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Creation and sharing of value in the telecoms sector. (How telecom operators' investments benefit content providers rather than themselves.)

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Creation and sharing of value in the telecoms sector.

(How telecom operators' investments benefit content providers rather than themselves.) *

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Work in progress

Abstract

The fast technical progress coupled with a fierce competition in telecommunication markets urge operators to invest year after year a huge amount of investment. However, the maturation and saturation of markets prevents them from reaping the benefits. While telecommunication infrastructure improves fastly, the revenues of telecommunication operators tend to stagnate. Content providers are taking advantage of increased network capacity to offer more content that they are able to monetize. It is therefore they who benefit from the increase in network capacities and not the operators who nevertheless financed it. This article develops a theoretical model which highlights this mechanism and corresponds perfectly to empirical observations.

Key Words: *Competition; Investment; Telecommunication Operators; Content Providers*

JEL Classification: D25, L51, L86, L96

*Any opinions expressed here are those of the authors and not those of Orange. All errors are our own.

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1 Introduction

Telecommunication sector enjoys a very fast technical progress. Data traffic flowed through telecommunications networks is increasing exponentially and requires year after year a huge amount of investment from telecommunication operators. At the same time, the revenues and profits of telecommunications operators tend to stagnate or even decline, particularly in developed countries and specifically in Europe where the fragmentation of telecommunications markets makes competition particularly fierce. But even if there are differences between countries, the increase in revenues of telecom operators is not commensurate with the investments made. On the contrary, driven by the growth in data traffic, content providers' revenues are increasing exponentially.

Figure.1 below shows the evolution of telcos investment, telcos capex, OTTs revenues and capex from 2011 to 2022 at worldwide level and Figure.2 shows the evolution of worldwide internet data traffic.

Figure 1: Worldwide quarterly Revenues and Capex of Telcos and OTTs (Source OMDIA)

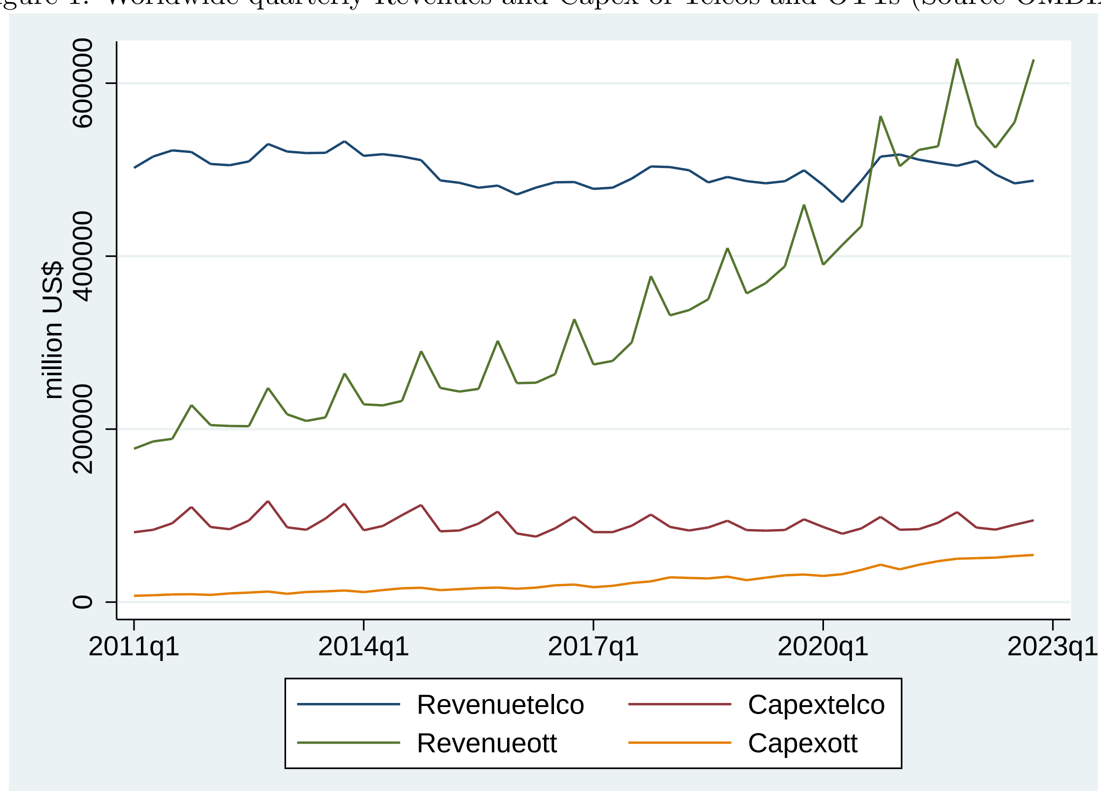
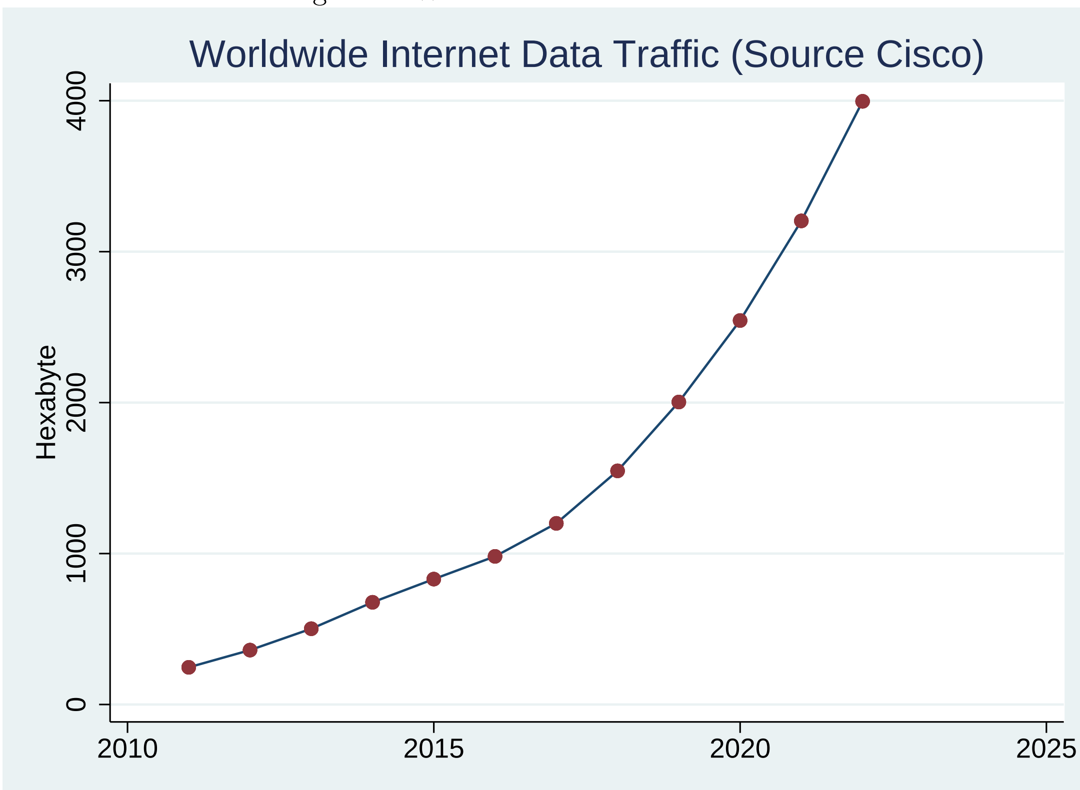


