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**Sector-specific productivity shocks in a matching model**

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Sector-Specific Productivity Shocks in a Matching Model
by Dennis Wesselbaum

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JEL classification: E24, J24, J41.

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Sector-Specific Productivity Shocks in a Matching Model*

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January 13, 2010

Abstract

Shocks driving the business cycle have different effects on low-skilled and high-skilled workers. This paper studies the effects of temporary and permanent sector-specific shocks in a New Keynesian matching model. We show that temporary sector-specific shocks have reallocation and aggregate effects. Permanent shocks explain wedges in real wages and different performances in labor markets. Furthermore, the model is able to replicate an aggregate Beveridge curve.

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1 Motivation

In this paper we develop an endogenous separation matching model with ex ante heterogeneous agents. This heterogeneity is founded within the existence of two types of workers: high-skilled and low-skilled, creating a two-sector production process. The firm searches on two distinct labor markets for low-skilled and high-skilled workers, since high-skill and low-skill employment are strict imperfect substitutes. The model allows us to study two types of shocks (i) temporary and (ii) permanent aggregate and sector-specific productivity shocks. With the former type we analyze business cycle fluctuations and with the latter we analyze inter- and intra-sectoral reallocation effects. Shocks driving the business cycle have different effects on low-skilled and high-skilled workers as for instance Keane and Prasad (1996) or Trefler (2004) show. Autor et al. (1998) find significant intra-sectoral employment shifts towards high-skill workers as a consequence of technological changes (computerization) and R&D investments. Along this line, Machin and Reenen (1998) show that the increase in skilled-workers in the OECD is caused by a broader technological change through R&D intensity. Berman et al. (1994) investigate the shift towards high-skilled labor in U.S. manufacturing over the 1980s and find that this shift is caused by biased technological change. Lilien (1982) focuses on the macroeconomic effects of reallocation shocks and founds the counterpart to regular business cycle analysis, in which aggregate shocks drive the cycle. This sectoral shift hypothesis highlights the allocative effects of shocks affecting the composition of demand. The analysis of temporary shocks is aimed to explain the stylized business cycle facts with a multi-sectoral matching model. In addition, we scrutinize steady state effects of permanent sector-specific productivity shocks. We show that temporary sector-specific shocks create an aggregate Beveridge curve but perform rather poor in explaining standard deviations. The aggregate, temporary productivity shock creates a much too volatile response of the economy. Permanent shocks cause sectoral shifts and explain wedges in real wages and different performances of sectoral labor markets. The paper is structured as follows. The next section analysis sectoral U.S. data, while section 3 develops the model. Section 4 discusses various shocks and section 5 concludes.
2 Data Analysis

This section analyses U.S. data to generate a transparent basis for the assessment of our model. We use monthly data provided by the U.S. Bureau of Labor Statistics from 2000:Q12 till 2009:Q10 (107 observations). The time series for aggregate vacancies is the JOLTS time series for total job openings. In order to generate empirical facts about the high- and low-skilled labor markets, we choose two sectors within the JOLTS database, namely the construction and the professional sectoral data. The construction data is related to the low-skilled sector and the professional and service data is applied to the high-skilled sector. The construction data set contains the subcategories (i) Construction of Buildings (NAICS 236000), (ii) Heavy and Civil Engineering Construction (NAICS 237000) and (iii) Specialty Trade Contractors (NAICS 238000). The second sector contains the subcategories (i) Professional, Scientific and Technical Services (NAICS 54), (ii) Management of Companies and Enterprises (NAICS 55) and (iii) Administrative and Support and Waste Management and Remediation Services (NAICS 56). We use log time series and detrend them with a Hodrick-Prescott filter with smoothing parameter $\lambda = 100,000$. Our results are presented in Table 1. We find an aggregate ($\text{corr}(u,v) = -0.94$) and sector-specific (-0.47 high / -0.38 low) Beveridge curves. Aggregate unemployment is positively correlated with sector-specific unemployment, as well as aggregate vacancies are positively correlated with sector-specific vacancies. Within the sectors, unemployment and vacancies are positively correlated. In both sectors, labor market tightness is negatively correlated with unemployment and positively correlated with vacancies. While separations in the high-skilled sector are positively correlated with vacancies, they are negatively correlated with unemployment. In the low-skilled sector the result holds vice versa for the low-skilled separation rate.

