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**Working Paper**

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ISER Discussion Paper, No. 1216

**Provided in Cooperation with:**

The Institute of Social and Economic Research (ISER), Osaka University

*Suggested Citation:* Ishida, Junichiro; Suen, Wing-chuen (2023) : Pecuniary emulation and invidious distinction: Signaling under behavioral diversity, ISER Discussion Paper, No. 1216, Osaka University, Institute of Social and Economic Research (ISER), Osaka

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**PECUNIARY EMULATION  
AND INVIDIOUS DISTINCTION:  
SIGNALING UNDER  
BEHAVIORAL DIVERSITY**

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October 2023

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# Pecuniary Emulation and Invidious Distinction: Signaling under Behavioral Diversity

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October 17, 2023

*Abstract.* We introduce behavioral diversity to an otherwise standard signaling model, in which a fraction of agents choose their signaling actions according to an exogenous distribution. These behavioral agents provide opportunities for strategic low-type agents to successfully emulate higher types in equilibrium, which in turn reduces the cost for strategic high-type agents to separate from lower types. Behavioral diversity thus improves the equilibrium payoffs to all types of strategic agents. The model also exhibits a convergence property which is intuitively more appealing than the least-cost separating equilibrium of the standard setting.

*Keywords.* least-cost separating equilibrium; insensitivity to prior; behavioral diversity; equilibrium emulation

*JEL Classification.* D82; Z13

## 1. Introduction

From the musical film *My Fair Lady* to the television drama *Inventing Anna*, emulating the ways of the upper class is often depicted as a means to improving one's social image and gaining acceptance into high society. In real life, conspicuous consumption is used by elites to signal their superior status (Veblen, 1899), but is increasingly also adopted by the aspiring middle class to emulate the elites. LVMH, the world's leading purveyor of luxury products, has experienced an almost fourfold increase in its revenue from 2010 to 2022.<sup>1</sup> Many of its luxury goods are no doubt bought by the truly affluent, but most of the revenue growth is probably driven by the merely well-to-do. Conspicuous consumption is often viewed in economics through the lens of the standard Spence (1973) signaling model (in which perception about one's type matters), or status signaling models such as Hopkins and Kornienko (2004) and Hoppe et al. (2009) (in which perception about one's relative rank matters). The focus, however, is usually given to separating equilibria: despite the incentive of lower types to mimic higher types, the higher types choose signaling actions just costly enough to deter imitation. Successful emulation, in other words, is not an equilibrium phenomenon in these models.

In reality, some low-type agents may have high gaming ability (witness the Elizas and Annas of the real world) that allows them to pool with high types. Frankel and Kartik (2019) show that in a signaling model with multi-dimensional heterogeneity, signaling actions are not fully revealing, and successful emulation can occur in equilibrium. Another possible deviation from the idealized signaling model is preference diversity. In his critique of Veblen's (1899) thesis, Mencken (1919) wrote, "Do I enjoy a decent bath because I know that John Smith cannot afford one—or because I delight in being clean? Do I admire Beethoven's *Fifth Symphony* because it is incomprehensible to Congressmen and Methodists—or because I genuinely love music?" If music lovers of high social status have preferences that do not have reputation concerns as described by signaling models, then these agents will not move on to more difficult repertoire even when the masses flock to Beethoven in emulation of the higher class. Finally, some of the higher-type agents may be financially constrained. The landed aristocracy of the nineteenth century might have preferred to choose even more conspicuous luxury items to differentiate themselves from bankers and industrialists, but many were not able to afford them.

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<sup>1</sup> Total revenue of the LVMH Group increased from 20 billion euros to 79 billion euros during this period. Source: <https://www.statista.com/statistics/245852/total-revenue-of-the-lvmh-group-worldwide/>.

In this paper, we capture these deviations due to varying signaling costs, preference heterogeneity, or financial constraints by positing that a fraction of the agents are behavioral in the sense that their choice of signaling action is exogenously given by a possibly type-dependent probability distribution. The remaining agents have standard single-crossing preferences and may potentially gain from emulating the behavioral agents or emulating other types of strategic agents. We use this simple model to explore how the possibility of emulation affects the signaling choices and welfare of strategic agents and to elucidate the underlying mechanism that leads to those changes.

**Main results.** Signaling models typically admit a plethora of equilibria. A number of refinement criteria have been proposed to narrow down the set of “reasonable equilibria.” Among those, the most widely adopted are the Intuitive Criterion and D1 of Cho and Kreps (1987) as they provide a sharp characterization of equilibria: the least-cost separating equilibrium (LCSE) is the only possible form of equilibrium that survives those criteria. This exclusive focus on LCSE, however, entails one undesirable feature that the equilibrium outcome is invariant to prior belief about the agent’s type. This feature precludes us from discussing, for instance, the agent’s incentive to build reputation in an earlier stage of a more dynamic setting. Moreover, insensitivity to the prior leads to an implausible implication as noted by Mailath et al. (1993): consider a two-type signaling model with almost complete information where there are very few low-type agents, say one in a million; even in this case, high-type agents are still forced to choose an action, which is bounded away from their complete information optimum, to separate from low-type agents that are almost non-existent. This argument suggests that there is a sharp discontinuity between complete information and near-complete information environments. Kreps and Sobel (1994, p. 860) note that the lack of continuity is “the intuitively least appealing aspect” of the refinement concepts.

In the model that admits behavioral diversity, we show that the equilibrium signaling level is decreasing in the agent’s prior reputation. As a consequence, all types benefit from having a better prior reputation. The driving force behind this result is the supply of emulation opportunities relative to the demand. An improvement in the agent’s prior reputation leads to more quality emulation opportunities, which directly benefit lower types who are in need of emulation. In addition, this reduces the need for lower types to mimic higher types, thereby enabling those higher types to separate more easily at lower signaling actions.

This finding leads to two important implications. On the empirical side, suppose we want to test the Veblen effect by examining the statistical relationship between conspicuous consumption and wealth. Regressing the level of consumption just on the level of wealth is misspecified because conspicuous consumption depends also on the individual’s prior reputation, which could well be correlated with the level of wealth. If we just look at the correlation between conspicuous consumption and wealth, we may end up finding a non-monotone relationship, because many truly wealthy individuals may have already established their status (i.e., a high prior belief) and feel less compelling need to engage in conspicuous consumption than the aspiring middle class who have no such status just yet.<sup>2</sup>

On the theoretical side, this finding indicates the possibility of bridging the gap between complete information and near-complete information environments. In Section 3, we consider a two-type example in which the equilibrium signaling level of the high type decreases and converges to the complete information optimum as the prior probability that the agent’s type is high tends to 1. Section 4 shows that this convergence result holds generally, for an arbitrary number of types and arbitrary decision rules of the behavioral types. Moreover, the extent of behavioral diversity required for the convergence can be arbitrarily small—the condition that should be satisfied in virtually any signaling situation. Our analysis thus suggests that the lack of convergence in the standard setup need not be viewed as a shortcoming of the refinement concepts, as it may well be a consequence of the theoretical simplification that excludes behavioral diversity from analysis.

