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## A present value concept for measuring welfare

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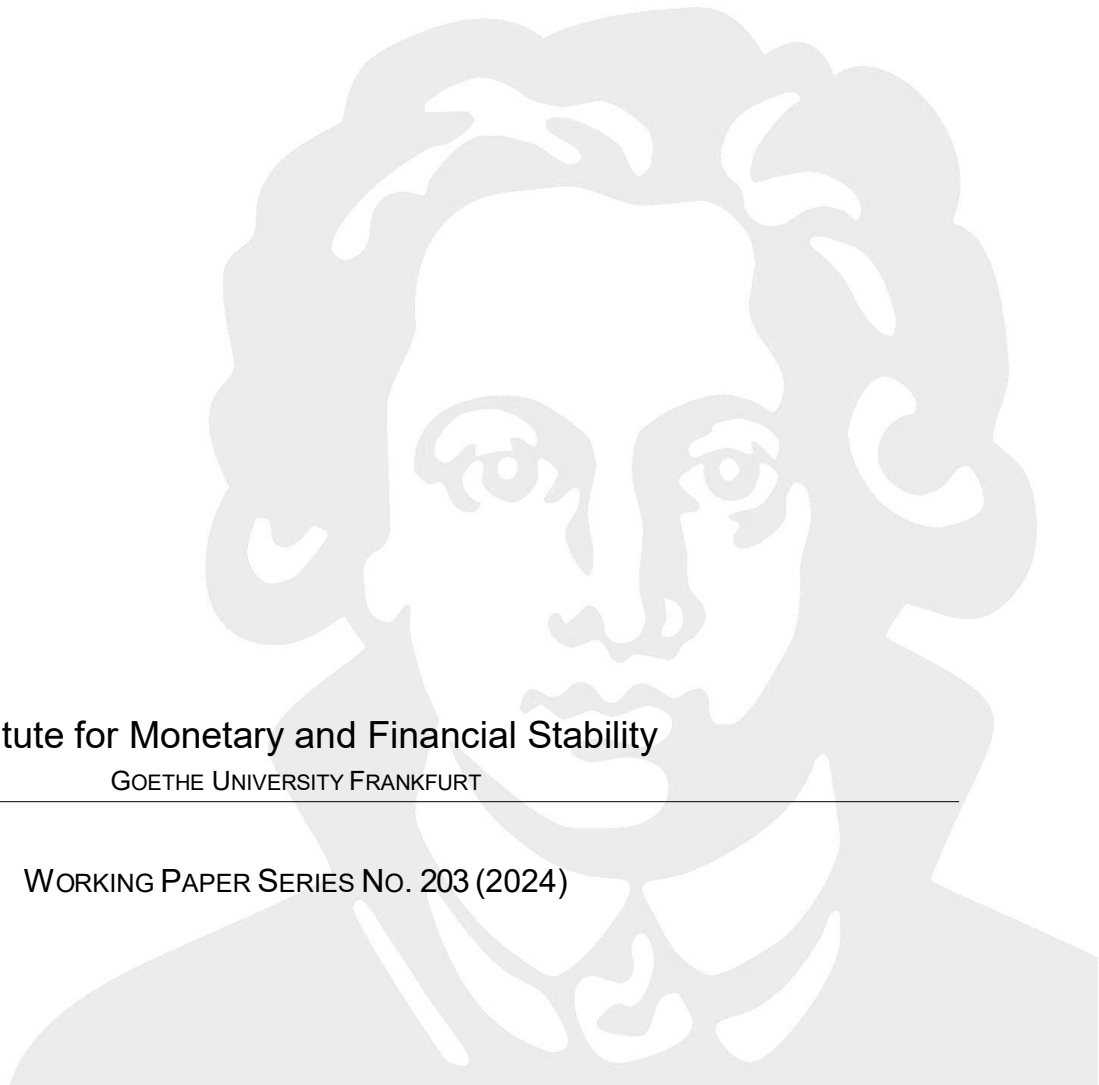
GERHARD RÖSL

A present value concept for measuring welfare

Institute for Monetary and Financial Stability  
GOETHE UNIVERSITY FRANKFURT

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# **A present value concept for measuring welfare**

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## **Abstract**

We create an alternative version of the present utility value formula to explicitly show that every store-of-value in the economy bears utility-interest (non-pecuniary income) for its holder regardless of possible interest earnings from financial markets. In addition, we generalize the well-known welfare measures of consumer and producer surplus as present value concepts and apply them not only for the production and usage of consumer goods and durables but also for money and other financial assets. This helps us, inter alia, to formalize the circumstances under which even a producer of legal tender might become insolvent. We also develop a new measure of seigniorage and demonstrate why the well-established concept of monetary seigniorage is flawed. Our framework also allows us to formulate the conditions for liability-issued money such as inside money and financial instruments such as debt certificates to become – somewhat paradoxically – net wealth of the society.

JEL: D14, D60, E41, E50

Keywords: Welfare, money, seigniorage, net wealth

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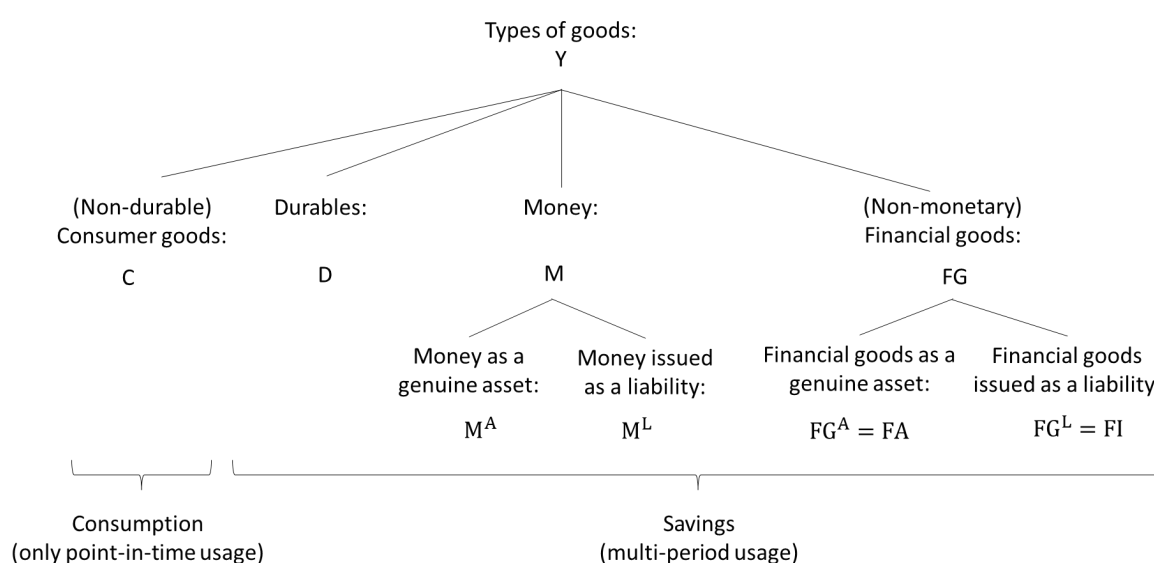
## 1. Introduction

Following the introduction, chapter 2 makes general remarks on our unconventional categorization of goods which overturns the usual hierarchy of consumer goods and durables (as goods that meet the final objective of the consumer, i.e., consumption in a narrow sense) over financial goods such as money and financial instruments that are usually considered as intermediate or transactional goods that only help to increase allocative efficiency. Based on this rigorously utility-based definition of goods, we demonstrate in chapter 3 that multi-period goods such as durables, money, and other financial products bear (unobservable) utility-interest (non-pecuniary income) regardless of possible financial market interest earnings. In chapter 4 we generalize the well-known contemporary measures of consumer and producer surplus as present value concepts. We apply them not only to the production and usage of consumer goods and durables but also to money and other financial assets. This helps us, inter alia, to formalize the circumstances under which even a producer of legal tender might become insolvent. In addition, we analyze the well-established measure of monetary seigniorage from a welfare-theoretical point of view and suggest that this concept should be replaced by our newly developed measure of “cumulated, compounded seigniorage profit”. In chapter 5, we use the framework developed in chapter 4 to examine the conditions for liability-issued money and financial instruments to be – somewhat paradoxically – net wealth of the society. Chapter 6 summarizes and concludes.

## 2. General remarks on the categorization of goods, production, and utility

We assume rational economic agents having four types of goods (Y) available: non-durable consumer goods (C), durables (D), money (M) and (non-monetary) financial goods (FG).

Figure 1: Categorization of goods



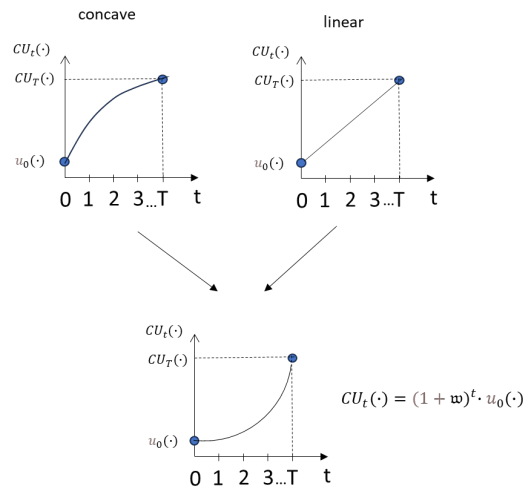
Since each type of good has its own unique characteristics, we do not consider them as perfect substitutes. Nonetheless, all of them render utility for its holder at any given time, and, hence, we consider them (only in this regard) similar.

