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Technology Adoption in Critical Mass Games:
Theory and Experimental Evidence

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Technology Adoption in Critical Mass Games:  
Theory and Experimental Evidence*

Claudia Keser†  Irina Suleymanova‡  Christian Wey§

December 2009

Abstract

We analyze the choices between two technologies A and B that both exhibit network effects. We introduce a critical mass game in which coordination on either one of the standards constitutes a Nash equilibrium outcome while coordination on standard B is assumed to be payoff-dominant. We present a heuristic definition of a critical mass and show that the critical mass is inversely related to the mixed strategy equilibrium. We show that the critical mass is closely related to the risk dominance criterion, the global game theory, and the maximin criterion. We present experimental evidence that both the relative degree of payoff dominance and risk dominance explain players’ choices. We finally show that users’ adoption behavior induces firms to select a relatively unrisky technology which minimizes the problem of coordination failure to the benefit of consumers.

JEL-Classification: C72, C91, D91, D84

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1 Introduction

In many parts of modern economies (e.g., in information and communications) users’ payoffs associated with a particular product (or service) depend positively on the total number of other users choosing the same product or service; a phenomenon, commonly termed as a positive network effect (see Shapiro and Varian, 1998, and Farrell and Klemperer, 2007, for surveys). Though products may be differentiated, its importance for buyers’ purchasing decisions is often negligible when compared with their preference for compatible products. A characteristic feature of markets with network effects is that users (which can be consumers or firms) typically face several incompatible technologies (so-called “standards”), while they can adopt only one of the available technologies.

It is well-known that the choice between incompatible standards that exhibit positive network effects typically leads to multiple equilibria (see, Farrell and Saloner, 1985, and Katz and Shapiro, 1985, for seminal contributions). Whether or not these equilibria will emerge, and if yes which equilibrium, depends on how well consumers cope with the coordination problem created by positive network effects. Successful coordination requires coordination on the Pareto-dominant equilibrium (if equilibria are Pareto-rankable) while coordination failure arises either if users coordinate on a Pareto-dominated Nash equilibrium or fail to coordinate altogether.

The general class of games which captures the coordination problem with Pareto-ranked equilibria is referred to as coordination games. The seminal work by Harsanyi and Selten (1988) proposes a theory of equilibrium selection based on two criteria; namely, payoff dominance and risk dominance. While the first criterion selects the equilibrium based on overall collective rationality, the second criterion is based on individual rationality and takes into account out-of-equilibrium payoffs. The results of many experiments in which the subjects played coordination games suggest that both criteria are important for predicting players’ decisions. For instance, Van Huyck, Battalio, and Beil (1990) report for their “minimum game” that “[...] coordination failure results from strategic uncertainty: some subjects conclude that it is too ‘risky’ to choose the payoff-dominant action.”

Moreover, Van Huyck, Battalio, and Beil (1990) express the idea that strategic uncertainty becomes more pronounced the larger the number of subjects involved.

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1 In the minimum game a subject’s payoff depends negatively on its own “effort” and positively on the minimum “effort” chosen by the other subjects.
becomes: “[...] when the number of players is large it only takes a remote possibility that an individual player will not select the payoff-dominant action [...] to motivate defection from the payoff-dominant equilibrium.”

In this paper we specify a critical mass game as a one-shot coordination game, where $N \geq 2$ (homogeneous) users decide simultaneously to adopt either standard $A$ or standard $B$. The utility of each user depends on the stand-alone value and the (linear and positive) network effect of the standard. In this setting, both the choice of standard $A$ and the choice of standard $B$ are affected by consideration of strategic uncertainty.

Moreover, if the environment is not perfectly symmetric, then standards are typically differentiated regarding their degree of “riskiness.” Intuitively, if a standard needs a relatively large market share when compared to the other standard so as to become a strictly profitable choice for a single user, then we should expect that choosing this standard involves a relatively high degree of “strategic uncertainty.” This reasoning carries us to a heuristic definition of the critical mass of a particular standard which we define as the minimum share of users necessary so as to make the adoption of this standard a best reply for any remaining user. Intuitively, a standard with a larger critical mass should be less likely to gain dominance in the market than a standard with a smaller critical mass.

In this paper we relate the critical mass concept to the criterion of risk dominance as proposed by Harsanyi and Selten (1988). The critical mass game has two pure strategy Nash equilibria ($A$-equilibrium and $B$-equilibrium). We assume that the $B$-equilibrium is payoff-dominant. Our game also has a unique symmetric equilibrium in mixed strategies. We show that the equilibrium in mixed strategies is inversely related to our critical mass concept. We also show that there is an unambiguous concordance between the critical mass of a standard and the risk dominance of a particular equilibrium. Similar results are obtained for the maximin criterion and the global game theory. Precisely, the equilibrium in which users adopt a standard with the smaller critical mass is also selected by the risk dominance criterion, the maximin criterion, and the global game theory.

In contrast to the mentioned selection theories, our heuristic of a critical mass allows us to analyze the influence of the riskiness of a particular standard (or, equivalently, of strategic uncertainty) on users’ behavior by varying the critical mass of a standard which we derive from
the primitives of users' utilities. By varying the difference in the maximum payoffs delivered by
the two standards, we are also able to identify the impact of the payoff dominance criterion on
users' adoption decisions.

We confront our theoretical analysis of the critical mass game with the experimental results
of a paper-and-pencil experiment performed at the University of Göttingen. In the experiment
we specify 16 different decision situations, which can be grouped into four different blocks.
Within each block we increase the relative riskiness of the payoff-dominant standard $B$ (by
increasing its critical mass), while the degree of standard $B$'s payoff dominance (measured by
the difference in the maximum payoffs of the two standards) is kept constant. By doing so we
can analyze how the riskiness of the payoff-dominant equilibrium influences subjects' decisions.
By re-grouping the 16 decision situations, we can also analyze how the relative degree of payoff
dominance affects players' choices, while keeping the relative riskiness of the standards constant
in each block.

The analysis of the experimental data reveals that both risk dominance and payoff dominance
considerations significantly affect players' adoption decisions. Precisely, we proxy the relative
degree of standard $B$'s payoff dominance by the difference of both standards' maximum payoffs
(relative to standard $B$'s maximum payoff). Similarly, we proxy the relative degree of standard
$B$'s riskiness by the difference of the standards' critical masses (relative to standard $B$'s critical
mass). Regression results (OLS and Logit) then show that both explanatory variables are signif-
icant drivers of users' adoption decisions such that the number of $B$ choices (or the probability
of a $B$-choice) increases, whenever the relative payoff dominance of standard $B$ increases while
the number of $B$-choices decreases if the relative riskiness of standard $B$ increases. In another
specification we use the difference in both standards' minimum values (again, relative to the
minimum value of standard $B$) as a proxy of standard $B$'s relative riskiness (an approach sug-
gested by the maximin criterion). Here, we obtain even more significant results which suggest
that users’ may very well refer to a simpler rule (than suggested by critical mass considerations)
to proxy the relative riskiness of a standard.

Taking our experimental results seriously, we are left with the observation that users’ choices,
and hence, a standard’s expected market share should be determined by its relative payoff
dominance and its relative riskiness. We postulate a simplified specification of users' aggregate
demands which incorporates both features. We further abstract from pricing problems and assume that firms maximize their market shares. Given the so specified expected demands, we analyze firms’ technology choices in a two-stage game. We suppose that firms choose between standard $A$ and standard $B$ in the first stage of the game, while buyers’ demand is realized in the second stage of the game. Our analysis shows, if demand is biased towards the risk-dominant standard, then both firms choose to supply the risk-dominant standard which ultimately benefits buyers in expected terms. Hence, users are on average better off if firms choose an inferior standard (i.e., a standard with a lower maximum payoff) in a world where a miscoordination is pervasive.

Our paper’s main intention is to contribute to the extensive industrial organization literature which has been analyzing network effects. The fundamental problem of choice in that literature is that between two competing standards exhibiting network effects (see Farrell and Saloner, 1985, and, for a recent survey, Farrell and Klemperer, 2007) which means that users essentially face a critical mass game. Interestingly, that literature has been (to our best knowledge) largely salient about the role of selection criteria as risk dominance and the maximin rule for predicting users’ choices and market outcomes.² Typically, that literature took the multiplicity of Nash equilibria for granted or simply assumed coordination on a Pareto-dominant standard, or even applied the mixed strategy equilibrium when highlighting coordination failure (as in Farrell and Saloner, 1988). Our analysis of firms’ technology choices is, therefore, the first analysis of the implications of user behavior in a critical mass game on firms’ technology choices when users have to solve a trade-off between payoff dominance and risk dominance.

From a global game theory perspective the analysis of the technology adoption problem under network effects is a natural application (see Myatt, Shin, and Wallace, 2002). Recently, the more traditional industrial organization literature on network effects and the global game theory was brought together in Argenziano (2008). She uses the theory of global games to find a unique equilibrium in a model where a continuum of consumers choose between the products of two firms both exhibiting network effects. In contrast to our model, consumers are assumed

²Incidentally, Liebowitz and Margolis (1996) also point out the importance of the critical mass (which they label differently) in their illustrative analysis of consumers’ choices between different standards. Besides several differences, our analysis gives theoretical support to their approach based on the risk dominance criterion.
to be heterogenous in her analysis leading to equilibria which are (from a social welfare point of view) “too balanced.”


The experimental part of our paper is closely related to Heinemann, Nagel, and Ockenfels (2009) who run a series of experiments to explore aspects of strategic uncertainty in one-shot coordination games with multiple Nash equilibria. They consider a multi-player coordination game of two choices, where one choice yields a “secure” payoff while the payoff of the “risky” choice depends positively on the other players’ choices. In contrast to their set-up, our critical mass games assumes that both choices depend positively on the adoption decisions of the other players (we provide a more precise comparison below).

Our work contributes to those experimental studies which elicit how players resolve the trade-off between payoff dominance and risk dominance (Van Huyck, Battalio, and Beil, 1991 and Straub, 1995). Similar to Schmidt et al. (2003) who examine a two-player coordination game, we also find that risk dominance has a significant influence on players’ ability to coordinate. However, in contrast to their study, we find that the degree of payoff dominance also impacts significantly on players’ choices.

We proceed as follows. In Section 2 we define the critical mass game and present the critical mass heuristic. In Section 3 we examine risk dominance, the maximin criterion, and the theory of global games within the critical mass game. Section 4 presents the design of the experiment while in Section 5 we report the results. In Section 6 we analyze firms’ technology choices when both risk dominance and payoff dominance drive users’ adoption decisions. Finally, Section 7 concludes.

