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## The World Gas Model: a multi-period mixed complementarity model for the global natural gas market

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# Discussion Papers

# 959

Ruud Egging • Franziska Holz • Steven A. Gabriel

**The World Gas Model**  
**A Multi-Period Mixed Complementarity**  
**Model for the Global Natural Gas Market**

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# **The World Gas Model**

## **A Multi-Period Mixed Complementarity Model**

### **for the Global Natural Gas Market**

**Ruud Egging<sup>^</sup>, Franziska Holz<sup>#</sup>, Steven A. Gabriel<sup>\*</sup>**

#### **Abstract**

We provide the description and illustrative results of the World Gas Model, a multi-period complementarity model for the global natural gas market. Market players include producers, traders, pipeline and storage operators, LNG liquefiers and regasifiers as well as marketers. The model data set contains more than 80 countries and regions and covers 98% of world wide natural gas production and consumption. We also include a detailed representation of cross-border natural gas pipelines and constraints imposed by long-term contracts in the LNG market. The Base Case results of our numerical simulations show that the rush for LNG observed in the past years will not be sustained throughout 2030 and that Europe will continue to rely on pipeline gas for a large share of its imports and consumption.

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# 1 Introduction

The World Gas Model (WGM) is a multi-period numerical equilibrium model of the global natural gas market covering the next three decades. It includes more than 80 countries and over 98% of global natural gas production and consumption (in 2005, BP 2008). The WGM allows for endogenous investments in pipelines and storage capacities, as well as for expansion of regasification and liquefaction capacities and considers demand growth, production capacity expansions and price and cost increases over time. Taking into account the game-theoretic aspects of the imperfectly competitive natural gas market, the model includes market power à la Cournot for some players participating in natural gas trade (i.e., traders and regasifiers.)

This paper documents the 2008 version of the World Gas Model that was used in Egging et al. (2009) and Huppmann et al. (2009). This model was based on the work of Gabriel et al. (2005a, b) which established existence and uniqueness results for a class of gas market models and then applied their model to the North American market.

Huppmann and Egging (2009) provide a more detailed programmers' and user manual. WGM is a deterministic model, assuming perfect information and foresight. A stochastic extension of the WGM was presented in Egging and Holz (2009).

Compared to earlier equilibrium models of international natural gas markets (e.g., Egging et al. 2008, Lise and Hobbs 2008, Holz et al. 2008, Zwart 2009), the World Gas Model is unique with its combination of:

- the level of detail for the market agents,
- the level of detail for the transport options (pipeline, LNG),
- the breadth of the regional coverage,
- the multi-period approach with endogenous capacity expansions,
- the inclusion of multiple seasons and seasonal arbitrage by storage operators,
- the representation of market power.

The World Gas Model is formulated as a mixed complementarity problem (MCP). The concept of MCP is briefly introduced in the following paragraph.

## 1.1 Mixed Complementarity Problems

Complementarity modeling provides a very general mathematical framework that can be applied in many different fields. Cottle et al. (1992) and Bazarara et al. (2004) provide extensive introductions on various variants of complementarity problems. In equilibrium modeling of energy markets mixed complementarity problems are increasingly used, implementing them through the Karush Kuhn Tucker (KKT) conditions and market-clearing conditions.

MCPs are a generalization of pure nonlinear complementarity problems (NCPs). MCPs also allow for other than zero lower bounds as well as upper bounds to the variables for which a solution must be determined.

In NCP a vector  $x$  must be determined, so that:  $0 \leq x \perp F(x) \geq 0$ .<sup>1</sup> To facilitate comparison with the MCP formulation, another way to put this is that for each element  $x_i$ :

$$x_i > 0 \quad \Leftrightarrow \quad F_i(x) = 0$$

In a MCP, however, a vector  $x$  must be found for which for each element  $x_i$ :

$$\text{i.} \quad l_i = x_i \quad \Leftrightarrow \quad F_i(x) \geq 0$$

$$\text{ii.} \quad l_i < x_i < u_i \quad \Leftrightarrow \quad F_i(x) = 0$$

$$\text{iii.} \quad x_i = u_i \quad \Leftrightarrow \quad F_i(x) \leq 0$$

where  $l_i$  and  $u_i$  are lower and upper bounds, respectively. The MCP formulation can represent characteristics prevailing in natural gas markets. From natural lower bounds such as non-negativity of volumes and contractual minimal deliveries, to upper bounds such as limits on daily production rates, or pipeline capacities. Moreover, the KKTs used in the MCP can be the optimality conditions of strategic players exerting market power which allows for the modeling of imperfect markets.

The World Gas Model is based on behavioral assumptions of representative players that are active in the global natural gas markets. The following section presents the optimization problems and constraints for all the player types represented in the model as well as the Karush Kuhn Tucker conditions and market-clearing conditions that together formulate the mixed complementarity model. In Section 3, the data set is described. We

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<sup>1</sup> This is shorthand for: all elements of vector  $x$  are non-negative  $x_i \geq 0$ ; all vector function values are non-negative:  $F_i(x) \geq 0$ ; and complementarity i.e.,  $x_i^T F_i(x) = 0$  for all indices  $i$ .

present illustrative results obtained with the WGM for a Base Case until 2030/2040 in Section 4 before we conclude and provide an outlook on further research.

## 2 Model Formulation

In this section, the deterministic multi-period MCP model for the global natural gas market is introduced. For each player type the objective function and constraints and the related Karush-Kuhn-Tucker conditions are presented as well as the market-clearing constraints, which are equations that tie the separate players' problems together into one MCP. While we take into account that there is strategic behavior and market power in parts of the natural gas market, we must limit this behavioral assumption to only certain market agents that sell gas to the final consumption sectors.

### 2.1 The World Gas Model

Natural gas consumption and production can be found in all world regions. However, there are big differences between the regions. North America and Eurasia have well-developed gas pipeline systems to transport the gas from suppliers to consumers, possibly crossing several country borders on the way. In other parts of the world, pipeline transmission systems are much less developed. Liquefied natural gas (LNG) is used to transport gas between geographically distant regions.

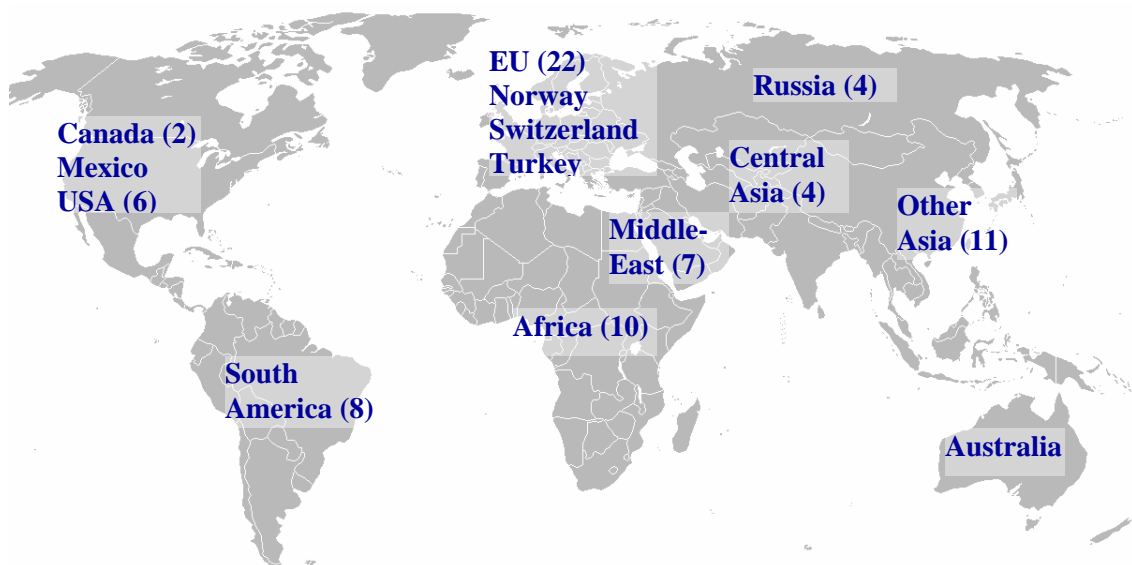


Figure 1: Country nodes included in WGM



**Table 1: Market participants represented in the WGM**

<b>Actor</b>	<b>Role</b>	<b>Comment</b>
<b>Producer</b>	Produces natural gas and supplies it to its trading arm and – if applicable – domestic liquefiers.	
<b>Trader</b>	Buys gas from producers and sells it to marketers and storage operators (in countries accessible by pipeline).	
<b>Pipeline Operator</b>	Assigns pipeline capacity to traders who need to transport gas from one country to another.	
<b>Liquefier</b>	Buys gas from the producer, liquefies it and sells it to regasifiers.	
<b>LNG shipping vessels</b>	Facilitate the oversea-transport of Liquefied Natural Gas from liquefiers to regasifiers.	Represented in the model by distance-dependent costs and losses.
<b>Regasifier</b>	Buys gas from liquefiers and sells it to the marketers and storage operators.	
<b>Storage Operator</b>	Buys gas in the low demand season from traders and – if applicable – regasifiers and sells it to the market in the high and peak demand season to take advantage of seasonal price differentials.	
<b>Transmission System Operator</b>	Responsible for pipeline network expansions.	
<b>Marketer</b>	Buys natural gas from traders, regasifiers and storage operators and distributes it to end-users.	Represented by the aggregate inverse demand curve of the consumer segments
<b>End users</b>	The three consumption sectors: power generation, industry and residential/commercial.	See marketer

The different aspects of individual regions must be addressed in the data set when setting up a model. Infrastructure and market characteristics must be represented at an adequately detailed level to be able to draw useful conclusions. However, many of the desired data are not publicly available, and thus there are limits to the level of detail that can be used.

