Wrede, Matthias

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Matthias Wrede

Asymmetric Tax Competition with Formula Apportionment

Matthias Wrede
University of Marburg and CESifo
Am Plan 2, 35032 Marburg, Germany
eMail: wrede@wiwi.uni-marburg.de

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Coordination: Bernd Hayo • Philipps University Marburg • Faculty of Business Administration & Economics • Universitätsstr. 24, D-35032 Marburg
Tel: ++49-6421-2823091, Fax: ++49-6421-2823088 e-mail: hayo@wiwi.uni-marburg.de
Asymmetric Tax Competition
with Formula Apportionment

Matthias Wrede*

University of Marburg and CESifo

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Abstract

This paper analyzes asymmetric tax competition under formula apportionment. It sets up a model with multinationals where two welfare-maximizing jurisdictions of different size levy source-based corporate taxes and allocate taxes using the formula approach. At the Nash equilibrium, tax rates are too low and public goods quantities are to small. The paper shows that the larger country levies a larger tax rate compared to the smaller country as it does under separate accounting. Citizens of the larger country are worse off than those of the smaller country.

JEL Classification: H25, H42, H73.

Keywords: Multinational enterprises, corporate taxation, formula apportionment, asymmetric tax competition.

*Address: University of Marburg, Faculty of Business Administration and Economics, Am Plan 2, 35032 Marburg, Germany, eMail: wrede@wiwi.uni-marburg.de
1 Introduction

Ever since Zodrow and Mieszkowski (1986), it is well known that tax competition leads to underprovision of public goods when jurisdictions cannot use the full set of tax instruments. In response to profit shifting and tax competition, the European Commission suggested a transition from separate accounting to a common tax base and formula apportionment (see European Commission, 2001). Although the idea seems like a good one at first glance, since its inception the proposed benefits, namely a reduction in compliance costs, tax planning, and tax competition, have been seriously challenged (see, for an overview, Fuest, 2008). However, at the subnational level, formula apportionment is common; examples are, corporate taxation in the US and Canada and the German local business tax (“Gewerbesteuer”).

Regarding tax competition, many scholars have shown that harmonizing the tax base and employing formula apportionment does not solve the problem of inefficient public good supply. Scholars reach various conclusions as to whether there is under- or overprovision under formula apportionment. According to Nielsen, Raimondos-Moeller, and Schjelderup (2009), the positive fiscal externality of taxation and the negative aggregate investment externality are responsible for this ambiguity. Pethig and Wagener (2007) argue that equilibrium tax rates are too low for property-share apportionment but tend to be too high for other formulas. Eichner and Runkel (2008) unambiguously find underprovision. Kolmar and Wagener (2007) claim that tax competition leads to suboptimally low tax rates if and only if the investment elasticity of the tax base is lower than the investment elasticity of the apportionment factor. Wrede (2009) shows that in the absence of profit shifting, even under formula apportionment, tax competition leads to underprovision of public goods. When multinationals are able to shift profits from high-tax to low-tax countries, overprovision cannot be ruled out. Finally, when jurisdictions can appropriately tax residents, tax competition does not distort the public good supply. This has been shown for the standard model of tax competition by Bucovetsky and Wilson (1991) and has been confirmed for formula apportionment by Eggert and Schjelderup (2003).

Surprisingly, the theoretical literature on formula apportionment and tax competition has only dealt with symmetric tax competition. Country size differences were completely
neglected. The reason is probably that asymmetry in models with decreasing returns to scale is rather difficult to handle. Nevertheless, this is disappointing from a practitioner’s point of view, since large and wealthy industrialized countries are particularly challenged by small tax havens. Also from a theorist’s perspective the focus on a very special case, namely symmetric tax competition, is not satisfying.