**Literature.** In his celebrated treatise, Veblen (1899) emphasized two fundamental motives for conspicuous consumption: *pecuniary emulation* and *invidious distinction*. Pecuniary emulation refers to the act of a member of a lower class to meet the consumption standards of a higher class to signal that he belongs to that class, whereas invidious distinction occurs when a member of a higher class consumes conspicuously to distinguish himself from the lower classes. Between the two notions, there is indication that Veblen saw pecuniary emulation as the more important force behind conspicuous consumption. He noted that “[a]mong the motives which lead men to accumulate wealth, the primacy, both in scope and intensity, therefore, continues to belong to this motive of pecuniary emulation” (Veblen, 1899, p. 40) and devoted an entire chapter to the treatment of this concept in his book. Despite his emphasis, however, the analytical focus of the signaling lit-

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<sup>2</sup> A standard approach to account for this potential non-monotonicity is counter-signaling; see Feltovich et al. (2002), Araujo et al. (2007), and Chen et al. (2022).

erature, ever since its inception by Spence (1973), has been primarily on the transmission of information via the act of separation, which can be seen as a manifestation of invidious distinction: in the standard setup, the agent’s behavior in equilibrium is driven solely by the desire to distinguish himself but not by the desire to emulate others.<sup>3</sup> In this paper, we revisit Veblen’s old insight and explore the role of emulation that has hitherto received scant attention in the literature.

Standard refinement criteria consistently select the LCSE in signaling models. Mailath et al. (1993) are among the first to point out the rather disturbing implication of this prediction, which promoted them to propose an alternative refinement criterion called *undefeated equilibrium*. In this paper, instead of modifying the ways to refine equilibria, we modify the model structure, by introducing potential behavioral diversity among agents of the same type. In our model, we posit that the agent’s signaling action may be determined by exogenous stochastic shocks. This approach is related to the literature on noisy signaling (Matthews and Mirman, 1983; de Haan et al., 2011; Heinsalu, 2018), in which the agent’s chosen action is subject to stochastic shocks. Also, closely related in this regard is Daley and Green (2014) who consider a setting where the receiver can observe both the sender’s action and a type-dependent noise. A crucial difference is that in this strand of models, a shock occurs after the signaling choice is made, whereas in our model with behavioral diversity, a shock occurs before the signaling choice.

There are several works that may fall under the category of “signaling under behavioral diversity.” Frankel and Kartik (2019) and Ball (2022) consider a signaling model in which agents have multidimensional attributes and may differ in gaming ability. Dilme and Li (2016) consider a dynamic signaling model with dropout risk. They study how the possibility of exogenous dropout affects the trading dynamics. Although we cast our model in a static setting, their model can be seen as a special case of ours; as a consequence, the two models share some technical properties.<sup>4</sup> The focus of our study is, however, different,

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<sup>3</sup> In equilibrium, it is not possible for any type to emulate a higher type. The desire to emulate only works behind the scenes to determine how far the higher type must go, but that itself does not impact his own signaling choice.

<sup>4</sup> Dilme and Li (2016) show that in the model with two types, the equilibrium signaling level decreases and coincides with the complete information outcome as the prior belief becomes sufficiently high. Daley and Green (2014) also obtain the convergence result under an extra condition called RC-informativeness, but their underlying reasoning is different: in their model, it is the lack of single crossing that prevents the high type from fully separating. This is manifested in the fact that pooling in their model occurs at the top while pooling in ours occurs at the bottom.

as we aim to make a broader point that the possibility of emulation is a ubiquitous and integral aspect of signaling in society.

## 2. A signaling model with behavioral diversity

Consider a signaling model with an agent (sender) and a market (receiver). The agent is characterized by his type  $i \in \{1, \dots, I\}$ . The agent's type is his private information and cannot be directly observed by the market. We denote the prior belief by  $P := (p_1, \dots, p_I)$ , where  $p_i \in (0, 1)$  is the probability that the agent's type is  $i$ . The prior belief is an important primitive of the model, which captures the agent's prior reputation.

The model is otherwise standard, except that the agent's action is determined exogenously with some positive probability. Specifically, with probability  $\delta \in (0, 1)$ , the agent is a behavioral agent whose action is drawn from a set  $D := [0, d]$  of “exogenous actions.” For each type, the distribution of exogenous actions is continuous on  $D$  with full support. We use  $\beta_i(a)$  to represent the density of type  $i$  choosing action  $a$ , which summarizes the decision rules of a behavioral agent in a reduced-form way.<sup>5</sup> Let  $q(a) := \sum_{i=1}^I p_i \beta_i(a)$  and define

$$\tau(a) := \frac{\sum_{i=1}^I p_i \beta_i(a) i}{q(a)} \in (1, I)$$

as the corresponding reputation at action  $a$  among the behavioral agents. With the remaining probability  $1 - \delta$ , the agent is strategic and chooses signaling action  $a \in \mathbb{R}_+$  to maximize his payoff. We view  $\delta$  as capturing the extent of behavioral diversity that may originate, for example, from differences in gaming ability, differences in preferences, or the presence of external factors that exogenously constrain the agent's signaling choice.

The payoff to type  $i$  is given by  $U_i(a, t)$  where  $t \in [1, I]$  denotes the market's belief (expectation) of the agent's type. The payoff function is continuously differentiable, strictly decreasing in  $a$ , and strictly increasing in  $t$ . It also satisfies the standard single-crossing property. Let

$$MRS_i(a, t) := -\frac{\partial U_i(a, t) / \partial a}{\partial U_i(a, t) / \partial t}$$

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<sup>5</sup> All of our results hold for any arbitrary set  $D$ , although the argument becomes more involved when  $D$  is finite and has probability masses. The simplest example of this situation is where the agent has no reputation concerns and chooses the complete information optimum with probability  $\delta$ , in which case  $D = \{0\}$ .



be the marginal rate of substitution at  $(a, t)$ . The single-crossing property is equivalent to  $MRS_{i'}(a, t) > MRS_{i''}(a, t)$  for all  $(a, t)$  and  $i'' > i'$ .

Let  $\mu(a)$  denote the market's belief (expectation) of the agent's type when his signaling action is  $a$ . The strategy of each type  $i$  is denoted by  $\pi_i$ , a probability distribution of actions on  $\mathbb{R}_+$ . Throughout the analysis, we focus on signaling equilibria defined as follows:

1. Given  $\mu(\cdot)$ ,  $a' \in \arg \max_a U_i(a, \mu(a))$  if  $a' \in \text{supp}(\pi_i)$ ;
2.  $\mu(\cdot)$  is consistent with  $\{\pi_i\}$  and Bayes' rule whenever applicable.

