Knapp, Fabian

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Optimal Time-Consistent Macroprudential Policy Revisited*

Fabian Knapp†

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Abstract

This paper studies optimal time-consistent macroprudential policy in a model with endogenous capital formation. Previous studies on optimal time-consistent macroprudential policy in economies where borrowing is limited by the value of collateral assume that aggregate capital is fixed or apply models where production does not depend on capital. I find that it is optimal to restrict borrowing in "good times" in an economy with endogenous capital formation, while borrowing should be supported in times of financial stress. Both of these results are consistent with related studies with fixed aggregate capital. However, the imposition of macroprudential regulation distorts capital formation, which makes a more cautious ex ante intervention compared to previous studies optimal. This is because less capital not only leads to lower production but also hampers borrowing during crises, as capital serves as collateral in debt contracts. As a main result, I find that the optimal macroprudential policy hardly prevents crises and that the induced welfare gains are minimal. If financial regulation is coordinated with an investment policy that addresses the adverse effects on capital formation, almost all crises are prevented and the welfare gains are substantially higher.

Keywords: Macroprudential Policy, Investment, Optimal Policy, Financial Crises, Collateral Constraint

JEL Codes: D62, E32, E44, F32, F41

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†University of Cologne, Department of Economics, Albertus-Magnus-Platz, 50923 Cologne, Germany. Email: knapp@wiso.uni.koeln.de. Website.

1 Introduction

Macroprudential policy - defined as measures to limit the build-up of excessive levels of private debt - aims to prevent the emergence of financial crises and to reduce the severity of crisis events once they occur.[1] Research on macroprudential policy confirms that these goals can be achieved to a high degree (see e.g. Bianchi 2011 and Bianchi and Mendoza 2018). Yet, limiting the buildup of debt comes at the cost of restricting aggregate spending, i.e. consumption and investment expenditures, in non-crisis times. Generally, less investment and hence capital not only results in lower production, but also hampers borrowing during crises, as capital serves as collateral in debt contracts. Ideally, regulation balances the costs and benefits of intervening in debt markets. Therefore, the extent to which a regulator chooses to intervene in debt markets critically depends on how strongly the costs of regulation in non-crisis times rise with reduced borrowing.

Previous studies that have provided quantitative analyses of optimal macroprudential regulation in terms of a state-contingent debt tax/subsidy, e.g. Bianchi 2011, Bianchi and Mendoza 2018, Jeanne and Korinek 2019, Benigno et al. 2016 and Ma 2020, include cost of macroprudential regulation in terms of lower consumption in favorable states. However, they assume that aggregate capital is fixed or apply models where production does not depend on capital, even though capital formation is typically considered as a fundamental factor for macroeconomic dynamics.[2] Therefore, they underestimate the total cost by neglecting from capital formation. These studies find that optimal macroprudential policy strongly reduces the number of crises and substantially increases welfare.

The main novel contribution of this paper is to provide a quantitative analysis of optimal time-consistent macroprudential policy in a model with endogenous capital formation, taking into account the effects of macroprudential policy not only on consumption but also on investment. Foremost, the magnitude of the optimal intervention as well as the effectiveness of optimal macroprudential policy with regard to the reduction in crisis probability and the impact on social welfare will be revisited. I find that under endogenous capital formation the optimal macroprudential policy is more cautious than in an economy with fixed aggregate capital, that it hardly prevents crises and that the induced welfare gains are minimal. Furthermore, the analysis provides insights into how investment and crisis dynamics change when the optimal

[1] See e.g. Buch 2019 or Neely and Dunn 2023.

[2] Bianchi and Mendoza 2018 set up a planner problem with endogenous capital formation in their appendix but do not derive or analyze the state-contingent debt tax, do not analyze the additional effects that come with endogenous capital formation and do not provide a quantitative exploration.

macroprudential policy is implemented.

The competitive equilibrium of the model applied in this paper is constrained inefficient due to a "collateral externality" (see Dávila and Korinek 2017). This type of pecuniary externality arises when agents do not take into account how their behavior affects the price of an asset that serves as collateral in debt markets. Importantly, both borrowing and capital formation are constrained inefficient. Since the political and academic debate on the prevention of financial crises revolves around the macroprudential regulation of debt, I analyze the inefficiencies of the competitive equilibrium with respect to the borrowing choice by solving for the constrained efficient allocation where a social planner chooses borrowing on behalf of agents, like Bianchi and Mendoza 2018, Bianchi 2011, Ma 2020, Jeanne and Korinek 2019, Benigno et al. 2013, among others. The type of intervention that is desirable is consistent with related studies: ex ante, there is overborrowing, i.e. it is optimal to restrict borrowing, whereas ex post borrowing should be supported. However, I show analytically that the size of the optimal intervention differs from economies with a fixed level of aggregate capital since regulation distorts capital formation. Quantitatively, I find that this distortion is harmful and increases the costs of macroprudential policy, which makes a more cautious ex ante regulation - compared to studies without endogenous capital formation - optimal.[3] Therefore, the crisis probability is reduced by 0.52 percentage points only,[4] which is about one eighth of what related studies with fixed capital find.[5] Put differently, despite an optimal intervention, severe crises occur approximately once every 25 years, as opposed to once every 22 years under laissez-faire. The welfare gains are negligibly small compared to studies with a fixed level of aggregate capital, ranging from one twentieth to one hundredth of the welfare gains in related studies, revealing the limited effectiveness of macroprudential regulation due to its distortionary effect on investment.[6]

To assess whether, in general, there is a constrained efficient allocation in this economy with substantial welfare improvements compared to laissez-faire, I construct a reference case: I solve for the constrained efficient allocation resulting from a social planner problem where a regulator chooses debt and investment on behalf of agents. This allocation can be implemented by combining macroprudential policy with investment policy, which addresses the inefficiencies in the investment decision as well as the distortionary impact of macroprudential policy on investment.

[3] The ex ante policy is defined as the imposed tax when the collateral constraint is not binding.

[4] The reduction of the crisis probability is due to the combination of ex post and ex ante policy. However, I will provide evidence that the role of the ex post tax is limited.

[5] Bianchi and Mendoza 2018 find a reduction of the crisis probability by 3.98 percentage points and Ma 2020 finds a reduction of 4.16 percentage points.

[6] Bianchi and Mendoza 2018 report a welfare gain of 0.3 percent and Ma 2020 reports a welfare gain of 0.06 percent, whereas I observe a welfare gain of 0.0022 percent.

I find that in this case welfare gains increase by a factor of thirteen, and crises become almost nonexistent.

My analysis closely follows Bianchi and Mendoza 2018, facilitating a direct comparison with their seminal study. Importantly, this paper's open economy DSGE model with an occasionally binding collateral constraint nests the model of Bianchi and Mendoza 2018, where aggregate capital is fixed. The additional element is the inclusion of endogenous capital formation.[7] Productive capital, which is traded by agents at an endogenous price, serves as collateral and can be increased by firms by using a standard investment technology with adjustment costs. Borrowing is restricted by a fraction of the market value of the pledgeable asset, i.e. capital. This collateral constraint is the root of strong financial amplification. In adverse states, capital is traded at a lower price. A lower price of capital reduces the value of collateral and the borrowing capacity via the collateral constraint. This, in turn, reduces aggregate spending and thereby asset prices in equilibrium, which is not internalized by private agents. This sets a downward spiral in motion which can cause severe crises.

For the analysis of optimal policy, I follow Bianchi and Mendoza 2018 and solve for the Markov perfect equilibrium of the constrained efficient planner problem, ensuring time consistency. The current planner takes the decision of future planners as given, but anticipates that his/her choices of the state variables for the subsequent period will influence the future planner. The planner's decisions can be decentralized with a state-contingent Pigouvian debt tax/subsidy.

I follow the calibration strategy of Bianchi and Mendoza 2018 and calibrate my model to the OECD member countries between 1984 and 2012. Due to endogenous capital formation some parameter values differ from theirs, while my calibration strategy shares many targets. If, for example, I use all of their parameter values, agents would rather save than borrow due to an extremely tight collateral constraint.[8] The parameter values from my study, however, would imply a fixed capital economy in which the collateral constraint is never binding.[9] To compare my results of an economy with endogenous capital formation to an economy with a fixed level of aggregate capital, I therefore repeatedly refer to their study. By endogenizing capital formation my model closely relates to Mendoza 2010, where capital adjustment is essential to reproduce

[7] Endogenous capital formation not only increases the number of state variables but also introduces an additional endogenous variable to the collateral constraint, thereby increasing the complexity of the optimal policy algorithms.

[8] This is because the endogenous level of capital resulting from their parameters would be much lower than one, the value at which Bianchi and Mendoza fix aggregate capital.

[9] This is because the price of capital varies around one in my model with capital adjustment costs but would vary around a multiple of one in a model with fixed capital, implying a very high collateral value.

the key features of sudden stops in Mexico. Therefore, my model also generates realistic financial crisis dynamics.[10]

I find that it is optimal to tax borrowing ex ante, which is consistent with Bianchi and Mendoza 2018, Benigno et al. 2016, Biljanovska and Vardoulakis 2024, Jeanne and Korinek 2019, Dávila and Korinek 2017, Bianchi 2011, and to subsidize borrowing ex post, which is consistent with Bianchi and Mendoza 2018 and Biljanovska and Vardoulakis 2024. However, the size of the optimal intervention differs substantially compared to studies without capital formation. In particular, the optimal ex ante Pigouvian debt tax of 0.53 percent is substantially lower than e.g. in the study of Bianchi and Mendoza 2018 (3.6 percent). This difference arises because, unlike in an economy with fixed aggregate capital, the social planner now considers the impact of the intervention on capital formation. He/she takes into account that financial regulation affects the price of capital and thereby distorts capital formation. Although the social planner takes this additional cost of regulation into account, the state-contingent optimal macroprudential regulation nevertheless reduces investment. The optimal policy leads to a reduction of the crisis probability by only 0.52 percentage points (e.g. Bianchi and Mendoza 2018: reduction of 3.98 percentage points). In total, the welfare gain induced by macroprudential regulation is only 0.0025 percent, which is about one hundredth of what Bianchi and Mendoza 2018 find. Another interesting finding is that the welfare effect of a pure ex ante policy is close to the welfare effect of a combined ex ante and ex post policy.[11] When the collateral constraint is binding, the amount of collateral is predetermined so that only a higher price of collateral can increase the borrowing capacity. In contrast to a model with fixed aggregate capital, where higher consumption is sufficient to increase the price of capital in equilibrium via the asset pricing equation, in my model a higher price of capital only emerges in the capital market if both consumption and investment increase. Thus, increasing the price of capital is more costly and the power of the ex post policy is limited.

In the reference case, i.e. the constrained efficient allocation where the social planner chooses borrowing and investment on behalf of agents, the allocation can be decentralized with a Pigouvian debt tax/subsidy and a Pigouvian investment tax/subsidy. The sign of the optimal debt tax/subsidy is identical to the case where the social planner can only choose borrowing opti-

[10]Without being specifically targeted, the response of price of capitals during average crises closely mirrors observed crises (see Mendoza 2010) as opposed to models with fixed collateral, which produce excessively strong decreases.

[11]This observation contributes to the discussion whether the ex ante or the ex post policy is the more relevant part of the optimal policy (see Benigno et al. 2024). Benigno et al. argue that the ex post policy is the main driver of welfare gains in the framework of Bianchi and Mendoza 2018. However, when capital formation is taken into account, the ex ante policy is the more important part, as I will demonstrate in chapter 5.

mally, but the average level of the optimal ex ante tax differs substantially: it is 2.5 times as high. This is because investment policy compensates for the effects of macroprudential policy on investment: the social planner chooses higher investment than under laissez-faire, despite less borrowing. Thus, macroprudential policy is used more intensively. Consequently, crises are almost fully prevented: the probability of crises reduces to 0.03 percent and welfare gains increase tenfold, i.e. to 0.0221 percent.

The remainder of the paper is structured as follows: Chapter 2 summarizes the related literature and chapter 3 describes the model and the competitive equilibrium. Chapter 4 analyzes the social planner problem and the optimal tax rates. Chapter 5 explains the calibration and summarizes the quantitative results. Chapter 6 concludes.

2 Literature Review

This paper is related to several studies on optimal macroprudential policy and on pecuniary externalities in models with collateral constraints. The literature in this area is divided into analytical papers, mainly using finite horizon models, and quantitative analyses. My paper contributes to this literature by providing the first quantitative study of optimal time-consistent macroprudential policy in a model where both components of the collateral value that limits borrowing, i.e. the price of capital and the level of capital, are endogenous.

My paper is most closely related to the seminal study by Bianchi and Mendoza 2018, which analyzes optimal time-consistent macroprudential policy in a model with a fixed level of aggregate capital. Their model is nested in my model with endogenous capital formation and the calibration is closely related. Bianchi and Mendoza show that a positive ex ante debt tax is optimal. Quantitatively, they find that the probability of crises decreases to almost zero when the optimal macroprudential policy is imposed. Ma 2020 provides a quantitative study of optimal macroprudential regulation, where productivity is endogenized and grows over time. It analyzes the inefficiencies associated with borrowing in an economy with positive growth rates. Households can invest in "growth-enhancing expenditures" (Ma 2020, p. 3) to increase future productivity, while the aggregate amount of collateral is fixed. Ma finds that the optimal time-consistent ex ante debt tax is positive and that the crisis probability is reduced by two-thirds when the optimal macroprudential tax is imposed. Jeanne and Korinek 2019 analyzes optimal time-consistent taxes on borrowing in an economy where income is exogenous, borrowing is limited by a collateral constraint and the amount of the collaterizeable asset is assumed to be fixed.

They find that it is optimal to tax debt when the collateral constraint is binding and that it can be set to zero when it is not. Biljanovska and Vardoulakis 2024 is a quantitative study that adds heterogeneity in the form of two agents (workers and entrepreneurs). They fix the aggregate level of capital that serves as collateral and equip the social planner with a debt tax as well as a payroll tax. They find that the optimal debt tax is positive on average but negative in sudden stops. Zaretski 2024a analyzes the non-time-consistent optimal policy mix in an economy with endogeneous capital formation, multiple financial frictions and sticky prices. Zaretski 2024b analyzes optimal time-consistent and non-time-consistent policy in an economy with endogeneous capital formation and heterogeneous bankers who can divert funds and are therefore subject to an enforcement constraint. The constraint limits the value of banks to be greater than or equal to a fraction of the value of bank assets. Zaretski decentralizes the Markov perfect constrained efficient allocation with a deposit tax and a bank asset tax or a capital requirement. The analysis differs from mine because in Zaretski's model the constrained efficient equilibrium indicates underborrowing (of banks) in some favorable states and because of different financial frictions: in his model, the value of a bank rather than borrowing is constrained by the value of capital.
Bianchi and Mendoza 2020 is - to my knowledge - the only quantitative analysis of pure macroprudential regulation where both components of the collateral value that limits borrowing, i.e. the price of capital and the level of capital, are endogenous. Their analysis differs from mine because, first, they restrict their analysis to constant macroprudential taxes and under this restriction they choose the optimal debt tax. In my study, however, a planner problem is solved to obtain the constrained efficient allocation and to analyze optimal state-contingent debt taxes. Second, the model structure differs substantially. Their model is a two-sector model with tradable goods and non-tradable goods where the price of capital is determined by the price of non-tradables and is furthermore directly influenced by an investment productivity shock. Bianchi and Mendoza find that the best constant macroprudential debt tax is positive and that it leads to small welfare gains. Consistent with findings in my study with optimal state-contingent taxes, macroprudential policy reduces the capital-to-GDP ratio in their framework. In a further analysis, the authors restrict their analysis of the interplay of macroprudential and investment policy to the best one-time debt and one-time investment tax rates. Both taxes are levied in a particular state for one period only, followed by zero tax rates in all subsequent periods, whereas I analyze optimal state-contingent debt and investment tax rates/subsidies that are not limited to one period. In this analysis they find that the sign of the best one-time investment tax depends on the sign of the debt tax.

My paper is also related to further papers on financial crises and pecuniary externalities within two-sector models with tradable and non-tradable goods, i.e. Bianchi 2011, Benigno et al. 2016, Benigno et al. 2013, Schmitt-Grohé and Uribe 2017. In this part of the literature borrowing is constrained by a share of income which is either assumed to be exogenous or independent of capital.

