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Increasing Block Tariffs in the Water Sector – A Semi-Welfarist Approach
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Increasing Block Tariffs in the Water Sector - A
Semi-Welfarist Approach

by

GEORG MERAN* and CHRISTIAN VON HIRSCHHAUSEN

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We analyze the properties of progressive water tariffs, that are often applied in the sector in the form of discretely increasing block tariffs (IBT). We are particularly interested in water tarification in a poverty context where a subsistence level of water has to be allocated to each household. Our approach is ”semi-welfarist” to the extent that we analyze second-best pricing schemes that may be applied in practice due to ”fairness” or other, non-welfarist considerations. In our theoretical model we compare a modified Coase-tariff and a progressively increasing block tariff with respect to water consumption, water expenses and utility levels. When we impose cost coverage on the water utility, there are clearly adverse effects on the ”almost poor” by introducing a progressive tariff. This result is supported with a numerical application using real data from Bangladesh: progressive tariffs may fail to achieve ”fair” cross-subsidization of low-income groups.

Keywords: water, tarification, prices, fairness, distribution, institutions (JEL: L51, L95, H21, D40).

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1 Introduction

The pricing of water is particularly akin to political, socio-economic, and cultural influences. Empirical evidence suggests that the level and the structure of water prices rarely correspond to welfare-optimality. Instead, they are heavily influenced by country- and sector specific considerations, and redistributive aspects. This applies to industrialized countries, but even more to emerging and developing countries. Increasing block tariffs (IBTs), by which higher-income consumers cross-subsidize poorer consumers, prevail (Whittington, 2003).

In this paper, we analyze the effects of increasing block tariffs in a development framework, e.g. where a certain group of consumers is allocated a subsistence amount of water free of charge. We adopt a "semi-welfarist" approach, i.e. we blend a welfare-maximizing approach with other institutional determinants of price-setting such as "fairness" and "transparency". One element of the fairness approach is that we include a Stone-Geary utility function that allocates a subsistence amount of water to each household. This is certainly the most specific aspect of water that distinguishes it from all other goods; without a minimum daily consumption, survival is impossible. In addition, we construct welfare transfers from the high-income to the lower-income households. We also introduce asymmetric information into the model (i.e. income is not observable) and construct incentive compatible constraints. From there we calculate the "fair" and "incentive-compatible" nonlinear expenditure function for water.

Roughly speaking, the literature on pricing and regulation in the water sector consists of "optimal" pricing schemes and others, closer to those observed in reality, that are "second-best" from a welfare perspective, if not third- or fourth-best. The new institutional economic approach adopts yet another perspective, by insisting on the importance of the institutional environment.

The "first-best" literature derives welfare-optimal nonuniform prices which are in general related to Ramsey-rules. A prominent example is Goldman, Leland, and Sibley (1984), who take into account income effects and the "optimal taxation" reasoning initially developed by Mirrlees (1971, 1976). Sharkey and Sibley (1993) develop optimal non-linear pricing schemes for an arbitrary number of customer types and general cost functions; the "benevolent" regulator can define welfare weights which
vary over the set of customer types. Interestingly, the marginal price can be below marginal cost if welfare weights increase with type (p. 228). Cowen and Cowen (1998) propose a radical form of price differentiation: the unregulated monopoly, that maximizes social surplus by maximizing producer rent, at the expense of consumer rent.

Authors that are closer to field work in the water sector generally argue in favour of second-best pricing schemes, that are more easily applied. Thus, Whittington (2003) reports about the wide-spread use of increasing-block tariffs in South Asia, but which do not accomplish their main objectives, e.g. revenue sufficiency, economic signals, and helping the poor. Boland and Whittington (2000) also present a critical view of IBTs. Dahan and Nisan (2007) insist on the "unintended" consequences of increasing block tariffs in urban water: since larger households, that are generally poorer, consume more water than smaller households, they are charged a higher price for water. This erodes the effectiveness of increasing block tariffs. Agthe and Billings (1987) analyze the relation between household income levels and residential water use for Tucson, Arizona. The demand models show that "under the existing increasing block rate pricing schedules, higher income households not only use more water, but have lower elasticities of demand" (p. 273). This implies that a uniform proportional increase will cause a larger percentage drop in water use among low income households than among high income households. Agthe and Billings (1987, p. 273) therefore argue in favour of substantially steeper block rates to improve interpersonal equity in water pricing.

Empirical work on the specifics of water demand is rare. Gaudin, Griffin, and Sickles (2001) analyze the "Stone-Geary" form of water demand, where a portion of water use is not responsive to price. Martinez-Espineira and Nauges (2004) also apply a Stone-Geary utility function to assess if water consumption is sensitive to price control; interestingly, they find a pattern of "path-dependent" water subsistence levels. Garcia-Valinas (2005) estimates urban water demand and water costs for the Spanish municipality of Seville; she finds that two-part tariffing could be a compromise between efficient-but-impossible Ramsey pricing, and inefficient-but-socially-acceptable free allocation of water to the poor.