Figure 1 shows an overview of the – nearly 80 - countries and regions that are used in the World Gas Model. For larger regions in Figure 1 the numbers in parentheses indicate the number of sub-regions in the model. For example, the US consists of six model nodes. Our data set covers 98% of the total production and consumption in 2005 (BP, 2008).

Many different types of agents are active in the natural gas sector and many possible interactions may occur among them. Table 1 details the agents that are separately represented in the World Gas Model (WGM). The interactions between the market participants are summarized in Figure 2. The consumer markets may include both, the storage operators and the end-users with final demand.

Producers sell gas to their trading arms and to domestic liquefiers. Traders ship gas to consumer markets, domestically via distribution networks (not represented in the model), and internationally via high pressure pipeline networks. Liquefiers ship gas to regasifiers in other countries. The regasifiers domestically sell gas to end user markets. Not shown separately in this picture are the deliveries from marketers to consumers and the deliveries in high and peak demand period by storage to marketers.

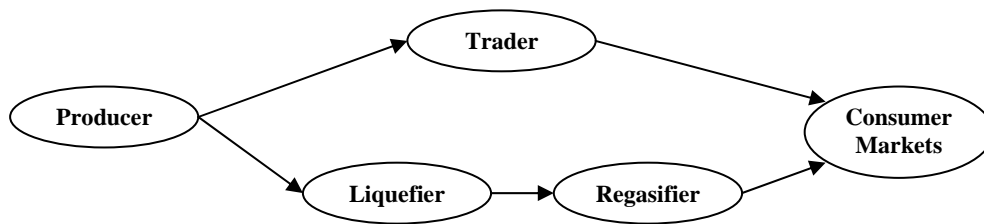


Figure 2: Natural gas export and supply chains

## 2.2 MCP Formulation of the World Gas Model

We tried to use notational conventions in the following model formulation that are mostly self-explanatory. Market player indices are the first letter of their full name. For example,  $SALES^X$  are the total sales of a market agent of type  $X$ . Also,  $SALES^{X \rightarrow Y}$  are the sales of an agent of type  $X$  to an agent of type  $Y$  and  $PURCH^{Y \leftarrow X}$  are the purchases of an agent of type  $Y$  from agents of type  $X$ . Country nodes are denoted by indices from the set  $N$ , and

subsets of nodes where a player  $X$  is present, by  $N(x)$ . To denote the subset of agents  $X$  present at node  $n$ , we use:  $X(n)$ . Where necessary and appropriate, more variable and parameter names will be introduced. Greek letters are used the dual variables for restrictions that are added in parentheses and will be used when deriving the KKTs.

### 2.2.1. Natural Gas Producers' Problem

The production of natural gas includes the well operation and the processing of the produced natural gas. We deal with the produced natural gas that is available for the market, i.e., without so-called “own use” or re-injection into gas fields. We consider one producer agent per production node (in general a country) that disposes of the aggregated production capacities in that node and decides on total production.

The producer maximizes his discounted profits, which are the result of revenues from sales  $SALES_{pdm}^P$  minus production costs. Cash flows in year  $m$  are discounted with a factor  $\gamma_m$ . We implicitly assume that the production exactly equals the sales by the producer. Since we compute daily production (sales) which may take different values in each season, the sales rates are multiplied by the number of days in each season  $d$ :  $days_d$

$$\max_{SALES_{pdm}^P} \sum_{m \in M} \gamma_m \left\{ \sum_{d \in D} days_d \left[ \pi_{n(p)dm}^P SALES_{pdm}^P - c_{pm}^P (SALES_{pdm}^P) \right] \right\} \quad (1)$$

The daily production (sales) rate is restricted by a production capacity  $\overline{PR}_{pm}^P$  (that can vary by year):

$$s.t. \quad SALES_{pdm}^P \leq \overline{PR}_{pm}^P \quad \forall d, m \quad (\alpha_{pdm}^P) \quad (2)$$

Due to reserve limitations or governmental restrictions the aggregate production over all years in a time period can be restricted by a production ceiling  $\overline{PROD}_p$

$$\sum_m \sum_d days_d SALES_{pdm}^P \leq \overline{PROD}_p \quad \forall m \quad (\beta_p^P) \quad (3)$$

$$\text{Non-negativity of sales: } SALES_{pdm}^P \geq 0 \quad \forall d, m \quad (4)$$

### 2.2.1.1 KKT conditions for the producer problem

To obtain the MCP model we take the first order conditions with respect to each decision variable (here:  $SALES_{pdm}^P$ ) of the profit maximization problem(s) to derive the Karush-Kuhn-Tucker (KKT) conditions. The following are the KKT conditions for the producer optimization problem which are necessary by the linearity constraint qualification and sufficient as long as the production cost function  $c_{pm}^P(\bullet)$  is convex (Bazaara et al., 1993).

$$0 \leq \left[ \begin{array}{l} days_d \left[ \gamma_m \left( -\pi_{n(p)dm}^P + \frac{\partial c_{pm}^P(SALES_{pdm}^P)}{\partial SALES_{pdm}^P} \right) \right] \\ + \alpha_{pdm}^P + days_d \beta_p^P \end{array} \right] \perp SALES_{pdm}^P \geq 0 \quad \forall d, m \quad (5)$$

$$0 \leq \overline{PR}_{pm}^P - SALES_{pdm}^P \perp \alpha_{pdm}^P \geq 0 \quad \forall d, m \quad (6)$$

$$0 \leq \overline{PROD}_p - \sum_m \sum_{d \in D} days_d SALES_{pdm}^P \perp \beta_p^P \geq 0 \quad (7)$$

The market-clearing conditions, and the market-clearing price  $\pi_{n(p)dm}^P$  that tie the producer optimization problem to the optimization problems of the traders and liquefiers are as follows:<sup>2</sup>

$$0 \leq SALES_{pdm}^P - PURCH_{i(p)n(p)dm}^{T \leftarrow P} - \sum_{l \in L(p)} PURCH_{ldm}^{L \leftarrow P} \perp \pi_{n(p)dm}^P \geq 0 \quad \forall d, p, m$$

### 2.2.1.2 Production input data and supply cost function

While we use a generic convex production cost function in the above optimization problem we detail our specific choice next. We assume a functional form following Golombek et al. (1995), including a steep increase of production costs close to the capacity limit  $Q$ .

$$C(q) = (\alpha - \gamma)q + \frac{1}{2}\beta q^2 - \gamma(Q - q) \ln\left(\frac{Q - q}{Q}\right), \alpha > 0, \beta \geq 0, \gamma \leq 0, \forall q: 0 \leq q < Q$$

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<sup>2</sup> In practice this inequality holds as an equality and should hold as shown in Zhuang (2005).

for which the marginal supply cost curve is:  $C'(q) = \alpha + \beta q + \gamma \ln\left(\frac{Q-q}{Q}\right)$  and where  $Q$  is the production capacity,  $\alpha$  is the minimum per unit cost term,  $\beta$  the per unit linearly increasing cost term, and  $\gamma$  a term that induces high marginal costs when production is close to full capacity. To derive the parameters  $\alpha$ ,  $\beta$  and  $\gamma$  we set the production rate  $q$  equal to the reference value of production in the base year.

### 2.2.2. Traders' Problem

The traders in the WGM have a simplified role: they buy gas from one or more producers and sell gas to one or more final consumption markets. Examples of traders in today's natural gas markets include Gazexport, the trading arm for Gazprom (Russia) and GasTerra for NAM (Nederlandse Aardolie Maatschappij). This modeling approach can represent both, a vertically integrated production and trading company (separate parts of the same overall organization with marginal cost internal accounting prices) as well as an independent trader that purchases gas from one or several producers. We distinguish two types of traders:

- A. Traders operating only at the domestic node of the producer in case it is a small producer that does not export any gas. Previous papers (e.g., Boots et al., 2004) usually refer to this production as exogenous production, and do not model these quantities endogenously.
- B. Traders that can operate at any consumption node that can be reached via pipelines through transit nodes from their own producer's node.

The trader maximizes profits resulting from selling gas to marketers ( $SALES_{indm}^{T \rightarrow M}$ ) and – in the low demand season,  $\delta_d^{low} = 1$  – to storage operators ( $SALES_{imm}^{T \rightarrow S}$ ), net of the gas +purchasing costs and the costs of using the transportation system ( $\tau_{nn, dm}^A + \tau_{nn, dm}^{Reg}$ ), a regulated fee plus congestion fee, for the gas flow ( $FLOW_{nn, dm}^T$ ) between  $(n, n_i)$ . The parameter  $\delta_{t, n}^C \in [0, 1]$  indicates the level of market power exerted by a trader  $t$  at a

consumption node  $n$ ; a value of 0 representing perfect competitive behavior and 1 Cournot (oligopolistic) behavior. The expression  $(\delta_{t,n}^C \Pi_{ndm}^W(\cdot) + (1 - \delta_{t,n}^C) \pi_{ndm}^W)$  can be viewed as a weighted average of market prices resulting from the inverse demand function  $\Pi_{ndm}^W(\cdot)$  and a perfectly competitive market clearing wholesale price  $\pi_{ndm}^W$ .

$$\max_{\substack{SALES_{ndm}^{T \rightarrow M} \\ SALES_{tnm}^{T \rightarrow S} \\ FLOW_{tn_i, dm}^T \\ PURCH_{ndm}^{T \leftarrow P}}} \sum_{m \in M} \gamma_m \left\{ \sum_{d \in D} days_d \left( \sum_{n \in N(t)} \left[ \begin{aligned} & (\delta_{t,n}^C \Pi_{ndm}^W + (1 - \delta_{t,n}^C) \pi_{ndm}^W) SALES_{ndm}^{T \rightarrow M} \\ & + \delta_d^{low} \pi_{ndm}^T SALES_{tnm}^{T \rightarrow S} \\ & - \pi_{n(p(t)), dm}^P PURCH_{ndm}^{T \leftarrow P} \end{aligned} \right] \right) \right. \quad (8) \\ \left. - \sum_{d \in D} days_d \left( \sum_{(n, n_i) \in A(t)} (\tau_{nn_i, dm}^A + \tau_{nn_i, dm}^{Reg}) FLOW_{tn_i, dm}^T \right) \right\}$$