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3 Related is also Carlsson and van Damme (1993a) who examine the stag hunt games which is a special case of our critical mass game.

4 There are also many other works which analyze the influence of several features on the likelihood of coordination. See, for instance, Van Huyck, Gillette, and Battalio (1990) who examine the role of an arbiter in two-person coordination games and Keser, Ehrhart, and Berninghaus (1998) who analyze the influence of local interaction on equilibrium outcomes in three-player coordination games. More recently, Crawford, Gneezy, and Rottenstreich (2008) examined the role of payoff asymmetries as a source of coordination failure.
2 The Critical Mass Game

Suppose $N \geq 2$ identical users (which can be consumers and/or firms) make simultaneously their choices between two standard technologies (or, in short, standards) $A$ and $B$. The payoff a user derives from standard $i = A, B$ depends on the total number of users choosing the same standard, $N_i \leq N$, and is assumed to be given by the linear function

$$U_i(N_i) = v_i + \gamma_i(N_i - 1).$$

(1)

We assume that users always find it optimal to adopt one of the standards, so that the market is always covered; i.e., $N_A + N_B = N$ holds. The parameter $v_i \geq 0$ can be interpreted as the “stand-alone value” a user derives from the standard technology when he is the only user of this standard. The term $\gamma_i(N_i - 1)$ measures positive network effects if $N_i \geq 1$ users choose the same standard $i = A, B$.

The game is parameterized such that it has two Nash equilibria in pure strategies in which either all players choose standard $A$ ($A$-equilibrium) or all players choose standard $B$ ($B$-equilibrium). In other words, users face a coordination problem in the game. This is ensured by the assumption $v_i < v_j + \gamma_j(N - 1)$ for any $i, j = A, B$ and $i \neq j$. Furthermore, we assume that the $B$-equilibrium is payoff-dominant; i.e., the utility of every user in the $B$-equilibrium is higher than in the $A$-equilibrium. Formally, $v_B + \gamma_B(N - 1) > v_A + \gamma_A(N - 1)$ holds.

To analyze the coordination problem we introduce a heuristic definition of the critical mass of a standard. We define the critical mass $m_i$ of a standard $i = A, B$ as the minimum share of users choosing standard $i$ necessary to make the choice of this standard a best reply for any remaining player. The following lemma provides the formal derivation of the critical mass and states its properties.

Lemma 1. The value of the critical mass $m_i$ is given by

$$m_i = \frac{v_j - v_i + \gamma_j(N - 1)}{(\gamma_i + \gamma_j)(N - 1)},$$

(2)

with $i, j = A, B$ and $i \neq j$. It holds that $m_A = 1 - m_B$. Moreover, $\partial m_i / \partial v_i < 0$, $\partial m_i / \partial \gamma_i < 0$, $\partial m_i / \partial v_j > 0$, and $\partial m_i / \partial \gamma_j > 0$.

\footnote{We assume that users do not create network effects for themselves.}
Proof. First note that all users are homogeneous. Consider the decision problem of a single user. Assume that \( \tilde{N} \) other users choose standard \( i \). If choosing standard \( i \) constitutes a best response for a user under the assumption that all the other, \( N - \tilde{N} - 1 \), users choose standard \( j \neq i \), then it also constitutes a best response in all the other cases (when less than \( N - \tilde{N} - 1 \) users choose standard \( j \)). Hence, it must hold that \( U_i(\tilde{N} + 1) > U_j(N - \tilde{N}) \) or

\[
v_i + \gamma_i \tilde{N} \geq v_j + \gamma_j(N - \tilde{N} - 1) .
\]  

(3)

The minimum value of \( \tilde{N} \), which satisfies Condition (3) is given by\(^6\)

\[
\tilde{N} = \frac{v_j - v_i + \gamma_j(N - 1)}{\gamma_A + \gamma_B}.
\]

Under the parameter restriction \( v_i < v_j + \gamma_j(N - 1) \) it holds that \( 0 < \tilde{N} < N - 1 \). Thus, \( \tilde{m}_i \) is given by

\[
m_i(v_i, \gamma_i, v_j, \gamma_j, N) = \frac{v_j - v_i + \gamma_j(N - 1)}{(\gamma_A + \gamma_B)(N - 1)},
\]

for \( i, j = A, B \) and \( i \neq j \). Adding up the critical masses of standards \( A \) and \( B \), we get \( m_A + m_B = 1 \). The signs of the derivatives \( \partial m_i / \partial v_i < 0 \), \( \partial m_i / \partial \gamma_i < 0 \) and \( \partial m_i / \partial v_j > 0 \) are straightforward, while

\[
\partial m_i / \partial \gamma_j = -\frac{v_j - [v_i + \gamma_i(N - 1)]}{(\gamma_A + \gamma_B)^2(N - 1)} > 0, \ i \neq j
\]

(4)

follows from our assumption that \( v_j < v_i + \gamma_i(N - 1) \) must hold. Q.E.D.

The value of the critical mass of a standard \( i \) decreases when the respective parameters \( v_i \) and \( \gamma_i \) of the payoff function increase, while the critical mass increases in the parameters \( v_j \) and \( \gamma_j \) of the rival standard \( j \neq i \). Those results are intuitive as with an increase of both the stand-alone value and the slope of the network effects function less adopters are needed to make the choice of this standard a best reply for the remaining users. With the increase of the parameters of the rival standard the attractiveness of the standard decreases, so that the value of the critical mass increases.

We are now in a position to define the critical mass game.

**Definition 1.** A critical mass game is a game in which \( N \geq 2 \) users simultaneously make their choices between two standard technologies, \( A \) and \( B \), such that:

---

\(^6\)If \( \tilde{N} \) is not an integer, then we take instead the next integer which fulfills (3).
(i) for each standard the payoff of an individual user from choosing this standard is given by Equation (1),

(ii) users face a coordination problem so that \( v_i < v_j + \gamma_j(N - 1) \) (or, equivalently, \( m_i \in (0, 1) \)) for \( i, j = A, B \) and \( i \neq j \) holds, and

(iii) the outcome where all users choose standard B is payoff-dominant; i.e., \( v_B + \gamma_B(N -1) > v_A + \gamma_A(N - 1) \) holds.

Assumption ii) assures that there are two Nash equilibria in pure strategies (A- and B-equilibrium), while assumption iii) implies that the B-equilibrium is payoff-dominant. The following proposition states that the critical mass game has in addition to the two Nash equilibria in pure strategies a unique equilibrium in (symmetric) mixed strategies.\(^7\)

**Proposition 1.** *The critical mass game has exactly two strict equilibria in pure strategies, the A- and the B-equilibrium, and a unique equilibrium in (symmetric) mixed strategies where each user chooses standard i, with probability \( p_i = m_i \ (i = A, B) \).*

**Proof.** We start with the pure strategy equilibria. An equilibrium in which every user chooses standard \( i \) is a strict equilibrium if \( U_i(N) > U_j(1) \) holds which is equivalent to \( v_j < v_i + \gamma_i(N -1) \). There cannot exist another equilibrium in pure strategies in which both standards are chosen. Assume to the contrary that there exists such an equilibrium with \( N_A < N \) users choosing standard \( A \) and \( N_B < N \) users choosing standard \( B \) with \( N_A + N_B = N \). Then it must hold that \( U_A(N_A) \geq U_B(N_B + 1) \) and \( U_B(N_B) \geq U_A(N_A + 1) \). From Equation (1) it follows that \( U_A(N_A + 1) > U_A(N_A) \), which together with the former inequality implies \( U_B(N_B) \geq U_A(N_A + 1) > U_A(N_A) \geq U_B(N_B + 1) \). From this it follows that \( U_B(N_B) > U_B(N_B + 1) \). Obviously, this is not consistent with (1). Hence, the condition \( -\gamma_A(N - 1) < v_A - v_B < \gamma_B(N - 1) \) assures that there are only two equilibria in pure strategies; namely, the A-equilibrium and the B-equilibrium.

We now turn to the mixed strategy equilibrium. In that equilibrium all users choose each standard with some probability such that the expected payoffs from choosing each standard are equal. Let \( p_i \) be the probability with which users choose standard \( i = A, B \) in the mixed strategy equilibrium. Users’ choices of a standard then give rise to a binomial distribution such that each

\(^7\)Kim (1996) derives similar results for a symmetric coordination game in which \( N \geq 2 \) players make binary choices.
user expects $p_i(N - 1)$ other users to choose standard $i$. In a mixed strategy equilibrium every player must be indifferent between choosing standard $A$ or standard $B$ which yields the condition

$$v_i + \gamma_i [1 + p_i(N - 1) - 1] = v_j + \gamma_j [1 + p_j(N - 1) + 1],$$

from which we obtain the equilibrium probability

$$p_i = \frac{v_j - v_i + \gamma_j(N - 1)}{(\gamma_A + \gamma_B)(N - 1)} = m_i,$$

where the latter equality follows from Lemma 1. As our parameter restrictions assure $m_i \in (0, 1)$ it also follows that $p_i \in (0, 1)$. Q.E.D.

In the proof of Proposition 1 we have shown that the equilibrium probability, $p_i$, with which each user chooses standard $i$, is equal to $m_i$, so that the comparative static results of Lemma 1 also apply to the mixed strategy equilibrium. Hence, an increase of standard $i$’s quality (in terms of $v_i$ and/or $\gamma_i$) implies that consumers reduce the probability with which to choose standard $i$ in the mixed strategy equilibrium. While it is well-known that a mixed strategy equilibrium may exhibit counter intuitive features, its inverse relationship to our critical mass heuristic adds to its curiosity. In sharp contrast to our heuristically defined critical mass, which suggests to favor the choice of the standard with the smaller critical mass, the mixed strategy equilibrium requires to favor the standard with the larger critical mass.

Proposition 1 states that the critical mass game has two Nash equilibria in pure strategies. Harsanyi and Selten (1988) propose two equilibrium selection criteria: namely, payoff dominance and risk dominance. In the critical mass game the $B$-equilibrium is by definition payoff-dominant. In the next section we show that our critical mass heuristic is closely related to the concept of risk dominance, the global game theory, and the maximin criterion.

