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## **Volatility Models with Innovations from New Maximum Entropy Densities at Work**

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# VOLATILITY MODELS WITH INNOVATIONS FROM NEW MAXIMUM ENTROPY DENSITIES AT WORK

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## SUMMARY

Generalized autoregressive conditional heteroskedasticity (GARCH) processes have become very popular as models for financial return data because they are able to capture volatility clustering as well as leptokurtic unconditional distributions which result from the assumption of conditionally normal error distributions. In contrast, Bollerslev (1987) and several follow-ups provided evidence that starting with leptokurtic and possibly skewed (conditional) error distributions will achieve better results. Parallel to these flexible but to some extent arbitrary chosen parametric distributions, recent years saw a rise in suggestions for maximum entropy distributions (e.g. Rockinger and Jondeau, 2002, Park and Bera, 2009 or Fischer and Herrmann, 2010). Within this contribution we provide a comprehensive comparison between both different ME densities and their parametric competitors within different generalized GARCH models such as APARCH and GJR-GARCH.

*Keywords and phrases:* GARCH; APARCH; Entropy density; Skewness; Kurtosis

## 1 Introduction

Introduced by Engle (1982) and extended by Bollerslev (1986), generalized autoregressive conditional heteroskedasticity (GARCH) processes have become very popular as models for financial return data because they are able to capture both "distributional" stylized facts (e.g. thick tails and high peakedness) and stylized facts concerning the time structure (e.g. volatility clustering). Succeeding generalizations can be divided into three major classes: First of all, models that take asymmetric behaviour of volatility into account. Secondly, models with time-varying skewness and kurtosis rather than just time-varying variance and volatility, respectively. Thirdly, models based on a residual distribution different from the Gaussian one. It was already detected by Black (1976) that stock return volatility is strongly asymmetric: Negative returns are followed by larger increases in volatility than equally large positive returns. Both APARCH specification of Ding et al. (1993) which includes, among others, the TS-GARCH model (see Taylor, 1990 and Schwert, 1989 and 1990), the GJR-GARCH model (see Glosten et al., 1993) and the T-GARCH (see Zakoian, 1994) as special cases, and the (exponential) EGARCH model of Nelson (1991) account for this effect. In addition, there is no reason to assume that higher moments – in particular skewness and kurtosis represented by the third and fourth standardized moments – should be time-invariant. Allowing them to be time-varying may improve the approximation of the

actual return distributions. Such types of models (so-called autoregressive conditional density (ARCD) models) were introduced by Hansen (1994). For other contributions see Harvey and Siddique (1999) or Hueng and McDonald (2005). Unfortunately, there is no systematic or significant evidence for time-varying higher moments. Finally, although GARCH models with conditionally normal errors imply leptokurtic unconditional distributions, Bollerslev (1987) found evidence that starting with leptokurtic and possibly skewed (conditional) error distributions will achieve better results. Bollerslev (1987), for instance, uses the Student- $t$  distribution whereas Mittnik et al. (1998) advocate the stable distribution and Fischer (2004, 2006) found evidence in favour of generalized hyperbolic secant families. Parallel to these parametric distributions, recent years saw a rise in suggestions for maximum entropy (ME) distributions examined by econometricians, see e.g. Rockinger and Jondeau (2002), Park and Bera (2009) or Fischer and Herrmann (2010), as well as by physicians, see. e.g. Quéiros (2005). All these approaches allow or can easily be extended to allow for kurtosis and skewness if presented in the unified framework following Herrmann (2009). Until now the ME approaches have only been compared with either the gaussian distribution (which for all suggestions appears as a special or limiting case) or with each other. The purpose of this contributions is to extend these new models to asymmetric volatility dynamics and to compare them to their most successful parametric peers. The presented approach is three-fold: Firstly, rather than compare plain GARCH models with MED density we use generalized variance specifications such as APARCH and GJR-GARCH. Secondly, a comprehensive comparison is provided between flexible parametric families and MED families. Thirdly, we compare the goodness-of-fit within the MED class.