In our framework, a non-durable consumer good (C) such as food shall yield utility only during its (point in) time of consumption whereas durables (D), money (M) and (non-monetary) financial goods (FG) can be used over a longer horizon. Those “multi-period goods” may be viewed in the broadest sense as different types of savings products which are from a utility perspective costly since the holders bear the opportunity costs of foregone consumption. Rational individuals, however, will only choose to hold such assets if their (expected) utility outweigh their opportunity costs. Hence, multi-period goods must bear some sort of “utility-interest” (non-pecuniary income), and our framework shows how to formalize it (see chapter 3). In case of durables (D), such as gold or real estate, owners get multi-periodic utility (non-pecuniary income) directly from the physical traits of these goods. In case of money (M), money holders are rendering money services from money balances (Sidrauski (1967)). From a utility perspective, however, it does not matter if money was produced as a genuine asset,  $M^A$ , such as commodity money, or issued based on a liability of the money producer,  $M^L$ , regardless of whether the money producer is a public institution or a private commercial bank. Non-monetary financial goods (FG) comprising genuine financial assets (representing equity),  $FG^A$ , and financial instruments (FI) which are created based on an effective liability of its issuer,  $FG^L$ , such as debt obligations also spent periodic utility for its owner as a store of value, and thus also bear “utility-interest”. But the respective utility-interest rates for the investor must be conceptually separated from dividends on equity and the financial market interest rate paid by the borrower in case of financial instruments, as shown in section 4.4.

Since a present-value welfare concept (see chapter 4) provides a point-in-time analysis, our framework commands just two assumptions for the utility function of the goods owners. First, utility at a given initial time (zero) is positive for every good,  $u_0(Y) > 0$ , and, second, in the case of multi-period goods (D, M, FG) the cumulated periodic utility over time,  $CU_T(\cdot) = \sum_{t=0}^T u_t(\cdot)$ , is larger than the initial utility at time zero,  $CU_T(\cdot) > u_0(\cdot)$ . We can then calculate for every multi-period good for the (intended) usage time T a utility growth rate  $\varpi$  without knowing the utility function specifically. To demonstrate, assume for sake of argument that the personal discount rate of the goods owner is zero:  $r = 0$ . The cumulated periodic utility level at time T together with the utility in the initial period 0, gives then the desired (constant) utility growth rate by  $\varpi = \sqrt[T]{CU_T(\cdot)/u_0(\cdot)} - 1$ .

Figure 2 illustrates that our framework does indeed not require the knowledge of the utility function regardless of the multi-period good in question. For instance, an assumed concave or linear cumulated periodic utility function gives the same transformed cumulated utility function,  $CU_T(\cdot) = (1 + \varpi)^T \cdot u_0(\cdot)$ , using the same given initial and cumulated future utility values of a certain multi-period good.

Figure 2: Transformation of different cumulated utility functions



Regarding the production of the different goods categorized, we also do not need to know the specific production functions. Again, we make only two assumptions: First, all types of goods shall be seen as a simple outcome of a costly production function with the usual production factors (e.g., labor, capital, and land) and second, and only for sake of simplicity, marginal production costs equal average production costs. Consumer goods and durables shall be produced by “normal” production firms. Somewhat unusual, we consider total money in circulation being produced and not lent by their respective issuers regardless of whether money is created as a genuine asset (such as commodity money) or issued based on a liability of either the private sector (such as transferable commercial bank deposits held by nonbanks) or of a public institution (such as central banks’ banknotes during the gold standard). Most uncommonly, we also look at the creation of non-monetary financial goods as a simple outcome of a costly production process. In particular, we interpret the signing of a debt contract as a creation of a financial instrument by a borrower who uses this self-created financial instrument to pay for the usage rights on the borrowed funds (see chapter 4.4). Of course, this view on financial instrument production implies that essentially every economic agent inside the economy can produce financial vehicles not only specialized financial institutions. This generalizes our approach to every form of lending/borrowing in the society.

As already indicated by the last paragraph, the role of credit is of special interest in our approach. Normally, credit is interpreted as a “use now, pay later” transaction where the lent resources are provided at the beginning of the investment period and the return of these resources (including interest payments) takes place later. In our framework, however, we consider lending/borrowing of funds as selling/buying of usage rights on that provided funds both taking place at same time at the beginning of the investment period. This interpretation of credit does, of course, not mean that it has no intertemporal dimension as shown by the present value consumer and producer surplus concepts of chapter 4.



### 3. On the utility-interest of multi-period goods

The present utility value (*PUV*) for an individual with a given positive subjective discount rate ( $r > 0$ ) who receives periodic utility from a certain quantity of good defined in the previous chapter,  $u_t$ , is usually expressed as the sum of the discounted periodic utilities,

$$(1) \quad PUV = \sum_{t=0}^T \frac{u_t}{(1+r)^t},$$

with  $T$  as the personal “usage-time”. To find an alternative version of this formula that seems to be more suited for our purpose, let us make use of the future utility value formula,

$$(2) \quad FUV = \sum_{t=0}^T u_t \cdot (1+r)^{T-t}$$

which describes the cumulated compounded periodic utilities at the end of the usage period  $T$ .<sup>2</sup> Together with the initial (uncompounded) periodic utility value,  $u_0$ , we can calculate for each type of good a constant periodic growth rate,  $\theta = \sqrt[T]{FUV/u_0} - 1$ , to obtain the following alternative present utility value formula,

$$(3) \quad PUV = \left[ \frac{1+\theta}{1+r} \right]^T \cdot u_0.$$

To get a better insight into the economic meaning of  $\theta$ , let us define for a fixed usage time  $T$  the constant growth factor for the initial utility value,  $u_0$ , as  $1 + \omega = \frac{1+\theta}{1+r}$ , rewrite eq 3 as

$$(3.1) \quad PUV = [1 + \omega]^T \cdot u_0 = \left[ \frac{1+\theta}{1+r} \right]^T \cdot u_0$$

and compare it to the well-known future market value (FMV) formula for initially invested capital,

$$(3.2) \quad FMV = [1 + \rho]^T \cdot K_0 = \left[ \frac{1+\varphi}{1+\pi} \right]^T \cdot K_0,$$

with  $\varphi$  as the nominal financial market interest rate,  $\pi$  as the goods market inflation rate,  $\rho$  the real financial market interest rate, and  $K_0$  as the capital available at the beginning ( $t = 0$ ) of the investment period  $T$ .

Analogy suggests that the initial utility value of a product available at time zero,  $u_0$ , being in use over multiple periods  $T$ , can be seen equivalent to a given initial capital stock, and the present utility formula of eq 3 can be reinterpreted as a future value formula for initial utility invested at time zero. Moreover, the constant periodic utility rate,  $\theta$ , can be viewed as the “nominal utility-interest rate” the holder receives from goods that can be used over multiple periods. This rate, however, is not a “pure” utility-interest rate since it also contains a waiting premium in form of the subjective discount rate  $r$ . To demonstrate, recall the definition of  $\theta$  that contains under its root the compounded utility future value which, in turn, depends on  $r$  (see eq 2). To obtain a periodic “pure” or “real” utility-interest rate ( $\omega$ ) for multi-period goods, we must therefore correct the nominal utility-interest factor,  $1 + \theta$ , by the subjective

<sup>2</sup> For convenience, see in case of  $T = 2$ :  $FUV_2 = u_0 \cdot (1+r)^2 + u_1 \cdot (1+r) + u_2$ .

discount factor,  $1 + r$ , just as we usually deflate with the inflation factor,  $1 + \pi$ , in investment theory (see eq 3.2). From a utility perspective, this is reasonable since not-consuming is expensive if the assumption of a positive time preference holds. Consequently, a rise in the individual discount rate leads to a decrease in the present utility value of multi-period goods as shown by eq 3.

A (non-durable) consumer good as defined in chapter 2 has from a pure utility perspective no intertemporal dimension ( $T = 0$ ) and its “pure” or “real” utility-interest rate is therefore zero,  $\omega(C) = 0$ ,<sup>3</sup> while the individual nominal utility-interest rate is identical to the personal discount rate,  $\theta(C) = r$ ,<sup>4</sup> which shows that postponing consumption is for the individual nonetheless costly. By contrast, multi-period goods ( $D, M, FG$ ) contain each a positive “pure” or “real” utility interest rate. It is a premium the holder receives in utility terms from the multi-period characteristic of that good to compensate for the opportunity costs of forgone consumption. For example, in case of durables such as gold, a positive real utility-interest rate,  $\omega(D)$ , explains why in practice investors hold seemingly “non-interest bearing” gold instead of consumption goods – because the investor attains (non-pecuniary) utility-interest income. The same considerations are also true for money, where  $\theta(M)$  can be interpreted as the nominal and  $\omega(M)$  as the “pure” or “real” utility-interest rate of money, the money holder receives from a given quantity on money  $M$  over the intended usage time  $T$ . The nominal and real utility-interest rates of a non-monetary financial goods,  $\theta(FG)$  and  $\omega(FG)$ , find their roots also in the multi-period characteristic of the financial good as a store-of-value and must be conceptually separated from the financial market interest rate  $r$  paid by the borrower. Since all nominal and real individual utility-interest rates are unobservable, they cannot be directly measured in practice.