3 Equilibrium Selection in the Critical Mass Game

Risk Dominance. To find the risk-dominant equilibrium in the critical mass game, we apply the tracing procedure as proposed by Harsanyi and Selten (1988). This procedure describes a process of converging expectations leading to a particular equilibrium which is coined as the risk-dominant equilibrium, in which every player adopts and expects the other players to adopt the standard that implies the risk-dominant equilibrium. To find this equilibrium, we first have to
determine bicentric priors for every player which represent a probability distribution over the two Nash equilibria, $A$ and $B$. To determine the bicentric prior for a particular player $l = 1, 2, ..., N$ we assume first that the player expects that either all other players choose standard $A$ or all other players choose standard $B$. The outcome in which all other players choose standard $i$ ($i = A, B$) is assumed to occur with probability $q_i$, while the opposite outcome is realized with counter probability $1 - q_i$. Second, a player plays a best response to his beliefs. And third, it is assumed that the beliefs are distributed uniformly over the unit interval. The tracing procedure consists then in finding a feasible path from the equilibrium in the starting point given by the bicentric priors to the equilibrium in the end point given by the original game. The equilibrium in the end point constitutes the risk-dominant equilibrium.

The next proposition states which equilibrium should be chosen in the critical mass game when we apply the risk dominance criterion.\footnote{Carlsson and van Damme (1993a) derive implicitly the condition of risk dominance for the stag hunt games. In that game $N \geq 2$ identical players make binary choices between two options, one if which delivers a secure payoff while the other delivers a risky payoff that is increasing in the share of the players opting for the risky choice. Kim (1996) derives the explicit condition of risk dominance for $N$-person coordination games.}

**Proposition 2.** In the critical mass game the equilibrium in which all players adopt standard $i$ is risk-dominant if and only if standard $i$ has a lower critical mass than the rival standard $j$, with $i, j = A, B$ and $i \neq j$. If $m_A = m_B$, then there exists no risk-dominant equilibrium.

**Proof.** We search for the risk-dominant equilibrium by applying the tracing procedure as proposed by Harsanyi and Selten (1988). We start with users’ bicentric priors. Let a user $l = 1, 2, ..., N$ attribute probability $q_l$ to the situation that all the others choose standard $B$ and, correspondingly, probability $1 - q_l$ to the situation that all the others choose standard $A$. Using (5), we can re-write user $l$’s indifference condition between choosing $A$ and $B$ to obtain

$$q_l = \frac{v_A - v_B + \gamma_A(N - 1)}{(\gamma_A + \gamma_B)(N - 1)} =: \bar{q}. \quad (6)$$

Note that the critical value $\bar{q}$ is the same for all users and that $\bar{q} \in (0, 1)$ holds. From Condition (6) we observe that user $l$’s best reply to his beliefs is as follows: play $A$ if $q_l < \bar{q}$ and play $B$ if $q_l > \bar{q}$. Now, recall that the bicentric belief $q_l$ is assumed to be uniformly distributed over the interval $[0, 1]$. Hence, the probability that $q_l < \bar{q}$ is given by $\bar{q}$ and the probability that $q_l > \bar{q}$ is...
given by $1 - \tilde{q}$. Hence, users choose $A$ with probability $\tilde{q}$ and $B$ with counter probability $1 - \tilde{q}$. As all users are identical in our case, the expected return of a user from choosing standard $A$ given that all the other players choose $A$ with probability $\tilde{q}$ is

$$v_A + \gamma_A(N - 1)\tilde{q}. \quad (7)$$

Similarly, the expected payoff from choosing standard $B$ is given by

$$v_B + \gamma_B(N - 1)(1 - \tilde{q}). \quad (8)$$

Combining (7) and (8) we obtain that a user chooses $B$ if and only if

$$v_B + \gamma_B(N - 1)(1 - \tilde{q}) > v_A + \gamma_A(N - 1)\tilde{q}$$

holds, which is equivalent to

$$2(v_A - v_B) + (N - 1)(\gamma_A - \gamma_B) < 0. \quad (9)$$

Comparing Condition (9) with the definition of the critical mass (2), it is obvious that Condition (9) holds if and only if $m_B < 1/2$ holds. From Condition (9) it is immediate that a user chooses $A$ if and only if

$$2(v_A - v_B) + (N - 1)(\gamma_A - \gamma_B) > 0. \quad (10)$$

If $m_B = 1/2$, then a user is indifferent between selecting $A$ or $B$ from a risk dominance perspective. By Lemma 4.17.7 of Harsanyi and Selten (1988, p. 183) the equilibrium of the game based on the bicentric priors is the outcome selected by the tracing procedure if the following conditions hold. First, the equilibrium must be a strong equilibrium point when each player behaves according to his prior beliefs, which is guaranteed for the $B$-equilibrium by Condition (9) and for the $A$-equilibrium by Condition (10). Second, the equilibrium must also be an equilibrium of the original game, which holds by Proposition 1. Hence, we obtain the result that a standard $i = A, B$ is risk-dominant if $m_i < m_j$, for $i, j = A, B$ and $i \neq j$. Q.E.D.

According to Proposition 2 the critical mass can be used as a measure to determine whether or not a standard is risk-dominant in the critical mass game. Precisely, the standard with the lower critical mass is risk-dominant. This result is intuitive as a larger critical mass of a

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9In a strong equilibrium each player has a strong best reply to the equilibrium strategies of the other players.
standard implies that relatively more users are needed to make the adoption of the standard surely profitable. As more users are needed to make the choice profitable, the choice of a standard with a larger critical mass involves a higher degree of strategic uncertainty. The risk dominance criterion then requires to select the payoff-inferior equilibrium in the critical mass game.

Re-writing Condition (9), we obtain that the equilibrium \( i = A, B \) is risk-dominant if

\[
U_i(N) - U_j(1) > U_j(N) - U_i(1)
\]

holds. Hence, the risk dominance criterion selects the equilibrium in which the loss from deviating from the equilibrium strategy \( i \) when all the others play \( i \) is larger than the loss from deviating from equilibrium strategy \( j \) when all the others play \( j \) (\( i, j = A, B \) and \( i \neq j \)). The Condition (9) can also be re-written in the following way

\[
U_i(1) + U_i(N) > U_j(1) + U_j(N),
\]

which implies that standard \( i \) constitutes a risk-dominant equilibrium outcome if the expected payoff from choosing standard \( i \) is larger than the payoff from playing \( j \), if a player expects all the others to behave as one player who chooses with equal probabilities either \( i \) or \( j \).

From Proposition 2 it follows that if \( m_B < 1/2 \), then both selection criteria (payoff dominance and risk dominance) are aligned and pick the same equilibrium \( B \), while for \( m_B > 1/2 \), both criteria favor different outcomes (namely, equilibrium \( A \) is favored by risk dominance and equilibrium \( B \) is selected by payoff dominance).\(^\text{10}\) Following Harsanyi and Selten, we expect that users should be able to coordinate successfully on \( B \) whenever both criteria are aligned, whereas in other instances players face a trade-off. Below we analyze experimentally users’ behavior in the critical mass game to better understand how subjects resolve the trade-off between payoff dominance and risk dominance when both criteria are not aligned.

\(^\text{10}\)There is an interesting connection between risk dominance in our critical mass game and cognitive hierarchy models. In a cognitive hierarchy model a type \( k \)-player anchors its beliefs in a nonstrategic 0-type and adjusts them by thought experiments with iterated best responses where a type 1 player chooses a best response to type 0, type 2 to type 1, and so on. In our critical mass game, then half of type 0 players choose either \( A \) or \( B \), while type 1 players choose \( A \) as a best response whenever the critical mass of standard \( A \) is smaller than the critical mass of standard \( B \). Accordingly, all higher types then also choose \( A \) (see Camerer, Ho, and Chong, 2004 for a similar observation for the stag hunt game).
Maximin criterion. The maximin criterion selects the choice which delivers the maximum payoff in the worst outcome. In the critical mass game the worst outcome a user can face is given by the standard’s stand-alone value, \( v_i \) (\( i = A, B \)). In the following Corollary we state how the maximin criterion relates to the criteria of payoff dominance and risk dominance.

**Corollary 2.** If in the critical mass game the equilibrium in which all players adopt standard \( i \) is payoff-dominant and the equilibrium in which all players adopt standard \( j \) is risk-dominant, then equilibrium \( j \) is also an equilibrium which is chosen by the maximin criterion, with \( i, j = A, B \) and \( i \neq j \).

**Proof.** If equilibrium \( i \) is payoff-dominant, then

\[
v_j - v_i + (N - 1)(\gamma_j - \gamma_i) < 0. \tag{11}\]

must hold. If equilibrium \( j \) is risk-dominant, then according to Proposition 2

\[
2(v_j - v_i) + (N - 1)(\gamma_j - \gamma_i) > 0 \tag{12}
\]

holds as well. Condition (11) can only be true if one of the three following parameter constellations holds: 

i) \( v_j < v_i, \gamma_j < \gamma_i \); ii) \( v_j < v_i, \gamma_j > \gamma_i \); or iii) \( v_j > v_i, \gamma_j < \gamma_i \). Assume that \( v_j < v_i \) is true, then it follows from Equation (11) that

\[
2(v_j - v_i) + (N - 1)(\gamma_j - \gamma_i) < 0
\]

must hold, which contradicts Inequality (12). Hence, it is only possible that \( v_j > v_i \) and case iii) applies. It is left to note that the stand-alone value of standard \( i, v_i, \) is the minimum payoff a player can get by choosing this standard. Hence, the risk-dominant equilibrium is selected by the maximin criterion. Q.E.D.

Corollary 1 states that for the particular case that payoff dominance and risk dominance select different standards that the risk dominant standard is then also the standard selected by the maximin rule. In those instances, the risk-dominant standard has a higher stand-alone value than the other standard picked by the payoff dominance criterion. As the maximin rule chooses a standard with the higher stand-alone value, it follows that the maximin rule coincides with the risk dominance criterion. Before we turn to our experiment (which focuses on the case where payoff dominance and risk dominance select different standards), we show in the following section that the critical mass is also closely related to the global game theory.
Global Game Theory. We now apply the theory of global games (see Carlsson and van Damme, 1993b, and Morris and Shin, 2002) to the critical mass game to select one of the two Nash equilibria in pure strategies. The global game specification requires to introduce incomplete information into the critical mass game. Let us now assume that every player $l$ gets a private signal, $v_l$, about the stand-alone value of the standard $B$, $v_B$, which is now assumed to be a random variable with an improper uniform distribution. The private signal of a player $l$ is given by $v_l = v_B + \varepsilon_l$, where $\varepsilon_l$ is independently normally distributed with $\varepsilon_l \sim N(0, \sigma^2)$. When a player observes signal $v$, then he expects $v_B$ to be normally distributed with $v_B \sim N(v, \sigma^2)$. Furthermore, he concludes that the signals of the other players, $v_{l'} (l \neq l')$, are also normally distributed with $v_{l'} \sim N(v, 2\sigma^2)$.\footnote{If $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ are two normally and independently distributed random variables, then $Z = X + Y$ is also normally distributed with $Z \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$.} Given those specifications of the information structure, we can now state in the next proposition which equilibrium is chosen by the global game theory.