In the standard framework of tax competition under separate accounting some progress on asymmetric tax competition has been made. Bucovetsky (1991) and Wilson (1991) show that population differences imply tax differences. The smaller jurisdiction levies a lower tax rate in equilibrium, and its residents are better off than those in the larger jurisdiction (see also Wellisch, 2000). Hwang and Choe (1995) consider differences in per-capita endowments. Depending on income effects of the public good, a poor large region may choose a lower tax rate. Kanbur and Keen (1993) set up a model with commodity taxation and cross-border shopping. They show that both the large and the small country may gain from minimum tax rates. Burbidge and Cuff (2005) come up with the result that the existence of increasing returns can reverse the result that small regions have higher per-capita utility in Nash equilibria with only capital taxes. According to Wrede (2008), in this setting the allocation of resources could be improved by a simple fiscal equalization scheme. Stoewhase (2005) considers asymmetric capital tax competition when profit shifting is feasible. Asymmetry is also studied in the literature on tax havens (see, e.g., Hong and Smart, 2007; Slemrod and Wilson, 2006).

This paper aims at analyzing tax competition under formula apportionment when countries differ in size. The paper is mainly inspired by Bucovetsky (1991) and Wilson (1991). However, it sets up a model with multinationals where two welfare-maximizing jurisdictions of different size levy source-based corporate taxes and allocate taxes using the formula approach. The framework is taken from Wrede (2009). In contrast to most papers on corporate taxation which assume revenue-maximizing governments (see, e.g., Pethig and Wagener, 2007; Kolmar and Wagener, 2007; Eichner and Runkel, 2008), this paper analyzes the strategies of welfare-maximizing governments. Private consumption effects, as well as revenue effects, are considered. Instead of assuming a decreasing returns to scale technology (see, e.g., Pethig and Wagener, 2007; Eichner and Runkel, 2008) this paper assumes linearly homogeneous production functions. Since corporate taxes are distorting
as long as equity is not deductible, even with constant returns to scale, economic profits
are non-zero. Following Eichner and Runkel (2008), the total stock of capital is fixed, but
the return to capital is endogenous. Most other papers consider the small-country case
where the return to capital is exogenous (see, e.g., Wellisch, 2004; Pethig and Wagener,
2007; Pinto, 2007; Riedel and Runkel, 2007; Nielsen, Raimondos-Moeller, and Schjelderup,
competition under formula apportionment in a small, open federation framework where
governments maximize the welfare of their citizens, but only in a symmetric setting.

The main results of this paper can be summarized as the following:

1. Symmetric tax competition under formula apportionment leads to underprovision of
   public goods when profit shifting is ruled out.

2. Even under formula apportionment, the smaller country undercuts the larger country.
   Residents of the smaller country are better off than residents of the larger country.

Hence, it shows that fundamental features of asymmetric tax competition discovered by

The paper is organized as follows. Section 2 develops the model and derives the results.
Section 3 concludes.

2 The model

I consider an economy that consists of 2 jurisdictions with population $L_1$ and $L_2$. Each
individual supplies one unit of labor in the country of residence. There are a great many
identical multinational enterprises (MNEs) operating a plant in each jurisdiction. These
firms produce a private good with a constant returns to scale technology. Since the pro-
duction function is linearly homogenous, the number of firms and output per firm are
indeterminate. Without loss of generality, I proceed as if the total output is produced by
a single representative MNE that behaves competitively. It employs $K_i$ units of capital
and $L_i$ units of labor in jurisdiction $i$ to produce $F(K_i, L_i)$ units of output whose price is
normalized to 1. Marginal productivity of any input is positive and decreasing: $F_K > 0,$
$F_L > 0$, $F_{KK} < 0$, and $F_{LL} < 0$.\(^1\) Since the production function is linearly homogenous, $F = F_K K + F_L L$ and $F_{KL} = -F_{KK} K/L > 0$. By assuming that the marginal product of capital becomes rather large when capital intensity approaches 0, it is ensured that the MNE will indeed produce in both jurisdictions. For example, the Inada conditions would guarantee this.