3 Model Derivation

We now present a New Keynesian model with search frictions in the spirit of Mortensen and Pissarides (1994) and Krause and Lubik (2007). Households maximize their utility by choosing the optimal consumption path of a CES aggregate of differentiated products. Firms maximize profits by setting prices and choosing the optimal mixture of high- and low-skilled workers subject to Rotemberg (1982) price adjustment costs and labor turnover costs. Separations are
driven by job-specific productivity shocks, that are drawn from a time-invariant distribution. These shocks generate a flow of workers into unemployment while the transition process from unemployment to employment is subject to search frictions, characterized by a matching function.

3.1 Preferences

Our economy is populated with two types of workers, named high- and low-skilled. Therefore, there are two types of representative households who maximize utility given by

$$U^x = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_{t+1}^{1-\sigma} - 1}{1 - \sigma} \right],$$  \hfill (1)

where $x \in h, l$ is the index for high- and low-skilled workers and the degree of risk aversion is given by $\sigma$. We assume that a household consists of a continuum of members, inelastically suppling one unit of labor and being represented by the unit interval. In addition, household members insure each other against income fluctuations.\(^1\) The intertemporal budget constraint can be written as

$$C_t + \frac{B_t}{P_t} = W_t^x + R_{t-1} \frac{B_{t-1}}{P_t} + b^x u_t + \Pi_t + T_t.$$  \hfill (2)

$b^x$ corresponds to unemployment benefits and $W_t^x$ is labor income. $B_t$ is bond holding which pays a gross interest rate $R_t$, $\Pi_t$ are aggregate profits and $T_t$ are lump sum transfers from the government. The FOC of the household problem is given by

$$C_t^{1-\sigma} = \beta R_t E_t \left[ \frac{P_t}{P_{t+1}} C_{t+1}^{1-\sigma} \right].$$  \hfill (3)

3.2 Search and Matching

The firm searches for high- and low-skilled workers on two discrete and closed markets.\(^2\) One market contains all high-skilled workers, as the other contains all low-skilled workers.\(^3\) For the sake of simplicity we assume that a low-skilled

\(^1\) See Merz (1995).
\(^2\) See Davis (2001) for ex ante labor sorting into separate search markets.
\(^3\) This assumption allows us to avoid an aggregate matching function and to consider a more general approach (see Tapp (2007)). As a consequence, we can account for differences in vacancy filling rates across sectors, found by Davis et al. (2007).
worker can not become a high-skilled worker. Labor market frictions are modeled via a Cobb-Douglas type matching function with constant returns to scale

\[ m(u^x_t, v^x_t) = m(u^x_t)^\mu (v^x_t)^{1-\mu}. \]

The function gives the number of new employment relationships at the beginning of the next period. Where \( u^x_t \) is the number of unemployed worker and \( v^x_t \) is the number of open vacancies, assumed to lie on the unit interval. Where \( \mu \in (0, 1) \) is the elasticity of the matching function with respect to unemployment and the matching efficiency is governed by \( m > 0 \). The underlying homogeneity assumption leads to the probability of a vacancy being filled in the next period, i.e. \( q(\theta^x_t) = m(\theta^x_t)^{-\mu}. \) Labor market tightness, given by \( \theta^x_t = v^x_t / u^x_t \) is a key point in explaining equilibrium unemployment, due to the fact that it contains the congestion externality, which follows from the fact that, if a firm posts a vacancy it decreases simultaneously the probability for other firms to fill a vacancy. Furthermore, an additional searcher causes negative search externalities for other searchers, i.e. reduces the job finding probability of all other searchers.