**Proposition 3.** In the critical mass game the global game theory chooses the $i$-equilibrium if standard $i$ has a lower critical mass than standard $j$, with $i, j = A, B$, $i \neq j$.

**Proof.** See Appendix.

Proposition 3 shows that the global game theory prediction coincides with the risk dominance selection criterion in the critical mass game.

We are now interested how users resolve the trade-off between payoff dominance and risk dominance in a one-shot critical mass game. We focus on those parameter constellations which guarantee that the $A$-equilibrium is risk-dominant and the $B$-equilibrium is payoff-dominant.

4 Design of the Experiment

Both concepts of payoff dominance and risk dominance predict that either all users choose the standard with the higher maximum payoff or the standard with the lower critical mass, respectively. We design an experiment where the risk dominance criterion selects the $A$-equilibrium while the payoff dominance criterion selects the $B$-equilibrium. In this setting, we expect that the exact values of the critical mass and of the relative payoff dominance matter. Precisely, we take the critical mass of standard $B$ as a measure of the relative risk dominance of standard
A. Holding everything else constant, a higher critical mass of standard $B$ should then induce more users to adopt standard $A$. Accordingly, we take the absolute difference in the standards’ maximum payoffs as a measure of the relative payoff dominance of standard $B$. Again, holding everything constant, we expect that an increase of the payoff dominance of standard $B$ should induce more users to select standard $B$.

The experiment consists of 16 decision situations. Every decision situation is based on a particular specification of the critical mass game. In every decision situation, each of the 17 players chooses between two alternatives: standard $A$ and standard $B$. The payoffs (which depend on the choices of the other players) were presented in a table to each player (see the Appendix for the tables of the 16 decision situations).

In Table 1 we present the most important parameters characterizing each decision situation; namely: the maximum possible payoff from choosing standard $i = A, B$, denoted by $U_i^\text{max}$ with $U_i^\text{max} = U_i(N)$, the difference in the maximum payoffs of the two standards given by $d^\text{max}$, with $d^\text{max} = U_B(17) - U_A(17)$, the minimum possible payoff from choosing standard $i$, denoted by $U_i^\text{min}$ with $U_i^\text{min} = U_i(1) = v_i$, the difference in the minimum payoffs of the two standards given by $d^\text{min} = U_A(1) - U_B(1)$, and the critical mass of standard $i$ multiplied with 16 (the number of the other players in a decision situation).

The decision situations consist of four different blocks. In each block we keep $U_A^\text{max}$ and $U_B^\text{max}$ constant. Hence, the difference $d^\text{max}$ which we interpret as a measure of the relative payoff dominance of standard $B$, remains constant within each block. Across blocks, we vary the payoff dominance of standard $B$. Precisely, we reduce the difference $d^\text{max}$ from 75 in the first block to 46 in the fourth block.

Within each block we have four decision situations which vary with respect to the critical mass of standard $B$. The critical mass of standard $B$ is assumed to be always larger than one-half which ensures that standard $A$ is risk-dominant. We increase the critical mass of standard $B$ (multiplied by 16) from 9 up to 12, so that within each block the degree of standard $A$’s risk dominance increases.

We hypothesize that for a given degree of payoff dominance of standard $B$, $d^\text{max}$, the number

---

12In the tables the critical mass game is stated as a discrete game where we rounded the payoffs if given by a non-integer.
Table 1: Parameters of the experiment

<table>
<thead>
<tr>
<th></th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
<th>Block 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_B^{\text{max}}$</td>
<td>325 325 325 325</td>
<td>300 300 300 300</td>
<td>280 280 280 280</td>
<td>310 310 310 310</td>
</tr>
<tr>
<td>$U_A^{\text{max}}$</td>
<td>250 250 250 250</td>
<td>245 245 245 245</td>
<td>229 229 229 229</td>
<td>264 264 264 264</td>
</tr>
<tr>
<td>$d^{\text{max}}$</td>
<td>75 75 75 75</td>
<td>55 55 55 55</td>
<td>51 51 51 51</td>
<td>46 46 46 46</td>
</tr>
<tr>
<td>$U_B^{\text{min}}$</td>
<td>5 5 5 5</td>
<td>60 60 60 60</td>
<td>133 104 64 4</td>
<td>164 134 92 30</td>
</tr>
<tr>
<td>$U_A^{\text{min}}$</td>
<td>134 178 214 243</td>
<td>156 189 216 238</td>
<td>205 205 205 205</td>
<td>232 232 232 232</td>
</tr>
<tr>
<td>$d^{\text{min}}$</td>
<td>129 173 209 238</td>
<td>96 129 156 178</td>
<td>72 101 141 201</td>
<td>68 98 140 202</td>
</tr>
<tr>
<td>$16 \cdot m_B$</td>
<td>9 10 11 12</td>
<td>9 10 11 12</td>
<td>9 10 11 12</td>
<td>9 10 11 12</td>
</tr>
<tr>
<td>$16 \cdot m_A$</td>
<td>7 6 5 4</td>
<td>7 6 5 4</td>
<td>7 6 5 4</td>
<td>7 6 5 4</td>
</tr>
</tbody>
</table>

of $B$-choices is lower the higher is the critical mass of standard $B$. Moreover, we hypothesize that for a given degree of risk dominance of standard $A$, $m_B$, the number of $B$-choices is higher the higher is the degree of standard $B$’s payoff dominance.

We ran two sessions of a paper-and-pencil experiment at the Georg-August-University of Göttingen in February, 2009. In both experimental sessions together there were 153 participants. We excluded five from the analysis, whose answers were incomplete. In the following, we analyze the decisions of the remaining 148 participants.

The experimental instructions were read aloud to guarantee that all the participants know that the conditions of the experiment are common knowledge. After the instructions were read the participants could ask questions which were answered individually.

In each session 16 participants were randomly chosen whose answers were analyzed in a pre-selected decision situation (decision situation 2). Out of these 16 participants one was randomly chosen for the final payment. In the first session the chosen participant got 83.00 Euro and in the second the payment was 114.00 Euro.
Table 2: Choices depending on the degree of payoff dominance

<table>
<thead>
<tr>
<th></th>
<th>$m_B = 9$</th>
<th>$m_B = 10$</th>
<th>$m_B = 11$</th>
<th>$m_B = 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d^{\text{max}}$</td>
<td>75 55 51 46</td>
<td>75 55 51 46</td>
<td>75 55 51 46</td>
<td>75 55 51 46</td>
</tr>
<tr>
<td>$N_B$</td>
<td>75 70 68 63*</td>
<td>65 76 68 59*</td>
<td>64 67 61* 62*</td>
<td>71 66 61* 61*</td>
</tr>
<tr>
<td>$N_A$</td>
<td>73 78 80 85</td>
<td>83 72 80 89</td>
<td>84 81 87 86</td>
<td>77 82 87 87</td>
</tr>
</tbody>
</table>

*Note: Significance level (binomial test, two-sided) is: *10%.

5 Experimental Results

In the following we report the main results of our experiment. Our first observation is that participants largely fail to coordinate.

**Result 1. Subjects fail to coordinate on a unique standard.**

Table 2 presents the total number of $A$-choices and $B$-choices in the 16 decision situations. It shows that subjects fail to coordinate on one of the standards. The difference between the number of $A$-choices and $B$-choices is statistically significant in the six decision situations (two-sided binomial test with 10% significance level), while in the ten decision situations the difference is not significant. The highest share a standard achieved is 60% which is the share of standard $A$ in the decision situation 14.

We observe that in most of the decision situations the number of $B$-choices is smaller than the number of $A$-choices. Only in the decision situations 1 and 6 the number of $B$-choices is larger. The average share of standard $B$ is given by 45%, while the average share of standard $A$ is equal to 55%. These results clearly suggest that the pure strategy Nash equilibria fail to predict players’ behavior. Similarly, neither the risk dominance and the payoff dominance criterion nor the global game theory are able to predict players’ aggregate adoption decisions.

Our next observation shows that an increase of standard $B$’s relative payoff dominance tends to increase the number of $B$-choices.

**Result 2. The number of $B$-choices ($A$-choices) tends to increase (decrease) as $d^{\text{max}}$ increases.**

In Table 2 we keep in each block the critical mass constant, while within each block the
Table 3: Choices depending on the degree of risk dominance

<table>
<thead>
<tr>
<th></th>
<th>$d_{\text{max}} = 75$</th>
<th>$d_{\text{max}} = 55$</th>
<th>$d_{\text{max}} = 51$</th>
<th>$d_{\text{max}} = 46$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$16 \cdot m_B$</td>
<td>1 2 3 4</td>
<td>5 6 7 8</td>
<td>9 10 11 12</td>
<td>13 14 15 16</td>
</tr>
<tr>
<td>$N_B$</td>
<td>75 65 64 71</td>
<td>70 76 67 66</td>
<td>68 68 61* 61*</td>
<td>63* 59* 62* 61*</td>
</tr>
<tr>
<td>$N_A$</td>
<td>73 83 84 77</td>
<td>78 72 81 82</td>
<td>80 80 87 87</td>
<td>85 89 86 87</td>
</tr>
</tbody>
</table>

*Note: Significance level (binomial test, two-sided) is: *10%.

The difference $d_{\text{max}}$ decreases and takes the values 75, 55, 51, and 46. From Table 2 we observe that in each block the number of $B$-choices tends to fall from the left to the right. In each block the number of $B$-choices becomes significantly lower than the number of $A$-choices if $d_{\text{max}}$ takes the smallest value 46. For the second smallest value of $d_{\text{max}} = 51$ the number of $B$-choices is still significantly lower than the number of $A$-choices in blocks 3 and 4. Interestingly, in blocks 1 and 4 the number of $B$-choices decreases monotonically when $d_{\text{max}}$ becomes smaller, whereas blocks 2 and 3 exhibit some irregularities.

In Table 3 we have re-arranged the columns of Table 2 such that each block represents a different value of $d_{\text{max}}$, while within each block the critical mass increases from 9, to 10, to 11, and finally, to 12. If we take the average number of $B$-choices in each block of Table 3, we obtain the (rounded) values 69, 70, 65, and 61 for blocks 1, 2, 3, and 4, respectively. Hence, at the aggregate level we also see that the number of $B$-choices tends to decrease when the relative payoff dominance of standard $B$ decreases.

Table 3 allows us to infer how the critical mass affects players' choices.

Result 3. The number of $B$-choices ($A$-choices) tends to decrease (increase) as the critical mass, $m_B$, increases.