While labor is perfectly immobile, capital is perfectly mobile. Each individual is endowed with $k$ units of capital. The common return to capital, $r$, is determined so as to clear the capital market in all jurisdictions; the wage in jurisdiction $i$, $w_i$, clears the labor market in this jurisdiction. The capital market clearing condition is

$$\sum_{i=1}^n (K_i - kL_i) = 0. \tag{1}$$

The firm’s investment is completely equity financed and equity is not deductible from tax liabilities in every jurisdiction. Equity financing is assumed just for convenience. An exogenously determined uniform debt-to-capital ratio would leave the results basically unaltered despite the fact that debt is fully tax deductible. The economic profit in jurisdiction $i$ is output minus labor costs and capital costs; taxable profits exceed economic profits:

$$\pi_i = \pi^i_t - rK_i, \text{ where } \pi^i_t = F(K_i, L_i) - w_i L_i \quad i = 1, 2. \tag{2}$$

Total profits net of corporate taxes are denoted by $\Pi$.

Each jurisdiction levies a source-based tax on corporate income while exempting foreign-source income of domestic residents, where jurisdiction $i$’s tax rate is $t_i$. Under formula apportionment, the MNE faces a uniform tax rate $\tau$ independent of investment location. Tax bases are consolidated and distributed to jurisdictions according to a formula based on the capital share $K_i/\sum_j K_j$, the sales share $F(K_i, L_i)/\sum_j F(K_j, L_j)$, and the payroll share $w_i L_i/\sum_j w_j L_j$. Jurisdiction $i$’s share in the total tax base is

$$S^i = \gamma \frac{K_i}{\sum_j K_j} + \sigma \frac{F(K_i, L_i)}{\sum_j F(K_j, L_j)} + \phi \frac{w_i L_i}{\sum_j w_j L_j}, \quad i = 1, 2. \tag{3}$$

The weights of the capital share, the sales share, and the payroll share sum up to 1: $\gamma + \sigma + \phi = 1$. Hence, the jurisdictions’ shares also sum up to 1: $\sum_j S^j = 1$. The MNE’s

\(^1\)Partial derivatives are indicated by a subscript.
The effective tax rate is
\[ \tau = \sum_{j=1}^{2} t_j S^j = t_i + \sum_{j \neq i} (t_j - t_i) S^j. \tag{4} \]

The representative individual in jurisdiction \( i \) derives utility from private consumption, \( X_i \), and a publicly provided private good, \( G_i \). The utility function, \( U(X_i, G_i) \), exhibits positive and diminishing marginal utilities and is strictly quasi-concave. To exclude corner solutions, I assume that marginal utilities are sufficiently large when private and public consumption approaches 0. The representative individual in jurisdiction \( i \) owns one share of the MNE, and earns capital and labor income. The budget constraint reads:
\[ X_i = \frac{\Pi}{L_1 + L_2} + rk + w_i, \quad i = 1, \ldots, n. \tag{5} \]

The government of jurisdiction \( i \) pays for the provision of good \( G_i \) with its tax revenue \( T_i \). The marginal rate of transformation between the private and the publicly provided private good is constant and normalized to 1: \( G_i = T_i/L_i \). National governments set tax rates non-cooperatively to maximize the welfare of their citizens \( U(X_i, G_i) \). The timing is as follows:

1. National governments simultaneously set tax rates \( t_i, 0 \leq t_i \leq 1, i = 1, \ldots, n. \)

2. National wages and the common interest rate are determined such that the MNE maximizes its profits through choice of labor and capital demand, and markets clear.