The firm’s exit site is characterized by endogenous separations. The total number of separations at firm \( i \) is given by \( \rho^x(\tilde{a}^x_{it}) = F(\tilde{a}^x_{it}), \) where \( \tilde{a}^x_{it} \) is the cut-off point and \( F(\cdot) \) is a time-invariant distribution with positive support \( f(\cdot). \omega^x \) is the mean of the distribution and \( \varsigma^x \) is the dispersion of the function. We assume \( \omega^l < \omega^h \), i.e. the average high-skilled worker has a higher idiosyncratic productivity as the average low-skilled worker. Connecting the results for job creation and the job destruction enables us to determine the evolution of employment at firm \( i \) as

\[ n^{x}_{it+1} = (1 - \rho^{x}_{it+1})(n^x_{it} + v^x_{it}q(\theta^x_t)). \]  

(4)

And finally

\[ n_{it+1} = n^h_{it+1} + n^l_{it+1}. \]  

(5)

The firm is able to control the evolution of employment by adjusting the number of posted vacancies and by setting the critical threshold, which then influences the separation rate.
3.3 Technology

If the matching process has been successful, production commences along the production function given by

$$y_{it} = A_{it} \left( y_{it}^h \right)^{\alpha} \left( y_{it}^l \right)^{1-\alpha}, \tag{6}$$

where the sector-specific production functions can be written as

$$y_{xit} = A_{xit} n_{xit} \int_{\tilde{a}_{xit}} a^x \frac{f(a^x)}{1 - F(\tilde{a}_{xit}^x)} da^x \equiv A_{xit}^x n_{xit}^x H(\tilde{a}_{xit}^x).$$

While aggregate productivity $A_t$ and sectoral "aggregate" productivities $A_{xit}^x$ are common to all firms, the specific idiosyncratic productivity $a_{xit}^x$ is idiosyncratic and every period it is drawn in advance of the production process from the corresponding distribution function.

The firm maximizes the present value of real profits given by

$$\Pi_{i0} = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \frac{P_{it}}{P_t} y_{it} - W_{it}^h - W_{it}^d - c^h v_{it}^h - c^l v_{it}^l - \frac{\psi}{2} \left( \frac{P_{it}}{P_{it-1}} - \pi \right)^2 Y_t \right]. \tag{7}$$

Where the first term in parenthesis is real revenue, the second and the third term is the wage bill, which is given by the aggregate of individual wages

$$W_{st}^x = n_{it}^x \int_{\tilde{a}_{it}^x} \tilde{a}_{it}^x w_t^x(a^x) \frac{f(a^x)}{1 - F(\tilde{a}_{it}^x)} da^x. \tag{8}$$

This follows from the fact that the wage is not identical for all workers, instead it depends on the idiosyncratic productivity and the skill level of the worker. The fourth and fifth term reflect the total costs of posting a vacancy, with $c^h \geq c^l > 0$ giving the real cost per vacancy. The latter term corresponds to Rotemberg (1982) price adjustment costs. The degree of these costs is measured by the parameter $\psi \geq 0$. The first-order conditions are

\footnote{See Acemoglu (2001) for a similar assumption.}
\[
\frac{\partial n^x}{\partial t} = \xi^x = \varphi t A^x_t H(\tilde{a}^x_t) - \frac{\partial W^x}{\partial n^x_t} + E_t \beta t+1 (1 - \rho^x t+1) \xi^x t+1, \quad (9)
\]

\[
\frac{\partial v_t^x}{\partial t} = c^x q(\theta^x_t) = E_t \beta t+1 (1 - \rho^x t+1) \xi^x t+1, \quad (10)
\]