The following mass balance equation ensures that the volumes bought from the producer and imported by pipeline must be enough to meet the total sales and the pipeline exports, for each node in each season.

$$\left[ \begin{aligned} & PURCH_{ndm}^{T \leftarrow P} \\ & + \sum_{n_i \in N} (1 - loss_{n_i, n}) FLOW_{tn_i, ndm}^T \end{aligned} \right] \geq \left[ \begin{aligned} & \delta_d^{low} SALES_{tnm}^{T \rightarrow S} \\ & + SALES_{ndm}^{T \rightarrow M} \\ & + \sum_{n_i \in N} FLOW_{tn_i, dm}^T \end{aligned} \right] \quad \forall n \in N(t), d, m \quad (\phi_{ndm}^T) \quad (9)$$

The remaining constraints enforce non-negativity of the decision variables:

$$s.t. \quad SALES_{ndm}^{T \rightarrow M} \geq 0 \quad \forall n, d, m \quad (10)$$

$$SALES_{tnm}^{T \rightarrow S} \geq 0 \quad \forall n, m \quad (11)$$

$$PURCH_{ndm}^{T \leftarrow P} \geq 0 \quad \forall n = n(p(t)), d, m \quad (12)$$

$$FLOW_{tn_i, dm}^T \geq 0 \quad \forall (n, n_i) \in A(t), d, m \quad (13)$$

Beside traders, regasifiers ( $SALES_{rdm}^{R \rightarrow M}$ ) and, in the high and peak demand seasons, storage operators ( $SALES_{sdm}^{S \rightarrow M}$ ) can sell gas to the marketers, too. Market clearing in the end-user market, at a wholesale price  $\pi_{ndm}^W$ , is enforced by the following inverse demand function:

$$\pi_{ndm}^W \geq INT_{ndm}^M - SLP_{ndm}^M \cdot \left( \begin{array}{l} \sum_{t \in T(n)} SALES_{ndm}^{T \rightarrow M} \\ + \sum_{r \in R(n)} SALES_{rdm}^{R \rightarrow M} \\ + (1 - \delta_d^{low}) \sum_{s \in S(n)} SALES_{sdm}^{S \rightarrow M} \end{array} \right) \quad \forall n, d, m \quad (\pi_{ndm}^W) \quad (14)$$

The market-clearing conditions between traders and storage operators in the low demand (injection) season are as follows:

$$0 \leq \sum_{t \in T(n)} SALES_{nm}^{T \rightarrow S} - \sum_{s \in S(n)} PURCH_{sm}^{S \leftarrow T} \perp \pi_{n1m}^T \geq 0 \quad \forall n \in \{N(t) \cap N(s)\}, m \quad (15)$$

KKT conditions for the trader and the following players' optimization problems can be found in the Appendix. They are derived in the same way as described above for the producer, that is by taking the first-order conditions with respect to each decision variable and including the constraints.

### 2.2.3. Liquefaction

In the model, export LNG terminals ("liquefiers") are represented as players that buy gas from a single producer (located in the same country node) and can sell it to regasifiers around the world. The liquefier player in the World Gas Model covers the liquefaction process including its internal optimization of LNG storage.

The LNG market today is characterized by a large amount of contracted sales that imply that liquefiers have committed to sell a minimum amount of natural gas in general to a specific LNG importing country (regasifier). Where available, we include the data for contracts as a constraint in the model.<sup>3</sup>

The liquefier maximizes his discounted net profits from selling gas to regasifiers  $SALES_{ldm}^L$ , minus costs to purchase the gas  $PURCH_{ldm}^{L \leftarrow P}$  and costs for liquefaction  $c_{lm}^L(SALES_{ldm}^L)$  and investment costs  $b_{lm}^L \Delta_{lm}^L$ .

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<sup>3</sup> We thank Sophia Rüster and Anne Neumann for sharing the contract information from their data base.

$$\max_{\substack{SALES_{ldm}^L \\ PURCH_{ldm}^{L \leftarrow P}, \Delta_{lm}^L}} \sum_{m \in M} \gamma_m \left\{ \sum_{d \in D} days_d \begin{bmatrix} \pi_{n(l)dm}^L SALES_{ldm}^L \\ -\pi_{n(l)dm}^P PURCH_{ldm}^{L \leftarrow P} \\ -c_{lm}^L (SALES_{ldm}^L) \end{bmatrix} - b_{lm}^L \Delta_{lm}^L \right\} \quad (16)$$

Sales rates in any year are restricted by liquefaction capacity. Liquefaction capacity can be expanded to be available in the following period. Therefore, the total liquefaction capacity in a certain year  $m$  is the sum of the initial capacity  $\overline{LQF}_l^L$  and the expansion investments in all former years  $\sum_{m' < m} \Delta_{lm'}^L$ .<sup>4</sup>

$$s.t. \quad SALES_{ldm}^L \leq \overline{LQF}_l^L + \sum_{m' < m} \Delta_{lm'}^L \quad \forall d, m \quad (\alpha_{ldm}^L) \quad (17)$$

Liquefaction losses are significant and have to be accounted for in the mass balance between purchases and sales:

$$(1 - loss_l) PURCH_{ldm}^{L \leftarrow P} - SALES_{ldm}^L \geq 0 \quad \forall d, m \quad (\phi_{ldm}^L) \quad (18)$$

There can be regulatory, technical or budget restrictions limiting the capacity expansions in specific periods:

$$\Delta_{lm}^L \leq \overline{\Delta}_{lm}^L \quad \forall m \quad (\rho_{lm}^L) \quad (19)$$

Non-negativity of the involved variables:

$$PURCH_{ldm}^{L \leftarrow P} \geq 0 \quad \forall d, m \quad (20)$$

$$SALES_{ldm}^L \geq 0 \quad \forall d, m \quad (21)$$

$$\Delta_{lm}^L \geq 0 \quad \forall m \quad (22)$$

The market-clearing conditions between liquefiers and regasifiers are as follows, where the index  $b$  denotes the LNG tanker, running from node  $n_s(b)$  to  $n_e(b)$ :

$$0 \leq \sum_{l \in L(n(l))} SALES_{ldm}^L - \sum_{b: n_s(b)=n(l)} PURCH_{bdm}^{R \leftarrow L} \perp \pi_{n(l)dm}^L \geq 0 \quad \forall d, m \quad (23)$$

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<sup>4</sup> Capacity expansions cannot be executed instantaneously. Typically a multi-period run contains years that represent every fifth year in the time horizon. Five years are generally enough for addressing the time lag between a capacity expansion decision and the expansion to be constructed.



## 2.2.4. Regasification

The following section describes the problem of the importing side in the liquefied natural gas market, the regasification.. Regasifiers can buy gas from liquefiers and sell it to domestic storage operators and to marketers. The regasifiers can exert market power relative to the marketers, thereby representing strategic behavior on the LNG market, similar to the traders for the pipeline market.

Contrary to liquefiers, we may include more than one regasifier in a country depending on the country's geography. This choice allows countries like Spain, France and Mexico to have LNG import capacity on their respective East and West coasts, thereby potentially providing interesting insights in the developments in the various global basins.<sup>5</sup>

The operational process of a regasifier includes the internal optimization of LNG storage in addition to the main activities of unloading the LNG vessels and bringing the vaporized (gaseous) natural gas into the pipeline system. Moreover, in the WGM the regasifier's problem includes the optimization of LNG shipment by tankers. It is represented by a distance-based shipping cost and a gas loss rate that allow the regasifier to determine the optimal transport from any liquefier.

The regasifier maximizes his discounted profits resulting from the sales to marketers  $SALES_{rdm}^{R \rightarrow M}$  and storage operators  $SALES_{rdm}^{R \rightarrow S}$  minus the costs to purchase and ship the gas  $(\pi_{n_s(b)dm}^L + u_b^{R \leftarrow L}) PURCH_{bdm}^{R \leftarrow L}$ , the re-gasification costs  $c_{rm}^R (SALES_{rdm}^{R \rightarrow M} + SALES_{rdm}^{R \rightarrow S})$  and investment costs  $b_{rm}^R \Delta_{rm}^R$ .

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<sup>5</sup> However, our simplified representation does not allow for representing recent developments such as co-ownerships of LNG terminals such as the majority share of the French company GDF Suez and minority shares of Italian Publigras and others in the Belgian Zeebrugge terminal.

$$\max_{\substack{SALES_{rdm}^{R \rightarrow M}, \\ SALES_{rdm}^{R \rightarrow S}, \\ PURCH_{bdm}^{R \leftarrow L}, \\ \Delta_{rm}^R}} \sum_{m \in M} \gamma_m \left\{ \sum_{d \in D} days_d \left[ \begin{aligned} & \left( \delta_r^C \Pi_{ndm}^W + (1 - \delta_r^C) \pi_{ndm}^W \right) SALES_{rdm}^{R \rightarrow M} \\ & + \pi_{n(r)dm}^R SALES_{rdm}^{R \rightarrow S} \\ & - \sum_{b: n_e(b)=n(r)} \left( \pi_{n_s(b)dm}^L + u_b^{R \leftarrow L} \right) PURCH_{bdm}^{R \leftarrow L} \\ & - c_{rm}^R (SALES_{rdm}^{R \rightarrow M} + SALES_{rdm}^{R \rightarrow S}) \end{aligned} \right] - b_{rm}^R \Delta_{rm}^R \right\} \quad (24)$$

Sales (i.e, regasification) rates in any year are restricted by the regasification capacity. Regasification capacity can be expanded by an endogenous investment decision. Hence, the total capacity in a certain year  $m$  is the sum of initial capacity  $\overline{REG}_r^R$  and the yearly expansions in previous years,  $\sum_{m' < m} \Delta_{rm'}^R$ .

