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Article

Analysis of Green Bonds

Junior Management Science (JUMS)

Provided in Cooperation with:

Junior Management Science e. V.

Suggested Citation: Fauß, Tobias Friedrich (2022) : Analysis of Green Bonds, Junior Management Science (JUMS), ISSN 2942-1861, Junior Management Science e. V., Planegg, Vol. 7, Iss. 3, pp. 668-689,
<https://doi.org/10.5282/jums/v7i3pp668-689>

This Version is available at:

<https://hdl.handle.net/10419/294999>

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Online-Appendix zu

„ Analysis of Green Bonds“

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Junior Management Science 7(3) (2022) 668-689

A Appendix

A.1 Derivation of Trinomial Tree Model

We derive the probabilities for the trinomial tree following the assumptions in ?. For the default branching method (i.e., $j_{min} < j < j_{max}$), the condition for the expected change notates as,

$$\begin{aligned} p_u \cdot \Delta s + p_m \cdot 0 + p_d \cdot (-\Delta s) &= \mathbb{E}[dLP_t] \\ &= -a \cdot j \cdot \Delta s \cdot \Delta t \end{aligned} \quad (26)$$

Dividing by Δs and solving for p_d yields

$$p_d = p_u + a \cdot j \cdot \Delta t. \quad (27)$$

We use this result in the condition for the variance

$$\begin{aligned} p_u \cdot \Delta s^2 + p_m \cdot 0^2 + p_d \cdot \Delta s^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2 \\ p_u \cdot \Delta s^2 + (p_u + a \cdot j \cdot \Delta t) \cdot \Delta s^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2 \end{aligned} \quad (28)$$

and solve for p_u to obtain

$$p_u = \frac{1}{2}\sigma^2 \cdot \frac{\Delta t}{\Delta s^2} + \frac{1}{2}a^2 \cdot j^2 \cdot \Delta t^2 - \frac{1}{2}a \cdot j \cdot \Delta t. \quad (29)$$

We use $\Delta s = \sigma\sqrt{3\Delta t}$ or $\Delta s^2 = \sigma^2 \cdot 3\Delta t$ to obtain

$$\begin{aligned} p_u &= \frac{1}{2}\sigma^2 \cdot \frac{\Delta t}{\sigma^2 \cdot 3\Delta t} + \frac{1}{2}(a^2 \cdot j^2 \cdot \Delta t^2 - a \cdot j \cdot \Delta t) \\ &= \frac{1}{6} + \frac{1}{2}(a^2 \cdot j^2 \cdot \Delta t^2 - a \cdot j \cdot \Delta t). \end{aligned} \quad (30)$$

We use this result to obtain the probability p_d as

$$\begin{aligned} p_d &= p_u + a \cdot j \cdot \Delta t \\ &= \frac{1}{6} + \frac{1}{2}(a^2 \cdot j^2 \cdot \Delta t^2 + a \cdot j \cdot \Delta t). \end{aligned} \quad (31)$$

Finally, we use $p_u + p_m + p_d = 1$ to obtain

$$\begin{aligned} p_m &= 1 - p_u - p_d \\ &= \frac{2}{3} - a^2 \cdot j^2 \cdot \Delta t^2. \end{aligned} \quad (32)$$

At the limits of the trinomial tree (i.e., j_{min} and j_{max}), the branching structure changes as displayed in Figure [5](#).

First, we compute the probabilities for the lower limit j_{min} . For this, we change the condition for the expected change to,

$$\begin{aligned} p_u \cdot 2\Delta s + p_m \cdot \Delta s + p_d \cdot 0 &= \mathbb{E}[dLP_t] \\ &= -a \cdot j \cdot \Delta s \cdot \Delta t, \end{aligned} \quad (33)$$

as the liquidity premium cannot decrease any further. We obtain

$$p_m = -2p_u - a \cdot j \cdot \Delta t, \quad (34)$$

which we substitute into the new condition for the variance

$$\begin{aligned} p_u \cdot 4\Delta s^2 + p_m \cdot \Delta s^2 + p_d \cdot 0^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2 \\ p_u \cdot 4\Delta s^2 + (-2p_u - a \cdot j \cdot \Delta t) \cdot \Delta s^2 + p_d \cdot 0^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2. \end{aligned} \quad (35)$$

We solve the equation for p_u and use $\Delta s^2 = \sigma^2 \cdot 3\Delta t$ to obtain

$$\begin{aligned} p_u &= \frac{1}{2}\sigma^2 \cdot \frac{\Delta t}{\sigma^2 \cdot 3\Delta t} + \frac{1}{2} (a^2 \cdot j^2 \cdot \Delta t^2 + a \cdot j \cdot \Delta t) \\ &= \frac{1}{6} + \frac{1}{2} (a^2 \cdot j^2 \cdot \Delta t^2 + a \cdot j \cdot \Delta t). \end{aligned} \quad (36)$$

