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## Trade and variety in a model of endogenous product differentiation

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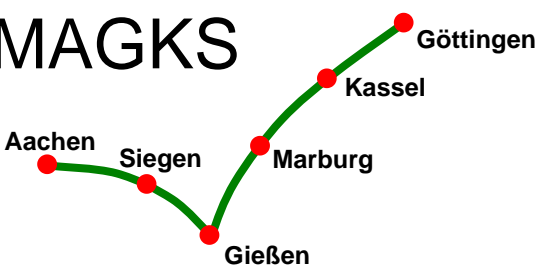
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**Trade and Variety in a Model  
of Endogenous Product Differentiation**

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# Trade and Variety in a Model of Endogenous Product Differentiation

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## Abstract

This paper sets up a model of endogenous product differentiation to analyze the variety effects of international trade. In our model multi-product firms decide not only about the number of varieties they supply but also about the degree of horizontal differentiation between these varieties. Firms can raise the degree of differentiation by investing variety-specific fixed costs. In this setting, we analyze how trade integration, i.e. an increase in market size, influences the number of firms in the market, the number of product varieties supplied by each firm, and the degree of differentiation.

*JEL-Classification:* D43, F12, L25.

*Keywords:* Product differentiation, multi-product firms, international trade.

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# 1 Introduction

If economists talk about the effects of international trade on product variety, they typically have in mind the influence of trade on the number of heterogeneous goods. Variety, however, can be much broader than this; it encompasses not only the mere number of goods but also their specification. Heterogeneous goods may differ only slightly from each other – for example just in their color – or substantially with respect to more important characteristics. Our analysis starts from this observation and endogenizes product differentiation in an otherwise standard “love of variety” model of intra-industry trade. We consider multi-product firms that decide not only about the number of varieties but also about the degree of horizontal product differentiation between these varieties. The equilibrium degree of product differentiation emerges from a trade-off between positive demand effects and additional product-specific fixed costs. International trade may then influence variety through three different channels: by changing *(i)* the number of firms on the market, *(ii)* the number of varieties firms supply, and *(iii)* the degree of product differentiation between these varieties.

Our model of endogenous product differentiation produces a couple of interesting insights: Firstly, we show that firms regard the number of varieties and the degree of product differentiation as complementary instruments if the cost function satisfies certain sufficient properties. An increasing scope then makes it more profitable for firms to raise the degree of product differentiation and vice versa. The reason is that consumers’ valuation of product differentiation increases in the number of varieties. Secondly, under the same sufficient conditions an increase in market size results in more varieties per firm and a higher degree of product differentiation. Thirdly, the activities of firms to differentiate their products horizontally have consequences for market entry which are comparable to the case of vertical product differentiation.

The theoretical literature on international trade has recently become increasingly engaged in analyzing the behavior of multi-product firms.<sup>1</sup> Feenstra and Ma (2008) consider multi-product firms in a Dixit-Stiglitz model of intra-industry trade with a uniform elasticity of substitution. In their paper, firms balance the additional sales generated from introducing new product varieties against a “cannibalization” of sales of their existing varieties. From this trade-off Feenstra and Ma (2008) determine the equilibrium number of varieties per firm, assuming that firms can not influence the degree of product differentiation between varieties. Allanson and Montagna (2005) consider a

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<sup>1</sup>In addition to the papers discussed below, see e.g. Brambilla (2006), Baldwin and Gu (2006), Bernard et al. (2006), Hansen and Jørgensen (2001), Hansen and Nielsen (2007) and Nocke and Yeaple (2006).

nested CES-utility with a lower elasticity of substitution between individual varieties supplied by a single firm compared to varieties supplied by different firms. Erkel-Rousse (1997) introduces vertical product differentiation in a model with multi-product firms and CES-preferences. Both, Allanson and Montagna (2005) and Erkel-Rousse (1997) neglect the cannibalization effect by assuming that firms are small and do not allow for endogenous product differentiation.