The institutional economic literature has also focussed on regulation and pricing in the water sector of developing and emerging countries. Mnard and Clarke (2000a,b),
Shirley (2002), Spiller (2005) and others analyze the nexus between private participation in water and the effects in terms of performance and pricing in emerging and developing countries. Biswas and Tortajada (2005) also check if water pricing is effected by public private partnerships. It seems that cost-coverage is now generally accepted as an side condition of water provision, be it from the private, the public, or the public-private sector.

We try to bridge between various approaches and analyze whether progressive tariffs, which are generally considered as "fair" really deliver on the promise that is associated with them. In fact, we find quite the opposite: progressive tariffs may significantly hurt the lower income groups, the "almost poor". The reason is that since the water company has to break even, the lower income groups have to pay a large share of the cross subsidy. We support this theoretical argument with numerical simulations, using real data from Bangladesh. We conclude that a traditional two-part Coase tariff may be less attractive politically, but closer to being "really" fair for the poor and the almost poor.

The rest of the paper is structured in the following way: in the next section, we develop a model where consumers choose between two goods: water, and "other" goods (basket). We include the specifics of the water sector, such as the Stone-Geary utility function, and the cost structure of the sector. In Section 3, the model is applied to two different pricing schemes: i) a "modified" Coase tariff, which is a two-part tariff with a break-even condition for the operator; and ii) a progressive tariff, the increasing block tariff.¹ The subsequent sections analyze whether the progressive tariff delivers what is generally promised. In Section 4, we compare the two tariffs with respect to water consumption, utility, and total welfare, for different income groups. We use real data from a case study of the water sector in Bangladesh. The analysis shows that some progression increases total welfare, but too strong a progression harms low income groups, rather than helping them. In Section 5 we insist on the necessity of analyzing the "unintended consequences" of progressive tariffs, i.e. the need to include the household size into consideration. Section 6 concludes.

¹The terms "progressive tariff" and "increasing block tariff" (IBT) are used interchangeably in this paper, the progressive tariff being the continuous form of the block tariff.
2 The Model

To construct water tariffs we assume that customers consume two goods: water and some other goods which are aggregated into a basket. Consumers differ with respect to income. We assume a continuum of income beginning with very poor households followed by a middle class and ended by rich customers. Income is distributed according to a density function $g(y) > 0$, $\forall y \in Y = [y, \bar{y}]$. The total number of people of income $y$ is $Pg(y)$, where $P$ is total number of customers. In the basic model we assume that households differ only with respect to income. Later we will include the household size into the analysis.

Each customer needs a subsistence level $w_s, x_s$ to survive where $w_s$ denotes the subsistence level of water and $x_s$ denotes the needed level of the other good, respectively. Without loss of generality we assume that $x_s = 0$. To capture the subsistence level into the analysis we introduce the following Stone-Geary-utility function:

$$U(w, x) = (w - w_s)^\alpha (x - x_s)^{(1-\alpha)}$$ (1)

The water tariff system is constructed such that water expenses depend on water consumption and income. This can be denoted by a tariff plan (TP)

$$TP := \{T(y), w(y)\}, \forall y \in Y = [y, \bar{y}]$$ (2)

where $y$ denotes income in the interval $Y$ and $T(y)$ is a continuous outlay function of customers to be determined subsequently. $w(y)$ is the respective profile for water consumption. Note that the usual tariff system $T(w)$ can be derived from (2).

Taking the tariff plan TP into account, the budget constraint of households can be derived:

$$T(y) + p_x x \leq y$$ (3)

where $p_x$ is the price of the other good. For simplicity we calibrate the measure of $x$ such that $p_x = 1$. If we insert (2) and (3) into (1) we have

$$U(w(y), y - T(y)) = (w - w_s)^\alpha (y - T(y))^{(1-\alpha)}$$ (4)

The tariff system should be affordable and "fair". Hence, it must depend on $y$. As water utilities cannot observe income (or are not allowed to ask for income details) the tariff system has to be built up in a way that customers have an incentive
to tell their true income. This requires that the tariff plan be constructed in an
incentive-compatible way. From the revelation principle we know that an incentive
compatibility for the continuous case satisfies the following incentive constraint:²

\[ y = \arg\max_y [U(w(y), y - T(y))] \]  

(5) requires that \( w(y) \) and \( T(y) \) be chosen such that customers do report their true
income to the water company. The respective properties can further be inspected if
we differentiate (5) with respect to \( \tilde{y} \) and set \( \tilde{y} = y \).

\[ U_w \dot{w}(y) - U_y \dot{T}(y) = 0, \quad \forall y \in Y \]  

(6)

where dots denote the derivatives with respect to \( y \).

