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Article

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Central Bank Review (CBR)

Provided in Cooperation with:

Central Bank of The Republic of Turkey, Ankara

Suggested Citation: Çakmaklı, Cem; Paap, Richard; van Dijk, Dick (2022) : Modeling and estimation of synchronization in size-sorted portfolio returns, Central Bank Review (CBR), ISSN 1303-0701, Elsevier, Amsterdam, Vol. 22, Iss. 4, pp. 129-140, <https://doi.org/10.1016/j.cbrev.2022.11.001>

This Version is available at:

<https://hdl.handle.net/10419/297953>

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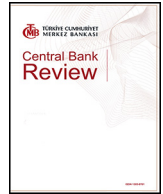
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Modeling and estimation of synchronization in size-sorted portfolio returns[☆]

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ARTICLE INFO

Article history:

Received 20 September 2022

Received in revised form

1 November 2022

Accepted 7 November 2022

Available online 8 December 2022

JEL classification:

C11

C32

C51

C52

Keywords:

Size-sorted portfolio returns

Regime-switching models

Imperfect synchronization

Phase shifts

Bayesian analysis

ABSTRACT

This paper examines the lead/lag relations between size-sorted portfolio returns through the lens of financial cycles governing these returns using a novel econometric methodology. Specifically, we develop a Markov-switching vector autoregressive model that allows for imperfect synchronization of cyclical regimes such as bull and bear market regimes in US large-, mid- and small-cap portfolio returns. This is achieved by characterizing the cycles of the mid- and small-cap portfolio returns in concordance with the cycle of large-cap portfolio returns together with potential phase shifts. We find that a three-regime model with distinct phase shifts across regimes characterizes the joint distribution of returns most adequately. These regimes are closely linked to the business cycle and small-cap portfolio returns are more sensitive to the cyclical phases than the large-cap portfolios. While all portfolios switch contemporaneously into boom and crash regimes, the large-cap portfolio leads the small-cap portfolio for switches to a moderate regime from a boom regime by a month. This suggests that small-cap portfolio adjusts with a delay to the relatively negative news compared to portfolios with larger market capitalization. We document that information diffusion accelerates in response to surprises related to the monetary policy. This reflects a link between financial returns and real economic activity from the viewpoint of 'financial accelerator theory' where portfolios with distinct size serve as a proxy for firm characteristics.

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1. Introduction

The synchronization of the cyclical behavior of financial returns has generated considerable interest in recent years. Typical examples include the synchronization of portfolios formed according to size (see Perez-Quiros and Timmermann, 2000; Guidolin and Timmermann, 2006, among others) or synchronization of bull and bear phases in financial markets (see Edwards et al., 2003; Bekaert et al., 2005, among others). Because

the size, described as the market capitalization of firms, serves as an essential proxy for firm characteristics, the synchronization of size-sorted portfolios is the focal point of these studies. Therefore, accurate estimation of the cyclical behavior paves the way for analysis of the link between firm characteristics and the real economy. Much attention in this research area has been focused on the extreme cases of independence and perfect synchronization of cycles characterized by these distinct phases. In the first case, the cycles embedded in multiple asset returns are purely idiosyncratic, while in the second case, all returns are driven by a single common cycle such that regime switches occur contemporaneously. A wide range of econometric techniques has been developed to examine these polar cases, ranging from (non-) parametric testing procedures (Artis et al., 1997; Pagan and Sossounov, 2003), to unobserved components models (Koopman and Harvey, 1997), to Markov-Switching Vector Autoregressive (MS-VAR) models (see Artis et al., 2004; Guidolin and Timmermann, 2006).

Perhaps not surprisingly, it is often found that neither independence nor perfect synchronization is adequate representations

[☆] We would like to thank Paolo Giordani, Nikolaus Hautsch, Mark Jensen, Gary Koop, John Maheu, Hashem Pesaran and Rodney Strachan for helpful comments and suggestions. An earlier version of this paper was circulated under the title 'Modeling and estimation of synchronization in multistate Markov-switching models'.

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Peer review under responsibility of the Central Bank of the Republic of Turkey.

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of the cyclical dynamics in portfolio returns. In particular, following the seminal paper of [Lo and MacKinlay \(1990\)](#) a substantial body of empirical research has documented the presence of lead/lag effects between different types of stocks essentially pointing an ‘imperfect synchronization’ of cycles embedded in portfolio returns. The typical finding on lead/lag effects, usually based on a VAR analysis of size-sorted portfolios, is that returns on (portfolios of) large stocks lead returns on (portfolios of) small stocks. These effects are usually linked to the differential speed of adjustment to information across stocks, see [Brennan et al. \(1993\)](#); [Chordia et al. \(2011\)](#); [Guo \(2023\)](#), among others.

Still, empirical models of portfolio returns explicitly accounting for the cyclical dynamics in returns have yet to incorporate the findings on lead/lag effects. Analysis of these effects could also potentially uncover the impact of policy changes across firms with distinct characteristics by capturing the differential speed of information diffusion among size-sorted portfolios. In this paper, we provide an econometric methodology, where we estimate the financial cycles embodied in size-sorted portfolio returns together with potential lead/lag relations between the cyclical phases of these returns to fill this gap. Specifically, we develop a framework in the context of MS-VAR models for describing imperfect synchronization between size-sorted portfolio returns due to different phase shifts of a single common cycle. To do this, we generalize the models of [Hamilton and Perez-Quiros \(1996\)](#) and [Paap et al. \(2009\)](#) in two critical directions to estimate the degree of synchronization of size-sorted portfolios. First, we allow for multiple regimes and consider the possibility of different phase shifts for the different regimes, i.e., regime-dependent phase shifts. Second, we allow for regime-dependent heteroskedasticity and correlations in addition to regime switching in the means. In this way, we obtain a general framework where we can simultaneously estimate the degree of synchronization together with the unobserved regimes in size-sorted portfolios. Our framework provides a new perspective on the lead/lag effects between size-sorted portfolio returns through the lens of cyclical dynamics beyond the conventional analysis based on cross-autocorrelations. This perspective, in turn, enables us to elaborate on the impact of policy changes on this cyclical dynamics.

Empirical evidence convincingly has demonstrated that asset returns are not normally distributed. Markov-switching models provide an intuitive approach for describing this non-normality, by means of a mixture of underlying normal distributions, see [Timmermann \(2000\)](#) and [Perez-Quiros and Timmermann \(2001\)](#), among others. A further attractive property of Markov-switching models is that they formalize the idea of cyclical variation in financial markets, with phases/regimes corresponding with periods of bull and bear sentiment, for example, during which asset prices have quite different characteristics. Therefore, Markov-switching models have become increasingly popular in modeling and forecasting asset prices, see [Ang and Timmermann \(2012\)](#), for a review on the use of these models in finance.

Allowing for multiple regimes is relevant for accurate representation of the cyclical behavior of many asset returns. Existing evidence suggests that three or four regimes are required to characterize the dynamics adequately in the context of Markov-switching models, see the evidence in [Turner et al. \(1989\)](#); [Chauvet and Potter \(2000\)](#); [Maheu and McCurdy \(2000\)](#), among others. Next to regimes representing bull and bear markets, additional regimes may be necessary to capture ‘crashes’ and ‘recoveries’ ([Guidolin and Timmermann, 2006](#)), or ‘bull market corrections’ and ‘bear market rallies’ ([Maheu et al., 2012](#)).

Allowing for regime-specific volatilities may be of crucial importance for reliable identification of the regimes of asset returns. The main identifying characteristic of bull and bear

markets is their different levels of volatilities (together with the sign of the returns), see, among others, [Maheu and McCurdy \(2000\)](#); [Guidolin and Timmermann \(2006\)](#). This gives rise to additional complications in case of imperfectly synchronized cycles, however, as there is no unique way to define a proper covariance matrix. We demonstrate that this complication may be overcome by using the decomposition of the covariance matrix in volatilities and correlations. It is fairly straightforward to guarantee that the resulting covariance matrix is positive definite, for example, while still allowing for a fair amount of flexibility in specifying the regime dynamics of the volatilities and correlations.

Allowing for intertemporal dynamics between the cyclical regimes of different types of portfolio returns may be essential for joint modeling of these returns in light of the empirical findings on the presence of lead/lag effects, see [Lo and MacKinlay \(1990\)](#) and [Hayashi and Koike \(2019\)](#) for a recent analysis. [Brennan et al. \(1993\)](#), among others, demonstrate that these lead/lag patterns among size-sorted portfolios arise due to differential speeds of adjustment of stocks to economy-wide shocks most notably monetary policy shocks, see [Gertler and Gilchrist \(1994\)](#). Various underlying mechanisms have been pointed out for explaining these differences of stock returns in incorporating new information. Specific features of stocks including analyst coverage, institutional ownership, trading volume, industry supply chains, market frictions, and liquidity are among prominent causes of these differences, see, [Badrinath et al. \(1995\)](#); [Richardson and Peterson \(1999\)](#); [Chordia et al. \(2005, 2011\)](#); [Cohen and Lou \(2012\)](#); [Chen et al. \(2020\)](#), among others. The common finding of these studies is that returns on (portfolios of) large stocks lead returns on (portfolios of) small stocks due to differences in these features.