We use this result to obtain the probability p_m as

$$\begin{aligned} p_m &= -2p_u - a \cdot j \cdot \Delta t \\ &= -\frac{1}{3} - a^2 \cdot j^2 \cdot \Delta t^2 - a \cdot j \cdot \Delta t - a \cdot j \cdot \Delta t \\ &= -\frac{1}{3} - a^2 \cdot j^2 \cdot \Delta t^2 - 2a \cdot j \cdot \Delta t. \end{aligned} \quad (37)$$

Finally, we use $p_u + p_m + p_d = 1$ to obtain

$$\begin{aligned} p_d &= 1 - p_u - p_m \\ &= \frac{7}{6} + \frac{1}{2} (a^2 \cdot j^2 \cdot \Delta t^2 + 3a \cdot j \cdot \Delta t). \end{aligned} \quad (38)$$

In the same fashion, we can compute the probabilities for j_{max} . In this case, we

change the condition for the expected change to,

$$p_u \cdot 0 + p_m \cdot (-\Delta s) + p_d \cdot (-2\Delta s) = \mathbb{E}[dLP_t] \quad (39)$$

and the condition for the variance to

$$p_u \cdot 0 + p_m \cdot \Delta s^2 + p_d \cdot 4\Delta s^2 = \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2. \quad (40)$$

In this case, we obtain

$$\begin{aligned} p_u &= \frac{7}{6} + \frac{1}{2} (a^2 \cdot j^2 \cdot \Delta t^2 - 3a \cdot j \cdot \Delta t) \\ p_m &= -\frac{1}{3} - a^2 \cdot j^2 \cdot \Delta t^2 + 2a \cdot j \cdot \Delta t \\ p_d &= \frac{1}{6} + \frac{1}{2} (a^2 \cdot j^2 \cdot \Delta t^2 - a \cdot j \cdot \Delta t) \end{aligned} \quad (41)$$

Based on the calibration result of the model in section [4.3](#), we use these formulas to compute the probabilities for the trinomial tree. The probabilities are displayed in [Table 8](#).

Table 8: Trinomial tree probabilities

	j	p_u	p_m	p_d
j_{max}	4	0.9008	0.0093	0.0900
	3	0.1058	0.6465	0.2477
	2	0.1238	0.6577	0.2184
	1	0.1441	0.6644	0.1914
	0	0.1667	0.6667	0.1667
	-1	0.1914	0.6644	0.1441
	-2	0.2184	0.6577	0.1238
j_{min}	-3	0.2477	0.6465	0.1058
	-4	0.0900	0.0093	0.9008

The table shows the probabilities for the trinomial tree using $\sigma = 0.0031$, $a = 11.9$, $T = 3.1$ years, $N = 791$ and assuming $\Delta s = \sigma\sqrt{3\Delta t}$.

A.2 Additional Robustness Tests for Switch Option Value

Table 9: Option value at execution for different interest rate r

r	LP	ST^{\max}
-100	12.06	4.06
0	12.06	4.06
100	12.06	4.06
200	12.06	4.06

The table shows the values of ST^{\max} for different r based on $GP = 8\text{bp}$, $\sigma = 0.0031$, $a = 11.9$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

Table 10: Option value at execution for different rounding precision

Precision	LP	ST^{\max}
4	8	0.35
5	10.31	2.36
6	12.06	4.06
7	12.35	4.35
8	12.35	4.35

The table shows the values of ST^{\max} for different rounding precisions, measured in digits after the decimal point. A precision of six digits equals 0.01bp. The further parameters are $GP = 8\text{bp}$, a risk-free rate of $r_f = 200\text{bp}$, $\sigma = 0.0031$, $a = 11.9$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

A.3 Additional Estimation Results and Statistics

Table 11: Summary statistics for BAS of German twin bonds

	Mean	SD	Min	p25	Median	p75	Max	N
BAS_{G2050}	0.67	0.31	0.00	0.50	0.60	0.90	1.50	131
BAS_{C2050}	0.57	0.27	0.10	0.40	0.50	0.90	1.00	131
BAS_{G2031}	0.57	0.33	0.20	0.30	0.50	0.90	1.30	47
BAS_{C2031}	0.29	0.11	0.20	0.20	0.30	0.30	0.80	47
BAS_{G2030}	0.70	0.36	0.20	0.40	0.60	0.90	1.70	305
BAS_{C2030}	0.29	0.24	0.10	0.20	0.20	0.30	1.70	305
BAS_{G2025}	1.27	0.85	0.20	0.60	1.15	1.40	3.30	262
BAS_{B2025}	0.57	0.26	0.30	0.40	0.50	0.70	2.40	262

The table shows the summary statistics (i.e., mean, **standard deviation (SD)**, minimum, **25th percentile (p25)**, median, **75th percentile (p75)**, maximum and **number of observations (N)**) for the bid-ask spread of the closing yields in basis points of the German twin bonds displayed in Table [1](#). The data is retrieved from Refinitiv Eikon (Accessed: 10.11.2021) and covers the period from 09.09.2020 to 10.11.2021.

