

Milaković, Mishael; Raddant, Matthias; Birg, Laura

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persistence of a network core in the time evolution of interlocking directorates

by Mishaël Milakovic, Matthias Raddant, and Laura Birg



Persistence of a Network Core in the Time Evolution of Interlocking Directorates*

Mishaël Milaković
Matthias Raddant
Laura Birg

Department of Economics
University of Kiel

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Abstract

We examine the bipartite graphs of German corporate boards in 1993, 1999 and 2005, and identify cores of directors who are highly central in the entire network while being densely connected among themselves. Germany's corporate governance has experienced significant changes during this time, and there is substantial turnover in the identity of core members, yet we observe the persistent presence of a network core, which is even robust to changes in the tail distribution of multiple board memberships. Anecdotal evidence suggests that core persistence originates from the board appointment decisions of largely capitalized corporations.

Keywords: Network core, density, centrality, interlocking directorates, corporate governance.

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1 Introduction

We study the time evolution of German corporate director interlocks between 1993 and 2005, and detect a persistent core of directors who are highly central in the network while being densely connected among themselves. The statistical properties of the network core show little variation over time in spite of considerable turnover in the identity of core directors, which has theoretical implications for the diffusion and co-ordination of attitudes among agents, and leads to questions about the mechanisms that are responsible for the origin of a persistent network core.

Traditional research in organization and management science has investigated the influence of shared directorships or ownerships on firm performance, profitability, and corporate strategy, including acquisition behavior, choice of financing, the magnitude and direction of political and charitable contributions, the adoption of poison pill practices, and many more, in fact generating such an abundance of results to warrant several recent review articles on different aspects of the subject (see, e.g., Borgatti and Foster, 2003; Brass et al., 2004; Galaskiewicz, 2007; Provan et al., 2007). Another strand of research, inspired by the interdisciplinary work of Barabási and Albert (1999), has emphasized the statistical properties of corporate networks (see, e.g., Battiston and Catanzaro, 2004; Caldarelli and Catanzaro, 2004; Newman et al., 2001) and concludes that director interlocks exhibit the small world *effect*, whereby the interpersonal distance between any two directors is several orders of magnitude smaller than the number of directors in the network.

Davis et al. (2003) have argued that director interlocks are also characterized by a high degree of clustering, as in the prototypical small world *networks* of Watts and Strogatz (1998).¹ Subsequently, however, Conyon and Muldoon (2006) and Robins and Alexander (2004) have shown from different yet complementary viewpoints that the high degree of clustering is present by construction and not an unexpected feature in director interlocks. Intuitively, the reason is that directors are linked by definition

¹See Uzzi et al. (2007) for a review of small world networks in the social sciences.

to all other colleagues on a company's board while the large majority of directors serves only on a single board in the network.

Conventional wisdom has it that the small world effect typically stems from the presence of 'hubs', i.e. nodes with a large number of links to other nodes in the network, and the degree distribution of nodes has been shown to obey a power law in many complex networks (see, e.g., the surveys by Newman, 2003; Schnettler, 2009). Still, in numerous social contexts the number of links and nodes exhibits a characteristic scale and does not span enough orders of magnitude to be a power law. In addition, the cost of forming a link on the World Wide Web or in an academic citation network is very small compared to the amount of resources that are involved in the hiring of corporate directors. Power-law degree distributions are, in a strong combinatorial sense, the most likely distributions when links are relatively costless, yet as links become costlier, the functional form of the most likely degree distribution becomes an exponential (see Venkatasubramanian et al., 2005). Thus it seems fair to say that power laws are an inadequate or at best imperfect explanation for the small world property of director interlocks.

To understand the origins of the small world effect in director interlocks, we find it instructive to start from the fact that the large majority of directors serves on a single board, and to ask whether the instances of multiple board membership deviate from a random benchmark. The benchmark assumes that board recruitment decisions are entirely random in the sense that each director is equally likely to obtain an additional board appointment, turning multiple board membership into a sequence of Bernoulli trials. The resulting binomial distribution and the empirically observed distribution display deviations over increasing orders of magnitude, and it turns out that directors with multiple board memberships are to a very large extent connected among themselves, which accordingly makes them highly central in the overall network. From this viewpoint, the small world property of director interlocks is due to a relatively small number of 'big players' who are densely connected among themselves, thereby forming a core network that substantially shortens the distance

between arbitrarily chosen directors in the entire network. An important point concerns the origins of a persistent network core since we observe considerable turnover in the identity of core directors over time. Our analysis suggests that both the reconstruction of broken ties among large corporations, as well as their preference for recruiting experienced directors with multiple board memberships, are responsible for the time persistence of a network core.

The idea that a core of director interlocks influences the degree of interest group formation has previously been put forward by Mintz and Schwartz (1981), and several authors have suggested procedures to classify or identify a core of key players in complex networks, both in the social sciences (see, e.g., Borgatti, 2006; Borgatti and Everett, 1999) and in interdisciplinary physics (see, e.g., Holme, 2005). The existence of a network core has implications beyond popular conspiracy theories, particularly for a class of processes that Borgatti (2005) categorizes as *duplication in walks*: Alfarano and Milaković (2009) show that a core is crucial for the system-wide propagation of fashions and fads in systems with a large number of heterogeneous agents. The upshot of their study is that network features like average clustering, diameter, small world coefficient, or degree distribution are not sufficient to warrant system-wide herding in the model. The presence of a hierarchical core-periphery structure, on the other hand, generates system-wide conformity in the model, including the possibility that the social interactions of core agents lead to the propagation of “animal spirits” across an entire system that is several orders of magnitude larger than the size of the core.