The next step in our analysis is to look at the utility-interest rates through the lens of the market. We start with the turnover ( $TO$ ) a producer receives from selling a certain quantity of a good  $Y$  at a market price (per unit)  $P$  at time zero,  $TO = P \cdot Y$ . For a given customers’ usage time  $T$  synchronized to eq 1, we can periodize this turnover analytically as a sum of discounted periodic turnovers,

$$(4) \quad TO = \sum_{t=0}^T \frac{to_t}{(1+r)^t} = to_0 + \frac{to_1}{1+r} + \frac{to_2}{(1+r)^2} + \dots + \frac{to_T}{(1+r)^T},$$

with  $r$  as the financial market interest rate (see figure 3).

<sup>3</sup> Set  $T = 0$  in eq 3.1  $\Rightarrow PUV(C) = [1 + \omega(C)]^0 \cdot u_0(C) \Rightarrow 1 + \omega(C) = 1 \Rightarrow \omega(C) = 0$ .

<sup>4</sup> Set  $u_0(C) > 0$  and  $u_1(C) + u_1(C) + \dots + u_T(C) = 0$ . Eq 2 then simplifies to  $U_T(C) = (1 + r)^T$  and  $\theta(C) = r$ .

Figure 3: Periodizing utility and turnover

Individual perspective

$$PUV = u_0 + \frac{u_1}{1+r} + \frac{u_2}{(1+r)^2} + \dots + \frac{u_T}{(1+r)^T} = (1+\omega)^T \cdot u_0$$
$$TO = to_0 + \frac{to_1}{1+r} + \frac{to_2}{(1+r)^2} + \dots + \frac{to_T}{(1+r)^T} = (1+\beta)^T \cdot to_0$$

Market perspective

To get a better understanding what this series does let us normalize the quantity sold at time zero to one unit ( $Y = 1$ ) which then allows us to express eq 4 as a simple sequence of periodized market prices,

$$(5) \quad P = \sum_{t=0}^T \frac{p_t}{(1+r)^t} = p_0 + \frac{p_1}{1+r} + \frac{p_2}{(1+r)^2} + \dots + \frac{p_T}{(1+r)^T},$$

to demonstrate that eq 4 effectively associates periodic market prices to each period of use for a given quantity of good available at time zero and for a fixed usage-time  $T$  and thus evaluates the individual periodic utilities shown in eq 1 from a market perspective.

If we now calculate analogously to eq 2 the corresponding “market-valued future value” of the turnover created by the producer at time zero,

$$(6) \quad FTO = \sum_{t=0}^T to_t \cdot (1+r)^{T-t},$$

we receive the corresponding nominal market-valued utility-interest rates by  $\nu = \sqrt[T]{FTO/to_0} - 1$  and express the (present market value of the) turnover at time zero as

$$(7) \quad TO = P \cdot Y = (1+\beta)^T \cdot to_0$$

with  $1+\beta = \frac{1+\nu}{1+r}$  as the real market-valued utility-interest factor.

If the present utility value of a certain quantity of a product  $Y$  is for an individual higher than its present market value,  $PUV > TO$ , the consumer realizes a corresponding present value consumer surplus (PVCS, see also eq 9). Its determinants can hence be decomposed into

$$(8) \quad PVCS = \left(\frac{1+\omega}{1+r}\right)^T \cdot u_0 - \left(\frac{1+\nu}{1+r}\right)^T \cdot to_0 \\ = (1+\omega)^T \cdot u_0 - (1+\beta)^T \cdot to_0.$$

Eq 8 shows that in case of (non-durable) consumer goods, the present value consumer surplus is identical to its contemporaneous version (set  $T = 0$ ),

$$(8.1.) \quad PVCS(C) = CS(C) = u_0(C) - to_0(C).$$

In case of durables, money, and non-monetary financial goods ( $D, M, FG$ ), however, the multi-period characteristic of these goods contains positive real utility-interest rates,  $\omega(\cdot)$ , which not only imply that their respective present utility values usually exceed those of (non-durable) consumer goods but also help to explain why there is normally a positive present value consumer surplus for the holders of multi-period goods: It can be due to differences in the individual nominal utility-interest rates to the nominal market-valued utility-interest rates,  $\theta(\cdot) > \nu(\cdot)$ , due to differing discount rates  $r(\cdot) < \mathfrak{r}(\cdot)$ , or differences between the initial periodized utility and market values  $u_0(\cdot) > to_0(\cdot)$ .

#### 4. A present value concept of consumer and producer surplus and its applications

By subtracting eq 4 from eq 3 we obtain the following generalized versions of the well-known consumer surplus which now also account for welfare gains buyers receive from the usage of goods that yield utility over a longer horizon than (non-durable) consumer goods. The corresponding present value producer surplus (PVPS), however, is always identical to its contemporaneous version (PS) since investing the net funds received by the producer,  $\Pi$ , leaves their present value unchanged.<sup>5</sup>

Both, the present value consumer surplus (PVCS) and the present value producer surplus (PVPS) refer to the time of buying/selling ( $t = 0$ ) as

$$(9) \quad PVCS = (1 + \omega)^T \cdot u_0 - TO,$$

$$(10) \quad PVPS = TO - \mathfrak{C} = \Pi,$$

where  $\mathfrak{C} = c \cdot Y$  represents the respective production costs, with  $c$  as marginal production costs and  $Y$  as the produced quantity sold at time zero. Since we assume that marginal production costs are equal to average production costs, eq 10 also measures producer's profit,  $\Pi$ . Adding eq 9 and 10 yields the present value welfare gain (PVWG) for the society from the production and usage of a given quantity of good as the difference between the cumulated present utility values of all buyers  $j$  and the associated cumulated production costs of all producers  $k$ ,

$$(11) \quad PVWG_S = \sum_j (1 + \omega)^T \cdot u_{j,0} - \sum_k \mathfrak{C}_{k,0}.$$

##### 4.1. (Non-durable) Consumer goods

To illustrate the difference between (non-durable) consumer goods that spent utility only in the (instant) period of consumption and goods that can be used over multiple periods regarding their corresponding nominal and real utility-interest rates and the contemporary and present value concepts of consumer and producer surplus, let us for sake of simplicity assume that there is only one buyer and one producer. For a numerical example, assume that a buyer purchases a certain quantity of consumer goods ( $C = 2$ ), say two "Hamburgers Royal

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<sup>5</sup> See  $PVPS = \Pi \cdot \left(\frac{1+\mathfrak{r}}{1+r}\right)^t = \Pi = PS$ .

with Cheese” (Pulp fiction (1994)) at a price of \$4.50 each at time zero. The subjective discount rate of the buyer  $r$  shall be 10% pa, time is measured in years.

Table 1: Consumer’s welfare in case of (non-durable) consumer goods in a numerical example

Time	Price per unit	Units	Turnover	Periodic utility	Present utility value	Contemporary consumer surplus	Present value consumer surplus
t	$P_t(C)$	$C$	$TO_t(C)$	$u_t(C)$	$PUV_t(C)$	$CS_t(C)$	$PVCS_t(C)$
0	\$4.50	2	\$9.00	\$10.00	\$10.00	\$1.00	\$1.00

Notes:  $\theta(C) = r = 0.10$ ,  $\omega(C) = 0$ .

Table 1 shows the market price per unit,  $P(C) = \$4.50$ , the quantity purchased, ( $C = 2$ ), and the corresponding turnover from buying the two hamburgers,  $TO(C) = \$9.00$ . If we assume the value of the utility of the buyer from consuming the burgers at time zero of  $u_0(C) = \$10.00$ , we get the “textbook” contemporary consumer surplus as the difference between the utility and the turnover at time zero,  $CS(C) = \$1.00$  (set  $T = 0$  in eq 9). In addition, the same calculation shows the equivalence of the present value producer surplus to its contemporaneous version:  $CS(C) = PVCS(C) = \$1.00$ .

Table 2: Producer’s welfare in case of (non-durable) consumer goods in a numerical example

Time	Price per unit	Units	Turnover	Average cost per unit	Total production costs	Contemporary producer surplus = Profit	Present value producer surplus
t	$P_t(C)$	$C$	$TO_t(C)$	$c_t(C)$	$\mathfrak{C}_t(C)$	$PS_t(C) = \Pi_t(C)$	$PVPS_t(C)$
0	\$4.50	2	\$9.00	\$2.00	\$4.00	\$5.00	\$5.00

On the producer side, we assume (marginal being equal to) average production costs of  $c = \$2.00$  each, leading to total production costs of  $\mathfrak{C}(C) = \$4.00$  and a profit of  $\Pi(C) = \$5.00$ . By adding the present value consumer surplus and the present value producer surplus we obtain the present value welfare gain for the society from the production and consumption of the hamburgers worth of  $PVWG_s(C) = PVCS(C) + PVPC(C) = \$1.00 + \$5.00 = \$6.00$ , which is in the case of (non-durable) consumer goods always identical to the sum of its contemporary welfare measure. As shown above, the nominal utility-interest rate for consumer goods is identical to the subjective discount rate,  $\theta(C) = r$ , reflecting the fact that postponing consumption (here for two periods,  $T = 2$ ) is from a utility point of view still expensive,  $\theta(C) = r = 0.1 > 0$ , although (non-durable) consumer goods do not contain a positive “real” or “pure” utility-interest rate,  $\omega(C) = 0$ , due to their nature as contemporaneous products.<sup>6</sup>

To sum up, as for a given quantity of a non-durable consumer goods,  $C$ , intended to be bought and sold at time zero the present value consumer surplus, producer surplus and the corresponding societal welfare gain are always identical to their contemporaneous versions.