Table 12: Summary statistics for yield of German twin bonds

	Mean	SD	Min	p25	Median	p75	Max	N
y_{G2050}	15.96	13.95	-9.80	4.70	17.00	26.70	41.40	127
y_{C2050}	6.12	17.73	-48.60	-8.00	4.30	20.60	44.80	566
y_{G2031}	-25.34	8.40	-40.70	-35.10	-23.70	-17.90	-13.50	43
y_{C2031}	-29.71	12.59	-49.80	-41.90	-30.10	-18.80	-9.70	103
y_{G2030}	-47.15	12.86	-66.90	-59.10	-48.50	-35.40	-21.20	301
y_{C2030}	-43.02	12.69	-64.10	-54.30	-45.50	-31.80	-15.70	358
y_{G2025}	-72.22	6.80	-86.70	-76.80	-72.20	-67.40	-54.50	258
y_{C2025}	-69.38	7.02	-83.70	-74.55	-69.50	-64.65	-46.60	344

The table shows the summary statistics for the yields in basis points of the German twin bonds displayed in Table [1](#). The data is retrieved from Refinitiv Eikon (Accessed: 04.11.2021).

Table 13: Summary statistics for Δy between German twin bonds

	Mean	SD	Min	p25	Median	p75	Max	N
Δy_{2050}	-3.95	0.45	-5.70	-4.30	-4.00	-3.60	-3.10	126
Δy_{2031}	-4.41	0.42	-5.50	-4.70	-4.40	-4.20	-3.20	43
Δy_{2030}	-4.76	1.64	-7.60	-6.30	-5.10	-3.30	-1.40	301
Δy_{2025}	-3.36	1.88	-8.50	-4.40	-3.10	-2.30	0.10	258

The table shows the summary statistics for the yield spread (i.e., $\Delta y = y_G - y_C$) in basis points between the German twin bonds displayed in Table 11. The data is retrieved from Refinitiv Eikon (Accessed: 04.11.2021).

Table 14: Summary statistics for liquidity proxy

	Mean	SD	Min	p25	Median	p75	Max	N
LP_{2050}	93.08	9.61	50.54	89.01	93.66	98.55	116.80	549
LP_{2031}	48.76	6.47	36.26	43.77	47.51	52.58	80.15	549
LP_{2030}	48.61	6.31	36.21	43.95	47.55	51.92	78.25	549
LP_{2025}	48.66	6.44	38.01	44.23	47.49	50.66	72.24	549

The table shows the summary statistics for the estimated proxy of the liquidity premium in basis points. The data is based on published yield curves by the ? and covers the period from 02.09.2019 until 01.11.2021.

Table 15: Summary of OLS estimation results for Vasicek process

	2050	2031	2030	2025
LP_{t-1}	0.7662*** (0.0537)	0.9023*** (0.0231)	0.9078*** (0.0223)	0.9534*** (0.0159)
Constant	0.0022*** (0.0005)	0.0005 * ** (0.0001)	0.0004*** (0.0001)	0.0002*** (0.0001)
R-squared	0.5863	0.8150	0.8250	0.9089
N	548	548	548	548

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

The table shows the estimation results for the process of the liquidity premium specified in Equation 22. The dependent variable is the daily measured liquidity proxy LP_t and the independent variables are a constant and the lagged value LP_{t-1} . The data is based on published yield curves by the ? and covers the period from 02.09.2019 until 01.11.2021.

A.4 List of German Twin Bonds

Table 16: German Twin Federal Securities

Name	First Issue Date	Last Issuance	Maturity Date	Coupon	Outstanding	Type	ISIN	RIC
2021 (2050) Bund/g	18.5.2021	11.05.2021	15.8.2050	0.00%	6.0 bn. EUR	Green	DE0001030724	DE103072=
2019 (2050) Bund	23.8.2019	18.08.2021	15.8.2050	0.00%	29.0 bn. EUR	Brown	DE0001102481	DE110248=
2021 (2031) Bund/g	10.9.2021	20.10.2021	15.8.2031	0.00%	6.5 bn. EUR	Green	DE0001030732	DE103073=
2021 (2031) II Bund	18.6.2021	10.11.2021	15.8.2031	0.00%	26.5 bn. EUR	Brown	DE0001102564	DE110256=
2020 (2030) Bund/g	9.9.2020	02.09.2020	15.8.2030	0.00%	6.5 bn. EUR	Green	DE0001030708	DE103070=
2020 (2030) II Bund	19.6.2020	18.11.2020	15.8.2030	0.00%	30.5 bn. EUR	Brown	DE0001102507	DE110250=
Bobl/g	6.11.2020	04.11.2020	10.10.2025	0.00%	5.0 bn. EUR	Green	DE0001030716	DE103071=
Bobl	10.7.2020	02.12.2020	10.10.2025	0.00%	25.0 bn. EUR	Brown	DE0001141828	DE114182=

Source: ? and Refinitiv Eikon (Accessed: 21.11.2021)