Inserting the Stone-Geary-utility function yields:

\[ \frac{\alpha \dot{w}(y)}{w(y) - w_s} - \frac{(1 - \alpha)\dot{T}(y)}{y - T(y)} = 0, \quad \forall y \in Y \]  

(7)

(7) implicitly determines some characteristics of the admissible tariff systems. From
the second order conditions³ it also follows that

\[ \dot{w}(y) > 0 \quad \text{and} \quad \dot{T}(y) > 0 \quad \forall y \in Y \]  

(8)

Finally, the cost structure of the water supply has to be captured in the model. We
assume the following simple cost function:

\[ C(W(y)) = F + cW(y), \]  

(9)

where \( F \) are fixed costs, \( c \) is a positive constant and

\[ W(y) = P \int_y^y w(v)g(v)dv \]  

(10)

is the aggregated water consumption for all incomes up to \( y \).

²See e.g. Mas-Colell et al. (1995, p. 492 ff.) or Wolfstetter (1999, p. 259 ff.).
³Strictly these condition must hold to guarantee a separating equilibrium, i.e. that \( w(y) \) and
\( T(y) \) vary with respect to \( y \), see appendix 7.1.
3 Tariff systems

In the following we turn to the issue of how to construct affordable and "fair" tariff systems. Here, we want to follow a semi-welfarist approach. This approach differs from a welfarist approach in that the optimal tariff is not the result of maximizing aggregate weighted utility\(^4\) of all customers but, instead, of introducing simple transparent rules which satisfy the notion of fairness and affordability. However, it remains welfarist by utilizing a utility function and by securing affordability, i.e. assuring the subsistence level of water consumption. We analyze the two archetypes of water tariffs: the Coase tariff and the increasing block tariff.

3.1 A modified Coase Tariff

If average costs are above marginal costs, a marginal cost-tariff is not economically viable. Either the price will be above marginal costs which leads to the Ramsey-Boiteux-pricing approach or non-linear pricing schedules are introduced. Coase (1946) first dealt with the latter. He proposed a uniform two-part tariff where the price for each unit is equal to marginal costs and an access fee is introduced such that fixed costs are covered. But this schedule can only be assured if no customers drop out of the market as a result of this two-part tariff. But it is exactly this case which is empirically relevant in developing and emerging countries. In the following we want to introduce a "modified" Coase tariff which takes into account that poor people cannot afford water supplied at marginal costs and, in addition, can not pay the access fee \(F/P\).

We assume that a fraction of consumers can not afford the subsistence level of water offered at marginal costs. Formally, there exists an interval \(I\), such that

\[
I = [y, y_s) \quad \text{where} \quad y_s = cw_s
\]

Since affordability must be secured water has to be provided below for the poor. This can be achieved by a non-exclusive two-part tariff, which we will call the "modified" Coase tariff. To begin with, for all income groups the subsistence level

\(^4\)In this respect the following approach differs from the classical literature on optimal tariff mentioned in the introduction.
\( w_s \) is guaranteed. Hence, the tariff starts with

\[
\{ T(y) = y, w(y) = w_s \} \tag{12}
\]

Expenses beyond the subsistence level increase according to

\[
\dot{T}(y) = (1 + m)c\dot{w}(y), \quad m > 0 \tag{13}
\]

where the uplift factor, \( m \), is chosen such that the water provider breaks even. Since poor people can not afford water provided at marginal costs and since fixed costs have to be taken into account, \( m \) has to be strictly positive to induce cost coverage. Solving the differential equation system (5) and (13) and inserting the starting condition (12) leads to the following tariff plan:\(^5\)

\[
T(y) = y + \alpha(y - \bar{y}) \tag{14}
\]

\[
w(y) = w_s + \frac{\alpha}{(1 + m)c}(y - \bar{y}) \tag{15}
\]

Inserting (15) into (14) yields the following outlay-schedule with respect to water consumption:

\[
T(w) = y + (1 + m)c(w - w_s) \tag{16}
\]

**Figure 1: Modified Coase tariff**

Figure 1 shows that the outlay schedule \( T(w) \) does follow the requirement of affordability. Those with low income get their subsistence level for \( y \), i.e. those who

\(^5\)For details see the appendix 7.2.
pay less than marginal costs are quantity rationed. The provision of water to the poor leads to deficits which have to be covered by customers with high consumption. Hence, the marginal price is above marginal costs. $m$ must be chosen such that the water provider breaks even, i.e.

$$\int_y^0 [T(y) - cw(y)]Pg(y)dy - F = 0$$

(17)

Inserting (14) and (15) yields:

$$(y - cw_s) + \alpha \frac{m}{1 + m} [E[y] - y] = F/P$$

(18)

where $E[y]$ is the average income. From (18) $m$ can be calculated.\(^6\)

The modified Coase tariff is not an increasing block tariff. Instead, it is a simple two-part tariff and similar to what has been proposed in the literature. Boland and Whittington (2000, p. 9 sq.) and Whittington (2003, p. 70) have critizised IBTs in many respects. As an alternative, they have proposed a 'Uniform Price with Rebate' (UPR) which is rather similar to the modified Coase tariff. In fact, the UPR is a two-part tariff where the volumetric charge is equal to marginal costs and a fixed monthly credit (fixed amount subtracted from the bill). The reason for marginal cost pricing follows from their assumption that, contrary to our model, average costs are below marginal costs, i.e. marginal costs are increasing. In fact, this is more in the spirit of Coase who thrived for marginal cost pricing.