There are also papers that use finite time horizon models where capital is endogenous for 1 or 2 periods. Dávila and Korinek 2017 analyzes collateral and distributive externalities in a 3-period model, where capital is endogenous only in the first period. The price of capital relevant to the collateral constraint is affected only by the capital decision in the period before the crisis, whereas in my model it is the current capital choice that affects the price of capital. They find that in the case of two instruments the social planner wants to tax debt and that the sign of the capital tax is ambiguous. They also show that in a version of their model without distributive externalities there is ex ante underinvestment, as in my model. To bring their results and mine together, I analyze a small open economy in the Appendix that builds on their model. Lorenzoni 2008 analyzes collateral and distributive externalities in a 3-period model, where capital is endogenous in the first and the second period. He does not solve for an optimal debt tax but discusses a capital requirement to implement constrained efficiency.

Some infinite horizon papers on optimal policy in economies with externalities and a collateral constraint focus on other instruments than macroprudential policy. Lanteri and Rampini 2023 builds a model with heterogeneous firms, old as well as new capital and distributive as well as collateral externalities. They eliminate the borrowing externality, which is present in my model as well as in Bianchi and Mendoza 2018, Jeanne and Korinek 2019, Ma 2020 and Biljanovska and Vardoulakis 2024, by assuming a linear utility function. Consequently, they focus on capital taxes. Lanteri and Rampini find that old capital should be taxed while new capital should be subsidized. Bianchi 2016 focuses on bailouts and develops a model with a collateral constraint, a constraint on dividends and endogenous capital formation. In contrast to the previous studies there is no collateral externality, i.e. no pecuniary externality with regard to prices in the collateral constraint.

3 Model

In this section, I present a small open economy DSGE model with endogenous capital formation and an occasionally binding collateral constraint. It contains several elements of the model by

Mendoza 2010, but differs in the way investment is modeled. Moreover, the model of Bianchi and Mendoza 2018 is nested in my framework.

There are two types of agents: households and investment firms. I follow Bianchi and Mendoza 2018 and model production within a household who also decides on the factors of production. There is a continuum of households of measure one, but since they behave identically I will use the term "the (representative) household" from here on. To integrate the supply side of capital, the model includes an investment firm that has access to a technology to increase the capital stock of the economy. Since capital serves both as a factor of production and as collateral, I will use the terms capital and collateral interchangeably in the following analysis.

Before proceeding with the details of the model, some general assumptions need to be made: Time is discrete and indexed by $t = 1, 2, \ldots$. All variables are in real terms. There are three shocks in the model - a real interest rate shock, a productivity shock and a financial shock. The shocks follow finite-state stationary Markov processes with compact support.

The following analysis begins with a description of the model setup and continues with the definition and interpretation of the decentralized competitive equilibrium without regulation.

3.1 Setup

The representative household derives utility from consumption c and disutility from supplying labor h. The utility function is a standard CRRA function[12]. Thus, preferences are given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - \chi \frac{h_t^{\omega}}{\omega}\right)^{1-\sigma} - 1}{1-\sigma} \right] , \tag{1}$$

where $\mathbb{E}$ is the expectations operator, β is the subjective discount factor, σ is the coefficient of constant relative risk aversion, χ is a weighting parameter of the disutility of labor and $\frac{1}{\omega-1}$ is the Frisch elasticity.

The household produces a final good by combining capital k, labor h and an intermediate good v. The production function is a standard Cobb-Douglas production function with total factor productivity A:

$$F(k_t, h_t, v_t) \quad = \quad A_t k_t^{\alpha_k} h_t^{\alpha_h} v_t^{\alpha_v} \, . \tag{2}$$

[12]The analytical results hold for every concave and twice continuously differentiable function that satisfies the Inada conditions. To make the equations easier to read, I decided to present the results with a CRRA function from the start.

A_t is an exogenous productivity shock and α_k, α_h as well as α_v denote parameters that determine the share of the respective input in gross output. The household faces the following trade-offs when choosing labor and intermediate inputs: Working time h increases production on the one hand, but decreases utility on the other hand. The intermediate good v also increases production, but costs p_v, which is an exogenous price, per unit. Furthermore, a share θ of the cost of the intermediate good must be paid before production and is financed by foreign debt. The household starts period t with an level of capital k_t and uses it for production. After production the household enters the capital market, where he/she can trade with other households or with investment firms. Since all households face the same (aggregate) shocks, all would like to increase their depreciated capital holdings by the same amount and there will be no trade between households in equilibrium. Unlike households, investment firms have a technology to increase the capital stock in the economy at the cost of one unit of the final good and some investment adjustment costs which are specified as

$$c(i_{F,t}) \quad = \quad \frac{a}{2}\left(i_{F,t}-\bar{i}\right)^2 \; . \tag{3}$$

$\bar{i}$ is a parameter that serves as the investment benchmark and a is an adjustment cost parameter that determines how strongly the adjustment costs react to deviations of investment from the benchmark level. After the investment decision the (representative) investment firm sells the amount of newly produced capital, i.e. $i_{F,t}$, to the households at price q_t. The static problem of the investment firm is therefore:

$$\max_{i_{F,t}} \quad \Pi_t \quad = \quad q_t i_{F,t} - i_{F,t} - \frac{a}{2}(i_{F,t}-\bar{i})^2 \; . \tag{4}$$

The following equation displays the investment firm's optimal investment choice:

$$i_{F,t} \quad = \quad \frac{q_t-1}{a} + \bar{i} \; . \tag{5}$$

Equation (5) shows that if the price of capital is greater than 1, the investment firm chooses to produce more than $\bar{i}$ and if the price of capital is less than 1, the investment firm produces less than $\bar{i}$. It also becomes clear that investment goes to zero for $\bar{i}=0$ and $a \to +\infty$. Investment firms are owned by the households so that profits/losses enter the household budget constraint. One-period, non-state contingent foreign bonds b are the second asset in the economy after capital. Negative values of b imply that households borrow at the gross real interest R which can be interpreted as the world interest rate that is taken as given by the domestic economy. The interest rate is modeled as an exogenous shock and each level of bonds/debt is supplied by the rest of the world.[13] Total borrowing in one period is the sum of borrowing $-b_{t+1}$ divided by the gross interest rate and working capital $\theta p_v v_t$, which is financed by intra-period loans at an interest rate of zero.

As the model's main financial friction the collateral constraint is at the heart of the model and generates the financial amplification mechanism mentioned above. As in Bianchi and Mendoza 2018, borrowing is limited by a fraction κ_t of the current value of collateral:[14]

$$\frac{b_{t+1}}{R_t} - \theta p_v v_t \quad \geq \quad -\kappa_t q_t k_t \ . \tag{6}$$

Bianchi and Mendoza 2018 show that this constraint can be microfounded by an incentive compatibility constraint on borrowers. The intuition is that borrowers can divert funds after borrowing, and due to limited enforcement, the lender sells the seized collateral in this case. In contrast to previous studies on optimal time-consistent macroprudential policy in economies with collateral constraints, the capital level is modeled endogenously and therefore varies over time. κ_t is a financial shock and can take two values. This reflects that borrowing tends to be more constrained during crises.

Households maximize utility subject to the collateral constraint and the following budget constraint:

$$A_t k_t^{\alpha_k} h_t^{\alpha_h} v_t^{\alpha_v} + b_t - \frac{b_{t+1}}{R_t} + \Pi_t \quad = \quad c_t + q_t k_{t+1} - q_t(1-\delta)k_t + p_v v_t \ . \tag{7}$$

Households spend the sum of production, profits of investment firms Π_t and net borrowing on consumption, capital formation and on intermediate goods. To sum up, the household's maximization problem looks as follows:

$$\max_{c_t, k_{t+1}, b_{t+1}, v_t, h_t} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - \chi \frac{h_t^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma} \right] , \tag{8}$$

s.t.

$$(\lambda) \qquad A_t k_t^{\alpha_k} h_t^{\alpha_h} v_t^{\alpha_v} + b_t - \frac{b_{t+1}}{R_t} + \Pi_t \quad = \quad c_t + q_t k_{t+1} - q_t(1-\delta)k_t + p_v v_t \ , \tag{Budget constraint}$$

[13] Borrowing is, however, constrained by the collateral constraint below.

[14] The constraint realistically reflects the fact that a significant part of corporate (bank) borrowing is asset-based (Ivashina et al. 2022).

$$(\mu) \qquad \frac{b_{t+1}}{R_t} - \theta p_v v_t \quad \geq \quad -\kappa_t q_t k_t \,. \qquad \text{(Collateral constraint)}$$

The first-order conditions of the household will be stated and interpreted in the next subsection. In equilibrium, the capital market has to clear so that $i_{F,t} = k_{t+1} - (1-\delta)k_t \ \forall\ t$ must hold. Furthermore, the final goods market must clear, leading to the following resource constraint:

$$A_t k_t^{\alpha_k} v_t^{\alpha_v} h_t^{\alpha_h} + b_t \quad = \quad \frac{b_{t+1}}{R_t} + c_t + k_{t+1} - (1-\delta)k_t + \frac{a}{2}(k_{t+1} - (1-\delta)k_t - \bar{i})^2 + p_v v_t \,. \tag{9}$$

3.2 Decentralized Equilibrium

The decentralized competitive equilibrium without regulation is defined as follows:

Definition 1 *A competitive equilibrium consists of a set of sequences* $\{c_t, h_t, v_t, k_{t+1}, b_{t+1}, q_t, \mu_t\}_{t=0}^{\infty}$ *satisfying*

$$\begin{aligned}
\left(c_t - \chi\frac{h_t^{\omega}}{\omega}\right)^{-\sigma} &= \beta R_t \mathbb{E}_t\left[\left(c_{t+1} - \chi\frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\right] + \mu_t \,, && (10)\\
\left(c_t - \chi\frac{h_t^{\omega}}{\omega}\right)^{-\sigma} q_t &= \beta \mathbb{E}_t\Bigg[\left(c_{t+1} - \chi\frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\Big(q_{t+1}(1-\delta) + \ldots \\
&\quad \alpha_k A_{t+1} k_{t+1}^{\alpha_k - 1} v_{t+1}^{\alpha_v} h_{t+1}^{\alpha_h}\Big) + \mu_{t+1} q_{t+1} \kappa_{t+1}\Bigg] \,, && (11)\\
\left(c_t - \chi\frac{h_t^{\omega}}{\omega}\right)^{-\sigma} p_v + \mu_t \theta p_v &= \left(c_t - \chi\frac{h_t^{\omega}}{\omega}\right)^{-\sigma} \alpha_v A_t k_t^{\alpha_k} v_t^{\alpha_v - 1} h_t^{\alpha_h} \,, && (12)\\
\chi h_t^{\omega - 1} &= \alpha_h A_t k_t^{\alpha_k} h_t^{\alpha_h - 1} v_t^{\alpha_v} \,, && (13)\\
1 + a\left(k_{t+1} - (1-\delta)k_t - \bar{i}\right) &= q_t \,, && (14)\\
A_t k_t^{\alpha_k} v_t^{\alpha_v} h_t^{\alpha_h} + b_t &= c_t + p_v v_t + k_{t+1} - (1-\delta)k_t + \frac{b_{t+1}}{R_t} + \ldots \\
&\quad \frac{a}{2}(k_{t+1} - (1-\delta)k_t - \bar{i})^2 \,, && (15)\\
\frac{b_{t+1}}{R_t} - \theta p_v v_t &\geq -\kappa_t q_t k_t \,, && (16)\\
\mu_t &\geq 0 \,, && (17)\\
\mu_t\left(\frac{b_{t+1}}{R_t} + \kappa_t q_t k_t - \theta p_v v_t\right) &= 0 \,, && (18)
\end{aligned}$$

given $\{A_t, R_t, \kappa_t\}_{t=0}^{\infty}$*,* b_0*,* k_0*, and the associated transversality conditions.*

I have used the capital market clearing condition to replace $i_{F,t}$ above. As already mentioned before, the model of Bianchi and Mendoza 2018 is nested in my model. For the following choice of parameters investment goes to zero, capital is fixed and the decentralized competitive equilibrium is the same as in Bianchi and Mendoza 2018: $\delta = 0$, $a \to +\infty$ and $\bar{i} = 0$.

Equations (10)-(13) are the optimality conditions of the household and equation (14) is the optimality condition of the investment firm. The left-hand side of these equations represents the cost of increasing the respective variable, while the right-hand side depicts the benefit from an increase.

Equation (10) is the Euler equation in which the household takes into account that less borrowing makes the collateral constraint less binding which is relevant when the collateral constraint is binding, i.e. $\mu_t > 0$. Equation (11) describes the household's capital decision: the cost of one additional unit of capital today is its price q_t, the benefit from one additional unit is that more depreciated capital can be sold tomorrow at price q_{t+1}, more is produced and more collateral is available, which is relevant if the constraint is binding in the subsequent period. Equation (12) is the optimal decision with regard to the intermediate good: on the one hand, it costs p_v and makes the collateral constraint tighter because more working capital is needed, on the other hand, production depends positively on the intermediate good. Equation (13) represents the optimal labor decision. One unit more labor means less leisure and therefore less utility, but also more production. Equation (14) is the optimal investment decision of investment firms. From their perspective more investment increases costs in terms of the final good, influences adjustment costs and increases revenues. Equation (15) represents the resource constraint, equation (16) is the collateral constraint and equations (17) to (18) are the remaining Karush-Kuhn-Tucker conditions.

4 Optimal Time-consistent Policy

In this section, I first define and qualitatively analyze the Markov perfect constrained efficient equilibrium, where the social planner chooses borrowing on behalf of agents. This means that he/she maximizes the household's utility while not being constrained by the household's first-order condition on borrowing but by all other conditions of the competitive equilibrium. As Bianchi and Mendoza 2018 show, there is a time consistency problem under commitment in models of the type used in this paper, because the social planner tends to announce low future consumption when the constraint is binding, which is no longer optimal ex post. Therefore, like most other related studies I focus on optimal time-consistent policies. Time consistency is ensured by solving for the Markov perfect equilibrium. For given policy functions of the future planner, the current planner chooses optimal values for all control variables and the subsequent states, taking into account that the choice of the subsequent states, i.e. bonds and capital, will change the future planner's choice of control variables. A Markov perfect equilibrium prevails when the policy functions of the current planner and those of the future planner are identical. This means that the future planner and the current planner will make the same choices if the states they are in are the same.

Furthermore, I analyze the state-contingent Pigouvian debt tax/subsidy, which decentralizes the resulting allocation, and compare it to the tax that prevails in an economy with fixed aggregate capital. Finally, I analyze the constrained efficient allocation of the reference case, i.e. the case where a social planner chooses borrowing and investment on behalf of agents. The resulting allocation can be decentralized with a state-contingent Pigouvian debt tax/subsidy and a state-contingent Pigouvian investment tax/subsidy. From here on, the notation changes to the recursive notation with $'$ denoting the subsequent state of the state variables.

4.1 Social Planner Problem

The social planner, who can choose borrowing on behalf of the representative household, maximizes the household's utility subject to the resource constraint, the collateral constraint and two implementability constraints that arise from the choice of capital by households and from the investment choice of the investment firm. In the Appendix I show that this reduced problem is equivalent to the maximization problem which also contains the optimality conditions with respect to the intermediate good v and labor h as well as the Karush-Kuhn-Tucker conditions of problem (8) as implementability constraints.

The constrained social planner's optimization problem can be summarized as follows:

$$\mathcal{V}(b,k,s) = \max_{c,k',b',q,v} \frac{(c-\chi\frac{h^\omega}{\omega})^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',k',s') \; , \tag{19}$$

s.t.

$$(\lambda^{SP}) \qquad Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} = \frac{b'}{R} - b + c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 + p_v v \; ,$$

(Resource constraint)

$$(\mu^{SP}) \qquad \frac{b'}{R} - \theta p_v v \geq -\kappa qk \; , \qquad \text{(Collateral constraint)}$$

$$(\xi) \qquad q\left(c-\chi\frac{h^\omega}{\omega}\right)^{-\sigma} = \beta\mathbb{E}_{s'|s}\Big[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^\omega}{\omega}\right)^{-\sigma} \dots$$

$$\Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A'k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\dots$$

$$\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\Big] + \beta\mathbb{E}_{s'|s}\Big[\boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\Big] \,,$$

(Household capital decision)

$$(\gamma) \qquad q = 1 + a(k' - (1-\delta)k - \bar{i}) \,. \qquad \text{(Firm investment decision)}$$

In order to facilitate notation, the following shortcuts of the current planner's policy functions have been used above: $b' = \mathcal{B}(b,k,s)$, $k' = \mathcal{K}(b,k,s)$, $q = \mathcal{Q}(b,k,s)$, $c = \mathcal{C}(b,k,s)$, $v = \boldsymbol{v}(b,k,s)$ and $h = \boldsymbol{h}(b,k,s)$. The subscript "$fp$" indicates that the respective policy function is a policy function of the future planner, which the current planner takes as given. There is an important difference between $\boldsymbol{\mu}/\boldsymbol{\mu}_{fp}$ and $\boldsymbol{\mu}^{SP}$: $\boldsymbol{\mu}/\boldsymbol{\mu}_{fp}$ denotes the household's multiplier of the collateral constraint under the optimal choices of the current/future social planner, which is defined by (12). $\boldsymbol{\mu}^{SP}$, however, denotes the social multiplier of the collateral constraint, which reflects the social benefit of marginally relaxing the collateral constraint.