Several refinements on the sluggish adjustment of the small stocks to the common information have been put forward. First, lead/lag patterns may alter over the type of information. [McQueen et al. \(1996\)](#), for example, find that large caps lead small caps only in response to good news, but not to bad news. By contrast, building on the gradual information diffusion model of [Hong and Stein \(1999\)](#), [Hong et al. \(2000\)](#) argues the slow diffusion of negative information due to behavioral biases. [Hou \(2007\)](#) reports a similar effect: small stocks adjust slowly to negative news but not to positive news. Therefore, allowing for the ‘directional asymmetry’ is pivotal when estimating these lead/lag patterns. Second, lead/lag effects may also be present for other features of returns, such as volatility and liquidity. [Chordia et al. \(2005, 2007\)](#) analyze the joint dynamics of returns, volatility, and liquidity of the stocks and find that volatility and liquidity are also informative in predicting these features. Therefore, allowing for lead/lag patterns also for volatility in addition to returns may be instrumental in capturing these effects.

Following existing studies, we estimate our model allowing for imperfect synchronization between the cyclical dynamics of financial returns using US size-sorted stock portfolio returns to facilitate comparison with earlier findings. Specifically, using monthly US large-, mid-, and small-cap portfolio returns from 1963 until 2017, we find a three-regime model with regime-dependent heteroskedasticity and correlations, in addition to the regime dependence in the means, captures the dynamics of portfolio returns most adequately. Results indicate that allowing for different volatilities and correlations is crucial for identifying multiple regimes. The regime-dependent characteristics of both the mean and volatilities link these regimes closely to the business cycle phases of recessions and expansions. The differences in conditional mean and volatility across size-sorted portfolios indicate that small-cap portfolio returns are more sensitive to the cyclical phases than the large-cap portfolios confirming the findings on the ‘financial

accelerator theory' of [Gertler and Gilchrist \(1994\)](#).³ The regimes can be characterized as 'boom' (high average returns with moderate volatility and low correlation), 'moderate' (mean returns close to zero, low volatility, and high correlations), and 'bear/crash' (negative mean return, high volatility, and high correlation). Interestingly, the model implies a uniform pattern in terms of the cyclical behavior of returns. While the bear market regime is mainly followed by a boom market regime reflecting a bounce-back effect, a moderate regime almost always follows the boom market regime. Crucially, we find heterogeneity in the phase shifts across different regimes of size-sorted portfolios. Our findings reveal that the large- and mid-cap portfolios lead the small-cap portfolio for switches to the moderate regime (following a boom regime) by a single month, while they switch contemporaneously into the boom and crash regimes. This delay suggests that negative news 'travels slowly' when transitioning from the times of recovery to moderate times, in the sense that it is incorporated only gradually into small stocks. Therefore, their prices adjust with a delay compared to the prices of medium and large stocks. Finally, we show that information diffusion accelerates in response to specific sorts of macroeconomic information, in particular, to surprises related to the monetary policy confirming the findings of [Chordia et al. \(2007\)](#), [Smales \(2017\)](#) and [Kontonikas and Zekaite \(2018\)](#).

We complement our statistical model evaluation with a utility based evaluation to examine whether our modeling framework allowing for imperfect synchronization of size-sorted portfolio returns translates into profitable investment strategies. Our results show that an investor with a power utility function who constructs a portfolio using size-sorted risky assets and a risk-free asset would be willing to pay an annual performance fee of several hundreds of basis points to switch from the predictions offered by the benchmark models with varying number of regimes and varying types of synchronization to those of the three-regime model and heterogeneous phase shifts.

The remainder of the paper is organized as follows. In Section 2 we describe the general Markov-switching VAR model with different phase shifts for multiple regimes and with regime-dependent heteroscedasticity. In Section 3 we discuss the empirical findings of our model framework with full details on estimation methodology provided in the supplementary material. Section 4 discusses the effect of various sorts of macroeconomic news on the speed of information diffusion among size sorted portfolios. Section 5 evaluates the economic value of our model specification. Finally, we conclude in Section 6.

2. Model specification

In this section, we put forward the MS-VAR model with imperfect synchronization due to phase shifts of a single common cycle for modeling the joint dynamics of size-sorted portfolio returns. The key feature of the model is that the amount of phase shift can be different across regimes allowing for potentially asymmetric lead-lag pattern between these returns. Furthermore, it permits quite a flexible specification of the financial cycle with multiple number of regimes.

Let $y_{1,t}$, $y_{2,t}$, and $y_{3,t}$ denote the observations of the large-, mid-, and small-cap portfolio excess returns, respectively, in period t . We assume that the J regimes are characterized by different means and variances of $y_{1,t}$, $y_{2,t}$, and $y_{3,t}$ and by contemporaneous correlations

between these returns. In case of first order autoregressive dynamics for the portfolio returns, our assumptions imply the model specification

$$Y_t - \mathcal{M}_{S_t} = \Phi(Y_{t-1} - \mathcal{M}_{S_{t-1}}) + \varepsilon_t, \quad \text{with } \varepsilon_t \sim N(0, \Sigma_t), \quad (1)$$

with

$$Y_t = \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix}, \quad S_t = \begin{pmatrix} S_{1,t} \\ S_{2,t} \\ S_{3,t} \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}, \quad \mathcal{M}_{S_t} = \begin{pmatrix} \mu_{1,S_{1,t}} \\ \mu_{2,S_{2,t}} \\ \mu_{3,S_{3,t}} \end{pmatrix},$$

$$\Phi = \begin{pmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} \\ \phi_{2,1} & \phi_{2,2} & \phi_{2,3} \\ \phi_{3,1} & \phi_{3,2} & \phi_{3,3} \end{pmatrix}.$$

Here $S_{l,t}$, $l = 1, 2, 3$ are latent multinomial variables taking the value j if $y_{l,t}$ is in regime j at time t for $j = 1, \dots, J$. Moreover, $\mu_{l,S_{l,t}} = E[y_{l,t} | S_{l,t}]$ denotes the mean of return, $y_{l,t}$, in the regime indicated by $S_{l,t}$ and the error terms follow a multivariate Normal distribution with covariance matrix Σ_t as

$$(\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t})' \sim NID(0, \Sigma_t). \quad (2)$$

For now, we do not specify Σ_t explicitly, but we return to this in detail below.

Next, we specify the dynamic properties of the unobserved regime indicators $S_{1,t}$, $S_{2,t}$, and $S_{3,t}$. We assume that $S_{1,t}$, $S_{2,t}$, and $S_{3,t}$ are first-order J -state homogenous Markov processes with transition probabilities

$$\Pr(S_{l,t} = j | S_{l,t-1} = i) = p_{ij,l}, \quad \text{for } i, j = 1, \dots, J \text{ and } l = 1, 2, 3.$$

Without loss of generality, we assume that $y_{1,t}$, i.e. the large-cap portfolio return, is the 'reference series'⁴ and for ease of exposition, here we only define the properties of $S_{2,t}$, the regime indicator of $y_{2,t}$, i.e. the mid-cap portfolio return, relative to $S_{1,t}$. For the lead/lag structure between $S_{1,t}$ and $S_{3,t}$, the exposition is identical except the fact that $S_{2,t}$ is replaced by $S_{3,t}$. Different specifications of the relation between the two Markov processes $S_{1,t}$ and $S_{2,t}$ imply different types of relations between the cycles of the two portfolio returns. Two extreme cases can be distinguished. First, we can assume that $S_{1,t}$ and $S_{2,t}$ are identical, that is,

$$S_{2,t} = S_{1,t}, \quad (3)$$

or, put differently, there is a single cycle governing both returns. Following [Harding and Pagan \(2006\)](#), we refer to this case as 'perfect synchronization' (PS). Second, we may assume that $S_{1,t}$ and $S_{2,t}$ are independent, that is, each portfolio return has its own idiosyncratic cycle. Note that this specification does not completely rule out the synchronization of the cycles in the two variables. In fact, letting p_{lj} denote the unconditional probability that $S_{l,t} = j$ for $l = 1, 2$ and $j = 1, \dots, J$, the two cycles are in the same regime with probability

$$\Pr(S_{2,t} = S_{1,t}) = \sum_{j=1}^J p_{1,j} p_{2,j} > 0.$$

In practice, the degree of synchronization between cycles in

³ The financial accelerator theory suggests that since large firms enjoy easier access to credit compared to small firms, their access to credit enables them to borrow and carry inventories, unlike small firms, even in the case of credit market shocks.