<sup>6</sup> From eq 3.1 we get  $1 + \omega(C) = \frac{1+\theta(C)}{1+r} = \frac{1+r}{1+r} = \frac{1+0.1}{1+0.1} = 1 \Rightarrow \omega(C) = 0$ .

Consequently, our framework can be seen as a generalization of the well-established textbook concept of consumer and producer surplus.

## 4.2. Durables

An intertemporal utility maximizing buyer will consider the present utility value instead of just looking at the initial utility in the period of purchase if he decides to buy a certain quantity of a durable ( $D$ ) at time zero as shown by eq 12 (see also eq 9):

$$(12) \quad PVCS(D) = (1 + \omega)^T \cdot u_0(D) - TO(D).$$

To illustrate, let us again refer to the numerical example above in which the buyer now decides to purchase an additional durable good in form of a sweater. Contrary to (non-durable) consumer goods, however, the sweater shall spend utility not only instantly at time of purchase ( $t = 0$ ) but also in the two following periods although its periodic utility decreases over time due to wear. Finally, after two years ( $T = 2$ ) it shall be not useable anymore as shown by table 3.

Table 3: Consumer's welfare in case of durables in a numerical example

Time	Price per unit	Units	Turnover	Periodic utility	Present utility value	Contemporary consumer surplus	Present value consumer surplus
$t$	$P_t(D)$	$D$	$TO_t(D)$	$u_t(D)$	$PUV_t(D)$	$CS_t(D)$	$PVCS_t(D)$
0	\$9.00	1	\$9.00	\$10.00	\$22.23	\$1.00	\$13.23
1	-	-	-	\$8.00	-	-	-
2	-	-	-	\$6.00	-	-	-

Notes:  $FUV_2 = \$26.90$ ,  $\theta(D) = 0.64$ ,  $r = 0.10$ ,  $\omega(D) = 0.49$ .

In our example, the consumer pays a price of \$9 for the sweater ( $D = 1$ ) creating a turnover of  $TO(D) = \$9.00$ . In exchange, the buyer gets a product that spends utility over two following periods and even exceeds the market price (turnover) immediately at time of purchase creating an contemporaneous consumer surplus of  $CS(D) = u_0(D) - TO(D) = \$10.00 - \$9.00 = \$1.00$ . The present utility value of the sweater,  $PUV(D) = (1 + \omega)^T \cdot u_0(D) = [1 + 0.49]^2 \cdot \$10 = \$22.23$ ,<sup>7</sup> however, is substantially higher than its contemporaneous utility value,  $u_0 = \$10$ , and shows the upper limit for the willingness to pay of buyer. In addition, it also implies that the present value consumer surplus is significantly larger than its contemporaneous measure,  $PVCS(D) = \$13.23 > CS(D) = \$1.00$ . Calculating the compounded utility future value of the sweater at the end of its lifetime (see, eq 2 for  $T = 2$ ),  $FUV_2(D) = \$26.90$  yields an annual nominal utility-interest rate of  $\theta(D) = 64.0\%$  and a "real" or "pure" utility-interest rate of  $\omega(D) = 49.1\%$  per year. As expected, this rate is positive due to the multi-period use characteristic of the sweater. Consequently, if the individual would have \$9 available at time zero and could either chose to buy the two cheeseburgers in our first example or the sweater, he would prefer the latter although the

<sup>7</sup> We get the same present utility value by adding the discounted periodic utilities using the usual present utility value formula, see eq 1:  $PUV = \sum_{t=0}^T \frac{u_t}{(1+r)^t} = u_0 + \frac{u_1}{1+r} + \frac{u_2}{(1+r)^2} = \$10.00 + \frac{\$8.00}{1.10} + \frac{\$6.00}{1.10^2} = \$22.23$ .

initial utility of both products are the same (see tables 1 and 3),  $u_0(D) = u_0(C) = \$10$ , but the present utility value of the sweater and, hence, its present value consumer surplus is comparatively higher  $PVCS(D) = \$13.23 > PVCS(C) = \$1.00$ .

From the perspective of a producer, the present value producer surplus is always identical to the contemporaneous producer surplus and to the profit if marginal production costs are equal to average production costs (see also eq. 10):

$$(13) \quad PVCS(D) = CS(D) = \Pi(D).$$

In other words, the multi-period use characteristic of a durable only affects the welfare of the consumers but not (the present value of) companies' profit. This is illustrated by table 4 in which we assume (marginal being equal to) average production costs of  $c = \$7.00$  to produce one sweater ( $D = 1$ ), implying in this case total production costs of  $\mathfrak{C}(D) = \$7.00$ , and together with the turnover,  $TO(D) = \$9.00$ , a (present value) producer surplus of  $PVPS(D) = \$2.00$ .

Table 4: Producer's welfare in case of durables in a numerical example

Time	Price per unit	Units	Turnover	Average cost per unit	Total production costs	Contemporary producer surplus = Profit	Present value producer surplus
t	$P_t(D)$	$D$	$TO_t(D)$	$c_t(D)$	$\mathfrak{C}_t(D)$	$PS_t(D) = \Pi_t(D)$	$PVPS_t(D)$
0	\$9.00	1	\$9.00	\$7.00	\$7.00	\$2.00	\$2.00

Adding eq 12 and 13 yields the corresponding present value welfare gain of the society (comprising all buyers  $j$  and all producers  $k$ ) from producing and buying additional durables as

$$(14) \quad PVWG_S(D) = \sum_j \{(1 + \omega)^T \cdot u_{j,0}(D)\} - \sum_k \{\mathfrak{C}_{k,0}(D)\},$$

and yields in our numerical illustration a present value of  $PVWG_S(D) = \$13.23 + \$2.00 = \$15.23$ .

### 4.3. Money

Money was created in its long history among various ethnicities and cultures, within different societies and legislations, and changed its appearance significantly over time (Kavuri et al. (2021)). In principle, money can take any shape – from a physical commodity to a digital string of data. All those different versions of money have in common that they are used as a means of exchange, unit of account, and as a store of value. From a utility perspective, the reason why money is demanded by economic agents is the pool of services money provides for its holders. Of these, the most important is the liquidity service (see, for instance, Holman (1998)) enabling current and future purchases at the goods markets and, thus, stabilizes consumption (Wen (2014)). But even if a certain stock of money would never be spent, money serves as an insurance against income fluctuations (Keynes (1936), Imrohoroglu (1992)) and financial market uncertainties (Tobin (1958)). In addition, money reduces search costs of finding trading

partners ("shopping time models", Walsh (2017)), thus saves time (Kimbrough (1986)) and fosters economic growth (Brock (1974)). This partial list of money services already indicates that money is highly effective in increasing allocative efficiency. But from a utility point of view, we claim that money is more than just a convenient means of exchange: It can increase personal welfare just like consumer goods and durables although "money can't be eaten", admittedly, but instead "lets you sleep well at night". The profit from money creation, in turn, finds its justification in the desire of money producers to meet the demand of their customers.

#### 4.3.1. Welfare gains for money holders and profit from money creation

In what follows, we assume that money is sold by its producer only in exchange for real resources.<sup>8</sup> The money holders are only willing to buy fresh money from the money producer if the present value of the current and future money services from the additional money received is at time of money issuance higher than the present market value of the resources to be handed over to the money producer. The turnover from selling real resources to the money producer is equivalent to the increase in the money stock,  $TO(M) = P(M) \cdot M$ , which, in turn, can be factored (Friedman & Schwartz (1969)) into the unit price of money,  $P(M)$  times the additionally provided units of money,  $M$ . Applying eq 9 for money issuance gives us the corresponding present value consumer surplus of the money holders:

$$(15) \quad PVCS(M) = [1 + \omega]^T \cdot u_0(M) - TO(M).$$

Eq 15 shows the extent to which the ability of a money producer to attract further real resources by issuing additional money is limited by the demand side. If the present utility value of money services is below the market value of the resources to be given up by the potential money holders in exchange, the money producer will not be able to finance his expenses through the "money press" anymore.

Table 5: Money holders' welfare in a numerical example

Time	Price per unit	Units	Turnover	Periodic utility	Present utility value	Contemporary consumer surplus	Present value consumer surplus
t	$P_t(M)$	$M$	$TO_t(M)$	$u_t(M)$	$PUV_t(M)$	$CS_t(M)$	$PVCS_t(M)$
0	\$1.00	50	\$50.00	\$70.00	\$113.80	\$20.00	\$63.80
1	-	-	-	\$30.00	-	-	-
2	-	-	-	\$20.00	-	-	-

Notes:  $FUV_2(M) = \$137.70$ ,  $\theta(M) = 0.40$ ,  $r = 0.10$ ,  $\omega(M) = 0.28$ .

To illustrate the welfare effects of money issuance (see table 5), assume that a money holder demands at time zero additional 50 units of money ( $M$ ) at a price per unit,  $P(M)$ , of \$1 and generates for the money producer a turnover from issuing additional money,  $TO(M)$ , of \$50.