At the first stage, to tackle asymmetry I focus on small deviations from the symmetric Nash equilibrium of the tax-competition game where all jurisdictions set the same tax rate. A symmetric equilibrium is characterized by \( K_i = K, L_i = L, w_i = w, t_i = \tau = t, X_i = X, \) and \( G_i = G \), for \( i = 1, 2. \)

**Market equilibrium** The MNE maximizes total profits net of corporate taxes:
\[ \max_{K_i, L_i} \Pi := \sum_{j=1}^{2} \left( \pi_j - \tau \pi^j \right), \quad i = 1, 2. \tag{6} \]
The first-order conditions of the MNE’s optimization problem are for \( i = 1, 2 \)

\[
(1 - \tau) [F_L(K_i, L_i) - w_i] + \sum_{j \neq i} (t_i - t_j) S_{L_i}^j \sum_{k=1}^2 \pi_k^i = 0, \tag{7}
\]

\[
(1 - \tau) F_K(K_i, L_i) - r + \sum_{j \neq i} (t_i - t_j) S_{K_i}^j \sum_{k=1}^2 \pi_k^i = 0. \tag{8}
\]

In its decision regarding labor and capital, the MNE takes into consideration that changes in employment and capital stock affect tax base shares and, therefore, the effective tax rate. High tax rates reduce marginal benefits of employment and investment. In a symmetric equilibrium \( S^i = 1/n, S_{L_i}^j = - (\phi / L + \sigma F_L / F) / n^2 < 0, \)

\( S_{K_i}^j = - (\gamma / K + \sigma F_K / F) / n^2 < 0, \)

\( S_{L_i}^i = - (n - 1) S_{L_i}^j, \) and \( S_{K_i}^i = - (n - 1) S_{K_i}^j. \) At a symmetric equilibrium, the marginal product of labor is equal to the wage rate and the user cost of capital exceed the interest rate:

\[ F_K = r / (1 - \tau). \]

Plugging first-order conditions into the definitions for profits and taking linear homogeneity into account, yields

\[
\pi_i = \tau F_{K_i}^i K_i + \sum_{j \neq i} (t_i - t_j) \left( \frac{S_{L_i}^j L_i}{1 - \tau} + \frac{S_{K_i}^j K_i}{1 - \tau} \right) \sum_{k=1}^2 \pi_k^i, \quad i = 1, \ldots, n, \tag{9}
\]

\[
\pi_i^i = F_{K_i}^i K_i + \sum_{j \neq i} (t_j - t_i) \frac{S_{L_i}^j L_i}{1 - \tau} \sum_{k=1}^2 \pi_k^i, \quad i = 1, \ldots, n.
\]

Economic and taxable profits are non-zero; outside a symmetric equilibrium, even net profits per country are not zero. However, it can be shown that total net profits \( \Pi \) are zero. Profits and losses cancel out. Hence, individual income consists only of capital and labor income.

First-order conditions and market-clearing conditions determine how unilateral tax rate changes affect capital, wages, and the interest rate. Starting at a symmetric equilibrium, the effects are following for \( i, j = 1, 2 \) and \( j \neq i \):

\[
\frac{dK_i}{dt_i} = \frac{FF_F K_j \gamma + F_k^2 k L \sigma}{2(1 - t)} < 0, \quad \frac{dK_j}{dt_i} = - \frac{dK_i}{dt_i} > 0, \tag{10}
\]

\[
\frac{dw_i}{dt_i} = - \frac{F_K k}{2(1 - t)} < 0, \quad \frac{dw_j}{dt_i} = - \frac{dw_i}{dt_i} > 0, \quad \frac{dr}{dt_i} = - \frac{F_K}{2} < 0.
\]

In response to an increase in one country’s tax rate, firms shift capital abroad, which, due to labor-capital complementarity, reduces wages in the country that raised taxes and
increases wages abroad. The increase in the tax rate also implies higher user cost of capital, which mitigates investment incentives and, eventually, reduces the return to capital.

Starting at the symmetric equilibrium, a small increase in populations size leads to according capital flows but leaves wages and interest rate unaltered:

\[
\begin{align*}
\frac{dK_i}{dL_i} &= k, \quad \frac{dK_j}{dL_i} = 0, \quad \frac{dw_i}{dL_i} = \frac{dw_j}{dL_i} = \frac{dr}{dL_i} = 0.
\end{align*}
\] (11)

Furthermore, independent of the formula, for identical tax rates country i’s share in tax revenue is \( S_i = \frac{L_i}{L_1 + L_2} \) such that at the symmetrical equilibrium \( dS_i/dL_i = 1/(4L) \).