Multi-product firms are also analyzed in models of spatial product differentiation, as for example with flexible manufacturing.<sup>2</sup> According to the flexible manufacturing approach firms cover a whole area around their “core competence” in a Salop-type circular market. Marginal costs increase with the distance from a firms’ core competence, which limits firms’ expansion over the product space in addition to the cannibalization effect. Other set-ups of product differentiation are chosen by Anderson and de Palma (1992 and 2006), who consider a nested logit-demand function, and Ottaviano et al. (2002) and Melitz and Ottaviano (2005), who assume linear-quadratic preferences. Our paper builds on a Dixit-Stiglitz model with multi-product firms, but in contrast to the literature mentioned above we let firms determine how strong cannibalization is by deciding about the degree of product differentiation.

Since we assume that product differentiation influences product-specific fixed costs, our model is related to the endogenous sunk-cost literature.<sup>3</sup> According to this literature firms can raise product quality or reduce variable costs by increasing fixed cost investments upon market entry. Shaked and Sutton (1987) show that under certain conditions a lower bound for individual market shares exists if fixed costs are endogenous. As market size grows, at least one firm then invests more to retain a certain market share. Eckel (2006b) incorporates endogenous sunk costs in an otherwise standard model of intra-industry trade with one product per firm. He assumes that firms can reduce variable costs by investing sunk costs, and he determines conditions under which the number of firms increases or decreases with market integration. We show that this result of a possible decline in the number of firms carries over to the case of multi-product firms with endogenous product differentiation. In our set-up, however, a decline in the number of firms is not equivalent with lower variety as it results from an increase in the number of product varieties each firm supplies and from a higher degree of horizontal

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<sup>2</sup>For trade models with flexible manufacturing see e.g. Eckel (2006a) and Eckel and Neary (2008).

<sup>3</sup>See e.g. Dasgupta and Stiglitz (1980), Shaked and Sutton (1984, 1987), or Sutton (1991).

differentiation between varieties.<sup>4</sup>

The remainder of the paper is organized as follows: Section 2 presents the baseline model. In section 3 we analyze the equilibrium and derive the influence of market size on product differentiation, the number of varieties per firm, and the number of firms in the market. Section 4 concludes.

## 2 The Model

We consider a market with  $m$  symmetric firms ( $i = 1 \dots m$ ). Each of these firms supplies a continuum of differentiated varieties with mass  $n > 1$  (the “number of varieties”). Free market entry determines  $m$  endogenously. On the demand side we assume  $L$  households which consume a differentiated manufacturing aggregate  $C$  and an agricultural good  $A$ , the numéraire. Preferences with respect to these two goods are Cobb-Douglas, such that households spend a constant share  $\mu$  of their income for  $C$  and  $1 - \mu$  for  $A$ . Labor productivity in the agricultural sector and thereby individual wage income is normalized to 1. A CES-function specifies the manufacturing aggregate:

$$C = \left( \sum_{i=1}^m \tilde{c}_i^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} . \quad (1)$$

In (1) the sub-utility  $\tilde{c}_i$  is an aggregate of all varieties supplied by firm  $i$ .

Firms not only decide about the number of varieties  $n_i$  they supply but also about the degree of horizontal differentiation  $v_i$  between these varieties. More specifically, we assume that each variety of firm  $i$  consists of a continuum of components – distributed uniformly between 0 and 1 – and that firms can decide whether to differentiate components or not. The fraction of differentiated components embodied in the varieties supplied by firm  $i$  then determines the degree of differentiation  $v_i$  ( $0 \leq v_i \leq 1$ ), and the sub-utility with respect to all varieties of firm  $i$  can be specified as:

$$\tilde{c}_i = v_i \left( \int_0^{n_i} c_i(k)^{(\sigma-1)/\sigma} dk \right)^{\sigma/(\sigma-1)} + (1 - v_i) \int_0^{n_i} c_i(k) dk .$$

The first term of  $\tilde{c}_i$  measures the part of the sub-utility which results from differentiated components and the second term is the part of the sub-utility derived from non-differentiated components. For simplicity, we assume perfect symmetry between all varieties of firm  $i$ . We can then set  $c_i(k) = c_i$  for

