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The Global Water Cycle and Climate Policies in a General Equilibrium Model*

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Abstract

Strong evidence suggests that the global economy disrupts the water cycle, and both are closely intertwined with the carbon cycle. However, leading economic analyses of climate change have paid little attention to the water cycle. To address this gap, our paper studies global freshwater resources and the water cycle within a comprehensive climate-economy model. Our model incorporates two distinct externality types: Firstly, fossil fuel use generates CO_2 emissions, leading to a standard climate externality. Secondly, freshwater resources are contaminated during economic activities and due to climate change, resulting in a secondary externality. We derive conditions under which global freshwater becomes a scarce resource and establish a Hotelling rule to determine optimal freshwater resource extraction in closed-form. Key result is an alternative form for the social cost of carbon, which comprises a direct impact component and a novel natural cycle interaction component. This indirect component contributes to a higher social cost of carbon compared to values reported in existing literature. Furthermore, we create a simplified yet comprehensive numerical model to quantitatively evaluate the social cost of carbon and the role of finite freshwater resources and water pollution in the transition to a 'clean' carbon-neutral economy. Our results reveal that emission taxation effectively reduces emissions but concurrently escalates water consumption and degrades water quality. If climate policy exclusively targets direct climate damages while disregarding indirect effects from and on the water cycle, overall welfare outcomes may be less favorable. This contradicts the prevailing perception of a 'clean' carbon-neutral economy as inherently environmentally friendly and socially optimal.

JEL classification: E62, H21, H23, Q25, Q32, Q54

Keywords: Exhaustible Resources, Water Pollution, Water Cycle, Carbon Cyle, Climate Policy, Social Cost of Carbon, Environmental Policy

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1 Introduction

Two natural cycles make our planet capable of sustaining life: the carbon cycle and the water cycle.[1] Together they form the basis for any form of eco-system, and thus for human civilization and economic development.

However, growth of the global economy disrupts both of these critical cycles significantly. Human activities, such as altering land use, burning fossil fuels for energy generation, and cement production, profoundly affect the carbon cycle, contributing to human-induced climate change. Simultaneously, the water cycle undergoes disturbances due to the escalating consumption of freshwater resources for agriculture, energy generation, manufacturing, and sanitation. This less conspicuous issue, while receiving less attention compared to climate change, is of paramount significance as it steadily intensifies the strain on global freshwater resources and worsens water pollution.[2]

The water cycle and the earth's climate are inextricably linked.[3] For instance, extreme weather events are making water more scarce and more polluted.[4] Also, climate change threatens sustainable development, biodiversity, and people's access to water through the water cycle, because flooding and rising sea levels can contaminate water resources with saltwater, and cause damage to water infrastructure.[5]

In order to study the interrelationships between the two natural cycles and their implications for the global economy and to analyze the net effects of alternative environmental policies, we propose a dynamic general equilibrium model. This model incorporates a simplified representation of the global water cycle, the global carbon cycle, and the inclusion of fossil fuel and fresh water as inputs in dirty, respectively clean energy production.

Given the link of the global economy and the water cycle, our first question that we will address in this paper is: *How are fresh water resources and the water cycle connected to the economy and what properties of the cycle determine whether fresh surface and groundwater is a scarce resource?*

One way to reduce disturbances in the carbon cycle is to levy a tax on fossil fuel. Given the intrinsic link between the climate issue and the water cycle, our second focal inquiry is as follows: *What is the influence of the water cycle on the optimal tax for fossil fuel*

[1] A short introduction of the carbon cycle is given in Archer (2010). For a detailed description of the hydrological cycle see for instance Inglezakis et al. (2016).

[2] A recent comprehensive report by The World Bank in 2023 delves into the role of freshwater use (both ground and surface water) in the economy and its environmental implications.

[3] See for instance Wiek & Larson (2012), Conca (2008), Overpeck & Udall (2010).

[4] Roderick et al. (2014) Oki & Kanae (2006), Lambert & Webb (2008), Allan & Soden (2008), Kundzewicz & Gerten (2015), Konapala et al. (2020), Ma et al. (2020), Chahine (1992)

[5] IPCC (2023).

emissions, or more precisely, the social cost of carbon?

To limit global warming, a complete phase-out of fossil fuel is inevitable. Substitutes for fossil fuel are carbon-neutral 'clean' energy technologies.[6] One characteristic of carbon neutral energy technologies is that large amounts of fresh water is needed during production.[7] From the perspective of natural resources involved, shifting from fossil fuel to carbon neutral energy then essentially is a substitution of fossil fuel by fresh water.[8] Since available fresh water resources are finite, a third question to be answered is: *What are the environmental effects of carbon taxation and do binding fresh water resource constraints induce limits to climate policy effectivety?*

In this paper, we develop a complete, yet analytically tractable reduced form model of the global water cycle and combine it with an otherwise standard single-region climate economy model closely related to Golosov et al. (2014). Similar to Nordhaus' (1992) carbon cycle model, our model simulates the movement of water through three primary reservoirs: oceans, freshwater, and atmospheric water vapor, facilitated by processes like evaporation and precipitation. Freshwater, a crucial resource for economic production, is considered quasi-renewable in our analysis.

Within our model, two types of externalities come into play: First, the utilization of fossil fuels leads to CO_2 emissions, resulting in a direct climate externality. Second, economic production contributes to the pollution of freshwater resources, both directly and indirectly through climate change. This creates a dual externality scenario in freshwater, encompassing direct pollution and an indirect climate externality. Consequently, it engenders a complex trade-off between freshwater pollution driven by climate change and freshwater pollution stemming from carbon-neutral energy production.

Our model relates to the large and growing literature on growth, resources, and climate change. One strand of climate-economy models are based on the famous DICE framework pioneered by Nordhaus (1977).[9] A typical feature of these models stressed in Hassler et al. (2016) is that solutions are derived as planning problems without explicit market structures and prices. Thus, these models make only limited use of dynamic general equilibrium theory which confines the class of policies that can be analyzed. Another strand of climate economy models builds on Golosov et al. (2014) whose framework takes full advantage of dynamic general equilibrium theory with explicitly defined markets and price formation. Examples are Rezai & van der Ploeg (2016) who study how the results reported in Golosov et al. (2014) change under more general prefer-

[6] Solar power, wind, hydro power, wood, biofuels, bioenergy, hydrogen, ammonia, synthetic fuels and also nuclear power etc.

[7] See Span et al. (2014).

[8] For instance hydrogen can be used as an energy carrier, industrial feedstock for products and transportation fuels or for long-duration energy storage. One kg hydrogen, however, needs between 20 and 120 litre fresh water, depending on the production technology (Xunpeng et al. (2020).

[9] Nordhaus (1977), Popp (2006), Nordhaus & Sztorc (2013)

ences and technologies, Gerlagh & Liski (2018) who include hyperbolic discounting and an alternative climate model. Cai & Lontzek (2019) develop a dynamic, stochastic general equilibrium framework to study the effects of risk, uncertainty and tipping points in the climate system on the social cost of carbon. Barrage (2020) explores how fiscal distortions affect the optimal climate policy.

As far as we are aware, none of the climate-economy models explicitly consider a global water cycle. The general contribution of our analysis to the literature thus are answers to the questions posed above. First, we derive a basic Hotelling rule for freshwater extraction in closed form, distinguishing between abundant and scarce resources. Second, we show that there is an efficient allocation which determines the optimal level of emissions and fresh water pollution for the global economy. We show that this efficient allocation can be implemented through environmental taxes on fossil fuels and freshwater pollution, for which we derive closed form solutions. The efficient emissions tax is given by the social cost of carbon (SCC), defined as the marginal economic loss caused by an extra metric ton of atmospheric carbon. We show that when considering the water cycle within optimal climate policy, the SCC comprises two components. The first component represents the conventional 'direct' climate impact, while the second component, a novel aspect, arises from the interplay between the carbon and water cycles, termed the 'interaction impact.'

While our model is stylized, it relies on widely accepted standard assumptions concerning preferences and technology. Consequently, we believe that a quantitative analysis remains valuable. In our numerical illustration, we observe that the SCC increases by as much as 20% compared to values found in existing literature, contingent upon specific parameter selections. Hence, incorporating insights from recent natural science literature leads to an upward adjustment of the optimal emissions tax. Our findings both qualitatively and quantitatively signify that, all else being equal, conventional climate-economy models tend to underestimate the appropriate carbon pricing. This complements the observations made by Dietz et al. (2021), who highlight the inconsistency between climate-economy models and natural science-based climate models, revealing a tendency to underestimate the genuine social cost of carbon.

Certainly, there are some important considerations to bear in mind regarding this article. Our model relies on a highly simplified representation of the global economy and employs reduced forms for both natural cycles. Additionally, it's crucial to emphasize a notable distinction: unlike the well-established concept of the 'social cost of carbon,' there isn't a direct equivalent for a 'social cost of water pollution.' A key disparity lies in the fact that the social cost of carbon tends to exhibit consistency across different countries, whereas the same may not hold true for freshwater resources.[10]

Given these considerations, it's essential to recognize that our findings are conceptual

[10]For a recent discussion on this topic, please refer to Rockström et al. (2023) and Wheeler et al. (2023).

in nature and should serve as a starting point for further research and exploration in this complex field.

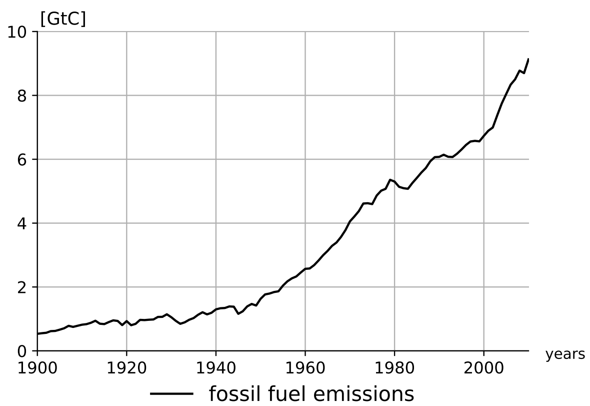
The rest of the paper is organized as follows. Section 2 presents an empirical motivation. Section 3 introduces the model. Section 4 computes the efficient solution. Results from a numerical simulation study are presented in Section 5. Section 6 concludes. Mathematical proofs and computational details are placed in the appendix.

2 Empirical motivation

Our primary focus lies within the context of the global economy. Our model encompasses two natural resources as production inputs, and we adopt a broad interpretation for each:

- Fossil Fuel: This category encompasses all goods and services whose production relies on energy derived from coal, oil, and natural gas.
- Freshwater: Within this category, we include all goods and services that necessitate the use of clean, fresh surface- or groundwater. Notably, this encompasses a range of carbon-neutral energy technologies, such as biofuels, biogas, hydrogen, ammonia, synthetic fuels, wood, nuclear power, and more.

By taking this comprehensive approach, we aim to capture the intricate interplay between these essential resources and their roles in the global economy.



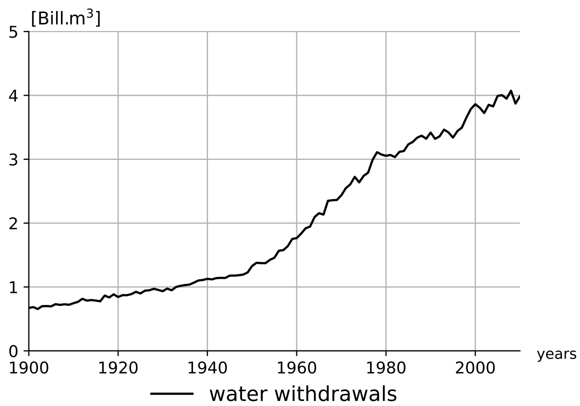


Figure 1: Fossil fuel emissions and fresh-water withdrawals 1900-2010

Figure 1 depicts time series for fossil fuel emissions and fresh water withdrawals for the years 1900 to 2010 according to data from the Worldbank (2020). The figure indicates a strong similarity between both time series: While global fossil fuel emissions increased by over 1000% relative to pre-industrial levels, global water demand has increased by more than 500% over the last 100 years.

Increasing global water demand leads to fresh water scarcity and water constraints. The UN (2021) reports that (i) 2.3 billion people live in water-stressed countries, of which 733 million live in high and critically water-stressed countries.[11]. (ii) 3.2 billion people live in agricultural areas with high to very high water shortages or scarcity, of whom 1.2 billion people live in severely water-constrained agricultural areas (FAO (2020)).[12]

Energy type	Energy carrier	Water input [m^3/GJ]		
		Estimate	Min	Max
FOSSIL FUEL				
	Coal	0.043	0.006	0.242
	Conventional oil	0.081	0.036	0.140
	Natural gas	0.004	0.001	0.027
NUCLEAR POWER		0.105	0.052	0.351
BIOFUELS				
	Sugarcane (ethanol)	24.550	0.000	152.000
	Maize (ethanol)	8.090	0.000	554.000
	Sugarbeet (ethanol)	9.790	0.000	157.000
	Rapeseed (biodiesel)	19.740	0.000	270.000
	Soybean (biodiesel)	11.260	0.000	844.000
HYDROGEN				
	Electrolysis	0.580	0.1410	1.001

Table 1: Fuel types and water consumption factors

Table 1 shows a summary of selected energy production technologies and estimates (minimum, maximum) for direct water consumption factors for each energy production type based on Span et al. (2014) and own calculations. The numbers show that energy production uses water, for instance fossil fuel needs between 0.040 and 0.081 m^3/Gigajoule [GJ] energy. Comparing, clean energy technologies' the specific water input per GJ energy is significantly higher. For instance biofuels such as ethanol need

[11]When a territory withdraws 25% or more of its renewable freshwater resources it is said to be 'water-stressed'. Five out of 11 regions have water stress values above 25%, including two regions with high water stress and one with extreme water stress. (UN-Water, 2021)

[12]Water scarcity is a relative concept. The amount of water that can be physically accessed varies as supply and demand changes. Water scarcity intensifies as demand increases and/or as water supply is affected by decreasing quantity or quality.

ca. 8 to 24 m^3/[GJ] energy and hydrogen produced from electrolysis has a specific water-consumption of 0.58 m^3/[GJ].