<sup>8</sup> This is the standard way of providing money in practice whereas money issuance based on transfer is more of theoretical interest (see, for instance, Friedman's famous "helicopter money" (Friedman (1969, 4); Buiter (2004)). Nonetheless, the latter has also been practiced in rare times of currency reforms when money was at least partly provided to the public for free (see, for instance, Bank deutscher Länder (1949, 14)).



Since money saves transaction costs (among others, search costs of finding appropriate trade partners with mutual demand for each other's products) and also decreases payment uncertainty for the foreseeable future ( $T = 2$ ), the money holder in our example would be willing to give up real resources worth of up to \$113.80 as shown by the respective present utility value of money (see eq 1 or eq 3). Consequently, the money holder not only realizes a contemporary consumer surplus at time of money purchase,  $CS(M) = \$20$ , but a considerably higher present value consumer surplus worth of \$63.80, due to the "real utility-interest" of money whose rate is in our example  $\omega = 28\%$  per year.<sup>9</sup> However, if the utility-return on money drops due to deterioration of trust as a means of exchange (for instance due to hyper-inflation, see Hanke & Kwok (2009), where immensely rising prices decrease the utility of money very quickly) the money producer will become eventually insolvent although from a technical point of view he can still print in terms of nominal value an "unlimited" quantity of (fiat) money.

Turning our analysis to the supply-side restriction of money issuance, the money producer will only be willing to produce new money and sell it for further real resources if the market value from the resources received in exchange for new money, i.e., the turnover from selling money,  $TO(M)$ , is higher than the associated money production costs,  $\mathfrak{C}(M)$ . Of course, such considerations become only relevant if the type of money in circulation is very costly to produce. Examples are commodity monies such as (full-bodied) gold or silver coins. If, for instance, the production of such a coin,  $\mathfrak{C}(M)$ , costs more than its face value,  $TO(M)$ , a profit maximizing money producer will not be willing to mint and sell the coins.<sup>10</sup> Eq 16 shows the present value producer surplus of the money producer at time of money issuance and formalizes the supply-side restriction of profit-maximizing money producers (see also eq 10):

$$(16) \quad PVPS(M) = TO(M) - \mathfrak{C}(M) = \Pi(M).$$

Since we assume marginal money production costs equal average money production costs, eq 16 also yields the traditional seigniorage profit measure,  $\Pi(M)$ , as the difference between the "face value" of money representing its purchasing power at given prices, i.e., the turnover from selling additional units of money,  $TO(M)$ , and the associated money production costs,  $\mathfrak{C}(M)$ , at time of money issuance (see, for instance, Buiter (2007), Culver (2022), Coinnews (2023), Deutsche Bundesbank (2023)). Please note, however, that the traditional seigniorage profit measure of eq 16 implicitly assumes that money is either produced as a genuine asset or issued as an "ineffective" liability of the money producer (see also figure 1), i.e., the money producer either has no effective take-back obligation or the money holders never have an incentive to give their money back to the money producer in exchange for real resources even if they could (see, for instance, Lange (1995) and chapter 5.1).

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<sup>9</sup> While the nominal utility interest rate of money is in our example  $\theta(M) = \sqrt[T]{FUV/u_0} - 1 = \sqrt[2]{\$137.7/\$70.00} - 1 = 0.403$ , the respective real rate is  $\omega = \frac{1+\theta(M)}{1+r} - 1 = \frac{1+0.403}{1+0.1} - 1 = 0.28$ .

<sup>10</sup> In practice, however, not all money producers such as the US Mint can be seen as pure profit maximizing institutions. According to Coinnews (2023) the unit production costs for USD 1-cent were reportedly USD 2.4 cents and for the 5-cent coin USD 9.2 cents in 2022. Higher denominations of coins, however, still led to a profit from total coin production of USD 310 million by the US mint in 2022.

As already demonstrated by eq 10 as a general case, eq 16 exemplifies that – contrary to the consumer side – the multi-period use characteristic of money does not affect the welfare of the producer implying that the present value producer surplus of the money producer is always identical to its contemporary measure:  $PVPS(M) = PS(M) = \Pi(M)$ .

Table 6: Producer’s welfare from money creation in a numerical example

Time	Price per unit	Units	Turnover (Face value)	Average cost per unit	Total production costs	Contemporary producer surplus = Profit	Present value producer surplus
T	$P_t(M)$	$M$	$TO_t(M)$	$c_t(M)$	$\mathfrak{C}_t(M)$	$PS_t(M) = \Pi_t(M)$	$PVPS_t(M)$
0	\$1.00	50	\$50.00	\$0.10	\$5.00	\$45.00	\$45.00

To illustrate, we assume in table 6 quite low unit-production costs of (fiat) money of \$0.10 which add up to total production costs of \$5.00 and leaves the money producer with a substantial profit of \$45.00 at time of money issuance.

By adding the welfare gain of all money holders, MH, and the surplus(es) of the money producer(s), MP, we obtain the present value welfare gain of the society due to the issuance of additional money at time of its creation as

$$(17) \quad PVWG_S(M) = \sum_{MH}\{(1 + \omega)^T \cdot u_0(M)\} - \sum_{MP}\{\mathfrak{C}_0(M)\}.$$

Eq 17 implicitly demonstrates that the desire to hold more money leads typically to a welfare-theoretical positive-sum game  $PVWG_S(M) = PVCS(M) + PVPS(M) = \$63.80 + \$45.00 = \$108.80$ . Hence, the real resources the money producer gets in exchange for its newly produced money are, as a principle, well earned. This changes, however, if money is supplied by a monopoly using its market power to create an inflation tax (Friedman (1971), Phelps (1973), Cooley & Hansen (1989), Correia & Teles (1999), Buiter (2007), Menna & Tirelli (2017)).

#### 4.3.2. Monetary seigniorage versus compounded cumulated seigniorage

In the literature, the profit from money creation is often approximated by the concept of “monetary seigniorage”.<sup>11</sup> It is a revenue concept that intends to measure the influx of real resources to the money producer by the change in the (real) value of the money stock over a given period and assumes money production costs being (effectively) zero. To emphasize the purchasing power of the newly issued money, the concept of monetary seigniorage usually deflates the increase in the money stock over time by a price level P such as the consumer

<sup>11</sup> See, for instance, Friedman (1953), Marty (1968), Calvo (1978), Fischer (1982), Klein (1989), Easterly et al. (1995), Schobert (2002), Chakraborty (2014), Buiter (2021). Neumann (1992) building on ideas of Drazen (1985) extended this concept by a separate “interest seigniorage” measure leading to his “extended monetary seigniorage” concept.

price index or by nominal GDP. A commonly used formula for monetary seigniorage measure is therefore (see, for instance, Lange (1995, 30), Schobert (2002,2), Walsh (2017, 173):<sup>12</sup>

$$(18) \quad S_{M,t} = \frac{\mathbb{M}_t - \mathbb{M}_{t-1}}{P_t} = \frac{\Delta \mathbb{M}_t}{P_t}.$$

This definition, however, is from a conceptual point of view problematic since it divides a flow variable, i.e., the change in the money stock over time,  $\Delta \mathbb{M}_t$ , by a point-in-time stock variable, the price level at time  $t$ ,  $P_t$ , and, hence, either effectively transforms the formula into a point-in-time measure or keeps the analytical focus on the time period and assumes stable prices. Moreover, from an economic point of view one can criticize that the definition of eq 18 implicitly assumes that during the defined period of money issuance, the money producer has no opportunity costs which is equivalent to the assumption of a financial market interest rate of zero,  $r = 0$ . In other words, it does not matter for the money producer getting the resources at the beginning, in the middle or at the end of the period of money issuance. To demonstrate, let us first define as usual an annual period,  $t = 1$ , which we now subdivide in  $N = 12$  number of months in which we assume that a money producer buys an increasing quantity of a consumer goods at the end of every month  $n$ ,  $C_n$ , at increasing prices,  $p_n(C)$ , as shown in table 7.

Table 7: Monetary seigniorage and CC seigniorage in a numerical illustration (I)

Time reference	Time reference	Index number month	Money stock	Price per unit of consumer good	Units of consumer goods bought	Turnover = increase in money stock	Monetary Seigniorage	CC Seigniorage
t (end of year)	n (end of month)	n	$\mathbb{M}_n$	$P_n(C)$	$C_n$	$TO_n(M) = \Delta \mathbb{M}_n$	$S_{M,n}$	$\Pi_{cc,n}$
0	Dec	0	\$1,000.00	\$1.00	-	-	-	-
	Jan	1	\$1,010.00	\$1.00	10	\$10.00	10	11.37
	Feb	2	\$1,030.20	\$1.01	20	\$20.20	20	22.47
	Mar	3	\$1,060.80	\$1.02	30	\$30.60	30	33.32
	Apr	4	\$1,102.00	\$1.03	40	\$41.20	40	43.91
	May	5	\$1,154.00	\$1.04	50	\$52.00	50	54.25
	Jun	6	\$1,217.00	\$1.05	60	\$63.00	60	64.34
	Jul	7	\$1,291.20	\$1.06	70	\$74.20	70	74.20
	Aug	8	\$1,376.80	\$1.07	80	\$85.60	80	83.82
	Sep	9	\$1,474.00	\$1.08	90	\$97.20	90	93.20
	Oct	10	\$1,583.00	\$1.09	100	\$109.00	100	102.36
	Nov	11	\$1,704.00	\$1.10	110	\$121.00	110	111.29
1	Dec	12	\$1,837.20	\$1.11	120	\$133.20	120	120.00
Sum					780	\$837.20	780	814.51

Notes:  $r = 0.15$  and  $l = (1 + r)^{\frac{1}{N}} - 1 = (1 + 0.15)^{\frac{1}{12}} - 1 = 0.01171492 \approx 0.0117$ .