\[
\frac{\partial \tilde{a}^x_t}{\partial t} = E_t \beta t+1 (1 - \rho^x t+1) \xi^x t+1 \left[ \varphi t A^x_t n^x_t \frac{\partial H(\tilde{a}^x_t)}{\partial \tilde{a}^x_t} - \frac{\partial W^x}{\partial \tilde{a}^x_t} \right], \quad (11)
\]

\[
\frac{\partial P_t}{\partial t} = 1 - \psi(\pi_t - \pi) \pi_t + E_t \beta t+1 \left[ \psi(\pi t+1 - \pi) \pi t+1 \frac{Y_{t+1}}{Y_t} \right] = \epsilon (1 - \varphi_t). \quad (12)
\]

The current period average value of workers across job-specific productivities and skill levels is given by \(\xi^x_t\) and \(\varphi_t\) reflects the real marginal costs. Combining (9) and (10) gives the job creation condition

\[
\frac{c^x}{q(\theta^x_t)} = E_t \beta t+1 (1 - \rho^x t+1) \left[ \varphi t A^x_t n^x_t \frac{\partial H(\tilde{a}^x_t)}{\partial \tilde{a}^x_t} - \frac{\partial W^x}{\partial \tilde{a}^x_t} + \frac{c^x}{q(\theta^x_{t+1})} \right]. \quad (13)
\]

This condition reflects the hiring decision as a trade-off between the cost of a vacancy and the expected return. Where \(1/q(\theta^x_t)\) is the duration of the relationship between the firm and worker. The lower the probability of filling a vacancy, the longer the duration of existing contracts, because the firm is not able to replace the worker instantaneously. By multiplying the duration of the relationship with the hiring costs we arrive at the costs of a vacancy. If expected productivity rises, the right-hand side rises while the left-hand side on impact remains unchanged. The rise in expected revenue causes an incentive for the firm to post more vacancies, which increases labor market tightness. Since the probability that an open vacancy is filled is decreasing in the degree of labor market tightness the cost of posting vacancies increases and coherently lowers the incentives to post new vacancies leading to the new equilibrium.

Log-linearizing the last FOC around a zero inflation steady state gives the New Keynesian Phillips curve

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{\varphi}_t, \quad (14)
\]

where \(\kappa = (\epsilon - 1)/\psi\) and \(\hat{\varphi}_t\) reflects the marginal costs.
3.4 Wage Determination

A match generates an economic rent which is splitted in individual Nash bargaining by maximizing the Nash product

$$w^x = \arg\max \left\{ (W^x_t - U^x_t)^\eta (J^x_t - V_t)^{1-\eta} \right\}$$  \hspace{1cm} (15)

Where the first term is the worker’s surplus, the latter term is the firm’s surplus and $0 \leq \eta \leq 1$ is the exogenously determined, constant relative bargaining power. $U^x_t$ and $V_t$ are the worker’s respectively the firm’s threat points. The asset value of a filled job for the firm and for the worker $W^x_t$ is the asset value of being employed and accordingly $U^x_t$ is the asset value of being unemployed. The individual real wage satisfies the optimality condition

$$W^x_t(a^x_t) - U^x_t = \frac{\eta}{1-\eta} J^x_t(a^x_t).$$  \hspace{1cm} (16)

To obtain an explicit expression for the individual real wage we have to determine the asset values and substitute them into the Nash bargaining solution (16). For the firm the asset value of the job depends on the real revenue, the real wage and if the job is not destroyed, the discounted future value. Otherwise the job is destroyed and hence has zero value. In terms of a Bellman equation the asset value is given by

$$J^x_t(a^x_t) = \varphi_t A^x_t a^x_t - w^x_t(a^x_t) + E_t \beta_{t+1} \left( (1 - \rho^x_{t+1}) \int_{\tilde{a}^x_{t+1}} J^x_{t+1}(a^x) \frac{f(a^x)}{1 - F(\tilde{a}^x_{t+1})} da^x \right).$$  \hspace{1cm} (17)