Additionally, transition to clean energy necessitates substantial quantities of critical minerals and rare earth metals, such as lithium, cobalt, and graphite, which are essential for manufacturing electric vehicles, solar panels, and other clean technologies. The current extraction processes for these raw materials, particularly lithium, are highly water-intensive. Depending on the deposit, between 100 and 800m^3 of fresh water is used per tonne of lithium extracted.[13]. Global lithium production tripled between 2010 and 2020.[14] The waste generated from mining and processing, including residual minerals and chemicals, contaminates both surface and groundwater resources.

In summary, the empirical evidence indicates two significant factors. Firstly, transitioning from fossil fuels to a low-carbon economy, a crucial step in combating climate change, is linked with an increase in fresh water consumption and pollution. This heightened demand has the potential to exacerbate pressure on already strained fresh water resources. Secondly, climate change itself contributes to the scarcity and contamination of fresh water. Factors such as flooding and rising sea levels can introduce saltwater contamination and inflict damage upon water infrastructure.[15] Given the uncertainty regarding these effects and the lack of an explicit water cycle formulation in the climate-economics literature, we believe that this makes the present study worthwhile.

3 The model

This section outlines our theoretical model, which combines a water pollution model akin to Acemoglu et al. (2012) with a single-region climate-economy model inspired by Golosov et al. (2014). A feature, distinguishing our model from standard Integrated Assessment Models (IAMs), is that agents exhibit preferences not only for consumption and climate but also for water quality. The notation is from the model presented in Hillebrand & Hillebrand (2024).

Our model encompasses three production sectors. The aggregate final consumption-investment good is manufactured using capital and two natural resource inputs: fossil fuel and freshwater. Therefore, the clean sector encompasses various forms of water consumption (e.g., farming) and carbon-neutral energy sources such as hydrogen, synthetic fuels, biofuels, biogas, and nuclear power, all of which require freshwater for energy generation. Both resource inputs are extracted from resource stocks and supplied to the final output firm. Fossil fuel usage results in CO_2 emissions, leading to

[13] Vera et al. (2023)

[14] Global EV Outlook (IEA, 2021); https://www.iea.org/reports/global-ev-outlook-2021.

[15] IPCC (2023).

climate changes. Carbon-neutral energy production and climate changes, in turn, impact freshwater quality as they contribute to water pollution. This water pollution effect accumulates and influences the overall quality of available freshwater.

The key innovation in comparison to modern IAMs is our incorporation of a reduced yet comprehensive description of the global water cycle and a water pollution externality into the model. This linkage connects the two global natural cycles—carbon and water—with the economic dynamics.

The world economy evolves in discrete time $t \in \{0,1,2,\ldots\}$. The main building blocks of our model are the consumption sector, the production stage, the water cycle, and the carbon cycle which we describe in detail next.

3.1 Consumption sector

Consumption is described by a single representative price taking consumer who supplies capital K_t^s to the capital market and takes initial capital stocks in period $t = 0$ as given. The representative consumer receives the return r_t on current net asset holdings K_t, profit income Π_t, and transfers G_t. All tax revenue is returned to consumers as a lump sum transfer, implying that the consumer receives a transfer in period t equal to

$$G_t = \tau_{x,t} X_t + \tau_{z,t} Z_t, \tag{1}$$

where $\tau_{x,t} X_t$ is revenue from taxing fossil fuel and $\tau_{z,t} Z_t$ is revenue from taxing water pollution. The budget constraint of the consumer can be written as

$$Y_t = K_{t+1} + c_x X_t + c_z Z_t + C_t. \tag{2}$$

Consumer's preferences are represented by a standard time-additive utility function over non-negative consumption sequences $(C_t)_{t\geq 0}$, the climate, described by global temperature $(T_t)_{t\geq 0}$ and fresh-water pollution $(P_t)_{t\geq 0}$. We assume that the climate and water pollution enter preferences additively separably from consumption

$$U\big((C_t, T_t, P_t)_{t\geq 0}\big) = \sum_{t=0}^{\infty} \beta^t \big(u(C_t) - v(P_t, T_t)\big) \tag{3}$$

The discount factor is $\beta \in]0,1[$. We assume that $\bar{P}$ denotes the natural level of fresh water pollution and T_0 denotes the pre-industrial equilibrium global mean surface temperature. The consumer takes the sequences $(T_t, P_t)_{t\geq 0}$ as additional exogenous parameters in the decision problem.[16] So climate change and water pollution are externalities.

[16] Models which adopt this idea of a direct negative environmental impact on utility include Acemoglu et al. (2012), Gerlagh & Liski (2018), or Rezai & van der Ploeg (2016). Models where climate change affects productivity of output negatively, are Golosov et al. (2014), Hassler & Sinn (2016), Hillebrand & Hillebrand (2019). For instance Barrage (2020) studies different types of 'climate damage' formulations, where climate change either affects utility or final output productivity or both.

The instantaneous utility function for consumption $u(C)$ is increasing in C, twice differentiable and concave. The instantaneous (dis-) utility function for environmental quality $v(P_t, T_t)$ is increasing in P and T, twice differentiable and jointly concave in (P,T). Moreover, we impose the following Inada-type conditions on $u(\cdot)$ and $v(\cdot,\cdot)$ for the rest of this paper

Assumption A 1

$$\lim_{C \searrow 0} u'(C) = \infty, \qquad \lim_{C \nearrow \infty} u'(C) = 0 \tag{4}$$

$$\lim_{P \searrow 0} \partial_P v(P,T) = 0, \qquad \lim_{P \nearrow \infty} \partial_P v(P,T) = \infty \tag{5}$$

$$\lim_{T \searrow 0} \partial_T v(P,T) = 0, \qquad \lim_{S \nearrow \infty} \partial_T v(P,T) = \infty \tag{6}$$

3.2 Production sectors

Production consists of three different sectors, a final output sector and two resource extraction sectors. Each sector is described by a standard single representative firm.

Final output

The representative final output firm produces a homogeneous final output commodity Y_t using capital K_t and two natural resources X_t, Z_t as production inputs according to the production technology

$$Y_t = F_t(K_t, X_t, Z_t), \tag{7}$$

where X_t represents fossil fuel and Z_t represents all goods and services based on fresh water.[17] Capital inputs K_t are rented in the capital market at price r_t.

We further impose the following standard restriction on production technology (7) for the rest of this paper. The Inada condition ensures that each factor is employed in production.

Assumption A 2 *The production function $F_t : \mathbb{R}^3_+ \longrightarrow \mathbb{R}_+$ is strictly increasing, strictly concave, and continuously differentiable on $\mathbb{R}^3_{++}$. The partial derivatives satisfy the Inada condition*

$$\lim_{z_n \to 0} \frac{\partial F_t(z_1, z_2, z_3)}{z_n} = \infty \text{ for } n = 1,2,3 \text{ and all } z = (z_1, z_2, z_3) \in \mathbb{R}^3_{++}.$$

[17] Our qualitative results remain unchanged if we assume climate change affects final output productivity as in Golosov et al. (2014). Barrage (2020) allows for both types of climate damages, in Hillebrand & Hillebrand (2019) we discuss these different formulations in a multi-region model.

Natural resource inputs

Each resource sector consists of a single firm which extracts the corresponding resource and supplies it to the global resource market. The amount of resource extracted and supplied in period t are denoted $X_t^s, Z_t^s \geq 0$. To keep the analysis simple, the following analysis assumes that resource firms face constant per unit extraction costs $c_i \geq 0$ and take the initial resource stock $R_{i,0} \geq 0$ as a given parameter for $i \in \{x, z\}$.

The two natural resources feature two key differences: first, consumption of X_t, i.e. burning fossil fuel, cause CO_2-emissions. Consumption of fresh water Z_t does not cause any emission but water use is associated with pollution that negatively affects consumers utility which is determined by water quality. Second, fossil energy and fresh water possess naturally different characteristics which lead to differences in resource stock dynamics and profit maximizing extraction plans.

Fossil fuel
Fossil fuel used as combustibles in energy generation are burned and thus can be consumed only once. Stocks of fossil fuel shrink due to extractions X_t. This gives a standard relationship for fossil fuel resource stock development

$$R_{x,t+1} = R_{x,t} - X_t. \tag{8}$$

Feasible fossil fuel extraction plans then are non-negative sequences $(X_t^s)_{t\geq 0}$ which respect the feasibility constraint

$$\sum_{t=0}^{\infty} X_t^s \leq R_{x,0}. \tag{9}$$

To avoid trivialities, we impose the initial conditions $R_{x,0} > 0$, i.e., initial world fossil fuel reserves are strictly positive.

Fresh water
Fresh water withdrawn from the stock and used in economic production partially becomes sewage and can be recycled to become usable fresh water again. Also, available fresh water resource stock changes due to witdrawals $Z_t \geq 0$ and due to the global hydrological cycle which is introduced next.

3.3 Water cycle

The global hydrological or water cycle consists of three major processes: evaporation, condensation, and precipitation. Liquid fresh or salt water enters the atmospheric reservoir through evaporation as a gas. After condensation, it returns from the atmosphere back to other reservoirs through precipitation. In this paper, we abstract from an explicit formulation of condensation and focus on the other two processes of the water cycle.

The amount of global water is finite and constant and only transitions through the different reservoirs. Water circles across three different reservoirs. Surface and groundwater reservoirs ('freshwater'), W_t^F, the oceans W_t^O, and an atmospheric reservoir for water vapor W_t^A. Water reservoirs contain physical water stocks measured in million cubic meter of water [Mill. cbm] and denoted by the (3×1)-vector W_t. Water is extracted from surface and groundwater reservoirs W_t^F and used in final output production.

We assume the following basic reduced form of the global water cycle, described by a three dimensional system of linear equations:

$$W_{t+1}^F = \omega_{11}W_t^F + \omega_{21}W_t^A + \omega_{31}W_t^O - (1-\xi_1)Z_t, \tag{10a}$$
$$W_{t+1}^A = \omega_{12}W_t^F + \omega_{22}W_t^A + \omega_{32}W_t^O + \xi_2 Z_t, \tag{10b}$$
$$W_{t+1}^O = \omega_{13}W_t^F + \omega_{23}W_t^A + \omega_{33}W_t^O + \xi_3 Z_t, \tag{10c}$$

$\omega_{i,j} \geq 0$ describes share of water going from state i to state j, where $i,j \in \{1,2,3\}$. For instance, ω_{21} is the share of condensed water vapor leaving the atmosphere as terrestrial precipitation and ω_{23} is the share of precipitation over sea. ξ_1 is (technological or natural) efficiency of 'fresh water recycling'. ξ_2 is a share of used water which enters the atmosphere, e.g. hydrogen. ξ_3 is a share of fresh water which enters the oceans as waste water. Write (10) as:

$$\begin{pmatrix} W_{t+1}^F \\ W_{t+1}^A \\ W_{t+1}^O \end{pmatrix} = \begin{bmatrix} \omega_{11} & \omega_{21} & \omega_{31} \\ \omega_{12} & \omega_{22} & \omega_{32} \\ \omega_{13} & \omega_{23} & \omega_{33} \end{bmatrix} \begin{pmatrix} W_t^F \\ W_t^A \\ W_t^S \end{pmatrix} - \begin{pmatrix} (1-\xi_1) \\ -\xi_2 \\ -\xi_3 \end{pmatrix} Z_t, \tag{11}$$

so we can rewrite (11) in compact form:

$$W_{t+1} = \Omega W_t - (e-\xi)Z_t, \tag{12}$$

where $W_t := (W_t^F, W_t^A, W_t^O)^\top$, $e = (1,0,0)^\top$, $\xi = (\xi_1, \xi_2, \xi_3)^\top$ and

$$\Omega := \begin{bmatrix} \omega_{11} & \omega_{21} & \omega_{31} \\ \omega_{12} & \omega_{22} & \omega_{32} \\ \omega_{13} & \omega_{23} & \omega_{33} \end{bmatrix}, \tag{13}$$

where Ω is constant in time and describes the circulation of water between the different reservoirs. Next definition and assumption capture key characteristics of the global hydrological cycle:

Definition 1 *The hydrological cycle* (10) *is closed if no water leaves the system*

$$\sum_{i\in\{1,2,3\}} \xi_i = 1 \tag{14}$$

and is semi-closed if water can leave the system:

$$\sum_{i\in\{1,2,3\}} \xi_i \leq 1. \tag{15}$$

Assumption A 3 *No direct circulation between the oceans W^O and freshwater W^F is allowed:*

$$\omega_{31} = 0. \tag{16}$$

Note that definition 1 in conjunction with assumption 3 imply: $\omega_{11} = 1 - \omega_{12} - \omega_{13}$, $\omega_{22} = 1 - \omega_{21} - \omega_{23}$, and $\omega_{33} = 1 - \omega_{32}$ so Ω finally reads:

$$\Omega := \begin{bmatrix} 1-\omega_{12}-\omega_{13} & \omega_{21} & 0 \\ \omega_{12} & 1-\omega_{21}-\omega_{23} & \omega_{32} \\ \omega_{13} & \omega_{23} & 1-\omega_{32} \end{bmatrix} \tag{17}$$

This is a rather parsimonious formulation of this complex natural cycle. It nevertheless captures two key characteristics that are most relevant for our analysis, i.e. differences in resource depletion dynamics of a quasi-renewable resource like fresh water compared to a non-renewable resource like fossil fuel and the interaction of the water cycle with the carbon cycle.