On the last day of January, for instance, the money producer shall purchase 10 units of consumer goods ( $C_1 = 10$ ) for a price per unit of  $P_{n=1}(C) = \$1.00$  and pays by newly issued money,  $TO_{n=1} = \$10.00$ . Therefore, our point in time measure shown in eq 18 gives a profit from money creation of equal value,  $\Pi_{n=1} = \$10.00$  if we set money production costs in this example to zero,  $\mathfrak{C}_n(M) = 0$ , for convenience. End of February, the monthly profit from money issuance accounts for  $\Pi_{n=2} = \$20.20$  which comparably higher than one month before due to the increase in the quantity of consumer goods bought (from  $C_1 = 10$  to  $C_2 = 20$ ), but also due to a rise in the goods prices (from  $P_1(C) = \$1.00$  to  $P_2(C) = \$1.01$ ), and hence, more money had to be issued. At the end of the year, the money producer increased

<sup>12</sup> Buiter (2007,5) prefers to “deflate” by nominal GDP.

its money supply by  $\Delta M_t = M_{t=1} - M_{t=0} = \sum_{n=1}^{N=12} TO_n(M) = \$837.20$ . Of course, this number can hardly account for the annual profit from money creation since the annual inflation rate also increased considerably by  $\pi = P_{12}(C)/P_0(C) - 1 = 11\%$ . The monetary seigniorage formula of eq 18 yields now at the end of the year  $S_M = \frac{M_t - M_{t-1}}{P_t} = \frac{\$1,837.20 - \$1,000.00}{\$1.11} = 754.23$  units of consumption which is in our example 3.3% below the total quantity of consumer goods actually bought by the money producer during the year:  $\sum_{n=1}^{12} C_n = 780$ . But even if prices had been stable, the monetary seigniorage measure would yield  $S_M = \frac{M_t - M_{t-1}}{P_t} = \frac{\$1780 - \$780}{\$1.00} = 780$  units of consumption, which still underestimates the economic value of real resources received over the year due to opportunity costs. For instance, instead of consuming 10 units of consumer goods at the end of January the money producer could have invested these funds for a period of 11 months and would then have been able to consume in our example 11.37 units of consumer goods<sup>13</sup> instead at the end of December if we assume an annual financial market interest rate of  $r = 0.15$  with a corresponding monthly interest rate of  $l = (1 + r)^{1/12} - 1 = (1 + 0.15)^{1/12} - 1 \approx 0.0117$ . The cumulated compounded periodic revenues from monthly money production over one year are therefore at the end of the year:

$$(19) \quad S_{cc} = \sum_{n=1}^N \frac{TO_n(M)}{P_n(C)} \cdot (1 + l)^{N-n},$$

yielding in our example 814.51 units of consumption which increases the measurement error of the monetary seigniorage concept in our example to -7.0%.<sup>14</sup>

By including money production costs into eq 19, we propose by eq 20 a new measure of seigniorage, which one might call “cumulated, compounded seigniorage profit”,

$$(20) \quad \Pi_{cc} = \sum_{n=1}^N \frac{TO_n(M) - \mathfrak{C}_n(M)}{P_n(C)} \cdot (1 + l)^{N-n},$$

that seems to be more appropriate than the well-established concept of monetary seigniorage since it gets rid of the conceptual problem of dividing a flow variable by a stock variable, it accounts for inflation, and takes opportunity costs and money production costs into consideration. If we assume, however, money production costs,  $\mathfrak{C}_n(M) = 0$ , the financial market rate ( $r = l = 0$ ) and the inflation rate ( $\pi = 0$ ) being zero, we are able to transform eq 20 to the established definition of monetary seigniorage (eq 18).<sup>15</sup>

As shown in Table 8, under these heroic conditions, the concept of monetary seigniorage becomes indeed an accurate measure of the (annual) profit from money creation. The cumulated real value of the (consumer) goods bought by the money producer during the year in exchange for newly issued money is in our new numerical example  $S_M = \frac{M_1 - M_0}{P_1} =$

<sup>13</sup> See  $\frac{TO_1}{P_1} \cdot (1 + l)^{(12-1)} = \frac{10}{1} \cdot (1 + 0.01171492)^{11} = 11.37$ .

<sup>14</sup> See  $(S_M - S_{cc})/S_{cc} = (754.23 - 814.51)/814.51 = -0.074 \approx -7\%$ .

<sup>15</sup> See  $\Pi_{cc}(M) = \sum_{n=1}^N \frac{TO_n(M) - \mathfrak{C}_n(M)}{P_n(C)} \cdot (1 + l)^{N-n} = \frac{\sum_{n=1}^N TO_n(M) - \mathfrak{C}_n(M)}{P_n(C)} = \frac{M_t - M_{t-1}}{P_t} = S_M$ , if  $\mathfrak{C}_n(M) = \pi = r = l = 0$ .

$\frac{\$1780 - \$1000}{\$1} = 780$  units of consumption being the same as our cumulated and compounded seigniorage measure,  $S_{cc} = \sum_{n=1}^N \frac{TO_n(M)}{P_n(C)} \cdot (1 + l)^{N-n} = 780$ .

Table 8 Monetary seigniorage and CC seigniorage in a numerical illustration (II)

Time reference	Time reference	Index number month	Money stock	Price per unit of consumer good	Units of consumer goods bought	Turnover = increase in money stock	Monetary Seigniorage	CC Seigniorage
t (end of year)	n (end of month)	n	$M_n$	$P_n(C)$	$C_n$	$TO_n(M) = \Delta M_n$	$S_{M,n}$	$\Pi_{cc,n}$
0	Dec	0	\$1,000.00	\$1.00	-	-	-	-
	Jan	1	\$1,010.00	\$1.00	10	\$10.00	10	10
	Feb	2	\$1,030.00	\$1.00	20	\$20.00	20	20
	Mar	3	\$1,060.00	\$1.00	30	\$30.00	30	30
	Apr	4	\$1,100.00	\$1.00	40	\$40.00	40	40
	May	5	\$1,150.00	\$1.00	50	\$50.00	50	50
	Jun	6	\$1,210.00	\$1.00	60	\$60.00	60	60
	Jul	7	\$1,280.00	\$1.00	70	\$70.00	70	70
	Aug	8	\$1,360.00	\$1.00	80	\$80.00	80	80
	Sep	9	\$1,450.00	\$1.00	90	\$90.00	90	90
	Oct	10	\$1,550.00	\$1.00	100	\$100.00	100	100
	Nov	11	\$1,650.00	\$1.00	110	\$110.00	110	110
1	Dec	12	\$1,780.00	\$1.00	120	\$120.00	120	120
Sum					780	\$780.00	780	780

Notes:  $r = l = \pi = \mathcal{C}(M) = 0$ .

The obvious question is now if the simple measure of monetary seigniorage of eq 18 can be seen as a proper approximation for the annual revenue a money producer gets from its money production?<sup>16</sup> The answer depends on how low inflation and financial market interest rates evolve in practice. At any rate, in an economic environment with very high inflation and interest rates, the concept of monetary seigniorage seems to severely underestimate the revenue from money creation.

#### 4.4. (Non-monetary) Financial goods

Regarding (non-monetary) financial goods (see figure 1), we distinguish between genuine financial goods,  $FG^A$ , and financial goods that are issued as a liability of a borrower,  $FG^L$ . The first category reflects equity and therefore represents a financial asset ( $FA$ ) with no payment obligations of another party. By contrast, financial goods that are created as a liability of a borrower,  $FG^L$ , usually bear financial market interest rates and imply a take-back obligation by the borrower once the credit expires. We refer to this second category of financial goods in the following as financial instruments ( $FI$ ).

<sup>16</sup> Since central banks become more transparent over time, weekly data are sometimes available, see for instance, ECB, weekly financial statements, <https://www.ecb.europa.eu/press/pr/wfs/html/index.en.html>.

#### 4.4.1. Genuine financial assets

A holder of a genuine financial asset (equity),  $FA$ , such as a share receives utility-interest due to its characteristic as a multi-period store-of-value even if no dividends are paid.<sup>17</sup> To demonstrate, let's assume as an example that a domestic investor buys such a (newly-issued foreign) share worth of \$150 with the intended holding time of two years due to fears of domestic political instabilities. Since our investor judges the possibility of domestic turmoil as increasing over time, the periodic utility of the share as a stable store-of-value shall also increase over time as shown in table 9.

