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# **A CONSISTENT ASSESSMENT OF SOCIAL WELFARE BY TWO METHODOLOGIES. THE THEORY AND EVIDENCE FROM THE LUXEMBOURG INCOME STUDY DATABASE**

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## A consistent assessment of social welfare by two methodologies. The theory and evidence from the Luxembourg Income Study database

**JEL Classification:** C10; D30; D60; I30, O15

**Keywords:** *social welfare; Atkinson Index; Generalised Gini Index; inequality; poverty*

### Abstract

**Research background:** Applied welfare economics offers various social welfare functions (SWF) for appraising income distributions. Social planners commonly use two SWFs:  $SWF_\varepsilon$  implied by income inequality aversion ( $\varepsilon$ ) and  $SWF_\nu$  implied by rank inequality aversion ( $\nu$ ). However, a voluntary choice of  $\varepsilon$  or  $\nu$  may result in inconsistent assessments of social welfare embodied in a given income distribution.

**Purpose of the article:** We search for the combinations of  $\varepsilon$  and  $\nu$  that guarantee consistent assessments of social welfare, inequality and poverty embodied in a given income distribution.

**Methods:** We propose estimating the pairs  $(\varepsilon, \nu)$  by solving the system of two nonlinear equations. The first equation comprises *the equally distributed equivalent incomes* derived from  $SWF_\varepsilon$  and  $SWF_\nu$ . The second equation comprises *the benchmark incomes*. A small increase in income below the benchmark income reduces inequality, whereas a small increase above the benchmark income increases inequality. The system of these equations can be solved numerically.

**Findings and Value added:** We have estimated  $\varepsilon$ ,  $\nu$ , and related characteristics for selected Latin America and Caribbean countries using Luxembourg Income study database data.

### Introduction

Measuring the level of social welfare embodied in a given income distribution is a challenging issue of applied welfare economics. Economic theory delegates such a measurement to an abstractive *social planner* who uses a specific Social Welfare Function (SWF). Every member of society may play the role of such a social planner. Thus, there can be as many different SWFs as society members (Champagnowne and Cowell, 1998, p.88).

In this paper, we confine ourselves to two widely used families of SWFs. The first family, denoted by  $\{SWF_\varepsilon\}_{\varepsilon \in (0, \infty)}$ , was originated by Atkinson (1970), where the normative parameter  $\varepsilon$  reflects *aversion to income inequality*. The second family, denoted by  $\{SWF_\nu\}_{\nu \in (1, \infty)}$ , is implied by the generalised Gini index  $G_\nu$  (Yitzhaki, 1983; Kakwani, 1980; Donaldson and Weymark, 1980) where the normative parameter  $\nu$  reflects *aversion to rank inequality*. For convenience, we shall refer to these theoretical approaches as “ $\varepsilon$ -methodology” and “ $\nu$ -methodology”, respectively. Similarly, we denote by  $SP_\varepsilon$  and  $SP_\nu$  the social planners who follow the  $\varepsilon$ -methodology and  $\nu$ -methodology, respectively.

A voluntary choice of  $\varepsilon$  or  $\nu$  may result in different assessments of social welfare embodied in a given income distribution. This circumstance raises the question: “For a given income distribution, do there exist pairs  $(SP_\varepsilon, SP_\nu)$  of social planners who consistently assess social welfare?” Searching for an answer to this question is the aim of this paper.

One can formulate the above question formally as follows. Let  $SWF_\varepsilon$  and  $SWF_\nu$  be the cardinal measured social welfare functions of  $SP_\varepsilon$  and  $SP_\nu$ , respectively<sup>1</sup>. We search for  $\varepsilon$  and  $\nu$ , which satisfy the condition:

$$SWF_\varepsilon = SWF_\nu \quad (1)$$

The choice of  $\varepsilon$  and  $\nu$  satisfying Eq. (1) has several advantages besides a consistent assessment of social welfare. Section 3 shows that such pairs of  $\varepsilon$  and  $\nu$  also guarantee consistent assessment of inequality and poverty in a given income distribution.

If we impose an additional condition on  $\varepsilon$  and  $\nu$ , we can get a *single pair*  $(\varepsilon^*, \nu^*)$ . The equality of *benchmark incomes*, proposed by Lambert and Lanza (2006), seems to be a promising supplement to Eq. (1).

We recommend the pair  $(\varepsilon^*, \nu^*)$  as a normative standard for analysts who assess social welfare, inequality, and poverty in a given income distribution. If an analyst respecting  $\varepsilon$ -methodology uses  $\varepsilon^*$ , he will get the same results as an analyst who respects  $\nu$ -methodology and uses  $\nu^*$ . One may say that the point  $(\varepsilon^*, \nu^*)$  reflects a unified  $\varepsilon$ -methodology and  $\nu$ -methodology.

We implement the method of estimating  $\varepsilon$  and  $\nu$  to selected Latin America and Caribbean countries. Income data come from the Luxembourg Income Study (LIS) database.

The rest of this paper has the following structure. Section 2 offers a brief literature review on the measurement of SWF. Section 3 presents the method of eliciting  $\varepsilon$  and  $\nu$  from statistical data. In Section 4, we present the estimates of the pairs  $(\varepsilon^*, \nu^*)$  for selected Latin American and Caribbean countries. Income data come from the Luxembourg Income Study (LIS) database. Section 5 concludes.

## Literature review concerning social welfare measurements

### Some preliminaries

Welfare economics seeks an answer to whether a given policy provides a higher (lower) economic welfare of society than an alternative policy (Kakwani and Son, 2022, p.95). A social welfare function describes how individuals' economic welfare is aggregated into the economic welfare of society. SWF specifies normative judgments by assigning weights to individuals (ibid, p. 95). As social planners may use different SWFs, various answers exist to the above question.