Figure 2: Worldwide internet data traffic



As mentioned above, we observe that worldwide internet data traffic and OTTs' revenues increase both exponentially while Telcos revenues and capex are steady.

How can we explain that the massive and regular investment flows of telecommunications operators do not have a significant impact on their revenues while they have a major impact on network capacities and data traffic?

How to explain that OTT revenues are growing exponentially like data traffic, although at a lower compound annual growth rate (12% and 28% respectively).

This paper shows with a theoretical model, that telcos can not reap the benefit from their investment because of the conjunction of three elements. First, telecommunication markets are matured and saturated. Second the fastness of technical advance and third the fierce competition. In a context of rapid technical progress, fierce competition between telecommunications operators forces them to invest massively in their network or risk being quickly overtaken. However, if competitors invest in the same technology simultaneously, they do not gain a significant competitive advantage and cannot substantially increase their revenues or profits.

Investments in telecommunications networks increase network capacities and enable content providers, such as OTTs, to offer both more and larger content. As long as the demand for content is not saturated, they are able to monetize it and thus increase both their revenue and their profits. In short, investments by telecommunications operators benefit content providers more than themselves. The paper shows that this situation is not optimal for investment and consumers and explores different ways to address it.

The paper is organized as follows: This section is the introduction, section 2 is a literature review, section 3 is the basic model, section 4 is the dynamic model, section 5 discuss the results and explores the different ways to improve investment and consumer surplus and section 6 concludes.

2 Literature review

The relationship between telecommunications operators and content providers, often known as the Over-The-Top (OTT)-Telcos relationship, has been the subject of academic literature in the last decade with an acceleration in the last 5 years.

Many papers highlight the impact of the rise and impact of OTTs on telecommunication markets in different countries, both in developed and developing countries all over the world. Ganuza & Viecens (2014) in Latin America, Osundolire (2018) in Nigeria, Farooq & Raju (2019) in Pakistan or Steingröver *et al.* (2019) in European Union, these examples are not exhaustive, all underline the challenge telcos face with the competition of OTT players on their national markets. They notice a fall in voice traffic, starting from Long Distance to domestic revenues and a fall in SMS and messaging revenues. They often highlight the high regulatory burden telco face compared to OTTs which result in unfair competition to the detriment of telcos. They are calling for a change in regulatory frameworks to ease the burden on telecom operators alone to make competition with OTTs more fair and more sustainable. Furthermore, they mention the significant investments made by telcos to allow the increase in traffic without having a corresponding increase in revenue, but without going into detail in the analysis of the phenomenon.

However, Wellmann (2019) finds that in Norway, OTT messaging is not a substitute but rather a complement of Telcos services, although does not constitute a join market from the Norwegian competition policy perspective. Interestingly, they show that the structure of telcos' tariffs tends to change from a pay per use based structure to a flat rate subscription based structure. This can be explained by Jirakasem & Mitomo (2019) which show that OTT services are substitute to Telco services when sold on a per-use basis and are complement when included in a flat-rate subscription. Indeed, while OTT services are often free for consumers, the subscription combining data, voice and messaging allows the services of telecom operators to become more attractive again and thus, less subtitutable by OTTs. This encourages telecom operators to favor subscription. However, subscription is less likely to allow operators to benefit from the growth of data volumes that require heavy investments.