**Tax competition** Governments maximize national welfare which will be written as as \( V_i = V(t_i, t_j, L_i, L_j) \), \( i = 1, 2, j \neq i \). Nash equilibria at the tax competition stage are determined by the government’s first-order conditions:

\[
\begin{align*}
\frac{dV_i}{dt_i} &= \frac{\partial U(X_i, G_i)}{\partial X_i} \frac{dX_i}{dt_i} + \frac{\partial U(X_i, G_i)}{\partial G_i} \frac{dt_i}{L_i} = 0, \quad i = 1, 2.
\end{align*}
\] (12)

The marginal rate of substitution between private and public consumption is equal to the perceived marginal rate of transformation:

\[
\begin{align*}
\frac{\partial U(X_i, G_i)}{\partial U(X_i, G_i)} / \partial X_i = - \frac{dX_i/dt_i}{dt_i/L_i}, \quad i = 1, 2.
\end{align*}
\] (13)

Since total profits net of taxes are zero, individual income effectively consists only of capital and labor income, \( X_i = rk + w_i \). Hence, the impact of a unilateral tax rate increase on private consumption is given by

\[
\begin{align*}
\frac{dX_i}{dt_i} &= \frac{dr}{dt_i} k + \frac{dw_i}{dt_i} = - \frac{F_k k (2 - t)}{2(1 - t)} < 0, \quad i = 1, 2.
\end{align*}
\] (14)

In response to a tax rate change, wages and capital income and, thus, private consumption shrinks. The private consumption externality is positive at the symmetric equilibrium:

\[
\begin{align*}
PCE &= \frac{dX_j}{dt_i} = \frac{\Delta r}{dt_i} k + \frac{\Delta w_j}{dt_i} = \frac{tF_k k}{2(1 - t)} > 0, \quad i = 1, 2, j \neq i.
\end{align*}
\] (15)

Rising wages more than compensate for declining capital income. Taking Equation (9) into account, tax revenue can be written as

\[
T_i = t_i S_i \sum_{j=1}^2 \pi_j = t_i S_i \frac{\sum_{j=1}^2 K_j F_k^j}{1 - \sum_{j=1}^2 \sum_{k \neq j} (t_k - t_j)^{s_k^j t_j^i}} \frac{s_k^j t_j^i}{1 - t}, \quad i = 1, 2.
\] (16)
At the symmetric equilibrium, unilateral tax rate changes imply

$$\frac{dT_i}{dt_i} = (S^i + t_i S^i) \sum_{j=1}^{2} \pi_j^i + t_i S^i \left[ (K_1 F_{KK}^i + F_K) \frac{dK_1}{dt_i} + (K_2 F_{KK}^2 + F^2_K) \frac{dK_2}{dt_i} \right]$$

(17)

$$= (S^i + t_i S^i) \sum_{j=1}^{2} K_j F^j_K, \quad i = 1, 2,$$

where at the symmetric equilibrium

$$S^i_{t_i} = \frac{\gamma}{(K_1 + K_2)^2} \left[ (K_1 + K_2) \frac{dK_1}{dt_i} - K_i \left( \frac{dK_1}{dt_i} + \frac{dK_2}{dt_i} \right) \right]$$

$$+ \frac{\sigma}{(F_1 + F_2)^2} \left[ (F_1 + F_2) F_K^i \frac{dK_1}{dt_i} - F_i \left( F_K^i \frac{dK_1}{dt_i} + F^2_K \frac{dK_2}{dt_i} \right) \right]$$

$$+ \frac{\phi}{(w_1 L_1 + w_2 L_2)^2} \left[ (w_1 L_1 + w_2 L_2) L_i \frac{dw_i}{dt_i} - w_i L_i \left( L_1 \frac{dw_1}{dt_i} + L_2 \frac{dw_2}{dt_i} \right) \right]$$