Definition 2 *The Markov perfect constrained efficient equilibrium is defined by the policy functions $\mathcal{B}(b,k,s)$, $\mathcal{K}(b,k,s)$, $\mathcal{Q}(b,k,s)$, $\mathcal{C}(b,k,s)$, $\boldsymbol{v}(b,k,s)$, $\boldsymbol{\mu}(b,k,s)$,[15] $\boldsymbol{h}(b,k,s)$ and the value function $\mathcal{V}(b,k,s)$, which, first, solve the social planner optimization problem (19) and, second, are equal to the future planner's policy functions: $\mathcal{B}(b,k,s) = \mathcal{B}_{fp}(b,k,s)$, $\mathcal{K}(b,k,s) = \mathcal{K}_{fp}(b,k,s)$, $\mathcal{Q}(b,k,s) = \mathcal{Q}_{fp}(b,k,s)$, $\mathcal{C}(b,k,s) = \mathcal{C}_{fp}(b,k,s)$, $\boldsymbol{v}(b,k,s) = \boldsymbol{v}_{fp}(b,k,s)$, $\boldsymbol{\mu}(b,k,s) = \boldsymbol{\mu}_{fp}(b,k,s)$ and $\boldsymbol{h}(b,k,s) = \boldsymbol{h}_{fp}(b,k,s)$.*

The optimization problem leads to the following first-order conditions after applying the envelope theorem:

$$\lambda^{SP} = \left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma} + \xi\sigma\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma-1} q \,, \tag{20}$$

$$\lambda^{SP} = \beta R\mathbb{E}_{s'|s}\left[\lambda^{SP'}\right] + \mu^{SP} + \beta R\mathbb{E}_{s'|s}[\xi\Omega'] \,, \tag{21}$$

$$\begin{aligned} \lambda^{SP}(1 + a(k' - (1-\delta)k - \bar{i})) &= \beta\mathbb{E}_{s'|s}\Big[\lambda^{SP'}\Big(\alpha_k A' k'^{\alpha_k - 1} v'^{\alpha_v} h'^{\alpha_h} \ldots \\ &\quad + (1-\delta)(1 + a(k'' - (1-\delta)k' - \bar{i}))\Big)\ldots \\ &\quad + \mu^{SP'} q'\kappa'\Big] + \gamma a + \beta\mathbb{E}_{s'|s}[\xi\Gamma' - \gamma'(1-\delta)a] \,, \end{aligned} \tag{22}$$

$$\chi h^{\omega-1}\left(c - \chi\frac{h^\omega}{\omega}\right)^{-\sigma} = \lambda^{SP}\alpha_h A k^{\alpha_k} h^{\alpha_h - 1} v^{\alpha_v} \ldots$$

[15] The household's multiplier on the collateral constraint is defined by equation (12) and is not binding for the social planner (see Appendix).

$$
\begin{aligned}
& - \xi\chi h^{\omega-1}\sigma\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1} q \ , && (23)\\
\mu^{SP}\kappa k &= \xi\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} + \gamma \ , && (24)\\
Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} - \frac{b'}{R} + b &= c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 - p_v v \ , && (25)\\
\lambda^{SP}p_v &= \lambda^{SP}\alpha_v A k^{\alpha_k}v^{\alpha_v-1}h^{\alpha_h} - \mu^{SP}\theta p_v \ , && (26)\\
\frac{b'}{R} - \theta p_v v &\geq -\kappa q k \ , && (27)\\
q\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} &= \beta\mathbb{E}_{s'|s}\Bigg[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma} \ldots \\
& \Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\ldots \\
& \boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big) + \boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\Bigg] \ , && (28)\\
q &= 1 + a(k'-(1-\delta)k-\bar{i}) \ , && (29)\\
\mu^{SP} &\geq 0 \ , && (30)\\
0 &= \mu^{SP}\left(\frac{b'}{R} - \theta p_v v + \kappa q k\right) \ . && (31)
\end{aligned}
$$

Ω captures the effects of the current planner's bond decision b' on the future planner's decisions and thus on the first implementability constraint, which is taken into account by the current planner:

$$
\begin{aligned}
\Omega' = & -\sigma\left(\mathcal{C}_{fp,b}(b',k',s') - \chi\boldsymbol{h}_{fp,b}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\omega-1}\right)\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma-1}\ldots \\
& \Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A'k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\ldots \\
& + \left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma}\Bigg((1-\delta)\mathcal{Q}_{fp,b}(b',k',s')\ldots \\
& + \alpha_k A'k'^{\alpha_k-1}\Big(\alpha_v\boldsymbol{v}_{fp,b}(b',k',s')\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v-1}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\ldots \\
& + \alpha_h\boldsymbol{h}_{fp,b}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\Big)\Bigg)\ldots \\
& + \mathcal{Q}_{fp,b}(b',k',s')\boldsymbol{\mu}(b',k',s')\kappa' + \mathcal{Q}_{fp}(b',k',s')\boldsymbol{\mu}_b(b',k',s')\kappa' \ . \qquad (32)
\end{aligned}
$$

Analogously, Γ captures the effect of the current capital decision k' on the future planner's decisions and thus on the first implementability constraint:

$$
\begin{aligned}
\Gamma' = & -\sigma\left(\mathcal{C}_{fp,k}(b',k',s') - \chi\boldsymbol{h}_{fp,k}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\omega-1}\right)\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma-1}\ldots \\
& \Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A'k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\ldots \\
& + \left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma}\ldots \\
& \Bigg((1-\delta)\mathcal{Q}_{fp,k}(b',k',s') + \alpha_k A'k'^{\alpha_k-1}\Big(\alpha_v\boldsymbol{v}_{fp,k}(b',k',s')\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v-1}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\ldots
\end{aligned}
$$

$$
\begin{aligned}
&+ \alpha_h \boldsymbol{h}_{fp,k}(b', k', s')\boldsymbol{h}_{fp}(b', k', s')^{\alpha_h - 1}\boldsymbol{v}_{fp}(b', k', s')^{\alpha_v}\Big)...\\
&+ \alpha_k(\alpha_k - 1)A' k'^{\alpha_k - 2}\boldsymbol{v}_{fp}(b', k', s')^{\alpha_v}\boldsymbol{h}_{fp}(b', k', s')^{\alpha_h}\Bigg)...\\
&+ \mathcal{Q}_{fp,k}(b', k', s')\boldsymbol{\mu}_{fp}(b', k', s')\kappa' + \mathcal{Q}_{fp}(b', k', s')\boldsymbol{\mu}_{fp,k}(b', k', s')\kappa' \ .
\end{aligned} \tag{33}
$$

There are two dimensions of a pecuniary externalitiy with regard to the price of capital in the model: First, households do not take into account in their borrowing choice that in equilibrium the price of capital adapts to their consumption choice. This externality is the force which makes an intervention desirable in related studies with a fixed level of aggregate capital, e.g. Bianchi and Mendoza 2018 and Jeanne and Korinek 2020. Second, households also do not take into account that the capital they demand changes the firm's costs and therefore the price of capital. This additional channel is novel compared to related literature with a fixed level of aggregate capital. To sum up, if the collateral constraint is not binding, households do take into account that additional capital might be helpful for borrowing within the next period when the constraint may be binding. However, they do not take into account how their choices of borrowing and capital influence the price of capital.

The most important equations for the interpretation of the results in the next sections are equations (20)-(22) and equation (24). Equation (20) is the first-order condition on consumption. The second term on the right-hand side relates to the dimension of the pecuniary externality that is driven by consumption and indicates that the social planner takes the effect of higher consumption on the first implementability constraint (Household capital decision) into account: a higher level of consumption reduces the marginal utility of consumption and therefore reduces the cost of capital in utility terms for households. This, however, increases capital demand and the price of capital as the Household capital decision reveals.

Equation (21) displays the optimal decision with regard to bonds. There are two differences compared to the laissez-faire optimality condition: First, the size of the social multipliers (in general) differs from the ones from section 3. Importantly, the multipliers λ^{SP} and $\lambda^{SP'}$ differ from λ and $\lambda^{'}$, as the social planner's first order condition with regard to consumption differs from the one of the household. Thus, the social planner takes into account in his intertemporal decision that borrowing influences the price of capital today and tomorrow via the household capital decision. Second, the social planner incorporates into his/her decision that the choice of b' affects the future planner's choices of c, h, q, v and μ, and thus the price of capital via the first implementability constraint.

Equation (22) is the first-order condition on capital. Apart from the different multipliers compared to the laissez-faire, the equation reveals that the current planner takes into account that k' influences the price of capital via the second implementability constraint (Firm investment decision) both today and tomorrow. This relates to the second dimension of the pecuniary externality. Moreover, it is incorporated that the choice of k' influences the future planner's decisions regarding c, h, q, v and μ and thus the first implementability constraint.

Finally, equation (24), which is the first-order condition on the price of capital, shows that the multiplier of the collateral constraint and of the two implementability constraints are directly linked. This is the case since on the one hand, a higher price of capital q makes the collateral constraint less binding and on the other hand, the planner has to take into account the social cost of reaching this price on the capital market.

4.2 Optimal Debt Tax

The constrained efficient equilibrium can be decentralized with a tax on debt τ_b^{SP1}. A positive debt tax reduces the funds the household receives for a given amount of newly issued debt. The revenues from the tax are remitted to the households via a lump-sum transfer T.[16] Consequently, the household's budget constraint changes to:

$$c + qk'_H + \frac{b'}{(1+\tau_b^{SP1})R} + p_v v \quad = \quad Ak^{\alpha_k} h^{\alpha_h} v^{\alpha_v} + q(1-\delta)k + b + \Pi + T \ .$$

The expression for the optimal state-contingent debt tax is as follows:

$$\tau_b^{SP1} \quad = \quad \frac{\beta R \mathbb{E}_{s'|s}\left[\sigma\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma-1}\xi' q' + \xi\Omega'\right] - \sigma\left(c - \chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1}\xi q + \mu^{SP} - \mu}{\beta R \mathbb{E}_{s'|s}\left[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\right]} \tag{34}$$

where ξ is defined as

$$\xi \quad = \quad \left(c - \chi\frac{h^{\omega}}{\omega}\right)^{\sigma}\left(\mu^{SP}\kappa k - \gamma\right) \ . \tag{35}$$

Combining the two equations above gives an expression for the tax, which is easier to interpret:

$$\begin{aligned}\tau_b^{SP1} \quad = \quad & \frac{\mathbb{E}_{s'|s}\left[\sigma\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-1}\left(\mu^{SP'}\kappa' k' - \gamma'\right)q' + \left(c - \chi\frac{h^{\omega}}{\omega}\right)^{\sigma}\left(\mu^{SP}\kappa k - \gamma\right)\Omega'\right]}{\mathbb{E}_{s'|s}\left[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\right]} \dots \\ & + \frac{-\sigma\left(c - \chi\frac{h^{\omega}}{\omega}\right)^{-1}\left(\mu^{SP}\kappa k - \gamma\right)q + \mu^{SP} - \mu}{\beta R \mathbb{E}_{s'|s}\left[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\right]} \ .\end{aligned} \tag{36}$$

[16] In the case of a subsidy, a lump-sum tax is levied.

Interestingly, the expression of the optimal tax in equation (34) is the same as in Bianchi and Mendoza 2018 and Biljanovska and Vardoulakis 2024, where capital is fixed. This is the case because the same externality on the pricing condition (28) is internalized by the social planner: whether there is endogenous capital formation or not, households do not take into account that their borrowing affects the price of capital via the asset pricing equation of capital. The impact on utility of slightly relaxing the first implementability constraint, i.e. the asset pricing equation of capital, is measured by ξ. Hence, also in the economy with endogenous capital formation it holds: if $\mu^{SP} - \mu$ as well as Ω' were zero, the social planner would impose a tax in case the benefits from marginally relaxing the first implementability constraint tomorrow is larger than today.

However, the benefits and cost from marginally relaxing the first implementability constraint differ from an economy with a fixed level of aggregate capital because in an economy with endogenous capital formation the multiplier of the asset pricing condition of capital, ξ, does not solely depend on the collateral constraint multiplier μ^{SP}, as can be seen from equation (35). There is also a second relevant component measured by the multiplier on the optimality condition of the investment firm, γ.

$\mu^{SP}\kappa k$ in equation (35) measures the benefit of a higher capital price induced by a less strongly binding first implementability constraint. Less borrowing and therefore more consumption decrease the marginal utility of consumption, making capital cheaper in utility terms for households. This, however, implies a higher price of collateral via the first implementability constraint, which makes the collateral constraint less binding, as measured by $\mu^{SP}\kappa k$. The multiplier γ, however, is determined by equation (22) and measures the cost/benefit of the distortion of capital formation initiated by a higher price of capital. If the multipliers γ and μ^{SP} were zero in the current state and γ' and $\mu^{SP'}$ were positive, the ex ante tax would be smaller than in an economy with fixed capital due to the effect of regulation on capital formation.

Analytically, the sign of γ is not clear. The quantitative analysis (see section 5) leads to the following conclusions: When the collateral constraint is binding, γ is negative in 8 states and positive in 9669 states. Moreover, it holds that the lower the capital level and the higher the initial debt, the higher is γ. When the constraint is not binding, γ is negative for most states but always close to zero. To sum up, the distortion of capital formation tends to reduce the optimal debt tax when the collateral constraint is not binding.

4.3 Reference Case

In the reference case, i.e. the constrained efficient allocation where the social planner chooses borrowing and investment on behalf of agents, the allocation can be decentralized with a Pigouvian debt tax/subsidy and a Pigouvian investment tax/subsidy. The planner's optimization problem is available in the Appendix.

The debt tax is implemented as in the case where the social planner only chooses borrowing optimally. The investment tax is levied on the costs in terms of the final good incurred by investment firms. A positive investment tax τ_i^{SP2} increases firms' cost and leads c.p. to lower investment. Consequently, the profit function of the investment firm changes to:

$$\Pi = qi_F - (1+\tau_i^{SP2})i_F - \frac{a}{2}(i_F - \bar{i})^2 \, .$$

The constrained efficient allocation implies the following investment tax/subsidy:

$$\tau_i^{SP2} = q - \frac{\beta \mathbb{E}_{s'|s}\left[\lambda^{SP2'}\left(\alpha_k A' k'^{\alpha_k - 1} v'^{\alpha_v} h'^{\alpha_h} + (1-\delta)(1 + a(k'' - (1-\delta)k' - \bar{i}))\right) + \mu^{SP2'} q' \kappa' + \xi^{SP2}\Gamma'\right]}{\lambda^{SP2}} \, . \tag{37}$$

The right part of the expression is the price that would make firms choose the socially optimal investment level, from here on called q^I. Investment is taxed if the actual price of capital is higher than q^I and subsidized if the price of capital is lower than q^I. Hence, the tax/subsidy changes the cost structure of firms so that they always choose the socially optimal investment level irrespective of the level of the capital price q.

Opposed to the case where the social planner only chooses borrowing on agents' behalf, increasing the capital price by relaxing the first implementability constraint in the reference case does not come at the cost of a distortion of investment. Still, a higher capital price makes the collateral constraint less binding so that ξ^{SP2} is defined as follows (via the planner's first order condition on q):

$$\xi^{SP2} = \left(c - \chi\frac{h^\omega}{\omega}\right)^\sigma \left(\mu^{SP2}\kappa k\right) \, . \tag{38}$$

For macroprudential policy this implies that there are no costs due to distorted investment any longer and the expression for the optimal debt tax/subsidy is the same as in an economy with a fixed level of aggregate capital:

$$\tau_b^{SP2} = \frac{\beta R \mathbb{E}_{s'|s}\left[\sigma\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma-1}\xi^{SP2'}q' + \xi^{SP2}\Omega'\right] - \sigma\left(c - \chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1}\xi^{SP2}q + \mu^{SP2} - \mu}{\beta R \mathbb{E}_{s'|s}\left[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\right]} \tag{39}$$

$$\Leftrightarrow = \frac{\beta R \mathbb{E}_{s'|s}\left[\sigma\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-1}\mu^{SP2'}\kappa' k' q' + \mu^{SP2}\kappa k \Omega'\right] - \sigma\left(c - \chi\frac{h^{\omega}}{\omega}\right)^{-1}\mu^{SP2}\kappa k q + \mu^{SP2} - \mu}{\beta R \mathbb{E}_{s'|s}\left[\left(c' - \chi\frac{h'^{\omega}}{\omega}\right)^{-\sigma}\right]}. \tag{40}$$

Thus, the interpretation of the tax is the same as in an economy with a fixed level of aggregate capital (e.g. Bianchi and Mendoza 2018 or Biljanovska and Vardoulakis 2024): If the collateral constraint is not binding today, i.e. $\mu^{SP2} = 0$, the ex ante tax is positive, so that households borrow less, which increases the price of capital and the value of collateral in the subsequent period. To sum up, due to the availability of an investment tax/subsidy, a change in the price of capital due to macroprudential policy no longer has a distorting effect on capital formation.