We shall specify some terms for a formal exposition of the SWF concept.

Let the positive valued random variable  $X$ , with the (cumulative) distribution function  $F(x)=P(X \leq x)$  describe the distribution of personal incomes.<sup>2</sup> For a discrete  $n$ -point random variable, the distribution function  $F(x_i)$  specifies the rank of an  $i$ th value in ascending ordered values  $x_1, x_2, \dots, x_n$ .

We assume that the mean  $\mu=E_F[X]$  exists and is finite, where the operator  $E_F[\cdot]$  is the mathematical expectation of  $X$  with respect to  $F(x)$ . Formally

$$\mu = \int_0^\infty x dF(x) \quad (2)$$

When  $X$  is continuous with a density function  $f(x)$  the Lebegue-Stiltes integral (2) will be a 'usual' Rieman integral.

The utility of income function  $u(x)$  transforms the income distribution  $X$  into the welfare distribution  $W$ , namely  $u: X \rightarrow W$ , provided the usual conditions of transforming random variables hold.<sup>3</sup> We denote the distribution function of  $W$  by  $H(w)$ . We assume that the mean  $\mu_w=E_H[W] = E_F[u(X)]$  exists and is finite, where  $E_H[\cdot]$  is the mathematical expectation of  $W$  with respect to  $H(x)$ . Formally

$$\mu_w = \int_0^\infty u(x) dF(x) = \int_0^\infty w dH(w) \quad (3)$$

In applied welfare economics,  $\mu_w$  plays the role of SWF, reflecting a social planner's judgement of welfare embodied in income distribution  $X$ .

<sup>1</sup> In Economics, the cardinal measurement correspond to the measurement on the interval scale (Moscati, 2019). Money-metric SWFs are examples of such easurements.

<sup>2</sup> We reserve upper case letters for random variables and lower case letters for the values of the random variables.

<sup>3</sup> If a parametric form of income distribution  $X$  is known, a parametric form of welfare distribution  $W$  could be obtained (see, e.g. Kot, 2012).

The *equally distributed equivalent income* (EDEI) is a cardinal representation of *SWF* (Kolm, 1969; Atkinson, 1970; Sen, 1973). Lambert (2001,p.95) explains EDEI as “...the level of income which, if distributed equally to all individuals, would generate the same welfare (average utility) as the existing distribution *F*.”

For a formal presentation of EDEI, suppose the random variable *K* describes the *egalitarian distribution of income* with the following probability distribution function:

$$P(K = \xi) = 1, \wedge P(K \neq \xi) = 0 \quad (4)$$

Let the random variable  $D = u(K)$  describe the welfare distribution implied by the egalitarian distribution (4). Then, the social welfare embodied in the egalitarian distribution is

$$E_K[D] = E_K[u(K)] = u(\xi), \quad (5)$$

where  $E_K[\cdot]$  denotes the mathematical expectation of *D* with respect to distribution (4).

The following equation defines EDEI:

$$E_K[D] = E_F[u(X)], \quad (6)$$

or equivalently

$$u(\xi) = \mu_w \quad (7)$$

If  $u^{-1}(\cdot)$  exists, one could calculate EDEI as

$$\xi = u^{-1}(\mu_w) \quad (8)$$

$\xi$  specifies the egalitarian distribution (4) such that its social welfare (5) is the same as in the current income distribution *F*.

#### The Atkinson SWF

To specify a form of utility function *u*, Atkinson (1970) used constructively the concept of relative (proportional) aversion to risk, namely  $r(x) = x \cdot u''(x) / u'(x)$  (Arrow, 1963; Pratt, 1964), where inequality plays the role of risk. If  $r(x) = \varepsilon = \text{const.}$ , then the utility function will have the following form:

$$u(x) = \begin{cases} \frac{x^{1-\varepsilon}}{1-\varepsilon}, & \text{for } \varepsilon \neq 1 \\ \ln x, & \text{for } \varepsilon = 1 \end{cases}, x > 0 \quad (9)$$

(Atkinson, 1970). The function (9) is called the *constant (relative) inequality aversion utility function* and characterises the welfare of an individual with income *x*.

Using the notations introduced above, the Atkinson social welfare function is

$$SWF_\varepsilon = E_F[u(X)] = \begin{cases} \frac{E_F[X^{1-\varepsilon}]}{1-\varepsilon}, & \text{for } \varepsilon \neq 1 \\ E_F[\ln X], & \text{for } \varepsilon = 1 \end{cases} \quad (10)$$

Parameter  $\varepsilon$  -hereafter called *inequality aversion*, for short- characterises the attitude to inequality of a society or a social planner  $SP_\varepsilon$  possessing  $SWF_\varepsilon$ .

If  $\varepsilon < 0$ , a social planner or society is *averse to equality*. Null inequality aversion, i.e.  $\varepsilon = 0$ , characterises an *inequality-neutral* society. In this case,  $SWF_0 = \mu$ . Value judgments of income distributions are based only on the mean incomes carrying nothing for income inequality. Thus, income distribution *X* with the mean  $\mu_x$  is preferred over *Y* with the mean  $\mu_y$  if and only if  $\mu_x > \mu_y$ . If  $\varepsilon > 0$ , society is *inequality averse*. Hereafter, we will assume  $\varepsilon \geq 0$ .