Fresh water pollution

In principle, fresh water pollution is not straightforward to measure as polluted water can contain particles from many sources and the effects are also difficult to measure. Common pollutants are sewage, industrial waste, thermal or heat pollution, sedimentary deposits and oil spills, saline groundwater intrusion and land subsidence but also ecosystem degradation, and water basin pollution due to climate change through floods, rising sea levels, droughts etc. To keep things simple, we account for these different types and sources in a parsimonious way and add a 'pollution index' P_t to consumer utility which measures decline in fresh water quality.[18] Formally,

$$P_t = \underbrace{(1-\delta)P_{t-1}}_{\substack{\text{natural water}\\ \text{quality regeneration}}} + \underbrace{\chi Z_t}_{\substack{\text{direct water}\\ \text{pollution}}} + \underbrace{\psi T_t}_{\substack{\text{indirect water pollution}\\ \text{through climate change}}} \tag{18}$$

is the change in water pollution over time. Parameter δ is the rate of natural fresh water regeneration. χ measures degradation in fresh water quality resulting from using water in economic production such as bioenergy, biofuels, hydrogen, ammonia, synthetic fuels, wood, and also nuclear power. ψ measures pollution in fresh water quality resulting from climate change through the channels described above. Equation (18) introduces the first negative externality into our model. Key here is the connection of the carbon cycle with the water cycle so fossil fuel use indirectly affects fresh water quality.

3.4 Carbon cycle

One unit of exhaustible resource X_t contains $\zeta \geq 0$ units of CO_2. We measure fossil fuel directly in units of CO_2 so global emissions in period t are then given by burned fossil

[18] A similar formulation for 'environmental quality' in the utility function is proposed in Acemoglu et al. (2012).

fuel input X_t.

We adopt the carbon cycle model from Geoffroy et al. (2013) which is also described in Folini et al. (2024). Similar to the water cycle, the amount of carbon is finite and constant and only transitions through different reservoirs. In the climate model employed here, carbon circles across three reservoirs. Atmospheric carbon, M_t^A, the upper oceans M_t^U, and the lower oceans M_t^L. The following three-dimensional system of linear equations describes the carbon cycle:

$$M_{t+1}^A = \phi_{11}M_t^A + \phi_{21}M_t^U + \phi_{31}M_t^L + X_t, \tag{19a}$$
$$M_{t+1}^U = \phi_{12}M_t^A + \phi_{22}M_t^U + \phi_{32}M_t^L \tag{19b}$$
$$M_{t+1}^L = \phi_{13}M_t^A + \phi_{23}M_t^U + \phi_{33}M_t^L. \tag{19c}$$

Write (19) in compact form:

$$M_{t+1} = \Phi M_t + eX_t, \tag{20}$$

where $M_t := (M_t^A, M_t^U, M_t^L)^\top$ represents carbon at time t in the three reservoirs, $e = (1,0,0)^\top$, and

$$\Phi := \begin{bmatrix} \phi_{11} & \phi_{21} & \phi_{31} \\ \phi_{12} & \phi_{22} & \phi_{32} \\ \phi_{13} & \phi_{23} & \phi_{33}. \end{bmatrix} \tag{21}$$

The matrix Φ is constant in time and describes carbon circulation between the different reservoirs. Emissions from burning fossil fuel X_t enter the atmospheric carbon reservoir and then diffuse into the other carbon reservoirs. Comparing the two natural cycle systems (10) and (19) already shows quite a remarkable symmetry. The global carbon cycle model features the following key characteristics:

Definition 2 *The carbon cycle* (19) *is closed, i.e. no carbon leaves the system*

$$\sum_{i\in\{1,2,3\}} \phi_{ij} = 1, \quad \textit{for } j = 1,2,3 \tag{22}$$

and is semi-closed if carbon can leave the system:

$$\sum_{i\in\{1,2,3\}} \phi_{ij} \leq 1, \quad \textit{for } j = 1,2,3. \tag{23}$$

Assumption A 4 *No direct circulation between the lower oceans 'L' and atmospheric carbon 'A' is allowed:*

$$\phi_{13} = \phi_{31} = 0. \tag{24}$$

Definition 2 in conjunction with assumption 4 imply: $\phi_{11} = 1 - \phi_{12}$, $\phi_{22} = 1 - \phi_{21} - \phi_{23}$, and $\phi_{33} = 1 - \phi_{32}$ so the matrix Φ finally reads:

$$\Phi := \begin{bmatrix} 1-\phi_{12} & \phi_{21} & 0 \\ \phi_{12} & 1-\phi_{21}-\phi_{23} & \phi_{32} \\ 0 & \phi_{23} & 1-\phi_{32}, \end{bmatrix} \tag{25}$$

implying that in matrix Φ four parameters are used to calibrate the carbon cycle model.

3.5 Temperature dynamics

The atmospheric CO_2-concentration M_t^A relative to its pre-industrial equilibrium-level $\bar{M}^A = 581$ GtC (Gigatonnen Carbon) affects global mean surface temperature T_t, i.e. atmospheric temperature and temperature of the upper oceans. This 'forcing' is described by

$$\Delta_t = \eta \log\left(M_t^A/\bar{M}^A\right)/\log(2). \tag{26}$$

In simple terms, radiative forcing which refers to the difference between the incoming solar radiation absorbed by the Earth and the outgoing thermal radiation emitted by the Earth, typically expressed in watts per square meter (W/m^2). Positive radiative forcing indicates a warming effect, while negative radiative forcing indicates a cooling effect. Equation (26) quantifies the relationship between changes in greenhouse gas concentrations and radiative forcing. As atmospheric CO_2-concentration increases, this traps more outgoing thermal radiation, leading to a positive radiative forcing and an overall warming effect on the Earth's climate.

Global surface temperature is also affected by lower ocean temperature, denoted T_t^L. The following temperature dynamic describes this development in a two-dimensional linear system, described in Geoffroy et al. (2013) and also used in Nordhaus (2018):

$$\begin{aligned} T_{t+1} &= T_t + \theta_1\left(\Delta_t - \theta_2 T_t - \theta_3(T_t - T_t^L)\right), && \text{(27a)} \\ T_{t+1}^L &= T_t^L + \theta_4(T_t - T_t^L). && \text{(27b)} \end{aligned}$$

Parameters $\theta_1, \theta_2, \theta_3, \theta_4$ in (27) can be interpreted as follows. $1/\theta_1$ determines effective heat capacity of the atmosphere and upper ocean layer, while θ_4 determines effective heat capacity of the lower layer. θ_3 determines heat exchange between the upper layer (atmosphere and upper oceans) and the lower layer (lower oceans). θ_2 is a radiative feedback parameter.[19] Equation (27) introduces the second negative externality into our model, as we assume that global warming, i.e. a larger T_t negatively affects consumer utility.

3.6 Summary of the economy

The economy $\mathscr{E}$ introduced in the previous sections can be summarized by its sectoral and resource structure, production technologies, hydrological cycle, climate parameters and consumer parameters.

$$\mathscr{E} = \left\langle (F_t)_{t\geq 0}, (c_i)_{i\in\{x,z\}}, (\xi_j, \omega_{i,j})_{i,j\in\{1,2,3\}}, \eta, (\theta_i)_{i\in\{1,2,3,4\}}, \beta, \chi, \psi, \delta \right\rangle. \tag{28}$$

[19]A fundamental concept in climate change modeling is the so-called Arrhenius relation, who proposed it in 1896. The relation describes how the concentration of greenhouse gases in the atmosphere affects Earth's temperature. The 'λ' in the Arrhenius-relation would be $\eta\theta_2$ in the model notation of our model.

In addition, initial value for capital supply K_0^s, exhaustible resource stock $R_{x,0}$, an initial state for water (W_0^F, W_0^A, W_0^O), are given as well as an initial climate state (M_0^A, M_0^U, M_0^L), temperature (T_0, T_0^L), and the initial level of water pollution $P_0 = \bar{P}$.

4 Planning problem

Our focus is on optimal environmental policies. It is clear that a decentralized solution is not Pareto-optimal due to the presence of the two externalities. However, the planning solution coincides with the decentralized equilibrium by setting taxes equal to zero. Therefore, we restrict our attention here to planning problems and derive the efficient solution which accounts for the different externalities as a solution to a planning problem which maximizes consumer utility subject to the feasibility constraints imposed by technology, resources, and the environmental externalities. A 'constrained optimal' and a 'non-optimal' solution then follow by ignoring the indirect climate externality in the planning problem, respectively both externalities in the planning problem while the rest of the program remains unchanged.[20]

4.1 Efficient solution

Consider a planner choosing an allocation $A = (C_t, K_t, X_t, Z_t)_{t\geq 0}$ which accounts for both externalities in consumer utility. This planner takes initial capital K_0, fossil resources $R_{x,0} \geq 0$, the initial water quality $\bar{P}$, and initial states of the two natural cycles W_0, $M_0 \geq 0$ as given. Also, the planner takes sequences of productivities $(Q_{n,t}, Q_{x,t})_{t\geq 0}$ as given. Formally, the planning problem reads:

$$\max_{A}\Big\{U\big((C_t,P_t,T_t)_{t\geq 0}\big)\;\Big|\;(2),(7),(12),(18),(20),(26),(27), K_t, C_t, X_t, Z_t \geq 0 \text{ hold } \forall t \geq 0\Big\} \tag{29}$$

Problem (29) is a constrained optimization problem. Adopting a standard infinite-dimensional Lagrangian approach, we can obtain explicit conditions which completely characterize the solution to (29) which essentially characterizes the social optimum. Detailed computations can be found in Section A.1 in the Appendix. The main findings are as follows.

4.1.1 Water withdrawals and the water cycle

We characterize the existence of an optimal fresh water extraction plan in the following Proposition:

[20] It seems less relevant to study a fourth hypothetical scenario where external costs of water pollution are internalized but climate change is ignored.

Proposition 1 *Let $\hat{v}_t$ denote the (shadow) price for fresh water and $\hat{p}_{z,t} := \hat{v}_{z,t} - c_z$ denote prices net of extraction costs. Then,* (29) *has an interior solution*

$$0 < Z_t^* < W_{t+1}^F \quad \textit{for all } t = 0, 1, 2, ... \tag{30}$$

if and only if fresh water resource prices satisfy

$$\hat{p}_{z,t+1} = r_{t+1} \hat{p}_{z,t} \quad \forall t \geq 0, \tag{31}$$

In this case, any sequence $(Z_t)_{t\geq 0}$ satisfying (30) *is a solution.*

(i) If the water system (12) *is closed ($\sum_{i\in\{1,2,3\}} \xi_i = 1$), the initial price satisfies*

$$\hat{p}_{z,0} = 0 \quad \Longleftrightarrow \quad \hat{v}_{z,t} = c_z.$$

(ii) If the water system (12) *is semi-closed ($\sum_{i\in\{1,2,3\}} \xi_i \leq 1$), the initial price satisfies*

$$\hat{p}_{z,0} > 0 \quad \Longleftrightarrow \quad \hat{v}_{z,t} > c_z.$$

The proof can be found in Appendix A. Condition (31) is the classical Hotelling rule stating that for the planner to be indifferent between extracting water resources in different periods, net resource prices must grow at the rate of interest. Note that when water is a closed system (case (i)), water has no scarcity rent, i.e., $p_{z,t} = 0$ for all t. The intuition is that freshwater extracted today will come back to the system in the future. Thus, the social planner just needs to wait. But at the same time, the Hotelling rule (31) ensures that the planner is indifferent between periods for extraction, so the timing is irrelevant. Therefore, extractions are not gone but still fully available (in the future). Consequently, fresh water resources are abundant, and so water does not carry a scarcity rent.

4.1.2 Water pollution externality

The total costs Λ_t^P of using one additional unit of fresh water in period t (measured in units of time t consumption) correspond to the discounted sum of all future marginal pollution costs and are given by:

$$\Lambda_t^P = \sum_{n=0}^{\infty} \frac{\beta^n (1-\delta)^n}{u'(C_t)} \frac{\partial v(P_{t+n}, T_{t+n})}{\partial P_{t+n}}. \tag{32}$$

The term (32) is the key quantity to incorporate the (direct) water pollution externality into the (shadow) price of used fresh water resources. Intuitively, the (direct) water pollution externality can be decomposed into three different factors: First a discount factor $\beta^n / u'(C_t)$ serving to measure pollution in $t+n$ in units of time t consumption. Second, the term $(1-\delta)^n$ measures the regeneration of fresh water quality after $t+n$

periods. Third, the marginal (dis-) utility of the consumer at time $t+n$ caused by an additional unit of pollution in fresh water. Summation of these factors over all periods $t, t+1, t+2, \ldots$ then gives total fresh-water pollution generated by one additional unit of fresh water used in period t. Water resource input in period t then earns a marginal product equal to its price net of extraction costs $p_{z,t} := v_{z,t} - c_z$ plus the total discounted marginal pollution in utility:

$$\partial_Z F(K_t, X_t, Z_t) = p_{z,t} + \chi \sum_{n=0}^{\infty} \frac{\beta^n (1-\delta)^n}{u'(C_t)} \frac{\partial v(P_{t+n}, T_{t+n})}{\partial P_{t+n}}. \tag{33}$$

4.1.3 Climate externality

The total costs Λ_t^{MA} of emitting one additional unit of CO_2 in period t (measured in units of time t consumption) into the atmosphere M_t^A corresponds to the discounted sum of all future marginal climate damages caused by this emission. Key difference compared to the standard literature is that the social cost of carbon now contains two externality components. A direct climate externality through fossil fuel emissions. And an indirect climate externality component given by water pollution through climate change. Formally, the social cost of carbon in our model then read:

$$\Lambda_t^{MA} = \sum_{n=0}^{\infty} \frac{\beta^n}{u'(C_t)} \Bigg(\underbrace{\hat{\theta}^n \frac{\partial v(P_{t+n}, T_{t+n})}{\partial T_{t+n}}}_{\text{direct climate impact}} + \underbrace{\frac{\psi (1-\delta)^n}{1-\hat{\theta}\beta} \frac{\partial v(P_{t+n}, T_{t+n})}{\partial P_{t+n}}}_{\text{natural cycles interaction impact}} \Bigg), \tag{34}$$

where $\hat{\theta} := 1 - \theta_1(\theta_2 + \theta_3)$. The term (34) is the key quantity to incorporate the total climate externality into the (shadow) price of fossil fuel resources. Besides the discount factor $\beta^n / u'(C_t)$, the climate externality consists of three additional components: First, the term $\hat{\theta}^n$ which measures the impact of CO_2 emitted at time t on atmospheric temperature at time $t+n$. Second, the term $\partial v(P_{t+n}, T_{t+n}) / \partial T_{t+n}$ which accounts for the marginal loss in consumer utility at time $t+n$ caused by an additional 'unit' of atmospheric temperature. Third the term $\psi (1-\delta)^n / (1-\hat{\theta}\beta) \partial v(P_{t+n}, T_{t+n}) / \partial P_{t+n}$ is the interaction term connecting the two natural cycles which gives the marginal loss in consumer utility $\partial v(P_{t+n}, T_{t+n}) / \partial P_{t+n}$ at time $t+n$ caused by an additional unit of polluted fresh water through climate change $\psi / (1-\hat{\theta}\beta)$ net of natural water regeneration $(1-\delta)^n$. We summarize our main analytical result in the following proposition:

Proposition 2 *If the interaction of the global carbon and water cycle is explicitly considered, the social cost of carbon given in equation* (34) *can be decomposed into two main components besides a discount factor: First, a standard term that quantifies the direct marginal loss in utility attributable to changes in temperature. Second, a term that measures the indirect marginal loss in utility resulting from climate change's impact on the global water cycle and resulting freshwater pollution.*

This result implies that due to the additional second component, the social cost of carbon in our model here is higher compared to values reported in the literature.