⁴ As large-cap portfolio closely track the returns on market index with a correlation coefficient of 0.97, selecting large-cap portfolios as reference series also measures the synchronization of size-sorted portfolios with respect to the market portfolio.

different portfolio returns may not be perfect but still higher than the expected level under independence as given above, see the empirical evidence in [Candelon et al. \(2008\)](#), among others. Hence, neither perfect synchronization nor independence may be adequate representations of the relation between cycles. A natural and elegant approach to accommodate this possibility is to assume that the cycle in large-cap portfolio returns, $y_{1,t}$, lead/lag the cycle in mid-cap portfolio returns, $y_{2,t}$, by $\kappa_{2,S_{1,t}}$ periods. This specification can be formulated as

$$S_{2,t-\kappa_{2,S_{1,t}}} = S_{1,t}. \quad (4)$$

(4) implies that there is a common cycle but it affects the different variables with a certain phase shift that are potentially different across regimes. The subscript $S_{1,t}$ to κ_2 indicates that the regime indicator is shifted by a possibly different number of time periods for each regime in the common cycle. Therefore, the specification involves regime-dependent phase shift parameters, $\kappa_{2,j}$ $j = 1, \dots, J$, in addition to regime dependence in the first and second moments of the return distributions. To put things differently, we assume that the lead/lag time is different per regime, such that each regime in the other series starts later or earlier by $\kappa_{2,j}$ periods. This specification is denoted as imperfect synchronization of the cycles with ‘asymmetric’ phase shifts (APS), where asymmetry refers to the fact we allow for distinct phase shifts across regimes. When $\kappa_{2,S_{1,t}} \equiv \kappa_2$, that is, when phase shifts are restricted to be identical across regimes, the model boils down to the case of imperfect synchronization with ‘symmetric’ phase shifts (SPS) in [Hamilton and Perez-Quiros \(1996\)](#). Notice that, when the number of regimes is two the specification in (4) encompass the specification in [Paap et al. \(2009\)](#) to capture the different lead time of the Conference Board leading indicator in leading US business cycle peaks and troughs. Obviously, if $\kappa_{2,j} = 0$ $j = 1, \dots, J$ equals zero, it reduces to the perfect synchronization case in (3). Detailed discussion of specification (4) is provided in Section A of the supplementary material.

The specification in (4) is not sufficient though, as it may lead to situations where for some time periods $S_{2,t}$ is assigned multiple values or is not defined at all. In these cases, when the regime with the larger amount of phase shift is assigned to such conflicting periods, it is guaranteed that $S_{2,t}$ is assigned only a single regime and each regime starts with a phase shift of $\kappa_{2,j}$ periods relative to $S_{1,t}$. To illustrate this explicitly, consider a transition from regime i to j that takes place in the large-cap portfolio return, $y_{1,t}$, in period t . The specification (4) implies that the regimes i and j in the mid-cap portfolio return, $y_{2,t}$, are shifted by $\kappa_{2,i}$ and $\kappa_{2,j}$ periods relative to $S_{1,t}$, respectively. In case of $\kappa_{2,i} > \kappa_{2,j}$, $S_{2,t}$ is not assigned any values for the time periods $t - \kappa_{2,i}, \dots, t - 1 - \kappa_{2,j}$. When regime i is assigned for these periods, as its phase shift parameter is larger than that of regime j , the final specification implies that regime j in the mid-cap portfolio return, $y_{2,t}$, starts $\kappa_{2,j}$ periods earlier (when $\kappa_{2,j}$ is positive) or later (when $\kappa_{2,j}$ is negative) than the regime j in the large-cap portfolio return, $y_{1,t}$. In case of $\kappa_{2,j} > \kappa_{2,i}$, $S_{2,t}$ is assigned multiple values for the time periods $t - \kappa_{2,j}, \dots, t - 1 - \kappa_{2,i}$. Assigning regime j for these periods, as its phase shift parameter is larger than that of regime i , we reach the same conclusion as in the initial case.⁵ Consequently, when regime j in the large-cap portfolio return,

$y_{1,t}$, is further followed by regime k , then regime j in the mid-cap portfolio return is $\kappa_{2,j} - \kappa_{2,k}$ periods longer (when $\kappa_{2,j} > \kappa_{2,k}$) or shorter (when $\kappa_{2,j} < \kappa_{2,k}$) than regime j in the large-cap portfolio return.

Finally, we return to the specification of the structure of the covariance matrix to finalize the general model framework. As discussed before, in many models of asset returns assuming homoskedasticity for the error terms may not be an appropriate assumption. In fact, regime-dependent variances and co-variances may be crucial for accurate identification of the regimes. In case of perfect synchronization, it is straightforward to specify a regime-dependent covariance matrix. The value of the variances and correlations just depend on the value of $S_{1,t}$. Imperfect synchronization, however, leads to the case that the different series are in different regimes in a given period due to phase shifts (either symmetric or asymmetric) of the common cycle for some of the series. To allow for this possibility we decompose the covariance matrix into the variances and correlations that is

$$\Sigma_t = D_t R_t D_t, \quad (5)$$

where $D_t = \text{diag}(\sigma_{1,t}, \sigma_{2,t}, \sigma_{3,t})$ is a diagonal matrix with standard deviations of the error terms as diagonal elements and R_t is a symmetric 3×3 matrix with ones on the diagonal and the correlations $\rho_{lm,t}$ as the off-diagonal elements for $l, m = 1, 2, 3$ and $l \neq m$, (see, for example, [Barnard et al., 2000](#)). The decomposition in (5) allows us to relate the variances of three series to their individual regime indicator $S_{1,t}$, $S_{2,t}$ and $S_{3,t}$. This leads to the following specification

$$\begin{aligned} \sigma_{l,t} &= \sigma_{l,S_{1,t}} & \text{for } l = 1, 2, 3 \\ \rho_{lm,t} &= \rho_{lm,S_{1,t}} & \text{for } l, m = 1, 2, 3 \text{ and } l \neq m \end{aligned} \quad (6)$$

where we use the regime indicator of the reference series, $S_{1,t}$, to describe the regime of the correlation parameters.⁶

3. Lead/lag effects in size-sorted portfolio returns

In this section, we use the trivariate MS-VAR model as given in (1) with (2), with imperfect synchronization due to asymmetric phase shifts as specified in (4) and covariance matrix as in (5)–(6) to analyze the degree of synchronization in monthly excess returns of size-sorted portfolios. For conducting a comprehensive analysis, we estimate MS-VAR models with two, three and four regimes. For the relation between the regime dynamics in the three portfolio returns series, we consider the three possibilities discussed in Section 2, namely (i) perfect synchronization (PS); (ii) Imperfect Synchronization with Symmetric Phase Shifts of the common cycle (IS-SPS); and (iii) Imperfect Synchronization with Asymmetric Phase Shifts (IS-APS).⁷

3.1. Data

The dataset of the size-sorted portfolios is based on all NYSE, AMEX, and NASDAQ stocks over the period July 1963–January 2017.

⁵ Alternatively, we could assign the regime with the ‘smaller’ amount of phase shift for the conflicting periods when $S_{2,t}$ is assigned multiple values or is not defined at all. In this case, each regime ‘ends’ with a phase shift of $\kappa_{2,j}$ periods relative to $S_{1,t}$. This is observationally equivalent to the previous case with simply another identification scheme and associated interpretation for the lead/lag parameters. Hence, the decision rule opted here does not impose any ad-hoc restriction on the final lead/lag structure.

⁶ We also estimate models with the correlation regimes being determined by the regimes of the mid- and small-cap portfolio's expected return and volatility. Overall these models perform worse in terms of marginal and predictive likelihood values. Results are available upon request.

⁷ We also estimate models where we assume that the regime dynamics of the covariance matrix is governed by the cycle embedded in the large-cap portfolio returns to examine the lead/lag effects only in expected excess return. Overall these models perform worse in terms of marginal and predictive likelihood values. Results are available upon request.

We consider monthly returns on equal-weighted large-, mid- and small-cap portfolios, consisting of the 20% largest, 20% at the center and 20% smallest stocks sorted according to market capitalization, respectively.⁸ We use the returns in excess of the risk-free rate.⁹

3.2. Estimation details

We use a Bayesian approach for estimation and inference in the MS-VAR model with imperfect synchronization due to asymmetric phase shifts using Markov Chain Monte Carlo (MCMC) techniques. Specifically, we use Metropolis within Gibbs sampling together with data augmentation (see Geman and Geman, 1984; Tanner and Wong, 1987) to obtain posterior results.¹⁰ The posterior distribution for the model parameters is proportional to the product of the likelihood function and the prior distributions for model parameters. While for the likelihood specification we make use of the multivariate Normal distribution following the convention, for the prior specifications of the model parameters, we specify mildly informative proper distributions. Here, we provide details on the prior specification of the lead/lag parameters as these are essential for estimation of the MS-VAR model with varying degrees of synchronization. Details on inference including likelihood function, prior specifications of remaining model parameters, and the resulting posterior distribution along with the simulation scheme is provided in Section B of the supplementary material.

