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## Article

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## Optimal Monetary Policy with Staggered Wage and Price Contracts\*

Christopher J. Erceg\*\*, Dale W. Henderson\*\*\* and Andrew T. Levin\*\*\*\*

### Abstract

We formulate an optimizing-agent model in which both labor and product markets exhibit monopolistic competition and staggered nominal contracts. The unconditional expectation of average household utility can be expressed in terms of the unconditional variances of the output gap, price inflation, and wage inflation. Monetary policy cannot achieve the Pareto-optimal equilibrium that would occur under completely flexible wages and prices; that is, the model exhibits a tradeoff in stabilizing the output gap, price inflation, and wage inflation. We characterize the optimal policy rule for reasonable calibrations of the model. We also find that strict price inflation targeting generates relatively large welfare losses, whereas several other simple policy rules perform nearly as well as the optimal rule.

*Keywords:* Monetary policy; Inflation targeting; Nominal wage and price rigidity; Staggered contracts

*JEL classification:* E31; E32; E52

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## I. Introduction

For several decades, economists have investigated the monetary policy tradeoff between price inflation variability and output gap variability, using a wide variety of theoretical and empirical models.<sup>1</sup> However, the existence of a variance tradeoff has been called into question in recent analysis of dynamic general equilibrium models with optimizing agents. In these models, staggered price setting is the sole form of nominal rigidity, and monetary policy rules that keep the inflation rate constant also minimize output gap variability.<sup>2</sup> The monetary authorities can achieve the Pareto-optimal welfare level (that is, the welfare level that would occur in the absence of nominal inertia and monopolistic distortions) through the remarkably simple policy of strict price inflation targeting, irrespective of the parameter values or other specific features of these models.

In this paper, we analyze an optimizing-agent model with staggered nominal wage setting in addition to staggered price setting. As in recent contributions, volatility of aggregate price inflation induces dispersion in prices across firms and hence inefficient dispersion in output levels. Similarly, with staggered wage contracts, volatility of aggregate wage inflation induces inefficiencies in the distribution of employment across households. Hence, achieving the Pareto-optimal equilibrium would require not only a zero output gap and complete stabilization of price inflation, but also complete stabilization of wage inflation.

These considerations lead directly to our main result: it is impossible for monetary policy to attain the Pareto optimum except in the special cases where either wages or prices are completely flexible. Nominal wage inflation and price inflation would remain constant only if the aggregate real wage rate were continuously at its Pareto-optimal level. Such an outcome is impossible because the Pareto-optimal real wage moves in response to various shocks, whereas the actual real wage could never change in the absence of nominal wage or price adjustment. Given that the Pareto optimum is infeasible, the monetary policymaker faces tradeoffs in stabilizing wage inflation, price inflation, and the output gap.

Under staggered wage and price setting, the optimal monetary policy rule depends on the specific structure and parameter values of the model. These features affect both the set of feasible monetary policy choices (the policy frontier) and the preferences of the policymaker (the indifference loci implied by the so-

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<sup>1</sup> The seminal papers include Phelps and Taylor (1977); Taylor (1979,1980). Some recent examples include Bryant *et al.* (1993); Henderson/McKibbin (1993); Tetlow/von zur Muehlen (1996); Williams (1999); Levin *et al.* (1999); Rudebusch/Svensson (1999).

<sup>2</sup> Contributions include Goodfriend/King (1997), King/Wolman (1999), Ireland (1997), Rotemberg/Woodford (1997,1999).

cial welfare function). For example, optimal monetary policy depends on the relative duration of wage and price contracts: the optimal rule induces greater variability in the more flexible nominal variable.

The welfare level under the optimal monetary policy rule provides a natural benchmark against which to measure the performance of alternative policy rules. We find that strict price inflation targeting can induce substantial welfare costs under staggered wage setting, due to excessive variation in nominal wage inflation and the output gap. This policy forces all adjustment in real wages to occur through nominal wages, which in turn requires variation in the output gap. We also analyze hybrid rules in which the nominal interest rate responds to either wage inflation or the output gap in addition to price inflation. The performance of each hybrid rule is virtually indistinguishable from that of the optimal rule for a wide range of structural parameters.