Some papers suggest some solutions for telcos to face this situation. Sujata *et al.* (2015) claims for a more fair regulation between OTTs and Telcos services arguing that OTTs do not bear, as Telcos, the burden of network costs and hence have a cost advantage that distort competition. Other solutions such as blocking, throttling, data capping or offering differentiated qualities of service (speed, bandwidth) depending on the amount paid by the OTTs have been suggested: Sujata *et al.* (2015) and Heuermann (2019), however they could rise concerns about net neutrality. These papers and Mohr & Meffert (2017) also suggest that telecom operators make bundle OTT services with their own services or partner with OTT services, or even develop their own OTT services.

Some recent papers claim for a cost sharing between OTTs and Telcos: Jullien & Bouvard (2022), Jeanjean (2022), Baranes & Vuong (2023) and Patel *et al.* (2023). Jullien & Bouvard (2022) and Jeanjean (2022) are theoretical models of competition that show that a network cost sharing by OTTs tends to decrease prices for consumers and increase total welfare.

Some consulting reports such as FEAR (2023) contest the fair cost sharing principle arguing that OTT apps boost demand for network capacity pushing Telcos to invest in network improvement to provide better and faster network. Improved network allows OTT to provide new services that increase demand both for OTT and Telco services and thus increases revenues both for OTTs and Telcos. This raises a virtuous circle of OTT services adoption and growth of Telco networks. This is almost true, but it does not take into account markets saturation which prevents the growth of Telcos' revenues (see figure 1) and breaks the virtuous circle.

3 The model

We assume a telecommunications market that is fully covered represented by a duopoly. All consumers subscribe to one of the two operators. We use the well known Hottelling's model which is well suited to represent a fully covered market. The consumers are uniformly distributed over a segment of length normalized to 1. The operators are located at each end of the segment. The first operator at abscissa 0 and the second at abscissa 1.

We assume that operators invest to improve their network capacity. They invest an amount $F = \dfrac{b^2}{2\tau}$ to increase network capacity by b. τ is a constant parameter that represents the technical progress. The higher the technical progress, τ, the lower the cost F to increase network capacity b.

We assume that contents are provided by a monopolistic content provider.

3.1 Basic model

The utility of a consumer located at abscissa $x \in [0, 1]$ to subscribe to operator_1 located at abscissa 0 or operator_2 located at 1 is: respectively

$$U_1 = \alpha q_1 - \frac{q_1^2}{2b_1} - p_c q_1 - tx - p_1 \tag{1}$$

$$U_2 = \alpha q_2 - \frac{q_2^2}{2b_2} - p_c q_2 - t(1 - x) - p_2 \tag{2}$$

Where q_1 and q_2 are the quantity of content consumed by subscribers of respectively operator_1 and operator_2. t is the "transportation cost" and represents the cost for the consumers of being distanced from their ideal offer. α is a positive coefficient. Equations (1) and (2) show that utility of consumers increase with the quantity of contents, but the marginal growth of utility decreases with the term $-\frac{q^2}{2b}$. This represents the disutility due to the limits of network capacity. For a low network capacity, the term $\frac{1}{b}$ is high and, therefore, the disutility increases steeply with content consumption while for a higher network capacity, the network is less limited and allow consumers to consume more content without experiencing network limitation. For $b = 0$, there is no network and therefore no more content consumption.

The indifferent consumer having the same utility for operator 1 or operator 2,($U_1 = U_2$) is located at:

$$x^* = \frac{1}{2} + \frac{1}{2t}\left[(\alpha - p_c)(q_1 - q_2) - \left(\frac{q_1^2}{2b_1} - \frac{q_2^2}{2b_2}\right) + p_2 - p_1\right] \tag{3}$$

Profit of operators are respectively:

$$\pi_1 = (p_1 - c)x^* - \frac{b_1^2}{2\tau} \tag{4}$$

$$\pi_2 = (p_2 - c)(1 - x^*) - \frac{b_2^2}{2\tau} \tag{5}$$

Where c are the cost of operating a subscriber.

The profit of the content provider is:

$$\pi_c = (p_c - c_c)(q_1 x^* + q_2(1 - x^*)) \tag{6}$$

Where c_c is the marginal cost of providing a content.

The timing of the game is as follows. The content provider set the price of content independently of the operators, then operators invest simultaneously and compete in price.

At equilibrium, the duopoly is symmetric, the results are as follows (see proofs in the annexe)

$x^* = \dfrac{1}{2}$

$p_1 = p_2 = p = c + t$

$b_1 = b_2 = b = \dfrac{(\alpha - c_c)^2\tau}{16}$

$q_1 = q_2 = q = \dfrac{(\alpha - c_c)^3\tau}{32}$

$F_1 = F_2 = F = \frac{(\alpha - c_c)^4 \tau}{512}$
$\pi_1 = \pi_2 = \pi = \frac{t}{2} - \frac{(\alpha - c_c)^4 \tau}{512}$
$p_c = \frac{(\alpha + c_c)}{2}$ and
$\pi_c = \frac{(\alpha - c_c)^4 \tau}{64}$

Consumer surplus is given by:

$$CS = \int_0^{x^*} U_1 dx + \int_{x^*}^1 U_2 dx = \frac{(\alpha - c_c)^4 \tau}{128} - c - \frac{5t}{4}$$

3.2 Discussion

It worth being noticed that the price, p and the revenue of operators $px^* = (c+t)/2$ do not depend on investment. When rivals are both equally efficient, they invest the same amount and increase quality and value to consumers equally. In this case, the investment does not bring them any profit. This is because market is fully covered, the size of the market is normalized to 1, therefore no more consumer enter the market following the increase in network capacity.