The asset value of being employed for the worker consists of the real wage, the discounted continuation value and in case of separation the value of being unemployed

$$W^x_t(a^x_t) = w^x_t(a^x_t) + E_t \beta_{t+1} (1 - \rho^x_{t+1}) \int_{\tilde{a}^x_{t+1}} W^x_{t+1}(a^x) \frac{f(a^x)}{1 - F(\tilde{a}^x_{t+1})} da^x$$

$$+ E_t \beta_{t+1} \rho^x_{t+1} V^x_{t+1}. \hspace{1cm} (18)$$

\(^5\)Due to a free entry condition the equilibrium value of $V_t$ is zero.
Analogously, the asset value of a job seeker is given by

\[ U_t^x = b^x + E_t \beta_{t+1} \theta_t^x q(\theta_t^x) (1 - \rho_{t+1}) \int_{\tilde{a}_{t+1}^x} W_{t+1}^x f(a^x) \frac{1 - F(a_{t+1}^x)}{1 - F(a_{t+1}^x)} da^x \]

\[ + E_t \beta_{t+1} (1 - \theta_t^x q(\theta_t^x) (1 - \rho_{t+1})) U_{t+1}^x. \]

(19)

Unemployed worker receive the unemployment benefit \( b^x \), the discounted continuation value of being unemployed and if he is matched he receives the value of future employment. Inserting these value functions into the Nash bargaining solution yields the individual real wage

\[ w_t^x(a_t^x) = \eta(\varphi_t A_t^x a_t^x + c^x \theta_t^x) + (1 - \eta) b^x. \]

(20)

The gap between the real wage and the reservation wage is increasing in every time-depending component and the worker’s bargaining power. The firm will endogenously separate from a worker if and only if

\[ J_t^x(a_t^x) < 0. \]

(21)

After some algebra, the threshold is given by

\[ \tilde{a}_t^x = \frac{1}{(1 - \eta) \varphi_t A_t^x} \left[ (1 - \eta) b^x + \eta c^x \theta_t^x - \frac{c^x}{q(\theta_t^x)} \right]. \]

(22)

3.5 Model Solution

The monetary authority targets the nominal interest rate by following a standard Taylor rule, given by

\[ \left( \frac{R_t}{\bar{R}_t} \right) = \left( \frac{\pi_t}{\bar{\pi}} \right) \phi_x \left( \frac{Y_t}{\bar{Y}} \right) \phi_y \]

(23)

where \( \phi_x \) and \( \phi_y \) are the respective weights. The aggregate productivity shock is formulated as

\[ A_t = A_{t-1}^A e^{\alpha \omega_t}. \]

(24)

The sector-specific shocks also follow a standard AR(1), i.e.

\[ A_t^x = A_{t-1}^x e^{\alpha A_t^x}. \]

(25)
The i.i.d. error terms are \( \alpha_{Ax,t} \sim N(0, \sigma_x) \) with \( \text{cov}(A_{t-1}^x, \alpha_{x,t}) = 0 \) \( \forall t \).

The resource constraint is given by

\[
Y_t = C_t + c^h v^h_t + c^l v^l_t. \tag{26}
\]

Then, the model is log-linearized around its deterministic steady state and simulated using Dynare.

### 3.6 Calibration

In this section, we set the deep parameters of our model economy. Values are contained in Table 2. The mean of the distribution functions are set to 0 for the low-skilled worker and 0.5 for the high-skilled workers. In plain words, the average low-skilled worker has a lower idiosyncratic productivity as the average high-skilled worker. The variance of the distribution functions is set to 0.12 for both types of workers as in Krause and Lubik (2007). We assume that unemployment is larger in the low-skilled sector and set \( n^l = 0.7 \) and \( n^h = 0.9 \) (see Albrecht et al. (2006)). The parameter in the aggregate production function is set to 0.75. Unemployment benefits also show a wedge, such that \( b^l = 0.4 \) and \( b^h = 0.7 \). Missing parameter values are computed from steady state equations.