Our tariff system can also be amended to allow for marginal cost pricing.\(^7\) This requires to introduce an additional fixed fee, say $A$, if customers consume more than the subsistence level. The tariff plan $TP^A$ has the following structure:

$$TP^A(y) = \begin{cases} 
    y & \text{if } y < y + A \\
    y + A + \alpha(y - A - y) & \text{if } y \geq y + A
\end{cases}$$

(19)

The corresponding water supply function is

$$w^A(y) = \begin{cases} 
    w_s & \text{if } y < y + A \\
    w_s + A + \frac{\alpha}{2}(y - A - y) & \text{if } y \geq y + A
\end{cases}$$

(20)

\(^6\)Note that for high fixed costs $F$ and/or a severe affordbility problem ($cw_s - y < < 0$) $m$ might be negative. In this case the water supply is economically not viable.

\(^7\)We also could have considered the more general case, where a low access fee is combined with a low mark up on marginal case. Here we confine ourselves to the two polar cases which allows to work out the distributional implications.
To compare the two tariff plans the following two figures display the outlay functions and the water supply functions.

Figure 2: Comparison of water consumption

Figure 3: Comparison of water expenses

A comparison of both figures shows that the introduction of an additional access fee $A$ leads to lower consumption of water for a certain range of lower income. Here, customers remain at the subsistence level until income $y$ covers both fix parts of the tariff $y + A$. This can be referred to as a cluster effect. Income rises and water consumption remains at the subsistence level. If income is sufficiently high ($y \geq y + A$), water consumption increases at a higher speed than under the tariff.
without access fee. This is due to marginal cost pricing under the A-tariff as opposed to the former tariff which covers costs by a mark up \( m \). On the other side, expenses under the A tariff are lower for low incomes than under the tariff without additional access fee. If we take into account both effects, one can show that for low income utility under an A-tariff is lower than under tariff with a mark up.

**Proposition 1** Define \( \tilde{y} \) as income where \( w(\tilde{y}) = w^A(\tilde{y}) \). Then there exists a \( y^U > \tilde{y}, \) such that \( U(w(y^U), T^A(y^U)) = U(w(y^U), T(y^U)) \). For all incomes \( y < y^U \) utility under the A-tariff is lower than under the tariff with a mark up and without an additional access fee \( A \).

Proof see appendix.

As a result, the Coase tariff with mark up favors low income groups; the A-tariff favors the higher income groups. The choice of the tariff is a political issue. The decision could also take into account the more general case, where \( m \) can be gradually reduced and at the same time \( A \) increased.

### 3.2 Increasing Block Tariffs

**3.2.1 The principle of increasing block tariffs**

The uniform linear two-part tariff resulted from the requirement of incentive compatibility and a simple rule of proportionality in the case of \( A = 0 \). Everyone that consumes more than the subsistence level should contribute to the coverage of costs according to his consumption. The proportionality rule complies with the notion of fairness. The introduction of an access fee \( A > 0 \), however, is a per-capita approach. Each customer who consumes more than the subsistence level should contribute to the coverage of costs in a lump sum manner. The contribution does not depend on consumption and, hence, income. This tariff typ secures affordability but denies fairness aspects within the income groups that can afford more than the subsistence level.

In this subsection we analyze a progressive tariff that is generally considered to resolve this issue, and to be "fair" from a distributional perspective. If fairness considerations play an important part in water demand management, the introduction
of cost coverage\(^8\) might be accompanied by an explicit distributional policy, i.e. by the introduction of a progressive tariff. In the case of block tariffs this is referred to as increasing block tariffs (IBT). In the continuous case this can be achieved by introducing a mark up which depends on income. We introduce the following cost coverage mechanism:

\[
T(y) = (1 + n)(y - y)\beta c\dot{w}(y), \quad 0 \leq \beta < 1
\]  

(21)

To meet the second order condition of the incentive compatibility constraint, \(\beta\) must be less than unity. Together with (7) this equation forms a system of non-linear differential equations which can be solved (see appendix):

\[
T^p(y) = \frac{y + \alpha((1 - \beta)y - w)}{1 - \alpha\beta}
\]  

(22)

\[
w^p(y) = w_s + \frac{\alpha(y - y)\beta}{(1 + n)c(1 - \alpha\beta)}
\]  