## 2 A primer on GARCH models and its generalizations

Let  $P_0, \dots, P_T$  denote the time-discrete prices of an arbitrary asset from time  $t = 0$  to  $t = T$ . Usually – rather than the prices themselves – the log-returns  $R_1, \dots, R_T$  defined by  $R_t \equiv \log(P_t/P_{t-1}) = \log(P_t) - \log(P_{t-1})$  are analyzed. In general, the standard models for the returns in financial econometrics are of the form

$$\Theta_m(L)R_t = \mu + U_t, \quad t = 1, \dots, T$$

with

$$U_t | \mathcal{F}_{t-1} \sim D(0, h_t^2, \eta) \text{ or } U_t = h_t \epsilon_t \text{ with } \epsilon_t \sim D(0, 1, \eta), \quad (2.1)$$

where  $\Theta_m(L)$  is a polynomial in the lag operator  $L$  of order  $m$  which allows to include linear dependence from the own history  $R_{t-1}, \dots, R_{t-m}$  and  $\mu \in \mathbb{R}$ . Moreover, the distribution of the residuals  $U_t$  (conditioned on the information set  $\mathcal{F}_{t-1}$  up to time  $t-1$ ) is assumed to follow a standardized<sup>1</sup> distribution  $D$  with shape parameter  $\eta$  and time-varying variance  $h_t^2$ . For reasons of simplicity, assume that  $\Theta_m(L) \equiv 1$  and  $\mu \equiv 0$ , i.e.  $R_t = h_t \epsilon_t$ . Otherwise,

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<sup>1</sup>This means in particular that scale parameter and variance parameter  $h_t$  are identically.

replace  $R_t$  by  $R_t^* \equiv \Theta_m(L)R_t - \mu$  in the sequel. In the GARCH(1,1) specification of Bollerslev (1986),

$$h_t^2 = \alpha_0 + \alpha_1 R_{t-1}^2 + \beta_1 h_{t-1}^2 = \alpha_0 + \alpha_1 h_{t-1}^2 \epsilon_{t-1}^2 + \beta_1 h_{t-1}^2, \quad \alpha_0 > 0, \alpha_1, \beta_1 \geq 0, \quad (2.2)$$

where the fundamental ARCH model of Engle (1982) is included for  $\beta_1 = 0$ . GARCH models have been generalized in many different ways: In order to capture leverage effects (i.e. asymmetric behaviour of volatility for positive or negative returns), Zakoian (1994) introduced the T-GARCH model with standard deviation defined by

$$h_t = \alpha_0 + \alpha_1^+ R_{t-1}^+ - \alpha_1^- R_{t-1}^- + \beta_1 h_{t-1}, \quad \alpha_1^+ \geq 0, \alpha_1^- \geq 0, \quad (2.3)$$

where  $R_t^+ \equiv \max\{R_t, 0\}$  and  $R_t^- \equiv \min\{R_t, 0\}$ . Imposing Box-Cox-transformations on both conditional standard deviation and asymmetric absolute returns essentially leads to the APARCH-specification of Ding et al. (1993), namely<sup>2</sup>

$$h_t^\lambda = \alpha_0 + \alpha_1 (|R_{t-1}| - c R_{t-1})^\lambda + \beta_1 h_{t-1}^\lambda, \quad |c| \leq 1, \lambda \geq 0. \quad (2.4)$$

Equation (2.4) reduces to (2.3) for  $\lambda = 1$ ,  $\alpha_1 = \alpha_1^- / (2 - \alpha_1^+)$  and  $c = 1 - \alpha_1^+ (2 - \alpha_1^+) / \alpha_1^-$ . Moreover, equation (2.2) is achieved for  $\lambda = 2$  and  $c = 0$ . Restricting  $\lambda = 2$  for the APARCH includes the GJR-GARCH model of Glosten, Jagannathan and Runkle (1993). Although there might be some further (theoretical) generalizations, using the APARCH specification for the variance equation will capture the conditional volatility sufficiently well.