s.t.

$$SALES_{rdm}^{R \rightarrow M} + SALES_{rdm}^{R \rightarrow S} \leq \overline{REG}_r^R + \sum_{m' < m} \Delta_{rm'}^R, \quad \forall d, m \quad \left( \alpha_{rdm}^R \right) \quad (25)$$

The purchased gas, corrected for shipment losses ( $loss_b$ ) and regasification losses ( $loss_r$ ), must be greater or equal to the total sales:

$$\sum_{b: n_e(b)=n(r)} (1 - loss_r)(1 - loss_b) PURCH_{bdm}^{R \leftarrow L} - \left( \begin{array}{l} SALES_{rdm}^{R \rightarrow M} \\ + SALES_{rdm}^{R \rightarrow S} \end{array} \right) \geq 0 \quad \forall d, m \quad \left( \phi_{rdm}^R \right) \quad (26)$$

Again, there can be regulatory, technical or budget restrictions limiting the capacity expansions in specific periods:

$$\Delta_{rm}^R \leq \overline{\Delta}_{ry(m)}^R \quad \forall m \quad \left( \rho_{rm}^R \right) \quad (27)$$

The presence of contracts may impose a lower bound on purchases from a specific liquefier in a certain year:

$$PURCH_{bdm}^{R \leftarrow L} \geq Contract_{bdy(m)}^R \quad \forall b: n_e(b) = n(r), d, m \quad \left( \varepsilon_{bdm}^R \right) \quad (28)$$

Non-negativity of the decision variables:

$$SALES_{rdm}^{R \rightarrow M} \geq 0 \quad \forall d, m \quad (29)$$

$$SALES_{rdm}^{R \rightarrow S} \geq 0 \quad \forall d = 1, m \quad (30)$$

$$PURCH_{bdm}^{R \leftarrow L} \geq 0 \quad \forall b: n_e(b) = n(r), d, m \quad (31)$$

$$\Delta_{rm}^R \geq 0 \quad \forall m \quad (32)$$

The market-clearing conditions between regasifiers and storage operators are as follows:

$$0 \leq \sum_{r \in R(n)} SALES_{r1m}^{R \rightarrow S} - \sum_{s \in S(n)} PURCH_{sm}^{S \leftarrow R} \perp \pi_{n1m}^R \geq 0 \quad \forall n, m \quad (33)$$

### 2.2.5. Storage

Natural gas storage can be used for a variety of reasons, including daily balancing and price arbitrage, seasonal balancing and as a strategic backup supply to overcome temporary supply disruptions or to meet peak demand on cold winter days. We focus on the seasonal arbitrage and assume the storage to be empty at the beginning and the end of each year. Storage can also be used to compensate disrupted supplies in our model.

There are various types of gas storages: depleted reservoirs in oil and gas fields, aquifers, and salt caverns. Each of them has different characteristics relative to the amount of gas that can be stored and the speed with which the gas can be injected and extracted. In most countries, one type of storage is prevailing and we include these country-specific characteristics.

The amount of gas available for operation is the working gas. Typically, gas installations have minimum and maximum injection and extraction rates. Compressors are used to generate pressure to be able to inject the gas in the storage. These compressors use some of the gas, therefore there is a loss rate associated with the operations. In the WGM, we assume that storage operators buy gas and inject it in the low demand season and extract gas and sell it in the high and peak demand seasons, as long as the seasonal price differential (corrected for the loss rate) is larger than the operational costs.

The storage operator sells gas to the domestic marketer in the high and peak demand season:  $SALES_{sdm}^{S \rightarrow M}$ . The gas is bought in the low demand season (of that same year) and injected into storage. Costs are made for purchasing the gas from the traders  $PURCH_{sm}^{S \leftarrow T}$  and regasifiers  $PURCH_{sm}^{S \leftarrow R}$ , and to inject the gas into storage  $c_{sm}^S (PURCH_{sm}^{S \leftarrow T} + PURCH_{sm}^{S \leftarrow R})$ . To expand capacity for injection, extraction or total working gas, the investment costs sum up to  $b_{sm}^{S,INJ} \Delta_{sm}^{S,INJ} + b_{sm}^{S,EXT} \Delta_{sm}^{S,EXT} + b_{sm}^{S,WG} \Delta_{sm}^{S,WG}$ .

$$\begin{aligned}
& \max_{\substack{PURCH_{sm}^{S \leftarrow T} \\ PURCH_{sm}^{S \leftarrow R} \\ SALES_{sdm}^{S \rightarrow M} \\ \Delta_{sm}^{S, INJ} \\ \Delta_{sm}^{S, EXT} \\ \Delta_{sm}^{S, WG}}} \sum_{m \in M} \gamma_m \left\{ \begin{aligned} & \sum_{d=2,3} days_d \pi_{n(s)dm}^W SALES_{sdm}^{S \rightarrow M} \\ & -days_1 \left[ \begin{aligned} & \pi_{n(s)1m}^T PURCH_{sm}^{S \leftarrow T} \\ & + \pi_{n(s)1m}^R PURCH_{sm}^{S \leftarrow R} \\ & + c_{sm}^S (PURCH_{sm}^{S \leftarrow T} + PURCH_{sm}^{S \leftarrow R}) \end{aligned} \right] \\ & -b_{sm}^{S, INJ} \Delta_{sm}^{S, INJ} -b_{sm}^{S, EXT} \Delta_{sm}^{S, EXT} -b_{sm}^{S, WG} \Delta_{sm}^{S, WG} \end{aligned} \right\} \quad (34)
\end{aligned}$$

Injection rates in any year are restricted by the injection capacity. Capacity can be expanded, therefore the total capacity in a year is the sum of initial capacity  $\overline{INJ}_s^S$  and the yearly expansions  $\sum_{m' < m} \Delta_{sm'}^{S, INJ}$ . Similar explanations apply to (36) and (37) for extraction and working gas limitations.

s.t.

$$PURCH_{sm}^{S \leftarrow T} + PURCH_{sm}^{S \leftarrow R} \leq \overline{INJ}_s^S + \sum_{m' < m} \Delta_{sm'}^{S, INJ} \quad \forall m \quad (\alpha_{sm}^S) \quad (35)$$

$$SALES_{sdm}^{S \rightarrow M} \leq \overline{EXT}_s^S + \sum_{m' < m} \Delta_{sm'}^{S, EXT} \quad d = 2, 3, \forall m \quad (\beta_{sdm}^S) \quad (36)$$

$$\sum_{d=2,3} days_d SALES_{sdm}^{S \rightarrow M} \leq \overline{WRKG}_s^S + \sum_{m' < m} \Delta_{sm'}^{S, WG} \quad \forall m \quad (\theta_{sm}^S) \quad (37)$$

Total purchases corrected for losses must be enough to cover the total sales:

$$\left[ days_1 (1 - loss_s) \left( \begin{aligned} & PURCH_{sm}^{S \leftarrow T} \\ & + PURCH_{sm}^{S \leftarrow R} \end{aligned} \right) \right] - \sum_{d=2,3} days_d SALES_{sdm}^{S \rightarrow M} \geq 0 \quad \forall m \quad (\phi_{sm}^S) \quad (38)$$

Limitations to the capacity expansions:

$$\Delta_{sm}^{S, INJ} \leq \overline{\Delta}_{sy(m)}^{S, INJ} \quad \forall m \quad (\rho_{sm}^{S, INJ}) \quad (39)$$

$$\Delta_{sm}^{S, EXT} \leq \overline{\Delta}_{sy(m)}^{S, EXT} \quad \forall m \quad (\rho_{sm}^{S, EXT}) \quad (40)$$

$$\Delta_{sm}^{S, WG} \leq \overline{\Delta}_{sy(m)}^{S, WG} \quad \forall m \quad (\rho_{sm}^{S, WG}) \quad (41)$$

Non-negativity of the decision variables:

$$SALES_{sdm}^{S \rightarrow M} \geq 0 \quad \forall d = 2, 3, m \quad (42)$$

$$PURCH_{sm}^{S \leftarrow T} \geq 0 \quad \forall m, t \in T(s(n)) \quad (43)$$

$$PURCH_{sm}^{S \leftarrow R} \geq 0 \quad \forall m \quad (44)$$

$$\Delta_{sm}^{S, INJ} \geq 0 \quad \forall m \quad (45)$$

$$\Delta_{sm}^{S, EXT} \geq 0 \quad \forall m \quad (46)$$

$$\Delta_{sm}^{S, WG} \geq 0 \quad \forall m \quad (47)$$

### 2.2.6. Pipeline operator

The pipeline operator is responsible for assigning available capacities of international high pressure pipelines to the traders needing transport capacity for exporting gas. Ownership, management and operation of the pipeline network are done differently in various countries. We assume in a simplified approach that the pipeline network is regulated such that the access to transport infrastructure for third parties is ensured and capacity is allocated on a willingness-to-pay basis. While this describes some markets realistically (e.g., in the USA), it is a hypothetical assumption for others (e.g., in Europe), albeit in line with the objectives of the European Commission (e.g., EC, 2003).

It is necessary to address pipeline capacities in an economic natural gas market model since they limit the supplied volumes from producers to end-users. We simplify from engineering considerations of the flow problem (e.g., pipeline friction, pressure differentials between two nodes) due to their nonlinear properties that usually are not included in a MCP model.<sup>6</sup> We include pipeline capacities in level at the cross-border points, using annualized data.

Some natural gas pipelines are bidirectional. In the model these pipelines are modeled with two separate capacities. Thus, there is no netting of flows. In a perfectly competitive setting this has no impact, since in an optimal solution (assuming strictly positive costs and/or losses) at most one direction will have positive flow. However, in a market power situation there is an incentive for traders to supply to other markets, often

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<sup>6</sup> Midthun (2007) presents a complementarity model taking into account the so-called *Weymouth* equation by using linearizing techniques. In contrast to our market model, he deals with the optimization of technical processes related to production and transport (in the Norwegian North Sea).

resulting in congested pipelines in both directions (see Egging and Gabriel, 2006, for a deeper analysis of this issue).