(23)

where \(p\) indicates that the tariff system follows progressivity. \(n\) has to be chosen such that the water company breaks even, i.e.

\[
\int_{y}^{\bar{y}} [T^p(y) - cw^p(y)]Pg(y)dy - F = 0
\]  

(24)

Both equations guarantee affordability. If \(y = y\) is inserted into (22) and (23) it follows that \(T^p(y) = y\) and \(w^p(y) = w_s\). If (23) is solved for \(y\) and inserted into (22) we obtain the outlay schedule:

\[
T^p(w) = y + \frac{\alpha(1 - \beta)}{1 - \alpha\beta} \left( \frac{(1 + n)(1 - \beta\alpha)}{\alpha} \right)^{1/(1 - \beta)} (w - w_s)^{1/(1 - \beta)}
\]  

(25)

(25) shows the continuous case of IBTs. Since \(\beta < 1\), the outlay function is convex. The philosophy of IBTs is to secure affordability of water and to implement the notion of fairness which implies the redistribution between the income groups. Those with high income should contribute to cost coverage relatively more than those with low income, thus cross-subsidizing the latter.

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\(^8\)In many developing and emerging countries the water supply is far from economically viable.
3.2.2 Choosing the degree of progression

Having introduced a device for cross subsidization it remains to determine the degree of progression (i.e. determining the level of $\beta$). Both tariffs, the Coase tariff and the IBT, imply cross-subsidization. Figure 1 showed that the Coase tariff not only guarantees affordability but also provides subsidies to the lower incomes consuming water less than $\widetilde{w}$. The degree of cross-subsidization in the IBT is governed by the progression parameter $\beta$ whereas the mark up $(1 + n)$ secures total cost coverage according to eq. (24). This, of course, requires that cost coverage can be achieved. The following proposition specifies the interrelation between progression and economic viability.

proposition 2 The economic viability of water supply decreases along with tightened progression.

Proof: Inserting (22) and (23) into (24) yields after some rearrangements:

$$\frac{\alpha(1-\beta)}{(1-\alpha\beta)}[E[y] - y] + (y - cw_s) - \frac{F/P}{1+n} = \frac{\alpha(y-y)^{1-\beta}}{(1+n)(1-\alpha\beta)}$$

Economic viability requires that $1 + n > 0$ (see (21)) which implies that the l.h.s. of the equation must be positive. The second and third term of the l.h.s. is negative. Hence, a positive sign requires the first term to be sufficiently above zero. The first term decreases in $\beta$ and, hence, reduces the economic viability of the water supply.

4 Numerical Analysis

4.1 Data sources for numerical example (Bangladesh)

Our numerical example is based upon data from one of the poorest countries in the world, Bangladesh. With a population of 153 mn., the small state is one of the most densely settled areas in the world. While this would seem to facilitate the development of a comprehensive water infrastructure, the very low average income in the country, and the steep income distribution point to financial bottlenecks of infrastructure development, to which political and institutional obstacles need to be added. Not even three fourth of the population has access to piped water (113 mn. of a total of 153 mn.). For this population, water consumption can be measured
and a tariff system can be implemented. Also, Bangladesh is a water-rich country, so that questions of long-distance water transportation and its pricing do not have to be taken into consideration.

Our data is collected from public sources covering the water sector. The relevant population in this case is 113.2 mn. people, i.e. those with access to piped water.\textsuperscript{9} The average income in this group is assumed to be USD 380 per month. However, income distribution is very skewed: the average income of the 10 per cent most wealthy people is USD 1,060,\textsuperscript{10} whereas the lowest income is in the range of USD 15 per month, i.e. USD 0.5 per day. Based on this, we set the parameters $y$ at USD 15 and $\bar{y}$ at USD 10,000.

The average household size is 4.9 persons per household\textsuperscript{11}, so that there are 23.1 mn. households. Fixed costs are estimated at approx. 20 mn. USD per month and variable costs are 1 USD per cubic meter.\textsuperscript{12} The subsistence level is assumed to be 20 m$^3$ per household and month.\textsuperscript{13} The average residential consumption of water is 87 l per day and capita.\textsuperscript{14}

We approximate the income distribution by a Pareto-function, with a relatively low $k$-parameter (1.05). This estimate is based on the idea of a steep distribution in many developing countries, and taking into account that in the year 2000, over a third of the population lived with incomes below the poverty line (USD 1/day).\textsuperscript{15}