Table 9: Financial asset holders' welfare in a numerical example

Time	Price per unit	Units	Turnover	Periodic utility	Present utility value	Contemporary consumer surplus	Present value consumer surplus
$t$	$P_t(FA)$	$FA$	$TO_t(FA)$	$u_t(FA)$	$PUV_t(FA)$	$CS_t(FA)$	$PVCS_t(FA)$
0	\$150.00	1	\$150.00	\$50.00	\$162.40	-\$100.00	\$12.40
1	-	-	-	\$60.00	-	-	-
2	-	-	-	\$70.00	-	-	-

Notes:  $FUV_2(FA) = \$196.50$ ,  $\theta(FA) = 0.98$ ,  $r = 0.10$ ,  $\omega(FA) = 0.80$ ,  $r = 0.00$

At time of purchase ( $t = 0$ ), however, the chance of an immediate outbreak of a domestic crisis seems to be rather low implying a quite low utility value of the financial asset of only \$50. This means that the immediate political risk alone would not be enough to justify the purchase of the share as demonstrated by the (negative) contemporary consumer surplus of -\$100. Since the share provides utility as a safe store-of-value also over the next two periods, however, the present utility value of the share (\$162.40) even surpasses the sales price (\$150) creating a present value consumer surplus for the investor of \$12.40. Applying eq 9 to financial assets,

$$(21) \quad PVCS(FA) = (1 + \omega)^T \cdot u_0(FA) - TO(FA),$$

demonstrates that this is mainly due to the enormous real utility-interest rate of  $\omega = 80\%$ .<sup>18</sup>

On the producer's side, the welfare gain from financial asset production amounts to (see also eq 10):

$$(22) \quad PVPS_0(FA) = TO_0(FA) - \mathfrak{C}_0(FA) = \Pi_0(FA).$$

Table 10: Producer's welfare from financial asset production in a numerical example

Time	Price per unit	Units	Turnover	Average cost per unit	Total production costs	Contemporary producer surplus = Profit	Present value producer surplus
$T$	$P_t(FA)$	$FA$	$TO_t(FA)$	$c_t(FA)$	$\mathfrak{C}_t(FA)$	$PS_t(FA) = \Pi_t(FA)$	$PVPS_t(FA)$
0	\$150.00	1	\$150.00	\$1.00	\$1.00	\$149.00	\$149.00

<sup>17</sup> For dividend bearing financial assets see the analogous argumentation for financial instruments in section 4.4.2.

<sup>18</sup> See  $PVCS(FA) = (1 + \omega)^T \cdot u_0(FA) - TO(FA) = (1 + 0.802202)^2 \cdot 50 - 150 = 12.396 \approx 12.40$

According to table 10, our example leaves the company with a net gain from share issuance of \$149.

Eq 23 yields the societal welfare gain from the issuance and purchase of genuine financial assets for all investors  $i$  and respective firms  $f$ :

$$(23) \quad PVWG_S(FA) = \sum_i \{ (1 + \omega)^T \cdot u_{i,0}(FA) \} - \sum_f \{ \mathfrak{C}_{f,0}(FA) \}.$$

In our example which assumes just one investor and one share-issuing firm, the welfare gain from issuing equity is therefore  $PVWG_S(FA) = PVCS(FA) + PVPS(FA) = \$161.40$ . In book accounting terms, however, attracting new investors who provide additional equity for the firm is usually considered as a financial market transaction whose net market value is zero.

#### 4.4.2. Financial instruments

Financial instruments render periodic utility for its holder as multi-period goods and thus also bear corresponding nominal and pure utility-interest rates,  $\theta_i(FI)$  and  $\omega_i(FI)$ . As already mentioned, these rates are conceptually different to the financial market-interest rates paid by the borrower to the financial instrument holder. To illustrate, let us first restrict for sake of simplicity the number of financial instruments issued to only one piece ( $FI = 1$ ), which then reduces the turnover from selling financial instruments to the market price of that single financial instrument,  $TO(FI) = P(FI) \cdot 1$ . By using eq 4, we can now periodize the market price of the financial instrument as  $P(FI) = \sum_{t=0}^T \frac{p_t}{(1+r)^T}$  (see eq 4\* in figure 4), and if we also assume a constant return on investment,  $R = p_t$ , eq 4 simplifies to the sum of discounted periodic returns on investment to  $P(FI) = \sum_{t=0}^T \frac{R}{(1+r)^T}$  (see eq 4\*\* figure 4).

Figure 4: Periodizing utility and turnover from financial instruments

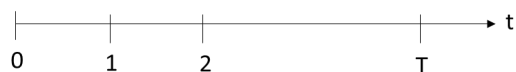
$$(Eq\ 1) \quad PUV(FI) = u_0 + \frac{u_1}{1+r} + \frac{u_2}{(1+r)^2} + \dots + \frac{u_T}{(1+r)^T} = (1+\omega)^T \cdot u_0 \quad (Eq\ 3.1)$$



$$(Eq\ 4) \quad TO(FI) = to_0 + \frac{to_1}{1+r} + \frac{to_2}{(1+r)^2} + \dots + \frac{to_T}{(1+r)^T} = (1+\beta)^T \cdot to_0 \quad (Eq\ 7)$$



$$(Eq\ 4^*) \quad P(FI) = p_0 + \frac{p_1}{1+r} + \frac{p_2}{(1+r)^2} + \dots + \frac{p_T}{(1+r)^T} \quad \text{if } FI = 1$$



$$(Eq\ 4^{**}) \quad P(FI) = R + \frac{R}{1+r} + \frac{R}{(1+r)^2} + \dots + \frac{R}{(1+r)^T} \quad \text{if } FI = 1 \text{ and } R = p_t = \text{const.}$$

Table 11 provides a numerical example corresponding to eq 4\*\* with an assumed investment period of  $T = 3$  years.

Table 11: Financial instrument holders' welfare in a numerical example

Time	Price per unit	Units	Turn-over	Periodized turnover/ market price	Periodic utility	Present utility value	Contemporary consumer surplus	Present value consumer surplus
t	$P_t(FI)$	$FI$	$TO_t(FI)$	$to_t(FI) = p_t(FI) = R$	$u_t(FI)$	$PUV_t(FI)$	$CS_t(FI)$	$PVCS_t(FI)$
0	\$34.87	1	\$34.87	\$10.00	\$15.00	\$46.49	\$5.00	\$11.62
1	-	-	-	\$10.00	\$12.00	-	-	-
2	-	-	-	\$10.00	\$12.00	-	-	-
3	-	-	-	\$10.00	\$12.00	-	-	-

Notes:  $FUV_3 = \$56.95$ ,  $\theta(FI) = 0.56$ ,  $\omega(FI) = 0.46$ ,  $r = 0.07$ ,  $FTO_3 = \$46.41$ ,  $v(FI) = 0.67$ ,  $\beta(FI) = 0.52$ ,  $r = 0.10$ .

In our example, the investor (lender) as the buyer of the financial instrument with a personal discount rate of  $r = 0.07$  values the periodized turnovers obviously higher than the market. Instead of investing funds worth of \$34.87 he would have been willing to invest \$46.49 for the same contractual agreement which is shown by the respective present utility value. One might ask, though, how the initial periodic utility value can be higher than the corresponding market value creating an instant contemporary consumer surplus for the lender,  $CS(FI) = u_0(FI) - p_0(FI) = \$15 - \$10 = \$5$ . The reasons might be manifold. For instance, the financial instrument can be a "safe haven", i.e., from the viewpoint of the investor a better store-of-value than the lent resources regardless of future interest income.<sup>19</sup> Table 11 also exemplifies that an investor can receive welfare gains from future interest earnings, whose present utility value is higher than its present market value,  $\sum_{t=1}^3 \frac{\$12}{(1+0.07)^t} = \$31.49 > \sum_{t=1}^3 \frac{\$10}{(1+0.1)^t} = \$24.87$ , which implies that the lender would have been willing to accept a lower interest rate than the actual market interest rate. In total, the present value consumer surplus of the lender as a buyer of the financial instrument (see also eq 9),

$$(24) \quad PVCS(FI) = (1 + \omega)^T \cdot u_0(FI) - TO(FI),$$

in our example accounts for \$11.62 and can mainly be attributed to the positive real utility-interest rate of  $\omega_l(FI) = 0.46$ .<sup>20</sup> This rate, however, differs from the financial market rate,  $r = 0.10$ , as does the corresponding nominal utility-interest rate,  $\theta_l(FI) = 0.56$ , as expected (see table 11). Eq 24 also shows the demand side restriction of the financial instrument purchase. If the real utility-interest rate of the financial instrument is not

<sup>19</sup> In addition, buyers of financial instruments such as institutional investors might get additional benefits from that asset to fulfil immediate regulatory requirements. Both factors seem to explain at least partly why in practice European commercial banks were willing to buy German government bonds even at negative nominal interest rates.

<sup>20</sup> If there weren't a contemporary consumer surplus,  $CS(FI) = u_0(FI) - p_0(FI) = \$0$ , the real utility-interest rate would be solely responsible for the welfare gain of the lender from investments in financial instruments.



sufficiently high to create a present utility value of the financial instrument that exceeds the turnover from lending funds to the borrower at market prices, the lender will not be willing to provide credit.