The pipeline operator provides an economic mechanism to efficiently allocate pipeline capacity to traders. The pipeline operator maximizes the discounted profit resulting from selling pipeline capacity to traders,  $SALES_{nn_i, dm}^A$ . The regulated fees collected from the traders are assumed to equal the operating costs, therefore the profit margin is equal to the congestion fee  $\tau_{nn_i, dm}^A$ .

$$\max_{SALES_{nn_i, dm}^A} \sum_{m \in M} \gamma_m \sum_{(n, n_i)} \left\{ \sum_{d \in D} days_d \tau_{nn_i, dm}^A SALES_{nn_i, dm}^A \right\} \quad (48)$$

The assigned pipeline capacity can be at most the available capacity. Available pipeline capacity on an arc  $(n, n_i)$  is the sum of initial pipeline capacity  $\overline{PL}_{nn_i}^A$  and capacity expansions in former years  $\sum_{m' < m} \Delta_{nn_i, m'}^O$ .

*s.t.*

$$SALES_{nn_i, dm}^A \leq \overline{PL}_{nn_i}^A + \sum_{m' < m} \Delta_{nn_i, m'}^O \quad \forall d, m \quad (\alpha_{nn_i, dm}^A) \quad (49)$$

Non-negativity of variables:

$$SALES_{nn_i, dm}^A \geq 0 \quad \forall d, m \quad (50)$$

Market-clearing conditions for pipeline capacity between pipeline operator and traders:

$$0 \leq SALES_{nn_i, dm}^A - \sum_{t \in T((n, n_i))} FLOW_{nn_i, dm}^T \perp \tau_{nn_i, dm}^A \geq 0 \quad \forall (n, n_i), d, m \quad (51)$$

### 2.2.7. Transmission System Operator Problem

The market agent that we assume to be responsible for expanding the pipeline network is the transmission system operator (TSO). The transmission system operator maximizes a function with revenues from congestion payments and investment costs of expansion of the pipeline network. This mechanism balances the pipeline investment costs and the added value to the market given by the added pipeline capacity. Hence, we represent the long-term optimization of the pipeline network.

The separation of short-term and long-term optimization ensures that there is no incentive to withhold long-term capacity expansion in order to increase congestion revenues in the short-term. However, given that the endogenous variables from one player (TSO) enter into the optimization problem (in the constraint set) of another player (the pipeline operator), this version of the World Gas Model is in fact an instance of a generalized Nash problem. As such, it is equivalent to a quasi-variational inequality. Under certain circumstances, one can solve an associated variational inequality (or mixed complementarity) problem to resolve it, as we do here. The optimization problem of the transmission system operator is given as follows:

$$\max_{\Delta_{nn,m}^O} \sum_{m \in M} \gamma_m \left\{ \left[ \sum_{(n,n_i)} \sum_{d \in D} \left( \text{days}_d \sum_{m' > m} \tau_{nn,dm'}^A \right) \Delta_{nn,m}^O \right] - \left( \sum_{(n,n_i)} b_{m,m}^O \Delta_{nn,m}^O \right) \right\} \quad (52)$$

There may be limitations to the allowable pipeline expansions

*s.t.*

$$\Delta_{nn,m}^O \leq \bar{\Delta}_{m,m}^O \quad \forall n, n_i, m \quad (\rho_{nn,m}^O) \quad (53)$$

Non-negativity of the expansion decision variables

$$\Delta_{m,m}^O \geq 0 \quad \forall n, n_i, m \quad (54)$$

### 2.2.8. Marketer, Distribution and Consumption Sectors

The KKT and market-clearing conditions presented in the above sections represent the World Gas Model. Some market aspects are indirectly accounted for in the model. The main one is the final consumption by three sectors (electricity generation, industry, residential) that are represented via an aggregation of their respective inverse demand functions into a single inverse demand function, which in turn represents the marketer.

For our analysis of the world gas market and the international trade flows, it is not necessary to include all different demand sectors in each country. Equation (14) in the trader problem (Section 2.2.2) shows the aggregate demand function. To simplify the model structure and limit the number of model variables, we include country aggregate inverse demand curves. However, the model is calibrated by sector level and the sector

level information is retained. Ex-post the demand for each sector can be calculated based on the individual inverse demand curves. As long as all sectors have positive consumption, we know that the aggregation to a single inverse demand curve does not change the obtained outcomes compared to sector-specific demand functions.

The combination of all KKTs and the market-clearing conditions form the market equilibrium (MCP) model. Due to concavity of the profit functions<sup>7</sup>, convexity of the cost functions and convexity of the feasible regions, the KKT points for this system are optimal solutions.

### 3 Data Set

We are interested in the international trade of natural gas, so most countries are represented as just one node. Large countries and/or countries active in several regional basins are split up into several nodes, such as the U.S.A., Canada, Russia, and Mexico. We deal with normalized units of natural gas (at 15°C temperature and 760 mmHG pressure as defined by the International Energy Agency, e.g., IEA, 2008a). The data set can be adapted for scenario runs (e.g., Huppmann et al., 2009); here we present the base case data set and assumptions. Our base year is 2005 and we need additional data input for the following model years 2010, 2015, 2020, 2025, 2030, 2035 and 2040.<sup>8</sup> We assume a 10% discount rate in the multi-period optimization. In general, we use data per day, distinguished by season (low, high and peak demand) where applicable.

On the supply side, we must realistically include limits on how much can be produced and transported. These capacity constraints are based on existing facilities for the base year and include projects currently under construction for the second model period (2010).<sup>9</sup> Starting in 2010, there can be endogenous investments in transport and storage infrastructure. In order to maintain a MCP we assume continuous capacity

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<sup>7</sup> Since we are minimizing the negative of a concave profit function, we are effectively minimizing a convex function.

<sup>8</sup> The last two model years are not reported in the model results, but they are necessary to have a sufficient payback period for the model-derived, endogenous investments.

<sup>9</sup> We also include one exogenous reduction of capacity in 2015, namely the LNG terminal in Alaska which will cease operations by 2012.



expansions. The investment is limited in each period; where available we use projections to determine these limits, otherwise we include our own assessment.

Production capacity data for the base year is based on information and forecasts in the technical literature (e.g., OME, 2005, *Oil and Gas Journal*). Production capacity is determined exogenously for all model periods (i.e., no endogenous investments). For future periods, we apply a growth rate to the base year capacity that is based on production growth projections with the PRIMES model for Europe (EC, 2008) and the POLES model for the rest of the world (EC, 2006).

International pipeline transport is limited at the cross-border points. When there are several cross-border points between two adjacent country nodes, we aggregate the capacities of these points to a single bound. We use capacity data from GTE (2005, 2008) for intra-European transport. Data on pipeline capacity between the North American nodes<sup>10</sup> was obtained from the Energy Information Agency. For all other pipelines, we use company reports and websites as well as technical literature. For given pipeline expansions between 2005 (first model period) and 2008 (time of our data base construction), we exogenously include the realized capacities in the model year 2010. For new greenfield pipeline projects that are planned but do not exist yet, e.g. the Nabucco pipeline, we include a zero capacity in the first model year and allow for positive investments in later periods (with the exact period depending on the project). Storage capacities are obtained from IEA (2007) and GSE (2008) for existing facilities. GSE (2008) also provides projected capacities in Europe.

The LNG transport value chain contains liquefaction, shipment and regasification, as explained above. Liquefaction and regasification capacity data for 2005 are from IEA (2007). For future capacity expansion limits, including new terminals, we use technical literature such as IEA (2008b), the *Oil and Gas Journal*, etc. For the downstream actor in the LNG chain, the regasifier, we additionally use GLE (2005) for Europe. Shipment is optimized by the regasifier, given the distance-based transport costs. Distances between each pair of liquefier and regasifier are obtained for the approximate location of the

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<sup>10</sup> North America is split into nine regions: Alaska, Canada-East and Canada-West, US-West, US-Rockies, US-Gulf, US-Midwest, US-East, and Mexico.

terminals using [www.distances.com](http://www.distances.com). There is no restriction on the trading pairs, and we do not include limits on the shipment capacity.

We assume linear cost functions for the construction of incremental capacity of transport or storage. The parameters are averages based on reported project costs in technical literature such the *Oil and Gas Journal* and company information. In the LNG value chain, the parameters are chosen such as to reflect the fact that the infrastructure for the regasification of gas is less capital intensive than the liquefaction. For pipelines, we determine a base cost of 50,000,000 US-\$ for a new capacity of 1 bcm/year between two nodes, based on industry cost reports. For each of the characteristics “greenfield project”, “very long pipeline” or “offshore pipeline”, this unit cost is doubled.

Storage expansions comprise expansion of injection, extraction and working gas capacity. Building extra injection capacity is costlier (our assumption: 3,000,000 US-\$/mcm/d) than building extra extraction (500,000 US-\$/mcm/d). For working gas the investment costs are 150,000 US-\$/bcm.

Short-run production costs and losses are similar to Egging et al. (2008) but have been updated. As explained in the model description above, the production cost function in the short term is a function of the produced quantity that increases strongly close to the production capacity limit (Golombek et al., 1995). The parameters for the cost function are derived from OME (2005) but had to be adjusted upwards in the calibration process.