\textsuperscript{10}Household income or consumption by percentage share: Highest 10 per cent: 27.9 per cent (2000), CIA World Fact Book, 2008. 100 per cent income= 153,000,000 * USD 380= USD 5.814*1010 USD 5.814*1010 * 0.279= USD 1.622*1010 ’ average income of highest 27 per cent: USD 1.622*1010/ (153,000,000*0.1)= USD 1060.20
\textsuperscript{11}Bangladesh Bureau of Statistics (2001): Bangladesh Census Results at a Glance.
\textsuperscript{12}Whittington (2003, p. 66) assesses a water price of USD 0.07/m$^3$ which appears to be rather low; most likely this price has been heavily subsidized, and it does not correspond to neither marginal cost pricing, or any other schemes discussed in the literature.
\textsuperscript{13}http://www.un.int/bangladesh/statements/55/poverty.htm
\textsuperscript{15}United Nations ESCAP Division, 2005.
4.2 Analysis of the progressive tariff

We start with an analysis of the progressive tariff, and the effect of choosing different degrees of progressiveness $\beta$. Inserting (22) and (23) into (24) allows to calculate for each $\beta$ the corresponding mark up $(1 + n)$. Figure 4 shows the graph of $(1 + n)$ as a function of $\beta$. The lowest $\beta$ is nil (Coase tariff) and the upper limit $^{16}$ is 0.9.

![Figure 4: Mark-up $(1+n)$](image)

The graph shows that at first $(1 + n)$ decreases due to the tightening of the progression forcing the upper income classes to more profit margin. Profit margin is defined as

$$T^p(y) - cw^p(y)$$

(27)

After $(1 + n)$ has reached the minimum it increases again, first only gradually, but then very steeply. At approximately $\beta = 0.78$ economic viability ceases. Tightening the progression does not only lead to a greater contribution to profit margin of the upper classes it also shifts the income threshold between lower incomes cross-subsidized and income ranges cross-subsidizing. Define the threshold as

$$y^\infty : T^p(y) - cw^p(y) = 0$$

(28)

Utilizing (22) and (23) we can calculate the threshold for all admissible $\beta$. This yields the following figure:

$^{16}$Recall, that $\beta$ must be strictly less than 1 to meet the second order conditions of the incentive compatibility constraints (See appendix 7.1).
Figure 5: threshold income

The figure shows that for lower values of $\beta$ the income threshold $y^{cc}$ increases leading to a more expanded income band which is cross-subsidized. In the course of further tightening the threshold reaches a maximum and then decreases. Obviously, a too strong progression lowers the income range receiving cross-subsidization. The underlying process can be described as follows:

Starting from $\beta = 0$ more progression allows to get more profit margin from the higher income range and, at the same time, more cross-subsidization for the lower incomes. This increases water demand and decreases water expenses of the lower income groups. In the course of increasing $\beta$ more customers are subsidized by the upper incomes. But, due to the Pareto distribution there are not many upper incomes and, as a result, a financing gap will occur which has to be covered by an increased mark-up. Finally, $(1 + n)$ will rise such that the threshold income $y^{cc}$ will decrease. The figure shows that certain levels of $y^{cc}$ can be achieved by, both, a low and a high progression coefficient.

The distributional impact of $\beta$ is not sufficiently described by $y^{cc}$. The extent of well being of the low income range is, of course, crucial. To what extent does a sharp progression alleviate the poor? Let us define low income as the half of average income, i.e. $y^{\text{poor}} = 0.5E[y]$. Inserting (22) and (23) into (4) yields

$$U^{pr}(y) = \left(\frac{1}{1-\alpha\beta}\right)(\frac{\alpha}{((1+n)c)})^{\alpha}(1-\alpha)^{(1-\alpha)}(y-y)^{(1-\alpha\beta)}$$

(29)

Total welfare of $y \in [y, 0.5E[y]]$ can be derived by integrating (29) with respect\(^1\) to $y$ over the relevant income intervall. The following figure shows the graph of total

\(^1\)Alternatively, we could calculate average utility of the relevant income range.
welfare\textsuperscript{18} as a function of the progression factor $\beta$.

![Total Utility](image)

**Figure 6: Total welfare of low income groups with varying progression rates**

The figure shows that welfare of low income groups is not a positive monotonic function of the degree of progression ($\beta$). There exists an optimal value of $\beta$ which maximizes total welfare of the low income range due to the budgetary constraint of water supply (economic viability).

### 4.3 Comparing the progressive tariff with the Coase tariff

We can now compare the effects of the two different tariff schemes directly. Figure 6 illustrates the trade-off for different low-income groups. It displays the utility difference between utility under a Coase tariff $U^C$ and utility under IBTs $U^{pr}$ for different values of $\beta$.

![Utility Difference](image)

**Figure 6: Welfare difference between the progressive and the Coase tariff**

\textsuperscript{18}The vertical axis measures Total Utility/10\textsuperscript{8}.
The curve with the flattest curvature is the utility difference for $\beta = 0.05$. It shows that utility is higher under weak progression than under a Coase tariff for all income less than approximately 225. The curve above is drawn for $\beta = 0.1$. It provides even more utility to all incomes less than approximately 182 (the intersection of the first two graphs). For very high values of $\beta$ utility of the low income range decreases. This shows that a too strong progression harms low income groups.