### 3 Time-varying volatility based on maximum entropy

Contrary to parametric volatility models the maximum entropy models are characterized by the dynamics of the conditional moments of  $U_t | F_{t-1}$  in time, e.g. as

$$E(U_t | F_{t-1}) = 0, \quad (3.1)$$

$$E(U_t^2 | F_{t-1}) = h_t^2 = \alpha_0 + \alpha_1 R_{t-1}^2 + \beta_1 h_{t-1}^2, \quad (3.2)$$

but might also assume the more sophisticated volatility dynamics as presented above. Following Jaynes' (1957) principle of maximum entropy, the conditional distribution is chosen as the distribution which has maximal entropy within the set of distributions consistent to these conditional moments. Formally this may be written as

$$U_t | F_{t-1} \sim ME, \quad \text{with } f_{ME} = \underset{f}{\operatorname{argmax}} (H(f), f \in \mathcal{D}), \quad (3.3)$$

where  $H$  denotes some entropy measure and  $\mathcal{D}$  the set of distributions consistent with equations 3.1 and 3.2. Entropy  $H$  may be measured using some differentiable, convex function  $\phi$  as

$$H(f) = - \int_{D(X)} \phi(f(x)) dx, \quad (3.4)$$

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<sup>2</sup>This model is sometimes also referred to as Power GARCH or P-GARCH model.

such that if  $\mathcal{D}$  admits a density function  $f_{ME}$  with

$$\phi'(f_{ME}(x)) = \sum_{i=0}^k \lambda_i g_i(x), \quad (3.5)$$

it is at the same time the solution to the entropy maximization task above, see Kesavan and Kapur (1989). In these cases the solution's functional form is defined, but the  $\lambda_i$  have to be derived using numerical algorithms for dual problems. Such dual problems are given in Kapur (1994), for efficient algorithms see e.g. Rockinger and Jondeau (2002) or for generalized entropy measures Herrmann (2009).

The main advantage of the maximum entropy approach is its flexibility. The first source of flexibility is the choice of the measure for entropy. For the measures examined in this work see table 1.<sup>3</sup>

Author	$\phi(f)$	$f_{ME}(x)$	Parameter	Example
Shannon	$-f \ln(f)$	$\exp\left(-\sum_{i=0}^k \lambda_i g_i(x)\right)$	-	Gaussian
Havrda-Charvat	$\frac{1}{1-\alpha}$	$\left(\sum_{i=0}^k \lambda_i g_i(x)\right)^{\frac{1}{\alpha-1}}$	$\alpha$	Student t
Kapur	$-f \ln(f) + \frac{1}{c}(1 + cf) \ln(1 + cf)$	$\frac{1}{\exp(-\sum_{i=0}^k \lambda_i g_i(x)) - c}$	$c$	-

Table 1: Suggestions for Measures of Entropy.

Using the Havrda-Charvat entropy leads to stronger tails for  $\alpha < 1$ , the Kapur entropy for  $c > 0$ . Such that letting these freely adjust to the data gives flexibility with respect to kurtosis. Another way to include higher moments is the inclusion of additional restrictions for  $\mathcal{D}$ , e.g. as

$$E(\tan^{-1}(U_t)|F_{t-1}) = m_3, \quad (3.6)$$

which implies a skewed set  $\mathcal{D}$  or

$$E(\ln(1 + U_t^2)|F_{t-1}) = m_4, \quad (3.7)$$

which implies a leptokurtic set  $\mathcal{D}$ .<sup>4</sup> Here  $m_3$  and  $m_4$  denote the target values for higher moments. Further suggestions for suitable moment functions have been compared in Park and Bera (2009), but we found that this combination performed best.

<sup>3</sup>The entropy measure suggested by Havdra-Charvat is a monotonic transformation of Rényi's entropy. In physics it is usually denoted as Tsallis entropy.