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<sup>4</sup>Ferguson (2008) also endogenizes fixed costs in a trade model with Dixit-Stiglitz preferences. As Eckel (2006b) he considers a setting with only one product per firm.

all  $k$  to obtain

$$\tilde{c}_i = \left[ v_i n_i^{1/(\sigma-1)} + 1 - v_i \right] n_i c_i. \quad (2)$$

To interpret the influence of  $v_i$  first consider the limit case  $v_i = 0$ . In this case all varieties of firm  $i$  are perfect substitutes and  $\tilde{c}_i = n_i c_i$ . The aggregate of firm  $i$ 's varieties is just equal to the total quantity produced, and our model does not differ from a standard Dixit-Stiglitz setting with one product per firm. In the other extreme  $v_i = 1$  and  $\tilde{c}_i = n_i^{\sigma/(\sigma-1)} c_i$ . In this case consumers can substitute different varieties supplied by firm  $i$  in the same way as varieties supplied by different firms. This is the setting analyzed by Feenstra and Ma (2008). Most interestingly for our paper are, of course, the intermediate cases in which  $0 < v_i < 1$ .

We can define  $x_i(n_i, v_i)$  by  $x_i \equiv \tilde{c}_i / (n_i c_i) = v_i n_i^{1/(\sigma-1)} + 1 - v_i$  as the sub-utility generated per unit of output supplied by firm  $i$ . This expression, which can be interpreted as a quality index for products of firm  $i$ , monotonically increases in  $n_i$  and in  $v_i$ , i.e.  $\partial x_i / \partial v_i > 0$  and  $\partial x_i / \partial n_i > 0$ . An increase in the number of varieties or in the degree of product differentiation makes the product range of firm  $i$  more attractive for consumers. We also have  $\partial x_i^2 / \partial n_i \partial v_i > 0$ . The marginal utility from raising the degree of product differentiation increases in the number of varieties. That is, the more varieties a firm supplies, the higher consumers value differentiation between these varieties. This property of the utility function will become important for the results of our subsequent analysis.

Utility maximization of consumers leads to an individual demand of

$$\tilde{c}_i = \mu \tilde{p}_i^{-\sigma} \left( \sum_{j=1}^m \tilde{p}_j^{1-\sigma} \right)^{-1}, \quad (3)$$

where  $\tilde{p}_i$  is the price index for the variety range supplied by firm  $i$ , i.e.  $\tilde{p}_i \tilde{c}_i = n_i p_i c_i$  or  $\tilde{p}_i = p_i / x_i$ . For aggregate demand  $Q_i = L n_i c_i$  we can write

$$Q_i = \mu L x_i^{\sigma-1} p_i^{-\sigma} \left( \sum_{j=1}^m x_j^{\sigma-1} p_j^{1-\sigma} \right)^{-1}.$$

Each firm decides about the number of varieties  $n_i$ , the degree of horizontal product differentiation between these varieties  $v_i$ , and their price  $p_i$ . With marginal costs of  $\phi$  and fixed costs of  $F$ , profits of firm  $i$  are  $\pi_i = (p_i - \phi) Q_i - F$ . We assume that fixed costs of the firm increase in the number of varieties and in the degree of product differentiation:  $F = F(n, v)$ . More specifically, we assume  $F_n > 0$ ,  $F_v > 0$ ,  $F_{vv} \geq 0$ ,  $F_{nn} \geq 0$ , and  $F_{vn} \geq 0$ . For later use we may define  $\epsilon^n \equiv F_n n / F$  and  $\epsilon^v \equiv F_v v / F$  as the elasticities

of  $F$  with respect to  $n$  and  $v$ . These elasticities can be interpreted as measures for economies of scope in the  $F(\cdot)$  function or economies of horizontal product differentiation, respectively, as  $\eta_{F/n,n} = \epsilon^n - 1$  and  $\eta_{F/v,v} = \epsilon^v - 1$ .