4.1.4 Intertemporal efficiency conditions

The remaining optimality conditions ensure intertemporal efficiency of final good allocation is ensured by the standard Euler equation which holds for all $t \geq 1$:

$$\frac{\beta u'(C_{t+1})}{u'(C_t)} \partial_K F_{t+1}\big(K_{t+1}, (X_{t+1}, Z_{t+1}\big) = 1. \tag{35}$$

Setting $\hat{\tau}_{x,t} := \Lambda_t^{MA}$ gives

$$\frac{u'(C_{t+1})}{u'(C_t)} \Bigg(\partial_X F_{t+1}(K_{t+1}, X_{t+1}, Z_{t+1}) - c_x - \hat{\tau}_{x,t+1}\Bigg) = \partial_X F_t(K_t, X_t, Z_t) - c_x - \hat{\tau}_{x,t} \tag{36}$$

Define a shadow price $\hat{v}_{x,t} := \partial_X F_t(K_t, X_t, Z_t) - \hat{\tau}_{x,t}$ corresponding to the marginal product of fossil fuels in production net of the total discounted damage captured by $\hat{\tau}_{x,t}$. Equation (36) defines the true shadow price as the marginal product in production minus the cost of emissions defined in (34). Compared to the planning allocation, the social planner includes a wedge between the marginal product of fossil fuel X_t and its (shadow) price which accounts for the externality cost of an additional unit of emissions.

Next, setting $\hat{\tau}_{z,t} := \Lambda_t^P$ gives

$$\frac{u'(C_{t+1})}{u'(C_t)} \Bigg(\partial_Z F_{t+1}(K_{t+1}, X_{t+1}, Z_{t+1}) - c_z - \hat{\tau}_{z,t+1}\Bigg) = \partial_Z F_t(K_t, X_t, Z_t) - c_z - \hat{\tau}_{z,t} \tag{37}$$

Define a shadow price $\hat{v}_{z,t} := \partial_Z F_t(K_t, X_t, Z_t) - \hat{\tau}_{z,t}$ corresponding to the marginal product of fresh water in production net of the total discounted marginal costs in utility captured by $\hat{\tau}_{z,t}$. (37) corresponds to the Hotelling rule given in (31) in Proposition 1 for the planning problem, ensuring efficient water resource extraction over time.

The following proposition summarizes the result of this section. The proof is given in Section A.1 in the appendix.

Proposition 3 *Let Assumption 1 hold and preferences given by* (3)*. Then, any feasible allocation A which satisfies conditions* (34) - (36) *for all $t \geq 0$ is efficient, i.e., solves* (29)*.*

5 Quantitative implications

This section provides a basic quantitative anlysis. Our results predict the evolution of the model economy under the three political scenarios. The first scenario is a 'no intervention' case where emissions and water pollution are not taxed and disturbances in both natural cycles are fully ignored. The second scenario assumes that the world introduces the optimal emissions tax in the first period $t = 2020$, but ignores the connection of the two cycles and the water pollution externality. The third scenario is the 'efficient' scenario, where all externalities and connections are considered and taxed such that they are fully internalized. We choose these three scenarios because they represent a best-case, a worst-case and a realistic outcome to highlight the role of the water cycle and water pollution. In this sense, our results define the range of possible outcomes under alternative environmental policies.

5.1 Functional forms

Preferences

We assume consumer's preferences given in (3) take the specific form

$$U(C_t, P_t, T_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \left(\alpha_T T_t^{1+\gamma} + \alpha_P P_t^{1+\gamma} \right) \right], \tag{38}$$

where the parameter γ measures the sensitivity of consumer utility to climate change and water pollution and α_T, α_P are positive coefficients. In standard climate-economy models α_T is typically calibrated to some postulated loss of consumption, say $\approx 2\%$ for $T \approx 2°C$.

Production technology

We assume a production function quite similar to Hassler, Krusell & Olovsson (2021) now extended with two natural resources. The production function $F_t : \mathbb{R}_+^3 \longrightarrow \mathbb{R}_+$ in (7) is of the form

$$F_t(K, X, Z) = \left[\kappa \left(K^{\alpha} (Q_{n,t} N_t)^{1-\alpha} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\kappa)(R(X,Z))^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \kappa, \alpha \in]0,1[, \varepsilon > 0. \tag{39}$$

Here, N_t and $Q_{n,t}$ denote population size and labor efficiency at time t. Parameter ε controls the elasticity of substitution between the the capital-labor aggregate and the two natural resources are given by $R(X,Z)$. Fossil fuel and fresh water resource inputs are aggregated according to the following technology:

$$R(X,Z) = \left(\kappa_x (Q_{x,t} X)^{\frac{\varepsilon_x - 1}{\varepsilon_x}} + (1-\kappa_x)(Q_{z,t} Z)^{\frac{\varepsilon_x - 1}{\varepsilon_x}} \right)^{\frac{\varepsilon_x}{\varepsilon_x - 1}}, \quad 0 < \kappa_x < 1, \varepsilon_x > 0. \tag{40}$$

$Q_{x,t}, Q_{z,t}$ represent resource productivity, so $Q_{x,t}, Q_{z,t}$ capture resource augmenting technical change and $Q_{n,t}$ captures labor-augmenting technical change, respectively. They all grow at constant exogenous rates $g_n, g_x, g_z \geq 0$ such that

$$Q_{i,t} = (1+g_i)^t Q_{i,0} \quad i \in \{n, x, z\} \quad \text{for } t \geq 0. \tag{41}$$

Resource sector
We abstract from a resource scarcity problem studied for instance in Hassler et al. (2021). and formally set initial fossil resources $R_{x,0} = \infty$. This assumption implies a constant resource price $v_{x,t} = c_x$.

Hydrological model
The hydrological cycle in our model is a closed system. Fresh water thus has no scarcity rent. This assumption implies a constant resource price $v_{z,t} = c_z$. Initial fresh water resources are nevertheless finite and equal $W_0^F << \infty$ and are specified below.

Climate model
We use the climate model stated in (20) which is described Geoffroy et al. (2013) and also used in Nordhaus (2018) and Folini et al. (2023). The pre-industrial level of atmospheric carbon is $\bar{M}^A = 581$ GtC.

5.2 Calibration

Next, we state the basic parametrization of our simulation model based on empirical observations and predictions. One time period t in our model equals ten years. Our initial model period $t = 0$ represents the year 2011-2020. The time horizon ranges from 2020 to 2120. Subsequent periods representing years 2021 - 2030, 2031 - 2040, etc. are referred to by their endpoints 2030, 2040, etc. Environmental taxes are introduced in the first period, i.e. 2030. We aggregate flow variables such as production output or water consumption over the entire period. Stock variables like capital or fresh water reserves usually refer to the beginning of the period.

Consumer sector
Parameter values for the representative consumer are close to the ones used by GHKT in their benchmark scenario. We set σ, γ in (3) equal to one $\sigma = \gamma = 1.0$ which implies that U is logarithmic in consumption and environmental damages are quadratic.[21] The 'environmental damage' parameters are set to $\alpha_T = 0.000022$ for climate change damages and $\alpha_P = 0.000102$ for water pollution in consumer utility.

The annual discount rate is 1.5% which implies a discount factor $\beta = 0.985^{10}$. We set the initial global capital stock to $\bar{K}_0 = 0.1$ in oder to avoid a transitory effect due to capital adjustments in the initial periods. Initial population level equals 7.432 Billion, we set $N_0 = 0.7423$ and assume that population grows with exogenous rate $g_n = 0.1046$. Resource efficiency g_x, g_z are chosen identical to imply an annual growth rate of 1% which is a conservative estimate. This implies $g_n = g_x = 0.1046 = 10.46\%$ per decade.

Final output production
The literature contains various estimates for the elasticity of substitution between the

[21] See for instance Weitzman (2010) on specific damage function choices and their effect on numerical results.

capital-labor aggregate and fossil fuel ε. For instance, Papageorgiou et al. (2017) argue that this elasticity 'significantly exceeds unity' and estimate it to be about 2. Hassler et al. (2021) obtain a much smaller value of about 0.02 in their estimation for the U.S. economy. Our choice of $\varepsilon = 0.825$ is somewhat in the middle of these estimates. It implies that natural resources are gross complements to other inputs. Note that for $\varepsilon = 1$, $1-\kappa$ is the value of fossil energy relative to GDP. Empirical numbers put this share at about 5%. As we do not only consider fossil fuel but also water in the composite natural resource good, we set a slightly higher value than Hassler et al. (2021) and set $\kappa = 0.88$, despite the fact that $\varepsilon > 1$ in our simulations. With regard to capital costs, we follow Hassler et al. (2021) by setting $\alpha = 0.2632$.

Resource sectors
We set the elasticity of substitution between our two natural resources, $\varepsilon_x = 1.25$, making fossil fuel and fresh water gross substitutes in economic production. κ_x can be interpreted as the cost share of fossil fuel relative to total resource cost in economic production. We set $\kappa_x = 0.90$. Initial fossil resources $R_{x,0} = \infty$ and we set $v_{x,t} = c_x = 0.000090$ implying an extraction cost of 90 \$/t for fossil fuel.

It is difficult to estimate extraction costs for fresh water because average cost of water extraction can vary significantly depending on various factors, including the source, location, technology used for extraction, and local regulatory and environmental conditions. However, we set extraction costs and thus fresh water prices equal to $v_{z,t} = c_z = 0.000010$ implying withdrawal cost of 10.0\$/t for water. Our aim here is not to provide a precise cost estimate, rather than having in mind that water extraction should be much cheaper compared to fossil fuel.

Climate and temperature
We use the parameter set from Nordhaus & Sztorc (2013) to calibrate the climate model, temperature, radiative forcing and initial conditions for the three-layer carbon cycle and the two-dimensional temperature model. We set $\theta_1 = 0.208$, $\theta_2 = 1.1875$, $\theta_3 = 0.31$ and $\theta_4 = 0.05$. The radiative feedback parameter equals $\eta = 3.681$. The initial atmospheric carbon concentration is $M^A_{-1} = 851$ GtC in 2020, the concentration in upper oceans is $M^U_{-1} = 460$ GtC, in the deep oceans $M^L_{-1} = 1740$ GtC and global emissions are set t $X_0 = 95$ GtC in this decade. Initial temperature in the upper layer is $T_{-1} = 0.85$ and in the deep oceans $T^L_{-1} = 0.0068$. The time-constant matrix Φ describes the mass transfer among carbon reservoirs and we use the following values:[22]

$$\Phi = \begin{pmatrix} 0.88 & 0.04700 & 0 \\ 0.12 & 0.94796 & 0.000750 \\ 0 & 0.00500 & 0.009925 \end{pmatrix}$$

[22]See Nordhaus & Sztorc (2013) or Folini et al. (2023) for details.

Water cycle

For the initial state of the hydrological cycle $W_0 := (W_0^F, W_0^A, W_0^O)$, we choose values consistent with empirical estimates on water reservoir quantities: We set initial fresh surface and groundwater equal to $W_0^F = 15.6$ billion cubic meters [bm^3], initial atmospheric water vapor to $W_0^A = 0.9847$ bm^3 , and salt water $W_0^O = 1361.421$ bm^3.[23]

The remaining parameters of the hydrological cycle are as follows. A share of fresh water moving from land to atmosphere (evaporation over land) is set to $\omega_{12} = 0.004678$. $\omega_{32} = 0.00030335$ is a share of fresh water moving from oceans to atmosphere (evaporation over sea). A share of atmospheric water vapor moving to fresh surface and groundwater (precipitation over land) equals $\omega_{21} = 0.114756$ and precipitation over sea is $\omega_{23} = 0.378796$. A share representing the flow from surface fresh and groundwater into the sea is set to $\omega_{13} = 0.0025641$. Finally, the reverse flow of sea water into fresh water reservoirs is $\omega_{31} = 0$.The time-constant matrix Ω describing the transfer among water reservoirs is then given by:[24]

$$\Omega = \begin{pmatrix} 0.99276 & 0.114760 & 0 \\ 0.00468 & 0.506449 & 0.00030 \\ 0.00255 & 0.378790 & 0.99970 \end{pmatrix}$$

Internal fresh water recycling or water resource efficiency is set to $\xi_1 = 0.35$ which implies that 50% of each tonne fresh water withdrawal is consumed of which $\xi_2 = 0.05$ enters the atmosphere (hydrogen) and $\xi_3 = 1 - \xi_1 - \xi_2 = 0.60$ of water withdrawals enter the sea as sewage.[25]

As $\sum \xi_j = 1$, the water cycle in our model simulations is a closed system and fresh water consequently has no scarcity rent.