5 Quantitative Analysis

In this section, I present a quantitative analysis of the decentralized equilibrium without regulation and of optimal policy. First, I describe the calibration of the model and briefly summarize the structure of the algorithms for laissez-faire and optimal policy. It is then shown that the model is able to generate financial crises that are consistent with related studies. Finally, I present and interpret the quantitative results. A description of the data that has been used to compute the targets can be found in the Appendix.

5.1 Calibration and Algorithms

I follow the calibration strategy of Bianchi and Mendoza 2018 and calibrate my model to the OECD member countries between 1984 and 2012. As in their calibration, data from all 34 OECD members (as of 2012) were used and aggregated. To calculate the targets, the individual statistics were weighted by the 2012 real GDP in purchasing power standards. The focus of my calibration was, on the one hand, to share many parameters/targets with Bianchi and Mendoza 2018 and, on the other hand, to match many moments related to investment. The parameters were determined partly by simulation and partly ex ante. In particular, those parameters that are difficult to observe but have a direct impact on certain statistics that can be more easily observed were chosen to be determined by simulation.

I set the constant relative risk aversion coefficient to one, which is standard. The shares of inputs and labor in gross output are taken from Bianchi and Mendoza 2018.[17] I choose a value of 0.198 for the capital share so that the exponents of the production function sum to one, i.e. there are constant returns to scale. This implies a capital share in GDP of 0.36, since GDP is defined as $F(k_t, h_t, v_t) - p_v v$. The labor disutility coefficient is normalized so that labor is equal to one third in the deterministic steady state without collateral constraint. For the parameter ω, which determines the Frisch elasticity $\frac{1}{\omega-1}$, I choose a value that is standard in macroeconomics. For the working capital coefficient I pick the same value as Bianchi and Mendoza, who compute it by using data from the US Flow of Funds data set. The logged interest rate follows the AR(1) process $\ln(R_t) = (1-\rho_R)\bar{R} + \rho_R \ln(R_{t-1}) + \varsigma_t$ with $\varsigma_t \sim N(0, \sigma_\varsigma)$. For the parameters of this process I follow Bianchi and Mendoza 2018.

The two possible realizations of κ, which determine the maximum loan-to-value ratio (LTV), are not easy to identify at the macroeconomic level. In this paper, capital is defined as total fixed capital. Because LTVs differ between debt contracts backed by different types of fixed assets, and because data on debt contracts tend to be highly selective, it is difficult to obtain an average value for the economy as a whole. However, a recent study by Kermani and Ma 2022 uses data on the liquidation value of assets on corporate balance sheets in the U.S. and finds that the average liquidation value of property, plant and equipment is 35% of its book value. This is a good approximation for κ, because it is thought to reflect the expected sale price of the collateral if the borrower is unable to repay. I therefore set the loan-to-value ratio in normal times κ_H to 0.35.[18] To determine the value in crisis periods, I apply the same relative reduction of κ as in Bianchi and Mendoza 2018.

There are eight parameters that are determined by simulation. The targets are mainly thought to represent several important moments of investment of the OECD member countries. The discount factor is chosen so that the ratio of capital to GDP in the stochastic steady state is equal to the capital-to-GDP ratio of OECD members of 2.89. Productivity A_t is defined as follows:

$$A_t = e^{z_t} , \tag{41}$$

$$z_t = \bar{z} + \rho_z z_{t-1} + \epsilon_t , \tag{42}$$

[17] For a detailed description of how these values are calculated I refer to their paper.

[18] Bianchi and Mendoza 2018 use higher values, since their price of capital is substantially lower than one in steady state.

$$\epsilon_t \sim N(0, \sigma_\epsilon) \ . \tag{43}$$

The mean of $\bar{z}$ is set to zero so that the mean of A is 1. To match the observed autocorrelation of GDP of 0.68, ρ_z is set to 0.52. The standard deviation σ_ϵ equals 2.25% to match the ratio of the standard deviation of investment to the standard deviation of GDP, which is 2.8 on average for OECD countries.

The parameter $\bar{i}$, which determines the level of investment at which adjustment costs are zero and the price of capital is 1, is set to the mean of investment in the stochastic steady state, which is 0.0248. As the price of capital, q, is also the price of investment in my model, the sensitivity of q to deviations of investment from $\bar{i}$ is equal to 6.2 in my calibration to match the standard deviation of the OECD investment price. Since the relative price of investment is not directly observable, I follow the method of studies on the relative price of capital (e.g. Lian et al. 2020) and compute it as the ratio of the investment price level to the consumption price level. This parameterization leads to an average adjustment cost relative to GDP of 0.13% under laissez-faire.

The transition probabilities of the loan-to-value ratio κ are set to match a crisis probability of 4% and an average crisis duration of one year. These targets as well as the definition of crises are taken from Bianchi and Mendoza 2018: "A financial crisis is defined as an event in which the linearly detrended current account is above two standard deviations from its mean".

Since there is evidence that local solution methods are imprecise for the type of model used in this paper (Groot et al. 2023), I use a global solution method for both laissez-faire and the social planner problems. In particular, I use fixed-point iteration on the model's Euler equations to compute policy functions on a discrete grid $b \times k \times z \times R \times \kappa$. The grid size is $60 \times 30 \times 3 \times 3 \times 2$. So there are 32400 states. For values that do not lie on the grid I use bilinear interpolation. In the case of laissez-faire, the algorithm is a modified version of the "FiPit-algorithm" by Mendoza and Villalvazo 2020, which was designed to solve the model of Mendoza 2010 very fast. I adapt the algorithm to the equations and shocks of my model. For the optimal policy problems I use two nested fixed-point algorithms, which differ from other optimal macroprudential policy papers in the solution method I use in the inner loop. In the outer loop I update the policy functions of the future planner. In the inner loop I solve for the policy functions given the policy functions of the future planner by using a fixed-point algorithm. A detailed description of the three algorithms can be found in the Appendix.

Table 1: Calibration

Parameter	Description	Value	Source/Target
σ	Risk aversion	1	Standard
α_v	Share of inputs in gross output	0.45	Bianchi and Mendoza 2018
α_h	Share of labor in gross output	0.352	Bianchi and Mendoza 2018
α_k	Share of capital in gross output	0.198	Constant returns to scale
χ	Labor disutility coefficient	0.49	Normalization so that $h = \frac{1}{3}$ in steady state without collateral constraint
$\frac{1}{\omega-1}$	Frisch elasticity	1	Standard
θ	Working capital coefficient	0.16	Bianchi and Mendoza 2018
κ_H	Normal credit regime	0.35	Average liquidation value of fixed assets (Kermani and Ma 2022)
κ_L	Tight credit regime	0.29	Procentual reduction of LTV observed for housing
$\bar{R}$	Mean of interest rate process	1.1%	Bianchi and Mendoza 2018
ρ_R	Autocorrelation of interest rate process	0.68	Bianchi and Mendoza 2018
σ_ς	Conditional SD of interest rate process	1.38 %	Bianchi and Mendoza 2018
Parameters determined by simulation			
β	Discount factor	0.97	Ratio of capital to GDP = 2.89
a	Adjustment cost parameter I	6.2	SD of investment price = 0.04
$\bar{i}$	Adjustment cost parameter II	0.0248	Mean of investment (stochastic steady state)
$\bar{z}$	Mean of TFP process	0	Normalization
ρ_z	Autocorrelation of TFP process	0.52	Autocorrelation of GDP of 0.68
σ_ϵ	Conditional SD of TFP process	2.25%	Ratio of SD of investment and SD of GDP = 2.8
$P_{H,L}$	Transition probability κ_H to κ_L	0.06	Crises probability of 4 %
$P_{L,L}$	Transition probability κ_L to κ_L	0	Average crises duration of 1 year

5.2 Crisis Events under Laissez-faire

To depict the dynamics around financial crises, I first simulated the laissez-faire economy for 100,000 periods and identified all events as a financial crisis where the current account is two standard deviations above its mean. Then, for each crisis event and for several variables, I created a time series from 5 years before to four years after the crisis. Finally, I computed for each period the mean value of all events. Figure 1 shows the dynamics around crises which occur in $T = 0$. All values are in levels - except for R and TFP which are in percentage deviations from the long run mean. The dotted line in each subfigure displays the long-run mean of the respective variable.

The credit-to-GDP ratio drops in a crisis because the collateral constraint is binding, the loan-to-value ratio is lower than usual due to a bad realization of the financial shock and the price of collateral is low due to the financial amplification mechanism. The fall in investment in a crisis is much stronger than the one in consumption. As the financial shock, κ, is at the low realization for only one period, the loan-to-value ratio will increase again in the next period and more will be borrowed and consumed. Therefore, the future marginal utility is quite low compared to today, making the purchase of capital expensive in utility terms. For that reason, households decide to buy less new capital and the investment level is low. Moreover, the sum of investment, consumption, and spending on intermediate goods must fall as lower GDP and less borrowing lead to a reduction in available funds. The current account increases strongly during a crisis as this is how a crisis is defined. Like in Mendoza 2010, crises in my model occur when the already low interest rate has decreased ex ante, stimulating overborrowing, and then rises again. Furthermore, in crisis periods the financial shock κ is at the lower level, while in the preceding periods it was at the high realization. The price of capital declines up to a level of 0.9, which is consistent with the magnitude of the decline observed in sudden stop events by Mendoza 2010. The price of capital is low because of financial amplification: since investment and consumption are low, both first-order conditions related to capital imply a lower price of capital, which further tightens the collateral constraint and Except for the financial shock, these pre-crisis and crisis dynamics are qualitatively in line with Mendoza 2010. In Mendoza's study, the third shock after the interest rate shock and the TFP shock is a shock to price of intermediate goods.

After the crisis the credit-to-GDP ratio increases because financial conditions improve. Consequently, the current account is negative. Capital and GDP are declining one period after the

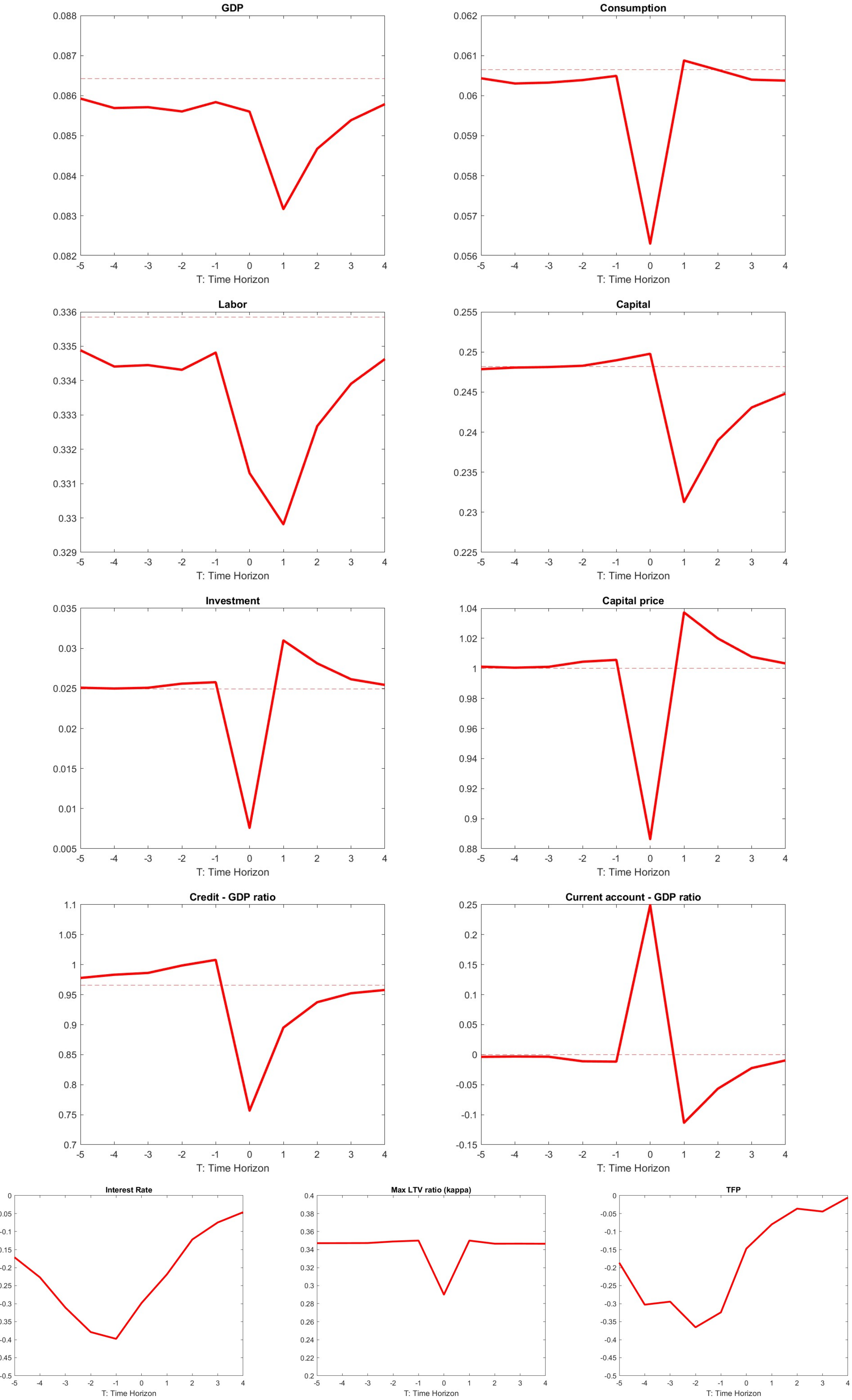

Figure 1: *Dynamics around crises (dotted line = long-run average)*

financial crisis due to the reduction of investment one period earlier. In Mendoza 2010 there is a stronger decline in GDP during the crisis and an increase in GDP in period T=1. This is due to the different crisis definition in his study, which requires not only a positive deviation of the current account from its long-run mean, but also a negative deviation of GDP from its mean. When I use the same crisis definition as Mendoza, the GDP dynamics are again in line. Labor shrinks in a crisis because the low demand for intermediate goods due to tight financing conditions reduces the return to labor. In the post-crisis period, the low level of capital induced by the low level of investment in $T = 0$ further reduces the return to labor and thus the hours worked. Investment rises strongly in the first period after the crisis because TFP has risen and the current level of capital is low, implying a high marginal product of capital.

Compared to Bianchi and Mendoza 2018, who show the crisis dynamics for the credit-to-GDP ratio, the price of capital, output, consumption and shocks, the figures look almost the same, except for production and the interest rate. For production, Bianchi and Mendoza do not observe a fall in the post-crisis period because capital is fixed. For the interest rate, the level hardly changes after the crisis period, while in my simulation the interest rate continues to rise on average after the crisis. To sum up, the model produces strong financial crises under laissez-faire that are in line with related studies.

5.3 Results for Optimal Policy

In this subsection, the constrained efficient equilibrium and the Pigouvian debt tax which replicates this equilibrium as a competitive equilibrium will be described and interpreted. First, the optimal state-contingent tax rates will be analyzed. A closer look at differences in borrowing and investment between laissez-faire and the constrained efficient equilibrium will follow. Furthermore, summary statistics will be presented and crisis dynamics under laissez-faire will be compared to those under constrained efficiency. Finally, the quantitative results of the reference case will be analyzed.