Although investment does not increase their profits, operators are forced to invest to maintain their market shares, in fact, not investing means a loss of network capacity compared to the rival and therefore a loss of market share. Investment decreases operators profits by $\frac{(\alpha - c_c)^4 \tau}{512}$ as it is spent without any counterpart in terms of Revenue growth. The term $\frac{t}{2}$ represents the profit without investment. If investment is too high, if $\tau > \frac{256t}{(\alpha - c_c)^4}$ profit is negative. This is the dynamic competition which is not always considered by authorities. This is in some way a hidden competition. Indeed, the profit of operators decreases with the amount of investment and with the technical progress, τ that boosts investment. Technical progress amplifies competition as it decreases operators profit, not by decreasing prices and margins, but by increasing investments.

However, these investments are not loss for everyone. As we can see, they increase consumer surplus by $\frac{(\alpha - c_c)^4 \tau}{256}$. Indeed, investment increase network capacities, and allow more services and more content with a subscription. In the model, investment increases b, which increases the demand for content.

Growth in network capacity can be used by content providers to deliver more content over networks. By doing so, they directly benefit from telcos investments. Moreover, the parameters of content demand, a and c_c appear explicitly in the expression of telcos' investment. Moreover, content provider profit is directly linked to telcos' investment. We can observe that $\pi_c = 16F$.

The symmetry between firms leads to the symmetric equilibrium where $x* = 1/2$. This symmetric equilibrium allows that market shares of the two operators remains 1/2 whatever the amount of investment. In this case, both operators invest the same amount, and therefore keep the same attractiveness for consumers and keep their market share.

If the symmetric hypothesis is relaxed (see in the annexes), then, there is a leader and a follower. The leader invests more than the follower, and, in this case, technical progress τ, increases the difference between the leader and the follower. As long as the follower has a positive market share, it remains in the market. However, if τ is too high, it is excluded from the market. In this paper, I have chosen the symmetric hypothesis to avoid the exclusion of the follower in the dynamic model, where technical progress increases regularly. This is a stylised fact which, I think, is consistent with the telecommunication markets where, exclusions remain scarce despite the tremendous pace of technical progress. Anyway, even in asymmetric markets, telcos' investments increase demand for content and contents revenues.

4 Dynamic model

We assume that the technical progress τ increases over time at the constant technical progress rate θ. At each period of time, i, operators invest and technical progress increases following: $\tau_{i+1} = \tau_i(1+\theta)$. At time $i = 0$, technical progress is τ_0. At time i, technical progress is:

$$\tau_i = \tau_0(1+\theta)^i \tag{7}$$

The growth of τ entails the growth of Investment $F = (a - c_c)^4\tau/512$, however, investment cannot exceed t/2 because profit must remain positive. As a result,

$$F_i = \begin{cases} \frac{(\alpha - c_c)^4 \tau_i}{512} & \text{if } F_i < \frac{t}{2} \\ \frac{t}{2} & \text{otherwise} \end{cases} \tag{8}$$

When i is sufficiently high, $F_i = \frac{t}{2}$ and in that case profits of the telcos is zero. This steady investment, thanks to the technical progress, allows a growing increase in the network capacities that translates into an increase in the growth of b over time. By convention, I choose the origin of times $i = 0$ from the moment when $F_i = t/2$

We know that $F_i = \frac{b_i^2}{2\tau}$, as a result $b_i = \sqrt{t\tau_0}(1+\theta)^{\frac{i}{2}}$. Denoting $b_0 = \sqrt{t\tau_0}$ and $\theta' = \sqrt{1+\theta}-1$, we can write $b_i = b_0(1+\theta')^i$

The evolution of the parameters of the model can be rewritten such as:

$x_i^* = \frac{1}{2}$

$F_i = \frac{t}{2}$

$p_i = c + t$

$\pi_i = 0$

$p_{ci} = \frac{(\alpha + c_c)}{2}$

$q_i = \frac{\sqrt{t\tau_0}(\alpha - c_c)}{2}(1+\theta')^i$

$\pi_c = \frac{\sqrt{t\tau_0}(\alpha - c_c)^2}{4}(1+\theta')^i$

$CS_i = \frac{\sqrt{t\tau_0}(\alpha - c_c)^2}{8}(1+\theta')^i - \frac{5t}{4} - c$

Over time, despite the growth of technical progress τ, although they invest regularly all what they have earned, $t/2$, prices and market shares of telcos remain unchanged. On the contrary, the dynamic model shows an exponential growth in the volume of content and the profit of content provider at the rate θ' while the price of content remains unchanged. Consumer surplus also increases at rate θ'.