5 Including the Household Size

A major shortcoming of IBTs in practice is that they do not take into account the size of households. Some tariff systems construct the first block taking into account a best guess of household size of the poor. In the following we want to explicitly introduce the number of household members into the tariff. Two methods are conceivable:

- The tariff is based on reported income and the reported number of household members. This requires a tariff schedule of the form $T(y, h), W(y, h)$, where $y$ is total household income and $h$ the household size. $T$ and $W$ are defined for households and not for individuals. This scheme is very difficult to design if there is no additional information available. It requires to solve partial differential equations. The resulting tariff schemes are sensitive with respect to the relevant parameters. Of course, if reliable information on income and household size is available, first best tariffs can be implemented.\(^{19}\)

- The tariff is based solely on reported income. This approach does not require households to report their size. The number of household members is estimated utilizing econometric methods. The resulting size function depends on income and is included in the tariff scheme.

\(^{19}\)In Chile, for example, some districts have implemented a so called "means-tested" approach where households have to verify their income and their size. If they fall short of certain social standards they receive water for a highly subsidized price, see Gomez-Lobo and Contreras (2003)
In the following we take the second approach and include the household size function into the tariff system. The size function is assumed as follows:\(^\text{20}\)

\[ h(y) = h + (\bar{h} - h) \left( \frac{y - \bar{y}}{y - y} \right)^\gamma, \quad \gamma < 1 \quad (30) \]

where \(\bar{h}\) is the household size of the lowest (highest) income group and \(h > \bar{h}\).

The incentive compatibility constraint can be derived by applying (5) to the whole household, i.e.

\[ y = \arg\max_\tilde{y} [h(y)U(W(\tilde{y}/h(y)) - w_s, (y - T(\tilde{y}))/h(y))] \quad (31) \]

Households try to maximize aggregate utility by choosing the optimal message \(\tilde{y}\). If the tariff system \(T(y), W(y)\) is incentive compatible \(\tilde{y} = y\). The first order condition requires:

\[ U_w W(y) - U_y T(y) = 0, \quad \forall \ y \in Y \quad (32) \]

The second order condition requires \(W(y) > 0\) and \(T(y) > 0\). Utilizing the Stone-Geary-utility function (32) can expressed as:

\[ \frac{\alpha W(y)h(y)}{W(y) - h(y)w_s} - \frac{(1 - \alpha)T(y)}{y - T(y)} = 0, \quad \forall \ y \in Y \quad (33) \]

To derive the tariff schedule we have to add a mechanism that prescribes how much households have to contribute to the coverage of costs. We confine our analysis to the Coase tariff.\(^\text{21}\) Hence, we assume

\[ T(y) = (1 + k)cW(y) \quad (34) \]

(30), (33) and (34) form a system of non-linear differential equations which can analytically be solved.

\[ T(y) = \alpha(y - y) + (1 - \alpha)(1 + k)c(h - \bar{h})w_s \left( \frac{y - \bar{y}}{y - y} \right)^\gamma + y \quad (35) \]

\[ W(y) = w_s + \alpha \left( \frac{1 + k}{c} \right)(y - y) + (1 - \alpha)(h - \bar{h})w_s \left( \frac{y - \bar{y}}{y - y} \right)^\gamma \quad (36) \]

Notice that (35) and (36) satisfy the second order conditions. Contrary to the single member household case (confer (14) and (15)), the tariff plan is degressive if the schedule controls for the household size.

\(^{20}\)In the following we disregard economies of scale of water consumption with respect to household size.

\(^{21}\)In a follow-up paper we will include progressive tariffs.
**proposition 3** The tariff functions $T(y)$ and $W(y)$ increase on a diminishing rate with respect to income.

Proof: It is straightforward to differentiate (35) and (36) twice and to assert that both second degree derivatives are negative.

Including the household size into the Coase tariff leads to a declining increase in both, outlay and consumption, of households. This is due to the assumed decrease in household size with respect to income. Under the Coase tariff for single member households higher income increases water consumption proportionally. If the tariffs allow for the household size, the lower household size increases consumption per capita leading to the diminishing increase of water consumption (and outlays).

Note however, that the outlay schedule $T(W)$ remains linear.

**proposition 4** The outlay schedule $T(W)$ is linear. From (35) and (36) it follows that $T'(W) = (1 + k)c = \text{constant}$.