As usual, the set of signaling equilibria can be quite large. The D1 refinement requires that, for any  $a_{i'} \in \text{supp}(\pi_{i'})$ ,  $a_{i''} \in \text{supp}(\pi_{i''})$ , and any off-equilibrium action  $a'$ , if for all  $t$ ,

$$U_{i'}(a', t) \geq U_{i'}(a_{i'}, \mu(a_{i'})) \implies U_{i''}(a', t) > U_{i''}(a_{i''}, \mu(a_{i''})),$$

then the off-equilibrium belief associated with action  $a'$  must assign zero probability to the agent being of type  $i'$ . In our setting, however, the standard D1 criterion is not directly applicable because there are no off-path actions on  $D$ . We thus introduce an extended version of the criterion. Consider a discretized space  $D^n := \{d_1^n, \dots, d_n^n\}$  where  $d_1^n = 0$  and  $d_j^n = (j-1)d/n$  for  $j = 1, \dots, n$ , and let  $\beta_i^n(d_j) := \int_{d_j^n}^{d_{j+1}^n} \beta_i(a) da$ , so that a behavioral agent of type  $i$  chooses  $d_j \in D^n$  with probability  $\beta_i^n(d_j)$ . We say that an equilibrium in our model with continuous  $D$  satisfies (the extended version of) D1 if it is the limit of equilibria with discrete  $D^n$  that satisfy D1 as  $n$  goes to infinity. We refer to such equilibrium simply as D1 equilibrium hereafter. In effect, this restriction requires that for any  $a' \in D$  which is not in the support of the equilibrium strategy of any type, no agent has an incentive to deviate to  $a'$  even when the belief  $\mu(a') = \tau(a')$  is replaced by some other “off-equilibrium” belief  $\hat{\mu}(a')$  that satisfies the D1 criterion.<sup>6</sup>

### 3. An example with two types

We may illustrate the main ideas by a simple example with two types. In this example, we refer to type 1 as the low type and to type 2 as the high type, and let the prior probabilities of these two types be  $1-p$  and  $p$  respectively. Also assume for simplicity that the distribution of exogenous actions is type-independent and uniform on  $D$ , so that  $q(a) = 1/d$  and  $\tau(a) = 1+p$  for all  $a \in D$ .

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<sup>6</sup> When there are only two types, we may require equilibrium to satisfy an extended version of the Intuitive Criterion defined in a similar manner, and all the arguments will continue to hold.

Let  $\underline{u}_i$  and  $\bar{u}_i$  be the payoff lower bound and upper bound, respectively, for type  $i$ . Since the highest reputation that can be achieved is  $t = 2$  and effort is costly, it is easy to see  $\bar{u}_i := U_i(0, 2)$  for both types. Note also that the low type can always choose  $a = 0$  and secure at least  $t = 1$ , so that  $\underline{u}_1 = U_1(0, 1)$ . Finally, define  $\bar{s}$  such that  $U_1(\bar{s}, 2) = U_1(0, 1)$ , which is the action chosen by the high type in the LCSE. Since the low type can secure at least a payoff of  $U_1(0, 1)$ , we have  $\underline{u}_2 = U_2(\bar{s}, 2)$  as the payoff lower bound for the high type. Note that  $\underline{u}_1$  and  $\underline{u}_2$  are the respective payoff that each type earns in the LCSE.

As we will detail in the general case, our model admits a unique D1 equilibrium. In this equilibrium, the high type always chooses a least-cost separating action and gets the reputation  $t = 2$ . This means that once we fix the high type's payoff  $u_2 \in \Upsilon := [\underline{u}_2, \bar{u}_2]$ , we can uniquely pin down his corresponding action,  $s_2(u_2)$ , from  $u_2 = U_2(s_2(u_2), 2)$ . Define  $\mathbf{x}_2(u_2) := (s_2(u_2), 2)$  as the low type's *reservation allocation*. We say that an action  $a$  is binding for the low type if choosing that action unilaterally gives him a higher payoff than the reservation allocation. Formally, let  $t = \phi_1(a; u_2)$  represent the indifference curve of type 1 that passes through  $\mathbf{x}_2(u_2)$ . Then,  $a$  is binding if  $\tau(a) > \phi_1(a; u_2) > 1$ .<sup>7</sup> Define

$$B_1(u_2) := \{a \in D : \tau(a) > \phi_1(a; u_2) > 1\}$$

as the *set of binding actions* for the low type when the high type's payoff is  $u_2$ .

In D1 equilibrium in which the high type's payoff is  $u_2$ , the low type randomizes over  $a \in B_1(u_2)$  with some density  $\pi_1(a; u_2)$ . Given this, all of the low type's on-path actions must give him the same payoff, i.e.,  $\mu(a) = \phi_1(a; u_2)$  for all  $a \in B_1(u_2)$ . Thus, for each  $u_2 \in \Upsilon$  and  $a \in B_1(u_2)$ , the density  $\pi_1(a; u_2)$  must satisfy Bayes' rule and the indifference condition:

$$\mu(a) = \frac{\delta(1+p)(1/d) + (1-\delta)(1-p)\pi_1(a; u_2)}{\delta(1/d) + (1-\delta)(1-p)\pi_1(a; u_2)} = \phi_1(a; u_2).$$

Note that we have  $\pi_1(a; u_2) = 0$  for  $a \notin B_1(u_2)$ . Solving the above equation yields

$$\pi_1(a; u_2) = \max \left\{ \frac{\delta(1+p - \phi_1(a; u_2))}{(1-\delta)(1-p)d(\phi_1(a; u_2) - 1)}, 0 \right\}. \quad (1)$$

The equilibrium value of  $u_2$  is determined by the requirement that the low type's mixed strategy is a valid probability distribution. Let

$$\Pi_1(u_2) := \int_{a \in (0, d]} \pi_1(a; u_2) da. \quad (2)$$

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<sup>7</sup> It is possible to have  $\tau(a) > \phi_1(a; u_2) = 1$ . In the general analysis, we treat this case separately because the mixed strategy (given below) is not well defined at  $a$  such that  $\phi_1(a; u_2) = 1$ .

If  $\Pi_1(u_2) \leq 1$ , then the equilibrium payoff to the high type is  $u_2^* = \underline{u}_2$ , and the low type's mixed strategy is given by the density  $\pi_1(\cdot; u_2^*)$ , with an atom of mass  $1 - \Pi_1(u_2^*)$  at  $a = 0$ .<sup>8</sup> If  $\Pi_1(u_2) > 1$ , then the equilibrium payoff  $u_2^*$  satisfies  $\Pi_1(u_2^*) = 1$ , and the low type's mixed strategy is given by the density  $\pi_1(\cdot; u_2^*)$ . Observe that  $\Pi_1(u_2)$  is strictly decreasing in  $u_2$  with  $\Pi_1(\bar{u}_2) = 0$ . Therefore,  $u_2^*$  exists and is unique.