The parameters of key interest are the lead/lag times or phase shifts κ_l , $l = 2, 3$ where $\kappa_l = (\kappa_{l,1}, \dots, \kappa_{l,J})'$. Specific values of those enable us to impose restrictions to obtain the model with PS and imperfect synchronization with SPS. We use informative priors to achieve this. Specifically, we use a discrete uniform prior where we assign equal probability to each possible value of $\kappa_{l,j}$ in a predefined set C_l , that is

$$f(\kappa_l) \propto \begin{cases} 1 & \text{for all } \kappa_l \in C_l, \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

The information content of the prior distribution is reflected through C_l , as specification of the set C_l determines the type of synchronization that is assumed. For example, when $C_l = \{\kappa_l \in \mathbb{Z}^J \mid -c_{l,j} \leq \kappa_{l,j} \leq c_{l,j}, j = 1, \dots, J\}$ for certain positive valued $c_{l,j}$, $j = 1, \dots, J$ and $l = 2, 3$, we allow for imperfect synchronization with the phase shifts being restricted only by the bounds $c_{l,j}$. Additional restrictions may be imposed such that, for example, the difference between the phase shifts of the distinct regimes cannot exceed a certain threshold d_l . In this case, C_l can be specified as $C_l = \{\kappa_l \in \mathbb{Z}^J \mid -c_{l,j} \leq \kappa_{l,j} \leq c_{l,j}, |\kappa_{l,i} - \kappa_{l,j}| \leq d_l, i, j = 1, \dots, J, l = 2, 3\}$. Note that setting d_l equal to zero (while $c_{l,j} > 0$ for $j = 1, \dots, J$) results in the IS-SPS model where phase shift parameters are identical across regimes, that is, the model corresponding with the specification of Hamilton and Perez-Quiros (1996). Setting $c_{l,j} = 0$ we obtain an MS-VAR model with perfect synchronization as in (3). In our empirical analysis, we set $c_{l,j} = 2$ and $d = 1$ for the models of imperfect

synchronization with asymmetric phase shifts.¹¹

Finally, we do not include autoregressive dynamics in any of the models following the evidence in Guidolin and Timmermann (2006). This may seem restrictive, especially in light of Boudoukh et al. (1994) who argue that lead/lag effects among size-sorted portfolios may be spurious as they are in fact, to a large extent, due to contemporaneous correlation and own-autocorrelation of small-caps. This suggests large-caps does not necessarily lead small-caps once lagged small-cap portfolio returns are included to account for these autocorrelation effects. With this in mind, we also estimate all models with the first lag of the small-cap returns (as well as with unrestricted first-order autoregressive dynamics) included. The results, especially those related to lead/lag parameters, are very similar to those reported here.¹²

3.3. Evaluation of models

The MS-VAR model of size-sorted portfolio returns with imperfect synchronization with asymmetric phase shifts provides a flexible and intuitive framework for characterizing partial synchronization of cycles embedded in these returns. However, moving away from more restricted but more parsimonious forms of synchronization may also result in overfitting. Therefore, we want to establish the potential of the unrestricted model by comparing it with more restricted (nested) alternative models with symmetric phase shifts and with perfect synchronization.

First, we focus on statistical evaluation of models and conduct marginal likelihood based comparisons, e.g. Bayes factors. A drawback of marginal likelihoods is its sensitivity to the specifications of prior distributions. Despite the fact that we use quite mildly informative priors, we still want to support our comparison with a more robust metric of model evaluation. An alternative way of model comparison is by means of predictive Bayes factors computed using predictive likelihoods. This approach has the advantage that it is not affected by the choice of prior distributions and overfitting, while it is directly related to the posterior model probabilities. For a given model, the predictive likelihood of the observation at t_0+1 , Y_{t_0+1} , conditional on the previous observations Y^{t_0} , is given by

$$f(Y_{t_0+1}|Y^{t_0}) = \int f(Y_{t_0+1}|\theta) f(\theta|Y^{t_0}) d\theta, \quad (8)$$

where $p(\theta|Y^{t_0})$ is the posterior distribution of the model parameters gathered in the parameter set, θ , given the observations until t_0 , and $p(Y_{t_0+1}|\theta)$ is the density of the observation Y_{t_0+1} , which can be written as

$$f(Y_{t_0+1}|\theta) = \sum_{j=1}^J f(Y_{t_0+1}|S_{1,t_0+1}=j, \theta) f(S_{1,t_0+1}=j|\theta, Y^{t_0}). \quad (9)$$

We can use the posterior simulator to obtain the distribution of the model parameters and estimate the predictive likelihood by $G^{-1} \sum_{g=1}^G f(Y_{t_0+1}|Y^{t_0}, \theta^{(g)})$, where G is a large number of draws from the posterior distribution, see Frühwirth-Schnatter (2006), for details. For evaluating the predictive likelihood, we use the sample

⁸ The data is obtained from Kenneth French's website data library <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.

⁹ For the risk-free rate we use Fama-Bliss risk free rates from CRSP (Center for Research in Security Prices). This series is discontinued in December 2001. For the remaining part of the sample period we use the 4-week T-Bill rate measured at close on the last trading day of each month from the St. Louis Federal Reserve (<http://research.stlouisfed.org/fred2/>).

¹⁰ The main reason for adopting a Bayesian approach is that we treat the lead/lag times between the cyclical components of the portfolio returns, $\kappa_{l,j}$, $j = 1, \dots, J$ and $l = 2, 3$, as unknown parameters to be estimated along with the other model parameters. Given that the $\kappa_{l,j}$'s are discrete, a frequentist approach (such as maximum likelihood combined with the EM algorithm) is infeasible for a joint inference of all model parameters.

¹¹ This setting facilitates the inference substantially, in particular, for the four-regime models where identification of the lead/lag parameters can be intractable for large values of $c_{l,j}$ due to insufficient number of transitions between regimes. We also estimate models with less restrictive sets of parameters as $c_{l,j} = 4$ and $d = 2$. Results are very similar and available upon request.

¹² Here we do not display these results for the sake of brevity but these are available upon request.

from July 1963 until December 2000 for estimation of the model and the remaining period from January 2001 until January 2017 to compute the predictive likelihood. For the complete out-of-sample period the predictive likelihood is computed using the posterior distribution based on the initial estimation sample, as recursive updating is not feasible in terms of computation time.

3.4. Empirical findings

In this section, we discuss the findings obtained by estimating the competing models, which are the MS-VAR model with perfect synchronization of cycles (PS); with imperfect synchronization with symmetric phase shifts of the common cycle (IS-SPS); and with imperfect synchronization with asymmetric phase shifts of the common cycle (IS-APS). The first two columns of Table 1 display the (log-) marginal likelihood and predictive likelihood values of the competing models.

The first two columns of Table 1 offer three clear-cut conclusions.

First, independent of the number of regimes, we find that the marginal and predictive likelihood values for the specifications with perfect synchronization and with imperfect synchronization with symmetric phase shifts are (almost) identical. This is due to the fact that the phase shift parameters in the IS-SPS models are estimated to be equal or very close to zero, which reduces this specification to the PS model.

Second, independent of the number of regimes, we find that the

Table 1
Comparison of the models estimated using LC, MC and SC Data.

	Statistical Evaluation		Economic Evaluation
	Marginal	Predictive	Performance
	Likelihood	Likelihood	Fee, Δ
2 Regimes			
PS	−5024.8	−1456.8	0.0
IS-SPS	−5024.8	−1456.8	0.0
IS-APS	−4987.4	−1458.2	197.7
3 Regimes			
PS	−5044.6	−1442.8	−13.4
IS-SPS	−5044.6	−1442.8	37.6
IS-APS	−4975.9	−1417.5	247.7
4 Regimes			
PS	−5098.3	−1525.6	58.8
IS-SPS	−5098.3	−1525.6	58.8
IS-APS	−5092.2	−1521.0	90.9

Note: The first two columns of the table presents statistical evaluation outcomes using marginal likelihoods and predictive likelihoods (in logarithms) of the competing models with (i) different number of regimes; (ii) different types of synchronization in expected returns and variances. The last column of the table presents economic values as measured by the performance fee (in basis points), Δ as shown in (14), for switching to competing models with (i) different number of regimes; (ii) different types of synchronization in expected returns and variances from the two-regime model with perfect synchronization. The models are applied to monthly excess returns on large-, mid- and small-cap portfolios for the sample period starting from July 1963 until January 2017. Marginal likelihoods are those obtained when the models are estimated for the complete sample period. Predictive likelihood values and performance fees are computed using the predictive distribution from estimation using the sample starting from the sample period July 1963–December 2000 and predicting the value of January 2001 and each time adding one observation until the prediction of the value at January 2017. The regime dynamics in the expected excess returns and variances correspond with (i) perfect synchronization (PS); (ii) imperfect synchronization with a single phase shift of the common cycle (IS-SPS); and (iii) imperfect synchronization with regime-specific phase shifts (IS-APS). Finally, correlations switch regimes according to the regime indicator for the expected excess returns of the large-cap portfolio. Posterior results are based on 20 000 simulations of which the first 10 000 are discarded for burn-in.

marginal and predictive likelihood values for the specifications with imperfect synchronization due to asymmetric phase shifts are higher than those for the PS and SPS specifications. This suggests that allowing for regime-specific lead/lag times is appropriate for size-sorted portfolios. These differences in likelihood values further increase when we switch from two-regime to three-regime models reflecting the importance of allowing for multiple regimes.