Technical progress incorporated into telcos' investment with growth at θ rate, (equation 7) is translated into the revenue and the profit of content provider at rate θ' while telcos earn no benefit from their investments. In other words, the value created by telcos' investments is entirely captured by the content provider.

5 Content provider investment

Investments by content providers do not increase operators' revenues and profits. They increase the revenues of content providers and the welfare of consumers, but they fail to increase the profits of operators, which always limits investments in improving networks. Let assume that content provider invests an amount $F_c(z)$ to increase by z the value consumers derive for contents. In such case, equations (1) and (2) can be rewritten:

$$U_1 = (\alpha + z)q_1 - \frac{q_1^2}{2b_1} - p_c q_1 - tx - p_1 \tag{9}$$

$$U_2 = (\alpha + z)q_2 - \frac{q_2^2}{2b_2} - p_c q_2 - t(1-x) - p_2 \tag{10}$$

The profit of content provider, equation (6) can be rewritten:

$$\pi_c = (p_c - c_c)(q_1 x^* + q_2(1 - x^*)) - F_c(z) \tag{11}$$

Equations of operators profits, equations (4) and (5) are unchanged.

These equations, in the context of dynamic model, lead to:

$x_i^* = \frac{1}{2}$

$F_i = \frac{t}{2}$

$p_i = c + t$

$\pi_i = 0$

$p_{ci} = \frac{(\alpha + z + c_c)}{2}$

$q_i = \frac{\sqrt{t\tau_0}(\alpha + z - c_c)}{2}(1+\theta')^i$

$\pi_{ci} = \frac{\sqrt{t\tau_0}(\alpha + z - c_c)^2}{4}(1+\theta')^i - F_c(z)$

$CS_i = \frac{\sqrt{t\tau_0}(\alpha + z - c_c)^2}{8}(1+\theta')^i - \frac{5t}{4} - c$

As investment of content provider increases consumers' demand, This could suggest an increase in operators investment, however, as the profit before investment remains equal to $t/2$, investment cannot increase above $t/2$. As a result investment of content provider increases consumer demand for content and consumer surplus but not operators' profits nor investments in network improvement.

6 Remedies

What type of remedy can be considered to avoid obstacles to investment due to operator capacity limits? A contribution of content providers for the use of network could solve the problem.

With a contribution a per content, equations (4), (5) and (6) are rewritten:

$$\pi_1 = (p_1 - c + aq_1)x^* - \frac{b_1^2}{2\tau} \tag{12}$$

$$\pi_2 = (p_2 - c + aq_2)(1 - x^*) - \frac{b_2^2}{2\tau} \tag{13}$$

$$\pi_c = (p_c - c_c - a)(q_1 x^* + q_2(1 - x^*)) \tag{14}$$

Consumer utility functions, equations (1) and (2), remain unchanged.

6.1 Model with contribution

With these equations, at equilibrium, as in previous section $x^* = \frac{1}{2}$ and:

$b_1 = b_2 = b = \frac{((\alpha - c_c)^2 - a^2)\tau}{16}$

$p_1 = p_2 = c + t - \frac{a(a - c_c + a)(a - cc - a)^2\tau}{32}$

$F = \frac{b^2}{2\tau} = \frac{((\alpha - c_c)^2 - a^2)^2\tau}{512}$

$\pi_1 = \pi_2 = \pi = \frac{t}{2} - \frac{((\alpha - c_c)^2 - a^2)^2\tau}{512}$

$\pi_c = \frac{(\alpha - c_c + a)(\alpha - c_c - a)^3\tau}{64}$

$CS = \frac{(\alpha - c_c + a)(\alpha - c_c - a)^3\tau}{128} - \frac{5t}{4} - c$

(See proofs in the appendix)

In these results, contribution a seems to reduce investment, profits and consumer surplus, however, this holds only if technical progress τ is not too high. If τ is high enough, like in previous section, those equations must be corrected to avoid a negative profit.

6.2 Dynamic model with contribution

Like in previous section, it is considered that τ increases over time following equation (7). Operators earn money from the use of the network for the contents. This reinforce competition because the value of each subscriber increases. As a result, operators reduce their prices.

In the dynamic model, τ increases regularly, which reduces price p, however, price cannot decrease under a certain limit, $p = 0$. In the following, we consider $p = 0$. Once the limit is reached , profit of operators must be rewritten from equations (12) and (13): $\pi = aq - \frac{b^2}{2\tau} = ab(\alpha - p_c) - \frac{b^2}{2\tau}$.

Parameter b increases over time with technical progress, τ. This tends to decrease the profit of operators, π. If the technical progress is sufficiently high, $\tau > max\{\frac{256t}{((\alpha - c_c)^2 - a^2)}, \frac{32t}{a(a - c_c)^2 - a^2)(a - cc + a)}\}$, the profit cannot be negative, like in the previous section, it is considered that profit cannot be lower than $\delta t/2$ as in previous section.

With these assumptions $b = \frac{a\tau(\alpha - c_c - a) \pm \sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}{4}$

(see proof in the appendix)

Which means that b increases at a higher rate than in the previous section. The contribution a increases the growth of investment and therefore the growth of b. Without contribution a, b is growing at rate θ'. With contribution a, b is growing at almost the technical progress rate θ which is higher as θ'.

