readme_longhor   August2016

Kenneth D. West, 2016, "Approximate Bias in Time Series Regression"

This write-up describes some RATS and MATLAB procedures that can be used to compute bias to order T^{-1} in a least squares regression of the form

(*) $y_t = \text{const.} + \beta_1 x_{t-1} + ... + \beta_k x_{t-k} + \eta_t = \text{const.} + X_{t-1}'\beta + \eta_t$, $\eta_t \sim \text{MA}(q)$.

In this equation, y_t, x_t and η_t are scalars, and dating is arbitrary–in many applications, y_t and η_t will be realized in period $t+q$ for some $q>0$. (See examples below.) The unobservable disturbance η_t follows a moving average process of known order q.

Note that the right hand side consists solely of lags of a single variable. Additional procedures are described in a companion document "readme_proc_vb.wpd" can be used to compute bias in equations in which right hand side variables include lags of two or more different variables.

1. Overview

2. Procedures **longhor** and **longhor1**

3. Schematic diagram of hierarchy of routines

4. Code illustrating the procedures

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1. Overview

The equation of interest is

(1.1) $$y_t = \text{const.} + \beta_1 x_{t-1} + ... + \beta_k x_{t-k} + \eta_t = \text{const.} + X_{t-1}'\beta + \eta_t,$$

The dating of y_t and η_t is arbitrary:

- The lhs variable y_t may be a cumulated sum–say, $y_t = x_t+...+x_{t+q}$ because one is forecasting a cumulated sum using the direct method. Or perhaps $y_t = x_{t+q}$ (again, direct method of forecasting). q=0 (one step ahead forecasting) is an allowable special case. Or perhaps y_t is the period t+q realization of a variable other than x_t. See next bullet point.

- In the empirical example described in the final section of this document, the regression is the interest parity regression: the q+1 period change on the interest rate is regressed on the corresponding q+1 period cross-country interest differential. Thus k=1 and

  - y_t is the q+1 period percentage change in the exchange rate, $y_t = s_{t+q}-s_{t-1} = \Delta s_{t+q}+...+\Delta s_t$; Δs_t is the percentage change in the exchange rate;
  - x_{t-1} is the corresponding q+1 period cross-country interest differential.

Let b be bias to order T^{-1} in the least squares estimator of β,

(1.2) $$E\hat{\beta} \approx \beta + \frac{b}{T}.$$

The code described here computes $k\times 1$ estimates $\hat{\beta}$, $\hat{b}$ and the bias adjusted estimate

$$\text{bias adjusted estimate} = \hat{\beta} - \frac{\hat{b}}{T}.$$

From West (2016), b is a function of: autocovariances of X_t, cross-covariances between X_t and η_t, and (possibly) fourth cumulants. The code supplied here assumes that

- η_t follows a moving average process of known order q (serially uncorrelated [q=0] a special case), with $E\eta_t X_{t-j}=0_{k\times 1}$ for all $j\geq 1$;
- fourth cumulants are zero (roughly, no conditional heteroskedasticity or conditional skew).

Section 3 of the West (2016) provides formulas when η_t has an autoregressive component, when $E\eta_t X_{t-j}\neq 0_{k\times 1}$ for $j\geq 1$ and when cumulants are not zero.

In computation of the required second moments (autocovariances of X_t and cross-covariances between X_t and η_t), the code relies in part on an autoregression or vector autoregression whose lag length must be specified by the user (details below). Other than that lag length, all that is required of the user is data and basic parameters such as k (the number of lags on the right hand side) and q (the order of the moving average of η_t).

2. Procedures **longhor** and **longhor1**

As noted above, bias b is a function of second moments such as the autocovariances of X_t,

longhor1: compute the relevant second moments relying in part on a univariate AR in x_t.

longhor: compute the relevant second moments relying in part on a VAR in x_t and an additional list of variables supplied by the user.

longhor1 is a special case of **longhor**, packaged as a separate routine because it can be invoked without having to pass a list of additional variables to be used to compute moments of x_t. If **longhor** is invoked with a null set of additional variables, it calls **longhor1**.

The additional list of variables used in **longhor** might be derived from a model in which (1.1) is but one equation. More generally, this list might include variables thought to be very informative about x_t. See discussion below.