The limiting case, i.e. when  $\varepsilon \rightarrow \infty$ , requires special attention. If income distribution is continuous, then  $SWF_{\varepsilon \rightarrow \infty}$  converges to the Dirac delta (Hazewinkel, 2011). The Dirac delta is not a function, at least not a usual one, with domain and range in the real numbers. For discrete income distributions,  $SWF_{\varepsilon \rightarrow \infty}$  approaches

Rawlsian leximin (Lambert, 2001, p. 99).<sup>4</sup> Roughly speaking, the Rawlsian criterion evaluates income distributions according to the incomes of the poorest in society. Thus, the greater inequality aversion  $\varepsilon$ , the more “Rawlsian” a society is.

Using Eq. (8) and (9), we can calculate EDEI, say  $\xi_\varepsilon$ :

$$\xi_\varepsilon = \begin{cases} \{E_F[X^{1-\varepsilon}]\}^{1/(1-\varepsilon)}, & \text{for } \varepsilon \neq 1 \\ \bar{g}, & \text{for } \varepsilon = 1 \end{cases} \quad (11)$$

where  $\bar{g}$  is the geometric mean of  $X$ ,

Recalling the interpretation of *EDEI* presented above,  $\xi_\varepsilon$  (11) is the cardinal measure of  $SWF_\varepsilon$ . For a given income distribution,  $\xi_\varepsilon$  is a diminishing function of  $\varepsilon$ . For a continuous income distribution,  $\xi_\varepsilon$  converges to the Dirac delta function when  $\varepsilon \rightarrow \infty$ . For any discrete income distribution with the smallest income  $x_{\min}$ ,  $\lim_{\varepsilon \rightarrow \infty} \xi_\varepsilon = x_{\min}$  (Lambert, 2001, p.101).

Blackorby and Donaldson (1978) demonstrated that any social evaluation function implies a family of indices of relative inequality. On the other hand, any of these families implies a family of social evaluation functions.<sup>5</sup>

Atkinson (1970) showed that  $SWF_\varepsilon$  (10) implies the following normative index of income inequality  $A_\varepsilon$ :

$$A_\varepsilon = \frac{\mu - \xi_\varepsilon}{\mu} \quad (12)$$

Multiplying the numerator and denominator of (12) by the total population size reveals that  $A_\varepsilon$  measures the fraction of total income that a society could sacrifice to eradicate inequality without social welfare loss (Lambert, 2001, p. 98).

From (12), we get:

$$\xi_\varepsilon = \mu(1 - A_\varepsilon) \quad (13)$$

$\xi_\varepsilon$  (13) is the cardinal measure of  $SWF_\varepsilon$  expressed in terms of the index of inequality  $A_\varepsilon$  and the mean income  $\mu$ . Function (13) is a particular case of the so-called “abbreviated social welfare functions” of form  $v(\mu, I)$ , where  $\mu$  is the mean income, and  $I$  is a measure of inequality (Lambert, 2001, 106).

#### *The social welfare function implied by the generalised Gini index*

The generalised Gini index,  $G_v$ , implies another widely used family of social welfare functions. This index has the following form:

$$G_v = 1 - v(v-1) \int_0^1 (1-p)^{v-2} L(p) dp, \quad v > 1, p \in [0,1] \quad (14)$$

in which  $v > 1$  is ‘the distributional judgement parameter’ and  $L(p)$  is the Lorenz curve (Yitzhaki, 1983; see also Donaldson and Weymark, 1980; Kakwani, 1980). For  $v=2$ ,  $G_2$  is the ordinary Gini index. Duclos and Araar (2005) interpret  $v$  as ‘the aversion to rank inequality’.

Denoting by  $c = v(v-1)(1-p)^{v-2}$ ,  $G_v$  (14) can be expressed as

$$G_v = 1 - \int_0^1 c L(p) dp \quad (15)$$

Thus,  $G_v$  is the weighted integrated area below the Lorenz curve, where  $c$  is a weight (Yitzhaki, 1983). The author notes that  $c$  is independent of a specific income distribution since  $(1-p) = (1-F(x))$  reflects the rank in this distribution. Only the Lorenz curve  $L(p)$  provides the information about the distribution (up to the mean).

The relationship between the weight  $c$  and the rank  $p = F(x)$  is visible from

<sup>4</sup> Rawlsian leximin ranks distribution  $X$  higher than distribution  $Y$  if the poorest income is greater than under  $Y$ , or under  $X$  is the same but occurs with a lower frequency (Lambert, 2001, p. 100).

<sup>5</sup> Actually, the cited authors operated with relative indices of *equality* for technical reasons. Subtracting such indices from one gives indices of inequality.

$$\frac{\partial c}{\partial F} = -v(v-1)(v-2)(1-F)^{v-3}, v > 1 \quad (16)$$

In the case  $1 < v < 2$ , the weight increases with the rank. In the case  $v = 2$ ,  $c$  is independent of the rank. When  $v > 2$ , the weight  $c$  decreases with the rank. In other words, changing  $v$  increases *weights* attached to the lower tail of the distribution and decreases those attached to the upper tail (Yitzhaki, 1983).

Yitzhaki (1983) observed that  $G_v$  has most of the properties of Atkinson's index (12). Indeed, at the extremes  $v \rightarrow 1$  and  $v \rightarrow \infty$ , the behaviour of  $G_v$  resembles that of the  $A_\varepsilon$  at the extremes  $\varepsilon \rightarrow 0$  and  $\varepsilon \rightarrow \infty$  of inequality aversion (Lambert, 2001, p. 115). As  $v \rightarrow \infty$ ,  $G_v \rightarrow 1 - L'(0)$ . For a discrete distribution of  $X$ ,  $G_v \rightarrow 1 - x_{\min}/\mu$  as  $v \rightarrow \infty$ .