5.3.1 Optimal Taxes

Figure 2 shows the optimal debt tax when the collateral constraint is not binding (ex ante tax) and figure 3 shows the optimal debt tax when the collateral constraint is binding (ex post tax). I plot the ex ante tax for intermediate values of the interest rate and the productivity shock and a high realization of the financial shock κ. For the ex post tax I choose the same values of the interest rate and the productivity shock but a low κ.[19]

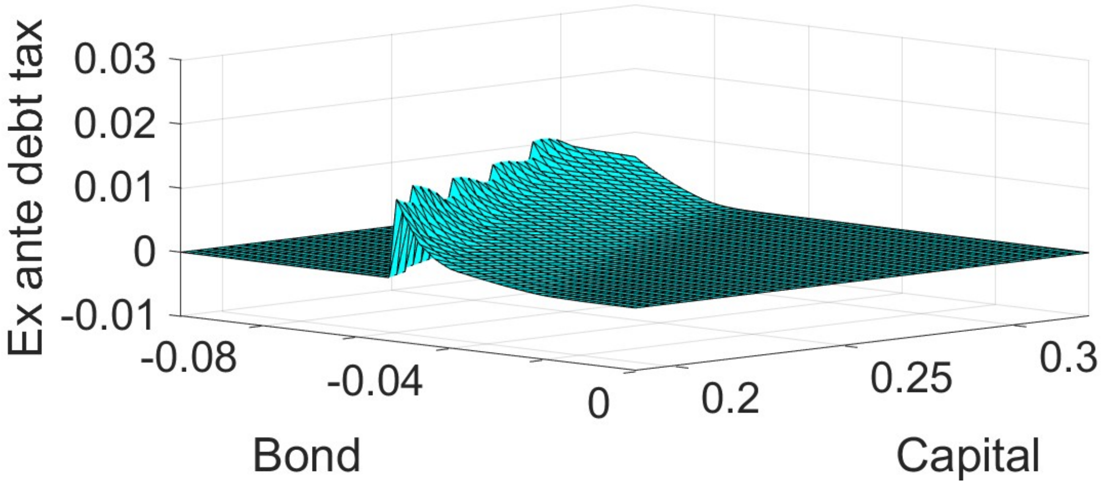

Figure 2: *Optimal Pigouvian ex ante debt tax*

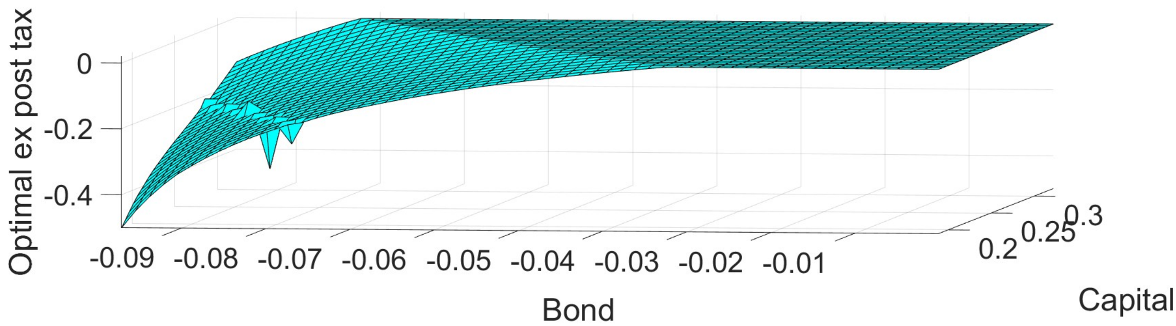

Figure 3: *Optimal Pigouvian ex post debt tax*

The optimal ex ante debt tax is zero when debt is low, i.e. the bond level is negative but close to zero, because the probability of being constrained in the subsequent state is very low. When debt is high and the constraint is not yet binding, debt is taxed by up to 1.3 percent. The closer the state is to the binding region (left part of the figure), the higher is the macroprudential tax. This is due to the high probability that the collateral constraint will be binding in the next period with the pecuniary externality having strong adverse effects. In the binding region the ex ante tax is zero, because the ex post tax is in place. It is also striking that the value of the ex ante debt tax depends on the level of capital. If the economy is in a state with a relatively high level of capital, the tax starts to be greater than zero for higher levels of debt, and the constraint starts to be binding for higher levels of debt. This is because a higher level of capital means a higher level of collateral.

[19] The figure only changes slightly when considering the same financial shock, but the probability that the constraint is binding, i.e. an ex post tax has to be imposed, is much higher when the financial shock is at the low realization.

The optimal ex post debt tax is slightly positive when the constraint is hardly binding and becomes more negative the higher the level of debt and the lower the level of capital. Since the price of capital is lower in the high debt, low capital region, the amplification effects are strong and it is optimal to increase the price of capital by reducing the amount of debt that has to be repayed via a subsidy. Households do not take into account that their consumption as well as their capital demand affect the price of capital, which means that they consume to less and demand too less capital implying a decreased collateral value. This, however, has adverse consequences, since the lower price of capital would induce an even lower level of consumption and an even lower price of capital. The social planner is aware of the amplification mechanism and subsidizes borrowing to increase aggregate expenditures and thus consumption and investment. The lower the level of debt and capital, the stronger is the reduction of the price of capital, making a very high debt subsidy (up to 50 percent at the corner of the grid) optimal.

5.3.2 Differences in Borrowing and Investment

Figures 4 and 5 show the bond and investment policy functions under laissez-faire and the constrained efficient allocation, respectively. To facilitate comparison, capital is fixed at an intermediate level. The interest rate shock and the productivity shock are also at intermediate levels, while the maximum loan-to-value ratio, κ, is high.

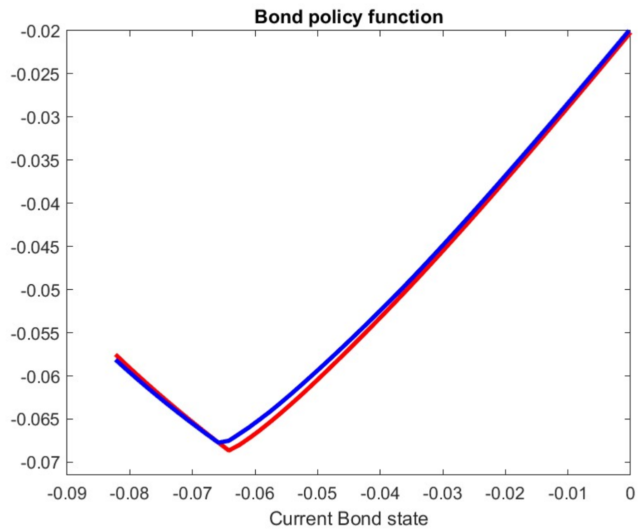

Figure 4: *Bond policy functions for a given level of capital and exogenous states: Laissez-faire (red) vs. Optimal macroprudential policy (blue)*

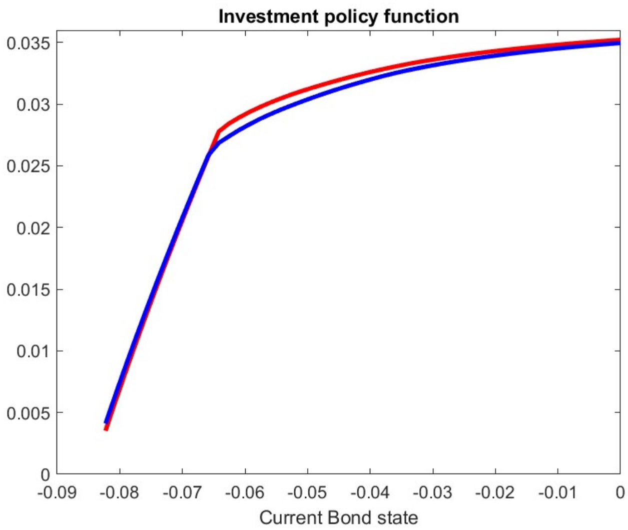

Figure 5: *Investment policy functions for a given level of capital and exogenous states: Laissez-faire (red) vs. Optimal macroprudential policy (blue)*

The social planner chooses less borrowing than under laissez-faire when the constraint is not binding. This is because the regulator internalizes the effect of high debt on the price of capital when there is a positive probability that the constraint is binding in the subsequent state. The difference increases with higher debt as long as the collateral constraint is not binding and peaks in the last state before the constraint binds, which is the point of the kink in the bond policy function. However, the difference is small because less borrowing also reduces investment. This correlation is evident in figure 5. The difference in investment between laissez-faire and the constrained efficient equilibrium also increases as one moves closer to the constrained region, meaning that investment is reduced most when debt is high and the collateral constraint is not yet binding. When the collateral constraint is binding, the social planner borrows slightly more than under laissez-faire and invests more. More borrowing under the constraint efficient allocation increases the price of capital and thus the borrowing limit, creating room for consumption and investment spending.

5.3.3 Summary Statistics and Welfare

The following table summarizes the main results of the quantitative analysis. Welfare is computed as the standard compensating consumption variations for each initial state that equates the current expected utility of laissez-faire and the economies of interest (with consumption c^*

and labor h^*). This means that welfare is computed as $\vartheta(b,k,s)$, which satisfies

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t^{LF}(1+\vartheta),h_t^{LF})\right]=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t^*,h_t^*)\right].$$

Table 2: Main Results

	Laissez-faire	**Optimal policy (Ex ante & ex post)**	**Ex ante debt tax only**
Average ex ante debt tax (in %)	–	0.53	0.52
Average ex post debt tax (in %)	–	-1.71	–
Probability of crises (in %)	4.47	3.95	4.06
Binding collateral constraint (in %)	39.7	25.62	24.28
Change of investment compared to laissez-faire (in %)	–	-0.16	-0.28
Average debt to GDP	0.836	0.82	0.815
Average capital to GDP	2.884	2.882	2.879
Average welfare gain (in %)	–	0.0025	0.0018

The optimal ex ante Pigouvian tax on debt is 0.53 percent on average, which is much lower than what other studies find (see table 3).[20] As the analytical analysis showed, the optimal ex ante tax is reduced when the multiplier of the firms' investment decision, γ, is positive in the subsequent state. Indeed, when the collateral constraint is binding, γ is positive, indicating that in a state where the collateral constraint is not binding but there is a positive probability that it will be binding in the subsequent state the ex ante tax is lower than if capital formation was not distorted. This finding is confirmed by the following experiment: if γ was zero in all states but all other variables remained unchanged, the ex ante tax would average 3.1 percent, which is very close to the value of Bianchi and Mendoza 2018. The optimal ex post Pigouvian tax on debt averages -1.71 percent. The optimal time-consistent debt policy, i.e. ex ante and ex post debt policy, reduces the probability of crises by 0.52 percentage points.[21] This means that there is one severe crises approximately once every 25 years under the constrained efficient

[20] The average is computed with the ergodic distribution.
[21] The current account values that define a crisis are those from laissez-faire.

allocation, as opposed to once every 22 years under laissez-faire. Nevertheless, the optimal debt tax is very effective at reducing the probability of being in a constrained state. This suggests that optimal macroprudential policy is effective at making states non-binding where the constraint is hardly binding under laissez-faire, but not effective at improving conditions when the constraint is strongly binding under laissez-faire. Average investment is reduced by 0.16% due to reduced borrowing. The reduction in investment is small because the social planner takes the distortionary effect on investment into account and changes borrowing only slightly compared to laissez-faire. Welfare increases by 0.0025 percent, which is a tiny value compared to related studies.

To assess the relative importance of the ex ante part of the optimal policy, I also compute policy functions and the ergodic distribution when only the optimal state-contingent ex ante tax is applied.[22] This experiment provides evidence that the major part of the welfare gain is due to the ex ante tax. When the collateral constraint is binding, the amount of collateral is predetermined and only a higher price of collateral can increase the borrowing capacity. In contrast to a model with fixed aggregate capital, where higher consumption increases the price of capital directly, a higher price of capital in my model can only arise in the capital market if both consumption and investment increase. Thus, increasing the price of capital is more costly in utility terms. The experiment also reveals that the ex post part of optimal policy also helps to prevent (some) crises because the probability of crises is lower compared to laissez-faire when the ex post tax is imposed.

Table 3: Results of Related Studies with Fixed Aggregate Capital

	Bianchi and Mendoza 2018	**Ma 2020**
Optimal ex ante debt tax (in %)	3.6	1.28
Average welfare gain (in %)	0.3	0.06
Reduction of crisis probability (in %)	4 to 0.02	6.23 to 1.89

5.3.4 Crises under Laissez-faire: What Would a Social Planner Do?

Figure 6 compares the average crises dynamics under laissez faire to dynamics of the constrained efficient allocation. For given bond and capital states five periods before the laissez-faire crisis and for given sequences of shocks from five period before until four periods after the crisis, I compute the optimal choice of the social planner. It is important to underline that the graphs

[22] Opposed to Bianchi and Mendoza 2018 the ex ante tax is not necessarily optimal without the ex post part. However, it is a good reference point.

cannot be interpreted as a time series, since they do not depict an example crisis but the average of all laissez-faire crisis situations. However, if the averages diverge, the differences point at systematical deviations of the constrained efficient equilibrium under unfavorable economic situations.

Before a crisis, the social planner chooses to borrow less than under laissez-faire. Lower borrowing reduces aggregate spending on consumption, investment and the intermediate good compared to laissez-faire. Reduced investment leads to a lower level of capital before the crisis which implies a lower return to labor and therefore less labor. Consequently, GDP is lower under the optimal policy compared to laissez-faire. Less borrowing also implies a slightly positive/less negative current account to GDP ratio. The optimal policy leads to slightly less consumption before the crisis compared to laissez-faire due to reduced borrowing. The price of capital is almost the same before the crisis, because the differences in investment and in consumption are small.

In the crisis period, i.e. $T = 0$, the lower initial level of debt chosen by the social planner compared to laissez-faire leads to a lower debt service and, furthermore, the planner borrows more. Thus, there are more funds available, which leads to a slightly less pronounced decrease in investment as well as a slightly higher consumption level than under laissez-faire, which stabilizes the price of capital and dampens financial amplification. The borrowing capacity does not drop as strongly as under laissez-faire, which is reflected by the less positive current account and the higher credit to GDP ratio. Labor falls more strongly compared to laissez-faire, because the lower capital level reduces the return to labor. Overall, the increase in households' utility merely comes from reduced working hours. After the crisis the average of most variables align, since the economies return to "normal times" and the probability of another crisis is low.

5.3.5 Reference Case

In the reference case, where the social planner optimally chooses borrowing and investment, the type of intervention that is desirable is the same as in the case, in which social planner only chooses borrowing on agents' behalf. It is also optimal to subsidize debt when the collateral constraint is binding and to tax debt if not. However, the ex ante debt tax is considerably higher in the reference case, both on average and in the individual states. Figure 7 compares the optimal state-contingent ex ante debt tax that result from the two analyzed social planner problems.

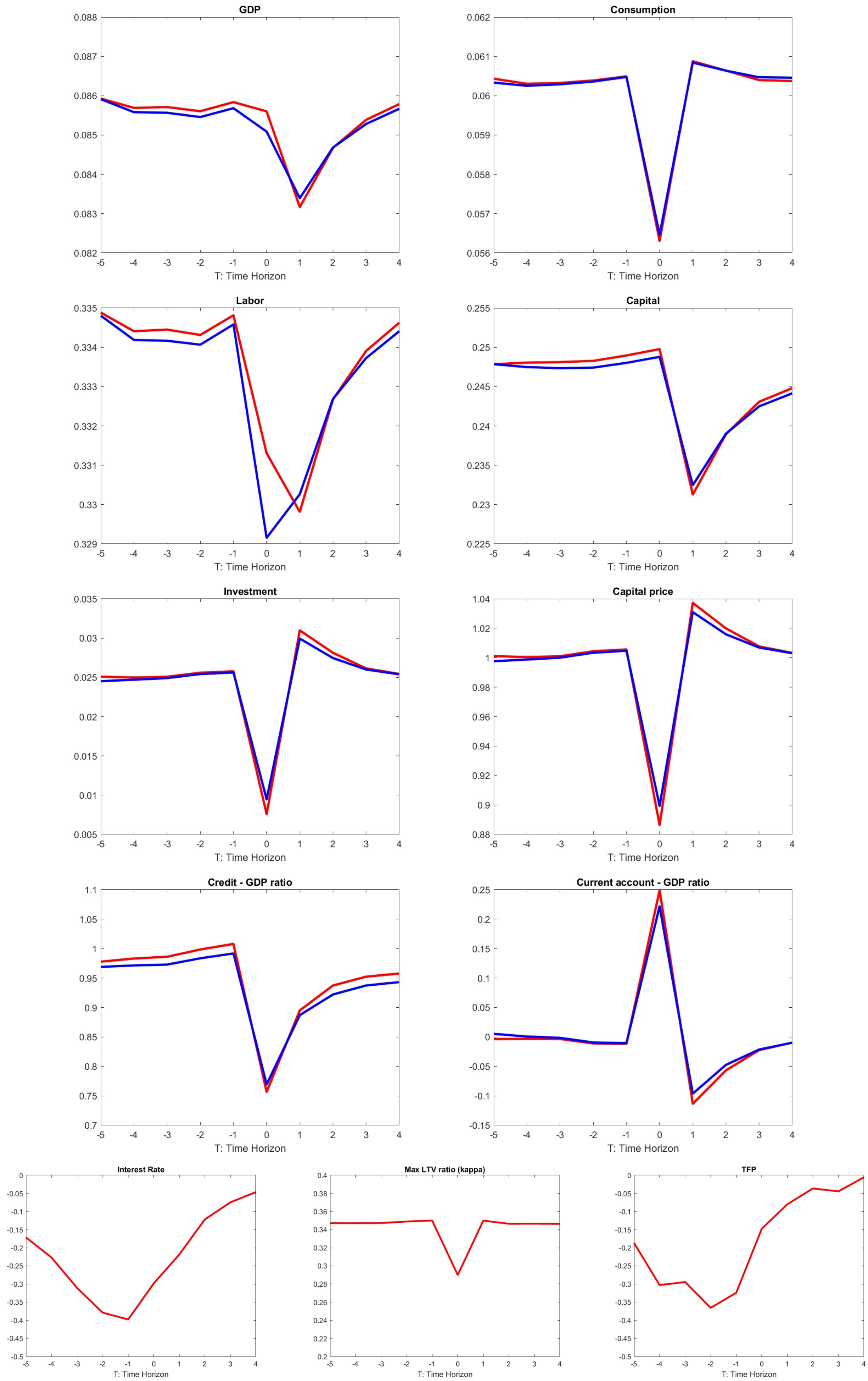

Figure 6: *Crises situations under laissez-faire: what would a social planner do? Red line = Laissez-faire, Blue line = Optimal macroprudential policy*