2 Data and Method

Our compilation of board composition data aimed for Germany’s one hundred largest publicly traded companies in 1993, 1999, and 2005. The thirty largest companies are listed in the German stock index DAX (*Deutscher Aktienindex*), while the next largest companies are listed in the Mid-Cap-

DAX, or MDAX. The MDAX was founded in 1996, containing the seventy largest companies that were not included in the DAX, which we also used in the 1993 sample. In 2003, the number of companies in the MDAX was reduced from seventy to fifty, so we used the survivors among the twenty companies that left the MDAX in the 2005 sample, or replaced those that no longer existed with the next largest companies in 2005.

For the purpose of our study, corporate boards consist of executive management (*Vorstand*) and supervisory board (*Aufsichtsrat*). According to the pertinent German legal code, they have to meet at least four times per year (§ 94(3) of *Aktiengesetz*, or *AktG*). Executives are appointed for a maximum of five years, and both appointment as well as potential reappointment need to be approved by the supervisory board (§ 84 *AktG*). In light of the five-year limit, we chose equally spaced intervals of six years to increase the likelihood of observing changes in the composition of corporate boards. We compiled the data by consulting various archives that keep records of the annual reports of these companies, and by writing to companies for whom we could not locate annual reports.² The descriptive statistics of our sample, reported in Table 1, show a decreasing average board size over time, which is mainly due to M&A activity among very large corporations (Dresdner Bank was acquired by Allianz, for instance, or VEBA and VIAG merged to EON), and also to the fact that 2005 additions had on average only about half the board size of other companies in the sample.

Let n be the number of directors in a year, and let c be the number of companies in that year. Then the *incidence matrix* \mathbf{M} of dimension $n \times c$, with $m_{ij} = 1$ if director i is on the board of company j and zero otherwise, describes the corporate network in each year. The projection onto directors, $\mathbf{D} = \mathbf{M}\mathbf{M}^T$, is the weighted adjacency matrix of director interlocks. Its diagonal entries equal the total number of board memberships of director i , while non-zero entries off the diagonal of \mathbf{D} represent the weight of a link, showing on how many boards two directors serve together. Sym-

²Three companies in the 1993 sample did not reply to our inquiry, all three of them with relatively minor market capitalization, leaving us with 97 companies in that year.

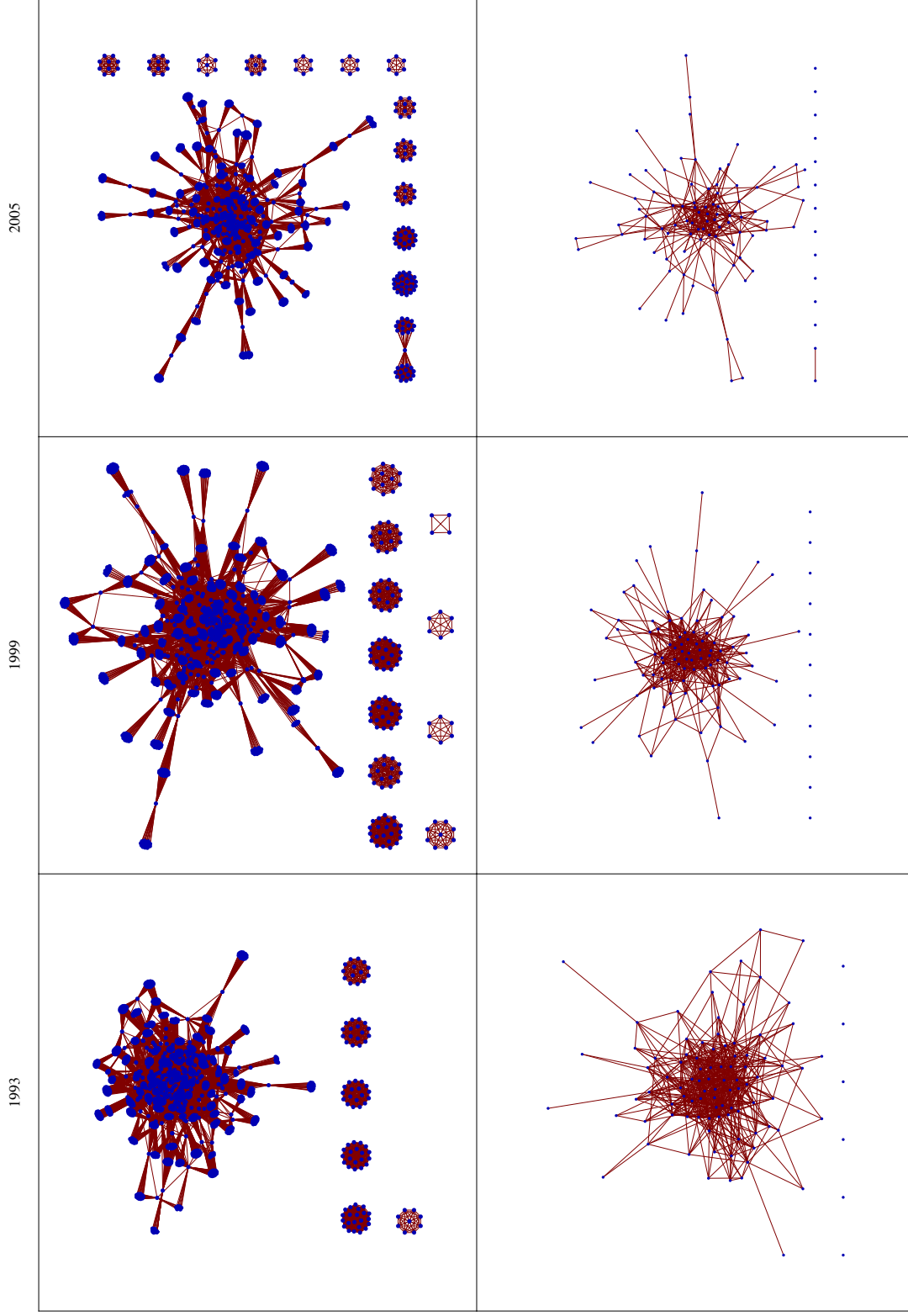


Figure 1: The network of German director (top panel) and company (bottom panel) interlocks in 1993, 1999 and 2005. Only a few companies are isolated from the large connected components, and a casual graphical inspection already suggests that each network has a core-periphery structure.