Water pollution

We set the rate of water pollution from water consumption $\chi = 0.0005$, the rate of water resource pollution from climate change $\psi = 0.006$, and the rate of natural water resource regeneration $\delta = 0.1$.[26]

[23]Details can be found in appendix B.2, in particular Table 2.

[24]See Appendix for detailed calculations.

[25]It is important to distinguish between 'water withdrawal' and effective 'water consumption': 'water withdrawals' define the total amount of water withdrawn from its source to be used. Water extracted may be used, recycled (or returned to rivers or aquifers) and reused several times over. 'Water consumption' defines the portion of water use that is not returned to the original water source after being withdrawn and can no longer be reused. Precise empirical data of effective water consumption are difficult to estimate and thus not available. We used World Bank data on water withdrawals and fresh water used for farming. The amount of water extracted and that actually consumed.

[26]The rate of natural water regeneration from pollution can vary significantly depending on several factors, including the type and extent of pollution, the specific water body, environmental conditions, and the presence of natural purification processes. In some cases, natural processes such as dilution, sedimentation, and microbial degradation can help to reduce or remove pollutants from water over time. For

Computation
We use a slightly adjusted single-region version of the algorithm developed in Hillebrand & Hillebrand (2023). Details can be found in Section A.1 in Appendix A.

5.3 Environmental taxes

With the specific functional forms (18), (20) and (38) for climate and water pollution impacts on consumer utility the efficient tax formulae (32) and (34) take the specific forms:

$$\tau_{x,t} := (1+\gamma)C_t^\sigma \sum_{n=0}^{\infty} \beta^n \left(\alpha_T \hat{\theta}^n T_{t+n}^\gamma + \alpha_P \frac{\psi(1-\delta)^n}{1-\hat{\theta}\beta} P_{t+n}^\gamma \right) \tag{42a}$$

$$\tau_{z,t} := (1+\gamma)C_t^\sigma \sum_{n=0}^{\infty} \beta^n \left(1-\delta\right)^n \alpha_P P_{t+n}^\gamma \tag{42b}$$

Environmental taxes in (42) grow with current consumption levels and depend on the structural parameters of the model and entire future paths of temperature and water pollution. In general, taxes thus can not be computed explicitly.

Next, we briefly study an impulse response experiment, i.e. the reaction of our hydrological system to some initial external change to gain more intuition of the dynamics of our reduced form water cycle model.

5.4 Water cycle impulse response

Figure 2 shows the circulation path of water caused by an impulse of one Gigaton of fresh water [Gt] withdrawn in the first period, contrasted with a counterfactual path without this extraction impulse. We use the calibrated hydrological cycle model and the standard parameter set described above to obtain the graphs We employed the calibrated hydrological cycle model, utilizing the standard parameter set mentioned earlier, to generate the graphs in Figure 2. i) The solid dark line in the figure represents the percentage change in the fresh water reservoir resulting from the initial extraction shock. Commencing at a reduction of -12%, this shock gradually diminishes to slightly above -5% after a span of 200 years, ultimately dwindling to less than 1% in absolute terms after 400 years. Remarkably, it requires over 1500 years for the fresh water reservoir to fully restore itself to its initial state.

example, certain types of pollutants may settle to the bottom of a water body, where they become less concentrated and may eventually be buried by sediments. Microorganisms can also break down some pollutants through biodegradation. However, the rate of natural water regeneration from pollution is often limited, especially when dealing with persistent or highly toxic contaminants. In many cases, pollutants can persist in the environment for extended periods, causing long-term harm to aquatic ecosystems and potentially impacting human health.

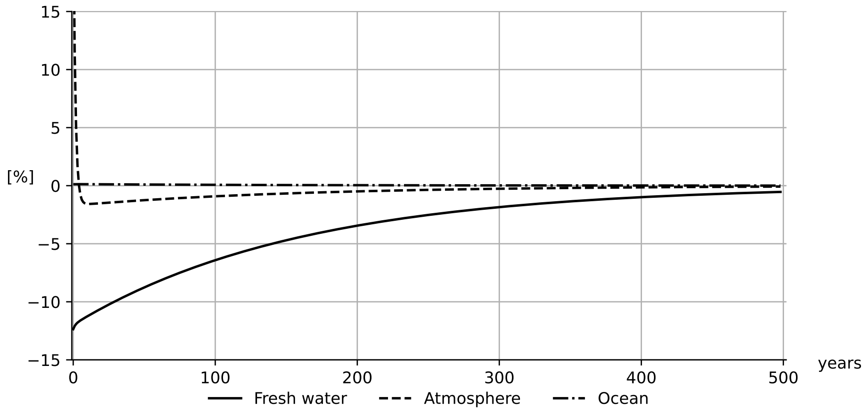


Figure 2: Response of an initial fresh water extraction impulse

The dashed line illustrates the evolution of water in the atmospheric reservoir over time. An initial withdrawal of fresh water results in an immediate and substantial percentage increase in atmospheric water vapor. According to our assumption, 2% of the consumed water enters the atmosphere. Consequently, this initial extraction shock elevates the water content in the atmosphere by more than 20%, with the entire increase being absorbed within a mere four years.

The dotted-dashed line represents the response of sea water reservoir over time. An initial fresh water withdrawal shock translates into an increase in the amount of sea water, since our parameter choices imply that 18% of water consumed enters the oceans as sewage. An initial extraction of fresh water, increases water in the sea by 0.13% immediately and permanently. It takes more than 1500 years for the sea water reservoir to reach its initial level.

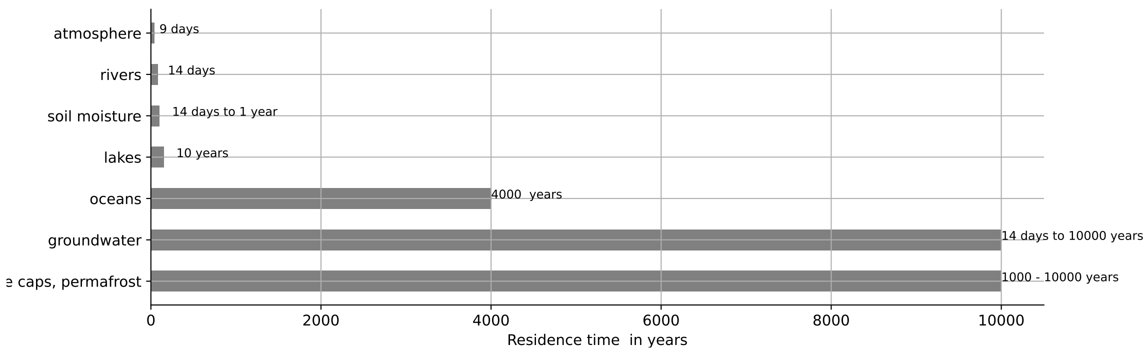


Figure 3: Estimated Residence time of water resources

The dynamics just described are due to finite water on earth, the water cycle being a closed system, and the 'residence time' which describes the amount of time, a water molecule spends in a reservoir during a cycle. Figure 3 reports empirically estimated residence times for water on the different reservoir types. Residence times for water in groundwater reservoirs, in the Antarctic ice sheet and in the oceans, are more than thousands of years. In contrast, water remains up to ten years in lakes and two weeks in rivers. The residence time of water in the atmosphere is the shortest of all, and estimated to be about nine days.

Summarizing, based on these empirical estimates, our calibrated water cycle model computes residence times within empirically plausible ranges. Although the water cycle is a closed system and water consequently has no scarcity rent at any given point in time, fresh water extraction can be ’too large’, given that used fresh water (waste water or vapor) needs time to complete the cycle to become liquid fresh water again.

5.5 Numerical simulation results

Given assumptions on functional forms, parameter sets and resource stocks, we now generate quantity paths –optimal and suboptimal ones– for fossil fuel and fresh water use thus for climate and water cycle disturbances and their economic implications. We then discuss potential limits to climate policy imposed by the hydrological cycle and fresh water resource stocks and propose solutions to overcome this via coordinated environmental economic policies.

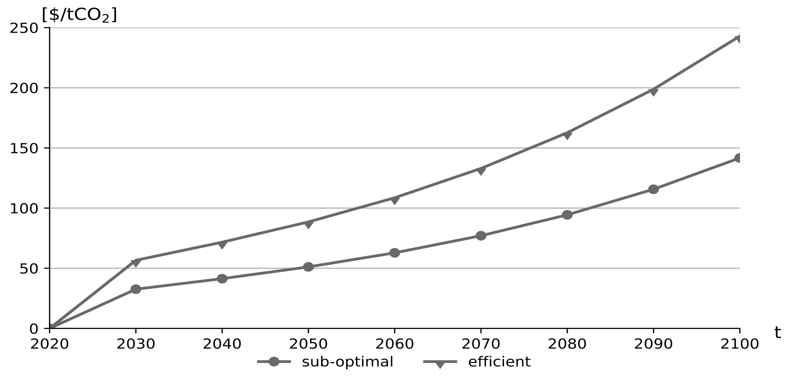


Figure 4: Social cost of carbon

5.5.1 Social cost of carbon

Figure 4 quantifies the social cost of carbon in $ per ton of CO_2 under optimal and sub-optimal taxation.[27] Our parametrization yields a suboptimal carbon tax equal to

[27]Throughout the rest of the numerical example, the optimal or efficient tax is the emissions tax in (42), while the sub-optimal tax is given by setting $\Psi = 0$ in (42). In the case of no intervention, both environmental taxes are equal to zero.

32\$/t CO_2 in 2030 if the interaction component of the two cycles is ignored and a higher efficient carbon tax equal to 59\$/t CO_2 in 2030. The former is in range of optimal emissions taxes reported, e.g., in Nordhaus (2007) and Golosov et al. (2014), the latter is much higher due to the additional climate interaction component in the social cost of carbon (42).

We also see that both climate taxes increase over time reflecting the growth trend of GDP, respectively consumption relative to climate and water pollution levels which must be accompanied by a corresponding increase in the price of carbon emissions for both optimal and sub-optimal environmental policies. Comparing sub-optimal and optimal carbon taxation, the gap increases due to the additional component in (42) and differences in endogenous consumption paths. Initially, the difference is 27\$/t CO_2 (82%) and rises over time such that in 2100, with 253 \$/t CO_2 the optimal tax is 112\$ (79%) higher than the sub-optimal tax (141\$/t CO_2).

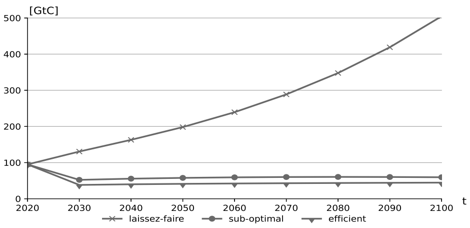
5.5.2 Equilibrium dynamics under different taxation regimes

Comparing the efficient and the sub-optimal solution, Figure 5 depicts the predicted evolution of selected economic and environmental variables over the next 100 years.

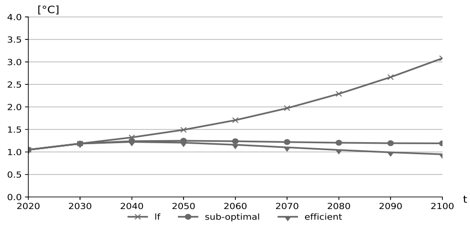
Emissions and global temperature
The main criterion to evaluate the success of climate policy is whether it reduces emissions. Figure 5 (a) shows how global emissions evolve over time under the three political scenarios. The result confirms that introducing a carbon tax in $t = 1$ leads to a substantial and permanent reduction in emissions for both the sub-optimal and the optimal tax scenario. Emissions in 2030 decline by about 70% under the optimal tax relative to the no-intervention equilibrium and by 59% under sub-optimal taxation. This is in stark contrast to the no-intervention scenario where emissions continue to grow without bounds due to a continuous increase in fossil fuel consumption. This translates into initially increasing global mean surface temperature for all scenarios, even for optimal taxation (cf. Figure 5 (b)). While the rise in temperature is ever increasing under laissez-faire, under both climate taxation scenarios, the increase is limited and reaches its maximum in 2040. In the sub-optimal tax scenario, temperature then remains almost constant at 1.3°C until 2100. Under efficient taxation, the model predicts a slight decline such that temperature in 2100 reaches initial levels.