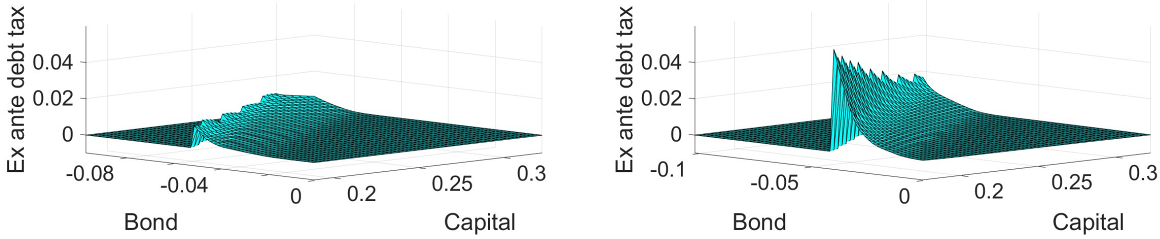

Figure 7: *Optimal Pigouvian ex ante debt tax: Pure macroprudential policy (left) vs. Reference case (right)*

In both cases the optimal ex ante tax increases the closer the constrained region is because the probability of being in a constrained state tomorrow is highest. However, the ex ante debt tax increases much more strongly in the reference case and reaches much higher levels. As the analytical expression for the optimal debt tax indicated, this is due to the fact the investment tax/subsidy counteracts distorting effects of the debt tax, reducing the costs of macroprudential policy. The difference is largest when the level of debt is high and the level of capital is low because in the case where the social planner only chooses borrowing optimally because the a further reduction of investment due to macroprudential policy is most costly. The ex post debt tax is on a comparable level in both analyzed cases.

The debt tax is accompanied by an investment tax that is negative when the collateral constraint does not bind and positive when it is binding. This implies that there is ex ante underinvestment under laissez-faire and ex post overinvestment. Intuitively, the ex ante investment subsidy counteracts the negative effect of the ex ante debt tax on capital formation and the ex post investment tax leads to high prices when the collateral constraint is binding despite of low investment.

The optimal policy in the reference case reduces the crisis probability to 0.03 percent, i.e. there is approximately one crisis every 3000 years. Consequently, welfare gains are ten times as high as in the case, in which the social planner cannot choose investment on behalf of firms.

Table 4: Results Reference Case

Average ex ante debt tax (in %)	1.34	Average ex post debt tax (in %)	-1.31
Average ex ante investment tax (in %)	-1.89	Average ex post investment tax (in %)	8.67
Probability of crises (in %)	0.03	Average welfare gain (in %)	0.0221

6 Conclusions

This paper presented an analysis of optimal time-consistent macroprudential policy in an economy with endogenous capital formation. The main difference from an economy with a fixed level of aggregate capital is that the price of capital is determined not only by an asset pricing equation but also by firms' decision on current investment. A macroprudential policy that stabilizes the value of collateral in crises by stabilizing the price of capital will therefore always affect capital formation, which is what a social planner takes into account. Thus, a cautious macroprudential policy that distorts capital only to a small amount is optimal ex ante. As a consequence, the optimal policy yields much smaller welfare gains and much less effective crisis prevention than related literature without endogenous capital formation. However, if the distorting effect of macroprudential policy on capital formation is counteracted by an investment tax/subsidy, almost all crises can be prevented and welfare gains are substantially higher.

My analysis raises two interesting questions that remain unanswered: First, what would optimal macroprudential policy look like if there were not only collateral externalities in the borrowing decision, but also distributional externalities arising from differences between old and new capital (like in Lanteri and Rampini 2023)? Second, this analysis aggregates across different types of capital. How would the results change if different types of capital with different prices and loan-to-value ratios were analyzed? These questions are left for future research.

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Appendix

Contents

A Reference case

The constrained social planner's optimization problem can be summarized as follows:

$$\mathcal{V}(b,k,s) \quad = \quad \max_{c,k',b',q,v} \frac{(c-\chi\frac{h^{\omega}}{\omega})^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',k',s') \ , \tag{App.1}$$

s.t.

$$(\lambda^{SP2}) \qquad Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} \quad = \quad \frac{b'}{R} - b + c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 + p_v v \ , \tag{Resource constraint}$$

$$(\mu^{SP2}) \qquad \frac{b'}{R} - \theta p_v v \quad \geq \quad -\kappa q k \ , \tag{Collateral constraint}$$

$$\begin{aligned}(\xi^{SP2}) \qquad q\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} \quad = \quad & \beta\mathbb{E}_{s'|s}\Big[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma} \dots \\ & \Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v} \dots \\ & \boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\Big] + \beta\mathbb{E}_{s'|s}\left[\boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\right] \ .\end{aligned} \tag{Household capital decision}$$

The optimization problem leads to the following first-order conditions after applying the envelope theorem:

$$\lambda^{SP2} \quad = \quad \left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} + \xi^{SP2}\sigma\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1} q \ , \tag{App.2}$$

$$\lambda^{SP2} \quad = \quad \beta R\mathbb{E}_{s'|s}\left[\lambda^{SP2'}\right] + \mu^{SP2} + \beta R\mathbb{E}_{s'|s}[\xi^{SP2}\Omega'] \ , \tag{App.3}$$

$$\begin{aligned}\lambda^{SP2}(1+a(k'-(1-\delta)k-\bar{i})) \quad = \quad & \beta\mathbb{E}_{s'|s}\Big[\lambda^{SP2'}\Big(\alpha_k A' k'^{\alpha_k-1}v'^{\alpha_v}h'^{\alpha_h} \dots \\ & + (1-\delta)(1+a(k''-(1-\delta)k'-\bar{i}))\Big) \dots \\ & + \mu^{SP2'}q'\kappa'\Big] + \beta\mathbb{E}_{s'|s}[\xi^{SP2}\Gamma'] \ ,\end{aligned} \tag{App.4}$$

$$\begin{aligned}\chi h^{\omega-1}\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} \quad = \quad & \lambda^{SP2}\alpha_h A k^{\alpha_k} h^{\alpha_h-1} v^{\alpha_v} \dots \\ & - \xi^{SP2}\chi h^{\omega-1}\sigma\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma-1} q \ ,\end{aligned} \tag{App.5}$$

$$\mu^{SP2}\kappa k \quad = \quad \xi^{SP2}\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} \ , \tag{App.6}$$

$$Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} - \frac{b'}{R} + b \quad = \quad c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 - p_v v \ , \tag{App.7}$$

$$\lambda^{SP2}p_v \quad = \quad \lambda^{SP2}\alpha_v A k^{\alpha_k} v^{\alpha_v-1} h^{\alpha_h} - \mu^{SP}\theta p_v \ , \tag{App.8}$$

$$\frac{b'}{R} - \theta p_v v \quad \geq \quad -\kappa q k \ , \tag{App.9}$$

$$\begin{aligned}q\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} \quad = \quad & \beta\mathbb{E}_{s'|s}\Bigg[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma} \dots \\ & \Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v} \dots\end{aligned}$$

$$\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big) + \boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\Bigg] \, , \tag{App.10}$$

$$\mu^{SP2} \geq 0 \, , \tag{App.11}$$

$$0 = \mu^{SP2}\left(\frac{b'}{R} - \theta p_v v + \kappa q k\right) \, . \tag{App.12}$$

Ω captures the effects of the current planner's bond decision b' on the future planner's decisions, which is taken into account by the current planner:

$$\begin{aligned}
\Omega' = &-\sigma\left(\mathcal{C}_{fp,b}(b',k',s') - \chi\boldsymbol{h}_{fp,b}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\omega-1}\right)\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma-1}\ldots \\
&\left((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\right)\ldots \\
&+\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma}\Bigg((1-\delta)\mathcal{Q}_{fp,b}(b',k',s')\ldots \\
&+\alpha_k A' k'^{\alpha_k-1}\Big(\alpha_v\boldsymbol{v}_{fp,b}(b',k',s')\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v-1}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\ldots \\
&+\alpha_h\boldsymbol{h}_{fp,b}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\Big)\Bigg)\ldots \\
&+\mathcal{Q}_{fp,b}(b',k',s')\boldsymbol{\mu}(b',k',s')\kappa' + \mathcal{Q}_{fp}(b',k',s')\boldsymbol{\mu}_b(b',k',s')\kappa' \, .
\end{aligned}$$

Analogously, Γ captures the effect of the current capital decision k' on the future planner's decision on the choice variables:

$$\begin{aligned}
\Gamma' = &-\sigma\left(\mathcal{C}_{fp,k}(b',k',s') - \chi\boldsymbol{h}_{fp,k}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\omega-1}\right)\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma-1}\ldots \\
&\left((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\right)\ldots \\
&+\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma}\ldots \\
&\Bigg((1-\delta)\mathcal{Q}_{fp,k}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\Big(\alpha_v\boldsymbol{v}_{fp,k}(b',k',s')\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v-1}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\ldots \\
&+\alpha_h\boldsymbol{h}_{fp,k}(b',k',s')\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\Big)\ldots \\
&+\alpha_k(\alpha_k-1)A' k'^{\alpha_k-2}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Bigg)\ldots \\
&+\mathcal{Q}_{fp,k}(b',k',s')\boldsymbol{\mu}_{fp}(b',k',s')\kappa' + \mathcal{Q}_{fp}(b',k',s')\boldsymbol{\mu}_{fp,k}(b',k',s')\kappa' \, .
\end{aligned} \tag{App.13}$$

Definition 3 *The Markov perfect constrained efficient equilibrium is defined by the policy functions $\mathcal{B}(b,k,s)$, $\mathcal{K}(b,k,s)$, $\mathcal{Q}(b,k,s)$, $\mathcal{C}(b,k,s)$, $\boldsymbol{v}(b,k,s)$, $\boldsymbol{\mu}(b,k,s)$,[23] $\boldsymbol{h}(b,k,s)$ and the value function $\mathcal{V}(b,k,s)$ that, first, solve the social planner optimization problem (App.1) and, second, are equal to the future planner's policy functions: $\mathcal{B}(b,k,s) = \mathcal{B}_{fp}(b,k,s)$, $\mathcal{K}(b,k,s) = \mathcal{K}_{fp}(b,k,s)$, $\mathcal{Q}(b,k,s) = \mathcal{Q}_{fp}(b,k,s)$, $\mathcal{C}(b,k,s) = \mathcal{C}_{fp}(b,k,s)$, $\boldsymbol{v}(b,k,s) = \boldsymbol{v}_{fp}(b,k,s)$, $\boldsymbol{\mu}(b,k,s) = \boldsymbol{\mu}_{fp}(b,k,s)$ and $\boldsymbol{h}(b,k,s) = \boldsymbol{h}_{fp}(b,k,s)$.*

[23] Analogously to the case where the social planner only chooses borrowing on agents' behalf, the household's multiplier on the collateral constraint is defined by equation (12) and is not binding for the social planner.

B Proof

The reduced social planner problem is (19) equivalent to the social planner problem incorporating all first-order conditions of the decentralized equilibrium (except for the first-order condition on bonds):

$$
\begin{aligned}
\mathcal{V}(b,k,s) &= \max_{c,k',b',q,v} \frac{(c-\chi\frac{h^{\omega}}{\omega})^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_{s'|s}\mathcal{V}(b',k',s')\ , \\
& \text{s.t.} \\
Ak^{\alpha_k}v^{\alpha_v}h^{\alpha_h} &= \frac{b'}{R} - b + c + k' - (1-\delta)k + \frac{a}{2}(k'-(1-\delta)k-\bar{i})^2 + p_v v\ , \\
& \qquad \text{(Resource constraint)} \\
\frac{b'}{R} - \theta p_v v &\geq -\kappa q k\ , \qquad \text{(Collateral constraint)} \\
q\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} &= \beta\mathbb{E}_{s'|s}\Big[\left(\mathcal{C}_{fp}(b',k',s') - \chi\frac{\boldsymbol{h}_{fp}(b',k',s')^{\omega}}{\omega}\right)^{-\sigma}\ldots \\
& \Big((1-\delta)\mathcal{Q}_{fp}(b',k',s') + \alpha_k A' k'^{\alpha_k-1}\boldsymbol{v}_{fp}(b',k',s')^{\alpha_v}\ldots \\
& \boldsymbol{h}_{fp}(b',k',s')^{\alpha_h}\Big)\Big] + \beta\mathbb{E}_{s'|s}\left[\boldsymbol{\mu}_{fp}(b',k',s')\mathcal{Q}_{fp}(b',k',s')\kappa'\right]\ , \\
& \qquad \text{(Household capital decision)} \\
q &= 1 + a(k'-(1-\delta)k-\bar{i})\ , \qquad \text{(Firm investment decision)} \\
\left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma} p_v + \mu\theta p_v &= \left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma}\alpha_v A k^{\alpha_k} v^{\alpha_v-1} h^{\alpha_h}\ , \qquad \text{(App.14)} \\
\chi h^{\omega-1} &= \alpha_h A k^{\alpha_k} h^{\alpha_h-1} v^{\alpha_v}\ , \qquad \text{(App.15)} \\
& \mu \geq 0\ , \qquad \text{(App.16)} \\
& \mu\left(\frac{b'}{R} - \theta p_v v + \kappa q k\right) = 0\ . \qquad \text{(App.17)}
\end{aligned}
$$

μ is defined via equation (App.14). Combining equations (26) and (App.14) leads to the following relation between μ and μ^{SP}:

$$
\mu = \left(c-\chi\frac{h^{\omega}}{\omega}\right)^{-\sigma}\frac{\mu^{SP}}{\lambda^{SP}}\ .
$$

Equation (App.16) is not binding since the equation above shows that it is positively related to μ^{SP}, which is either positive or zero. If the collateral constraint is not binding, μ^{SP} and - as can be seen in the equation above - also μ are equal to zero. Thus, equation (App.17) is not binding. Equation (App.15) is not binding because the combination of equations (20) and (23) of the reduced planner problem yields the same condition. Finally, equation (App.14) is not binding because it only defines μ, which is not relevant for any binding constraint.

C Algorithms

Laissez-faire (for an even more detailed description of the updating steps see Mendoza and Villalvazo 2020[24])

1. Uniformly spaced discrete grids for the state variables bond b (60 nodes) and capital k (30 nodes) as well as a grid for the shock state space are created. The interest rate shock and the productivity shock are discretized by Tauchen's method with 3 realizations each. The financial shock has two realizations so that there are 18 different possible combinations of shocks. Thus, the state space has 60 $\times$ 30 $\times$ 18 elements. The interpolation scheme is bilinear interpolation.
2. Guess the ratio of the collateral constraint multiplier μ to λ which is denoted by $\hat{\boldsymbol{\mu}}_0^{Guess}$, the bond policy function $\mathcal{B}_0^{Guess}$ and the price of capital policy function $\mathcal{Q}_0^{Guess}$. I used $\mathcal{Q}_0^{Guess} = \text{ones}(b, k, s)$, $\mathcal{B}_0^{Guess} = b(b, k, s)$ and $\hat{\boldsymbol{\mu}}_0^{Guess} = \text{zeros}(b, k, s)$ as initial guesses.
3. Use guesses $\hat{\boldsymbol{\mu}}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\mathcal{Q}_j^{Guess}$ to compute guesses $\mathcal{K}_j^{Guess}$, $\mathcal{C}_j^{Guess}$, $\boldsymbol{h}_j^{Guess}$ and $\boldsymbol{v}_j^{Guess}$.
4. Assume that the collateral constraint is not binding and use the equilibrium conditions to update all policy functions except $\mathcal{Q}$.
5. Check whether the collateral constraint is binding.
6. Solve for $\hat{\mu}_j$ in the binding states. Update all other policy functions except for $\mathcal{Q}$.
7. Use all updated policy functions and equation (11) to compute $\mathcal{Q}_j$.
8. Check convergence. If $sup_{B,K,S}\|x_j(b, k, s) - x_j^{Guess}(b, k, s)\| \geq \epsilon$ for $x = \mathcal{Q}$, $\mathcal{B}$, $\hat{\boldsymbol{\mu}}$, compute $\mathcal{Q}_{j+1}^{Guess}$, $\mathcal{B}_{j+1}^{Guess}$ and $\hat{\boldsymbol{\mu}}_{j+1}^{Guess}$ as weighted sums of $\mathcal{Q}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\hat{\boldsymbol{\mu}}_j^{Guess}$ and $\mathcal{Q}_j$, $\mathcal{B}_j$ and $\hat{\boldsymbol{\mu}}_j$. Then go to step 3. Else stop.