Table 1: The descriptive statistics of our sample illustrate a slight decrease in the number of directors and mandates over time, and a highly nonlinear decrease in the number of links between companies that are formed by multiple board membership. The non-linearity is caused by the fact that a director with b mandates creates $l(b) = b!/2(b-2)!$ links among companies. If, for instance, a director with $b = 6$ mandates retires and is replaced by single-mandate directors, then $l(6) = 15$ links are removed from the network.

	1993	1999	2005
# of companies (distinct: 176)	97	100	100
# of directors (distinct: 3884)	1744	1711	1593
# of mandates	2143	2044	1833
average board size	22.9	20.4	18.3
average mandates per director	1.23	1.17	1.15
Company links (total)	803	657	375
Company links (unweighted)	597	490	291

metrically the projection onto boards, $\mathbf{B} = \mathbf{M}^T \mathbf{M}$, yields the weighted adjacency matrix of company interlocks, its diagonal entries correspond to the board size of company j , and off-diagonal non-zero elements indicate the number of directors that two companies have in common. The resulting networks are displayed in Figure 1 and readily reveal the existence of a core in each period, but the figure also suggests that the number of core companies and directors decreases over time. The question is then whether, as one might intuitively expect, core directors also become less influential in the sense that they are less central or less densely connected among themselves.

2.1 Random benchmark

To tackle the issue, we first need to identify a core of directors. For this purpose, it is instructive to consider the frequency of multiple board memberships, shown in Figure 2.

We start from the observation that the vast majority of directors serves on just one board, and conduct a simple thought experiment. Suppose that

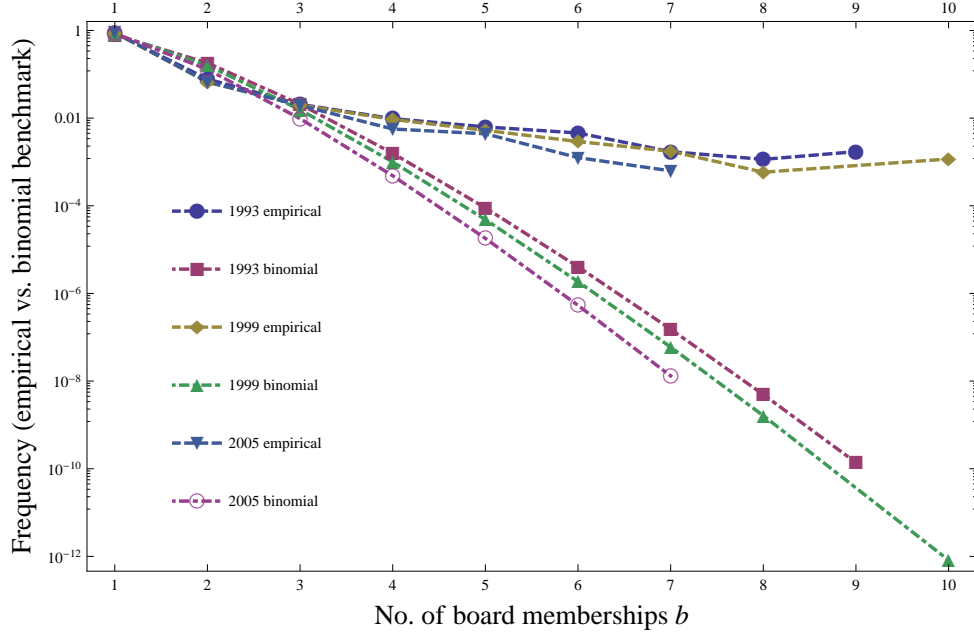


Figure 2: The top curves show the empirical relative frequency of multiple board membership in each of the three years, while the bottom curves illustrate the binomial probability of observing multiple board membership in the respective sequences of independent Bernoulli trials described in the text. The semi-log scale reveals deviations on increasing orders of magnitude for $b > 3$.

the directors in each sample are indistinguishable; then we can determine the probability of observing multiple board membership as a sequence of k independent Bernoulli trials, resulting in a binomial distribution for observing $B = b$ additional board memberships,

$$Pr[B = b] = \binom{k}{b} p^b (1 - p)^{k-b},$$

where p is the probability of success, i.e. of obtaining an additional board membership. To illustrate the procedure, consider for example the year 1993: there are 1744 directors in total, and the number of mandates is 2143, yielding $k = 2143 - 1744 = 399$, and $p = 1/1744$. Figure 2 illustrates the resulting binomial distributions and compares them to the empirical