### 3 Equilibrium and Trade Integration

This section first characterizes the equilibrium of our model. From this equilibrium we can determine the effects of trade integration by raising the size of the relevant market  $L$ . We consider a market with at least 2 active firms, i.e.  $m \geq 2$ . Assuming symmetry of all competitors of firm  $i$  (denoted by  $-i$ ) we can write the profit of firm  $i$  as follows:

$$\pi_i = \mu L \cdot \frac{p_i - \phi}{p_i} \cdot \frac{x_i^{\sigma-1} p_i^{1-\sigma}}{x_i^{\sigma-1} p_i^{1-\sigma} + (m-1)x_{-i}^{\sigma-1} p_{-i}^{1-\sigma}} - F. \quad (4)$$

In equilibrium, the first order conditions for  $p_i$ ,  $v_i$ , and  $n_i$  have to be satisfied. In addition, because of free entry firms make zero profits. The first order condition for  $p_i$  determines the mark-up rule

$$\frac{p - \phi}{p} = \frac{m}{1 + (m-1)\sigma} \quad (5)$$

in the symmetric equilibrium. This rule determines the equilibrium price as a function of the number of firms  $m$ . According to (5), the mark-up declines in  $m$ . If there are more firms in the market, demand becomes more elastic and as a consequence firms reduce their mark-up. Using (5), the first order conditions for  $n_i$  and  $v_i$  and the zero-profit condition in the symmetric equilibrium can be written as

$$\mu L \cdot \frac{m-1}{m[1 + (m-1)\sigma]} \frac{vn^{1/(\sigma-1)}}{nx} = F_n, \quad (6)$$

$$\mu L \cdot \frac{m-1}{m[1 + (m-1)\sigma]} \cdot \frac{(\sigma-1)(n^{1/(\sigma-1)} - 1)}{x} = F_v, \quad (7)$$

$$\mu L \cdot \frac{1}{1 + (m-1)\sigma} = F. \quad (8)$$

Were fixed costs independent of the number of varieties and the degree of differentiation, firms would supply as many varieties as possible and differentiate them completely. However, cost considerations lead to less differentiation and to a finite number of varieties. The zero-profit condition determines the number of firms  $m$  as a function of market size  $L$ . Not surprisingly,



$dm/dL > 0$  for a given  $n$  and  $v$ . Using (8), the equilibrium can be written as a system in  $n$  and  $v$  only:

$$\frac{\mu L - F}{\mu L - F(1 - \sigma)} \cdot \frac{vn^{1/(\sigma-1)}}{x} = \epsilon^n, \quad (9)$$

$$\frac{\mu L - F}{\mu L - F(1 - \sigma)} \cdot \frac{(\sigma - 1)v(n^{1/(\sigma-1)} - 1)}{x} = \epsilon^v. \quad (10)$$

Dividing both equations and rearranging yields

$$(\sigma - 1)(1 - n^{1/(1-\sigma)}) = \frac{\epsilon^v}{\epsilon^n}. \quad (11)$$

Equation (11) determines the relationship between  $n$  and  $v$ , which is independent of market size  $L$ . From totally differentiating (11) we obtain

$$\frac{dv}{dn} = \frac{\left(\frac{\epsilon^n}{\epsilon^n} - \frac{\epsilon^v}{\epsilon^v}\right) \frac{\epsilon^v}{\epsilon^n} + n^{\sigma/(1-\sigma)}}{\left(\frac{\epsilon_v^v}{\epsilon^v} - \frac{\epsilon_v^n}{\epsilon^n}\right) \frac{\epsilon^v}{\epsilon^n}}. \quad (12)$$

The following lemma describes a sufficient condition for a positive relationship between  $n$  and  $v$ , which follows from (12).

**Lemma.** *An increase in the number of varieties  $n$  causes the firm also to raise the degree of differentiation  $v$  if the following two conditions are satisfied:*

$$\frac{\epsilon_v^v}{\epsilon^v} > \frac{\epsilon_v^n}{\epsilon^n} \quad \text{and} \quad \frac{\epsilon_n^n}{\epsilon^n} \geq \frac{\epsilon_n^v}{\epsilon^v}. \quad (13)$$

Assuming stability of the system (9) and (10) further analysis shows that (see appendix)

$$\frac{dn}{dL} > 0 \quad \text{iff} \quad \frac{\epsilon_v^v}{\epsilon^v} > \frac{\epsilon_v^n}{\epsilon^n}. \quad (14)$$

An increase in market size then raises the number of varieties supplied by each firm and the degree of horizontal differentiation between these varieties.