(18)

gives the impact of a country’s tax rate on its share in the tax base. Any unilateral increase in the tax rate reduces the jurisdiction’s share in the global tax base no matter what the weights in the formula are. Ceteris paribus, the magnitude of $S^i_{t_i}$ depends positively on each weight. At the equilibrium of the tax competition game, the tax rate must be on the upward-sloping part of the country’s perceived Laffer curve, i.e., $dT_i/dt_i > 0$. The public consumption externality is also positive at the symmetric equilibrium:

$$\text{PGE} = \frac{dT_i}{dt_i} \frac{1}{L_i} = t_j S^j_{t_j} \sum_{k=1}^{2} K_k F^k_K \frac{1}{L_j} > 0, \ i = 1, 2, \ j \neq i.$$

(19)

Since both the private consumption externality and the public good externality are positive, the deviation from the Pareto optimum, at which $\left[ \partial U(X_i, G_i) / \partial G_i \right] / \left[ \partial U(X_i, G_i) / \partial X_i \right] = 1$ should hold, could be unambiguously signed.

**Proposition 1** Under formula apportionment and equity financing, the symmetric Nash equilibrium of tax competition is characterized by underprovision of publicly provided goods. Both jurisdictions would benefit from small increases in tax rates and public good quantities.

Even under formula apportionment, non-cooperatively taxing governments perceive higher marginal costs of tax rate increases, since they expect capital flight in response to unilateral
Figure 1: Underprovision of publicly provided goods

tax changes. Figure 1 shows the Nash equilibrium, $A$, where the perceived production possibility curve is steeper than the production possibility curve under coordination. The formula approach does not solve the inefficiency problem. In contrast to the case where profit shifting is present, overprovision could also be ruled out (see Wrede, 2009).

**Asymmetry** In order to tackle asymmetric tax competition, I consider small deviations from the symmetric tax competition equilibrium. Without loss of generality, I analyze a marginal shift of workers from country 2 to country 1. It turns out that, starting at the symmetric equilibrium, relocating workers from one country to the other has no direct impact on private and public consumption. When labor is relocated from one country to the other, capital moves accordingly, leaving wages and capital returns and, hence, private consumption, $X_i$, untouched. Moreover, for identical tax rates, aggregate taxable profits, $\sum_{j=1}^{2} \pi_j$, remain unaffected, too, since profit decreases in the source country and profit increases in the destination country cancel out. Since the share of the destination country in total tax revenue $S^1$ increases by $1/(2L)$, public consumption, $G_1 = T_1/L_1$, is also
unchanged. Only \( dX_i / dt_i \) and \((dT_i / dt_i)(1/L_i)\) change:

\[
\frac{d^2X_1}{dt_1dL_1} - \frac{d^2X_1}{dt_1dL_2} = \frac{F_Kkt}{2(1-t)L} > 0, \tag{20}
\]
\[
\frac{d^2T_1}{dt_1dL_1} - \frac{d^2T_1}{dt_1dL_2} = F_Kk > 0.
\]

Using these calculations, the impact of a relocation of workers on the perceived production possibility curve in the destination country could be unambiguously signed:

\[
d\left( -\frac{dX_1}{dt_1} \right) \frac{dL_1}{L_1} - d\left( -\frac{dX_1}{dt_1} \right) \frac{dL_2}{L_2} = F_Kk \left( 1 - \partial U_1 / \partial G_1 \right) < 0. \tag{21}
\]