From Table 3 we observe that in every block the number of $B$-choices almost monotonically decreases as the value of the critical mass of standard $B$ increases from 9 up to 12. For example, in the first decision block we get 75, 65, 64, and 71 $B$-choices for the critical masses of 9, 10, 11, and 12, respectively. Only in the last decision situation of the first block with the critical mass of 12 we see an irregularity. Turning back to Table 2, we can calculate in each block the average
number of B-choices for a given value of the critical mass. The (rounded) average number of B-choices for the critical mass of 9 is 69, for the critical mass of 10 it is 67, for the critical mass of 11 it is 64, and for critical mass of 12 the average value is 65. Again, we see that the average number of B-choices almost monotonically decreases when standard B’s critical mass increases.

While these results suggest that risk dominance tends to affect players’ choices, they also show that the mixed strategy equilibrium performs poorly.\(^{13}\)

Results 2 and 3 show that both the relative degree of payoff dominance of standard B (as measured by \(d_{\text{max}}^{\text{max}}\)) and the relative degree of risk dominance of standard A (as measured by \(m_B\)) affect players’ choices. To better understand the trade-off between them we next examine the combined effect by using regression analysis.

**Result 4.** Payoff dominance and risk dominance measured by the relative difference in the maximum payoffs and the relative difference in the critical masses, respectively, jointly explain the choice of a standard.

Table 4 presents the results of a simple OLS regression with the number of B-choices as the dependent variable. We checked several specifications for the explanatory variables. We finally decided to relate our measure of the relative payoff dominance of standard B (\(d_{\text{max}}^{\text{max}}\)) to the absolute value of the maximum utility of standard B (\(U_{B_{\text{max}}}^{\text{max}}\)). Similarly, we related the relative degree of riskiness of standard B (\(m_B - m_A\)) to the critical mass of standard B (\(m_B\)). That

\(^{13}\) A similar result is obtained in Heinemann, Nagel, and Ockenfels (2009) who point out that a Bayesian game specification is not helpful because of its similarity to the mixed strategy equilibrium.
Table 5: Logit regression explaining the probability of B-choices

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Coefficients (p-Values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-1.1^{**}$ (0.045)</td>
</tr>
<tr>
<td>$(U_B^{max} - U_A^{max})/U_B^{max}$</td>
<td>$5.72^{***}$ (0.009)</td>
</tr>
<tr>
<td>$(m_B - m_A)/m_B$</td>
<td>$-0.75^*$ (0.054)</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>$-940$</td>
</tr>
<tr>
<td>Number of observations (number of groups)</td>
<td>2368(148)</td>
</tr>
</tbody>
</table>

Note: Significance levels are: ***1%, **5%, *10%.

specification turned out to yield the most significant results.

Table 4 shows that both the degree of payoff dominance and the degree of risk dominance influence subjects’ choices. The regression results imply that the number of B-choices increases when the relative payoff dominance of standard B increases. The respective parameter estimate is significant at the 5% significance level. Our measure of the relative riskiness of standard B is negatively correlated with the number of B-choices. The respective parameter estimate is still significant at the 10%-significance level. In line with findings in Heinemann, Nagel, and Ockenfels (2009) the minimum number of players necessary to make a risky choice profitable helps to explain players’ choices.

In the Table 5 we present the results of a Logit model with random effects with the probability of choosing standard B as the dependant variable. The results of the Logit regression support our previous conclusions that both the relative difference in the maximum payoffs as well as the relative difference in the critical masses significantly impact on the probability of choosing standard B. Similar to the OLS regression, the significance level of the parameter estimate which measures the influence of payoff dominance is higher than the one which measures the influence of risk dominance.

Result 5. The choice of a standard can be explained jointly by the degree of payoff dominance and the relative difference in the minimum payoffs of the two standards.

Above we stated in Corollary 1 that a risk-dominant standard which is not at the same a payoff-dominant standard also has a larger stand-alone value. Therefore, the risk-dominant
standard is also the standard selected by the maximin criterion. We measure the influence of the maximin criterion on players’ choices by the difference of the minimum payoffs of standard A and standard B (again, relative to the minimum payoff of standard B). We incorporate that measure into our regression analysis as an explanatory variable. The results are presented in Table 6.

Table 6 shows that both the relative difference in the maximum payoffs as well as the relative difference in the minimum payoffs of the standards explain players’ choices of standard B. Again, the larger the relative difference in the maximum payoffs becomes, the more players adopt standard B. The respective parameter estimate is significant at the 1% significance level. We also see that a widening of the relative difference of the minimum payoffs reduces the number of B adoptions. The respective parameter estimate is significant at the 5% significance level. When we compare Table 6 with with Table 4 (where we used the relative difference in the standards’ critical masses as an explanatory variable), we see that the “maximin” specification performs better in terms of the significance level of the parameter estimates as well as in terms of the overall explanatory power. We speculate that the maximin criterion is easier to apply than a calculation of the critical mass as it only requires to compare the save payoffs (i.e., the minimum payoffs of each standard). In other words, the critical mass seems to be a more sophisticated concept for the subjects than the maximin criterion which relies on the standards’ minimum payoffs.

In Table 7 we present the parameters of the Logit regression with random effects explaining
Table 7: Logit regression explaining the probability of B-choices

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Coefficients (p-Values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-2.26^{***}$ ($&lt; 0.001$)</td>
</tr>
<tr>
<td>$(U_B^{\text{max}} - U_A^{\text{max}})/U_B^{\text{max}}$</td>
<td>$10.9^{***}$ (0.001)</td>
</tr>
<tr>
<td>$(U_A^{\text{min}} - U_B^{\text{min}})/U_B^{\text{min}}$</td>
<td>$-0.01^{**}$ (0.022)</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>$-940$</td>
</tr>
<tr>
<td>Number of observations (number of groups)</td>
<td>2368 (148)</td>
</tr>
</tbody>
</table>

*Note:* Significance levels are: $^{***}1\%$, $^{**}5\%$.

the probability of choosing standard $B$. These results support again the results of the simple OLS regression.

We can summarize our experimental results now as follows. First, the Nash equilibrium predictions (both in pure strategies and in mixed strategies) fail to explain players’ adoption behavior. Secondly, and accordingly, the global game theory and the maximin criterion which both select the $A$-equilibrium fail to predict players’ choices. Third, as suggested by Harsanyi and Selten (1988) both the payoff dominance and risk dominance refinements together help to explain the aggregate adoption behavior of players.

With regard to the global game prediction our results are also supportive to Heinemann, Nagel, and Ockenfels’ (2009) findings who analyze the choice between a certain payoff and a risky payoff where the risky payoff depends as in our critical mass game positively on the other players’ choices. While their approach helps to elicit the role of the certainty equivalent in a context of strategic uncertainty, our results show that in a setting where both choices are associated with strategic uncertainty, the role of payoff dominance becomes important. Heinemann, Nagel, and Ockenfels (2009) consider a game in which players choose between a secure payoff and a risky payoff. If a certain number of players choose the risky choice, then the payoff of the risky choice is higher when compared with the choice of the certain payoff. If the number of players choosing the risky choice falls short of a certain value, then the certain choice implies a higher payoff. This setting is similar to our critical mass game as both games highlight a trade-off between a relatively high certain payoff (standard $A$) and a relatively high uncertain payoff (standard $B$).
Heinemann, Nagel, Ockenfels (2009) increase the secure payment stepwise and also consider three different values for the coordination requirement $k$. The coordination requirement $k$ is similar in spirit to our concept of a critical mass: if a share $k$ of the other players chooses $B$, then it is the best reply for each of the remaining players to choose also $B$.

There are, however, important differences between their experiment and ours. First, both strategies $A$ and $B$ in the critical mass game deliver risky payoffs in the sense that they always depend on the other players’ choices. Second, in their study the coordination requirement $k$ is given exogenously while we derive the value of the critical mass endogenously (from the parameters of the payoff functions associated with the two standards). Hence, we show how the critical mass naturally emerges in the presence of network effects and how it is related to the secure payoffs of each standard.\footnote{An important consequence for the experimental design of our approach is that subjects had to infer the value of the critical mass from the presented payoff tables (see the Appendix), while in Heinemann, Nagel, and Ockenfels (2009) the critical value, $k$, was stated explicitly in the decision situations.} Third, in their experiment the decision situations were displayed on a screen ordered by the coordination requirement. Our experiment instead placed all the decision situations in the questionnaire in a random order so that the subjects were not explicitly framed to follow threshold strategies according to the riskiness of the uncertain choice. Besides those differences, our experimental results by large do not contradict the results obtained in Heinemann, Nagel, and Ockenfels (2009).\footnote{A difference noteworthy though, is the relatively large degree of coordination failure and the lower statistical significance of our regressions. However, those differences can be easily explained by both the limits of a paper-and-pencil experiment and the absence of framing devices (as the ordered presentation of all decision situations on a single screen and the explicitly stated critical values of $k$; see Heinemann, Nagel, and Ockenfels 2009, Figure 1), which naturally increases the consistency of subjects’ choices (namely, to adhere to threshold strategies). In fact, as decision situation were not ordered in our experiment, we observe a considerable portion of subjects behaving inconsistently.} We note, however, that our analysis helps to elicit the role of payoff dominance, when both choices involve strategic uncertainty.

We finally note that our results stand in contrast to Schmidt et al. (2003) who showed within their setting that players were following the risk dominance criterion while the payoff dominance criterion did not considerably affect players’ choices.

The payoff dominance and risk dominance concepts as such are “discrete” concepts in the sense that they select one of the two standards with probability one. Our experiment suggests
that subjects resolve the trade-off between both forces differently, so that in the aggregate changes in the relative risk dominance and the relative payoff dominance only affect players’ choices at the margin. In the following section we turn to the industrial organization implications of our findings, where we analyze how users’ behavior affects firms’ technology choices.

6 Technology Choice

In this section we analyze how our experimental results may impact on firms’ technology choices. This means, we postulate that the expected demand for a certain standard is driven by the joint impact of its relative payoff dominance and its relative risk dominance.

We assume two firms $k = 1, 2$ which maximize their market shares. Each firm has to decide whether to adopt technology $A$ or technology $B$. Both technologies give rise to network effects as specified in Equation (1). We assume a critical mass game as stated in Definition 1. In addition, we suppose that standard $A$ is risk-dominant; i.e., $m_A < m_B$ holds.

We analyze a two-stage game, where in the first stage firms simultaneously and noncooperatively decide which technology to adopt. In the second stage, $N \geq 2$ users simultaneously and independently make their choices. Users’ utility functions are given by Equation (1). Hence, the utility a consumer realizes only depends on the standard $i = A, B$ and is independent of the firm.