[24] Please note that I use a different notation.

Optimal policy:

Outer loop

1. Equivalent to the decentralized equilibrium, uniformly spaced discrete grids for the state variables bond (60 nodes) and capital (30 nodes) as well as a grid for the shock state space are created. The interest rate shock and the productivity shock are discretized by Tauchen's method with 3 realizations each. The financial shock has two realizations so that there are 18 different possible combinations of shocks. Thus, the state space has 60 $\times$ 30 $\times$ 18 elements. The interpolation scheme is bilinear interpolation.
2. Guess the policy functions of the future planner $\mathcal{Q}_{fp,0}^{Guess}$, $\boldsymbol{\mu}_{fp,0}^{Guess}$, $\boldsymbol{h}_{fp,0}^{Guess}$, $\boldsymbol{v}_{fp,0}^{Guess}$ and $\mathcal{C}_{fp,0}^{Guess}$. Compute derivatives of these policy functions with respect to b and k.

   Inner Loop (iteration of current planner's policy functions given the policy functions of the future planner):

   (a) Guess the ratio of the collateral constraint multiplier μ to λ, which is denoted by $\hat{\boldsymbol{\mu}}_0^{Guess}$, the bond policy function $\mathcal{B}_0^{Guess}$, the price of capital policy function $\mathcal{Q}_0^{Guess}$ and the investment implementability constraint multiplier $\boldsymbol{\gamma}_0^{Guess}$.
   (b) Use guesses $\hat{\boldsymbol{\mu}}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\mathcal{Q}_j^{Guess}$ to compute guesses $\mathcal{K}_j^{Guess}$, $\mathcal{C}_j^{Guess}$, $\boldsymbol{h}_j^{Guess}$ and $\boldsymbol{v}_j^{Guess}$.
   (c) Eliminate ξ in all equations by using equation (25). Assume that the collateral constraint is not binding and update all policy functions of the current planner except $\mathcal{Q}$.
   (d) Check whether the collateral constraint is binding.
   (e) Solve for $\hat{\boldsymbol{\mu}}_j$ in the binding states. Update all other policy functions except for $\mathcal{Q}$.
   (f) Use updated policy functions and equation (28) to compute $\mathcal{Q}_j$.
   (g) Check convergence. If $sup_{B,K,S}\|x_j(b,k,s) - x_j^{Guess}(b,k,s)\| \geq \epsilon$ for $x = \mathcal{Q}$, $\mathcal{B}$, $\hat{\boldsymbol{\mu}}$, $\boldsymbol{\gamma}$, compute $\mathcal{Q}_{j+1}^{Guess}$, $\mathcal{B}_{j+1}^{Guess}$, $\hat{\boldsymbol{\mu}}_{j+1}^{Guess}$ and $\boldsymbol{\gamma}_{j+1}^{Guess}$ as weighted sums of $\mathcal{Q}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\hat{\boldsymbol{\mu}}_j^{Guess}$, $\boldsymbol{\gamma}_j^{Guess}$ and $\mathcal{Q}_j$, $\mathcal{B}_j$, $\hat{\boldsymbol{\mu}}_j$, $\boldsymbol{\gamma}_j$. Then go to step b. Else stop.
3. Check convergence. $\mathcal{Q}_i(b,k,s)$, $\boldsymbol{\mu}_i(b,k,s)$, $\boldsymbol{h}_i(b,k,s)$, $\boldsymbol{v}_i(b,k,s)$ and $\mathcal{C}_i(b,k,s)$ are the converged policy functions of the inner loop given the guessed future planner's policy functions of the outer loop's iteration i. If $sup_{B,K,S}\|x_i(b,k,s) - x_{fp,i}^{Guess}(b,k,s)\| \geq \epsilon^*$ for $x = \mathcal{Q}$, $\boldsymbol{\mu}$, $\boldsymbol{h}$, $\boldsymbol{v}$, $\mathcal{C}$, compute new guesses $\mathcal{Q}_{fp,i+1}^{Guess}$, $\boldsymbol{\mu}_{fp,i+1}^{Guess}$, $\boldsymbol{h}_{fp,i+1}^{Guess}$, $\boldsymbol{v}_{fp,i+1}^{Guess}$ and $\mathcal{C}_{fp,i+1}^{Guess}$ as weighted sums of old guesses and converged policy functions from the inner loop. Compute derivatives of these policy functions with respect to b and k. Then go to the inner loop. Else stop.

Optimal policy (reference case):

Outer loop

1. Equivalent to the decentralized equilibrium, uniformly spaced discrete grids for the state variables bond (60 nodes) and capital (30 nodes) as well as a grid for the shock state space are created. The interest rate shock and the productivity shock are discretized by Tauchen's method with 3 realizations each. The financial shock has two realizations so that there are 18 different possible combinations of shocks. Thus, the state space has 60 $\times$ 30 $\times$ 18 elements. The interpolation scheme is bilinear interpolation.
2. Guess the policy functions of the future planner $\mathcal{Q}_{fp,0}^{Guess}$, $\boldsymbol{\mu}_{fp,0}^{Guess}$, $\boldsymbol{h}_{fp,0}^{Guess}$, $\boldsymbol{v}_{fp,0}^{Guess}$ and $\mathcal{C}_{fp,0}^{Guess}$. Compute derivatives of these policy functions with respect to b and k.

   Inner Loop (iteration of current planner's policy functions given the policy functions of the future planner):

   (a) Guess the ratio of the collateral constraint multiplier μ to λ, which is denoted by $\hat{\boldsymbol{\mu}}_0^{Guess}$, the bond policy function $\mathcal{B}_0^{Guess}$ and the capital policy function $\mathcal{K}_0^{Guess}$.
   (b) Use guesses $\hat{\boldsymbol{\mu}}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\mathcal{K}_j^{Guess}$ to compute guesses $\mathcal{C}_j^{Guess}$, $\boldsymbol{h}_j^{Guess}$, $\boldsymbol{v}_j^{Guess}$ and $\hat{\boldsymbol{q}}_j^{Guess}$. $\hat{q}$ is an auxiliary variable defined as follows: $\hat{q} := q\left(c - \chi\frac{h^{\omega}}{\omega}\right)^{-\sigma}$.
   (c) Eliminate ξ in all equations by using equation (App.6). Assume that the collateral constraint is not binding and update all policy functions of the current planner.
   (d) Check whether collateral constraint is binding.
   (e) Solve for $\hat{\boldsymbol{\mu}}_j$ in the binding states. Update all other policy functions.
   (f) Check convergence. If $sup_{B,K,S}\|x_j(b,k,s) - x_j^{Guess}(b,k,s)\| \geq \epsilon$ for $x = \mathcal{K}$, $\mathcal{B}$, $\hat{\boldsymbol{\mu}}$, compute $\mathcal{K}_{j+1}^{Guess}$, $\mathcal{B}_{j+1}^{Guess}$ and $\hat{\boldsymbol{\mu}}_{j+1}^{Guess}$ as weighted sums of $\mathcal{K}_j^{Guess}$, $\mathcal{B}_j^{Guess}$, $\hat{\boldsymbol{\mu}}_j^{Guess}$ and $\mathcal{K}_j$, $\mathcal{B}_j$ and $\hat{\boldsymbol{\mu}}_j$. Then go to step b. Else stop.
3. Check convergence. $\mathcal{Q}_i(b,k,s)$, $\boldsymbol{\mu}_i(b,k,s)$, $\boldsymbol{h}_i(b,k,s)$, $\boldsymbol{v}_i(b,k,s)$ and $\mathcal{C}_i(b,k,s)$ are the converged policy functions of the inner loop given the guessed future planner's policy functions of the outer loop's iteration i. If $sup_{B,K,S}\|x_i(b,k,s) - x_{fp,i}^{Guess}(b,k,s)\| \geq \epsilon^*$ for $x = \mathcal{Q}$, $\boldsymbol{\mu}$, $\boldsymbol{h}$, $\boldsymbol{v}$, $\mathcal{C}$, compute new guesses $\mathcal{Q}_{fp,i+1}^{Guess}$, $\boldsymbol{\mu}_{fp,i+1}^{Guess}$, $\boldsymbol{h}_{fp,i+1}^{Guess}$, $\boldsymbol{v}_{fp,i+1}^{Guess}$ and $\mathcal{C}_{fp,i+1}^{Guess}$ as weighted sums of old guesses and converged policy functions of the current planner. Compute derivatives of these new guesses with respect to b and k. Then go to the inner loop. Else stop.

D Data

Table 5: Data Sources

Data	Measure	Unit	Source	URL / Reference	Usage
Annual National Accounts, 9B. Balance sheets for non-financial assets: Fixed assets & GDP (expenditure approach)	Current prices	Local currency, Millions	OECD	https://stats.oecd.org/	Computation of capital-to-GDP ratio
Annual National Accounts, 1. Gross domestic product (GDP): Gross fixed capital formation & GDP (expenditure approach)	VOB: Constant Prices: OECD base year	Local currency, Millions	OECD	https://stats.oecd.org/	Logarithmized and linearly detrended time series of investment and GDP per worker used to compute standard deviation (sd) of investment relative to sd of to GDP and autocorrelation of GDP
Annual National Accounts, 1. Gross domestic product (GDP): GDP (expenditure approach)	Constant Prices, Constant PPPs, OECD base year	US Dollar Millions, 2015	OECD	https://stats.oecd.org/	Computation of country weights (2012 share of summed real GDP)
Annual Labor Force: Statistics: ALFS Summary tables	–	Persons, Thousands	OECD	https://stats.oecd.org/	Computation of investment and GDP per worker
Penn World Table, version 10.01, Price level of capital formation & of consumption	Price level relative to price level of US GDP in 2017	–	University of Groningen	https://dataverse.nl/api/access/datafile/354095 Feenstra et al. 2015	Logarithmized and HP-filter detrended time series of relative price of capital used to compute sd

E 3-period model

As mentioned in the literature review, in this section I analyze optimal policy in a 3-period model that builds on Dávila and Korinek 2017. Above all, I will make two points. First, also in a 3-period model, the optimal borrowing tax when there is only macroprudential policy takes into account that the imposed tax reduces capital. Second, it is optimal to tax debt ex ante and subsidize investment ex ante in the case where macroprudential policy and investment policy are combined, which is in line with the results of the main part of this paper.
Different from Dávila and Korinek 2017, the following model builds on an open economy setting instead of a two-agent setting. Like in Dávila and Korinek 2017, ex ante instruments will be used to implement the social planner allocations.

E.1 Set-up

Agents/households live for three periods $t = 1, 2, 3$. There is a continuum of agents of mass one. They derive utility from consumption of a non-durable good c_t. Agents maximize their lifetime utility $\sum_{t=1}^{3} \beta^t u(c_t)$ and their preferences satisfy $u_t = \log c_t$ for $t = 1, 2$ and $u_t = c_t$ for $t = 3$. There is no uncertainty and only one type of agent.
In the first period, the household receives an exogenous income y and can borrow $-b_2$ at the interest rate R_1 in this period.[25] Income and borrowed funds are spent on consumption c_1 and on investment. The investment technology is given by a quadratic cost function as in chapter 4 of Dávila and Korinek 2017. Thus, higher investment increases the stock of capital in the next period k_2, but also increases investment cost in the current period.
Production in period 2 is given by the product of productivity A_2 and capital k_2. In the same period households are subject to a collateral constraint and can only borrow up to a fraction ϕ of the value of their collateral, which is given by end-of-period capital holdings times the price of capital q_2. Income plus borrowing $-b_3$ at the rate R_2 can be spent on consumption c_2 and on capital accumulation: households can trade non-depreciating capital among each other at price q_2. In the last period the household receives income $A_3 k_3$, repays its debt $-b_3$ and consumes c_3.

Table 6: Summary of household income and constraints

	Period I	**Period II**	**Period III**
Income	y	$A_2 k_2$	$A_3 k_3$
Budget constraint	$y - \frac{b_2}{R_1} - c_1 - \frac{a}{2} k_2^2$	$A_2 k_2 - \frac{b_3}{R_2} - c_2 - q_2(k_3 - k_2) + b_2$	$A_3 k_3 + b_3 - c_3$
Collateral constraint	–	$\frac{b_3}{R_2} \geq -\phi q_2 k_3$	–

Borrowed funds are assumed to be supplied in any amount by a foreign country. Aggregate capital is fixed from period in period $t = 2$ on so that $k = k_2 = k_3$ holds.

[25] b denotes bonds. So a positive amount of b means saving and a negative amount means borrowing.

E.2 Laissez-faire

The Lagrangian of the household's maximization problem looks as follows:

$$\begin{aligned}
L = &\log c_1 + \beta \log c_2 + \beta^2 [A_3 k_3 + b_3] \\
&+ \lambda_1 [y - \frac{b_2}{R_1} - c_1 - \frac{a}{2} k_2^2] \\
&+ \beta \lambda_2 [A_2 k_2 - \frac{b_3}{R_2} - c_2 - q_2 (k_3 - k_2) + b_2] \\
&+ \beta \mu [\frac{b_3}{R_2} + \phi q_2 k_3] \; .
\end{aligned} \tag{App.18}$$

Parameters are restricted to the subspace that leads to a binding collateral constraint under laissez-faire. Thus, the problem above leads to the following first-order conditions:

$$c_1 : \quad \frac{1}{c_1} - \lambda_1 = 0 \tag{App.19}$$

$$c_2 : \quad \frac{1}{c_2} - \lambda_2 = 0 \tag{App.20}$$

$$b_2 : \quad -\frac{\lambda_1}{R_1} + \beta \lambda_2 = 0 \tag{App.21}$$

$$b_3 : \quad \beta - \frac{\lambda_2}{R_2} + \frac{\mu}{R_2} = 0 \tag{App.22}$$

$$k_2 : \quad -a k_2 \lambda_1 + \beta (q_2 + A_2) \lambda_2 = 0 \tag{App.23}$$

$$k_3 : \quad \beta A_3 + \mu \phi q_2 - q_2 \lambda_2 = 0 \; . \tag{App.24}$$

Equilibrium:
Capital is assumed to be fixed so that capital market clearing implies

$$k := k_2 = k_3 \; . \tag{App.25}$$

The combination of the capital market clearing condition and budget constraints yields the following resource constraints:

$$t = 1 : \; y - \frac{b_2}{R_1} - c_1 - \frac{a}{2} k^2 = 0 \tag{App.26}$$

$$t = 2 : \; A_2 k - \frac{b_3}{R_2} - c_2 + b_2 = 0 \tag{App.27}$$

$$t = 3 : \; A_3 k + b_3 - c_3 = 0 \; . \tag{App.28}$$

Furthermore, the collateral constraint is binding:

$$\frac{b_3}{R_2} = -\phi q_2 k \; . \tag{App.29}$$

The household's optimality conditions can be rearranged in the following way:

$$c_2 = c_1 \beta R_1 \tag{App.30}$$

$$\mu = \frac{1}{c_2} - \beta R_2 \tag{App.31}$$

$$k = \beta \frac{(A_2 + q_2)c_1}{ac_2} \tag{App.32}$$

$$q_2 = \frac{\beta A_3 c_2}{1 - \phi + \beta\phi R_2 c_2} \ . \tag{App.33}$$

Equations (App.32) and (App.33) will be taken into account by the social planner who is equipped with a debt tax and an investment subsidy. The social planner who is equipped with a debt tax only just takes equation (App.33) into account.