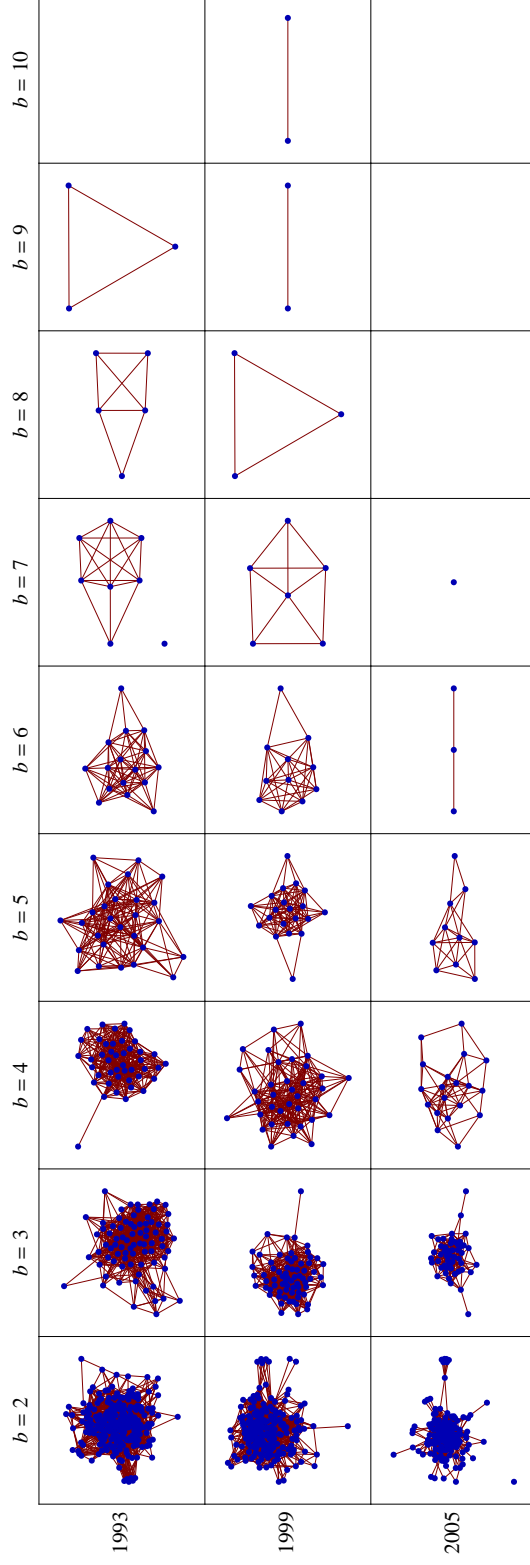


Figure 3: Network structures formed by considering directors with an increasing threshold of board memberships $B \geq b$. Notice that there is only one instance of an isolated director with $b \geq 3$ mandates in any of the years (Alfred Pfeiffer in the 1993 $b = 7$ core). The size of cores and the fraction of companies in the respective cores both decrease over time, shown in more detail in Figure 4.

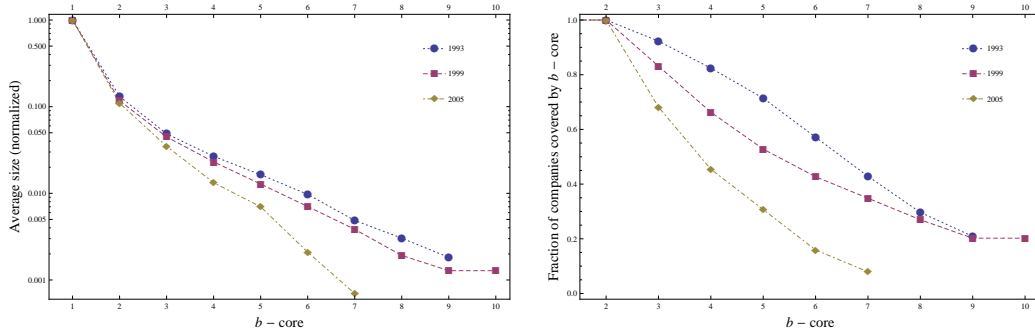


Figure 4: The number of directors in the respective b -cores (left panel) decreases over time, as well as the fraction of companies that are linked by the respective b -core directors (right panel).

relative frequencies of multiple board membership.

For $b > 3$, the incidence of multiple board membership is several orders of magnitude higher than we would expect in a sequence of independent Bernoulli trials, which suggests that directors with three or more mandates are probabilistically distinct and in some sense special. One would expect to observe a network core if these directors were connected among themselves, thus we plot the network structure among directors with $B \geq b$ board memberships in Figure 3, which reveals that the resulting sub-graphs, or b -cores,³ are indeed to a very large extent connected. We also observe that both the number of directors and the fraction of companies that are connected by the respective core directors decrease over time in our sample, shown in Figure 4. A major contributing factor to this development is certainly the recent reform of Germany's corporate governance code (DCGK).⁴ The reform deals with a number of national and international criticisms that have been leveled against Germany's traditional corporate governance, mostly concerning the inadequate focus on shareholder interests, and the inadequate independence of supervisory boards, addressed for instance in DCGK paragraphs 5.1.2 (age limit for management board service), 5.4.2 (independence of supervisory board members),

³Notice that our b -cores are different from so-called k -cores, which are constructed using a node's minimum degree (see, e.g., Seidman, 1983).

⁴See <http://www.corporate-governance-code.de/index-e.html>.

and 5.4.4 (deterrence of the hitherto custom that former executives serve as supervisory board chairmen). While the new code aims at standardizing best practices in corporate governance, it does not have the status of a formally binding law. Nevertheless, deviations from DCGK rules have to be explicitly justified and publicized on an annual basis (§ 161 *AktG*), and the observed decrease in the average number of board memberships is not an unexpected feature from this perspective. It is noteworthy that the code took effect in early 2002, while the pronounced decrease in average mandates indeed occurs between the 1999 and 2005 samples. One can only speculate whether the DCGK is the ultimate cause of these developments or not, but over the years we do in fact observe a pronounced decline in executive managers' supervisory board memberships:⁵ Table 4 shows that in 1993 (1999, 2005), the 569 (441, 457) directors with executive positions additionally served on 228 (164, 83) supervisory boards of other corporations. The drop in the ratio of supervisory board memberships per executive ($228/569 = 0.4$ in 1993, 0.3 in 1999 and 0.17 in 2005) illustrates that corporate governance practices have indeed changed over the investigated time period. This brings us back to the question whether shrinking *b*-core sizes also imply that core directors become less influential over time, and how to formally define a network core in the first place.