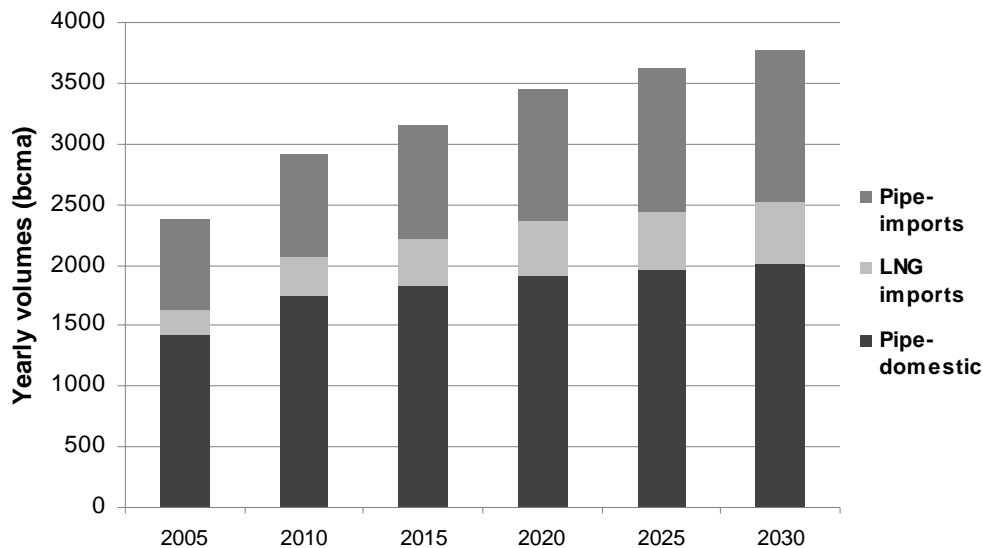
Short-term transport costs per pipeline are a linear function, related to the distance to be traveled and including royalties where applicable (e.g. for the pipeline through Tunisia). Similarly, losses for pipeline transportation are assumed to be higher for long-distance pipelines, following Oostvoorn (2003). For LNG transport, we apply linear cost functions for liquefaction and for regasification. In the absence of detailed data, we use the same parameters for all countries. Shipment costs and losses, that are added to the regasification costs, are distance-based.

The total demand function for natural gas is obtained from aggregating sector-specific consumption for each country. The International Energy Agency, in its Monthly Natural Gas Survey (<http://www.iea.org/Textbase/stats/surveys/archives.asp>) reports consumption levels for the power sector, industry, residential/households and other categories for each month. We aggregate these data by season (low, high and peak

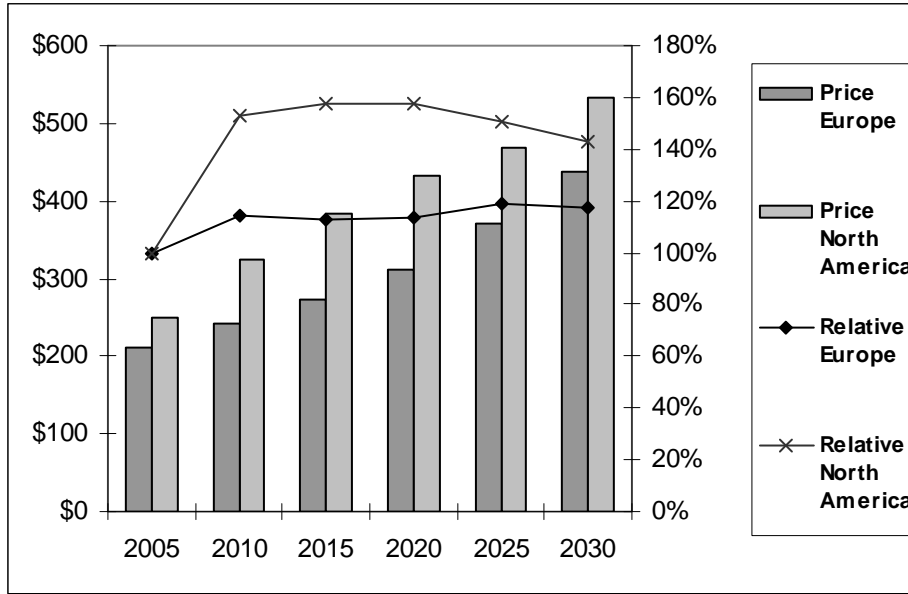
demand), with the monthly distribution depending on the geographic location of each country (with differences, e.g., between the Northern and the Southern hemisphere) and determine a parameter reflecting the intensity of seasonal change of demand. For each sector-specific demand, another price elasticity is assumed (between -0.25 and -0.75). For the construction of the demand function for each period, we also need a reference price. The 2005 prices are based on IEA (2007) and BP (2008). For future periods, we assume an annual growth rate in the willingness to pay of 3%, based on EC (2008). Total demand is then an aggregated function of the linear functions for each sector.

## 4 Base Case Results

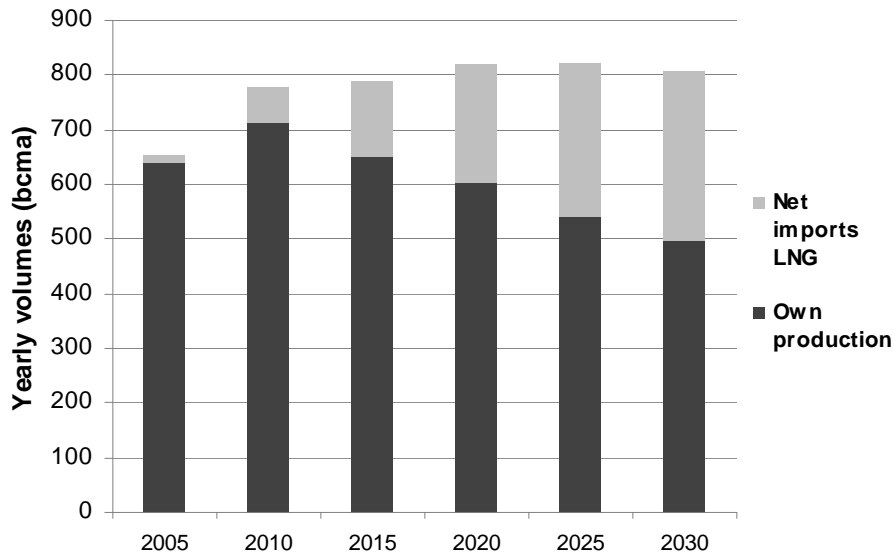
The Base Case shows a steady increase of natural gas production over the whole period that results in a total global production level of about 3,900 bcm/y (3,700 bcm/y. of consumption after the subtraction of losses) in 2030 (Figure 3). LNG trade grows until 2020 and then reaches a plateau close to 600 bcm/y. At that moment, LNG will account for approximately 15% of total natural gas production. The amount of natural gas domestically consumed in the producing countries drops from 60% to about 50% of total consumption, while the share of natural gas exported by pipeline remains relatively stable (30%). In other words, the international trade of natural gas and in particular the share of LNG in the trade volumes will increase.



**Figure 3: Global consumption obtained from domestic production and imports per pipeline and LNG, in bcm per year**



**Figure 4: Development of wholesale prices in Europe and North America, in absolute levels (in 2005 US-\$) and in growth rates (in percentage)**

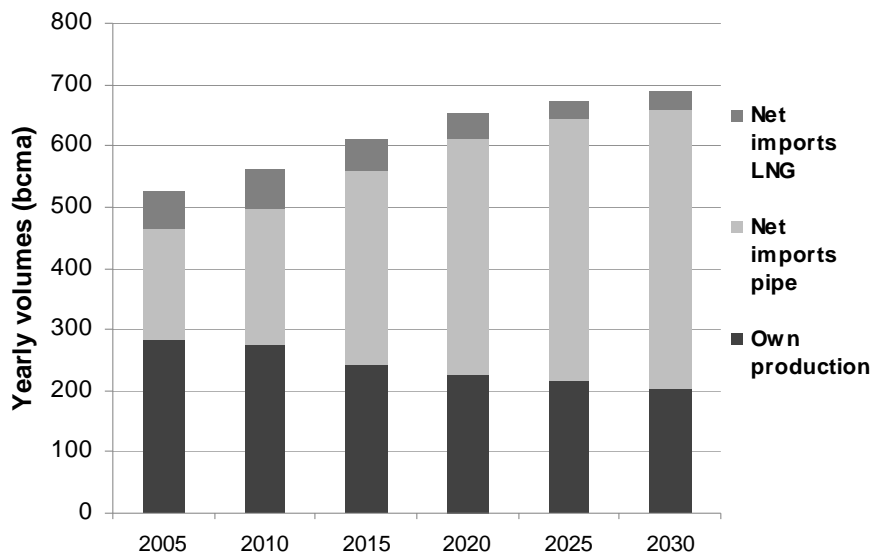


**Figure 5: Breakdown of the sources of North American consumption for all model years, in bcm per year**

As explained above, we assume a yearly price increase of 3% (in real 2005 US-Dollars) for the construction of the demand functions. This price increase is reflected in the results, albeit with varying intensities. As shown in Figure 4, the price increase in North America is considerably more pronounced than in Europe, especially in the first model years. This is due to the strong increase in imports, in particular of LNG, due to the increasing demand and insufficient own production capacities in North America

before alternative domestic supply sources (from Alaska) come on-stream (Figure 5).<sup>11</sup> In 2030, North America produces about 60% of its consumption domestically with the remaining 40% satisfied by LNG imports.

In 2030, the Middle East, Russia and the Caspian region split the major part of their sales between Europe and Asia, with small amounts sold as LNG to North America. Total consumption in Europe in 2030 amounts to 667 bcm/y.; of this, 27 bcm/y. are supplied in the form of LNG, which accounts for 4% of total consumption, and 200 bcm/y. are produced domestically. A large share of European consumption is imported from Russia and the Caspian region, but also from North Africa as pipeline gas (Figure 6). Hence, in the competition for LNG in the Atlantic basin, North America would be able to take the lead because of its higher willingness to pay in the absence of other local sources. Europe, in contrast, can continue to rely on a number of pipeline import options with LNG playing the role of marginal supplier with important diversification impacts.