Fresh water use and water stress
Figure 5 (c) and (d) depicts fresh water consumption in absolute terms and relative to initial available fresh and ground water resources for the three scenarios. Note that the latter describes 'water stress' which measures to what extend water demand exceeds available internal resources during a certain period.[28] The dotted horizontal line shows

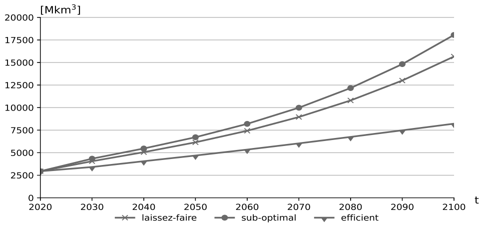
[28]Water stress causes deterioration of fresh water resources in terms of quantity (aquifer over-exploitation, dry rivers, etc.) and quality (eutrophication, organic matter pollution, saline intrusion,



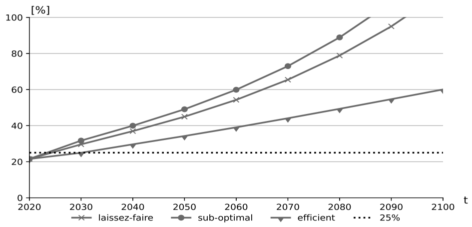
(a) Emissions



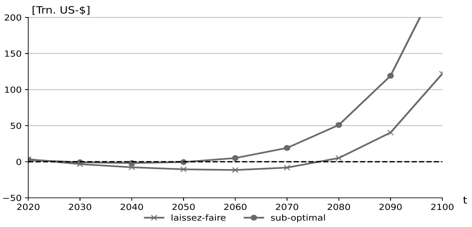
(b) Atmospheric temperature



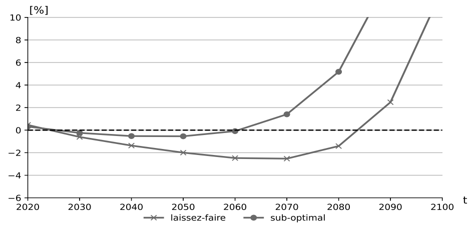
(c) Water use



(d) Water stress



(e) Equivalent compensation in Trn. $



(f) Equivalent compensation in %

Figure 5: Evolution under optimal and sub-optimal taxation

the UN water stress indicator (UN (2021)), which defines a territory to be 'water-stressed, if water withdrawals exceed 25 per cent of its renewable freshwater resources.[29]

Shifting from fossil fuel based energy production towards a low carbon economy through imposing the sub-optimal tax on fossil fuel consumption only, results in significantly higher levels of fresh water use and ceteris paribus water pollution, even though water pollution resulting from climate change is reduced. Water consumption is 2948 Billion cubic meter (Bm^3) and doubles within the next 30 years in both, the no-tax and the sub-optimal tax scenario. 80 years from now in 2100, water use under laissez-faire and sub-optimal carbon taxation is about five times higher than optimally. Compared to this, in the efficient solution, water consumption 'only' doubles at ca.7000 Bm^3 until the end of the century.

A result of this water consumption pattern is increasing water stress such that the UN's

etc.). See EEA (1999): Environment in the European Union at the turn of the century. Page 155. Environmental assessment report No 2.

[29]At the global level, the UN reports a value of 19.5% between 2015 and 2019 for the water stress indicator.

critical water stress threshold of 25% is reached already between 2020 and 2030 in the sub-optimal tax scenario and the fossil fuel dominated no-intervention equilibrium. This trend continues with water stress levels reaching 100% within the 21st century. Compared to this, under efficient taxation, water stress reaches 60% at the end of the century. This result indicates that the water cycle and the effects of climate change and climate policy on fresh water resources should be considered in more integrative economic analyses of climate change.

Equivalent consumption compensation

To compare welfare levels, the representative consumer receives additional consumption levels ΔC_t lump-sum per period such that welfare in the laissez-faire and sub-optimal tax scenarios are equal to efficient welfare levels:

$$\Delta(C_t^{pol}) = \left[(1-\sigma)\left(\bar{U}_t^{eff} + 1 + \alpha_T (T_t^{pol})^2 + \alpha_P (P_t^{pol})^2\right)\right]^{\frac{1}{1-\sigma}} - C_t^{pol} \quad \text{for} \quad pol \in \{\text{lf, so}\}. \tag{43}$$

Figure 5 (e) and (f) depict the evolution of welfare compensations measured in Trn. US-$ respectively in percent. According to the model's predictions, welfare levels are anticipated to be temporary higher under laissez-faire conditions which can be seen by the negative values of welfare compensation which mean that welfare levels are higher under no environmental taxes compared to the efficient scenario, implying that we would have to reduce consumption in the no-tax and sub-optimal tax scenario relative to efficient taxation.

Comparing laissez-faire and optimal taxation, consumption would need to be curtailed by as much as 18 trillion dollars, equivalent to 2.5% of 2020 consumption levels, for a period spanning five decades after the introduction of the optimal tax. Afterwards, consumer in laissez-fire would need continous and rising compensation, ultimately reaching levels as high as 186 trillion dollars or 15% of initial 2020 consumption levels. Similarly, adjusting for lower welfare under efficient environmental taxation relative to sub-optimal taxation is projected to be up to four trillion $ over the next 40 years before the efficient solution reaches beneficial welfare levels. In 2100, lump-sum compensation reaches 31 trillion $ or 4% of 2020 consumption levels.

6 Conclusions

In this article, we introduce a unified global general equilibrium model that integrates two fundamental natural cycles – the carbon cycle and the water cycle – along with two finite natural resources: fossil fuels and fresh water. These cycles are disrupted by economic activities, leading to negative external costs, here measured in diminishing consumer welfare. This includes a standard climate externality arising from fossil fuel combustion and a water pollution externality a water pollution externality, where water quality deteriorates due to both the consumption of fresh water and the influence of climate change.

Our paper is a first step toward a comprehensive framework that can be used to analyze alternative climate policies taking into account the intricate interplay between the carbon cycle and the water cycle. Our findings provide insights into the effects of various climate policies and the challenges of transitioning to carbon-neutral technologies if climate damages measures are interpreted broader than GDP damages only. This is especially relevant when taxing fossil fuel emissions while low-carbon energy technologies and climate change both adversely impact finite freshwater resources which are fundamental to any form of ecosystem.

First, we developed a reduced form model of the global water cycle and established criteria for the emergence of a scarcity rent associated with fresh water. Our analysis revealed that when the global water cycle operates as a closed system, fresh water resources are abundant and do not yield a scarcity rent. Conversely, when the global water cycle is conceptualized as a semi-closed system, fresh water becomes a finite resource and results in a scarcity rent.

Second, we characterized the structure of equilibria and the dynamic environmental tax policies that maximize intertemporal consumer welfare. Our examination underscores the importance of climate policy aimed at mitigating climate change impacts. This extends beyond the direct economic consequences of carbon emission taxation and encompasses the secondary effects arising from the interplay between climate and the Earth's water cycle, as posited in natural science literature.

Formally, we demonstrate that when the two cycles are intertwined, the social cost of carbon can be divided into two components. The first component arises from the direct consequences of climate change. The second component stems from an indirect interaction between the two natural cycles. The second component contributes to a higher social cost of carbon compared to values typically found in existing literature.

Third, to quantitatively assess the influence of the water cycle on optimal climate policies, we developed a concise yet comprehensive numerical model. This model was fine-tuned to align with critical empirical data points, such as fossil fuel emissions and freshwater withdrawals. The model predicts a significant reduction in emissions due to fossil fuel taxation, yet it highlights a simultaneous increase in water consumption. Since our accessible freshwater resources are limited, this shift from fossil fuels to carbon-neutral energy production results in elevated water stress, as measured by the ratio of water consumption to available internal freshwater resources. This trend suggests that freshwater scarcity could potentially impede the growth of a low-carbon economy in the future, unless we see improvements in internal water recycling rates or the widespread adoption of large-scale seawater desalination.

Our analysis estimated the global welfare costs associated with either not taxing carbon at all or inadequately taxing carbon over the next century. The results indicate that under laissez-faire conditions, the equivalent impact on consumption ranges from -11 to 121 trillion dollars (equivalent to -1.6% to 18% of 2020 consumption levels). Under sub-optimal taxation, this impact ranges from -1.8 to 250 trillion dollars (equivalent to -0.25% to 37% of 2020 consumption levels). Furthermore, adjusting for lower welfare under optimal taxation is projected to be on average 10 trillion dollars over 40 years under laissez-faire conditions and more than 1 trillion dollars over 30 years under sub-optimal taxation.

Several directions of future research appear fruitful. One intriguing direction is the incorporation of endogenous and directed technological change, akin to the approaches of Acemoglu et al. (2012) and Hassler et al. (2021). Additionally, it appears crucial to explicitly model an energy sector, including hydrogen production, following the framework of Golosov et al. (2014), where energy production relies on inputs such as capital, labor, and natural resources.

Given the uneven global distribution of available ground and surface freshwater, it would be beneficial to develop a multi-country model that integrates a global water cycle alongside environmental constraints. This would provide a more comprehensive understanding of how regional disparities in freshwater resources impact economic and environmental dynamics.

A Proofs and mathematical derivations

A.1 The social planning problem

The boundary behavior (2) of F_t and of the utility function (3) ensures that any solution to (29) satisfies $K_t > 0$, $X_t > 0$, $Z_t > 0$, and $C_t > 0$ for all t. Thus, we can neglect non-negativity constraints. Define remaining non-negative Lagrange variables μ_x and for each $t \geq 0$, $\mu_t := (\mu_{F,t}, \mu_{A,t}, \mu_{U,t}, \mu_{L,t}, \mu_{\Delta,t})$, $\lambda_t := (\lambda_{0,t}, \lambda_{F,t}, \lambda_{A,t}, \lambda_{O,t}, \lambda_{P,t}, \lambda_{T,t}, \lambda_{L,t})$ and the Lagrange function:

$$
\begin{aligned}
\mathscr{L}\Big(\big(C_t, K_t, X_t, Z_t, W_{t+1}, M_t, P_t, T_t\big)_{t\geq 0}, \big(\lambda_t, \mu_t\big)_{t\geq 0}, \mu_x\Big) := \sum_{t=0}^{\infty} \beta^t \Big(u(C_t) - v(P_t, T_t)\Big) \\
\mu_x\Big(R_{x,0} - \sum_{t=0}^{\infty} X_t\Big) + \sum_{t=0}^{\infty} \mu_{F,t}\Big((1-\xi_1)Z_t - W_{t+1}^F\Big) \\
+ \sum_{t=0}^{\infty} \lambda_{0,t}\Big(F_t(K_t, X_t, Z_t) - C_t - K_{t+1} - c_x X_t - c_z Z_t\Big) \\
+ \sum_{t=0}^{\infty} \lambda_{F,t}\Big((1-\omega_{12}-\omega_{13})W_t^F + \omega_{21} W_t^A - (1-\xi_1)Z_t - W_{t+1}^F\Big) \\
+ \sum_{t=0}^{\infty} \lambda_{A,t}\Big(\omega_{12} W_t^F + (1-\omega_{21}-\omega_{23})W_t^A + \omega_{32} W_t^O + \xi_2 Z_t - W_{t+1}^A\Big) \\
+ \sum_{t=0}^{\infty} \lambda_{O,t}\Big(\omega_{13} W_t^F + \omega_{23} W_t^A + (1-\omega_{32})W_t^O + \xi_3 Z_t - W_{t+1}^O\Big) \\
+ \sum_{t=0}^{\infty} \lambda_{P,t}\Big(P_t - (1-\delta)P_{t-1} - \chi Z_t - \psi T_t\Big) \\
+ \sum_{t=0}^{\infty} \lambda_{T,t}\Big(T_t + \theta_1\big(\Delta_t - \theta_2 T_t - \theta_3(T_t - T_t^L)\big) - T_{t+1}\Big) \\
+ \sum_{t=0}^{\infty} \lambda_{L,t}\Big(T_t^L + \theta_4\big(T_t - T_t^L\big) - T_{t+1}^L\Big) \\
+ \sum_{t=0}^{\infty} \mu_{A,t}\Big((1-\phi_{12})M_t^A + \phi_{21} M_t^U + X_t - M_{t+1}^A\Big) \\
+ \sum_{t=0}^{\infty} \mu_{U,t}\Big(\phi_{12} M_t^A + (1-\phi_{21}-\phi_{23})M_t^U + \phi_{32} M_t^L - M_{t+1}^U\Big) \\
+ \sum_{t=0}^{\infty} \mu_{L,t}\Big(\phi_{23} M_t^U + (1-\phi_{32})M_t^L - M_{t+1}^L\Big) \\
+ \sum_{t=0}^{\infty} \mu_{\Delta,t}\Big(\eta \log\big(M_t^A / \bar{M}^A\big)/\log(2) - \Delta_t\Big)
\end{aligned}
$$

Any solution to (29) has to satisfy the first order and complementary slackness conditions.