2.2 Density and corporate reach

Intuitively a network core consists of directors that are highly central in the network and densely connected among themselves. The density of the (unweighted) graph \mathbf{D} is given by the ratio of the existing number of links, denoted $|L|$, to the number of links in a complete graph of the same size, denoted $|N|$,

$$\text{density}_D = 2|L|/|N|(|N| - 1),$$

⁵Current members of the management board must not simultaneously serve on the company's supervisory board (§ 105 *AktG*), but have been routinely allowed to serve as supervisory board members at other companies.

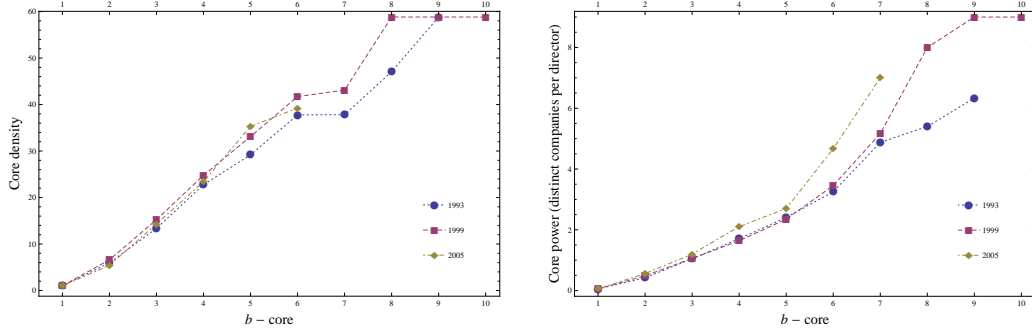


Figure 5: The density of b -cores (left panel) remains fairly stable over time, as well as the ratio of distinct core companies to core directors (right panel). If anything, core density and core power increase despite the decrease in the average number of mandates over time.

which is by construction confined to the interval $[0, 1]$. The left panel of Figure 5 illustrates that (i) the density of b -core sub-graphs increases with b , and that (ii) the density of the respective b -cores remains fairly constant over time in spite of decreasing b -core size. In addition, we can assess the corporate reach per core director by the ratio of distinct core companies to the number of core directors, the rationale being that a core of densely connected directors probably yields the more institutional power the fewer individuals constitute the core (), and the more companies they span. It is noteworthy that this measure of core power, shown in the right panel of Figure 5, actually increases over time, so the recent decline in the number of board memberships does not necessarily mean that core directors in Germany's corporate board network also become less influential.

2.3 Centrality

A complementary approach to measuring the importance or influence of directors is to consider their centrality in the overall network of director interlocks. Let \mathbf{C} denote the respective adjacency matrices of the large connected components of \mathbf{D} in the respective years, and let V denote the set of directors contained in \mathbf{C} . A shortest path between two directors $u, v \in V$ is known as a graph *geodesic*, which is not necessarily unique,

and the length of the geodesic $d_C(u, v)$ is known as the graph *distance* between the pair (u, v) . The first centrality measure we consider is *closeness centrality*, which measures the distance of a node to all other nodes in the network, and is typically defined as the reciprocal of the sum of geodesics to all other nodes in the network,

$$\text{closeness}_u = 1 / \sum_{v \in V} d_C(u, v).$$

Since we would like to compare the centrality of directors across years, we divide by the closeness score of the director with maximal closeness centrality in each year in order to normalize the scores. Directors who are more central in this sense should in principle be better able to reach out into the entire network or be faster in doing so.

Another measure of the centrality of node u is *degree centrality*, constructed by summing the number of links that each node has, $\text{degree}_u = \sum_{v \in V} C_{uv}$. Intuitively, directors who have many links compared to their peers are in an advantageous position if they are able to influence many of their peers, or if they have better access to resources through their many links. But degree centrality only takes immediate ties of directors into account, and lacks information about the distance to directors that are not immediate neighbors. Moreover, directors with many board memberships have a relatively large degree by construction since the board size distribution has a characteristic scale that is well captured by its mean.

Therefore, we also compute the eigenvector centrality (see Bonacich, 1972) for all nodes in V . *Eigenvector centrality* assigns scores of relative importance to directors in the network, based on the principle that connections to high-scoring directors contribute more to a director's score than equal connections to low-scoring peers. Hence the idea behind eigenvector centrality is that the quality of links is important, because directors who are connected to many influential peers can be expected to be important themselves. Suppose the eigenvector centrality score of node u , denoted

e_u , is proportional to the centrality score of its neighbors,

$$e_u = \frac{1}{\lambda} \sum_{v \in V} C_{uv} e_v,$$

where λ is a constant. Then we can write the vector of centrality scores in matrix notation as $\lambda \mathbf{e} = \mathbf{C} \cdot \mathbf{e}$, which shows that \mathbf{e} is an eigenvector of \mathbf{C} with corresponding eigenvalue λ . It is convenient to consider the eigenvector corresponding to the largest eigenvalue of \mathbf{C} since its elements are all non-negative according to the Perron-Frobenius theorem. Again, we divided all scores by the maximum score in each year to normalize the data. Figure 6 shows that core directors are not only densely connected among themselves, but that they are also increasingly central in the entire network, which is another characteristic that one intuitively expects in the definition of a network core.