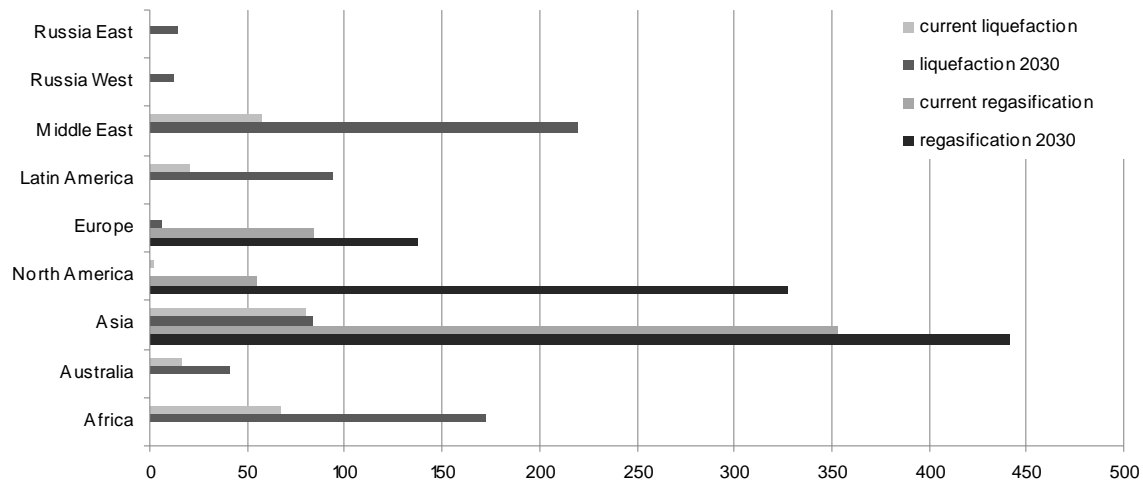
**Figure 6: Breakdown of European consumption for all model years, in bcm/y.**

<sup>11</sup> The Base Case does not include unconventional resources that have recently been added to the North American reserves. We explore the impact of the large increase in North American production capacities that may result from shale gas production in a scenario in Huppmann et al. (2009). and Gabriel, S.A., R. Egging, H. Avetisyan, “An Analysis of the North American Natural Gas Market Using the World Gas Model.” (working title, forthcoming).

“

Asia consumes almost 850 bcm/y. in 2030 with Japan and Taiwan continuing to rely heavily on LNG imports that come to a large extent from the Middle East. China and India each produce half of their consumption domestically and import another 40% by pipeline from Russia, Myanmar, and the Caspian region.

Liquefaction and regasification capacities over time are shown in Figure 7. While liquefaction capacities increase from 242 bcm in 2005 to 652 bcm/y. in 2030, regasification capacities expand even further from 491 to 945 bcm/y. Thus, we continue to observe proportionally higher regasification capacity than liquefaction capacity reflecting the flexible spot LNG trade that we assume at least for later model runs. There are certain spare capacities in order to meet seasonal demand or to benefit from the option of importing additional volumes of liquefied natural gas. Investment in LNG infrastructure is strongest at the beginning of the time horizon (where it is to some extent driven by the inclusion of projects currently under construction) and again in 2020. After 2020, investments slow down due to the assumption of demand stagnation in many developed markets.



**Figure 7: Liquefaction and regasification capacities; in bcm/y.**

The pipeline capacity development is reported in Table 2 for those regions where pipeline trade plays an important role. One can see that Russia as well as the Caspian and the Middle Eastern regions are considerably expanding their pipeline capacities to Asia, in

particular after 2015, in many places with construction of new, greenfield pipeline projects. These new pipeline capacities can accommodate the large exports of natural gas to satisfy the strong Asian demand for natural gas. Europe continues to be an important pipeline market with decreasing domestic production and a stable demand for natural gas. In line with the minor role for LNG on the European market, some substantial pipeline capacity expansions are coming forward: above all from North Africa, but also from the Caspian region and the Middle East.

**Table 2: Pipeline capacities over time between selected world regions, in bcm/y.**

Outgoing	Year	Incoming				
		Europe	Ukraine, Belarus	Caspian	Middle East	Asia-Pacific
Africa	2005	49				
	2015	92				
	2030	130				
Ukraine, Belarus	2005	208	29			
	2015	212	29			
	2030	213	29			
RUSSIA	2005	40	207	13		0
	2015	87	230	13		0
	2030	183	233	13		91
Caspian	2005	7		118	8	0
	2015	25		215	45	33
	2030	52		247	45	117
Middle East	2005	10		0	2	0
	2015	26		2	6	0
	2030	54		2	6	38
Asia-Pacific	2005					20
	2015					44
	2030					148

## 5 Conclusions

We have presented an extensive model of the global natural gas markets, the flows and the infrastructure, called the World Gas Model. This multi-period model allows to take

into account endogenous investment decisions over the next decades while at the same time including market power in the pipeline and the LNG market.

Our Base Case results confirm the results by larger energy system models and exhibit an increase in global natural gas trade in the next decades. However, the strong rise, in particular of LNG trade will not be sustained after 2020. The largest increase in natural gas consumption and imports will come from Asia where, consequently, the biggest expansion of infrastructure capacity takes place.

The World Gas Model can be used for a variety of analyses of trends in the international natural gas and energy markets. In Huppmann et al. (2009) we presented several development scenarios until 2030, including such intriguing questions as the unconventional resource base in the U.S. which may trigger considerably less LNG demand and the advent of an alternative “clean technology” that would gradually replace natural gas.

In Egging et al. (2009) we discussed the possibility and effects of a cartelization of the natural gas markets within the Gas Exporting Countries Forum. A simplifying representation of the cartel was achieved by modifying the model structure in order to incorporate a single trader of pipeline gas and a single LNG supplier for the cartel countries.

Another extension of the model is the inclusion of stochastic aspects, that is to allow for several scenarios to realize with a certain probability. In such a model, the optimal reaction by the players is different to deterministic simulations because they have to prepare for all possible events. For example, a pipeline from Iran to Europe may be necessary in the future or not, depending on whether Iran and the Gas Exporting Countries Forum will be able to implement an effective cartel withholding strategy. If the probability of such a cartel is less than 100% it may still be optimal to built a pipeline, maybe with a smaller capacity, in case the Iranian gas will not be withheld. Some first stochastic WGM results are presented in Egging and Holz (2009) which complement the work in Gabriel et al. (2009) for scenario reduction methods applied to small natural gas networks.



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## Appendix: Karush-Kuhn-Tucker Conditions

The KKT conditions of the producer problem are described above in Section 2.2.1.1. In the following we detail the KKT conditions derived from the optimization problems presented in Section 2.2. The combination of all the KKT conditions of all players and the market-clearing conditions form the World Gas Model as it is programmed in GAMS.

### A.1 The KKT conditions for the trader's problem

$$0 \leq \text{days}_d \left[ \gamma_m \left( \begin{array}{c} \delta_{t,n}^C SLP_{ndm}^M SALES_{ndm}^{T \rightarrow M} \\ - \left( \delta_{in}^C \Pi_{ndm}^{W(T)} + (1 - \delta_{in}^C) \pi_{ndm}^W \right) \end{array} \right) \right] + \phi_{ndm}^T \quad (\text{A.1})$$

$$\perp SALES_{ndm}^{T \rightarrow M} \geq 0, \quad \forall t, n, d, m$$

$$0 \leq -\text{days}_d \left[ \gamma_m \pi_{n1m}^T \right] + \phi_{n1m}^T \quad \perp SALES_{nm}^{T \rightarrow S} \geq 0, \quad \forall t, n, m \quad (\text{A.2})$$

$$0 \leq \text{days}_d \left[ \gamma_m \pi_{ndm}^P \right] - \phi_{ndm}^T \quad \perp PURCH_{ndm}^{T \leftarrow P} \geq 0, \quad \forall n \in N(p(t)), d, m \quad (\text{A.3})$$

$$0 \leq \text{days}_d \left[ \gamma_{y(m)} \left( \tau_{nn, dm}^A + \tau_{nn, dm}^{Reg} \right) \right] + \phi_{ndm}^T - (1 - \text{loss}_{n,n}) \phi_{ni, dm}^T \quad \perp FLOW_{nn, dm}^T \geq 0, \quad \forall n, n_i, d, m \quad (\text{A.4})$$

$$0 \leq \left[ \begin{array}{c} PURCH_{ndm}^{T \leftarrow P} \\ + \sum_{n_i \in N} (1 - \text{loss}_{n_i, n}) FLOW_{ni, ndm}^T \end{array} \right] - \left[ \begin{array}{c} \delta_d^{low} SALES_{nm}^{T \rightarrow S} + SALES_{ndm}^{T \rightarrow M} \\ + \sum_{n_i \in N} FLOW_{nn, dm}^T \end{array} \right] \quad (\text{A.5})$$

$$\perp \phi_{ndm}^T \geq 0 \quad \forall n \in N(t), d, m$$

The equations for enforcing market-clearing at a wholesale price  $\pi_{ndm}^W$  are:

$$0 \leq \pi_{ndm}^W - INT_{ndm}^M + SLP_{ndm}^M \cdot \left( \begin{array}{c} \sum_{t \in T(n)} SALES_{ndm}^{T \rightarrow M} \\ + \sum_{r \in R(n)} SALES_{rdm}^{R \rightarrow M} \\ + (1 - \delta_d^{low}) \sum_{s \in S(n)} SALES_{sdm}^{S \rightarrow M} \end{array} \right) \perp \pi_{ndm}^W \geq 0 \quad \forall n, d, m \quad (\text{A.6})$$