Notice that if τ is very high, the term $16(\delta t + c)$ is negligible in front of $a^2\tau^2(a - c_c - a)^2$ and therefore: $b_i \approx \frac{a\tau(\alpha - c_c - a)}{2} = \frac{a\tau_0(\alpha - c_c - a)}{2}(1 + \theta)^i$

b is maximum if $a = \frac{\alpha - c_c}{2}$ (see proof in the appendix)

With this value of b, investment increases over time.

$F = \frac{[a\tau(\alpha - c_c - a) \pm \sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}]^2}{8\tau}$

The quantity of content becomes:

$q = \frac{a\tau(\alpha - c_c - a)^2 \pm (\alpha - c_c - a)\sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}{8}$

Profit of telecom operators remains steady:

$\pi = \frac{\delta t}{2}$

Profit of content provider and consumer surplus increase over time:

$$\pi_c = \frac{a\tau(\alpha - c_c - a)^3 \pm (\alpha - c_c - a)^2 \sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}{16}$$

$$CS = \frac{a\tau(\alpha - c_c - a)^3 \pm (\alpha - c_c - a)^2 \sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}{32} - \frac{5t}{4} - c$$

A contribution of content providers to the use of network increases investment, consumer surplus and content providers profits.

7 Conclusion

This model highlights the fact that the value created by telecom operators' investment is captured by content providers and do not benefit those who are behind it. The saturation of telecommunication markets (Mobile and fixed) , under fierce competition, prevents telecom operators to monetize the growing network capacities generated by their investment. They are nevertheless forced to invest to defend their market share. Content providers benefit this growing network capacities to provide more contents they are able to monetize, as the content market is not yet saturated.

The technical progress in telecommunication sector decreases the investment cost of a given network capacity. This encourage telecom operators to invest more and more, however, their investment capacities are limited by their financial capacities. As their revenues are not growing, they cannot invest more and more. This limitation slows down the growth of network capacities. This penalizes consumers and could also slow down innovation and the growth in other sectors. By allowing telecom operators to benefit from their investments, a contribution from content providers to the use of the network could remove this limitation and revitalize investment in the sector.

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8 appendix

Equilibrium in the basic model:

Investment stage: Maximisation of operator 1's profit (First order condition)

$$\frac{\partial \pi_1}{\partial b_1} = (p_1 - c)\frac{\partial x^*}{\partial b_1} - \frac{b_1}{\tau} = \frac{(p_1 - c)q_1^2}{4tb_1^2} - \frac{b_1}{\tau} = 0$$

$$b_1^3 = \frac{(p_1 - c)q_1^2 \tau}{4t}$$

Investment stage: Maximisation of operator 2's profit (First order condition):

$$\frac{\partial \pi_2}{\partial b_2} = -(p_2 - c)\frac{\partial x^*}{\partial b_2} - \frac{b_2}{\tau} = \frac{(p_2 - c)q_2^2}{4tb_2^2} - \frac{b_2}{\tau} = 0$$

$$b_2^3 = \frac{(p_2 - c)q_2^2 \tau}{4t}$$

Competition stage: Maximisation of operator 1's profit (First order condition):

$$\frac{\partial \pi_1}{\partial p_1} = x^* + (p_1 - c)\frac{\partial x^*}{\partial p_1} = x^* - \frac{(p_1 - c)}{2t} = 0$$

$$p_1 = c + 2tx^*$$

Competition stage: Maximisation of operator 2's profit (First order condition):

$$\frac{\partial \pi_2}{\partial p_2} = (1 - x^*) - (p_2 - c)\frac{\partial x^*}{\partial p_2} = (1 - x^*) - \frac{(p_2 - c)}{2t} = 0$$

$$p_2 = c + 2t(1 - x^*)$$

Maximisation of consumer utility:

$$\frac{\partial U_1}{\partial q_1} = (\alpha - p_c) - \frac{q_1}{b_1} = 0$$

$$\frac{\partial U_2}{\partial q_2} = (\alpha - p_c) - \frac{q_2}{b_2} = 0$$

$q_1 = b_1(\alpha - p_c)$ and $q_2 = b_2(\alpha - p_c)$

therefore, $\frac{q_1}{b_1} = \frac{q_2}{b_2}$

Maximisation of content provider profit (first order condition):

Equation (6) can be rewritten:

$$\pi_c = (p_c - c_c)(b_1 x^* + b_2(1 - x^*))(\alpha - p_c)$$

$$\frac{\partial \pi_c}{\partial p_c} = (b_1 x^* + b_2(1 - x^*))(\alpha - p_c) - (p_c - c_c)(b_1 x^* + b_2(1 - x^*)) = 0$$

$$\alpha - p_c = p_c - c_c$$

$$p_c = \frac{\alpha + c_c}{2}$$

Using the results of the first order conditions and the fact that $\frac{q_1}{b_1} = \frac{q_2}{b_2}$, we can rewrite equation (3)

$$x^* = \frac{1}{2} + \frac{1}{2t}\left[(\alpha - p_c)(q_1 - q_2) - \frac{q_1}{2b_1}(q_1 - q_2) + 2t - 4tx^*\right]$$
$$x^* = \frac{1}{2} + \frac{(\alpha - p_c)^2(b_1 - b_2)}{12t}$$
$$b_1 = \frac{(p_1 - c)\tau}{4t}\frac{q_1^2}{b_1^2} = \frac{(p_1 - c)\tau(\alpha - p_c)^2}{4t}$$
$$b_2 = \frac{(p_2 - c)\tau}{4t}\frac{q_2^2}{b_2^2} = \frac{(p_2 - c)\tau(\alpha - p_c)^2}{4t}$$
$$b_1 - b_2 = \frac{(\alpha - p_c)^2\tau}{4t}(p_1 - p_2) = \frac{(\alpha - p_c)^2\tau(2x^* - 1)}{2}$$
$$x^* = \frac{1}{2} + \frac{(\alpha - p_c)^4(2x^* - 1)\tau}{24t}$$
and finally $x^* = \frac{1}{2}$