<u>longhor1</u> <u>is invoked via</u>

(2.1) RATS: execute **longhor1** *yseries xseries first last nq nk narlag vbias betahat betahat_adj*
MATLAB: [*vbias betahat betahat_adj*] = **longhor1**(*yseries, xseries, first, last, nq, nk, narlag*)
<u>Passed by user</u>:

yseries univariate series (RATS) / vector (MATLAB) for lhs variable
xseries univariate series RATS) / vector (MATLAB) for rhs variable.
first integer start date for the left hand side variable in regression (1.1)
last integer end date for the left hand side variable in regression (1.1)
nq integer horizon, called q in the present document; **nq**$\geq$0
nk integer number of lags k of **xseries** to include on the rhs of the regression (1.1)
narlag integer number of lags to include in estimating an AR model for **xseries** (needed to compute the bias). The user should insure that **narlag** is sufficient to produce a white noise residual in this autoregression.

<u>Returned to user</u>:

vbias **nk**×1 vector: b = k×1 numerator of bias to order T
betahat **nk**×1 vector of regression coefficients
betahat_adj **nk**×1 vector, bias adjusted **betahat**, **betahat_adj** = **betahat** - (**vbias**/T), T=**last**-**first**+1

<u>longhor</u> <u>is invoked via</u>

(2.2) RATS:
execute **longhor** *yseries xseries Wseries nWseries first last nq nk narlag vbias betahat betahat_adj*
MATLAB:
[*vbias betahat betahat_adj*] = **longhor**(*yseries, xseries, Wseries, nWseries, first, last, nq, nk, narlag*)

Parameters are as above, with the two additional parameters defined as

Wseries vector of series (RATS) / matrix (MATLAB) containing variables in addition to *xseries* to be used in the VAR that will be used to compute autocovariances of x_t
nWseries the number of series or columns in **Wseries**. If **nWseries**=0, the procedure calls **longhor1** to compute b.

•Thus, the regression of interest is **yseries**, **nq** periods ahead, on lags 1 to **nk** of **xseries**:

$$\textbf{yseries}(t+\textbf{nq}) = \text{const.} + \beta_1\textbf{xseries}(t\text{-}1) + ... + \beta_k\,\textbf{xseries}(t\text{-}\textbf{nk}) + \text{disturbance}(t+\textbf{nq}),\quad t+\textbf{nq}=\textbf{first}, ..., t+\textbf{nq}=\textbf{last} \tag{1.1$'$}$$

N.B.: Since the regression is run with **yseries** dates running from **first** to **last**, the dates on $\textbf{xseries}_{t\text{-}1}$ go from t-1 = **first-nq-**1 to t-1 = **last-nq-**1.

Internally to the routine, and invisibly to the user, the code estimates an AR of order **narlag** in **xseries** (if **longhor1** is invoked) or a VAR of order **narlag** in **xseries** and **Wseries** (if **longhor** is invoked). Bias is computed under the assumption that the residuals to this AR or VAR are white noise.

Example of **longhor1**: Suppose that **yseries** includes 90 observations. For simplicity of exposition, assume data are annual and run from 1901 to1990. Thus **yseries**(4) is data from 1904, for example.

Example with **first**=10, **last**=90, **nq**=7, **nk**=2, **narlag**=4.

(RATS) execute **longhor1** *yseries xseries* 10 90 7 2 4 *vbias betahat betahat_adj*
(MATLAB) [*vbias betahat betahat_adj*] = longhor1(*yseries*, *xseries,* 10, 90, 7, 2, 4)

This tells the code: estimate

$$\textbf{yseries}_{t+7} = \text{const.} + \beta_1\textbf{xseries}_{t\text{-}1} + \beta\textbf{xseries}_{t\text{-}2} + \text{disturbance},\quad t+7=1910, ..., 1990 \tag{1.1$'$}$$

and uses an internally generated estimate of an AR(4) in **xseries** to compute autocovariances of **xseries**. Thus, in (1.1)′, the vector of the left hand side variable and the matrix of stochastic right hand side variables are

$$\begin{pmatrix} yseries_{1910} \\ yseries_{1911} \\ ... \\ yseries_{1990} \end{pmatrix}, \begin{pmatrix} xseries_{1902} & xseries_{1901} \\ xseries_{1903} & xseries_{1902} \\ ... & ... \\ xseries_{1982} & xseries_{1981} \end{pmatrix}$$