In this environment, the low type can potentially choose one of the binding actions to emulate the behavioral agents, or the separating action to mimic the high type. These two choices are substitutes for the low type and must yield the same payoff. The cost of mimicking the high type is given by the cost of the separating action  $s_2(u_2)$ , which we call the “price of signaling.” The price of signaling is determined by the supply of emulation opportunities relative to the demand, which is captured by (2). For a given  $u_2$ , if  $\Pi_1(u_2) > 1$ , emulation opportunities are in excess supply, and the price  $s_2(u_2)$  must go down; if  $\Pi_1(u_2) < 1$ , they are in excess demand, and the price must go up.

The magnitude of the relative supply of emulation opportunities depends on the balance between the numerator and the denominator of  $\pi_1(\cdot; u_2)$  in (1). Suppose, for example, that the agent's prior reputation  $p$  increases. Because a behavioral agent on average has a higher type, emulating a behavioral agent leads to a higher inference about one's type. In a sense this effect represents an increase in the supply of emulation opportunities, and is reflected in an increase in the numerator of  $\pi(\cdot; u_2)$ . At the same time, a higher  $p$  means that there are fewer low types who are attempting to emulate the behavioral agents. The demand for emulation decreases, and this effect is reflected in a decrease in the denominator of  $\pi(\cdot; u_2)$ . Both effects raise the overall relative supply  $\Pi_1(u_2)$ , making emulation a more attractive choice. In equilibrium, the price of signaling  $s_2(u_2)$  has to fall to compensate for the difference. Since  $s_2(u_2)$  is decreasing in  $u_2$ , this shows that  $u_2^*$  must increase with  $p$ . Of course, a lower price of signaling benefits the low type, and  $u_1^*$  also increases with  $p$ .

Since  $\phi_1(a; u_2) \geq 1$  for any  $u_2 \in \Upsilon$ , (1) shows that the supply of emulation opportunities goes to 0 in the limit as  $p$  approaches 0, which raises the price of signaling to the maximum level. The equilibrium then converges to the LCSE where the maximum level of signaling is required, and the payoffs to both types approach their lower bounds  $\underline{u}_i$ . At

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<sup>8</sup> It is possible to have  $\Pi_1(u_2)$  converge to a finite number if the indifference curves are concave in  $a$ , in which case there could be an equilibrium in which the low type chooses  $a = 0$  with strictly positive probability. This possibility can be ruled out and the equilibrium mixed strategies are always smooth if the indifference curves are weakly convex. See Online Appendix.

the other end, as  $p$  approaches 1, the demand for emulation goes to 0. The relative supply thus diverges to infinity, which makes emulation almost free. The equilibrium converges to the complete information outcome that requires no signaling effort from the high type, and the payoffs to both types approach their upper bounds  $\bar{u}_i$ .

## 4. General analysis

### 4.1. Equilibrium existence and characterization

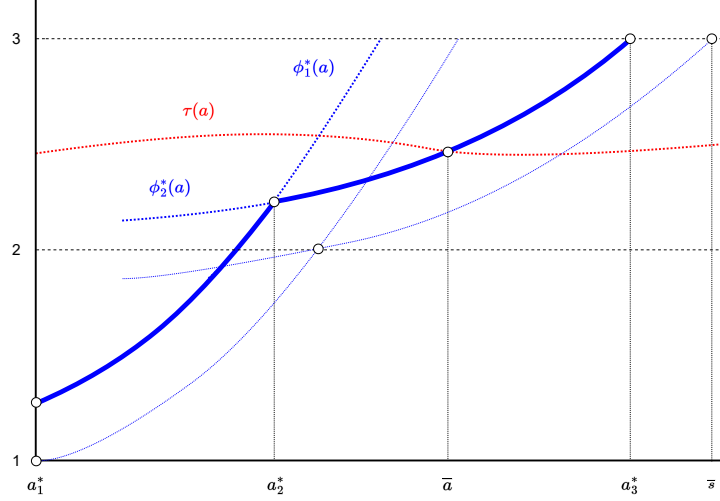
We now extend the analysis to incorporate an arbitrary number of types and an arbitrary distribution of exogenous actions. We propose an algorithm to find the set of binding actions and the corresponding mixed strategy recursively for each type, from the second highest type (type  $I - 1$ ) to the lowest (type 1). Since the essence of this procedure is already described in the previous section, we only provide a brief overview of the algorithm and relegate its details to Appendix A.

In any D1 equilibrium, the highest type must choose a least-cost separating action and get reputation  $t = I$ . The algorithm thus starts with the highest type and his payoff  $u_I \in \Upsilon := [\underline{u}_I, \bar{u}_I]$ .<sup>9</sup> Once we fix  $u_I \in \Upsilon$ , the fact that the highest type is getting reputation  $t = I$  allows us to uniquely pin down the fully separating action. Denote this action by  $s_I(u_I)$ . In the following, we use  $a_i(u_I)$  to denote the lower bound of the support of type  $i$ 's strategy when the payoff to type  $I$  is fixed at  $u_I$ . Consider the indifference curve of type  $I - 1$  that passes through the reference allocation  $\mathbf{x}_{I-1}(u_I) := (a_I(u_I), I)$ , which we denote by  $\phi_{I-1}(\cdot; u_I)$ . We derive the set of binding actions and the corresponding mixed strategy for type  $I - 1$ , in a manner similar to that described in the two-type example. We repeat this process to obtain  $a_i(u_I)$  recursively for each  $i$ . A necessary condition for finding an equilibrium is that we reach the final round to determine  $a_1(u_I)$  for the lowest type. An equilibrium is obtained if  $a_1(u_I)$  is well defined, and the mixed strategy of type 1 on  $[a_1(u_I), a_2(u_I))$  is a well-defined probability distribution.

According to this algorithm, each type  $i \leq I - 1$  adopts a possibly mixed strategy on  $[a_i(u_I), a_{i+1}(u_I))$ . For each  $u_I \in \Upsilon$ , we can identify one indifference curve for each type on which the allocations of that type lie. We refer to the lower envelope of those indifference curves as the *allocation path* and denote it by  $\Phi(\cdot; u_I)$ ; see Figure 1 for a graphical illustration. For brevity, if  $u_I^*$  is the equilibrium payoff to type  $I$ , we let  $\phi_i^*(\cdot) := \phi_i(\cdot; u_I^*)$

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<sup>9</sup> The upper bound is  $\bar{u}_I = U_I(0, I)$ . If we let  $\bar{s}$  be the action chosen by type  $I$  in the LCSE, the lower bound is given by  $\underline{u}_I = U_I(\bar{s}, I)$ .