As a consequence, $p_1 = p_2 = c + t$
$$b_1 = \frac{(\alpha - p_c)^2\tau}{4} = \frac{(\alpha - c_c)^2\tau}{16} = b_2$$
$$q_1 = b_1(\alpha - p_c) = \frac{(\alpha - c_c)^3\tau}{32} = q_2$$
$$F_1 = \frac{b_1^2}{2\tau} = \frac{(\alpha - c_c)^4\tau}{512} = F_2$$
$$\pi_1 = (p_1 - c)x^* - F_1 = \frac{t}{2} - \frac{(\alpha - c_c)^4\tau}{512} = \pi_2$$
$$\pi_c = (p_c - c_c)\frac{(q_1 + q_2)}{2} = \frac{(\alpha - c_c)^4\tau}{64}$$
$$CS = \int_0^{\frac{1}{2}} U_1 + \int_{\frac{1}{2}}^1 U_2 = \frac{(\alpha - c_c)^4\tau}{256} - \frac{c + t}{2} - \frac{t}{8} + \frac{(\alpha - c_c)^4\tau}{256} - \frac{2t + c}{2} + \frac{3t}{8}$$
$$CS = \frac{(\alpha - c_c)^4\tau}{128} - \frac{5t}{4} - c$$

Dynamic Model:

We know that $q_i = b_i(\alpha - p_c)$, therefore $q_i = b_0\frac{(\alpha - c_c)}{2}(1 + \theta')^i = \frac{\sqrt{t\tau_0}(\alpha - c_c)}{2}(1 + \theta')^i$
$$\pi_{ci} = (p_c - c_c)q_i = \frac{\sqrt{t\tau_0}(a - c_c)^2}{4}(1 + \theta')^i$$
$$CS = \int_0^{\frac{1}{2}} U_1 + \int_{\frac{1}{2}}^1 U_2 = \frac{\sqrt{t\tau_0}(\alpha - c_c)^2}{16}(1+\theta')^i - \frac{c + t}{2} - \frac{t}{8} + \frac{\sqrt{t\tau_0}(\alpha - c_c)^2}{16}(1+\theta')^i - \frac{2t + c}{2} + \frac{3t}{8}$$
$$CS = \frac{\sqrt{t\tau_0}(\alpha - c_c)^2}{8}(1 + \theta')^i - \frac{5t}{4} - c$$

Remedies:

Investment stage: Maximisation of operator 1's profit (First order condition)
$$\frac{\partial \pi_1}{\partial b_1} = a(\alpha - p_c)x^* + ((p_1 - c + a(\alpha - p_c)b_1)\frac{\partial x^*}{\partial b_1} - \frac{b_1}{\tau} = 0$$
$$b_1 = \frac{[a(\alpha - p_c)(4tx^* + (\alpha - p_c)^2 b_1) + (p_1 - c)(\alpha - p_c)^2]\tau}{4t}$$

$b_1 = \dfrac{(p_1 - c)(\alpha - p_c)^2\tau + 4at(\alpha - p_c)\tau x^*}{4t - a(\alpha - p_c)^3\tau}$

Investment stage: Maximisation of operator 2's profit (First order condition)

$\dfrac{\partial \pi_2}{\partial b_2} = a(\alpha - p_c)(1 - x^*) + ((p_2 - c + a(\alpha - p_c)b_2)\dfrac{\partial x^*}{\partial b_2} - \dfrac{b_2}{\tau} = 0$

$b_2 = \dfrac{(p_2 - c)(\alpha - p_c)^2\tau + 4at(\alpha - p_c)\tau(1 - x^*)}{4t - a(\alpha - p_c)^3\tau}$

Competition stage: Maximisation of operator 1's profit (First order condition)

$\dfrac{\partial \pi_1}{\partial p_1} = x^* + (p_1 - c + a(\alpha - p_c)b_1)\dfrac{\partial x^*}{\partial p_1} = x^* - \dfrac{(p_1 - c + a(\alpha - p_c)b_1)}{2t} = 0$

$p_1 = c + 2tx^* - a(\alpha - p_c)b_1$

Competition stage: Maximisation of operator 2's profit (First order condition)

$\dfrac{\partial \pi_2}{\partial p_2} = (1 - x^*) + (p_2 - c + a(\alpha - p_c)b_2)\dfrac{\partial x^*}{\partial p_2} = (1 - x^*) - \dfrac{(p_2 - c + a(\alpha - p_c)b_2)}{2t} = 0$

$p_2 = c + 2t(1 - x^*) - a(\alpha - p_c)b_2$

Maximisation of consumer utility:

idem as previous sections $q_1 = b_1(\alpha - p_c)$ and $q_2 = b_2(\alpha - p_c)$