We suppose that the relative degree of risk dominance and the relative degree of payoff dominance jointly determine the market demand. To keep the analysis simple, we use the absolute difference in the standards’ minimum payoffs as a proxy for the degree of risk dominance. Similarly, we take the absolute difference in the maximum payoffs as a proxy for payoff dominance.

Given those assumptions, we can formulate the expected probability that a user chooses the standard of a firm $k = 1, 2$ when firm $k$ has chosen technology $i = A, B$ and the rival firm $k'$ ($k' \neq k$) has chosen standard $j = A, B$ as

$$P^{i,j}_k = 1/2 + \alpha(U^{\text{max}}_i - U^{\text{max}}_j) + \beta(U^{\text{min}}_i - U^{\text{min}}_j),$$  \hspace{1cm} (13)

16This is, we abstract from firms’ pricing decisions. One application may be the market for online search engines which are offered at a price of zero and where profits are generated by advertisements which are typically proportional to the number of users.
where $\alpha > 0$ measures the impact of payoff dominance and $\beta > 0$ measures the impact of risk dominance.\textsuperscript{17,18} Equation (13) mirrors the qualitative results of our experiment, such that the expected adoptions of a standard increase when its relative payoff dominance and/or its relative risk dominance increases. As every user chooses the standard of firm $k$ with probability $P_{k}^{i,j}$, the expected demand of firm $k$ is given by $P_{k}^{i,j}N$.

Given the demands, we can calculate firms’ equilibrium technology choices in the first stage of the game which yields the following proposition.

**Proposition 4.** For any given parameters $\alpha$ and $\beta$ the following equilibria emerge:

i) If $\beta/\alpha > (U_{B}^{\text{max}} - U_{A}^{\text{max}})/(U_{A}^{\text{min}} - U_{B}^{\text{min}})$, then in the only equilibrium (which is also an equilibrium in dominant strategies) both firms choose standard $A$.

ii) If $\beta/\alpha < (U_{B}^{\text{max}} - U_{A}^{\text{max}})/(U_{A}^{\text{min}} - U_{B}^{\text{min}})$, then in the only equilibrium (which is also an equilibrium in dominant strategies) both firms choose standard $B$.

iii) If $\beta/\alpha = (U_{B}^{\text{max}} - U_{A}^{\text{max}})/(U_{A}^{\text{min}} - U_{B}^{\text{min}})$, then four equilibria emerge, in which every firm chooses either standard $A$ or standard $B$.

Each firm’s expected market share is one-half in any equilibrium.

**Proof.** Note first that a firm’s market share is one-half if both firms adopt the same technology. Assume that firm 1 opts for standard $A$. Then if firm 2 also chooses standard $A$, its market share is $1/2$. If it chooses standard $B$, then its market share is $1/2 + \alpha(U_{B}^{\text{max}} - U_{A}^{\text{max}}) + \beta(U_{B}^{\text{min}} - U_{A}^{\text{min}})$, which is larger than $1/2$ if $\beta/\alpha < (U_{B}^{\text{max}} - U_{A}^{\text{max}})/(U_{A}^{\text{min}} - U_{B}^{\text{min}})$ holds and smaller otherwise.

Assume now that firm 1 opts for standard $B$, then by choosing standard $B$ firm 2, again, gets half of the market. If, in contrast, it chooses standard $A$, then its market share is equal to $1/2 + \alpha(U_{A}^{\text{max}} - U_{B}^{\text{max}}) + \beta(U_{A}^{\text{min}} - U_{B}^{\text{min}})$, which is larger than $1/2$ if $\beta/\alpha > (U_{B}^{\text{max}} - U_{A}^{\text{max}})/(U_{A}^{\text{min}} - U_{B}^{\text{min}})$ and smaller otherwise. Hence, it is a dominant strategy for a firm to choose standard $A$ if $\beta/\alpha > (U_{B}^{\text{max}} - U_{A}^{\text{max}})/(U_{A}^{\text{min}} - U_{B}^{\text{min}})$. If to the contrary $\beta/\alpha < (U_{B}^{\text{max}} - U_{A}^{\text{max}})/(U_{A}^{\text{min}} - U_{B}^{\text{min}})$ holds, then for both firms the dominant strategy is to choose standard $B$. Finally, if $\beta/\alpha =$

\textsuperscript{17}Note that $p_{1}^{i,j} + p_{2}^{i,j} = 1$, so that market shares sum up to one. Note also that (13) implies $p_{1}^{A,A} = p_{2}^{A,A} = p_{1}^{B,B} = p_{2}^{B,B} = 1/2$. Hence, if both firms opt for the same standard, then every firm’s standard is chosen with equal probabilities.

\textsuperscript{18}Equation (13) can easily be rewritten to account for the relative differences in the minimum and maximum payoffs by introducing new parameters $\tilde{\alpha} = \alpha(U_{B}^{\text{max}})$ and $\tilde{\beta} = \beta(U_{B}^{\text{min}})$, such that $p_{k}^{i,j} = 0.5 + \tilde{\alpha}(U_{i}^{\text{max}} - U_{j}^{\text{max}})/(U_{B}^{\text{max}} + \tilde{\beta}(U_{i}^{\text{min}} - U_{j}^{\text{min}})/(U_{B}^{\text{min}})$.
Proposition 4 shows that firms tend to choose the risk-dominant standard $A$ if the impact factor $\beta$ is large enough. If, however, the impact factor $\beta$ becomes relatively small, then firms are more likely to adopt the payoff-dominant standard $B$.

We finally examine the welfare consequences of firms’ technology choices. We abstract from producer surplus and focus on consumer surplus. We suppose that firms’ products stay incompatible even if they adopt the same technology. For simplicity, we also disregard the non-generic case $\text{iii)}$ of Proposition 4, so that either both firms choose standard $A$ or standard $B$ in the technology choice game. The following result is then immediate.

**Proposition 5.** For any $\alpha$ and $\beta$, expected consumer surplus is maximized when both firms choose the risk-dominant standard $A$.

**Proof.** From Equation (13) it follows that the market is shared equally if both firms adopt the same technology. As we assumed $m_B > m_A$ it follows that $NU_A(N/2) > NU_B(N/2)$ holds. Q.E.D.

Proposition 5 is an astonishing result which shows that consumers can be better off if firms choose an inferior standard; i.e., a standard which has a lower maximum value than the rival standard. When firms correctly expect that buyers’ choices are driven by considerations of risk dominance, then firms tend to favor the less risky technology, which ultimately benefits consumers. The reason behind this result is the prevalence of coordination failure. If both firms select the same technology, then the expected market share of each firm is one-half. If miscoordination is an overwhelming problem (as observed in one-shot experiments), then consumers are better off in expected terms, if firms choose the less risky technology $A$ which yields an expected aggregate consumer surplus of $NU_A(N/2)$. That expected consumer surplus is necessarily larger when compared with the consumer surplus, $NU_B(N/2)$, which can be expected if both firms choose the payoff-dominant technology $B$. This comparison follows immediately from our assumption that standard $A$ is risk-dominant.
7 Conclusion

We introduced a critical mass game in which $N \geq 2$ identical users make simultaneously their adoption decisions where they have to choose between two standards that exhibit positive network effects. The critical mass game gives rise to a coordination problem as it has two strict Nash equilibria in pure strategies. One of those equilibria is assumed to be payoff-dominant. We introduced heuristically the concept of a critical mass which we defined as the minimum share of users adopting a certain standard so as to make the choice of this standard a best response for any of the remaining users. In the theoretical part we showed that the equilibrium in which all users adopt the standard with the lower critical mass is risk-dominant according to Harsanyi and Selten, is chosen by the global game theory, and is also selected by the maximin criterion. Our critical mass heuristic, therefore, is theoretically instructive. It gives additional intuitive appeal to the risk-dominance criterion and the global game theory within the context of a critical mass game.

In the experimental part we showed that subjects’ choices depend on the degree of payoff dominance (measured by the relative difference in the standards’ maximum payoffs) and the degree of risk dominance (measured by the relative difference in the standards’ critical masses or the standards’ minimum payoffs). Our experimental results suggest that an increase of the relative degree of a standard’s payoff dominance tends to increase users’ adoptions. Similarly, we showed that an increase of a standard’s relative degree of risk dominance tends to increase adoptions.

We also analyzed how consumer behavior affects firms’ technology decisions. We showed, if the impact factor for risk dominance is sufficiently large, then both firms choose the risk-dominant standard. Quite surprisingly, those decisions lead to technology choices which tend to benefit users. The reason behind this result is that the choice of a less risky technology minimizes the (almost sure) losses from coordination failure.

There are many possible directions for further research. One route is to generalize the concept of the critical mass to games with nonlinear network effects and to establish its relationship to the concept of risk dominance. Another direction would be to analyze how the relative riskiness of a standard affects adoption decisions in a dynamic setting. Presumably, a standard with a larger critical mass may need more time to gain dominance (if at all) in a dynamic setting when
compared with a less riskier technology.

Appendix

In this Appendix we first present the proof of Proposition 3, then the instructions of the experiment, and finally, the participants’ decision situations in the experiment.