Since the country considers itself being on the upward-sloping part of the Laffer curve and underprovides the public good, inward labor flows reduces the slope of the perceived production possibility curve (in absolute terms). Without any tax response, the new curve is somewhat like the dashed curve in figure 1. By the same token, the perceived curve of the source country is steeper than in the symmetric equilibrium. In response, the larger country would raise its tax rate and the smaller country reduces it. As a consequence, production possibility curves would move in opposite directions. Inwards for the larger country that taxes capital more heavily, outwards for the smaller one. Presumably, the outcome of tax rate adjustment is an equilibrium where the larger country levies a higher tax rate and is worse off than in the symmetric equilibrium while the smaller country benefits from being smaller. That this is indeed the case under conditions of stability could be shown analytically. From the first-order conditions of the Nash equilibrium (12), the impact of changes in the labor force on tax rates could be calculated:

\[
\frac{dt_i}{dL_k} = -\frac{\partial^2 V_i}{\partial t_i \partial L_k} \frac{dt_i}{dt_j} \frac{\partial^2 V_j}{\partial t_j \partial L_k} \Delta, \quad i = 1, 2, j \neq i, \tag{22}
\]

where

\[
\Delta = \frac{d^2V_i}{dt_i^2} \frac{d^2V_j}{dt_j^2} - \frac{d^2V_i}{dt_i dt_j} \frac{d^2V_j}{dt_j dt_i}. \tag{23}
\]
Hence, moving workers from country 2 to country 1 leads to

\[
\frac{dt_1}{dL_1} - \frac{dt_1}{dL_2} = -\left( \frac{d^2V_1}{dt_1dL_1} - \frac{d^2V_2}{dt_1dL_2} \right) \frac{d^2V_2}{dt_2} - \left( \frac{d^2V_2}{dt_2dL_1} - \frac{d^2V_2}{dt_2dL_2} \right) \frac{d^2V_1}{dt_1dt_2}. \tag{24}
\]

Starting at the symmetric equilibrium,

\[
\frac{dt_1}{dL_1} - \frac{dt_1}{dL_2} = -\left( \frac{d^2V_1}{dt_1dL_1} - \frac{d^2V_2}{dt_1dL_2} \right) \frac{d^2V_2}{dt_2} + \left( \frac{d^2V_1}{dt_1dL_1} + \frac{d^2V_2}{dt_1dL_2} \right). \tag{25}
\]

Assuming stability, a simple relationship emerges. The symmetric Nash equilibrium is stable if and only if \(d^2V_i/dt_i^2 < 0\), \(i = 1, 2\), and \(\Delta > 0\). Taking symmetry explicitly into account, stability implies \(|d^2V_i/dt_i|^2 > |d^2V_i/dt_j|\), \(j \neq i\). Hence, at the stable symmetric Nash equilibrium of the tax competition game,

\[
\frac{dt_1}{dL_1} - \frac{dt_1}{dL_2} > 0 \quad \text{if and only if} \quad \frac{d^2V_1}{dt_1dL_1} - \frac{d^2V_2}{dt_1dL_2} > 0 \tag{26}
\]

holds. Shifting workers from country 2 to country 1 gives incentives for country 1 to raise its tax rate if and only if it increases the marginal benefit of its tax rate, i.e., if it raises \(dV_1/dt_1\). Under the same conditions, country 2 will lower its tax rate in response to a shrinking population size.

It is possible to sign the impact of the relocation of workers on \(dV_1/dt_1\). Somewhat lengthy calculations\(^2\) lead to

\[
\frac{d^2V_1}{dt_1dL_1} - \frac{d^2V_1}{dt_1dL_2} = \frac{\partial U_1}{\partial X_1} \left( \frac{d^2X_1}{dt_1dL_1} - \frac{d^2X_1}{dt_1dL_2} \right) + \frac{\partial U_1}{\partial G_1} \left[ \left( \frac{d^2T_1}{dt_1dL_1} - \frac{d^2T_1}{dt_1dL_2} \right) \frac{1}{L_1} - \frac{dT_1}{dt_1L_1} \right]
\]

\[
= \frac{\partial U}{\partial G} \left[ \frac{F^2K^2t(1-\sigma-\gamma)}{(F - F_KkL)(1-t)} - \frac{F^2Kt}{F_KK(1-t)} \left( \frac{\gamma^2}{2L^2} + \frac{\sigma^2F^2K^2}{2F^2} + \frac{\gamma\sigma F_Kk}{LF} \right) \right]
\]

\[
+ \frac{\partial U}{\partial X} \frac{F_kkt}{2L(1-t)} > 0.
\tag{27}
\]