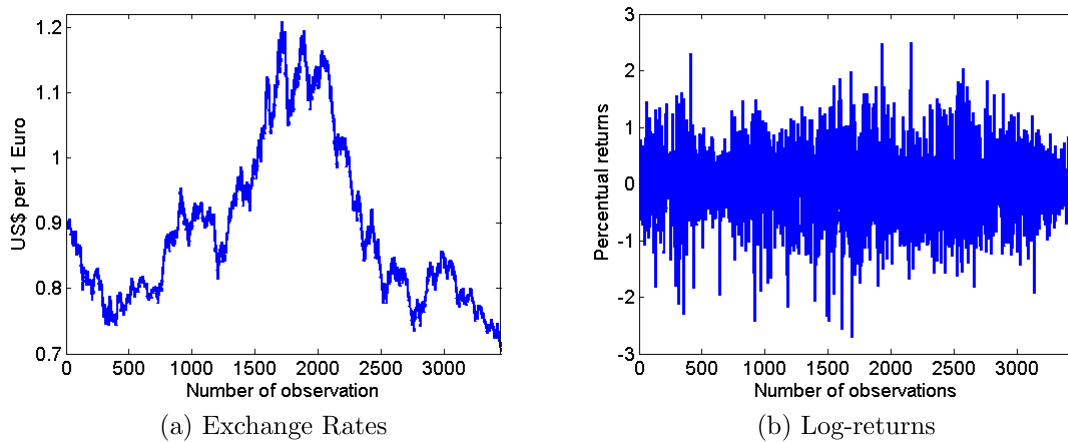
<sup>4</sup>For restrictions on possible moment combinations see e.g. Fischer and Herrmann (2010).

## 4 Application to fx data

### 4.1 Data set

We focus on the daily noon spot US dollar exchange rates (\$/local currency) for the Euro (EUR) over the period 3 January 1994 to 30 September 2007 (3453 observations)<sup>5</sup>. In a first step, the data are transformed to percentual log-returns defined as  $R_{i,t} \equiv \ln(S_{i,t}/S_{i,t-1}) \cdot 100$ . Both prices and log-returns can be seen in figure 1, below. To get some information on the

Figure 1: Prices and log-returns



underlying data set, table 2 summarizes some basic descriptive statistics.

	$\hat{\mu}$	$\hat{s}$	$\mathbb{S}$	$\mathbb{K}$	$q_{0.25;0.75}$	$q_{0.4;0.6}$	JB	$Q(30)$	$Q^2(30)$
EUR	-0.0071	0.575	-0.1067	4.238	0.6619	0.2411	227.1*	26.51	291.4*

Table 2: Descriptive statistics of the data

In particular, the USD/EURO data set exhibits some skewness and leptokurtosis, measured by the third and fourth standardized moments  $\mathbb{S}$  and  $\mathbb{K}$ . Moreover, the Jarque-Bera ( $\mathcal{JB}$ ) test indicates non-normality. Finally, application of the Ljung-Box test to the returns ( $Q$ ) and the squared returns ( $Q^2$ ), respectively, suggests the presence of GARCH effects but no significant correlation between different returns.

<sup>5</sup>The data are available from <http://www.econ.queensu.ca/jae>.

## 4.2 Goodness-of-fit statistics

As a natural candidate for the goodness-of-fit we consider the maximum log-likelihood value ( $\mathcal{LL}$ ) obtained from the ML estimation. It may be viewed as an overall measure of goodness-of-fit and allows to judge which candidate is more likely to have generated the data. To account for the different numbers of parameter  $k$ , we also calculate the Akaike information criterion (AIC), given by

$$\mathcal{AIC} = -2 \cdot \mathcal{LL} + \frac{2N(k+1)}{N-k-2},$$

where  $N$  denotes the number of data. An alternative penalization of additional parameter is given by the Bayesian information criterion as

$$\mathcal{BIC} = -2 \cdot \mathcal{LL} + k \cdot \ln N.$$

However, Boothe and Glassman (1987) presented arguments that non-nested model comparisons based on log-likelihood values may lead to spurious conclusions.