Proof of Proposition 3

We first define a switching strategy \( s(v) \) which prescribes which standard to choose depending on the value of the private signal a player receives:

\[
s(v) = \begin{cases} 
B & \text{if } v > \bar{v} \\
A & \text{if } v \leq \bar{v}.
\end{cases}
\]  

(14)

Assume that it is common knowledge that all users’ standard choices are given by such a switching strategy. Then each user \( l \) knows that the probability that a user \( l' \) with \( l \neq l' \) observes a signal smaller than \( \bar{v} \), and hence, chooses standard \( A \) is given by \( \Phi\left(\frac{\bar{v} - v}{\sqrt{2}\sigma}\right) \), where \( \Phi(x) \) is c.d.f. of the standard normal distribution.\(^{19}\) The probability that another player chooses standard \( B \) is then given by \( 1 - \Phi\left(\frac{\bar{v} - v}{\sqrt{2}\sigma}\right) \). The expected payoff of a player \( l \) if he chooses standard \( B \) after observing signal \( v \) is then given by

\[
v + \gamma_B \left[ 1 - \Phi\left(\frac{\bar{v} - v}{\sqrt{2}\sigma}\right) \right] (N - 1).
\]

The expected payoff of choosing standard \( A \) is given by

\[
v_A + \gamma_A \Phi\left(\frac{\bar{v} - v}{\sqrt{2}\sigma}\right) (N - 1).
\]

Hence, a player \( l \) will choose standard \( B \) if

\[
v + \gamma_B \left[ 1 - \Phi\left(\frac{\bar{v} - v}{\sqrt{2}\sigma}\right) \right] (N - 1) > v_A + \gamma_A \Phi\left(\frac{\bar{v} - v}{\sqrt{2}\sigma}\right) (N - 1)
\]

\(^{19}\)After observing signal \( v \) the user knows that the signals of the other players are distributed with \( v_j \sim N(v, 2\sigma^2) \). Hence, the probability that a user \( j \) observes a signal smaller than \( \bar{v} \) is given by \( P(v_j \leq \bar{v}) \). Note finally, that \( P(v_j \leq \bar{v}) = P\left(\frac{v_j - v}{\sqrt{2}\sigma} \leq \frac{\bar{v} - v}{\sqrt{2}\sigma}\right) = \Phi\left(\frac{\bar{v} - v}{\sqrt{2}\sigma}\right) \). The last equality follows from the fact that if \( v_j \sim N(v, 2\sigma^2) \), then \( z_j = \frac{v_j - v}{\sqrt{2}\sigma} \) is normally distributed with \( z_j \sim N(0, 1) \).
holds, which can be re-written as

$$v > v_A - \gamma_B(N - 1) + (\gamma_A + \gamma_B)\Phi\left(\frac{\tilde{v} - v}{\sqrt{2}\sigma}\right)(N - 1). \quad (15)$$

Hence, according to decision rule (14) a user’s signal must be high enough to induce him to choose standard $B$. Let us define the right-hand side of (15) by the function $f(\tilde{v}) := v_A - \gamma_B(N - 1) + (\gamma_A + \gamma_B)\Phi\left(\frac{\tilde{v} - v}{\sqrt{2}\sigma}\right)(N - 1)$ so that for $v = f(\tilde{v})$ a user is indifferent between standards $A$ and $B$. To proceed with the proof, we have to analyze the main properties of the function $f(\tilde{v})$.

**Ancillary Claim 1.** The function $f(\tilde{v})$ has the following properties:

i) $f(\tilde{v})$ is well-defined,

ii) $f(\tilde{v})$ is strictly increasing in $\tilde{v}$,

iii) $f(\tilde{v})$ has a unique fixed point, $\hat{v}$, with $\hat{v} = f(\hat{v}) = (v_A + (\gamma_A - \gamma_B)(N - 1))/2$,

iv) $f(\tilde{v})$ is concave for $v > \tilde{v}$ and convex for $v < \tilde{v}$.

**Proof.** We prove each property one after the other.

i) Suppose to the contrary that the function is not well-defined. Then, for some $\tilde{v}_1$ there are two values $v_1$ and $v_2$ such that

$$v - v_A + \gamma_B(N - 1) - (\gamma_A + \gamma_B)\Phi\left(\frac{\tilde{v}_1 - v}{\sqrt{2}\sigma}\right)(N - 1) = 0 \quad (16)$$

holds if either $v = v_1$ or $v = v_2$. Let us now introduce the function $G(v, \tilde{v}) := v - v_A + \gamma_B(N - 1) - (\gamma_A + \gamma_B)\Phi\left(\frac{\tilde{v} - v}{\sqrt{2}\sigma}\right)(N - 1)$. Assume without loss of generality that $v_1 > v_2$. Note next that $G(v, \tilde{v})$ strictly increases in $v$ as $\partial G(v, \tilde{v})/\partial v < 0$. It then follows that $G(v_1, \tilde{v}_1) > G(v_2, \tilde{v}_1)$, so that $G(v_1, \tilde{v}_1) = G(v_2, \tilde{v}_1) = 0$ cannot be true.

ii) Consider the values $\tilde{v}_1$, $\tilde{v}_2$, $v_1$ and $v_2$ such that $v_1 = f(\tilde{v}_1)$, $v_2 = f(\tilde{v}_2)$ and $\tilde{v}_1 > \tilde{v}_2$. We have to show that $v_1 > v_2$. Note that $G(v, \tilde{v})$ is strictly decreasing in $\tilde{v}$ since $\partial G(v, \tilde{v})/\partial \tilde{v} > 0$.

Hence, it holds that $G(v_1, \tilde{v}_1) < G(v_1, \tilde{v}_2)$. Note, moreover, that $G(v_1, \tilde{v}_1) = G(v_2, \tilde{v}_2) = 0$ and we obtain $G(v_2, \tilde{v}_2) = G(v_1, \tilde{v}_1) < G(v_1, \tilde{v}_2)$. As $G(v, \tilde{v})$ strictly increases in $v$, it follows from $G(v_2, \tilde{v}_2) < G(v_1, \tilde{v}_2)$ that $v_2 < v_1$.

iii) A fixed point requires $\tilde{v} = f(\tilde{v})$. Note, if $\tilde{v} = v$, then $\Phi\left(\frac{v - \tilde{v}}{\sqrt{2}\sigma}\right) = 1/2$. Hence, the fixed point $\hat{v}$ solves

$$\hat{v} = v_A - \gamma_B(N - 1) + \frac{\gamma_A + \gamma_B}{2}(N - 1), \quad (17)$$
which gives
\[ \tilde{v} = v_A + \frac{\gamma A - \gamma B}{2} (N - 1). \]

As \( \tilde{v} \) is uniquely determined by \( \tilde{v} = f(\tilde{v}) \), we conclude that there is only one fixed point.

\( iv) \) As \( G(v, \tilde{v}) \equiv 0 \) holds for any \( v \) and \( \tilde{v} \) we obtain the expression
\[ \left( 1 + A \frac{\partial \Phi(x)}{\partial x} \right) dv = A \frac{\partial \Phi(x)}{\partial x} d\tilde{v}, \]
with \( A \equiv \frac{\gamma A + \gamma B}{\sqrt{2\sigma}} (N - 1) \). Hence, we get that
\[\frac{\partial l(\tilde{v})}{\partial \tilde{v}} = \frac{A \frac{\partial \Phi(x)}{\partial x}}{1 + A \frac{\partial \Phi(x)}{\partial x}} > 0.\]

We next have to determine the second derivative
\[ \frac{\partial^2 f(\tilde{v})}{(\partial \tilde{v})^2} = \frac{A \frac{\partial^2 \Phi(x)}{\partial x^2} \left( 1 + A \frac{\partial \Phi(x)}{\partial x} \right) - A^2 \frac{\partial \Phi(x)}{\partial x} \frac{\partial^2 \Phi(x)}{\partial (\partial x)^2}}{(1 + A \frac{\partial \Phi(x)}{\partial x})^2} = \frac{A \frac{\partial^2 \Phi(x)}{\partial x^2}}{(1 + A \frac{\partial \Phi(x)}{\partial x})^2}. \tag{18} \]

We have to consider two cases: \( \tilde{v} \leq \hat{v} \) and \( \tilde{v} > \hat{v} \). If \( \tilde{v} \leq \hat{v} \), then \( \frac{\partial^2 \Phi(x)}{\partial x^2} > 0 \), hence, it follows from Expression (18) that \( \frac{\partial^2 f(\tilde{v})}{(\partial \tilde{v})^2} > 0 \) and \( l(\tilde{v}) \) is a convex function. If \( \tilde{v} > \hat{v} \), then \( \frac{\partial^2 \Phi(x)}{\partial x^2} < 0 \), hence, it follows from Expression (18) that \( \frac{\partial^2 f(\tilde{v})}{(\partial \tilde{v})^2} < 0 \) and \( f(\tilde{v}) \) is a concave function. This completes the proof of the claim.

In the next claim we show that there is the only strategy which survives the iterated deletion of strictly dominated strategies.

**Ancillary Claim 2.** The only switching strategy which survives the iterated elimination of strictly dominated strategies is given by:

\[ s(v) = \begin{cases} B & \text{if } v > \hat{v} = v_A + \frac{(\gamma_A - \gamma_B)}{2} (N - 1) \\ A & \text{if } v \leq \hat{v} = v_A + \frac{(\gamma_A - \gamma_B)}{2} (N - 1). \end{cases} \]

**Proof.** If a player \( l \) observes a signal \( v \) with \( v > v_A + \gamma_A (N - 1) \), then it is a dominant strategy for him to choose standard \( B \).\(^{20}\) If, to the contrary, player \( l \) observes a signal \( v \) such

\(^{20}\)This means, even if all the other players decided to choose \( A \) (depending on the signals they receive), it is still optimal for a player \( i \) to choose standard \( B \).
that \( v \leq v_A - \gamma_B(N - 1) \), then it is a dominant strategy for him to choose standard \( A \). Hence, every user infers that all the other users follow a switching strategy

\[
s(v) = \begin{cases} B & \text{if } v > v_A + \gamma_A(N - 1) \\ A & \text{if } v \leq v_A - \gamma_B(N - 1). \end{cases}
\] (19)

The strategy (19) follows from applying the first step of the iterated elimination of strictly dominated strategies. The best response to this strategy is then given by

\[
s(v) = \begin{cases} B & \text{if } v > f(v_A + \gamma_A(N - 1)) \\ A & \text{if } v \leq f(v_A - \gamma_B(N - 1)). \end{cases}
\]

The strategy we obtain in the \( n \)-th step of the iterated elimination of strictly dominated strategies is then given by

\[
s(v) = \begin{cases} B & \text{if } v > f^{n-1}(v_A + \gamma_A(N - 1)) \\ A & \text{if } v \leq f^{n-1}(v_A - \gamma_B(N - 1)). \end{cases}
\]

Using the properties of the function \( f(\bar{v}) \), which is concave for \( v > \bar{v} \), convex for \( v < \bar{v} \) and has a unique fixed point at \( \bar{v} \), we obtain

\[
\lim_{n \to \infty} f^{n-1}(v_A + \gamma_A(N - 1)) = \lim_{n \to \infty} f^{n-1}(v_A - \gamma_B(N - 1)) = \bar{v}.
\]

This completes the proof of the claim.

It is only left to note that the condition \( v > v_A + \gamma_A(N - 1) \) is equivalent to \( m_B \in (0, 1/2) \), while \( v < v_A - \gamma_B(N - 1) \) is equivalent to \( m_B \in (1/2, 1) \). Q.E.D.

**Instructions**

In the following we present the English translation of the instructions of our experiment which were handed out in German.

**Instructions.** Please do not communicate with other participants! If you have questions please raise your hand so that we can answer your question individually!

You are participating in a decision experiment in which you can earn money. With 16 other randomly chosen participants which will not be known to you, you build up a group. How much you earn depends on your own decisions and decisions of the other participants of your group. Every participant makes his (her) decisions independently of the others.

