shares and function locations. In each region i , the firm chooses a vector of N inputs, $\mathbf{k}_i = (k_{i1}, \dots, k_{iN})$. Let $\mathbf{k} = (\mathbf{k}_1, \dots, \mathbf{k}_I)$ denote the vector of regional input vectors. The inputs determine an output $y = f(\mathbf{k})$ and an associated input cost $c(\mathbf{k}, \cdot)$. A fraction s_i of the output is sold in region i at a price of p_i with $\sum s_i \leq 1$. The selling price may depend on quantity sold, i.e. $p_i = p_i(s_i y)$ with $p'_i \leq 0$. In addition, there may be a cost to

expanding sales in a given region, i.e. $c = c(\mathbf{k}, \mathbf{s} \cdot y, \cdot)$ with $\mathbf{s} = (s_1, \dots, s_I)$ and $\frac{\partial c(\cdot)}{\partial s_i} = c_{s_i y} y$. Finally, the firm has different functions (e.g. risk management, marketing etc.), which can be codified in a vector $\mathbf{q} = (\mathbf{q}_1, \dots, \mathbf{q}_I)$ where \mathbf{q}_i denotes the vector of functions located in region i . I assume that the relocation of functions has, for a given level of inputs, no effect on output. However, there may be a cost involved, i.e. $c = c(\mathbf{k}, \mathbf{s} \cdot y, \mathbf{q})$. The relocation of firm functions will usually involve a discrete jump in activity in two locations. For notational convenience, I treat functions as a continuous variable and assume that the cost function is twice differentiable in functions \mathbf{q} . For simplicity, I assume that functions are measurable on a one-dimensional scale, which gives rise to a vector $\mathbf{q} = (q_1, \dots, q_I)$.

Net profit is given by

$$\pi(1 - T) = (\bar{p}f(\mathbf{k}) - c(\mathbf{k}, \mathbf{s} \cdot f(\mathbf{k}), \mathbf{q}))(1 - T)$$

with $\bar{p} = \sum s_i p_i$. Let $R_i f(\cdot) \equiv p_i s_i f(\cdot)$ denote regional revenue and $R'_i f(\cdot) = (p_i + p'_i s_i f(\cdot)) f(\cdot)$ marginal regional revenue.

The first-order conditions with respect to s_i and k_{in} are:

$$\begin{aligned} s_i &: (R'_i - c_{s_i y}) f(\mathbf{k})(1 - T) - \pi \frac{dT}{ds_i} + \lambda^s = 0 \\ k_{in} &: \left(\sum s_j R'_j f_{k_{in}} - c_{k_{in}} \right) (1 - T) - \pi \frac{dT}{dk_{in}} + \lambda^{k_n} = 0 \\ q_i &: -c_{q_i} (1 - T) - \pi \frac{dT}{dq_i} + \lambda^q = 0 \end{aligned}$$

In the absence of taxes, the firm equates the marginal regional revenues (net of marginal sales cost) across locations: $R'_i - c_{s_i y} = R'_j - c_{s_j y}$ for all i, j . Moreover, with unbound inputs, $\lambda^{k_n} = 0$, it sets the marginal return to input equal to marginal input cost (weighted by the average marginal regional revenue, $\sum s_j R'_j = \sum s_j p_j \left(1 + \frac{1}{\varepsilon_j}\right)$, where $\varepsilon_j = \frac{ds_j f}{dp_j} \frac{p_j}{s_j f}$ denotes the price-elasticity of demand in region j), i.e. $f_{k_{in}} = \frac{c_{k_{in}}}{\sum s_j p_j \left(1 + \frac{1}{\varepsilon_j}\right)}$. If the firm is a price-taker, this condition is $f_{k_{in}} = c_{k_{in}} / \bar{p}$. With limited inputs, $\lambda^{k_n} > 0$, the gap between marginal return and weighted marginal cost is equalized across locations. Finally, it minimizes the cost by allocating functions such that the marginal cost of function location is equalized across locations.

With taxes, it depends on how the tax burden is affected by marginal variations in the choice variables. Thus, it depends on the formula.