In response to a relocation of workers, the larger country raises its tax rate, the smaller country reduces it. Still, the basic elasticity argument holds. The larger country faces a lower tax-rate elasticity than the smaller country.

\(^2\)Calculations are available from the author upon request.
Interestingly enough, these changes in size and tax rates hurt the larger country and benefit the smaller one. Taking first-order conditions (12) into account and the fact that neither $X_i$ nor $G_i$ changes, the welfare effect can be calculated as:

$$
\frac{dV_1}{dL_1} - \frac{dV_1}{dL_2} = \left( \frac{\partial U_1}{\partial X_1} \frac{dX_1}{dt_1} + \frac{\partial U_1}{\partial G_1} \frac{dT_1}{dt_1} \frac{1}{L_1} \right) \left( \frac{dt_1}{dL_1} - \frac{dt_1}{dL_2} \right)
+ \left( \frac{\partial U_1}{\partial X_1} \frac{dX_1}{dt_2} + \frac{\partial U_1}{\partial G_1} \frac{dT_1}{dt_2} \frac{1}{L_1} \right) \left( \frac{dt_2}{dL_1} - \frac{dt_2}{dL_2} \right)
+ \left[ \frac{\partial U_1}{\partial X_1} \left( \frac{dX_1}{dL_1} - \frac{dX_1}{dL_2} \right) + \frac{\partial U_1}{\partial G_1} \left( \frac{dT_1}{dL_1} L_1 - \frac{dT_1}{dL_2} L_1 - \frac{T_1}{L_1} \right) \right]
\left( \frac{dt_1}{dL_1} - \frac{dt_2}{dL_2} \right)
= \left( \frac{\partial U_1}{\partial X_1} \frac{dX_1}{dt_2} + \frac{\partial U_1}{\partial G_1} \frac{dT_1}{dt_2} \frac{1}{L_1} \right) \left( \frac{dt_2}{dL_1} - \frac{dt_2}{dL_2} \right)
= L \left( \frac{d^2V_1}{dt_1 dL_1} - \frac{d^2V_1}{dt_1 dL_2} \right) \left( \frac{dt_2}{dL_1} - \frac{dt_2}{dL_2} \right) < 0.
$$

Moving an infinitesimal number of workers from country 2 to country 1 gives incentives to the now smaller country 2 to reduce its tax rate. As a consequence, capital flows out of country 1 inducing lower tax revenue and lower wages. Private and public good externalities of taxation imply a utility loss in the high tax country. By similar reasoning it could be shown that the smaller country benefits.

The following proposition summarizes the main results of the paper:

**Proposition 2** *Even under formula apportionment and equity financing, moving an infinitesimal number of workers from one country to the other leads to divergence in tax rates and welfare. Compared to the smaller country, the larger country sets the higher tax rate and its representative individual is worse off.*

### 3 Concluding remarks

This paper analyzed asymmetric tax competition under formula apportionment. Employing a model with multinationals where two welfare-maximizing jurisdictions of different size levy source-based corporate taxes and allocate taxes using the formula approach, it was shown that the Nash equilibrium is characterized by tax rates and public good quantities being too low. The main result of the paper was that the larger country levies a larger tax rate than the smaller country and that inhabitants of the larger country are worse off...
than those of the smaller country. Hence, it showed that under formula apportionment and separating accounting differences in population size have similar effects.

The model could be extended in various ways. In particular, profit shifting could be incorporated which possibly would make results somewhat ambiguous.
References