Proof:

From (32) and (34) it follows:

$$\frac{dT}{dW} = \frac{U_w}{U_y} = (1 + k)c$$

(37)

### 6 Conclusions

In this paper we have analyzed different pricing schemes for water in a development context, where ”fairness” is supposed to play an important role. Our assumption was that welfare-optimal pricing of water does not exist anywhere around the world, and that understanding the trade-offs between second-best approaches is a useful exercise. Affordability of water consumption can be obtained via a ”modified” Coase tariff and progressive increasing block tariffs (IBTs). We find that - contrary to common belief - progressive tariffs do not generally fulfill the ”fair” cross-subsidization of subsistence levels. It is not certain that the increasing block tariff yields a higher utility under a progressive tariff than the Coase tariff. Hence the argument in favour of a strong progression can backfire and hurt the almost poor very strongly. This recalls the old saying that ”the opposite of a good tariff is a tariff full of good intentions”.
Further research should address the sensitivity of the welfare of different income
groups to the selected parameters, and confirm (or reject) our proposals on the role
of household size. The household size can be included in the analysis, either through
direct reporting, or through an econometric estimation of a relation between income
and household size. More empirical work on Stone-Geary demand for water is nec-
essary to underpin the theoretical findings, both for water-poor areas in developed
countries, and in emerging and developing countries.

7 Appendix

7.1 Second-order conditions to the IC-constraint

Define

\[ G_\tilde{y}(\tilde{y}, y) = U_w(w(\tilde{y}), y - T(\tilde{y})) \frac{d\tilde{y}}{dy} - U_y(w(\tilde{y}), y - T(\tilde{y})) \frac{dT}{dy} = 0, \]  

(38)

From (38) the optimal message \( \tilde{y} \) can be derived. A comparative static analysis yields:

\[ G_{\tilde{y}\tilde{y}}(\tilde{y}, y) \frac{d\tilde{y}}{dy} + G_{\tilde{y}y}(\tilde{y}, y) = 0 \]  

(39)

where \( G_{\tilde{y}\tilde{y}} < 0 \) to secure sufficiency of the first order conditions and \( \frac{d\tilde{y}}{dy} = 1 \) by construction
of the incentive compatible functions \( w(y), T(y) \). Hence, \( G_{\tilde{y}\tilde{y}}(\tilde{y}, y) = -U_{yy} \frac{dT}{dy} > 0. \) Since
\( U_{yy} < 0 \) it follows that \( \frac{dT}{dy} > 0 \) and hence, by (38) \( \frac{d\tilde{y}}{dy} > 0. \)

7.2 Coase-Tariff

The differential equation system (7) and (13) can be solved by inserting the latter equation
into the former. This yields

\[ \alpha(y - T(y)) = (1 - \alpha)(w(y) - w_s)(1 + m)c \]  

(40)

Solving (40) and (13) yields

\[ T(y) = (1 - \alpha)c(1 + m)w_s + \alpha(y - M) + M \]  

(41)

\[ w(y) = (1 - \alpha)w_s + \frac{\alpha(y - M)}{c(1 + m)} \]  

(42)

where \( M \) is an integration constant to be determined. Inserting the initial conditions (12)
yields \( M = y - c(1 + m)w_s \). Re-inserting into (41) and (42) yields the solution (14) and
(15).
7.3 Proof of proposition 1

$y^U$ is determined by the equation $U(w^A(y^U), T^A(y^U)) = U(w(y^U), T(y^U))$. Inserting the Stone-Geary-utility function yields

$$\left(\frac{\alpha}{(1+m)c}\right)^\alpha (1-\alpha)^{1-\alpha}(y-y) = \left(\frac{\alpha}{c}\right)^\alpha (1-\alpha)^{1-\alpha}(y-y-A)$$ (43)

and after some rearrangements

$$y^U - y = \frac{-A}{1-(1+m)^\alpha}$$ (44)

From $w(y) = w^A(y)$ it follows

$$\tilde{y} - y = \frac{1+m}{m}A$$ (45)

Comparing (44) and (45) and recalling $\alpha < 1$ shows that $y^U > \tilde{y} > y$.

7.4 Deriving the continous IBT

The differential equation system (7) and (21) can be solved by inserting the latter equation into the former. This yields

$$\alpha(y - T(y)) = (1-\alpha)(w(y) - w_s)(1+n)c(y - \tilde{y})^\beta$$ (46)

Differentiating (46) with respect to $y$ yields

$$\alpha(1 - \tilde{T}(y)) = (1-\alpha)\tilde{w}(y)(1+n)c(y - \tilde{y})^\beta + (1-\alpha)\beta (w(y) - w_s)(1+n)c\tilde{y}^{\beta - 1}$$ (47)

Solving (47) and (21) yields

$$T(y) = \frac{(1-\alpha)y + \alpha(1-\beta)y}{1-\alpha\beta} - \frac{(1-\alpha)c(1+n)(y-y)\alpha^\beta M1}{\alpha} + M2$$ (48)

$$w(y) = w_s + \frac{\alpha(y-y)^{1-\beta}}{c(1+m)(1-\alpha\beta)} + (y-y)^{\beta(\alpha-1)}M1$$ (49)

where $M1$ and $M2$ are integration constants. Recalling the initial conditions (12) and inserting into (48) and (49) allows to determine both constants. Re-inserting into the latter two equations yields (22) and (23).
References