$G_v$  implies the  $SWF_v$  cardinalised by the following abbreviated social welfare function:

$$\xi_v = \mu(1 - G_v) = \int_0^\infty vx[1 - F(x)]^{v-1} dF(x) \quad (17)$$

(Lambert, 2001, p. 125). For a given income distribution with the mean  $\mu$ ,  $\xi_v$  is driven by  $SP_v$ 's attitude to rank inequality measured by the ethical parameter  $v$ . The social planner  $SP_v$  assesses income distribution with  $SWF_v$  (17).

### *Eliciting $\varepsilon$ and $v$ from income data*

A practical problem which analysts meet is what range of  $\varepsilon$  and  $v$  should be used in empirical investigations. The literature has devoted most of its efforts to recovering  $\varepsilon$  from various sources of empirical data. Recommendations concerning a choice of  $v$  are sparse.

The *leaky bucket experiment* (LBE) is a popular method of eliciting  $\varepsilon$  (Okun, 1975). The participants of LBE assess a tolerable money loss ('leakage'), which inevitably occurs during discrete transfers among society members. The higher leakage a participant permits, the greater his/her aversion to inequality.

Clark and D'Ambrosio (2015) notice that LBE data have produced quite an extensive range for the estimated level of inequality aversion. In relatively infrequent studies, assuming  $\varepsilon$  as invariant over time and space is a rule. However, little theoretical or empirical ground supports such homogeneity (Aristei and Pugini, 2016). This obstacle limits their usage in worldwide analyses of inequality aversion across countries and over time.

In another strand of literature,  $\varepsilon$  is derived from the relationship between income and happiness (e.g. Layard et al., 2008) or indirect behavioural evidence about consumption patterns (Attanasio and Browning, 1995; Blundell et al., 1994). In yet another approach,  $\varepsilon$  is estimated as the ratio of the income elasticity of demand to the compensated own-price elasticity (Evans, 2005). Kot (2017) estimates  $\varepsilon$  using data from the survey in which respondents evaluate income thresholds, delimiting the just perceptible changes in the household's welfare.

One can also elicit inequality aversion,  $\varepsilon$ , from the equal sacrifice model (Richter, 1983; Vitaliano, 1977; Young, 1987). This model assumes that income taxes yield the same loss in individual utility across all income levels.

Kot (2020) proposes estimating  $\varepsilon$  assuming incomes obey the generalised beta distribution of the second kind (McDonald, 1984). The author derived the estimator of  $\varepsilon$  from the mathematical condition of the existence of the social welfare function. Using this method, Kot and Paradowski (2022) elaborate on the world atlas of inequality aversion.

However, we found no practical recommendation concerning  $v$ , except Duclos (2000). The authors recommend the leaky bucket experiment for this purpose.

Kot (2022) empirically analysed the relationship between  $G_v$  and some Italian inequality indices using data on household disposable income for Poland 2000-2017. He found that applying the Pietra index (Pietra, 1915) corresponds to  $G_v$  with  $v = 1.5$ . For the Bonferroni index (Bonferroni, 1930) and the 'new' Zenga index (Zenga, 2007), he got  $v$  equal to 3 and 11, respectively.

## Research methodology

### *Estimating the pairs $(\varepsilon, \nu)$*

Let us consider the social welfare assessments of a given income distribution performed by  $SP_\varepsilon$  and  $SP_\nu$ .  $SP_\varepsilon$  possesses  $SWF_\varepsilon$  represented by  $\zeta_\varepsilon$  (13), whereas the  $SP_\nu$  possesses  $SWF_\nu$  represented by  $\zeta_\nu$  (17). Condition (1) selects the pairs of  $SP_\varepsilon$  and  $SP_\nu$  who consistently assess social welfare, namely:

$$\mu(1-A_\varepsilon)=\mu(1-G_\nu) \quad \varepsilon \geq 0 \text{ and } \nu > 1, \quad (18)$$

or in alternative notation

$$\zeta_\varepsilon = \zeta_\nu, \quad (19)$$

For a given income distribution with the mean  $\mu$ , Eq. (18) is equivalent to

$$A_\varepsilon = G_\nu, \quad (20)$$

Kot and Paradowski (2024) argue that EDEI is an upper limit of all poverty lines  $z$ . If  $z$  were greater than EDEI, attaining an egalitarian income distribution would be possible only at the cost of total poverty. Arguably, no reasonable society and economic theory would accept such a peculiar poverty line.

The above shows that all pairs  $(\varepsilon, \nu)$  satisfying Eq. (18), or its equivalent versions (19) or (20), guarantee consistent social welfare assessment, inequality, and poverty in a given income distribution.

One may apply the method of eliciting the pairs  $(\varepsilon, \nu)$  in various ways. For instance, assuming *exogenous* values of  $\nu_1, \nu_2, \dots, \nu_m$ , one can get a corresponding sequence of  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m$  by solving equation (20). Fig.1 illustrates this procedure for Brazil 2016. Assuming twenty levels of  $\nu$  from 1.1 to 5, we numerically solve the nonlinear equation (20) for  $\varepsilon$ .

For instance, if a social planner were averse to rank inequality with  $\nu=2$ , i.e., if he used the ordinary Gini index, the corresponding social planner averse to income inequality ought to have  $\varepsilon=1.42161$  for consistent assessments of welfare, inequality and poverty.