The experiment consists of 16 different decision situations. In every decision situation every experiment participant makes the choice between two alternatives, \( X \) and \( Z \). The participant’s
payoff in a particular decision situation depends on the own choice and the number of the other participants of the group who have made the same choice. The payoff is higher the more other participants have chosen the same alternative. The payments in all the 16 decision situations are independent of each other and are given in fictitious monetary units.

The fictitious monetary units will be converted into Euros for one randomly chosen experiment participant such that one monetary unit will be worth 50 Cents. Before the Experiment we have randomly chosen one of the 16 decision situations, the number of this decision situation is kept in an envelope. At the end of the experiment first a group of 17 participants will be picked up, whose decisions in this decision situation will be analyzed. From this group then one participant will be randomly chosen for the cash payment. Please notice that in the left upper corner of this page as well as on the attached sheet you find your individual participation number. We ask you to keep the attached sheet with which we can identify you for the possible cash payment.

Every decision situation will be presented in a table. In this table you see how your individual payoff in fictitious monetary units depends on your choice and the choices of other participants. On the next page we give you an example.

Example:

Assume that your payoff in one given decision situation depends on your individual choice (alternative X or Z) and choices of the other participants of your group in the way presented in the following Table:

| Number of others who choose Z | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9  | 8  | 7  | 6  | 5  | 4  | 3  | 2  | 1  | 0  |
|------------------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Number of others who choose X| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Your choice                  | X  | 20 | 25 | 30 | 50 | 60 | 65 | 70 | 90 | 120| 125| 130| 140| 160| 165| 172| 180| 190|
|                             | Z  | 170| 150| 145| 130| 125| 120| 115| 90 | 80 | 75 | 70 | 65 | 60 | 55 | 50 | 45 | 40 |

According to this Table your payment is:

- 20, when you choose X and none of the other participants chooses X, what means that all the other 16 participants choose Z,
• 170, when you choose Z and none of the other participants chooses X, what means that all the other 16 participants choose Z,

• 30, when you choose X, two of the other participants choose X and 14 of the other participants choose Z,

• 145, when you choose Z, two of the other participants choose X and 14 of the other participants choose Z,

• 165, when you choose X, 13 of the other participants choose X and three of the others choose Z,

• 55, when you choose Z, 13 of the other participants choose X and 3 of the others choose Z,

• 190, when you choose X, all the other 16 participants choose X and none of the others chooses Z,

• 40, when you choose Z, all the other participants choose X and none of the others chooses Z.

We ask you now to analyze the following decision situations and mark your choice, alternative X or Z. For this you find a box under every decision situation.

When all the experiment participants are ready with their choices, we will collect the questionnaires and establish the person who will be paid in cash.

**Decision Situations**

In this section we present the decision situations in which experiment participants had to make their choices. Decision situations were handed out in a random order. We presented two decision situations on a single sheet of paper. Below, on top of each decision situation table, we also provide the assumed utility functions $U_A(N_A)$ and $U_B(N_B)$ from which we calculated the (rounded) payoffs which were presented to the participants in the tables.
### Decision Situation 1: $U_A = 134.44 + 7.22(N_A - 1)$ and $U_B = 5 + 20(N_B - 1)$

<table>
<thead>
<tr>
<th>Number of others who choose B</th>
<th>16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
</tr>
<tr>
<td>Your choice</td>
<td>A: 134, 142, 149, 156, 163, 171, 178, 185, 192, 199, 207, 214, 221, 228, 236, 243, 250</td>
</tr>
<tr>
<td></td>
<td>B: 325, 305, 285, 265, 245, 225, 205, 185, 165, 145, 125, 105, 85, 65, 45, 25, 5</td>
</tr>
</tbody>
</table>

### Decision Situation 2: $U_A = 178 + 4.5(N_A - 1)$ and $U_B = 5 + 20(N_B - 1)$

<table>
<thead>
<tr>
<th>Number of others who choose B</th>
<th>16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
</tr>
<tr>
<td>Your choice</td>
<td>A: 178, 183, 187, 192, 196, 201, 205, 210, 214, 219, 223, 228, 232, 237, 241, 246, 250</td>
</tr>
<tr>
<td></td>
<td>B: 325, 305, 285, 265, 245, 225, 205, 185, 165, 145, 125, 105, 85, 65, 45, 25, 5</td>
</tr>
</tbody>
</table>

### Decision Situation 3: $U_A = 213.64 + 2.27(N_A - 1)$ and $U_B = 5 + 20(N_B - 1)$

<table>
<thead>
<tr>
<th>Number of others who choose B</th>
<th>16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
</tr>
<tr>
<td></td>
<td>B: 325, 305, 285, 265, 245, 225, 205, 185, 165, 145, 125, 105, 85, 65, 45, 25, 5</td>
</tr>
</tbody>
</table>

### Decision Situation 4: $U_A = 243.33 + 0.42(N_A - 1)$ and $U_B = 5 + 20(N_B - 1)$

<table>
<thead>
<tr>
<th>Number of others who choose B</th>
<th>16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
</tr>
<tr>
<td>Your choice</td>
<td>A: 243, 244, 244, 245, 245, 246, 246, 247, 247, 247, 248, 248, 249, 249, 250, 250</td>
</tr>
<tr>
<td></td>
<td>B: 325, 305, 285, 265, 245, 225, 205, 185, 165, 145, 125, 105, 85, 65, 45, 25, 5</td>
</tr>
</tbody>
</table>
**Decision Situation 5:** \( U_A = 156.11 + 5.56(N_A - 1) \) and \( U_B = 60 + 15(N_B - 1) \)

<table>
<thead>
<tr>
<th>Number of others who choose B</th>
<th>16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
</tr>
<tr>
<td>Your choice</td>
<td>A 156 162 167 173 178 184 189 195 201 206 212 217 223 228 234 239 245</td>
</tr>
<tr>
<td></td>
<td>B 300 285 270 255 240 225 210 195 180 165 150 135 120 105 90 75 60</td>
</tr>
</tbody>
</table>

**Decision Situation 6:** \( U_A = 189 + 3.5(N_A - 1) \) and \( U_B = 60 + 15(N_B - 1) \)

<table>
<thead>
<tr>
<th>Number of others who choose B</th>
<th>16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
</tr>
<tr>
<td>Your choice</td>
<td>A 189 193 196 200 203 207 210 214 217 221 224 228 231 235 238 242 245</td>
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<tr>
<td></td>
<td>B 300 285 270 255 240 225 210 195 180 165 150 135 120 105 90 75 60</td>
</tr>
</tbody>
</table>

**Decision Situation 7:** \( U_A = 215.9 + 1.8(N_A - 1) \) and \( U_B = 60 + 15(N_B - 1) \)

<table>
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<th>16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</th>
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<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
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<tr>
<td>Your choice</td>
<td>A 216 218 220 221 222 223 224 227 229 230 232 234 236 238 240 241 243 245</td>
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<tr>
<td></td>
<td>B 300 285 270 255 240 225 210 195 180 165 150 135 120 105 90 75 60</td>
</tr>
</tbody>
</table>

**Decision Situation 8:** \( U_A = 238.3 + 0.42(N_A - 1) \) and \( U_B = 60 + 15(N_B - 1) \)

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</tr>
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<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
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<tr>
<td>Your choice</td>
<td>A 238 239 239 240 240 240 241 241 242 242 242 243 243 244 244 245 245</td>
</tr>
<tr>
<td></td>
<td>B 300 285 270 255 240 225 210 195 180 165 150 135 120 105 90 75 60</td>
</tr>
</tbody>
</table>
### Decision Situation 9: \( U_A = 205 + 1.5(N_A - 1) \) and \( U_B = 132.57 + 9.2(N_B - 1) \)

<table>
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<th>16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</th>
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</thead>
<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
</tr>
<tr>
<td>Your choice</td>
<td>A: 205 207 208 210 211 213 214 216 217 219 220 222 223 225 226 228 229</td>
</tr>
<tr>
<td></td>
<td>B: 280 271 262 252 243 234 225 216 206 197 188 179 169 160 151 142 133</td>
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</tbody>
</table>

### Decision Situation 10: \( U_A = 205 + 1.5(N_A - 1) \) and \( U_B = 104 + 11(N_B - 1) \)

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<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
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<tr>
<td>Your choice</td>
<td>A: 205 207 208 210 211 213 214 216 217 219 220 222 223 225 226 228 229</td>
</tr>
<tr>
<td></td>
<td>B: 280 269 258 247 236 225 214 203 192 181 170 159 148 137 126 115 104</td>
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</tbody>
</table>

### Decision Situation 11: \( U_A = 205 + 1.5(N_A - 1) \) and \( U_B = 64 + 13.5(N_B - 1) \)

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<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
</tr>
<tr>
<td>Your choice</td>
<td>A: 205 207 208 210 211 213 214 216 217 219 220 222 223 225 226 228 229</td>
</tr>
<tr>
<td></td>
<td>B: 280 267 253 240 226 213 199 186 172 159 145 132 118 105 91 78 64</td>
</tr>
</tbody>
</table>

### Decision Situation 12: \( U_A = 205 + 1.5(N_A - 1) \) and \( U_B = 4 + 17.25(N_B - 1) \)

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</thead>
<tbody>
<tr>
<td>Number of others who choose A</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16</td>
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<tr>
<td>Your choice</td>
<td>A: 205 207 208 210 211 213 214 216 217 219 220 222 223 225 226 228 229</td>
</tr>
<tr>
<td></td>
<td>B: 280 263 246 228 211 194 177 159 142 125 108 90 73 56 39 21 4</td>
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</tbody>
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37
### Decision Situation 13: $U_A = 232 + 2(N_A - 1)$ and $U_B = 164 + 9.1(N_B - 1)$

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<tbody>
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<td>3</td>
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<td>234</td>
<td>236</td>
<td>238</td>
<td>240</td>
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<td>262</td>
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<td>B</td>
<td>310</td>
<td>301</td>
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<td>283</td>
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### Decision Situation 14: $U_A = 232 + 2(N_A - 1)$ and $U_B = 134 + 10.97(N_B - 1)$

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<tbody>
<tr>
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<td>234</td>
<td>236</td>
<td>238</td>
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</tr>
<tr>
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### Decision Situation 15: $U_A = 232 + 2(N_A - 1)$ and $U_B = 93 + 13.58(N_B - 1)$

<table>
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### Decision Situation 16: $U_A = 232 + 2(N_A - 1)$ and $U_B = 30 + 17.5(N_B - 1)$

<table>
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References


Van Huyck, J.B., Battalio, R.C., and Beil, R.O. (1990), Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure, American Economic Review 80, 234-248.


Van Huyck, J.B., Gillette A.B., and Battalio, R.C. (1990), Credible Assignments in Coordination Games, Games and Economic Behavior 4, 606-626.