### 4 Discussion

#### 4.1 Temporary Shocks

We begin our analysis with the consideration of a temporary one percent shock to high-skilled productivity. The response of our economy is represented in Figure 1. We observe a sectoral shift towards high-skilled workers. Unemployment in the high-skilled sector decreases, while it increases in the low-skilled sector. Unfortunately, and as common in endogenous separation models, the main adjustment process works along the exit margin of the firm. The firm increases employment by slightly increasing vacancies in the high-skilled sector and by mainly reducing job destruction. The opposite pattern is visible in the low-skilled sector. As a consequence, real wages increase/decrease in the high-/low-skilled sector since labor market tightness increases/decreases.\(^6\) Aggregate effects are relatively small. We obtain large shifts in the composition in output,\(^6\)This implies an increase in the hiring costs.
while aggregate output stays rather constant over the cycle. Since marginal costs decrease, inflation falls and converges from below to the steady state. Aggregate unemployment increases, since the increase in low-skilled unemployment is larger than the decrease in unemployment in the high-skilled sector. The response of aggregate vacancies is quite small, since the sector-specific responses off-set each other. The second moments of selected labor market variables can be found in Table 3. The model is able to create a quite strong Beveridge curve, i.e. \( \text{corr}(u, v) = -0.64 \), which is remarkable, since standard endogenous separation models without firing costs are not able to replicate this stylized fact.\(^7\) In general, the model is able to replicate the empirical correlations with the exceptions being high-skilled labor market tightness and high-skilled unemployment.

Now, we consider a temporary shock to low-skilled productivity. As before, unemployment decreases in the shock sector while it increases in the other sector (see Figure 2). Firms adjust mainly along the separation margin. In contrast to the high-skilled shock, aggregate unemployment decreases in this case since the effects in the low-skilled sector dominate. Real wages in the low-skilled sector increase, while they decrease in the high-skilled sector. The different size of the reaction of real wages, viz. the smaller reaction of wages in the low-skilled sector, is caused by the labor market environment induced by the calibration. As we have seen, the reaction of aggregate output is rather small, due to the large reallocation effects. Again, marginal costs decrease and inflation falls. However, the model replicates the Beveridge curve (\( \text{corr}(u, v) = -0.85 \)) (see Table 4). We infer that the model is able to replicate the same pattern as before. Consistently, the model performs relatively weak in explaining the correlations with respect to high-skilled labor market tightness and high-skilled unemployment.

Finally, we briefly analyze a temporary aggregate shock to our economy. This shock affects the sectors in a similar way and leads to effects working in the same direction. The response of the economy is far too volatile compared with the data (see Figure 3 and Table 5). We have to conclude that the model is not able to replicate the stylized facts in response to an aggregate productivity shock.

### 4.2 Permanent Shocks

After the consideration of temporary shocks, we turn to permanent productivity shocks. Let us begin with a permanent increase in the productivity of high-

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\(^7\) See Wesselbaum (2009) for an endogenous separation paper that considers firing costs.
skilled worker (Figure 4). Aggregate and high-skilled output increase, while low-skilled output decreases. Since productivity increases persistently, marginal costs decrease persistently such that inflation stays on a lower steady state. Unemployment in the low-skilled sector increases, while it drops in the high-skilled sector such that aggregate unemployment decreases only slightly. Since high-skilled workers are more productive, the firm has an incentive to hire more high-skilled workers. The sectoral shift towards the high-skill sector leads output in the low-skill sector to decrease. The additional demand caused by lower prices can be matched by the increase in the high-skill sector. The shock drives a wedge between the real wages in the two sectors. The main driving factor for this result is the behavior of the sectoral labor markets. Tightness increases/decreases in the high-/low-skilled sector, therefore persistently changing the hiring costs and consecutively the hiring incentives.