3 Core Persistence and Individual Turnover

We have argued that the structural characteristics of director interlocks are stable over time in spite of changes in corporate governance and a decrease in the average number of mandates. Motivated by the persistence in network structure, we want to investigate in how far the core structure depends on the destinies of particular agents or not. An important aspect here concerns the links in the company network. Their time evolution over consecutive periods reveals some noteworthy patterns: in 1993 (1999) the company network had 597 (490) unweighted links, 290 (195) of which were with companies that remained in the sample in the next period (and had not merged in the meantime), while 141 (95) of these links were still in place in the following period. In 78 (62) cases, at least one director was constantly part of both boards. In addition, 24 (10) directors had been recruited to reinforce existing links, then serving on average on 4.38 (4.04) boards. Out of these 24 (10) directors, 18 (7) were already serving on at least one board in the previous period, when they held 2.22 (2.68)

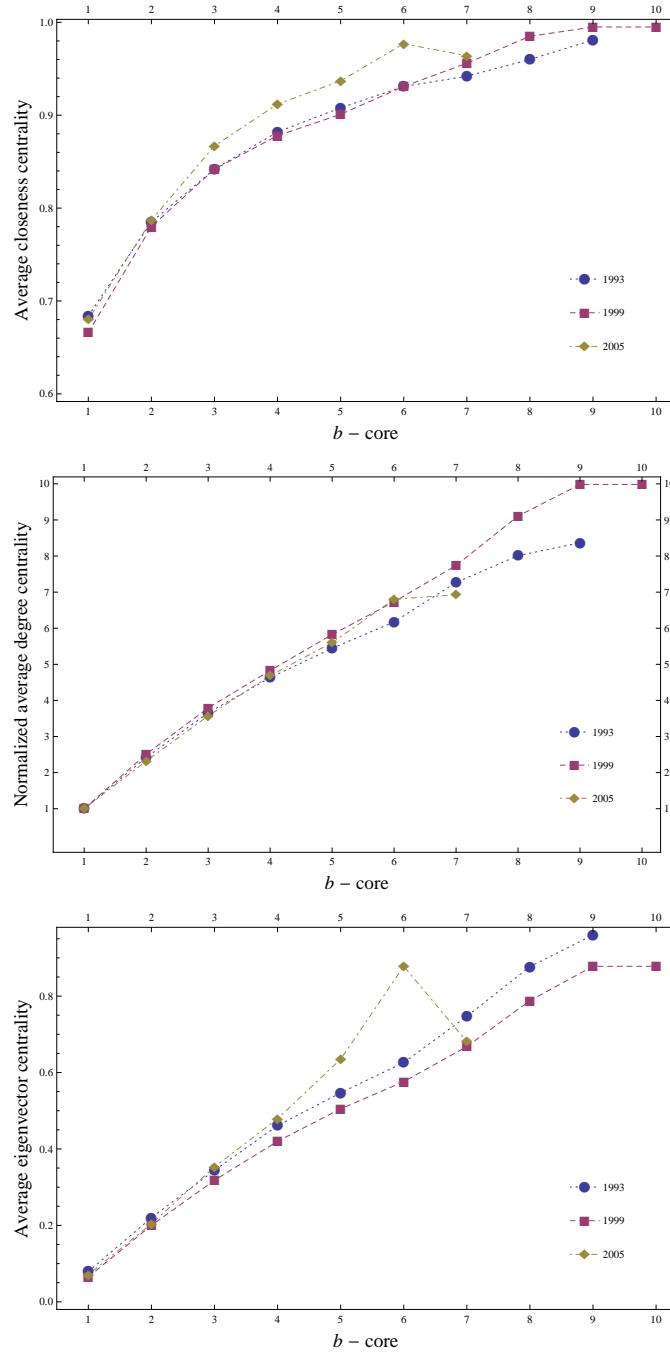


Figure 6: Normalized average centrality measures of b -core directors in the respective years: closeness (top), degree (center) and eigenvector (bottom) centrality remain fairly stable over time.

appointments on average. In the remaining 63 (33) cases, where an existing link was maintained through the recruitment of new directors, a total of 50 (26) directors was appointed to the 63 (33) positions, and these directors then served on average on 4.04 (3.77) boards. Out of these 50 (26) directors, 34 (20) had at least one mandate in the previous period, when they served on average on 2.68 (2.75) boards. So we observe that about half of the company links are being maintained in one way or the other between periods, which is consistent with earlier findings by Schreyögg and Papenheim-Tockhorn (1995) on the reconstitution of German corporate interlocks.⁶ Keeping in mind that a substantial number of links in the (initial) company network might be unintended,⁷ and given that the samples are six years apart, the reconstitution of links speaks in favor of planned or strategic connectivity among German corporations.

Secondly, these figures suggest that companies seem to prefer the appointment of directors who already serve on several other boards, which is particularly true for the maintenance of institutional links over time. It is rather doubtful that these directors were appointed for purely supervisory purposes since the effort involved in monitoring a handful of DAX companies is surely considerable, and in all likelihood becomes increasingly prohibitive if one of the appointments is an executive position. The frequency distribution of executives' supervisory board memberships in Table 4 nevertheless shows that some executives additionally served on up to ten other boards.