We may further specify the condition in (13) by looking more closely at the fixed cost function  $F(n, v)$ . Taking the partial derivatives of  $\epsilon^v$  and  $\epsilon^n$  yields

$$\frac{\epsilon_v^v}{\epsilon^v} - \frac{\epsilon_v^n}{\epsilon^n} = \frac{1}{v} (1 + \eta_{F_v, v} - \eta_{F_n, v}). \quad (15)$$

This implies

$$\frac{\epsilon_v^v}{\epsilon^v} > \frac{\epsilon_v^n}{\epsilon^n} \quad \text{iff} \quad \eta_{F_v, v} + 1 > \eta_{F_n, v}.$$

Similarly, we can show that

$$\frac{\epsilon_n^n}{\epsilon^n} \geq \frac{\epsilon_n^v}{\epsilon^v} \quad \text{iff} \quad \eta_{F_n,n} + 1 \geq \eta_{F_v,n} .$$

Hence:

**Proposition.** *An increase in market size  $L$  raises the degree of product differentiation  $v$  and the number of varieties per firm if the fixed cost function satisfies the following sufficient properties:*

$$\eta_{F_v,v} + 1 > \eta_{F_n,v} \quad \text{and} \quad \eta_{F_n,n} + 1 \geq \eta_{F_v,n} . \quad (16)$$

An example for a cost function that satisfies (16) is  $F = f + gn \exp(\theta v)$ . The term  $f$  then stands for firm-specific fixed costs and  $g \exp(\theta v)$  are variety-specific fixed costs per variety. Variety-specific fixed costs increase in  $v$  as fewer components can be standardized and used for all varieties. The cost function exhibits economies of scope as

$$\epsilon^n = \frac{gn \exp(\theta v)}{f + gn \exp(\theta v)} < 1 . \quad (17)$$

An increase in the number of varieties or in the degree of product differentiation lowers the economies of scope:  $\epsilon_v^n > 0$  and  $\epsilon_n^n > 0$ . Similarly, we have  $\epsilon_v^v > 0$  and  $\epsilon_n^v > 0$ . Differentiating the cost function, we can easily show that both inequalities in (16) are satisfied. The number of varieties and the degree of differentiation then both increase in the size of the market  $L$ . Another example is the cost function  $F = f + gn v^\theta + hn$ . This function has the interesting property that for the limit of  $h \rightarrow 0$  we obtain  $\epsilon^v/\epsilon^n = \theta$ , such that the number of varieties is constant in this special case.

The increase in  $n$  and  $v$  raises product specific fixed costs  $F(n, v)$ . Consequently, the influence of market size on the number of firms is ambiguous in this setting. More precisely, the zero profit-condition (8) implies

$$m = \frac{\mu L}{\sigma F} + \frac{\sigma - 1}{\sigma} . \quad (18)$$

Since

$$\frac{dm}{dL} = -\frac{\mu L}{\sigma F^2} \left( F_n \frac{dn}{dL} + F_v \frac{dv}{dL} \right) + \frac{\mu}{F\sigma} , \quad (19)$$

we obtain

$$\frac{dm}{dL} > 0 \quad \text{iff} \quad \epsilon^n \eta_{L,n} + \epsilon^v \eta_{L,v} < 1 . \quad (20)$$

Trade integration raises the total number of firms only if the elasticities of the cost function with respect to product variation times the market size

elasticity of product variation add up to less than one. This result is related to findings of the endogenous sunk-costs literature mentioned in the introduction where the number of firms not necessarily increases as the market becomes larger. Although the influence of trade integration on the total number of firms is indeterminate, the number of firms per country clearly declines if (16) is satisfied. Using (19), this follows immediately from the derivative  $d(m/L)/dL = (dm/dL - m/L)/L$ . The number of firms relative to  $L$  then declines in  $L$ , and each firm produces on a larger scale.