Similarly, assuming exogenous values of  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m$ , one can get a corresponding sequence of  $\nu_1, \nu_2, \dots, \nu_m$  (see Fig.2). These procedures may be applied in particular cases when  $\varepsilon$  or  $\nu$  are known, e.g. obtained by methods described in the previous section. For instance, in Fig.2 figure, we insert  $\nu=1.7745$  as corresponding to  $\varepsilon=1.2558$  from Kot and Paradowski (2022).

### *.Estimating a unique pair $(\varepsilon^*, \nu^*)$ .*

As (18) contains two unknowns,  $\varepsilon$  and  $\nu$ , we need an additional equation for uniquely identifying these normative parameters. For this purpose, we propose the use of the *benchmark incomes* originated by Hoffman (2001) and independently put forward by Lambert and Lanza (2006) and Corvalan (2014).

Hoffman (2001) recalls a well-known fact that in a two-person society with unequal incomes  $x_1 < x_2$ , a small rank-preserving subsidy given to  $x_1$  reduces inequality, whereas given to  $x_2$  raises inequality. Thus, for an  $n$ -member society, a specific income level, say  $x^*$ , dividing these effects, must exist. The level of income  $x^*$  depends on an inequality measure applied. The author shows the existence of  $x^*$  for Theil's (1967) entropy indices and the Gini index.

Lambert and Lanza (2006) prove the existence of  $x^*$  -called by the authors *the benchmark income*- for a general class of inequality measures. For the Atkinson index (10), the benchmark income,  $x_\varepsilon^*$ , has the following form:



$$x_\varepsilon^* = \begin{cases} \mu(1 - A_\varepsilon)^{(\varepsilon-1)/\varepsilon}, & \text{for } \varepsilon \neq 1 \\ \mu, & \text{for } \varepsilon = 1 \end{cases} \quad (21)$$

Lambert and Lanza (2006).

For the generalised Gini index  $G_v$  (14), finding the benchmark income, say  $x_v^*$ , is more complicated. Lambert and Lanza (2006) seek the *position*,  $k_v^*$ , of the benchmark income rather than for  $x_v^*$ . The authors assume the distribution of incomes in the sample of size  $n$  has the form  $P(X=x_i)=1/n$ . For incomes arranged in ascending order, the *benchmark position*,  $k_v^*$ , satisfies the nonlinear equation:

$$[n - k_v^* + 1)/n]^v - [n - k_v^*)/n]^v = [1 - G_v]/n \quad (22)$$

As obtaining an explicit solution to Eq. (22) is difficult, Lambert and Lanza (2006) propose the following approximation to  $k_v^*$ , for large  $n$ :

$$k_v^* \approx n\{1 - [(1 - G_v)/v]^{1/(v-1)}\} \quad (23)$$

For the ordinary Gini index, i.e.  $G=G_2$ ,  $k_2^*$  is

$$k_2^* \approx n \frac{G+1}{2} \quad (24)$$

For instance, if  $G=0.4$  in a sample of  $n=100$ , the benchmark income will be the income in the 70-*th* position, i.e.,  $x_2^* = x_{(70)}$ . In general, however, the benchmark position,  $k_v^*$  (20) may not be an integer.

Applying the formula (22) in practice will be problematic if weighted income data are available. In practice, weighted data are a rule rather than an exception. For instance, adjusting household incomes by an equivalence scale needs weighting equivalent incomes by household size.<sup>6</sup> Also, survey weights should be applied to properly replicate the global population of households.

Let income distribution in a sample of size  $n$  have the form:

$$P(X=x_i)=p_i, \quad i=1, \dots, n \quad (25)$$

where  $p_i = w_i / \sum_{j=1}^n w_j$ , and  $w_i$  are weights. In this distribution, there are  $n$  *distinct* observations.

**Proposition 1.** Suppose incomes  $x_1, \dots, x_n$  are arranged in ascending order. Let the income probability distribution function be of the form (25), and the cumulative distribution function at  $x_i$  be  $F(x_i)=p_1 + \dots + p_i$ . Then, the benchmark income  $x_v^*$  -implied by the generalised Gini  $G_v$ - satisfies the following equation

$$x_v^* = F^{-1}\left(1 - [(1 - G_v)/v]^{1/(v-1)}\right) \quad (26)$$

(for proof, see Appendix).

In other words, the benchmark income  $x_v^*$  is the quantile of order  $1 - [(1 - G_v)/v]^{1/(v-1)}$  in income distribution (25). Notice that the order of this quantile multiplied by  $n$  equals the *approximate* formula (23).

Knowledge of benchmark incomes is essential for various economic reasons. For a broader explanation, see Lambert and Lanza (2006).

The following equality will guarantee a consistent evaluation of benchmark incomes by social planners  $SP_\varepsilon$  and  $SP_v$ :

$$x_\varepsilon^* = x_v^* \quad (27)$$

If we complement Eq. (20) with Eq. (27), we get the system of nonlinear equations:

$$\begin{cases} A_\varepsilon = G_v \\ x_\varepsilon^* = x_v^* \end{cases}, \text{ for } \varepsilon \geq 0 \text{ and } v > 1 \quad (28)$$

---

<sup>6</sup> For other weighting see, among others, Ebert(1999)

which can be solved numerically. If a solution to (28) exists, we shall get a single pair  $(\varepsilon^*, v^*)$  of inequality aversion and aversion to rank inequality.

The pair  $(\varepsilon^*, v^*)$  may serve as a *standard of value judgements*. Analysts should use normative parameters  $\varepsilon^*$  and  $v^*$  when assessing social welfare, inequality and poverty in a given income distribution. This ethical recommendation guarantees a consistent assessment of social welfare inequality and poverty within the two methodologies mentioned in Section 1.