Third, for the four-regime models, we find that the differences in likelihood values between the specification with imperfect synchronization due to asymmetric phase shifts and the PS (and SPS) specifications are quite minor compared to those for the two and three-regime models. It seems that the phase shifts captured by IS-APS specification with three regimes are replaced by the additional fourth regime in the PS specification with four regimes. However, this additional flexibility, which may lead to overparameterization of the model, is penalized by both of the likelihood based metrics as these take the parameter uncertainty into account.

In sum, the results in Table 1 lead us to select the model with three regimes and imperfect synchronization with asymmetric phase shifts in the expected excess returns and the volatilities as the preferred specification. In the remainder of this section, we, therefore, focus on the estimation results for this model specification.

Table 2 summarizes the posterior results of the model parameters.

These indicate that the three regimes in the model have distinct features. A general pattern is that in each regime the volatility of the portfolios exhibits a monotonic decline with the small-cap portfolio having the highest volatility and the large-cap portfolio having the lowest, as expected. However, this higher volatility of the small-cap portfolio comes with a higher mean return (indicating a risk-return trade-off) only for the first regime. Regime 1 is characterized by a large positive mean return for all portfolios together with low volatility (especially for large caps), capturing a ‘boom’ market. For all portfolios regime 2 has mean return close to zero and relatively low volatility. An interesting result is that the volatility in the boom market is lower than the moderate regime for the large- and mid-cap portfolios. Regime 3 can be characterized as a ‘bear’ market, as it shows a substantially negative mean return accompanied by high volatility, indicating the large uncertainty during these time periods. The estimates of transition probabilities between these three regimes are displayed in Table 3.

Table 2
Posterior results for three-regime MS-VAR model allowing for imperfect synchronization with asymmetric phase shift.

		Regime 1	Regime 2	Regime 3	
Large Caps	Mean- $\mu_{1,j}$	2.45 (0.40)	0.37 (0.18)	−0.89 (0.55)	
	Volatility- $\sigma_{1,j}$	3.31 (0.33)	3.42 (0.14)	7.71 (0.56)	
Mid Caps	Mean- $\mu_{2,j}$	3.91 (0.42)	0.30 (0.22)	−1.13 (0.63)	
	Volatility- $\sigma_{2,j}$	3.66 (0.34)	4.13 (0.16)	9.29 (0.63)	
Small Caps	Mean- $\mu_{3,j}$	4.66 (0.43)	−0.28 (0.24)	−1.30 (0.67)	
	Volatility- $\sigma_{3,j}$	4.97 (0.51)	4.16 (0.21)	10.15 (0.86)	
		Correlation- $\rho_{12,j}$	0.89 (0.03)	0.91 (0.01)	0.92 (0.01)
		Correlation- $\rho_{13,j}$	0.51 (0.11)	0.78 (0.02)	0.73 (0.04)
		Correlation- $\rho_{23,j}$	0.73 (0.06)	0.91 (0.01)	0.88 (0.02)

Note: The table presents posterior means and standard deviations (in parentheses) of parameters in the MS-VAR model with three regimes allowing for imperfect synchronization with asymmetric phase shifts estimated for monthly excess returns of large-cap, mid-cap and small-cap portfolios over the period July 1963–January 2017. The volatilities $\sigma_{l,j}$ have the same regime dynamics as the corresponding expected excess returns $\mu_{l,j}$, $l = 1, 2, 3$. The correlations $\rho_j = (\rho_{12,j}, \rho_{13,j}, \rho_{23,j})'$ have the same regime dynamics as the mean and volatility of the large-cap portfolio. Posterior results are based on 20 000 simulations of which the first 10 000 are discarded for burn-in.

Table 3
Posterior results for transition probabilities and lead/lag parameters.

	Transition probabilities p_{ij}		
	Regime 1	Regime 2	Regime 3
Regime 1	0.554 (0.080)	0.416 (0.080)	0.030 (0.027)
Regime 2	0.083 (0.022)	0.870 (0.023)	0.047 (0.015)
Regime 3	0.158 (0.052)	0.026 (0.024)	0.816 (0.053)
Phase shifts κ_j			
Mid-Caps	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Small-Caps	0.000 (0.000)	−1.000 (0.000)	−0.037 (0.188)

Note: The table presents posterior means and standard deviations (in parentheses) of transition probabilities and lead/lag parameters in the MS-VAR model with three regimes allowing for imperfect synchronization with asymmetric phase shifts. See Table 2 for details.

The transition probability estimates reveal an interesting pattern in the relative duration of the different regimes and the cyclical dynamics in the portfolio returns. The probability of staying in the boom regime is 0.55 and hence the duration of this regime is short, on average only two months. As the transition probability of 0.87 indicates, the average duration of the moderate regime 2 is longer than the other regimes with almost eight months. The probability of staying in the bear regime is 0.82 implying an average duration of about five months. The transition from the bear regime is much more likely to happen to the boom market regime than to the moderate regime, with transition probabilities of 0.16 and 0.03, respectively. This suggests that bear markets are most often followed by a (short) period with high returns, providing evidence of a ‘recovery’ or ‘bounce-back effect’ in these returns, similar to findings for the US business cycle in, for example, Kim et al. (2005). By contrast, when the bull regime is left, this is more than ten times more likely to occur to the moderate regime than to the bear regime, with transition probability of 0.42 and 0.03, respectively. Hence, the boom regime typically does not end with a crash, but is followed by a period with moderate returns. These findings suggest a uniform cyclical pattern of size-sorted portfolio returns in the sense that the bear markets regime is often followed by a boom regime which in turn is succeeded by a moderate regime.

We display the posterior regime probabilities of large- and small-cap portfolio returns in Fig. 1.¹³ We also include recessions as dated by the NBER in shaded areas to elaborate the links between the regime timings with the business cycle.

First, from the graphs, we can infer that the regimes are estimated quite precisely. In general, the regime probabilities are close to zero or one, although there is some uncertainty about the regime assignment in a few periods. The close link between the regime probabilities and the fluctuation of the business cycle is evident in Fig. 1. The bear market regime, for example, mostly occurs before and during the start of the recessions. As is well known, returns, especially return on the market index, reflecting the market expectations serve as an important leading indicator of the business cycle. In addition, this regime captures all stock market crashes that occurred during the sample period, such as ‘Black Monday’ (October 1987), the Russia crises (August–September 1998), the collapse of the dot-com bubble (2000–1), 9/11 (September 2001) and European sovereign debt crisis (August 2011). The boom regime mostly occurs at the end of recessions (among others). In addition, except for the recession in 1981–2, the moderate regime appears considerably less frequently during recessions than during

expansions.

The evidence in Table 2 and Fig. 1 shows that the estimated cycle nicely captures the financial cycle embedded in size-sorted portfolios. This financial cycle, in turn, is closely connected to the business cycle because distinct phases of the financial cycle are often associated with the business cycle phases of expansion and recession. The estimates in Table 2 indicate that the responses of size sorted portfolios to business cycle are distinct depending on their market capitalization. Specifically, our findings suggest that the sensitivity of the portfolios to the business cycle increases with the decreasing size in all phases of the cycle with the more significant returns and volatility of small-cap portfolio returns compared to large and mid-caps, confirming the findings of Gertler and Gilchrist (1994). In addition, the difference in sensitivity is most prominent for the third regime, which is closely connected to recessions. This finding contrasts with those provided in Chari et al. (2013) and Kudlyak and Sánchez (2017) reporting that this picture reversed during the great recession of 2008.¹⁴ The difference might be because we endogenously estimate the financial cycle and link it to the business cycle rather than using NBER recession dates as exogenously determined recessions. Endogenizing cyclical patterns results in differences in the timing of the third regime of the financial cycle and the NBER recession dates.

As an alternative way of assessing the regime dynamics, Fig. 2 shows the implied model volatilities, obtained as the weighted average of the regime-specific volatilities with the posterior regime probabilities as weights, together with the monthly realized volatility constructed from the daily portfolio returns.¹⁵

It is seen that the patterns in realized volatility are captured fairly well by the model, in the sense that periods with a higher level of realized volatility generally are matched by increased ‘model-implied’ volatility.