## A.2 KKT conditions for the liquefier optimization problem

$$0 \leq \text{days}_d \left[ \gamma_m \left( -\pi_{n(l)dm}^L + \frac{\partial c_{lm}^L(\text{SALES}_{ldm}^L)}{\partial \text{SALES}_{ldm}^L} \right) \right] + \alpha_{ldm}^L + \phi_{ldm}^L \quad (\text{A.7})$$

$$\perp \text{SALES}_{ldm}^L \geq 0 \quad \forall d, m$$

$$0 \leq \gamma_m b_{ly(m)}^L - \sum_{d \in D} \sum_{m' > m} \alpha_{ldm'}^L + \rho_{lm}^L \quad \perp \quad \Delta_{lm}^L \geq 0 \quad \forall m \quad (\text{A.8})$$

$$0 \leq \overline{\text{LQF}}_l^L + \sum_{m' < m} \Delta_{lm'}^L - \text{SALES}_{ldm}^L \quad \perp \quad \alpha_{ldm}^L \geq 0 \quad \forall d, m \quad (\text{A.9})$$

$$0 \leq (1 - \text{loss}_l) \text{PURCH}_{ldm}^{L \leftarrow P} - \text{SALES}_{ldm}^L \quad \perp \quad \phi_{ldm}^L \geq 0 \quad \forall d, m \quad (\text{A.10})$$

$$0 \leq \bar{\Delta}_{lm}^L - \Delta_{lm}^L \quad \perp \quad \rho_{lm}^L \geq 0 \quad \forall m \quad (\text{A.11})$$

## A.3 KKT conditions for the regasifier problem

$$0 \leq \text{days}_d \left[ \gamma_m \left( \begin{array}{c} \delta_r^C \text{SLP}_{ndm}^M \text{SALES}_{rdm}^{R \rightarrow M} \\ - \left( \delta_r^C \Pi_{ndm}^{W(T)} + (1 - \delta_r^C) \pi_{n(r)dm}^W \right) \\ + \frac{\partial c_{rm}^R(\text{SALES}_{rdm}^{R \rightarrow M} + \text{SALES}_{rdm}^{R \rightarrow S})}{\partial \text{SALES}_{rdm}^{R \rightarrow M}} \end{array} \right) \right] + \alpha_{rdm}^R + \phi_{rdm}^R \quad (\text{A.12})$$

$$\perp \text{SALES}_{rdm}^{R \rightarrow M} \geq 0, \quad \forall d, m$$

$$0 \leq \text{days}_d \left[ \gamma_m \left( -\pi_{n(r)dm}^R + \frac{\partial c_{rm}^R(\text{SALES}_{rdm}^{R \rightarrow M} + \text{SALES}_{rdm}^{R \rightarrow S})}{\partial \text{SALES}_{rdm}^{R \rightarrow S}} \right) \right] + \alpha_{rdm}^R + \phi_{rdm}^R \quad (\text{A.13})$$

$$\perp \text{SALES}_{rdm}^{R \rightarrow S} \geq 0, \quad d=1, \forall m$$

$$0 \leq \text{days}_d \left[ \gamma_m \left( \pi_{n_s(b)dm}^L + u_b^{R \leftarrow L} \right) \right] - (1 - \text{loss}_r)(1 - \text{loss}_b) \phi_{rdm}^R - \epsilon_{bdm}^R \quad (\text{A.14})$$

$$\perp \text{PURCH}_{bdm}^{R \leftarrow L} \geq 0, \quad \forall d, m$$

$$0 \leq \gamma_m b_{ry(m)}^R - \sum_{d \in D} \sum_{m' > m} \alpha_{rdm'}^R + \rho_{rm}^R \quad \perp \quad \Delta_{rm}^R \geq 0, \quad \forall m \quad (\text{A.15})$$

$$0 \leq \bar{\Delta}_{rm}^R - \Delta_{rm}^R \quad \perp \quad \rho_{rm}^R \geq 0 \quad \forall m \quad (\text{A.16})$$

$$0 \leq \overline{\text{REG}}_r^R + \sum_{m' < m} \Delta_{rdm'}^R - \text{SALES}_{rdm}^{R \rightarrow M} - \text{SALES}_{rdm}^{R \rightarrow S} \quad \perp \quad \alpha_{rdm}^R \geq 0 \quad \forall d, m \quad (\text{A.17})$$

$$0 \leq \sum_{b:n_e(b)=n(r)} (1-loss_r)(1-loss_b)PURCH_{bdm}^{R \leftarrow L} - (SALES_{rdm}^{R \rightarrow M} + SALES_{rdm}^{R \rightarrow S}) \perp \phi_{rdm}^R \geq 0 \quad \forall d, m \quad (\text{A.18})$$

$$0 \leq PURCH_{bdm}^{R \leftarrow L} - Contract_{bdy(m)}^{R, DS} \perp \epsilon_{bdm}^R \geq 0, \quad \forall b : n_e(b) = n(r), d, m \quad (\text{A.19})$$

#### A.4 KKT conditions for the storage operator problem

$$0 \leq -days_d \left[ \gamma_m \pi_{n(s)dm}^W \right] + \beta_{sdm}^S + days_d \phi_{sm}^S + days_d \theta_{sm}^S \perp SALES_{sdm}^{S \rightarrow M} \geq 0, \quad d = 2, 3, \forall m \quad (\text{A.20})$$

$$0 \leq days_d \left[ \gamma_m \left( \pi_{n(s)dm}^T + \frac{\partial c_{sm}^S (PURCH_{sm}^{S \leftarrow T} + PURCH_{sm}^{S \leftarrow R})}{\partial PURCH_{sm}^{S \leftarrow T}} \right) \right] + \alpha_{sm}^S - days_d (1-loss_s) \phi_{sm}^S \perp PURCH_{sm}^{S \leftarrow T} \geq 0, \quad d = 1, \forall m \quad (\text{A.21})$$

$$0 \leq days_d \left[ \gamma_{y(m)} \left( \pi_{n(s)dm}^R + \frac{\partial c_{sm}^S (PURCH_{sm}^{S \leftarrow T} + PURCH_{sm}^{S \leftarrow R})}{\partial PURCH_{sm}^{S \leftarrow R}} \right) \right] + \alpha_{sm}^S - days_d (1-loss_s) \phi_{sm}^S \perp PURCH_{sm}^{S \leftarrow R} \geq 0, \quad d = 1, \forall m \quad (\text{A.22})$$

$$0 \leq \gamma_m b_{sm}^{S, INJ} - \sum_{m' > m} \alpha_{sm'}^S + \rho_{sm}^{S, INJ} \perp \Delta_{sm}^{S, INJ} \geq 0, \quad \forall m \quad (\text{A.23})$$

$$0 \leq \gamma_m b_{sm}^{S, EXT} - \sum_{d=2,3} \sum_{m' > m} \beta_{sdm'}^S + \rho_{sm}^{S, EXT} \perp \Delta_{sm}^{S, EXT} \geq 0, \quad \forall m \quad (\text{A.24})$$

$$0 \leq \gamma_m b_{sm}^{S, WG} - \sum_{m' > m} \theta_{sm'}^S + \rho_{sm}^{S, WG} \perp \Delta_{sm}^{S, WG} \geq 0, \quad \forall m \quad (\text{A.25})$$

$$0 \leq \left[ days_1 (1-loss_s) \left( PURCH_{sm}^{S \leftarrow T} + PURCH_{sm}^{S \leftarrow R} \right) \right] - \sum_{d=2,3} days_d SALES_{sdm}^{S \rightarrow M} \perp \phi_{sm}^S \geq 0 \quad \forall m \quad (\text{A.26})$$

$$0 \leq \overline{INJ}_s^S + \sum_{m' < m} \Delta_{sm'}^{S, INJ} - PURCH_{sm}^{S \leftarrow T} - PURCH_{sm}^{S \leftarrow R} \perp \alpha_{sm}^S \geq 0 \quad \forall m \quad (\text{A.27})$$

$$0 \leq \overline{EXT}_s^S + \sum_{m' < m} \Delta_{sm'}^{S, EXT} - SALES_{sdm}^{S \rightarrow M} \perp \beta_{sdm}^S \geq 0 \quad \forall d = 2, 3, m \quad (\text{A.28})$$

$$0 \leq \overline{WRKG}_s^S + \sum_{m' < m} \Delta_{sm'}^{S, WG} - \sum_{d=2,3} days_d SALES_{sdm}^{S \rightarrow M} \perp \theta_{sm}^S \geq 0 \quad \forall m \quad (\text{A.29})$$

$$0 \leq \bar{\Delta}_{sy(m)}^{S,INJ} - \Delta_{sm}^{S,INJ} \perp \rho_{sm}^{S,INJ} \geq 0 \quad \forall m \quad (\text{A.30})$$

$$0 \leq \bar{\Delta}_{sy(m)}^{S,EXT} - \Delta_{sm}^{S,EXT} \perp \rho_{sm}^{S,EXT} \geq 0 \quad \forall m \quad (\text{A.31})$$

$$0 \leq \bar{\Delta}_{sy(m)}^{S,WG} - \Delta_{sm}^{S,WG} \perp \rho_{sm}^{S,WG} \geq 0 \quad \forall m \quad (\text{A.32})$$

### **A.5 KKT conditions for the pipeline operator problem**

$$0 \leq -days_d \gamma_m \tau_{nn,dm}^A + \alpha_{nn,dm}^A \perp SALES_{nn,dm}^A \geq 0 \quad \forall d, m \quad (\text{A.33})$$

$$0 \leq \bar{PL}_{nn_i}^A + \sum_{m' < m} \Delta_{nn_i, m'}^O - SALES_{nn_i, dm}^A \perp \alpha_{nn_i, dm}^A \geq 0 \quad \forall d, m \quad (\text{A.34})$$

### **A.6 KKT conditions for the transmission system operator**

$$0 \leq b_{nn_i, m}^O - \gamma_m \left( \sum_{d \in D} days_d \sum_{m' > m} \tau_{nn_i, dm'}^A \right) + \rho_{nn_i, m}^O \perp \Delta_{nn_i, m}^O \geq 0 \quad \forall n, n_i, m \quad (\text{A.35})$$

$$0 \leq \bar{\Delta}_{nn_i, m}^O - \Delta_{nn_i, m}^O \perp \rho_{nn_i, m}^O \geq 0 \quad \forall n, n_i, m \quad (\text{A.36})$$