## Results

### Statistical data

We estimate  $\varepsilon^*$  and  $v^*$  for ten Latin America and Caribbean countries, namely Brazil (2016), Chile (2017), Colombia (2016), Dominican Rep. (2007), Guatemala (2014), Mexico (2016), Panama (2016), Paraguay (2016), Peru (2016), and Uruguay (2016).

We use statistical data from the LIS database on household yearly disposable income  $y_j$ , household size  $s_j$  (the number of household members), and household survey weights  $c_j$ , where  $j=1, \dots, N$ , and  $N$  is the sample size (the number of households) for a given country and year. Income data are in international US\$ adjusted by PPP. We calculate the household equivalent income  $x_j$  by applying the squared equivalence scale:

$$x_j = y_j / \sqrt{s_j}, i=1, \dots, N \quad (29)$$

We round off the data to the nearest dollar.

We weigh equivalent incomes by the product of household size and survey weight. Next, we group multiple income observations by aggregating their probabilities. Such a grouping is necessary since the formula (26) for the benchmark income  $x_v$  requires *distinct* income values.<sup>7</sup> Notably, grouping identical incomes with different probabilities does not lose information on the original sample data.

After grouping, we get a sample of size  $n \leq N$  of  $n$  distinct income values  $(x_1, \dots, x_n)$  and corresponding (aggregated) probabilities  $(p_1, \dots, p_n)$ . We arrange incomes in ascending order.

Table 1 presents the basic descriptive statistics for chosen countries.

Examining Table 1 shows large numbers of multiple income observations within country datasets. After grouping identical incomes and corresponding probabilities, the number of initial cases  $N$  decreases remarkably. Such a grouping preserves the initial estimates of descriptive statistics.

The pair  $(\varepsilon, v)$ , i.e. the solution to the system of two nonlinear equations (28), is obtained numerically using the IMLS Fortran subroutine NEQNF. This subroutine uses a modification of Powell's hybrid algorithm. This algorithm is a variation of Newton's method, which uses a finite-difference approximation to the Jacobian. (Moré et al., 1980).

### The estimates of $\varepsilon^*$ and $v^*$

We use the statistical data described above to solve the system of nonlinear equations (28). Table 2 presents the results.

One can see in Table 2 that the estimates of aversion to income inequality,  $\varepsilon^*$ , and to rank inequality,  $v^*$ , vary among countries. For Columbia, Mexico, Peru, and Uruguay, the estimates of  $v^*$  are less than 2. It means an increasing concern to more affluent people than poor ones. For remaining countries,  $v^* > 2$ . It means an increasing concern for poor people.

Aversion to income inequality,  $\varepsilon$ , also varies across countries. Mexico and Peru are the countries most tolerant of income inequality. On the other extreme are Panama and Paraguay, the countries most averse to income inequality. Notice that high aversion to income inequality does not necessarily imply a lower income inequality level. Kot and Paradowski (2022) also observed such a situation and proposed some explanations.

The numerical solution to the system of equations (28) performs excellently concerning the equality of the Atkinson and generalised Gini indices. However, the fulfilment of the second equation, i.e. the equality of benchmark incomes  $x_\varepsilon^*$  and  $x_v^*$ , is not entirely satisfactory. It is worth adding that numerical solutions of the nonlinear equations in question have been remarkably sensitive to initial guessing.

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<sup>7</sup> Actually, distinct observations are also necessary for estimating the generalised Gini index by the *covariance estimator* (Chotikapanich and Griffiths, 2001; Lerman, R. I., and Yitzhaki, S., 1989).

## Conclusions

A voluntary choice of aversion to income inequality or rank inequality enables flexibility in assessing social welfare. An obvious consequence of such a choice is inconsistency in value judgements of  $SP_\varepsilon$  and  $SP_\nu$  concerning the same income distribution. It raises the question: what range of  $\varepsilon$  and  $\nu$  do the social planners apply to attain consistency in social welfare evaluations?

A numerical solution to Eq. (18), or its equivalent formulations (19) or (20), can provide an answer to the central question of this paper. For a given  $\nu$  (resp.  $\varepsilon$ ), it is possible to obtain  $\varepsilon$  (resp.  $\nu$ ) guaranteeing identical assessment of social welfare, inequality and poverty. Similarly, a numerical solution to Eq. (27) guarantees identical assessments of benchmark incomes. Figures 1 and 2 show a robust numerical link between  $\nu$  and  $\varepsilon$ . In the literature, linking  $\varepsilon$  and  $\nu$  through equality of social welfare has not been analysed yet.

This paper also attempts to get a single pair  $(\varepsilon^*, \nu^*)$ , a solution to the system of equations (28), guaranteeing consistent evaluations of social welfare and benchmark incomes. The pair may serve as a *standard of value judgements*. This ethical recommendation could guarantee a consistent assessment of social welfare and other distributional issues by a ‘unified’  $\varepsilon$ - $\nu$  methodology’.

Implementing such a unified methodology to selected Latin American and Caribbean countries seems partially successful. The solutions to the system of nonlinear equations (28) give consistent estimates of  $A_\varepsilon$  and  $G_\nu$ . However, the equality of benchmark incomes is less satisfactory. Thus, the estimates of the pairs  $(\varepsilon^*, \nu^*)$  and resulting quantities should be taken cautiously.

In further research, improving the numerical algorithms for solving nonlinear equations seems necessary. For instance, a user might support an analytical form of Jacobian and the scaling matrix. Also, searching for an alternative second equation in (28) seems desirable.