In this example, the procedure returns three 2×1 vectors: **betahat** ≡ $\hat\beta \equiv (\hat\beta_1\ \hat\beta_2)'$, **vbias** = $\hat b$, **betahat_adj** ≡ $\hat\beta\text{-}\hat b/T$, where T=81. The procedure returns no information related to the autoregression in $\textbf{xseries}_t$; that regression is run purely to obtain some moments needed to compute $\hat b$.

Example of **longhor**: Same as previous, except suppose that second moments necessary to compute bias are obtained from a bivariate VAR of order 4 in (x_t, z_t) for some variable z_t. Then **nWseries**=1 and **Wseries** is set to z_t.

•Notes (see also the comments in the code itself):

1. Partly redundant clarification:

(a)**narlag** should be chosen to insure white noise residuals in the AR or VAR used to compute autocovariances of x_t.

(b)**longhor** might be used instead of **longhor1** if one thinks data in addition to x_t is informative about the autocovariances of x_t.

> Example: In the interest parity regression given in the empirical example section below, x_t is a cross-country interest rate differential and the additional variable is the change in the exchange rate: as a forward looking variable, the exchange rate plausibly has considerable information about x_t beyond what is contained in interest rate differentials themselves.

(c)I expect **longhor1** to be the default choice in computation of bias, particularly when one is using the direct method to make long horizon forecasts.

2. The following routines must be accessible from the directory in which the user calls **longhor** or **longhor1**: **proc_vb_ma0**, **proc_vb_maq**, and **proc_vbias**.

3. The code does not do error checks for missing data. So, suppose in the example above, where data runs from 1901-1999, that the user passes **first**=5 along with **nq**=7. Then the routine would assume the first observation on the left hand side variable is 1905 and the first observation on **xseries** is 8 years earlier (8=**nq**+1), i.e., 1897–a date that is not in the sample. Results are unpredictable if, as in this illustration, parameters point to data that are not available.

4. To map the example (1.1)′ into the notation of the part of West (2016) that presents a closed form solution for the bias when $X_t = P_X Z_t$ for a vector Z_t that follows a companion form VAR(1): In the notation of (*), $X_t = (x_t, x_{t-1})'$. Then for the **longhor1** example

$$Z_t = (x_t, x_{t-1}, x_{t-2}, x_{t-3})' \text{ and } P_X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

For the **longhor** example

$$Z_t = (x_t, x_{t-1}, x_{t-2}, x_{t-3}, z_t, z_{t-1}, z_{t-2}, z_{t-3})' \text{ and } P_X = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

3. A schematic of the hierarchy in the code

Applications in which (1.1) is the regression of interest–the only regressors are a set of lags of a single variable. User does not need to estimate the regression or compute moments. user calls **longhor** to get $\hat{\beta}$, $\hat{b}$ and bias adjusted $\hat{\beta}$-$\hat{b}/T$. See section 2.

Applications involving regressors other than a set of lags of a single variable. User estimates $\hat{\beta}$ and computes certain moments. User calls **proc_vb_ma0** or **proc_vb_maq** to get $\hat{b}$. user then computes bias adjusted $\hat{\beta}$-$\hat{b}/T$. See the separate document “readme_ proc_vbias”.

longhor and **longhor1** invoke **proc_vb_ma0** or **proc_vb_maq** to compute $\hat{b}$. They return $\hat{\beta}$, $\hat{b}$ and bias adjusted $\hat{\beta}$-$\hat{b}/T$.

proc_vb_ma0 and **proc_vb_maq**: take as input certain moments, invoke **proc_vbias**, return $\hat{b}$. See the separate document “readme_ proc_vbias”. This code must be accessible when **longhor1** or **longhor** are invoked.

proc_vbias: code that computes $\hat{b}$. See the separate document “readme_proc_vbias”. This code must be accessible when the user invokes any of the routines above.

4. Code illustrating the procedures

This section describes RATS and MATLAB programs to illustrate the code.