If the underlying model is correctly specified, the zero-mean and unit-variance returns  $\hat{\epsilon}_t | \mathcal{F}_{t-1} = (R_t - \hat{\mu}_t) / \hat{h}_t$  can be assumed to be independent realizations of the distribution  $D(0, 1, \hat{\eta})$ , where  $\hat{\eta}$  denotes the ML estimator of the shape vector  $\eta$ . Such that we can measure the distance between the fitted theoretical distribution and the empirical distribution of  $\epsilon$ .

Wang et al. (2001) suggested the  $\chi^2$  goodness-of-fit statistic which can be calculated by

$$\chi^2 = \sum_{i=1}^{N_c} \frac{(H_i - F_i)^2}{F_i},$$

where  $H_i$  is the observed count frequency of  $\epsilon$  in the  $i$ -th data class,  $F_i$  is the predicted count frequency under the assumed theoretical model and  $N_c$  is the number of classes.

Alternatively the Kolmogorov distance may be used as

$$\mathcal{KS} = 100 \cdot \sup_{x \in \mathbb{R}} |F_D(x) - \hat{F}(x)|,$$

where  $F_D$  denotes the cumulative distribution function of  $D(0, 1, \hat{\eta}_t)$  and  $\hat{F}$  the empirical distribution of  $\epsilon$ .

Whereas  $\mathcal{KS}$  emphasizes deviation around the median of the distribution,  $\mathcal{AD}_0$  defined by

$$\mathcal{AD}_0 = \sup_{x \in \mathbb{R}} \frac{|F_D(x) - \hat{F}(x)|}{\sqrt{F_D(x) \cdot (1 - F_D(x))}}$$

emphasizes discrepancies in the tails of the distribution. Instead of just the maximum deviation, one should also have a look at the second and the third largest value, denoted by  $\mathcal{AD}_1$  and  $\mathcal{AD}_2$ .



### 4.3 Parametric competitors

For a general comprehensive overview on successful parametric distribution family we refer to Bao, Lee and Saltoglu (2004) or Fischer (2010). Within our empirical analysis we focus on popular multi-parametric distributions which have already been successfully applied to financial return data. Among them, the Student-t distribution (T) and the generalized Student-t (GT) distribution of McDonald and Newey (1988) together with its skew counterpart (SGT2) developed by Grottko (2001). In contrast, McDonald (1991), and McDonald and Bookstaber (1991) used the exponentially generalized beta of the second kind (EGB2) distribution which generalizes the logistic distribution in a natural way. Similarly, the less-known but rather flexible generalized secant hyperbolic (GSH) distribution of Vaughan (2002) and its skew generalization of Fischer (2004) is taken into consideration. In addition, Theodossiou's (2000) skewed family of the generalized error distribution (SGED). Last, but not least we included the inverse hyperbolic sine (IHS) distribution which is used successfully, for instance, in Choi (2001) to model asymmetric and fat-tailed distributions.

### 4.4 Empirical Results

The results of the maximum likelihood estimations are summarized in table 3 and 4. For reasons of brevity, only the different goodness-of-fit measures were reported herein. Results for the specific parameter estimators for the different distribution families are available from the authors by request. First of all, the highest log-likelihood  $LL$  and the lowest of the other goodness-of-fit measures (representing the most favourable choice) were marked bold for each of the four generalized GARCH models (plain GARCH, T-GARCH, APARCH and GJR-GARCH). Across all variance specification, the results concerning the order of the distribution families remains nearly constant.

Within the classical parametric families, both SGT2 and SGSH are pre-dominant if only likelihood is taken into account. The logistic distribution gives by far the best results if additional parameters are penalized. For the likelihood-based measures all three MED densities are outperformed. However, if we focus on the tail-related Anderson-Darling statistics MEHC dominates all other competitors while MEK exhibits the minimal  $\chi^2$ -value and Kolmogorov-Smirnov distance, respectively.