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## Appendix

In this Appendix, we present the proof of Proposition 1. For the distribution (22) in a sample of  $n$  observations, Chotikapanich and Griffiths (2001) propose estimating the generalised Gini index according to the formula:

$$G_v = 1 - \frac{v}{\bar{x}} \sum_{i=1}^n x_i [1 - F(x_i)]^{v-1} p_i \quad (A1)$$

where  $\bar{x}$  is the mean, and  $F(x_i)$  is the cumulative distribution function. For convenience, (A1) can be expressed as:

$$G_v = 1 - \frac{v}{\bar{x}} \sum_{i \neq k}^n x_i [1 - F(x_i)]^{v-1} p_i - \frac{v}{\bar{x}} x_k [1 - F(x_k)]^{v-1} p_k \quad (A2)$$

where  $i, k=1, \dots, n$ .

Suppose a small income  $\theta$  is added to  $k$ -th person's income  $x_k$ , which becomes  $x_k + \theta$ . This operation gives a new distribution  $Y \sim H(y)$  where:

$$y_i = \begin{cases} x_i, & \text{for } i \neq k \\ x_k + \theta, & \text{for } i = k \end{cases} \quad (A3)$$

$$H(y) = \begin{cases} F(y_i), & \text{for } i \neq k \\ \frac{\theta p_k}{x_{k+1} - x_k}, & \text{for } i = k \end{cases} \quad (A4)$$

$$P(Y = y_i) = q_i = \begin{cases} p_i, & \text{for } i \neq k \\ H(y_k) - H(y_{k-1}), & \text{for } i = k \end{cases} \quad (A5)$$

and

$$\bar{y} = \bar{x} + \theta p_k \quad (A5)$$

In the new distribution  $Y$ , the generalised Gini index is

$$G_v^* = 1 - \frac{v}{\bar{y}} \sum_{i=1}^n y_i [1 - H(y_i)]^{v-1} q_i \quad (A6)$$

Expressing  $G_v^*$  in terms of  $x_i$  gives:

$$G_v^* = 1 - v[\bar{x} + \theta p_k]^{-1} \sum_{i \neq k}^n x_i [1 - F(x_i)]^{v-1} p_i - v[\bar{x} + \theta p_k]^{-1} (x_k + \theta) \cdot [1 - F(x_k + \theta)]^{v-1} [F(x_k + \theta) - F(x_k)] \quad (A7)$$

In (A7),  $F(x_k + \theta)$  is established by a linear interpolation within  $[x_k, x_{k+1}]$  interval. Then we can calculate  $P(X = x_k + \theta) = F(x_k + \theta) - F(x_k)$ .

The increase  $\theta$  in  $x_k$  will have caused the change in the generalised Gini indices:

$$\Delta G = G_v^* - G_v$$

This change is a function of  $x_k$  and  $\theta$ .

Following Hoffman (2001), we define the effect of the increase in income  $x_k$  on inequality as

$$\delta = \lim_{\theta \rightarrow 0} \frac{\Delta G}{\theta} = \lim_{\theta \rightarrow 0} \frac{dG_v^*}{d\theta} \quad (A8)$$

The calculation of  $dG_v^*/d\theta$  gives:

$$\begin{aligned} \frac{dG_v^*}{d\theta} &= \frac{vp_k}{[\bar{x} + \theta p_k]^2} \sum_{i \neq k}^n x_i [1 - F(x_i)]^{v-1} p_i - v \frac{\bar{x} - x_k p_k}{[\bar{x} + \theta p_k]^2} [1 - F(x_k + \theta)]^{v-1} \cdot \\ &[F(x_k + \theta) - F(x_k)] - v \frac{x_k + \theta}{\bar{x} + \theta p_k} \{ (v-1) [1 - F(x_k + \theta)]^{v-2} P(X = x_k + \theta) \cdot [F(x_k + \theta) - F(x_k)] - \\ &v [1 - F(x_k + \theta)]^{v-1} P(X = x_k + \theta) \} \quad (A9) \end{aligned}$$

Therefore:

$$\lim_{\theta \rightarrow 0} \frac{dG_v^*}{d\theta} = p_k \left\{ \frac{v}{\bar{x}} \sum_{i \neq k}^n x_i [1 - F(x_i)]^{v-1} p_i - v [1 - F(x_k)]^{v-1} \right\} \quad (\text{A10})$$

Note that

$$\frac{v}{\bar{x}} \sum_{i \neq k}^n x_i [1 - F(x_i)]^{v-1} p_i = 1 - G_v \quad (\text{A11})$$

Substituting (A11) into (A10) gives

$$\lim_{\theta \rightarrow 0} \frac{dG_v^*}{d\theta} = p_k \{1 - G_v - v[1 - F(x_k)]^{v-1}\} \quad (\text{A12})$$

A benchmark income  $x_v^*$  is inequality-neutral. In other words,  $x_v^*$  would be a benchmark if and only if  $\lim_{\theta \rightarrow 0} \frac{dG_v^*}{d\theta} = 0$ , namely

$$1 - G_v = v[1 - F(x_v^*)]^{v-1}$$

Therefore

$$F(x_v^*) = 1 - \left[ \frac{1 - G_v}{v} \right]^{\frac{1}{v-1}} \quad (\text{A13})$$

and

$$x_v^* = F^{-1} \left( 1 - [(1 - G_v)/v]^{1/(v-1)} \right) \quad (\text{A14})$$

Eq. (A14) equals Eq. (23).

QED.

## Annexe

**TABLE 1.** Descriptive statistics of equivalent disposable income distributions in selected Latin America and Caribbean countries.