Finally, we turn to the estimates of the phase shift parameters to assess the implications of the model concerning the lead-lag effects of the size-sorted portfolios. The posterior results of the phase shift parameters κ_{lj} for $l = 2, 3$ are shown in the bottom panel of Table 3. In addition, in Fig. 3 we display the (bivariate) histogram of the lead/lag parameters related to small caps, $\kappa_{3,j}$.¹⁶

The estimates provide convincing evidence for the presence of asymmetric phase shifts only for small-cap portfolio returns but the cyclical behavior of large- and mid-cap portfolios are perfectly synchronized. When we consider small-cap portfolio returns, we find large and small stocks to enter into the boom regime contemporaneously. For this regime, we find that all posterior probability mass is located at $\kappa_{2,1} = 0$. For the bear regime, there is only a modest probability that $\kappa_{2,3} = -1$. In contrast, for the moderate regime, the large-cap portfolio leads the small-cap portfolio by one month. The posterior uncertainty for this parameter is very small.¹⁷

¹⁴ We thank the anonymous referee for pointing out this critical link of ‘financial accelerator theory’.

¹⁵ Specifically, the monthly realized variance is computed as the sum of squared daily returns plus twice the sum of the cross-product of returns on consecutive trading days. The latter provides a correction for the presence of autocorrelation in the daily returns, see Zhou (1996).

¹⁶ As can be seen from the bottom panel of Table 3, for mid-cap portfolio returns, all posterior probability mass is gathered around 0 for all lead/lag parameters. Therefore, we do not display the posterior distribution of these parameters for the sake of brevity.

¹⁷ Note that we impose strong priors to deal with the uncertainty embodied in the data and to facilitate inference. Still, the prior specification assigns equal probability to all types of synchronization. Hence, the finding is not affected by this reasonably strong prior, though the amount of variation may be affected by its tightness. Nevertheless, results with weaker priors are very similar to the results shown here as discussed before and are available upon request.

¹³ As the posterior regime probabilities of mid-cap portfolio returns are identical to those of the large-cap portfolio returns we do not include these graphs in Fig. 1.

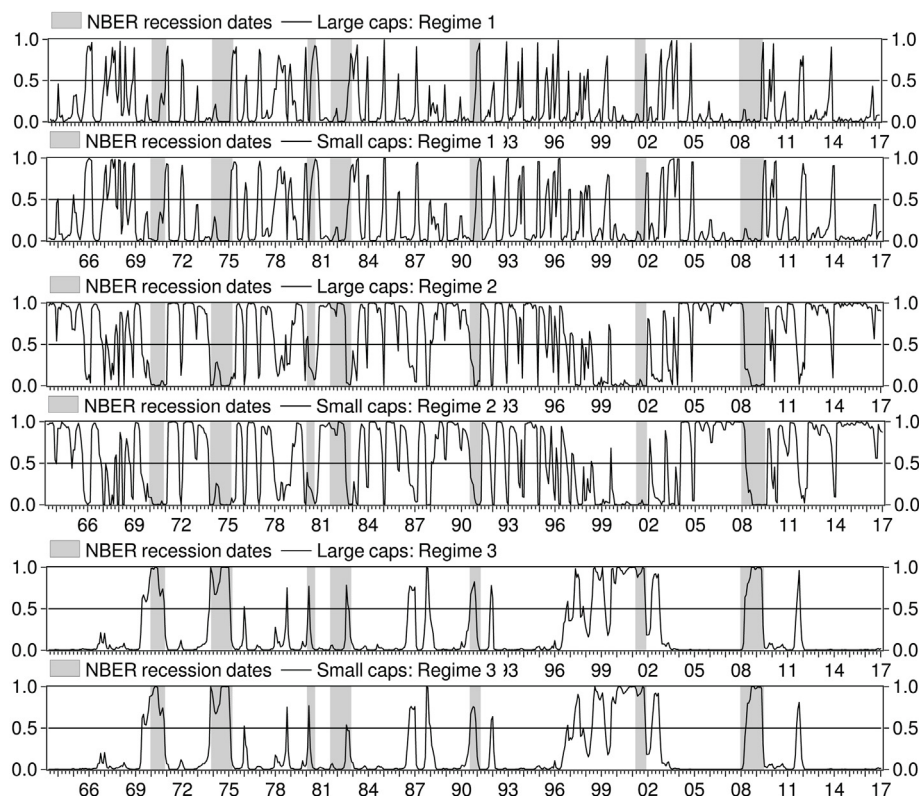


Fig. 1. Posterior probabilities of regimes of LC and SC.

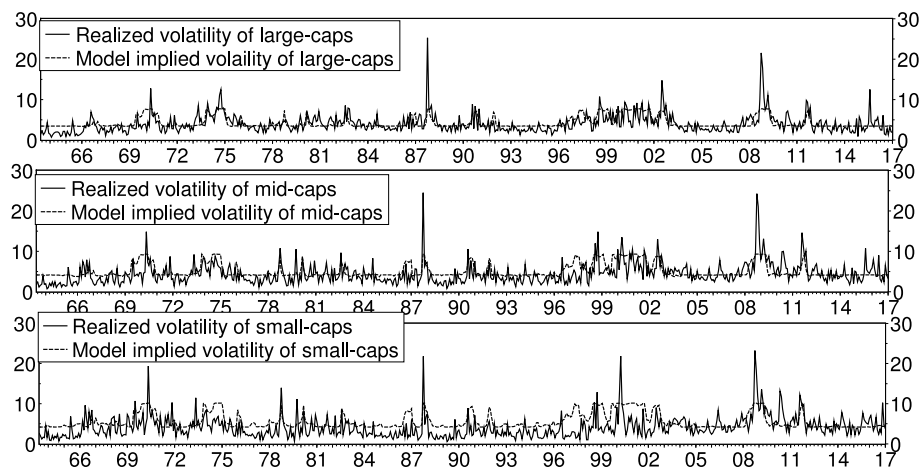


Fig. 2. Comparison of volatilities implied by the model and realized volatilities.

We interpret our findings concerning the phase shift parameters as follows. During transitions from a boom market regime to the moderate times, the relatively negative news signaling the end of the boom market regime is incorporated immediately into the large (and medium) stocks where the speed of adjustment to the newly emerging conditions is fastest. However, this negative news ‘travels slowly’ to the small caps, in the sense that it is incorporated only gradually into small stocks, such that their prices adjust with a delay compared to large (and medium) stocks. This sluggish adjustment of small stocks to the negative news confirms the findings of [Hong et al. \(2000\)](#) and [Hou \(2007\)](#) who report a similar behavior of stocks with differential size and related characteristics. Specifically, [Hong et al. \(2000\)](#) suggest a strong asymmetry in that

small stocks seem to react more sluggishly to negative news compared to good news. Likewise, [Hou \(2007\)](#) documents that small stocks adjust slowly to negative news but not to positive news. As discussed earlier, the moderate regime is mostly preceded by a bull market regime. Therefore, this sluggish adjustment of small stocks to negative news is only present when there is a transition from the bull market regime to the moderate times. However, as displayed in the upper panel of [Table 3](#), there is a negligible but still nonzero probability of 0.03 (0.05) for transitions from the bull market regime (moderate regime) to the bear market regime, which also implies the newly arising negative news leading to bear markets. In this case, the delay time of the small stocks reduces to only 0.04 month as indicated by the estimate of $\kappa_{3,3}$ in

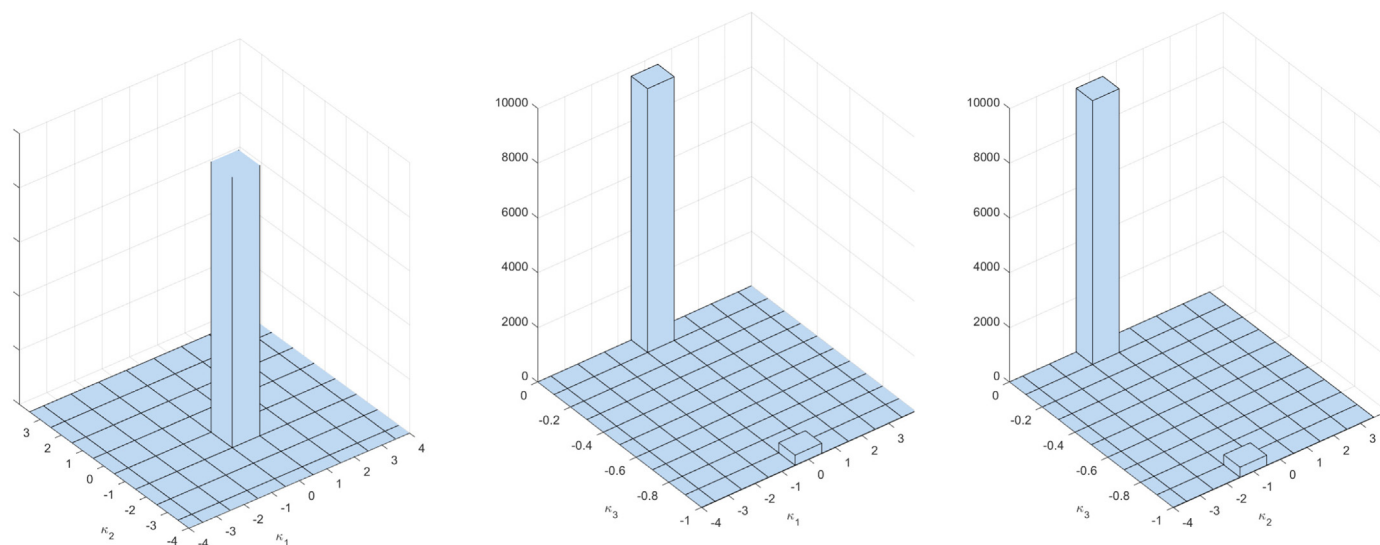


Fig. 3. Bivariate histograms of lead/lag times for LC and SC regimes.