Now, we consider a permanent shock to low-skilled productivity (Figure 5). The shock decreases real marginal costs and leads inflation to converge towards a lower steady state. Consistently, prices fall and demand increases such that aggregate output raises. The firm increases supply by producing more units of output in the low-skill sector. As a consequence, unemployment decreases in the low-skilled sector and increases in the high-skilled sector. As before, the shock drives a wedge between the sectoral real wages. The drop in wages in the high-skill sector is larger than in the low-skilled sector, due to the different labor market conditions. The effect on aggregate unemployment is larger as in the precedent case since the effects are larger for this shock.

Finally, we discuss the response of our economy to a permanent shock to aggregate productivity (Figure 6). We obtain similar responses of aggregate and sectoral variables. The basic mechanism is the same as above: marginal costs decrease and inflation falls. Lower prices induce higher demand and output increases. Unemployment in the low-skilled sector increases much stronger than in the high-skill sector. Aggregate output increases, even if sectoral outputs decrease. Again, we observe a wedge between real wages, as high-skill wages decrease.

## 5 Final Remarks

This paper develops an endogenous separation New Keynesian matching model with a two-sector production process. High- and low-skilled workers are sepa-
rated into two distinct markets. We study two types of shocks (i) temporary and (ii) permanent aggregate and sector-specific productivity shocks. We find that temporary sector-specific productivity shocks explain stylized labor market facts such as the Beveridge curve on an aggregate basis. While the model is able to match the standard deviations of unemployment on a quarterly basis, it is rather not able to fit the volatility on a monthly basis. Vice versa holds for the volatility of vacancies. The volatility of the separation rate is too high compared with the data which is a standard problem of endogenous separation models, because firms prefer to adjust along the exit site. Furthermore, the aggregate shock is not able to replicate the stylized facts.

In line with the empirical evidence found by Autor et al. (1998) or Berman et al. (1994), sector-specific shocks in this model can explain the different performances of particular labor markets. Sectoral shocks cause unemployment in the shock sector to decrease while unemployment in the other sector increases. Aggregate unemployment decreases more strongly in the case of the shock to low-skilled productivity. In both cases we observe that there is a positive wedge in real wages in favor of the worker’s in the shock sector. Furthermore, our results are robust to alternative calibrations. The most influential parameter is $\alpha$ within the aggregate production function. Changes to this parameter affect the reallocation effects largely but cause relatively small changes to the dynamic patterns. The permanent aggregate shock causes aggregate unemployment and aggregate output to increase. Furthermore, we obtain a wedge in the real wages. In the spirit of Lilien (1982) we conclude that sector-specific shocks have reallocation effects and can account for aggregate fluctuations. The model is able to explain short-run business cycle fluctuations and the different development of sectoral labor markets caused by technological change.
A References


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# B Tables and Figures

### Table 1: Business Cycle Statistics - U.S. Economy.

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| Autocorrelation    | 0.94| 0.83| 0.89| 0.89| 0.38| 0.62| 0.75    | 0.80    | -0.12 | 0.23  |

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| Notes: We use monthly, HP filtered ($\lambda = 10^5$) data from 2000:Q12 to 2009:Q10 provided by the BLS. All variables respond to log deviations. High-skilled: Professional and business services, low-skilled: Construction. |

### Table 2: Calibration.

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Table 3: Theoretical Moments - Productivity Shock High-skilled.

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Table 4: Theoretical Moments - Productivity Shock Low-skilled.

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Figure 1: Productivity Shock High-skilled
Figure 2: Productivity Shock Low-skilled
Table 5: Theoretical Moments - Productivity Shock Aggregate.

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Figure 3: Productivity Shock Aggregate.
Figure 4: Permanent Productivity Shock High-skilled.
Figure 5: Permanent Productivity Shock Low-skilled.
Figure 6: Permanent Productivity Shock Aggregate.