**Figure 1.** The equilibrium allocation path (with three types) is the lower envelope of the two equilibrium indifference curves,  $\phi_1^*(\cdot)$  and  $\phi_2^*(\cdot)$ , depicted by the thick blue line. The thin blue line represents the LCSE, whose allocation path stays strictly below the equilibrium allocation path.

be the equilibrium indifference curve of type  $i$  and  $\Phi^*(\cdot) := \Phi(\cdot; u_l^*)$  be the equilibrium allocation path. In equilibrium, the action space is divided into  $I - 1$  intervals,  $[a_i^*, a_{i+1}^*)$  for  $i = 1, \dots, I - 1$ , where  $a_i^* := a_i(u_l^*)$ . The equilibrium set of binding actions is:

$$B_i^* := \{a \in [a_i^*, a_{i+1}^*) : \tau(a) > \phi^*(a) > i\}.$$

Also define  $s_i^* := s_i(u_l^*)$  such that  $\Phi^*(s_i^*) = i$  if it exists, which is the fully separating action for type  $i$ .

**Proposition 1.** For any  $\delta > 0$  and  $\{\beta_i\}$ , there exists a unique D1 equilibrium characterized by  $\Phi^*(\cdot)$ . In the equilibrium,

1. Type  $I$  chooses a fully separating action  $s_I^*$  with probability 1;
2. Type  $i = 1, \dots, I - 1$  randomizes over  $B_i^* \cup \{s_i^*\}$  if  $B_i^*$  is nonempty, and chooses the fully separating action  $s_i^*$  with probability 1 if it is empty.

*Proof.* See Appendix A. ■

## 4.2. Comparative statics

Fix some equilibrium  $\Phi^*(\cdot)$ . For  $a \in B_i^*$ , when type  $i$  adopts an equilibrium mixed strategy with density  $\pi_i^*(\cdot)$ , the equilibrium strategy must satisfy

$$\Phi^*(a) = \frac{\delta q(a)\tau(a) + (1 - \delta)p_i\pi_i^*(a)i}{\delta q(a) + (1 - \delta)p_i\pi_i^*(a)}.$$

Define an indicator function  $\iota(\cdot)$  such that  $\iota(a) = i$  if  $a \in [a_i^*, a_{i+1}^*)$ . Let  $\bar{a} := \sup\{a : \tau(a) > \Phi^*(a)\}$  be the largest binding action and  $S^* := \{s_i^*\}$  be the set of fully separating actions under  $\Phi^*(\cdot)$ . This formulation allows us to express the mixed strategy by

$$\pi^*(a) = \max \left\{ \frac{\delta q(a)(\tau(a) - \Phi^*(a))}{(1 - \delta)p_{\iota(a)}(\Phi^*(a) - \iota(a))}, 0 \right\}, \quad (3)$$

for  $a \in [0, \bar{a}) \setminus S^*$ ,<sup>10</sup> which can be interpreted as the equilibrium relative supply of emulation opportunities at action  $a$ .

Aside from the preferences, the primitives of the model are summarized by  $\Theta := (\delta, P, \{\beta_i\})$ . Now consider an alternative environment characterized by  $\hat{\Theta} := (\hat{\delta}, \hat{P}, \{\hat{\beta}_i\})$  and define

$$\hat{\pi}(a) = \max \left\{ \frac{\hat{\delta}\hat{q}(a)(\hat{\tau}(a) - \Phi^*(a))}{(1 - \hat{\delta})\hat{p}_{\iota(a)}(\Phi^*(a) - \iota(a))}, 0 \right\},$$

where  $\hat{q}(a) := \sum_{i=1}^I \hat{p}_i \hat{\beta}_i(a)$  and  $\hat{\tau}(a) := (\sum_{i=1}^I \hat{p}_i \hat{\beta}_i(a)i) / \hat{q}(a)$ .

**Lemma 1.** *The equilibrium payoffs to all types increase weakly if  $\hat{\pi}(a) \geq \pi^*(a)$  for all  $a \in [0, \bar{a}) \setminus S^*$ .*

*Proof.* See Appendix B. ■

A change in parameters that changes  $\pi^*(\cdot)$  to  $\hat{\pi}(\cdot)$  increases the relative supply of emulation opportunities. Lemma 1 states that such a change increases the equilibrium payoffs to all types.<sup>11</sup> Propositions 2–3 below follow directly from this lemma.

**Proposition 2.** *Suppose the type distribution changes from  $P$  to  $\hat{P}$ , where (a)  $\hat{p}_i \leq p_i$  for all  $i \leq \iota(\bar{a})$  (with at least one strict inequality); and (b)  $\hat{p}_i \geq p_i$  for all  $i > \iota(\bar{a})$ . Then, the equilibrium payoffs to all types are weakly higher under  $\hat{P}$  than under  $P$ .*

<sup>10</sup> As in the two-type example, we exclude the fully separating actions because the mixed strategy is not well defined at  $a \in S^*$ .

<sup>11</sup> The result holds strictly if the indifference curves are weakly convex. See Online Appendix.

The change from  $P$  to  $\hat{P}$  described in Proposition 2 implies stochastic dominance. Thus the supply of emulation opportunities (numerator of  $\pi^*(\cdot)$ ) increases. Further, condition (a) of the proposition implies that the demand for emulation from types below  $\iota(\bar{a})$  (denominator of  $\pi^*(\cdot)$ ) decreases. This proposition broadly suggests that better prior reputation is beneficial for all types: lower types gain from emulating the behavioral agents, which enables higher types to separate at lower costs.

**Proposition 3.** *For any  $\delta$  and  $\hat{\delta} > \delta$ , the equilibrium payoffs to all types are weakly higher under  $\hat{\delta}$  than under  $\delta$ .*

Proposition 3 concerns the impact of a change in the extent of behavioral diversity. From (3), an increase in  $\delta$  raises the relative supply  $\pi^*(\cdot)$  of emulation opportunities, which benefits all types by Lemma 1. Note that this result implies that all types are weakly better off compared to the LCSE which prevails when  $\delta = 0$ . Although the standard analysis of signaling focuses on one particular dimension of reputation, there are typically many different dimensions along which an agent can gain or lose his reputation. Which dimension to focus on clearly depends on the agent's preferences about what should constitute an important quality of life. In a homogeneous society where people have similar values and aspire to achieve similar goals, more resources must be expended for signaling. A society with diverse values and aspirations would muddle information and make signaling less informative, but that can lead to a Pareto improvement as it reduces the intensity of wasteful signaling.