Then, for each $t = 0, 1, 2, \ldots$ the first order conditions are:

$$\frac{\partial \mathscr{L}(-)}{\partial C_t}: \qquad \beta^t u'(C_t) - \lambda_{0,t} = 0 \tag{A.2a}$$

$$\frac{\partial \mathscr{L}(-)}{\partial K_t}: \qquad \lambda_{0,t}\partial_K F_t(K_t, X_t, Z_t) - \lambda_{0,t-1} = 0 \tag{A.2b}$$

$$\frac{\partial \mathscr{L}(-)}{\partial X_t}: \qquad \lambda_{0,t}\partial_X F_t(K_t, X_t, Z_t) - \lambda_{0,t}c_x - \mu_x + \mu_{A,t} = 0, \tag{A.2c}$$

$$\begin{aligned}\frac{\partial \mathscr{L}(-)}{\partial Z_t}: \qquad & \lambda_{0,t}\partial_Z F_t(K_t, X_t, Z_t) - \lambda_{0,t}c_z + \mu_{F,t}(1-\xi_1) \\ & - \lambda_{F,t}(1-\xi_1) + \lambda_{A,t}\xi_2 + \lambda_{O,t}\xi_3 - \chi\lambda_{P,t} = 0\end{aligned} \tag{A.2d}$$

$$\frac{\partial \mathscr{L}(-)}{\partial W^F_{t+1}}: \qquad -\lambda_{F,t} + \lambda_{F,t+1}(1-\omega_{12}-\omega_{13}) + \lambda_{A,t+1}\omega_{12} + \lambda_{O,t+1}\omega_{13} = 0 \tag{A.2e}$$

$$\frac{\partial \mathscr{L}(-)}{\partial W^A_{t+1}}: \qquad -\lambda_{A,t} + \lambda_{F,t+1}\omega_{21} + \lambda_{A,t+1}(1-\omega_{21}-\omega_{23}) + \lambda_{O,t+1}\omega_{23} = 0 \tag{A.2f}$$

$$\frac{\partial \mathscr{L}(-)}{\partial W^O_{t+1}}: \qquad -\lambda_{O,t} + \lambda_{A,t+1}\omega_{32} + \lambda_{O,t+1}(1-\omega_{32}) = 0 \tag{A.2g}$$

$$\frac{\partial \mathscr{L}(-)}{\partial P_t}: \qquad -\beta^t\partial v_P(P_t, T_t) - \lambda_{P,t} + \lambda_{P,t+1}(1-\delta) = 0 \tag{A.2h}$$

$$\frac{\partial \mathscr{L}(-)}{\partial T_t}: \qquad -\beta^t\partial v_T(P_t, T_t) - \psi\lambda_{P,t} + \lambda_{T,t}\big(1-\theta_1(\theta_2+\theta_3)\big) - \lambda_{T,t-1} + \theta_4\lambda_{L,t} = 0 \tag{A.2i}$$

$$\frac{\partial \mathscr{L}(-)}{\partial T^L_t}: \qquad \theta_3\lambda_{T,t} + (1-\theta_4)\lambda_{L,t} - \lambda_{L,t-1} = 0 \tag{A.2j}$$

$$\frac{\partial \mathscr{L}(-)}{\partial M^A_{t+1}}: \qquad (1-\phi_{12})\mu_{A,t+1} + \phi_{12}\mu_{U,t+1} + \eta/\log(2)\mu_{A,t+1}/M^A_{t+1} - \mu_{A,t} = 0 \tag{A.2k}$$

$$\frac{\partial \mathscr{L}(-)}{\partial M^U_{t+1}}: \qquad \phi_{21}\mu_{A,t+1} + (1-\phi_{21}-\phi_{23})\mu_{U,t+1} + \phi_{23}\mu_{L,t+1} - \mu_{U,t} = 0 \tag{A.2l}$$

$$\frac{\partial \mathscr{L}(-)}{\partial M^L_{t+1}}: \qquad \phi_{32}\mu_{U,t+1} + (1-\phi_{32})\mu_{L,t+1} - \mu_{L,t} = 0 \tag{A.2m}$$

$$\frac{\partial \mathscr{L}(-)}{\partial \Delta_t}: \qquad \theta_1\lambda_{T,t} - \mu_{\Delta,t} = 0 \tag{A.2n}$$

Solving (A.2a) gives

$$\lambda_{0,t} = \beta^t u'(C_t) \tag{A.3}$$

Using (A.3) in (A.2b) gives

$$\frac{u'(C_{t-1})}{u'(C_t)} = \beta\partial_K F_t\big(K_t, (X_t, Z_t)\big),$$

which is a standard Euler equation.

A.1.1 Water pollution externality

Equation (A.2h) implies that the shadow value of water pollution at time t is equal to the marginal (dis-)utility that it generates in this period plus the shadow value of $(1-\delta)$ units of water pollution at time $t+1$. Assuming that $\lim_{n\to\infty} \beta^{n+1} \lambda_{P,t+n} = 0$ (A.2h) can be solved forward to obtain the shadow value of water pollution at time t as:

$$\frac{\lambda_{P,t}}{\lambda_{0,t}} = \sum_{n=0}^{\infty} \frac{\beta^n (1-\delta)^n}{u'(C_t)} \frac{\partial v(P_{t+n}, T_{t+n})}{\partial P_{t+n}} \tag{A.4}$$

which is (32) in the text.

A.1.2 Determining the social cost of carbon

To derive the expression for the social cost of carbon given in (34) take the first order conditions for the two-layer temperature model given in (A.2i) and (A.2j) and solve for $\lambda_{T,t-1}, \lambda_{L,t-1}$ gives:

$$\lambda_{T,t-1} = \hat{\theta}\lambda_{T,t} + \theta_4 \lambda_{L,t} - \beta^t \partial v_T(P_t, T_t) - \psi \lambda_{P,t} \tag{A.5a}$$

$$\lambda_{L,t-1} = \theta_3 \lambda_{T,t} + (1-\theta_4)\lambda_{L,t} \tag{A.5b}$$

where $\hat{\theta} := \big(1-\theta_1(\theta_2+\theta_3)\big)$. First, insert (A.5b) into (A.5a) and iterate forward $j = 1,2,...,n$ periods to eliminate $\lambda_{L,t}$:

$$\lambda_{T,t} = \hat{\theta}\lambda_{T,t+1} + \theta_3\theta_4 \sum_{j=0}^{n} (1-\theta_4)^j \lambda_{T,t+2+j} - \beta^t \partial v_T(P_{t+1}, T_{t+1}) - \psi \lambda_{P,t+1} \tag{A.6}$$

Iterate (A.6) forward for $k = 0,1,2,..,n$, insert the expressions for $\lambda_{T,t+1}$, $\lambda_{T,t+2}$, ..., $\lambda_{T,t+n}$ and rearange terms yields:

$$\begin{aligned}\lambda_{T,t} = {} & \hat{\theta}^n \lambda_{T,t+n} + \theta_3\theta_4 \sum_{k=0}^{n} (1-\theta_4)^k \lambda_{T,t+2+k} \sum_{j=0}^{k} \left(\frac{\hat{\theta}}{1-\theta_4}\right)^j \\ & - \sum_{k=0}^{n} \hat{\theta}^k \Big(\beta^{t+k} \partial v_T(P_{t+1+k}, T_{t+1+k}) - \psi \lambda_{P,t+1+k}\Big)\end{aligned} \tag{A.7a}$$

Finally, use (A.2h) for $j = 1,2,...,n$ to eliminate $\lambda_{P,t+n}$ from (A.7a):

$$\lambda_{T,t} = \sum_{n=1}^{\infty} \hat{\theta}^n \left(\beta^{t+n} \frac{\partial V(P_{t+n}, T_{t+n})}{\partial T_{t+n}} + \psi \lambda_{P,t+n}\right) \tag{A.8}$$

Assuming that $\lim_{n\to\infty} \hat{\theta}^n \lambda_{T,t+n} = \lim_{n\to\infty} \theta_3\theta_4 \sum_{n=0}^{\infty} (1-\theta_4)^n \lambda_{T,t+2+n} \sum_{j=0}^{n} \big(\hat{\theta}/(1-\theta_4)\big)^j = 0$

$$\lambda_{T,t} = \sum_{n=0}^{\infty} \frac{\beta^n}{u'(C_t)} \left(\hat{\theta}^n \frac{\partial v(P_{t+n}, T_{t+n})}{\partial T_{t+n}} + \frac{\psi(1-\delta)^n}{1-\beta\hat{\theta}} \frac{\partial v(P_{t+n}, T_{t+n})}{\partial P_{t+n}}\right) \tag{A.9}$$

gives equation (34) in the text.

Next, using (A.3) in (A.2c) and defining

$$\hat{\tau}_{x,t} := \sum_{n=0}^{\infty} \frac{\beta^n}{u'(C_t)} \left(\hat{\theta}^n \frac{\partial v(P_{t+n}, T_{t+n})}{\partial T_{t+n}} + \frac{\psi(1-\delta)^n}{1-\beta\hat{\theta}} \frac{\partial v(P_{t+n}, T_{t+n})}{\partial P_{t+n}} \right),$$

(A.2c) reads:

$$\mu_x/\lambda_{0,t} = \partial_X F_t(K_t, X_t, Z_t) - c_x - \hat{\tau}_{x,t}. \tag{A.10}$$

Equation (A.10) summarizes the costs and benefits of extracting a unit of fossil fuel. On the left-hand side is the scarcity cost $\mu_x/\lambda_{0,t}$ which is positive if the resource is exhaustible. On the right-hand side is the net benefit of its use in production given by the term $\partial_X F_t(K_t, X_t, Z_t) - c_x$ and the marginal externality damage $\hat{\tau}_{x,t}$. The term $\hat{\tau}_{x,t}$ can be decomposed into two factors. $\partial v(P_{t+n}, T_{t+n})/\partial T_{t+n}$ accounts for the direct marginal loss in consumer utility at time $t+n$ caused by an additional unit of carbon in the atmosphere and the corresponding change in temperature. The term $\psi(1+n)(1-\delta)^n \partial v(P_{t+n}, T_{t+n})/\partial P_{t+n}$ is the interaction term connecting the two natural cycles and gives the marginal loss in consumer utility at time $t+n$ caused by an additional unit of polluted fresh water through climate change ψ net of natural water regeneration $(1-\delta)^n$.

A.1.3 Proof of Proposition 1

Suppose water is never fully extracted: $(1-\xi_1)Z_t < W_{t+1}^F$ for all $t = 0, 1, 2, \ldots$. Then $\mu_{F,t} = 0$. Using (A.4) and define

$$\hat{\tau}_{z,t} := \sum_{n=0}^{\infty} \frac{\beta^n (1-\delta)^n}{u'(C_t)} \frac{\partial v(P_{t+n}, T_{t+n})}{\partial P_{t+n}},$$

(A.2d) reads:

$$\frac{\lambda_{F,t}}{\lambda_{0,t}}(1-\xi_1) = \partial_Z F_t(K_t, X_t, Z_t) - c_z + \xi_2 \frac{\lambda_{A,t}}{\lambda_{0,t}} + \xi_3 \frac{\lambda_{O,t}}{\lambda_{0,t}} - \chi \hat{\tau}_{z,t}. \tag{A.11}$$

On the left-hand side is the scarcity cost $\lambda_{F,t}/\lambda_{0,t}$ which is positive if the water cycle is closed. On the right-hand side is the net benefit of its use in production $\partial_Z F_t(K_t, X_t, Z_t) - c_Z$, the influence of the hydrological cycle is $\xi_2 \lambda_{A,t}/\lambda_{0,t} + \xi_3 \lambda_{O,t}/\lambda_{0,t}$, and the marginal externality pollution $\chi \hat{\tau}_{z,t}$.

From (A.2e), (A.2f), (A.2g) we get dynamics of Lagrange multipliers::

$$\lambda_{F,t} = (1-\omega_{12}-\omega_{13})\lambda_{F,t+1} + \omega_{12}\lambda_{A,t+1} + \omega_{13}\lambda_{O,t+1} \tag{A.12a}$$

$$\lambda_{A,t} = \omega_{21}\lambda_{F,t+1} + (1-\omega_{21}-\omega_{23})\lambda_{A,t+1} + \omega_{23}\lambda_{O,t+1} \tag{A.12b}$$

$$\lambda_{O,t} = \omega_{32}\lambda_{A,t+1} + (1-\omega_{32})\lambda_{O,t+1}. \tag{A.12c}$$

Rewrite (A.12) as

$$\begin{pmatrix} \lambda_{F,t} \\ \lambda_{A,t} \\ \lambda_{O,t} \end{pmatrix} = \begin{bmatrix} 1-\omega_{12}-\omega_{13} & \omega_{12} & \omega_{13} \\ \omega_{21} & 1-\omega_{21}-\omega_{23} & \omega_{23} \\ 0 & \omega_{32} & 1-\omega_{32} \end{bmatrix} \begin{pmatrix} \lambda_{F,t+1} \\ \lambda_{A,t+1} \\ \lambda_{O,t+1} \end{pmatrix} \tag{A.13}$$

Defining $\hat{\lambda}_t := (\lambda_{F,t}, \lambda_{A,t}, \lambda_{O,t})^\top$ for all $t = 0, 1, 2, \ldots$ gives:

$$\hat{\lambda}_t = \Omega^\top \hat{\lambda}_{t+1} \tag{A.14}$$

with $\Omega \in \mathbb{R}^{3\times 3}$ defined as in (17). Equation (A.14) suggest that either $\hat{\lambda}_t = 0$ or $\hat{\lambda}_t \neq 0$ but constant. The first case $\hat{\lambda}_t = 0$ is excluded by (A.11). Therefore, suppose

$$\lambda_{F,t} = \lambda_F, \quad \lambda_{A,t} = \lambda_A, \quad \lambda_{O,t} = \lambda_O \qquad \text{for all } t = 0, 1, 2, \ldots \tag{A.15}$$

Using (A.15) in (A.12c) gives $\lambda_O = \lambda_A$. Use this result in (A.12b) yields $\lambda_A = \lambda_F$. Also, (A.12a) is satisfied. Hence, hypothesis (A.15) is valid. It follows that again $\lambda_{F,t} = \lambda_{A,t} = \lambda_{O,t} = \lambda_F > 0$ for all $t \geq 0$. Insert this result into (A.11) gives

$$\lambda_{F,t}(1-\xi_1-\xi_2-\xi_3) = q_t(\hat{v}_{z,t} - c_z) \qquad \text{for all } t = 0, 1, 2, \ldots \tag{A.16}$$

Observe finally that if the global water cycle is a closed system, i.e.

$$\sum_{j\in\{x,a,s\}} \xi_j = 1 \quad \text{and} \quad \text{for all } t = 0, 1, 2, \ldots$$

(A.16) requires $\hat{v}_{z,t} = c_z$ for all t, so fresh water is abundant and has no scarcity rent. Otherwise, if the global water cycle is semi-closed, i.e.

$$\sum_{j\in\{1,2,3\}} \xi_j < 1 \qquad \text{for all } t = 0, 1, 2, \ldots$$

(A.16) requires $\hat{v}_{z,t} > c_z$ for all t, so fresh water is an exhaustible resource and has a scarcity rent. The shadow price of fresh water in the efficient solution reads:

$$\frac{\lambda_{F,t}(1-\xi_1-\xi_2-\xi_3)}{\lambda_{0,t}} = \partial_Z F_t(K_t, X_t, Z_t) - c_z - \chi \hat{\tau}_{z,t} \qquad \text{for all } t = 0, 1, 2, \ldots \tag{A.17}$$

■

B Computational details

B.1 Numerical algorithm

Golosov et al. (2014) show that for standard assumptions on preferences and technology, if the economy is on a balanced growth path on which output and consumption grow at constant and identical rates, optimal taxes can be approximated by a constant share of final output. We show in Hillebrand & Hillebrand (2023) in a multi-region setting that the approximation formula provides excellent results even if the equilibrium is not exactly on a balanced path. In the model here, however, climate change and water pollution directly enter consumer utility and the approximation 'constant share'-formula for environmental taxes cannot be applied. Instead we use the following 'forward-shooting' numerical algorithm to solve the model and approximate environmental taxes. The algorithm is a modified, single-region version of the general algorithm developed in Hillebrand & Hillebrand (2023). We illustrate the computational algorithm for an iteration of the model of length $T^{\max} > 0$.