the lower panel of Table 3. This indicates that information diffusion in fact occurs rapidly also for bad news items that have considerable impact on stock prices. Hence, all portfolio returns switch to the extreme bear market regime contemporaneously during rather turbulent times. These findings suggest that our model is capable to capture the differential speed of information diffusion depending both on the type of information but also on the size of the impact of the information. We elaborate this finding further in the next section when we analyze the macroeconomic sources of these phase shifts.

4. Macroeconomic sources of phase shifts

In the previous section, we document evidence on the presence of imperfect synchronization with asymmetric phase shifts between the cycles embodied in size-sorted portfolio returns. We further document that the estimated financial cycle is closely linked to the business cycle. Therefore, in this section, we analyze potential (macroeconomic) sources of these delays of adjustment in response to negative news. To do this, we first construct a measure of the speed of adjustment to the new information for the small-cap portfolio returns. In light of our finding documenting the sluggish adjustment of small caps compared to large caps during transitions to the second regime and to the third regime (with a minor probability), we compute the probability of small caps to transition to the second regime or the third regime one month later than large caps.¹⁸ We denote these as ‘delay probabilities’. This can be obtained by computing the differences between the probabilities of being in the second regime or being in the third regime for the large- and small-cap portfolio returns using the posterior output of regime indicator functions.¹⁹ We display these probabilities of delay in Fig. 4.

From Fig. 4 it is seen that in many periods the probability of observing a delay in adjustment of the small caps is larger than 0.5.

As expected, the frequency of these delays surges mostly during expansions when the bull market rallies are replaced with moderate swings in returns. We, further, construct proxies of macroeconomic news/surprises to analyze the sources of information that accelerate the information diffusion and thus reduce the delay probabilities. Specifically, we estimate a small-scale VAR that involves the Conference Board coincident indicator, as a monthly measure of economic activity, inflation, computed using the consumer price index and FED funds rate. We, then, compute squared residuals from this VAR as a proxy of ‘unanticipated surprises’ to capture various sources of newly arising macroeconomic information. If the adjustment of small caps to the new information is more sluggish when transitioning from the bull market regime to moderate times or for the transitions to the bear market, then new information, put differently, unanticipated surprises should be impotent for the delay probabilities. If, on the other hand, a specific source of information plays an influential role for the speed of adjustment of the prices for both large and small caps, then an increase in the size of the specific surprise would decrease the delay probabilities.²⁰ With this in mind, we run regressions of delay probabilities on proxies of macroeconomic surprises. Table 4 presents the results of these regressions.

As seen in Table 4 for many of the coefficients, specifically for the coefficients related to the surprises to CEI and for the coefficient of the surprises to current inflation, posterior standard deviations are very large indicating that zero is inside the 95% Highest Posterior Density Interval (HPDI). Hence, we conclude that unanticipated surprises to the current and lagged economic activity as well as to the current inflation do not affect the delay probabilities. For these specific sources of information, the adjustment of small caps to the new information seems to be sluggish. The coefficients, for which, zero is outside the 95% HPDI are those of the current and lagged surprises to the policy rate and the lagged inflation surprise. As a robustness check, we run a more parsimonious regression specification including only the current and lagged surprises to the policy rate and the lagged inflation surprise. Posterior means and standard deviations of the parameters estimated using this

¹⁸ Since large and medium caps are perfectly synchronized, we only use large caps together with small caps to facilitate our analysis.

¹⁹ We compute these two cases separately as two different cases of delays. This means that we first compute the differences between the probabilities of being in the second regime and then those for the third regime. The second case almost uniformly yields zero probabilities of delays as also suggested by the very low delay time for this regime displayed in the bottom panel of Table 3.

²⁰ Here, we prefer to proceed with the surprises rather than underlying economic shocks as the surprises are observed in contrast to the latter which should be deduced from observed surprises.

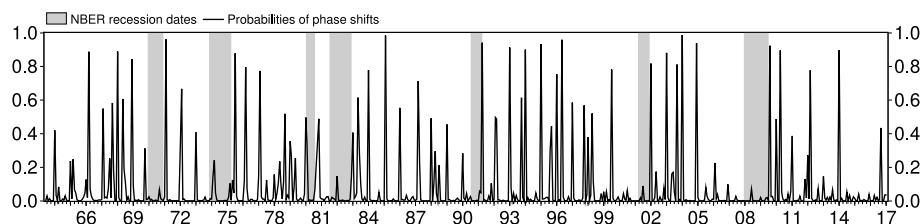


Fig. 4. Probabilities of the small caps to transition with a delay.

Table 4

The effect of macroeconomic surprises on delay probabilities.

	$\varepsilon_{t,CEI}^2$	$\varepsilon_{t-1,CEI}^2$	$\varepsilon_{t,INF}^2$	$\varepsilon_{t-1,INF}^2$	$\varepsilon_{t,FFR}^2$	$\varepsilon_{t-1,FFR}^2$
Model 1	0.0808 (0.0812)	0.0281 (0.0719)	−0.0050 (0.0331)	−0.0578 (0.0312)	−0.0042 (0.0019)	−0.0038 (0.0015)
Model 2				−0.0558 (0.0259)	−0.0036 (0.0018)	−0.0037 (0.0015)

Note: The table presents posterior means and standard deviations (in parenthesis) of the coefficients from regressions of the probabilities that the small-cap portfolio returns adjust to the various sources of macroeconomic surprises with a delay when switching from turbulent to normal times. ε_t^2 stands for the squared residuals, in period t , from a VAR model estimated using the growth rate of Coincident Economic Indicator (CEI), inflation (INF), as measured using percentage change in consumer price index and policy rates (FFR), as measured by the FED funds rate. Models are estimated using the sample period July 1963–January 2017. We use Newey–West HAC covariance matrix for capturing any remaining autocorrelation and heteroskedasticity in the error terms.

specification are displayed in the second row of Table 4. The models uniformly indicate that unanticipated surprises to the ‘monetary policy’ related variables, i.e. FED funds rate and inflation, accelerate the speed of information diffusion as delay probabilities decrease with the increasing size of these monetary policy surprises. This finding is in line with the ‘financial accelerator’ view of Gertler and Gilchrist (1994) in the sense that size serves as an important proxy for financial constraints especially during the times of financial distress. Similarly, Chordia et al. (2007) and Smales (2017) also points the link between the news related to monetary policy changes and the lead/lag effects between the different types of portfolio returns.

Next, we analyze the relation between probabilities of delay of small-cap portfolio returns in adjusting to new negative information and financial conditions for measuring the relation between the delay probabilities and the relative impact of information. As information flow accelerates during turbulent times with financial distress we would expect an increasing impact of information during these periods. Therefore, during times of tight financial conditions it is more likely that small caps adjust to the new information swiftly similar to large caps leading a reduction in delay probabilities. To see this, we perform regressions of delay probabilities on default spread (DS) computed as the difference between yields of Moody’s BAA and AAA corporate bond indexes for measuring the default risk, TED spread (TedS) computed as the difference between 3 months LIBOR and 3 months treasury bill rate for measuring the credit risk, national financial conditions index (FCI) published by the Federal Reserve Bank of Chicago for measuring financial conditions and term spread (TermS) computed as the difference between yields of 10 years treasury bonds and 3 months treasury bills, which is closely linked to the changes in monetary policy. If the adjustment of small caps to the new negative information is sluggish then delay probabilities should not be related to changing spreads during financial turmoil. If, on the other hand, a specific source of information plays an influential role for the speed of adjustment of the prices for both the large and small caps, then an increase in the specific spreads would decrease the delay probabilities. While DS and TermS data are available for the complete sample period, FCI (TedS) is available only for the period

Table 5

The effect of changing financial conditions on delay probabilities.