Within the entropy density, MED minimizes the log-likelihood and maximizes AIC and BIC. Above that conditional non-normality in the sense of Bollerslev (1987) is rejected for nearly all multi-parametric families. Finally, the close theoretical relation between the Student-t distribution and the MEHC distribution is recovered for the underlying data set.

## 5 Summary

This work extends the new approaches of maximum entropy volatility models to more sophisticated volatility dynamics and compares these to the the to most flexible parametric models known in econometrics. Using maximum entropy specifications with similar flexibility with respect to skewness and kurtosis we find that the maximum entropy densities give results similar to their parametric peers as far as overall fit is concerned. But if we compare the fit in the tails, only the Havrda-Charvat density – that may be related to t-type distributions – competes with its parametric peers.

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**GARCH(1,1)**

Distribution	$k$	$\mathcal{LL}$	$AIC$	$BIC$	$KS$	$\chi^2$	$AD_0$	$AD_1$	$AD_2$
NORM	4	-2877.5	5765	5787.6	2.94	48.023	3.061	0.29	0.264
t	5	-2830.9	5673.8	5702.5	1.43	8.693	0.04	0.039	0.038
GT	6	-2829.1	5672.2	5707.1	1.27	6.115	0.052	0.05	0.05
SGT2	7	<b>-2828.2</b>	5672.4	5713.4	1.09	6.848	0.051	0.045	0.044
LOG	4	-2829.9	<b>5669.8</b>	<b>5692.4</b>	1.42	6.968	0.057	0.046	0.045
EGB2	6	-2829.6	5673.2	5708.1	1.38	7.23	0.051	0.041	0.04
SGED	6	-2829.8	5673.6	5708.5	1.1	10.187	0.122	0.06	0.058
IHS	6	-2830.3	5674.6	5709.5	1.4	7.552	0.04	0.038	0.038
GSH	5	-2829.8	5671.6	5700.3	1.4	6.631	0.054	0.044	0.043
SGSH	6	-2828.3	5670.6	5705.5	1.01	6.272	0.043	<b>0.033</b>	<b>0.032</b>
MED	6	-2829.4	5672.8	5707.7	1.01	5.383	0.139	0.05	0.048
MEHC	6	-2829.9	5673.8	5708.7	1.09	8.038	<b>0.035</b>	0.034	0.034
MEK	6	-2830.7	5675.4	5710.3	<b>0.93</b>	<b>4.776</b>	0.237	0.064	0.06

**Threshold GARCH**

Distribution	$k$	$\mathcal{LL}$	$AIC$	$BIC$	$KS$	$\chi^2$	$AD_0$	$AD_1$	$AD_2$
NORM	5	-2878.7	5769.4	5798.1	2.92	47.426	2.402	0.368	0.298
t	6	-2832.5	5679	5713.9	1.54	11.252	0.05	0.045	<b>0.039</b>
GT	7	-2830.6	5677.2	5718.2	1.43	<b>7.912</b>	0.057	0.052	0.046
SGT2	8	-2830.1	5678.3	5725.4	1.24	8.563	0.049	0.047	0.045
LOG	5	-2831.5	<b>5675</b>	<b>5703.7</b>	1.53	8.84	0.052	0.049	0.047
EGB2	7	-2831.4	5678.8	5719.8	1.52	9.629	0.051	0.048	0.045
SGED	7	-2831.6	5679.2	5720.2	1.18	9.766	0.111	0.057	0.057
IHS	7	-2831.8	5679.6	5720.6	1.59	10.532	0.051	0.046	0.042
GSH	6	-2831.4	5676.8	5711.7	1.51	8.511	0.049	0.047	0.045
SGSH	7	<b>-2829.7</b>	5675.4	5716.4	1.16	10.777	0.041	0.041	0.041
MED	7	-2830.5	5677	5718	1.22	9.176	0.116	0.05	0.047
MEHC	7	-2831.5	5679	5720	1.25	12.233	<b>0.04</b>	<b>0.04</b>	<b>0.039</b>
MEK	7	-2831.9	5679.8	5720.8	<b>1.14</b>	8.585	0.197	0.071	0.063