As can be seen in equation (App.33), the price of capital crucially depends on the loan to value ratio ϕ. If the borrowing limit is equal to the entire collateral value, i.e. $\phi = 1$, the price of collateral no longer depends on the level of consumption and there is no externality. That is because the collateral constraint becomes the natural borrowing limit if $\phi = 1$. I define the price of capital that emerges for $\phi = 1$ as q^*:

$$q^* = \frac{A_3}{R_2} \ . \tag{App.34}$$

It is now possible to split the price of capital into the price without externality q^* and the distorting externality $\Lambda(\underset{+}{c_2})$, which will contribute to better understand the optimal policies in the next subsections:[26]

$$q_2 = \Lambda(\underset{+}{c_2})q^* \ , \tag{App.35}$$

$$\text{with} \quad \Lambda(\underset{+}{c_2}) = \frac{\beta R_2 c_2}{1 - \phi + \beta\phi R_2 c_2} \ . \tag{App.36}$$

E.3 Optimal policy: macroprudential policy combined with investment policy

In this subsection, it is solved for the macroprudential and investment policy that decentralize the constrained efficient allocation, where a social planner chooses investment and borrowing in period 1 on agent's behalf.

The social planner is constrained by the collateral and resource constraint as well as the second period's capital optimality condition and borrowing condition. The allocation that results from the optimization problem can be decentralized with a Pigouvian debt tax and a Pigouvian investment tax. A positive debt tax τ_b^{SP2} would mean that the amount the household receives for a given level of newly issued debt in period 1 was reduced. A positive investment tax τ_i^{SP2} would mean that the quadratic investment cost in the first period was increased. Thus, the household's budget constraint in period 1 changes to:

$$y - \frac{(1 - \tau_b^{SP2})b_2}{R_1} - c_1 - (1 + \tau_i^{SP2})\frac{a}{2}k^2 \ . \tag{App.37}$$

Moreover, the household's decisions on borrowing and investment (App.30) and (App.32) change to:

$$c_2(1 - \tau_b^{SP2}) = c_1 \beta R_1 \tag{App.38}$$

$$k = \beta \frac{(A_2 + q_2)c_1}{(1 + \tau_i^{SP2})ac_2} \ . \tag{App.39}$$

[26]The plus sign under c_2 indicates that Λ is a function of c_2 which positively depends on c_2.

The social planner maximizes households' consumption subject to the resource constraints as well as to the collateral constraint. The implementability conditions have been combined to receive equation (App.33) which is used to replace q_2 below. Consequently, the planner takes into account that the price of collateral in the collateral constraint is directly influenced by the level of consumption in the second period. The Lagrangian below follows from the maximization problem:

$$\begin{aligned} L = &\log c_1 + \beta \log c_2 + \beta^2 [A_3 k + b_3] \\ &+ \lambda_1^{SP2} [y - \frac{b_2}{R_1} - c_1 - \frac{a}{2} k^2] \\ &+ \beta \lambda_2^{SP2} [A_2 k - \frac{b_3}{R_2} - c_2 + b_2] \\ &+ \beta \mu^{SP2} [\frac{b_3}{R_2} + \phi q^* \Lambda(\underset{+}{c_2}) k] \, . \end{aligned} \tag{App.40}$$

The maximization problem above leads to the following first-order conditions:

$$c_1 : \frac{1}{c_1} - \lambda_1^{SP2} = 0 \, , \tag{App.41}$$

$$c_2 : \frac{1}{c_2} - \lambda_2^{SP2} + \phi \mu^{SP2} q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k = 0 \, , \tag{App.42}$$

$$b_1 : -\frac{\lambda_1^{SP2}}{R_1} + \beta \lambda_2^{SP2} = 0 \, , \tag{App.43}$$

$$b_2 : -\frac{\lambda_2^{SP2}}{R_2} + \beta + \frac{\mu^{SP2}}{R_2} = 0 \, , \tag{App.44}$$

$$k : -ak\lambda_1^{SP2} + \beta \lambda_2^{SP2} A_2 + \beta^2 A_3 + \beta \mu^{SP2} \phi q^* \Lambda(\underset{+}{c_2}) = 0 \, . \tag{App.45}$$

It is possible to write down all multipliers as functions of c_1:

$$\lambda_1^{SP2} = \frac{1}{c_1} \tag{App.46}$$

$$\lambda_2^{SP2} = \frac{1}{\beta c_1 R_1} \tag{App.47}$$

$$\mu^{SP2} = \frac{1}{\beta c_1 R_1} - \beta R_2 \, . \tag{App.48}$$

The combination of equations (App.42) and (App.47) leads to the following equation:

$$\frac{1}{c_2} - \frac{1}{\beta c_1 R_1} + \phi \mu^{SP2} q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k = 0 \, . \tag{App.49}$$

Furthermore, the investment decision can be rewritten by using equations (App.45), (App.46), (App.47) and (App.48):

$$-ak\frac{1}{c_1} + \beta \frac{1}{\beta c_1 R_1} A_2 + \beta^2 A_3 + \beta \left(\frac{1}{\beta c_1 R_1} - \beta R_2 \right) \phi q^* \Lambda(\underset{+}{c_2}) = 0 \, . \tag{App.50}$$

It is now possible to use the equations above as well as equations (App.20), (App.22), (App.24), (App.38) and (App.39) to solve for the optimal debt and investment tax:

$$\begin{aligned} \frac{1-\tau_b^{SP2}}{\beta c_1 R_1} - \frac{1}{\beta c_1 R_1} + \phi \mu^{SP2} q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k &= 0 \\ \Leftrightarrow \beta c_1 R_1 \phi \mu^{SP2} q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k &= \tau_b^{SP2} \end{aligned} \tag{App.51}$$

$$\begin{aligned} \left(\tau_i^{SP2} ak\frac{1}{c_1} - \beta^2 A_3 - \beta\phi(\frac{1}{c_2} - \beta R_2) q^* \Lambda(\underset{+}{c_2}) - \beta\frac{A_2}{c_2}\right) + \ldots \\ \frac{1}{c_1 R_1} A_2 + \beta^2 A_3 + \beta\left(\frac{1}{\beta c_1 R_1} - \beta R_2\right)\phi q^* \Lambda(\underset{+}{c_2}) &= 0 \\ \Leftrightarrow \beta\left(\frac{1}{c_2} - \frac{1}{\beta c_1 R_1}\right)\left(\phi q^* \Lambda(\underset{+}{c_2}) + A_2\right) &= \tau_i^{SP2} ak\frac{1}{c_1} \\ \Leftrightarrow -\frac{\beta\frac{\tau_b}{\beta c_1 R_1}\left(\phi q^* \Lambda(\underset{+}{c_2}) + A_2\right)}{ak\frac{1}{c_1}} &= \tau_i^{SP2}\ . \end{aligned} \tag{App.52}$$

The optimal debt tax τ_b^{SP2} has a positive sign, which means that under laissez-faire there is overborrowing and the social planner incentivizes the household to issue less debt. This is due to the fact that less debt to be repaid in period 2 leads to a higher level of consumption, which in turn increases the price of capital and loosens the collateral constraint. Furthermore, it can be seen that the tax is a function of the externality $\frac{\partial \Lambda}{\partial c_2}$ and is zero if there is no externality, i.e. $\frac{\partial \Lambda}{\partial c_2} = 0$.
The optimal investment tax has a negative sign which means that a subsidy is paid. Thus, the social planner incentivizes the household to invest more. If the debt tax τ_b^{SP2} was zero, i.e. there was no externality, the optimal investment tax would also be zero.

E.4 Optimal policy: pure macroprudential policy

In this subsection, it is solved for the macroprudential policy that decentralizes the constrained efficient allocation, where a social planner chooses borrowing in period 1 on agent's behalf.
The social planner is constrained by the collateral and resource constraint as well as all household's optimality conditions but the borrowing condition of period 1. The allocation that results from the optimization problem can be decentralized with a Pigouvian debt tax. A positive debt tax τ_b^{SP1} would mean that the amount the household receives for a given level of newly issued debt in period 1 was reduced. Thus, the household's budget constraint in period 1 changes to:

$$y - \frac{(1-\tau_b^{SP1})b_2}{R_1} - c_1 - \frac{a}{2}k_2^2\ . \tag{App.53}$$

Moreover, the household's decision on borrowing (App.30) changes to:

$$c_2(1-\tau_b^{SP1}) = c_1 \beta R_1\ . \tag{App.54}$$

The social planner maximizes household's consumption subject to the resource constraints as well as to the collateral constraint. The implementability conditions have been combined to express q_2 as function of c_2 and k as function of c_1 and c_2. Consequently, the planner takes into account that the price of collateral in the collateral constraint is directly influenced by the level of consumption in the second period and that capital is a function of the level of consumption in period 1 and 2. The Lagrangian below follows from the maximization problem:

$$
\begin{aligned}
L = & \log c_1 + \beta \log c_2 + \beta^2 \left[y_3 + b_3 + A_3 k(\underset{+}{c_1}, \underset{-}{c_2}) \right] \\
& + \lambda_1^{SP1} \left[y_1 - c_1 - a \frac{k(\underset{+}{c_1}, \underset{-}{c_2})^2}{2} - \frac{b_2}{R_1} \right] \\
& + \beta \lambda_2^{SP1} \left[y_2 + b_2 + A_2 k(\underset{+}{c_1}, \underset{-}{c_2}) - c_2 - \frac{b_3}{R_2} \right] \\
& + \beta \mu_2^{SP1} \left[\frac{b_3}{R_2} + \phi q^* \Lambda(\underset{+}{c_2}) k(\underset{+}{c_1}, \underset{-}{c_2}) \right] .
\end{aligned}
$$

The maximization problem above leads to the following first-order conditions:

$$
b_1 : \quad - \frac{\lambda_1^{SP1}}{R_1} + \beta \lambda_2^{SP1} = 0 \,, \qquad \text{(App.55)}
$$

$$
b_2 : \quad \frac{\mu_2^{SP1}}{R_2} + \beta - \frac{\lambda_2^{SP1}}{R_2} = 0 \,, \qquad \text{(App.56)}
$$

$$
\begin{aligned}
c_1 : \quad & \frac{1}{c_1} + \beta^2 A_3 \underset{+}{\frac{\partial k}{\partial c_1}} - \lambda_1^{SP1} \left(a \underset{+}{\frac{\partial k}{\partial c_1}} k(\underset{+}{c_1}, \underset{-}{c_2}) + 1 \right) + \beta \lambda_2^{SP1} A_2 \underset{+}{\frac{\partial k}{\partial c_1}} \ldots \\
& + \beta \mu_2^{SP1} \phi q^* \Lambda(\underset{+}{c_2}) \underset{+}{\frac{\partial k}{\partial c_1}} = 0 \,, \qquad \text{(App.57)}
\end{aligned}
$$

$$
\begin{aligned}
c_2 : \quad & \frac{1}{c_2} + \beta A_3 \underset{-}{\frac{\partial k}{\partial c_2}} - \frac{1}{\beta} \lambda_1^{SP1} \left(a \underset{-}{\frac{\partial k}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2}) \right) + \lambda_2^{SP1} \left(A_2 \underset{-}{\frac{\partial k}{\partial c_2}} - 1 \right) \ldots \\
& + \mu_2^{SP1} \left(\phi q^* \Lambda(\underset{+}{c_2}) \underset{-}{\frac{\partial k}{\partial c_2}} + \phi q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2}) \right) = 0 \,. \qquad \text{(App.58)}
\end{aligned}
$$

Equation (App.55) can be used to eliminate λ_2^{SP1} in equations (App.57) and (App.58):

$$
\begin{aligned}
& \frac{1}{c_1} + \beta^2 A_3 \underset{+}{\frac{\partial k}{\partial c_1}} - \lambda_1^{SP1} \left(a \underset{+}{\frac{\partial k}{\partial c_1}} k(\underset{+}{c_1}, \underset{-}{c_2}) + 1 \right) + \frac{\lambda_1^{SP1}}{R_1} A_2 \underset{+}{\frac{\partial k}{\partial c_1}} \\
& + \beta \mu_2^{SP1} \phi q^* \Lambda(\underset{+}{c_2}) \underset{+}{\frac{\partial k}{\partial c_1}} = 0 \qquad \text{(App.59)}
\end{aligned}
$$

$$
\frac{1}{c_2} + \beta A_3 \underset{-}{\frac{\partial k}{\partial c_2}} - \frac{1}{\beta} \lambda_1^{SP1} \left(a \underset{-}{\frac{\partial k}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2}) \right) + \frac{\lambda_1^{SP1}}{\beta R_1} \left(A_2 \underset{-}{\frac{\partial k}{\partial c_2}} - 1 \right) \ldots
$$

$$+\mu_2^{SP1}\left(\phi q^* \Lambda(\underset{+}{c_2}) \underset{-}{\frac{\partial k}{\partial c_2}} + \phi q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})\right) = 0\,. \tag{App.60}$$

The combination of the two equations above and equation (App.54) make it possible to derive a first expression of the optimal tax:

$$\begin{aligned}\frac{\tau_b^{SP1}}{c_1} = & \left(R_1 \underset{-}{\frac{\partial k}{\partial c_2}} - \underset{+}{\frac{\partial k}{\partial c_1}}\right)\left(\beta^2 A_3 - \lambda_1^{SP1} a k(\underset{+}{c_1}, \underset{-}{c_2}) + \frac{\lambda_1^{SP1}}{R_1} A_2 + \beta \mu_2^{SP1} \phi q^* \Lambda(\underset{+}{c_2})\right)\ldots \\ & + \beta R_1 \mu_2^{SP1} \phi q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})\,. \end{aligned} \tag{App.61}$$

The forces that determine the optimal tax can be interpreted in an intuitive way. The second line is - despite the size of the collateral constraint multiplier - exactly the same expression as the optimal debt tax in chapter E.3. Thus, this part is a point in favor of a positive debt tax, since a higher level of consumption in period 2 increases the price of collateral, which is not taken into account by the agents.
The first line incorporates the effects that arise because of the distorting effect of the debt tax on the level of capital. The expression inside the second bracket reflects the marginal social gain of having an additional unit of capital. Since a higher tax reduces capital, i.e. $R_1 \frac{\partial k}{\partial c_2} - \frac{\partial k}{\partial c_1} < 0$, a positive expression in the second bracket implies that the tax foregoes gains from a higher stock capital, resulting in a lower optimal debt tax. The social gain of increasing capital is not positive per construction, however, the case in chapter E.3 has shown that this value is positive for the optimal debt tax τ_b^{SP2} (otherwise an investment subsidy would not be optimal).
To show that the tax is zero when there is no externality, I rewrite the expression for the tax. I use the fact that $\lambda_1^{SP1} = \beta R_1(\beta R_2 + \mu_2^{SP1})$ and substitute the $\frac{c_1}{c_2}$ in equation (App.32) by $\frac{1-\tau_b^{SP1}}{\beta R_1}$ to take into account that the marginal costs of capital, $ak(c_1, c_2)$ are directly influenced by the tax:

$$\tau_b^{SP1} = \frac{\beta c_1 \left(R_1 \underset{-}{\frac{\partial k}{\partial c_2}} - \underset{+}{\frac{\partial k}{\partial c_1}}\right)\left(\beta R_2 \left(1 - \Lambda(\underset{+}{c_2})\right) q^* + \mu_2^{SP1} q^* (\phi - 1)\Lambda(\underset{+}{c_2})\right) + \beta c_1 R_1 \phi \mu_2^{SP1} q^* \underset{+}{\frac{\partial \Lambda}{\partial c_2}} k(\underset{+}{c_1}, \underset{-}{c_2})}{\left(1 - c_1 \left(R_1 \underset{-}{\frac{\partial k}{\partial c_2}} - \underset{+}{\frac{\partial k}{\partial c_1}}\right) \beta\left(\beta R_2 + \mu_2^{SP1}\right)\right)\left(A_2 + \Lambda(\underset{+}{c_2}) q^*\right)}\,. \tag{App.62}$$

If there is no externality, i.e. $\phi = 1$, $\Lambda(c_2) = 1$ and $\frac{\partial \Lambda}{\partial c_2} = 0$, the optimal tax is zero. In this case the constrained social planner cannot improve the allocation as he is also constrained by the collateral constraint.
It is important to underline that in this simplified 3-period model there is no further capital accumulation in period 2, the price of capital/collateral in period 2 hence does not depend on investment in that period and there is no depreciation. Furthermore, the stock of capital is only productive for 3 periods. Thus, some important effects influencing the allocation as well as the sign and size of the debt tax are missing. Nevertheless, this simple model makes it easier to understand some of the effects that are incorporated in a more complex model and it already shows that even without the effects outlined above the level of the optimal debt tax depends on

how strongly it distorts capital formation.
A more complex model containing all these capital effects is needed to fully evaluate the interaction of a debt tax and capital accumulation and is analyzed in the main part of this paper.