	DS	TermS	FCI	TedS
Model 1 (July 1986–Jan 2017)	−0.0233 (0.0422)	0.0234 (0.0075)	−0.0216 (0.0414)	0.0091 (0.0285)
Model 2 (Jan 1973–Jan 2017)	−0.0195 (0.0211)	0.0137 (0.0061)	0.0038 (0.0117)	
Model 3 (Jan 1963–Jan 2017)	−0.0184 (0.0135)	0.0120 (0.0039)		

Note: The table presents coefficient estimates and their standard deviations (in parenthesis) from regressions of the probabilities that the small-cap portfolio returns adjust to change in (various measures of) financial conditions with a delay when switching from turbulent to normal times. DS stands for the default spread computed as the difference between yields of Moody’s BAA and AAA corporate bond indexes for measuring default risk. TermS stands for the term spread computed as the difference between yields of 10 years treasury bonds and 3 months treasury bills which is used frequently as a predictor of future business conditions. FCI stands for financial conditions index published by the Federal Reserve Bank of Chicago for measuring financial conditions. Finally, TedS stands for the TED spread computed as the difference between 3 months LIBOR and 3 months treasury bill rate for measuring credit risk. Models are estimated using the different sample periods due to data availability indicated in parenthesis in the first column. We use Newey–West HAC covariance matrix for capturing any remaining autocorrelation and heteroskedasticity in the error terms.

following January 1973 (January 1986). Therefore, we run three regressions with all available data during these sample periods. Table 5 presents the results of these regressions.

Considering DS, TedS and FCI, for all models posterior distributions of the parameters contain sizable probability mass around zero indicating that zero is inside the 95% HPDI. For the TermS, however, for all three regressions we find positive estimates of coefficients with an enhanced effect during the later period after January 1986 as seen from Model 1. Note that a decreasing term spread, i. e., a decreasing slope of the yield curve implies tighter financial conditions. Therefore, a positive sign of this coefficient indicates decreasing delay probabilities with the tightening financial conditions. These results show that small caps adjust sluggishly to various sources of negative information. However, the delay probabilities reduce, or in other words, information diffusion accelerates with the negative news related to term spread which is

closely related to the surprises to monetary policy supporting the evidence in Table 4. This effect is further enhanced in the period starting from 1986. This period coincides with the periods of great moderation when the volatility of many macroeconomic variables have declined. It seems that news related to monetary policy have larger impact during this period compared to periods before great moderation.

5. Economic evaluation

The analysis so far reveals that the model with three regimes and imperfect synchronization with asymmetric phase shifts in the expected returns, the volatilities and the correlations has favorable statistical properties as displayed in the first two columns of Table 1. However, economic agents are often interested in whether capturing these phase shifts could provide economic gains. Therefore, we conduct a utility based evaluation of models for measuring the (realized) utility an investor perceives when using competing models. Specifically, a representative investor is assumed to maximize an expected power utility function with the coefficient of relative risk aversion, γ

$$u(W_{t+1}) = \frac{W_{t+1}^{1-\gamma}}{1-\gamma}, \quad \gamma > 0 \quad (10)$$

to determine portfolio weights $w_{t+1} \equiv (w_{LC,t+1}, w_{MC,t+1}, w_{SC,t+1})'$ using the predicted excess returns of large-, mid- and small-cap portfolios, $\hat{r}_{t+1} \equiv (\hat{r}_{LC,t+1}, \hat{r}_{MC,t+1}, \hat{r}_{SC,t+1})'$. Using these weights the wealth at the end of the period $t+1$, W_{t+1} , is equal to

$$W_{t+1} = W_t \left((1 - w'_{t+1} \iota_3) \exp(r^f_{t+1}) + w'_{t+1} \exp(r_{t+1} + r^f_{t+1} \iota_3) \right), \quad (11)$$

where ι_l is the vector of ones of size l and $r_t + 1^f$ is the risk-free rate that is known in period t . Without loss of generality we normalize the current wealth, W_t , as 1 every period, such that the investor's optimization problem is given by

$$E_t(u(W_{t+1})) = E_t \left(\frac{\left((1 - w'_{t+1} \iota_3) \exp(r^f_{t+1}) + w'_{t+1} \exp(r_{t+1} + r^f_{t+1} \iota_3) \right)^{1-\gamma}}{1-\gamma} \right) \quad (12)$$

Here, E_t is the conditional expectation given information at time t . The investor computes the expectation using the predictive distribution of the returns, r_{t+1} , obtained from model (1). Hence, the investor's optimization problem becomes

$$\max_{w_{t+1}} \int u(W_{t+1}) p(r_{t+1} | \Omega_t) dr_{t+1}, \quad (13)$$

where Ω_t is the information set in period t . The integral can be approximated by generating G independent draws from $\{r_{t+1}\}_{g=1}^G$ using the predictive density $p(r_{t+1} | \Omega_t)$ computed in (9). This framework allows us to conduct a utility based comparison of the competing models. Specifically, consider the two portfolios A and B constructed using the predictions of the competing models. We can compute the maximum performance fee the investor would be willing to pay to switch from strategy A to strategy B, which is

denoted as Δ , using the fact that the utility perceived by employing the strategy B after paying the performance fee should be equal to the utility that would be perceived if strategy B were to be used. We can compute this by solving the following equation

$$\sum_{T_0}^{T-1} u(W_{A,T+1}) = \sum_{T_0}^{T-1} u(W_{B,T+1} / \exp(\Delta)). \quad (14)$$

The last column of Table 1 displays the performance fees as computed in (14) using the predictions of the portfolio (excess) returns from competing models. We set the relative risk aversion parameter, γ , to be equal to 5, in convention to existing studies, see Barillas and Shanken (2018) for example. We provide our comparisons of competing models with a two-regime model with identical regimes for all size based portfolios. That is, Δ is computed for switching to competing models with (i) different number of regimes; (ii) different types of synchronization in expected returns and variances from the two-regime model with perfect synchronization.

On the one hand, focusing on the models with perfect synchronization we observe that the performance fee of the four-regime model is largest with an annual fee of 59 basis points. This shows that increasing flexibility may indeed yield higher returns.

On the other hand, independent of the number of regimes, the models that allow for imperfect synchronization with asymmetric phase shifts have higher performance fees compared to their counterparts with perfect synchronization and imperfect synchronization with symmetric phase shifts. In this case, for the four-regime models, the difference is negligible similar to the statistical finding indicating that the fourth regime may be replacing the effects of the phase shifts. Interestingly, while the performance fee is largest for the three-regime model, even the model with only two regimes but allowing for imperfect synchronization with asymmetric phase shifts has quite a sizable performance fee compared to any of the models with four regimes. This implies that accurate timing of the phase shifts indeed pays off. Specifically, a representative investor would be willing to pay an annual performance

fee of 248 basis points to switch from the predictions offered by the two-regime model with perfect synchronization to the three-regime model with imperfect synchronization due to asymmetric phase shifts.

6. Conclusion

In this paper, we estimate the cyclical regimes of size-sorted portfolio returns along with the lead/lag relations between their cyclical components using a novel econometric framework. Specifically, we develop a Markov-Switching vector autoregressive model that allows for imperfect synchronization of cyclical regimes in size-sorted portfolios, due to phase shifts of the cycle governing the large-cap portfolio returns. The model has three key features. First, it can capture the well documented lead/lag effects in size-sorted portfolios through phase shifts that can be different across

regimes and portfolios. This feature is empirically relevant given the evidence on the presence of the lead/lag effects in size-sorted portfolios that can be distinct in different phases of the financial cycle. Second, the cycle can consist of multiple regimes allowing a refined specification of cyclical dynamics in financial asset returns. This is essential for capturing the key phases of the cycle especially related to bull market rallies and bear markets. Third, the model allows for regime-dependent volatilities and correlations in addition to mean. This is crucial for identification of regimes in asset prices, where differences in volatilities often are more pronounced than those in mean returns.

Using monthly returns on US size-sorted stock portfolios, we found that a three-regime model that allows for imperfect synchronization with asymmetric phase shifts characterizes the joint distribution of returns most adequately. While all portfolios switch contemporaneously into boom and crash regimes, the large- and mid-cap portfolio lead the small caps for switches from the bull market regime to a moderate regime by a month. This suggests that relatively negative news signaling the end of the boom market regime is incorporated immediately into the large and medium stocks where the speed of adjustment to the newly emerging conditions is fastest. However, this negative news ‘travels slowly’, in the sense that it is incorporated only gradually into small stocks, such that their prices adjust with a delay compared to large (and medium) stocks. Our analysis using the probabilities of these delays show that information diffusion accelerates mostly in response to monetary policy surprises.

Using a utility based metric we further show that these delays can be translated into profitable strategies in the sense that the model economically outperforms many other competitors with varying number of regimes as well as varying structure of synchronization of these regimes. Specifically, a representative investor with power utility would be willing to pay an annual performance fee of 248 basis points to switch from the predictions offered by the two-regime model with perfect synchronization to the model with three regimes and with imperfect synchronization together with asymmetric phase shifts.

Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.cbrev.2022.11.001>.

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