In our framework, the borrower receives a welfare gain in terms of a present value producer surplus. It is the difference between the turnover from producing financial instrument,  $TO_0(FI)$ , and the respective production costs  $\mathfrak{C}_0(FI)$  which are usually very low in practice (see also eq 10):

$$(25) \quad PVPS_0(FI) = TO_0(FI) - \mathfrak{C}_0(FI) = \Pi_0(FI),$$

If marginal production costs equal average production costs of the newly created financial instrument, eq 25 can also be interpreted as the profit the borrower obtains as a producer from financial instrument production,  $\Pi_0(FI)$ , i.e., the “financial seigniorage profit”. It finds its root in a somewhat unusual interpretation of a borrowing transaction. Normally, borrowing is seen as a “use now – pay later”-transaction. But such a view is incompatible with our point-in-time framework which commands instant payment. We therefore interpret the signing of a debt contract as a creation of the financial instrument by the borrower who uses this financial instrument to pay for the usage-rights on the resources at time of their receipt: It is just as the familiar look at the balance of payments statistics of countries with current account deficits. In such a year, the net importing economy issued (produced) additional financial instruments (in the form of bank deposits, CDOs, ABS, bonds, bills, and so forth) and with it realized the associated financial seigniorage profit which then was instantly used to pay for net imports, or more precisely, the usage rights from net imported resources provided by the rest of the world on a credit basis. Eq 25 does, however, include but not limit our analysis to borrowing from abroad.

To illustrate, let us assume borrowing of resources to the amount of  $TO(FI) = \$34.87$  with very low financial-instrument production costs of  $\mathfrak{C}(FI) = \$0.87$  implying a (present value) producer surplus or profit of \$34.00.

Table 12: Borrower’s welfare from financial instrument production in a numerical example

Time	Price per unit	Units	Turnover	Average cost per unit	Total production costs	Contemporary producer surplus = Profit	Present value producer surplus
T	$P_t(FI)$	$FI$	$TO_t(FI)$	$c_t(FI)$	$\mathfrak{C}_t(FI)$	$PS_t(FI) = \Pi_t(FI)$	$PVPS_t(FI)$
0	\$34.87	1	\$34.87	\$0.87	\$0.87	\$34.00	\$34.00

Eq 25 can also be interpreted as the supply-side restriction from financial instrument production. In our example, the borrower would refrain from borrowing if the costs from producing the financial instrument would be higher than the market value (turnover) of the resources received – an occurrence probably never being observed in practice.

By adding eq 24 to eq 25 we get the present value welfare gain for the society from financial instruments production by borrowers ( $b$ ) and its usage by lenders ( $l$ ) as,

$$(26) \quad PVWG_S(FI) = \sum_l \{(1 + \omega)^T \cdot u_{l,0}(FI)\} - \sum_b \{\mathfrak{C}_{b,0}(FI)\},$$

which accounts in our numerical example for  $\$11.62 + \$34.00 = \$45.62$ . Unsurprisingly, credit is in utility terms at least in principle an (intended) positive-sum game and not, as suggested by book accounting, a zero-sum game.

#### 4.5. Is our framework at odds with models of dynamic optimization?

Extending the well-known contemporary measure of consumer and producer surplus to a present value concept for multi-period goods shows that not only durables and money but also the issuance of other financial goods such as equity and financial instruments typically increase the welfare of both market sides. This result, however, seems at first sight to be at odds with MUI models (Sidrauski (1967), Blanchard & Fischer (1989), Rösler et al. (2019)) and other models of dynamic optimization (see, for instance, Walsh (2017)) where only (consumer) goods and outside money are arguments in the utility function whereas inside money (if any) and financial instruments are usually part of the budget constraint.<sup>21</sup> But such a comparison would be an exaggeration. First, our framework itself is not a full-fledged model: It does not explain how market prices are determined but provides instead an ex-post view on successful market transactions (including the production and usage of money and other financial assets) and their welfare implications for both market sides. It nonetheless shows, however, that there are implicit boundaries for the market prices in the sense that the resulting present value consumer and producer surplus should not be negative (see eqs 9 and 10). Second, our framework emphasizes that inside money and liability-issued financial instruments also increase the welfare in the economy although in pure book accounting terms they do not create any net-wealth for the society. This problem is addressed in the following chapter.

### 5. Liability-issued money and financial instruments as net wealth for the society

What net wealth consists of and how it should be measured might to be two of the most fundamental questions in economic theory. In accounting, net wealth is equal to equity which is calculated by subtracting liabilities from assets both (usually) valued at market prices. This approach, however, tends to underestimate the respective utility-values of the resources available to the economic agents as shown in chapter 4 and, hence, net wealth of the society becomes also undervalued because according to Lucas (1994) “this is real money”. In addition, the established accounting approach cancels out societal net wealth through balance sheet consolidation if goods created as issuers’ liabilities can be – somewhat paradoxically – net wealth for the society at the same time. Using our framework, we can now show the conditions under which liability-issued money and financial instruments become indeed societal net wealth.

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<sup>21</sup> Feenstra (1986) shows the equivalence of putting money in the budget constraint and in the utility function.

### 5.1. Liability-issued money as societal net wealth

In the literature, there is a long debate if money can be considered net wealth for the society (see, for instance, Gurley & Shaw (1960), Buchanan (1969), Friedman (1969), Fischer (1972), Thornton (1983), Weil (1991), Andolfatto (2018), Brunnermeier & Niepelt (2019)). Gurley & Shaw (1960) suggested in their seminal book to distinguish here between “outside and inside money”. Outside money is characterized that the process of money creation does not imply an increase in private sectors’ debt. An example is, for instance, the issuance of fiat money by the central bank in exchange for gold with no take-back obligation of the monetary authority. Inside money such as transferable bank deposits held by nonbanks, however, are created based on private banks’ debt of equal size and should therefore not account as net wealth of the society (Lagos (2006)). Pesek & Saving (1967) criticized early on that the economic advantages of money for the user do not depend on how money is created. Therefore, inside money can be considered net wealth to the extent resources are saved by the society compared to a barter economy (Issing (2011)). In addition, differences in interest rates received by banks and paid (if any) on transferable deposits also indicate net wealth of inside money and should therefore be accounted for by means of capitalization. Others remark that even if inside money contains an actual take-back obligation of the money producer, it should not be seen as an effective liability but an equity instead if the money is constantly used as a means of payment (Beard (1968)). Bossone & Costa (2021) extend this notion in their “accounting view of money” (AVM) also to liability-issued outside money and effectively claim that liability-issued monies should be regarded equity regardless of its issuer as long as there is no incentive for the money holder to exchange it back for other real resources with the money producer. Our framework fully supports this view.

For that we slightly modify eq 15 in a way that it now refers to an existing money stock  $\mathbb{M}$  instead of its increase ( $M$ ):

$$(27) \quad PVCS_t(\mathbb{M}) = [1 + \omega(\mathbb{M})]^T \cdot u_t(\mathbb{M}) - TO_t(\mathbb{M}) > 0, \forall t.$$

Eq 27 demonstrates that money will never be given back to its issuer by utility maximizing money holders (even if it would be legally and practically always possible) if the present utility value of money services is continuously higher over time than the possible turnover that could be received by the money holders if they gave the money back to the issuer in exchange for real resources at market prices. Let us emphasize, that this statement is completely independent of who the money producer (private or public) is, if money is legally treated as an official liability or not (pure gold standard vs modern fiat money issuance), or what kind of money (banknotes or digital currency) is in circulation.

### 5.2. Financial instruments as societal net wealth

Of course, the idea that liability-issued money can be net wealth for the society can also be applied for financial instruments. To show, we repeat the same routine by applying eq 24 for

the outstanding stock of financial instruments in the market,  $\mathbb{FI}$ , to get the following condition for financial instruments being net wealth for the society:

$$(28) \quad PVCS_t(\mathbb{FI}) = [1 + \omega(\mathbb{FI})]^T \cdot u_t(\mathbb{FI}) - TO_t(\mathbb{FI}) > 0, \forall t.$$

The present value consumer surplus of the lenders must remain always positive for any future time  $t$ . In that case, creditors will continuously prolong their lending, or from the borrowers' view, old debt can always be rolled over when it is due at time  $T$ . Although the creation of financial instruments comes along with a return for the society as demonstrated by eq 26, this societal welfare gain is, of course, again limited by the demand-side and supply-side restrictions shown by eq 24 and 25. As a consequence, creating net wealth for the society by producing financial instruments is also limited.

### 3. Summary and conclusions

Our alternative version of the present utility value formula shows how multi-period goods such as durables, money, and non-monetary financial assets bear utility-interest (non-pecuniary income) for their holders to compensate for foregone consumption even if some of these goods do not generate interest income. In other words, there is in utility terms no such thing as a non-interest-bearing store-of-value in the economy.

Furthermore, we extended the well-known measures of consumer and producer surplus as present value concepts to be applied not only for (non-durable) consumer goods but also for multi-period goods such as durables, i.e., goods that meet the final objective of the consumer directly, but also for “financial goods” such as money and financial assets/instruments that are usually considered as mere financial vehicles to increase allocative efficiency. Although far from being a full-fledged model, our concept enabled us to find at least the respective supply- and demand-side restrictions for all goods in question. In case of money, for instance, we demonstrated that even fiat money producing central banks can go effectively bankrupt if the present utility value of money services from newly issued money is below the market value of the resources to be given up by the potential money holders in exchange. We also analyzed the well-established measure of monetary seigniorage from a welfare-theoretical point of view and concluded that one should replace this concept by our newly developed measure of “cumulated, compounded seigniorage profit” to properly measure money producer's profit from money creation.

In addition, our framework also helped us to formulate the precise conditions for liability-issued money and financial instruments to become net wealth for the society: The respective present utility value must be continuously higher than the market value of the turnover the holders could obtain by giving back their claim to the respective issuer instead. The suggestion of Bossone & Costa (2021) to accordingly adjust the respective calculations of net wealth in the national accounts seems to be well-founded.

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