Table 3: Comparison of goodness-of-fit measures (I)

**APARCH**

Distribution	$k$	$\mathcal{LL}$	$AIC$	$BIC$	$KS$	$\chi^2$	$AD_0$	$AD_1$	$AD_2$
NORM	6	-2876.6	5767.2	5802.1	2.89	48.863	2.958	0.293	0.284
t	7	-2830.2	5676.4	5717.4	1.52	9.971	0.042	0.041	0.04
GT	8	-2828.4	5674.9	5722	1.34	7.227	0.05	0.05	0.05
SGT2	9	<b>-2826.7</b>	5673.5	5726.7	1.19	8.758	0.05	0.046	0.045
LOG	6	-2829.2	<b>5672.4</b>	<b>5707.3</b>	1.5	7.878	0.055	0.046	0.046
EGB2	8	-2829.1	5676.3	5723.4	1.46	<b>7.053</b>	0.051	0.042	0.042
SGED	8	-2829.4	5676.9	5724	1.17	10.846	0.122	0.058	0.058
IHS	8	-2829.5	5677.1	5724.2	1.49	8.498	0.042	0.041	0.041
GSH	7	-2829.1	5674.2	5715.2	1.49	7.613	0.053	0.044	0.044
SGSH	8	-2827.5	5673.1	5720.2	1.09	7.652	0.044	<b>0.037</b>	<b>0.037</b>
MED	8	-2828.6	5675.3	5722.4	1.15	7.36	0.134	0.047	0.046
MEHC	8	-2829.2	5676.5	5723.6	1.15	9.123	<b>0.038</b>	<b>0.037</b>	<b>0.037</b>
MEK	8	-2830	5678.1	5725.2	<b>1.06</b>	7.231	0.231	0.061	0.058

**GJR\_GARCH**

Distribution	$k$	$\mathcal{LL}$	$AIC$	$BIC$	$KS$	$\chi^2$	$AD_0$	$AD_1$	$AD_2$
NORM	5	-2877.2	5766.4	5795.1	2.97	47.667	3.074	0.283	0.257
t	6	-2830.5	5675	5709.9	1.49	9.214	0.044	0.038	0.037
GT	7	-2828.7	5673.4	5714.4	1.34	6.938	0.057	0.051	0.05
SGT2	8	-2828.1	5674.3	5721.4	1.15	7.438	0.05	0.05	0.043
LOG	5	-2829.5	<b>5671</b>	<b>5699.7</b>	1.47	7.488	0.056	0.052	0.046
EGB2	7	-2829.4	5674.8	5715.8	1.44	7.642	0.051	0.046	0.04
SGED	7	-2829.7	5675.4	5716.4	1.15	11.262	0.122	0.064	0.057
IHS	7	-2829.9	5675.8	5716.8	1.48	8.42	0.046	0.04	0.038
GSH	6	-2829.5	5673	5707.9	1.45	7.146	0.054	0.05	0.043
SGSH	7	<b>-2827.9</b>	5671.8	5712.8	1.06	6.755	0.043	0.037	<b>0.034</b>
MED	7	-2829.1	5674.2	5715.2	1.11	7.273	0.138	0.056	0.048
MEHC	7	-2829.6	5675.2	5716.2	1.13	8.295	<b>0.039</b>	<b>0.035</b>	<b>0.034</b>
MEK	7	-2830.4	5676.8	5717.8	<b>1.02</b>	<b>6.115</b>	0.237	0.068	0.061

Table 4: Comparison of goodness-of-fit measures (II)

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