Since all of our preceding results suggest that multiple board memberships are essential for both the existence and the persistence of a core network, we also investigate the survival of directors over time. Overall,

⁶The percentage of reconstructed ties among German companies is four to five times higher than previously observed in the US (see Stearns and Mizruchi, 1986). From a network core perspective, it would be interesting to clarify whether the percentage of reconstituted ties among the very largest (core) corporations in the US is substantially higher than in the original Stearns and Mizruchi sample.

⁷Imagine a director with three mandates and suppose that she is on the board of company A, which manages to place her on the board of company B for strategic reasons, e.g. to oversee A's interests. If she also serves on the board of a third company C, we consider the link between A and B intentional, while the links AC and BC are unintentional byproducts.

out of the 1517 people in the 1993 sample, 518 are still present in 1999, which is a survival rate of 34%. For the 1999-2005 transition, this figure is 32%. During the first (second) transition period, 12.6% (13.1%) of the survivors gained mandates, while 12.4% (11.3%) lost at least one mandate. But these percentages conceal that directors with multiple memberships have a markedly higher survival probability than the vast majority of directors with a single mandate: while the survival probability conditional on the number of existing mandates is 31% (28%) for board members with a single mandate, it is 51% (47%) for directors with two mandates, 69% (76%) for those with three mandates, and 70% (78%) for those with four or more mandates.⁸ It seems fair to say that the persistent structure of Germany's corporate network is driven by the recruitment decisions of large companies, which are characterized by a process of selective replacement given the figures on the maintenance of links among companies and the conditional survival of directors.

While the vast majority of directors enters and exits the corporate network without ever being particularly central in it, a small number of highly connected key directors warrants a persistent network core over time. Moreover, fluctuations in the destinies of key players are mitigated by the reconstitution of ties among large corporations, who favor directors with multiple memberships for this purpose. To corroborate this claim, we consider the turnover in the centrality of companies and directors between periods. We start by calculating the change in each of the three centrality measures which is only sensible for surviving nodes in the connected component. Table 2 illustrates that about two thirds of the companies but only one third of the directors survive consecutive periods. The life span of directors is biologically limited while the same is obviously not true for corporations, thus the fact that about 70% of directors but less than 40% of companies drop out between periods is by itself not unexpected. In spite of the expected difference, we find that the *mean absolute change in centrality*, as a measure of variability among survivors, has the same level

⁸These figures are easily calculated from the transition matrices in Appendix B, which contain the detailed transitions for all directors between the various periods.

of magnitude for both companies and directors, and that absolute changes in centrality are rather small in both cases.⁹

In order to properly compare the turnover activity between companies and directors, we need to scale the absolute changes in centrality with a benchmark measure of persistence in centrality that accounts for the different scales of the company and director networks. In our benchmark case, we assume that each node's centrality could change to every observed centrality in next period's sample with equal probability, corresponding to a uniformly random rewiring of nodes. Thus the average absolute change in centrality would be zero in the case of a perfect conservation of the relative position of nodes, and would be equal to the benchmark value in case of a completely random rewiring of surviving nodes. For all centrality measures, we observe that the ratio of the benchmark to the observed value is larger for companies than for directors (about 3:1 vs 2:1), showing that surviving companies exhibit significantly less churning in their centrality than surviving directors. Comparing dropouts with survivors, both for companies and directors, we find that survivors are always substantially more central than dropouts (also reported in Table 2), and that the normalized degree is about one order of magnitude higher for survivors than for dropouts. Moreover, the very low average eigenvector centrality of dropouts further implies that the importance of the dropouts' few neighbors is also very low on average. In summary, both highly central companies as well as directors tend to stay central, while dropouts are located in more peripheral positions of the network. Company networks exhibit less turnover activity than director networks in the share of surviving nodes but also in the centrality changes among survivors.

⁹All three measures of centrality exhibit a slight decrease in *average centrality* between 1993 to 1999, and an increase between 1999 to 2005, as reported in Table 2. This is in line with the visual inspection of the network structure in Figure 1, which shows an increasing number of peripheral nodes in 1999, and denser cores in 2005.

Table 2: Turnover activity in company and director networks. The notation for average centrality measures refers to averages for dropouts (out), and the centrality of survivors in the current (in (t)) and next (in $(t + 1)$) period. The benchmark values for the changes in centrality have been calculated according to the procedure described in this section.

		companies			directors		
		1993 – 1999	1999 – 2005	1993 – 2005	1993 – 1999	1999 – 2005	1993 – 2005
average centrality	survivors	91	89		1645	1562	
	dropouts	34	38		1087	1081	
	out	0.5034	0.4438		0.4950	0.4450	
	closeness						
	in (t)	0.7789	0.7317		0.7014	0.6850	
	in $(t + 1)$	0.7521	0.7497		0.6845	0.7082	
	eigenvalue						
	out	0.0023	<0.001		0.0513	<0.001	
	in (t)	0.2766	0.2339		0.1040	0.0926	
	in $(t + 1)$	0.2458	0.3301		0.0866	0.0978	
	normalized						
	degree						
	out	0.0250	0.0270		0.0313	0.0207	
	in (t)	0.3636	0.3121		0.1509	0.1358	
	in $(t + 1)$	0.3295	0.3431		0.1352	0.1857	
change in centrality	avg. $ \Delta $	0.0547	0.0730		0.0521	0.0684	
	avg. Δ	-0.0268	0.0100		-0.0169	0.0233	
	benchmark	0.1348	0.1598		0.1057	0.1208	
	closeness						
	avg. $ \Delta $	0.0883	0.1161		0.0513	0.0571	
	avg. Δ	-0.0308	0.0962		-0.0175	0.0052	
	benchmark	0.2702	0.2912		0.1174	0.1199	
	eigenvalue						
	avg. $ \Delta $	0.0953	0.1039		0.0543	0.0855	
	avg. Δ	-0.0341	0.0310		-0.0157	0.0499	
	benchmark	0.2651	0.2838		0.0954	0.1143	

4 Discussion and Conclusion

We have argued that Germany’s corporate network exhibits a pronounced core structure that persists over time in spite of detectable changes in corporate governance, and substantial churning and entry-and-exit dynamics among corporate directors. An important issue concerns the validity of our findings beyond Germany’s one hundred largest publicly traded companies. So can we reasonably expect our results to apply to the entire corporate network?