### 4.3. Convergence

We now examine the convergence property as the underlying information asymmetry disappears. To this end, we pay attention to the largest binding action and show that it converges to 0 as the prior type distribution converges to a degenerate distribution at some type  $j$ .

**Proposition 4.** *For any  $\varepsilon > 0$ , there is  $\bar{p}_\varepsilon < 1$  such that  $\bar{a} < \varepsilon$  for all  $p_j > \bar{p}_\varepsilon$ .*

*Proof.* Observe that  $\lim_{p_j \rightarrow 1} \tau(a) = j$  for all  $a \in D$ . Also, since  $\lim_{p_j \rightarrow 1} = \infty$  for  $i \neq j$ ,  $a_j^*$  converges to 0. If  $\tau(a)$  converges from below in a neighborhood of  $\bar{a}$ ,  $a_j^* \geq \bar{a}$ , and this completes the proof. If  $\tau(a)$  converges from above, type  $j$  randomizes over  $[a_j^*, a_{j+1}^*)$ . As  $\tau(a)$  converges to  $j$ ,  $\bar{a}$  converges to 0 ■

Given that  $\lim_{p_j \rightarrow 1} \tau(a) = j$  for all  $a \in D$ , Proposition 4 implies that the allocations of type  $j$  collapse to a single point  $(0, j)$ , and so do the allocations of all types below  $j$ . Types lower than  $j$  receive  $u_i(0, j)$ , which is higher than their respective complete information payoff. For types above  $j$ , the allocations converge to the LCSE of the standard setting where the lowest type is type  $j$  (rather than type 1). Their payoff is less than the complete information payoff but is higher than their payoff from the LCSE of the standard setting.

## 5. Conclusion

The signaling literature often focuses on separating equilibria where the agent successfully conveys his private information via costly action. This paper looks at the flip side of signaling, with emphasis on the possibility of emulation (pooling) in equilibrium, by introducing behavioral diversity into the standard setup. The possibility of emulation changes the structure of incentives, not only for those who emulate higher types but also for those who attempt to distinguish themselves from lower types. The extended model delivers new insights into the role of external factors that are absent in the standard setup. We also note that with the slightest degree of behavioral diversity, the model exhibits a convergence property which is intuitively more appealing than the LCSE of the standard setting.



## Appendix A: Proof of Proposition 1

**Characterization.** In any discretized game, we can apply the standard D1 refinement to rule out any pooling at  $a \notin D^n$ . We first argue that any D1 equilibrium in a discretized game must have the following properties:

- The highest type chooses a fully separating action with probability 1;
- The lowest action chosen by type  $i$  is on the equilibrium indifference curve of type  $i - 1$ .

To show the first claim, suppose type  $I$  adopts a mixed strategy. For some action  $a'$  in the support of this mixed strategy such that  $\mu(a') < I$ , type  $I$  must be pooled with lower types. Type  $I$  must then have an incentive to deviate slightly upward from those actions because D1 would assign probability 1 to type  $I$  (observe that no type can randomize continuously over an interval in a discretized game). This shows that type  $I$  must adopt a pure strategy. The action chosen by type  $I$  must be fully separating because D1 assigns probability 1 to any upward deviation, giving type  $I$  an incentive to deviate slightly above. To show the second claim, suppose that there is some type  $i$  whose lowest action is bounded away from the equilibrium indifference curve of type  $i - 1$ . Type  $i$  then has an incentive to deviate slightly below because D1 assigns probability 1 to such a deviation.

**Algorithm.** In each round  $k \geq 1$ , we determine the allocations of type  $I - k$ . Let

$$\mathbf{x}_{I-k}(u_I) := (a_{I-k+1}(u_I), t_{I-k+1}(u_I))$$

be the reservation allocation of type  $I - k$ , where  $a_i(u_I)$  is the lowest action chosen by type  $i$  and  $t_i(u_I) = \mu(a_i(u_I))$  is the corresponding reputation for that action. In round 1, for each  $u_I \in \Upsilon$ , we obtain  $a_I(u_I) = s_I(u_I)$  such that  $U_I(s_I(u_I), I) = u_I$  and  $t_I(u_I) = I$ . For  $k \geq 2$ , the algorithm inherits a reservation allocation  $\mathbf{x}_{I-k}(u_I)$  from the previous round, and we let  $\phi_{I-k}(\cdot; u_I)$  represent the indifference curve of type  $I - k$  that passes through this reservation allocation. Define  $s_{I-k}(u_I)$  such that

$$I - k = \phi_{I-k}(s_{I-k}(u_I); u_I),$$

if  $\phi_{I-k}(0; u_I) \leq I - k$ , and let  $\underline{a}_{I-k}(u_I) = s_{I-k}(u_I)$ . If  $\phi_{I-k}(0; u_I) > I - k$ ,  $s_{I-k}(u_I)$  is not well defined, and we let  $\underline{a}_{I-k}(u_I) = 0$ . Define the tentative set of binding actions for type  $I - k$  by

$$\hat{B}_{I-k}(u_I) := \{a \in (\underline{a}_{I-k}(u_I), a_{I-k+1}(u_I)) : \tau(a) > \phi_{I-k}(a; u_I)\}.$$

For each  $a \in \hat{B}_{I-k}(u_I)$ , find  $\pi_{I-k}(\cdot; u_I)$  such that

$$\phi_{I-k}(a; u_I) = \frac{\delta q(a)\tau(a) + (1-\delta)p_{I-k}\pi_{I-k}(a; u_I)(I-k)}{\delta q(a) + (1-\delta)p_{I-k}\pi_{I-k}(a; u_I)}.$$

For  $a \notin B_{I-k}(u_I)$ , we let  $\pi_{I-k}(a; u_I) = 0$ . Solving the above equation yields

$$\pi_{I-k}(a; u_I) = \max \left\{ \frac{\delta q(a)(\tau(a) - \phi_{I-k}(a; u_I))}{(1-\delta)p_{I-k}(\phi_{I-k}(a; u_I) - I + k)}, 0 \right\}.$$

Let

$$\Pi_{I-k}(u_I) := \int_{a \in (\underline{a}_{I-k}, a_{I-k+1})} \pi_{I-k}(a; u_I) da.$$

There are three possibilities.

- If  $\Pi_{I-k}(u_I) > 1$ , there must be some  $a_{I-k}(u_I) \in \hat{B}_{I-k}(u_I)$  such that

$$\int_{a \in (a_{I-k}(u_I), a_{I-k+1}(u_I))} \pi_{I-k}(a; u_I) da = 1,$$

and we set  $t_{I-k}(u_I) = \phi_{I-k}(a_{I-k}(u_I); u_I)$ .