Country	Code	The number of cases		Mean	Median	V [%]
		Initial $N$	Grouped $n$			
Brazil	BR	144192	26421	10252	7088	120.42
Chile	CL	70574	22448	14419	9636	138.87
Colombia	CO	228929	31881	8577	5972	153.10
Dominican Republic	DO	8321	5715	7100	4393	151.00
Guatemala	GT	11512	6676	6101	4466	153.40
Mexico	MX	70109	20297	9382	6364	535.43
Panama	PA	11541	7800	16961	11992	106.50
Paraguay	PE	10219	7233	11870	7995	145.82
Peru	PY	35267	15319	8579	6532	104.76
Uruguay	UY	45114	15114	14634	11544	84.62

Note: The symbol V denote the coefficient of variation [%], i.e.  $V=100*(Std.Dev./Mean)$ .

Source: Authors' calculations.

**TABLE 2.** Estimates  $\varepsilon^*$  and  $\nu^*$  for selected Latin America and Caribbean countries.

Country	$\nu^*$	$\varepsilon^*$	$G_\nu=A_\varepsilon$	$x_\varepsilon^*$	$x_\nu^*$	EDEI
Brazil	4.03220	1.90247	0.67169	6044	7843	3366
Chile	2.73539	2.04873	0.55466	9530	12641	6421
Colombia	1.53212	1.01833	0.34062	8513	11393	5656
Dominican Republic	2.79187	1.83958	0.62272	4551	6432	2679
Guatemala	2.30098	2.08389	0.45999	4428	5904	3294
Mexico	1.28024	0.69100	0.24828	10660	13679	7053
Panama	7.79278	3.28296	0.78488	5826	9864	3649
Paraguay	6.52714	2.50534	0.76891	4923	7083	2743
Peru	1.31889	0.66664	0.22655	9756	12654	6636
Uruguay	1.88697	1.92266	0.34416	11952	15975	9598

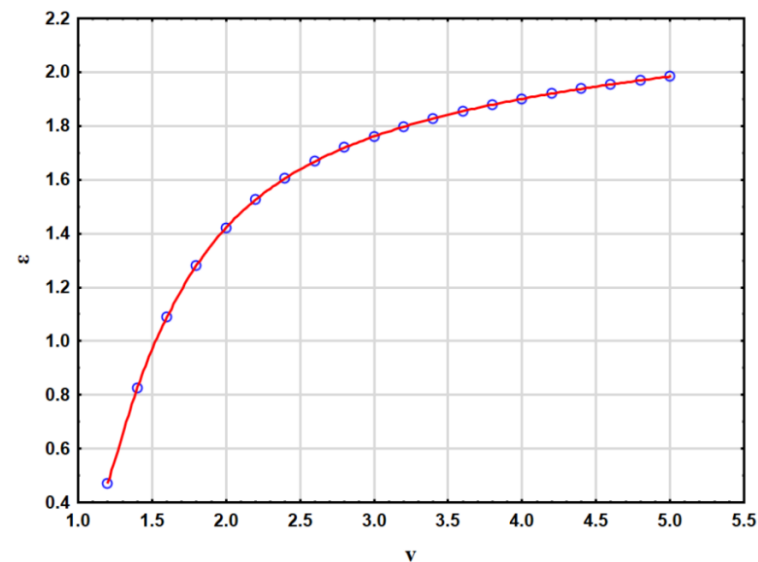
Notes:

The symbol  $x_\varepsilon^*$  denotes the benchmark incomes derived from  $A_\varepsilon$

The symbol  $x_\nu^*$  denotes the quantile of rank  $\alpha$ , i.e. the benchmark income derived from  $G_\nu$

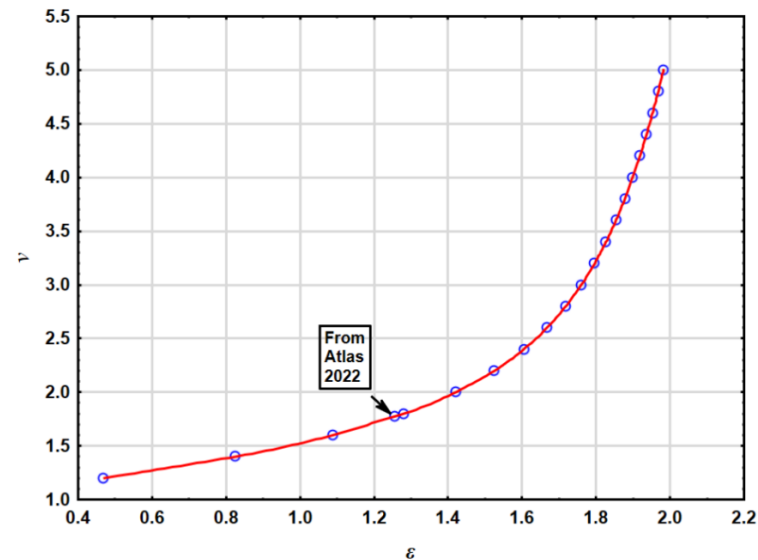
Source: Authors' calculations.

**Figure 1.** The pairs  $(v,\varepsilon)$  yielding a consistent assessment of social welfare for Brazil in 2016.



Source: own elaboration using data from the LIS database.

**Figure2.** The pairs  $(\varepsilon,v)$  of a consistent assessment of social welfare for Brazil in 2016. Note: “From Atlas 2022” indicates  $\varepsilon=1.2558$  from Kot and Paradowski (2022).



Source: own elaboration using data from the LIS database.



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