Milaković et al. (2008) have performed an analysis of Germany’s corporate network that is very close in spirit to our study, focusing on 284 publicly traded German companies in 2008. In that year, the sample accounted for more than 95% of the market capitalization of Germany’s stock exchange *Deutsche Börse*, and therefore represents an almost complete sample of the corporate network. Two aspects of their findings are encouraging news for the validity of our results. First, despite their larger sample, they observe very similar magnitudes in many variables, for instance in the maximum number of board memberships, in the increasing deviations in the binomial benchmark, and in the density and average centrality of successive *b*-cores. Second, they find that the pronounced core structure in Germany’s corporate network is formed by mega capitalized companies, which absolves our focus on the one hundred largest companies, since it is the largest three to four dozen of those that are responsible for the *existence* and, as our results show, also the *persistence* of a core.

Furthermore, Milaković et al. consider the probability of obtaining an additional mandate conditional on the number of already existing mandates b , $P(b + 1|b)$, and find a peaked parabolic shape for the conditional probabilities as a function of b .¹⁰ An initially increasing (conditional) probability is compatible with our claim that directors with multiple mandates are preferably selected by large corporations, and our findings on the survival probabilities of key players add an important dimension to this as-

¹⁰We observe the same shape for $P(b + 1|b)$ in each year of our sample (material upon request).

pect. The existence of a maximum that is followed by decreasing (conditional) probabilities, on the other hand, indicates that recruitment decisions of large companies are also subject to a trade-off mechanism, for instance between social capital and monitoring ability, or between various stakeholder interests (see Milaković et al., 2008, for details), the exact nature of which we can currently only speculate about.

While our results show that the presence and persistence of a core certainly originates from the appointment decisions of mega corporations, our findings leave little room for the relevance of individual directors from a systemic viewpoint. The churning and entry-and-exit dynamics of individual directors rather indicate that the only relevant characteristic of a director is the number of mega corporation boards that she or he serves on.

Ultimately, however, we would like to understand the *extent* to which selective recruitment and replacement of directors by mega corporations is responsible for the core characteristics of the corporate network, for instance how the average number of mega appointments influences core density and average centrality. If policymakers were to aim for a reduction of pressure group influence, we believe these questions would be crucial for the corresponding policy design.

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A Distribution of board membership

Table 3: Overall frequency distribution of mandates.

	1	2	3	4	5	6	7	8	9	10
1993	1528	136	36	17	11	8	3	2	3	0
1999	1528	113	34	16	9	5	3	1	0	2
2005	1436	107	31	9	7	2	1	0	0	0

Table 4: Frequency of executives' supervisory board memberships.

year	executives	# of additional mandates									
		0	1	2	3	4	5	6	7	8	9
1993	565	456	58	21	12	8	5	2	1	2	0
1999	539	469	29	19	9	5	4	1	1	0	2
2005	456	401	37	11	3	4	0	0	0	0	0

B Transition matrices for board membership

Table 5: Transition matrix for board membership during 1993-1999.

		# of mandates in 1999										
		0	1	2	3	4	5	6	7	8	9	10
# of mandates in 1993	0	906	911	39	7	4	0	0	0	0	0	0
	1	920	369	30	10	4	1	1	2	0	0	0
	2	59	33	14	4	1	0	0	0	0	0	0
	3	9	8	6	3	1	2	0	0	0	0	0
	4	4	7	2	1	1	1	0	0	0	0	0
	5	4	1	2	0	1	1	1	0	0	0	0
	6	1	0	0	0	0	2	2	1	0	0	1
	7	1	0	1	0	1	0	0	0	0	0	0
	8	0	0	0	0	0	1	0	0	0	0	0
	9	1	0	0	2	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0

Table 6: Transition matrix for board membership during 1999-2005.

		# of mandates in 2005										
		0	1	2	3	4	5	6	7	8	9	10
# of mandates in 1999	0	977	883	34	10	1	0	0	0	0	0	0
	1	961	328	32	6	2	0	0	0	0	0	0
	2	49	24	16	3	1	1	0	0	0	0	0
	3	6	9	3	4	1	3	1	0	0	0	0
	4	4	5	3	1	0	0	0	0	0	0	0
	5	3	2	2	0	0	0	0	1	0	0	0
	6	0	0	1	1	1	1	0	0	0	0	0
	7	0	1	1	0	0	1	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	1	0	0	0	0	0	0	0

Table 7: Transition matrix for board membership during 1993-2005.

		# of mandates in 2005										
		0	1	2	3	4	5	6	7	8	9	10
# of mandates in 1993	0	662	1125	58	16	4	2	0	0	0	0	0
	1	1197	104	25	7	1	3	0	0	0	0	0
	2	94	11	4	0	1	0	1	0	0	0	0
	3	21	4	4	0	0	0	0	0	0	0	0
	4	12	2	0	1	0	0	0	1	0	0	0
	5	8	1	0	0	0	1	0	0	0	0	0
	6	2	2	1	2	0	0	0	0	0	0	0
	7	2	1	0	0	0	0	0	0	0	0	0
	8	0	1	0	0	0	0	0	0	0	0	0
	9	2	1	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0