- If  $\Pi_{I-k}(u_I) \leq 1$  and  $s_{I-k}(u_I)$  is well defined, we let type  $I-k$  choose  $s_{I-k}(u_I)$  with probability  $1 - \Pi_{I-k}(u_I)$  and set  $(a_{I-k}(u_I), t_{I-k}(u_I)) = (s_{I-k}(u_I), I-k)$ .
- If  $\Pi_{I-k}(u_I) \leq 1$  but  $s_{I-k}(u_I)$  is not well defined,  $u_I$  is too high, and we terminate the algorithm.

Given a well-defined  $a_{I-k}(u_I)$ , define  $B_{I-k}(u_I) := \hat{B}_{I-k}(u_I) \cap [a_{I-k}(u_I), a_{I-k+1}(u_I))$  as the set of binding actions.

Suppose that  $u_I$  is set in the right range, and the algorithm reaches the final round. In round  $I-1$ , given  $\mathbf{x}_1(u_I)$ , we derive  $s_1(u_I)$  if it is well defined. If  $s_1(u_I) > 0$ ,  $u_I$  is too low, and we terminate the algorithm. A necessary condition for finding an equilibrium is therefore  $\phi_1(0; u_I) \geq 0$ , so that  $\underline{a}_1 = 0$ . Given this, we obtain  $B_1(u_I)$  and  $\Pi_1(u_I)$  as above. There are now four possibilities.

- If  $\Pi_1(u_I) > 1$ ,  $u_I$  is too low, and we terminate the algorithm.
- If  $\Pi_1(u_I) = 1$ , we have an equilibrium.
- If  $\Pi_1(u_I) < 1$  and  $s_1(u_I) = 0$ , we let type 1 choose  $s_1(u_I) = 0$  with probability  $1 - \Pi_1(u_I)$  and we have an equilibrium.

- If  $\Pi_{I-k}(u_I) < 1$  but  $s_{I-k}(u_I)$  is not well defined,  $u_I$  is too high, and we terminate the algorithm.

**Existence.** The characterization result suggests that the algorithm described above is the only way to construct a D1 equilibrium, if any. The algorithm produces an allocation path for each  $u_I \in \Upsilon$ . Observe that  $a_i(\cdot)$  is continuous and strictly decreasing at  $u'_I$  if  $a_i(u'_I) > 0$ . First, by definition,  $a_I(\cdot)$  is continuous and strictly decreasing. Now consider how a change in  $a_{i+1}(u_I)$  affects  $a_i(u_I)$ . Suppose there is an infinitesimal increase in  $u_I$  from  $u'_I$  to  $u''_I$ , which in turn implies  $a_I(u'_I) < a_I(u''_I)$ . Then,  $\phi_{I-1}(a; u'_I) < \phi_{I-1}(a; u''_I)$  for all  $a$ . If  $B_{I-1}(u'_I)$  is empty, we have  $a_{I-1}(u'_I) > a_{I-1}(u''_I)$ , again by definition. If  $B_{I-1}(u'_I)$  is nonempty,  $\pi_{I-1}(a; u'_I) > \pi_{I-1}(a; u''_I)$  for all  $a \in B_{I-1}(u'_I)$ . Since  $\sup B_{I-1}(u'_I) > \sup B_{I-1}(u''_I)$ , we must have

$$\int_{a \in (a_{I-1}(u'_I), \sup B_{I-1}(u''_I))} \pi_{I-1}(a; u''_I) da < 1,$$

which proves  $a_{I-1}(u'_I) > a_{I-1}(u''_I)$ . The claim is established because this argument holds for any  $i < I - 1$  as long as  $a_i(u'_I) > 0$ .

This fact suggests that the allocation path shifts up continuously as  $u_I$  increases. We thus need to find an allocation path that can satisfy the equilibrium condition by adjusting  $u_I$ . Observe that for any  $\delta > 0$  and  $\{\beta_i\}$ , we have  $a_{I-1}(u_I) > 1$  and  $\Pi_{I-k}(u_I) < 1$  if  $u_I$  is close enough to  $\bar{u}_I$ . Therefore,  $u_I$  is too high when it is close to the upper bound. At the other end, as  $u_I$  gets close to  $\underline{u}_I$ , there is  $u'_I \geq \underline{u}_I$  such that  $\phi_1(0; u'_I) = 1$ . By continuity, there must be  $u''_I > u'_I$  such that  $\phi_2(0; u''_I) = 2$ . For  $u_I \in (u'_I, u''_I)$ ,  $\Pi_1(u_I)$  is well defined and strictly decreasing in  $u_I$ . Note that  $\lim_{u_I \uparrow u''_I} \Pi_1(u_I) = 0$ . If  $\Pi_1(u'_I) \geq 1$ , we can find a unique  $u_I^*$  such that  $\Pi_1(u_I^*) = 1$  and this constitutes an equilibrium. If  $u'_I = \underline{u}_I$  and  $\Pi_1(\underline{u}_I) < 1$ , we let type 1 choose  $a = 0$  with probability  $1 - \Pi_1(\underline{u}_I)$ , and this is a unique equilibrium.

## Appendix B: Proof of Lemma 1

We first show that  $u_I^*$  increases if  $\hat{\pi}(a) \geq \pi^*(a)$  for all  $a \in [0, \bar{a}] \setminus S^*$ . Suppose we run the algorithm under the new set of parameters  $\hat{\Theta}$  with initial value  $u_I^*$  for type  $I$ . Let  $\tilde{\Phi}(\cdot)$  be the corresponding allocation path and  $[\tilde{a}_i, \tilde{a}_{i+1})$  be the corresponding partition. Define

$$\tilde{\pi}(a) = \max \left\{ \frac{\hat{\delta} \hat{q}(a)(\hat{\tau}(a) - \tilde{\Phi}(a))}{(1 - \hat{\delta}) \hat{p}_{\tilde{\tau}(a)}(\tilde{\Phi}(a) - \tilde{\tau}(a))}, 0 \right\},$$

where  $\tilde{\iota}(a) = i$  if  $a \in [\tilde{a}_i, \tilde{a}_{i+1})$ . Also let  $\bar{i} := \iota(\bar{a})$ . Because  $\hat{\pi}(a) \geq \pi^*(a)$  for  $a \in [0, \bar{a}) \setminus S^*$ , we have  $\tilde{a}_{\bar{i}+1} \geq a_{\bar{i}+1}^*$ , which also implies that  $\tilde{a}_{\bar{i}} \geq a_{\bar{i}}^*$  and  $\tilde{\Phi}(a) \leq \Phi^*(a)$  for  $a$  such that  $\iota(a) = \tilde{\iota}(a) = \bar{i} - 1$ . The last fact ensures that  $\tilde{\pi}(a) \geq \pi^*(a)$  for this interval of  $a$ . Together with  $\tilde{a}_{\bar{i}} \geq a_{\bar{i}}^*$ , this implies  $\tilde{a}_{\bar{i}-1} \geq a_{\bar{i}-1}^*$ . We can repeat this argument back to type 1 to show that  $\tilde{a}_1 \geq a_1^*$ . Thus, the equilibrium payoff to type  $I$  must increase weakly.