To determine the welfare effects of trade integration we define the aggregate price index as  $\tilde{P} = \left( \sum_{j=1}^m \tilde{p}_j^{1-\sigma} \right)^{1/(1-\sigma)}$ . Then  $C = \mu/\tilde{P}$  and aggregate welfare declines in  $\tilde{P}$ . Symmetry implies  $\tilde{P} = (m)^{1/(1-\sigma)} p x^{-1}$ . Welfare depends on the number of firms, the price, and the valuation index  $x$ , which in turn is determined by the product scope and the degree of differentiation. We know already that  $x$  increases in the size of the market. From the mark-up formula (5) we obtain

$$p = \frac{1 + \sigma(m-1)}{(m-1)(\sigma-1)} \phi.$$

The price  $p$  declines in the number of firms. Thus, if  $m$  increases or if the decline in  $m$  is sufficiently weak, then welfare clearly increases in  $L$ . Although trade integration raises cost efficiency since firms produce on a larger scale, it will only be welfare enhancing as long as competition intensity does decline to strongly.

## 4 Concluding Remarks

Because of its analytical tractability and the strong results it delivers, the Dixit-Stiglitz model has become a workhorse of modern trade theory. This paper proposes an approach to endogenize horizontal product differentiation in this framework. We formulate a setting in which multi-product firms not only decide about entry and about the number of varieties they supply but also about the degree of differentiation between these varieties. This approach provides us with a much richer picture of firm behavior than the standard model. In this concluding section we do not review our results in detail. Instead, we emphasize what we regard as the central message from our paper: To evaluate the influence of trade on product variety it is not enough to look at the mere number of products. Instead, trade may also change the degree of horizontal differentiation between products. The next step toward a better understanding the variety effects of trade would be an empirical test of the effects of trade on the degree of product differentiation.

## Appendix

This appendix derives the comparative static results underlying Proposition 1. The two equilibrium conditions (9) and (10) can be written as

$$M \cdot U - \epsilon^n = 0 , \quad (21)$$

$$M \cdot V - \epsilon^v = 0 , \quad (22)$$

where

$$\begin{aligned} M(L, n, v) &= \frac{\mu L - F(n, v)}{\mu L - F(n, v) (1 - \sigma)} , \\ U(n, v) &= \frac{v n^{1/(\sigma-1)}}{x(n, v)} , \quad \text{and} \\ V(n, v) &= \frac{v (\sigma - 1) (n^{1/(\sigma-1)} - 1)}{x(n, v)} . \end{aligned}$$

Totally differentiating (21) and (22) yields

$$\begin{pmatrix} a_{1n} & a_{1v} \\ a_{2n} & a_{2v} \end{pmatrix} \cdot \begin{pmatrix} dn \\ dv \end{pmatrix} = \begin{pmatrix} -U M_L dL \\ -V M_L dL \end{pmatrix} . \quad (23)$$

The terms in (23) are the first derivatives of the implicit functions (21) and (22) with respect to  $n$  and  $v$ , i.e.

$$a_{1n} = M \cdot U_n + U \cdot M_n - \epsilon_n^n$$

etc. The effect of an increase in  $L$  on  $n$  is given by

$$\frac{dn}{dL} = \frac{(V \cdot a_{1v} - U \cdot a_{2v}) M_L}{|J|} \quad (24)$$

For meaningful comparative static results we assume a positive sign of the Jacobian, i.e.  $|J| > 0$ . The partial derivative of  $M$  with respect to  $L$  is positive:  $M_L > 0$ . Thus,  $dn/dL > 0$  iff  $V \cdot a_{1v} > U \cdot a_{2v}$ . Determining  $a_{1v}$  and  $a_{2v}$  from (21) and (22) yields:

$$V \cdot a_{1v} > U \cdot a_{2v} \quad \text{iff} \quad \epsilon_v^v \cdot U > \epsilon_v^n \cdot V \quad (25)$$

$$\text{or} \quad \frac{\epsilon_v^v}{\epsilon_v^n} > (\sigma - 1) (1 - n^{1/(1-\sigma)}) . \quad (26)$$

Inserting from (11) then yields (14).

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