This paper is organized as follows. We outline the model in Section 2. In Section 3, we derive the social welfare function using essentially the same methods as *Rotemberg/Woodford* (1997). In Section 4, we present key results concerning the policy frontier. We use numerical methods to characterize optimal monetary policy in Section 5, and investigate the welfare costs of alternative policy rules in Section 6. Conclusions and directions for future research are given in Section 7.

## II. The Model

Our model is similar in many respects to recent optimizing-agent models with nominal price inertia. Monopolistically competitive producers set prices in staggered contracts with timing like that of *Calvo* (1983).<sup>3</sup> This price-setting behavior implies an equation linking price inflation to the gap between the real wage and the marginal product of labor. In contrast to most recent contributions, we assume that monopolistically competitive households set nominal wages in staggered contracts.<sup>4</sup> Household wage-setting behavior implies an equation linking wage inflation to the gap between the real wage and the marginal rate of substitution of consumption for leisure.

Under monopolistic competition, output and labor supply would be below their Pareto-optimal levels in the absence of government intervention, even with perfectly flexible wages and prices. We assume that the central task of monetary policy is to mitigate the effects of nominal inertia, while fiscal policy is responsible for offsetting distortions associated with imperfect competition.

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<sup>3</sup> *Kimball* (1995) and *Yun* (1996) pioneered the use of price contracts with Calvo-style timing in stochastic, optimizing-agent models.

<sup>4</sup> *Solow/Stiglitz* (1968), *Blanchard/Kiyotaki* (1987), *Kollmann* (1997), *Erceg* (1997), and *Kim* (2000) also incorporate both nominal price and wage inertia.

Therefore, following recent studies, we assume that output and labor are each subsidized at fixed rates to ensure that the equilibrium would be Pareto optimal if wages and prices were completely flexible.

### 1. Firms and Price Setting

We assume a continuum of monopolistically competitive firms (indexed on the unit interval), each of which produces a differentiated good that is consumed solely by households.<sup>5</sup> Because households have identical preferences, it is convenient to abstract from the household's problem of choosing the optimal quantity of each differentiated good  $Y_t(f)$  for  $f \in [0,1]$ . Thus, we assume that there is a representative 'output aggregator' who combines the differentiated goods into a single product that we refer to as the 'output index'. The output aggregator combines the goods in the same proportions as households would choose, and then sells the output index to households. Thus, the aggregator's demand for each differentiated good is equal to the sum of household demands.

The output index  $Y_t$  is assembled using a constant returns to scale technology of the *Dixit/Stiglitz* (1977) form (which mirrors the preferences of households):

$$(1) \quad Y_t = \left[ \int_0^1 Y_t(f)^{\frac{1}{1+\theta_p}} df \right]^{1+\theta_p}$$

where  $\theta_p > 0$ . The output aggregator chooses the bundle of goods that minimizes the cost of producing a given quantity of the output index  $Y_t$ , taking as given the price  $P_t(f)$  of the good  $Y_t(f)$ . The aggregator sells units of the output index at their unit cost  $P_t$ :

$$(2) \quad P_t = \left[ \int_0^1 P_t(f)^{-\frac{1}{\theta_p}} df \right]^{-\theta_p}.$$

It is natural to interpret  $P_t$  as the aggregate price index. The aggregator's demand for each good  $Y_t(f)$  – or equivalently total household demand for this good – is given by

$$(3) \quad Y_t(f) = \left[ \frac{P_t(f)}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}} Y_t$$

<sup>5</sup> Monopolistic competition rationalizes the assumption that firms are willing to satisfy unexpected increases in demand even when they are temporarily constrained not to change their prices.