We now argue that the equilibrium payoffs to all types weakly increase. Use  $\hat{\Phi}^*(\cdot)$  to denote the equilibrium allocation path under  $\hat{\Theta}$ . Define  $\hat{\iota}(\cdot)$  analogously and let

$$\hat{\pi}^*(a) = \max \left\{ \frac{\hat{\delta}\hat{q}(a)(\hat{\tau}(a) - \hat{\Phi}^*(a))}{(1 - \hat{\delta})\hat{p}_{\hat{\iota}(a)}(\hat{\Phi}^*(a) - \hat{\iota}(a))}, 0 \right\}.$$

Let  $\hat{i} \geq \bar{i}$  be the new  $\bar{i}$  and  $[\hat{a}_i^*, \hat{a}_{i+1}^*)$  be the new equilibrium partition under  $\hat{\Theta}$ . Suppose the equilibrium payoff to some type  $k$  is strictly lower under  $\hat{\Theta}$ . If there are multiple types that are made worse off by the change in distribution, let  $k$  represent the lowest such type. Note that  $k$  cannot be greater than  $\hat{i}$ , because this would imply that the least-cost separating solution starting from type  $k$  would be entirely below the original allocation path, violating the previously established result that the equilibrium payoff to type  $I$  increases. Because the equilibrium indifference curve of type  $k - 1$  under  $\hat{\Theta}$  is weakly above that under  $\Theta$ , while the equilibrium indifference curve of type  $k$  under  $\hat{\Theta}$  is below that under  $\Theta$ , the single-crossing property requires that  $\hat{a}_k^* < a_k^*$ . If  $k = 1$ , we must have  $\hat{a}_k^* \leq a_k^*$ . Therefore, in either case,  $\hat{a}_k^* \leq a_k^*$ . Furthermore,  $\hat{\Phi}^*(a) \leq \Phi^*(a)$  implies  $\hat{\pi}^*(a) > \pi^*(a)$  for  $a$  such that  $\iota(a) = \hat{\iota}(a) = k$ . Together with  $\hat{a}_k^* \leq a_k^*$ , this implies  $\hat{a}_{k+1}^* < a_{k+1}^*$ . This in turn implies that  $\hat{\Phi}^*(a) < \Phi^*(a)$  and therefore  $\hat{\pi}^*(a) > \pi^*(a)$  for  $a$  such that  $\iota(a) = \hat{\iota}(a) = k + 1$ . Together with  $\hat{a}_{k+1}^* < a_{k+1}^*$ , this implies  $\hat{a}_{k+2}^* < a_{k+2}^*$ . Repeating this argument shows that  $\hat{\Phi}^*(a) < \Phi^*(a)$  for  $a$  such that  $\iota(a) = \hat{\iota}(a) = \bar{i}$ . But a strictly lower equilibrium indifference curve  $\hat{\Phi}^*(a)$  for type  $\bar{i}$  would imply a strictly lower equilibrium indifference curve for type  $I$ , contradicting the result that the equilibrium payoff to type  $I$  must increase.

# Online Appendix to Pecuniary Emulation and Invidious Distinction: Signaling under Behavioral Diversity

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October 17, 2023

In the baseline model, we cannot rule out the possibility that some type  $i$  may choose a fully separating action (i.e., the action that gives him the reputation of his own type) with strictly positive probability, even when the set of binding actions is nonempty and this type adopts a mixed strategy. This is because  $\Pi_{I-k}(a; u_I) = \int_{a \in (s_{I-k}(u_I), a_{I-k+1})} \pi_{I-k}(a; u_I) da$  may converge to a number less than 1, even though  $\lim_{a \downarrow s_{I-k}(u_I)} \pi_{I-k}(a; u_I) = \infty$ . Below, we show that this possibility can be ruled out, and the mixed strategies are always smooth with no probability masses if the indifference curves are weakly convex.<sup>12</sup> Moreover, under this condition, Lemma 1 and Propositions 2 and 3 hold strictly.

Let  $\phi_i(\cdot)$  be the indifference curve of type  $i$  that passes through  $(s_i, i)$  for some  $s_i$ . Given that  $s_i$  is binding (i.e.,  $\tau(s_i) > \phi_i(s_i)$ ), it suffices to show that

$$\lim_{s \downarrow s_i} \int_s^{s_i + \varepsilon} \frac{1}{\phi_i(a) - i} da$$

diverges for any  $\varepsilon > 0$ . To show this, consider a linear function  $\eta(a - s_i) + i$  that passes through  $(s_i, i)$ , where  $\eta$  is set to satisfy  $\eta\varepsilon + i = \phi_i(s_i + \varepsilon)$ . Observe that this linear function always stays (weakly) above  $\phi_i(a)$  for  $a \in [s_i, s_i + \varepsilon]$  if  $\phi_i(\cdot)$  is weakly convex. This in turn implies that for any  $\varepsilon > 0$ ,

$$\lim_{s \downarrow s_i} \int_s^{s_i + \varepsilon} \frac{1}{\phi_i(a) - i} da \geq \lim_{s \downarrow s_i} \int_s^{s_i + \varepsilon} \frac{1}{\eta(a - s_i)} da = \infty.$$

This ensures that  $\Pi_{I-k}(u_I)$  is always greater than 1 and hence  $a_{I-k}(u_I) > s_{I-k}(u_I)$ , so that  $S^*$  is always empty.

Lemma 1 states that the equilibrium payoffs to all types increase weakly if  $\hat{\pi}(a) \geq \pi^*(a)$  for all  $a \in [0, \bar{a}] \setminus S^*$ . This is because even if  $\hat{\pi}(a) > \pi^*(a)$  for some  $a$ , this increase in the relative supply of emulation opportunities may be entirely absorbed by the corresponding

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<sup>12</sup> If the reputation payoff is additively separable and linear in  $t$ , this is equivalent to assuming that the cost of signaling effort is weakly convex in  $a$ , as often assumed in applications.

decrease in the probability of choosing the fully separating action, in which case the equilibrium allocation path would not shift up. Under the assumption that the indifference curves are weakly convex, this can be restated as follows.

**Lemma 1.A.** *The equilibrium payoffs to all types increase strictly if  $\hat{\pi}(a) \geq \pi^*(a)$  for all  $a \in [0, \bar{a})$  with strict inequality for some  $a \in [0, \bar{a})$ .*

Given this result, both Propositions 2 and 3 can also be restated accordingly, where the equilibrium payoffs are strictly higher under  $\hat{\Theta}$  in each instance.

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