Model solving

Step 1: Initialization:

a. Choose arbitrary sequences of environmental taxes $\left(\hat{\tau}_{x,t}, \hat{\tau}_{z,t}\right)_{t\in[0,T^{\max}]}$

b. Choose candidate initial values for consumption $C_{-1} > 0$ and resource prices for fossil fuel and freshwater $v_{x,-1} \in [c_x, \infty[$ and $v_{z,-1} \in [c_z, \infty[$. If $R_{x,0} = \infty$, set $v_{x,-1} = c_x$, otherwise $v_{x,-1} > c_x$, If $W^F_{z,0} = \infty$, set $v_{z,-1} = c_z$, otherwise $v_{z,-1} > c_z$

c. Use initial values together with the given states for the climate $(M^A_{-1}, M^U_{-1}, M^L_{-1})$ temperature (T_{-1}, T^L_{-1}), water cycle $(W^F_{-1}, W^A_{-1}, W^O_{-1})$ and $K_0 > 0$ to determine the remaining endogenous model variables. Set $t = 0$.

Step 2: Inner iteration for $0 \leq t \leq T^{\max}$:

Compute solution to planning problem for each t given $\left(\hat{\tau}_{x,t}, \hat{\tau}_{z,t}\right)_{t\in[0,T^{\max}]}$ using the forward shooting algorithm described in Hillebrand & Hillebrand (2023).

Step 3: Update environmental tax policies:

Compute updated sequences of environmental taxes $\left(\tau_{x,t}, \tau_{z,t}\right)_{t\in[0,T^{\max}]}$ using optimal tax formulae

$$\tau_{x,t} := 2C_t^{\sigma} \sum_{n=0}^{\infty} \beta^n \left(\alpha_T \hat{\theta}^n T_{t+n} + \alpha_P \frac{\psi(1-\delta)^n}{1-\hat{\theta}\beta} P_{t+n}\right)$$

$$\tau_{z,t} := 2\alpha_P C_t^{\sigma} \sum_{n=0}^{\infty} \beta^n (1-\delta)^n P_{t+n}$$

and sequences of states $(P_t, T_t)_{t\in[0,T^{\max}]}$ determined in step 2.

Step 4: Outer Iteration:

Repeat steps 3 and 4 until $||\hat{\tau}_{x,t} - \tau_{x,t}|| <$ error tolerance.

Optimality conditions

Consider an arbitrary period $t \geq 0$. Let exogenous population and productivity variables $(N^s_t, Q_{n,t}, Q_{R,t})$, the climate state $(M^A_{t-1}, M^U_{t-1}, M^L_{t-1})$, temperature (T_{t-1}, T^L_{t-1}) the state of the water cycle $(W^F_{t-1}, W^A_{t-1}, W^O_{t-1})$, the state of fresh water pollution P_{t-1} and the capital stock K_t from the previous period be given. Suppose the environmental taxes are determined by (42) and read:

$$\tau_{x,t} := 2C_t^\sigma \sum_{n=0}^{\infty} \beta^n \Big(\hat{\theta}^n \alpha_T T_{t+n} + \alpha_P \frac{\psi(1-\delta)^n}{1-\hat{\theta}\beta} P_{t+n}\Big)$$

$$\tau_{z,t} := 2\alpha_T C_t^\sigma \sum_{n=0}^{\infty} \beta^n (1-\delta)^n P_{t+n}.$$

Using the CES-form of production (39) and defining cost shares

$$\eta_{K,t} = \kappa \left(\frac{K_t^\alpha (Q_{n,t} N_t)^{1-\alpha}}{F_t(K_t,X_t,Z_t)}\right)^\rho, \tag{A.18a}$$

$$\eta_{x,t} = (1-\kappa)\left[\frac{R(X_t,Z_t)}{F_t(K_t,X_t,Z_t)}\right]^\rho \kappa_x \left(\frac{X_t}{R(X_t,Z_t)}\right)^{\rho_r}, \tag{A.18b}$$

$$\eta_{z,t} = (1-\kappa)\left[\frac{R(X_t,Z_t)}{F_t(K_t,X_t,Z_t)}\right]^\rho (1-\kappa_x)\left(\frac{Z_t}{R(X_t,Z_t)}\right)^{\rho_r}. \tag{A.18c}$$

one can write the planner's optimality conditions (A.2b), (A.2c) and (A.2d) as

$$r_t = \alpha \frac{Y_t}{K_t}\eta_{K,t}, \qquad v_{x,t} + \tau_{x,t} = \frac{Y_t}{X_t}\eta_{x,t} \qquad \text{and} \qquad v_{z,t} + \chi\tau_{z,t} = \frac{Y_t}{Z_t}\eta_{z,t}. \tag{A.19}$$

Final output in period t can be written as

$$Y_t = F_t(K_t,X_t,Z_t) \tag{A.20}$$

The temporary planning problem is to determine the resource factor allocation (X_t,Z_t) consistent with optimality conditions (A.19). Output Y_t determined by (A.20) and cost shares as in (A.18). Using (36), (37), re-arranging the second two conditions in (A.19) and denoting $\hat{p}_{i,t} := v_{i,t} - c_i$ resource price net of extraction costs gives

$$X_t = \frac{Y_t \eta_{x,t}}{c_x + r_t p_{x,t-1} + \tau_{x,t}} \tag{A.21a}$$

$$Z_t = \frac{Y_t \eta_{z,t}}{c_z + r_t p_{z,t-1} + \chi\tau_{z,t}} \tag{A.21b}$$

Next we show how the temporary planning problem can be computed numerically.

Factor allocation computation

Denote by $\eta_t = (\eta_{K,t},\eta_{x,t},\eta_{z,t})$ factor cost shares such that η_t takes values in the positive unit simplex $\Delta_+^3 := \{(\eta_{K,t},\eta_{x,t},\eta_{z,t}) \in \mathbb{R}_+^3 | \sum_{i\in\{K,x,z\}} \eta_i = 1\}$. Now, let arbitrary values $\hat{H} := (\hat{Y}_t,\hat{\eta}_t) \in \mathbb{H} := (\mathbb{R}_{++} \times [0,1[^3)$ be given. Using $\hat{H}$ in (A.21) determines the implied factor allocation $\hat{G} := (\hat{X}_t,\hat{Z}_t) \in \mathbb{G} := W_{++}^2$. Substituting the values $\hat{G}$ back into (A.18) and (A.20) yields the updated values $\tilde{H} := (\tilde{Y}_t,\tilde{\eta}_t) \in \mathbb{H}$ defining a second mapping $\Phi_H : \mathbb{G} \to \mathbb{H}$, $\hat{G} \mapsto \Phi_H(\hat{H}) := \tilde{H}$. The composition $\Phi_H \circ \Phi_G : \mathbb{H} \to \mathbb{H}, \hat{H} \mapsto (\Phi_H \circ \Phi_G)(\hat{H}) := \tilde{H}$

is a self-map on $\mathbb{H}$ and the equilibrium solution $H := (Y_t, \eta_t)$ is a fixed point of $\Phi_H \circ \Phi_G$. The composition $\Phi_H \circ \Phi_G$ is globally asymptotically stable such that simply iterating $\Phi_H \circ \Phi_G$ forward, starting with an arbitrary guess H_0 yields the equilibrium solution $\lim_{n\to\infty}(\Phi_H \circ \Phi_G)_n(H_0) = H = (Y_t, \eta_t)$. The implied factor allocation then obtains as $G = (X_t, Z_t) = \Phi_G(H)$.[30]

B.2 Data

Determining productivity parameters
Let the initial climate state $M_{-1} := (M^A_{-1}, M^U_{-1}, M^L_{-1})$, temperatures $T_{-1} := (T_{-1}, T^L_{-1})$, initial fresh water pollution P_{-1}, initial capital supply K^s_0 and initial state of the hydrological cycle $W_{-1} := (W^F_{-1}, W^A_{-1}, W^O_{-1})$ in $t = 0$ be given. Set output, exhaustible fossil fuel consumption and water consumption to target levels i.e, $Y^{target}_0 = Y_0$, $Z^{target}_0 = Z_0$ and $X^{target}_0 = X_0$. Use Y_0, Z_0, and $v_{z,0} - c_z$ in (A.19) to compute $\eta_{z,0}$ as

$$\eta_{z,0} = \frac{v_{z,0} Z_0}{Y_0}. \tag{A.22}$$

Combine cost shares in (A.18) to get $\eta_{x,0}$ as a function of $\eta_{z,0}$:

$$\eta_{x,0} = \frac{\kappa_x}{1-\kappa_x}\left(\frac{X_0}{Z_0}\right)^{\rho_x} \eta_{z,0} \tag{A.23}$$

Then, $\eta_{K,0} = 1 - \eta_{x,0} - \eta_{z,0}$. Use this result to compute initial labor productivity $Q_{n,0}$ from $\eta_{K,t}$ in (A.18):

$$Q_{n,0} = \left[\left(\frac{\eta_{K,0}}{\kappa}\right)^{1/\rho} \frac{Y_0}{K^\alpha_0}\right]^{\frac{1}{1-\alpha}} N_0^{-1} \tag{A.24}$$

Next, use (A.24) in (39) to determine R_0 in (40):

$$R_0 = \left[\frac{Y^\rho_0 - \kappa K^\alpha_0 (Q_{n,0} N_0)^{1-\alpha}}{1-\kappa}\right]^{\frac{1}{\rho}} \tag{A.25}$$

Finally, use the first order conditions and previous results to compute initial resource productivities $Q_{x,0}, Q_{z,0}$:

$$Q_{x,0} = \left[\frac{\eta_{x,0}}{(1-\kappa)\kappa_x}\left(\frac{Y_0}{R_0}\right)^\rho \left(\frac{R_0}{X_0}\right)^{\rho_x}\right]^{1/\rho_x} \tag{A.26a}$$

$$Q_{z,0} = \left[\frac{\eta_{z,0}}{(1-\kappa)(1-\kappa_x)}\left(\frac{Y_0}{R_0}\right)^\rho \left(\frac{R_0}{Z_0}\right)^{\rho_x}\right]^{1/\rho_x}. \tag{A.26b}$$

Determining hydrological cycle parameters
Table 2 reports estimated water cycle, i.e. global water resource stocks and flows across

[30] For a detailed description of the *recursive structure of the planning solution* see Hillebrand & Hillebrand (2023).

different reservoirs (Trenberth et al. (2007)). Accordingly, only 3.0% of global water resources are fresh water and only about 1% of fresh water is directly accessible by humans for consumption. The rest is either locked away in the form of ice in glaciers and polar ice caps (ca. 63%) and deep under the surface in the form of groundwater (36%).

Table 2: Estimated global hydrological cycle

Type of water	Reservoir type	Volume [billion m^3]	Share of total volume [%]
SALT WATER			96.93
	oceans	1335.04	
FRESH WATER			3.00
	ice	26.35	
	permafrost	0.022	
	lakes & rivers	0.18	
	soil moisture	0.12	
	groundwater	15.30	
ATMOSPHERIC WATER			0.07
	water vapor	0.013	
	water vapor from sea to land	0.04	
	fresh water flow from land to sea	0.04	
	land evaporation	0.07	
	sea evaporation	0.41	
	terrestrial precipitation	0.11	
	marine precipitation	0.37	
ROUNDED TOTAL		1397.00	100.0

Source: Trenberth et al. (2007)

We set initial values for fresh water, ocean water and water vapor in the atmosphere as follows: W_0^F contains all fresh water which is accessible for consumption, i.e. ground water, lakes & rivers, and soil moisture:

$$W_0^F = 15.30 + 0.18 + 0.12 = 15.60.$$

W_0^A contains water vapor in the atmosphere and initial flows from and into the atmosphere through marine and terrestrial precipitation and evaporation:

$$W_0^A = 0.0127 + 0.413 + 0.073 + 0.373 + 0.113 = 0.9847$$

W_0^O contains salt water (oceans), and all fresh water which is in solid state and not directly available for consumption, i.e. ice and permafrost:

$$W_0^O = 1335.04 + 26.35 + 0.022 = 1361.42.$$

Water flow rates in percent, such as precipitation over sea and over land are then calculated through division of flows by the corresponding quantity in each reservoir type:

$\omega_{12} = 0.073/15.6 = 0.004678$ is share of fresh water evaporation from land into the atmosphere. Evaporation from ocean water into the atmosphere is $\omega_{32} = 0.00030335$. Precipitation over land is $\omega_{21} = 0.113/0.9847 = 0.114756$ and precipitation over sea is $\omega_{23} = 0.373/0.9847 = 0.378796$. The share of surface fresh and groundwater into the sea is set to $\omega_{13} = 0.04/15.60 = 0.0025641$. Finally, the reverse flow of sea water into fresh water reservoirs is $\omega_{31} = 0$.

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