Let

s_t = 100×log exchange rate, US dollars per foreign currency unit
Δs_t = quarterly percentage change in bilateral U.S. dollar exchange rate
x_t = q+1 period interest differential, U.S. minus foreign, safe debt

Strictly speaking, x_t should be indexed by q but is not for simplicity.

The empirical example estimates the interest parity regression, regressing a q+1 period change in the exchange rate on the corresponding interest differential:

$$y_t \equiv s_{t+q}\text{-}s_{t-1} = \text{const.} + \beta x_{t-1} + \eta_t$$

for q=0 (one quarter), q=1 (6 month), q=3 (one year), q=19 (five year) and q=39 (10 years), for the U.S. dollar vs. the Canadian dollar, Japanese yen and U.K. pound, 1979-2011. Thus there are 15 sets of estimates: 5 horizons × 3 currency.

•Data are in **empirical_data.xlsx**. Kindly supplied by Menzie Chinn. A subset of the data used in Chinn and Quayyam (2012). The spreadsheet includes some data prior to 1979, but such data are not used in the example.

•**empirical_longhor.prg** (RATS) and **empirical_longhor.m** (MATLAB) obtain a bias adjusted estimate of β using (2.2) **longhor**, with **nWseries**=1, **Wseries** set to the quarterly exchange rate Δs_t, and **narlag**=2. These programs produce estimates for a given horizon and currency. Comments within the programs describe how to select horizon and currency.

•In addition to **empirical_data.xlsx**, the following files need to be accessible from the directory in which these programs are run:

RATS: **proc_vb_ma0.src**, **proc_vb_maq.src**, **proc_vbias.src**, **nwbandwidth.src**; a ***.inp** file as described in the comments in **empirical_longhor.prg**
MATLAB: **proc_vb_ma0.m, proc_vb_maq.m, proc_vbias.m**, **nwbandwidth.m**, **lagmatrix.m**, **NeweyWest1994.m**

The next page has results.

Empirical Estimates

q+1	Canada			Japan			U.K.		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\hat{\beta}$	$\frac{\hat{b}}{T}$	$\hat{\beta}-\frac{\hat{b}}{T}$	$\hat{\beta}$	$\frac{\hat{b}}{T}$	$\hat{\beta}-\frac{\hat{b}}{T}$	$\hat{\beta}$	$\frac{\hat{b}}{T}$	$\hat{\beta}-\frac{\hat{b}}{T}$
1	-0.29	-0.02	-0.27	-2.50	-0.01	-2.49	-1.51	0.04	-1.55
	(0.59)			(0.74)			(0.97)		
2	-0.15	0.00	-0.15	-2.66	-0.00	-2.66	-1.31	0.09	-1.40
	(0.57)			(0.54)			(0.95)		
4	-0.06	0.00	-0.06	-2.33	0.07	-2.39	-0.87	0.17	-1.04
	(0.68)			(0.45)			(0.82)		
20	0.74	-0.14	0.89	0.48	0.27	0.21	0.70	0.31	0.39
	(0.45)			(0.58)			(0.51)		
40	1.50	-0.47	1.98	0.79	0.10	0.73	0.48	-0.05	0.53
	(0.61)			(0.41)			(0.12)		

Notes:

1. The least squares regression is y_t = const. + $\beta x_{t-1}+\eta_t$, where y_t is change in US dollar exchange rate from t-1 to $t+q$ and x_{t-1} is the interest rate differential on q+1 quarter government debt. Sample period is 1979:2-(2011:2-q) for Canada, 1979:2-(2011:3-q) for Japan and the U.K.. This implies T=129, 128, 126, 110 and 90 for q=0,1,3,19 and 39 for Canada, with T one observation greater for Japan and the U.K.. HAC standard errors in parentheses.

2. The variable b is the bias, $\lim_{T\to\infty} T(E\hat{\beta}-\beta) = b$. Computation of $\hat{b}$ relies in part on moments computed from a VAR(2) in x_t and the one quarter change in the exchange rate.

3. Columns (3), (6) and (9) presented bias adjusted estimates of $\hat{\beta}$, with $\hat{\beta}$ itself reported in columns (1), (4) and (7) and the bias $\hat{b}/T$ reported in columns (2), (5) and (8).