Maximisation of content provider profit:

$\dfrac{\partial \pi_c}{\partial p_c} = (b_1x^* + b_2(1 - x^*))(\alpha - p_c) - (p_c - c_c - a)(b_1x^* + b_2(1 - x^*)) = 0$

$p_c - c_c - a = \alpha - p_c$

$p_c = \dfrac{\alpha + c_c + a}{2}$

The difference between p_1 and p_2 is:

$p_1 - p_2 = 2t(2x^* - 1) - a(\alpha - p_c)(b_1 - b_2)$

$b_1 - b_2 = \dfrac{(p_1 - p_2)(\alpha - p_c)^2\tau + 4at(\alpha - p_c)\tau(2x^* - 1)}{4t - a(\alpha - p_c)^3\tau}$

replacing $p_1 - p_2$ by its value yields:

$b_1 - b_2 = (2x^* - 1)\dfrac{(\alpha - p_c)\tau(\alpha - p_c + 2a)}{2(4t - a(\alpha - p_c)^3\tau}$

the location of the indifferent consumer is:

$x^* = \dfrac{1}{2} + \dfrac{1}{2t}\left(\dfrac{(b_1 - b_2)(\alpha - p_c)^2}{2} + p_2 - p_1\right)$

$x^* = \dfrac{1}{2} + \dfrac{(\alpha - p_c)(\alpha - p_c + 2a)(b_1 - b_2)}{12t}$

$x^* = \dfrac{1}{2} + (2x^* - 1)\dfrac{(\alpha - p_c)^2(\alpha - p_c + 2a)^2\tau}{24t(4t - a(\alpha - p_c)^3\tau)}$

and finally:

$x^* = \dfrac{1}{2}$

Therefore, $p_1 = c + t - \dfrac{ab_1(\alpha - c_c - a)}{2}$ and $p_2 = c + t - \dfrac{ab_2(\alpha - c_c - a)}{2}$

$$b_1 = b_2 = b = \frac{((\alpha - c_c)^2 - a^2)\tau}{16}$$
$$p_1 = p_2 = c + t - \frac{a(a - c_c + a)(a - cc - a)^2\tau}{32}$$
$$F = \frac{b^2}{2\tau} = \frac{((\alpha - c_c)^2 - a^2)^2\tau}{512}$$
$$\pi_1 = \pi_2 = \frac{t}{2} - \frac{((\alpha - c_c)^2 - a^2)^2\tau}{512}$$
$$\pi_c = \frac{(\alpha - c_c + a)(\alpha - c_c - a)^3\tau}{64}$$
$$CS = \frac{(\alpha - c_c + a)(\alpha - c_c - a)^3\tau}{128} - \frac{5t}{4} - c$$

Dynamic model with remedies:

If technical progress τ is sufficiently high, $\tau > \frac{256t}{((\alpha - c_c)^2 - a^2)}$, then the profit of operators turns to be negative, therefore, as in previous section, in this case, it is assumed that investment F is limited to avoid a negative profit. Moreover, the growth of τ decreases prices of operators. It is assumed that operators do not provide negative prices, therefore,

if $\tau > \frac{32(c+t)}{a(a - c_c)^2 - a^2)(a - cc + a)}$ then $p_1 = p_2 = 0$

If τ is sufficiently high: $\tau > max\{\frac{256t}{((\alpha - c_c)^2 - a^2)}, \frac{32(c+t)}{a(a - c_c)^2 - a^2)(a - cc + a)}\}$

then $\pi_1 = \pi_2 = \pi = \frac{\delta t}{2}$ and $p_1 = p_2 = p = 0$

Assuming $p = 0$ and $\pi = \frac{\delta t}{2}$, equations (12) and (13) can be rewritten:

$\pi = \frac{-2c + ab(\alpha - c_c - a)}{4} - \frac{b^2}{2\tau} = \frac{\delta t}{2}$ which yields:

$b^2 - \frac{a\tau(\alpha - c_c - a)}{2}b + (\delta t + c) = 0$ that is solved:

$$b = \frac{a\tau(\alpha - c_c - a) \pm \sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}{4}$$
$$F = \frac{b^2}{2\tau} = \frac{[a\tau(\alpha - c_c - a) \pm \sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}]^2}{8\tau}$$
$$q = (\alpha - p_c)b = \frac{a\tau(\alpha - c_c - a)^2 \pm (\alpha - c_c - a)\sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}{8}$$
$$\pi = \frac{\delta t}{2}$$
$$\pi_c = \frac{a\tau(\alpha - c_c - a)^3 \pm (\alpha - c_c - a)^2\sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}{16}$$
$$CS = \frac{a\tau(\alpha - c_c - a)^3 \pm (\alpha - c_c - a)^2\sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}{32} - \frac{5t}{4} - c$$

b is maximum if $\frac{\partial b}{\partial a} = 0$

$$\frac{\partial b}{\partial a} = ((\alpha - cc)\tau - 2a\tau)\frac{\sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau} \pm 1}{4\sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau}}$$

This expression is maximum if $\sqrt{a^2\tau^2(\alpha - c_c - a)^2 - 16(\delta t + c)\tau} \pm 1 = 0$, or if $a = \dfrac{\alpha - c_c}{2}$