Each differentiated good is produced by a single firm that hires capital services  $K_t(f)$  and a labor index  $L_t(f)$  defined below. Every firm faces the same Cobb-Douglas production function, with an identical level of total factor productivity  $X_t$ :

$$(4) \quad Y_t(f) = X_t K_t(f)^\alpha L_t(f)^{1-\alpha}.$$

The aggregate capital stock is fixed at  $\bar{K}$ , and capital and labor are perfectly mobile across firms. Each firm chooses  $K_t(f)$  and  $L_t(f)$ , taking as given both the rental price of capital and the wage index  $W_t$  defined below. The standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output ( $MC_t$ ). Marginal cost can be expressed as a function of the wage index, the aggregate labor index  $L_t$ , the aggregate capital stock, and total factor productivity, or equivalently, as the ratio of the wage index to the marginal product of labor ( $MPL_t$ ):

$$(5) \quad MC_t = \frac{W_t L_t^\alpha}{(1-\alpha)\bar{K}^\alpha X_t} = \frac{W_t}{MPL_t},$$

$$(6) \quad MPL_t = (1-\alpha)\bar{K}^\alpha L_t^{-\alpha} X_t.$$

Producers set prices in staggered contracts with random duration: in any given period, the firm is allowed to reset its price contract with probability  $(1 - \xi_p)$ . Note that the probability that a firm will be allowed to reset its contract price in any period does not depend on how long its existing contract has been in effect, and this probability is invariant to the aggregate state vector. Thus, a constant fraction  $(1 - \xi_p)$  of firms reset their contract prices each period.

When a firm is allowed to reset its price in period  $t$ , the firm chooses  $P_t(f)$  to maximize the following profit functional:

$$(7) \quad \mathcal{E}_t \sum_{j=0}^{\infty} \xi_p^j \psi_{t,t+j} \left( (1 + \tau_p) \Pi^j P_t(f) Y_{t+j}(f) - MC_{t+j} Y_{t+j}(f) \right).$$

The operator  $\mathcal{E}_t$  represents the conditional expectation based on information through period  $t$  and taken over states of nature in which the firm is not allowed to reset its price.<sup>6</sup> The firm's output is subsidized at a fixed rate  $\tau_p$ . The firm discounts profits received at date  $t + j$  by the probability that the firm will not have been allowed to reset its price ( $\xi_p^j$ ) and by the discount factor  $\psi_{t,t+j}$ .<sup>7</sup> Note

<sup>6</sup> For simplicity, the variables in Eq. (7) are not explicitly indexed by the state of nature.

<sup>7</sup> The state-contingent discount factor  $\psi_{t,t+j}$  indicates the price in period  $t$  of a claim that pays one dollar in a given state of nature in period  $t + j$ , divided by the probability that state will occur.

that whenever the firm is not allowed to reset its contract, the firm's price is automatically increased at the unconditional mean rate of gross inflation,  $\Pi$ . Thus, if firm  $f$  has not adjusted its contract price since period  $t$ , then its price in period  $t + j$  is  $P_{t+j}(f) = \Pi^j P_t(f)$ .

The first-order condition for a price-setting firm is

$$(8) \quad \varepsilon_t \sum_{j=0}^{\infty} \xi_p^j \psi_{t,t+j} \left( \frac{(1+\tau_p)}{(1+\theta_p)} \Pi^j P_t(f) - MC_{t+j} \right) Y_{t+j}(f) = 0.$$

Thus, the firm sets its contract price so that discounted real marginal revenue (inclusive of subsidies) is equal to discounted real marginal cost, in expected value terms. We assume that production is subsidized to eliminate the monopolistic distortion associated with a positive markup; that is,  $\tau_p = \theta_p$ . Thus, in the limiting case in which all firms are allowed to set their prices every period ( $\xi_p \rightarrow 0$ ), Eq.(8) reduces to the familiar condition that price equals marginal cost, or equivalently, that the real wage equals the marginal product of labor:

$$(9) \quad \frac{W_t}{P_t} = MPL_t$$

## 2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household's labor services  $N_t(h)$ ,  $h \in [0,1]$ , as an imperfect substitute for the labor services of other households. It is convenient to assume that a representative labor aggregator (or 'employment agency') combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The labor index  $L_t$  has the Dixit-Stiglitz form:

$$(10) \quad L_t = \left[ \int_0^1 N_t(h)^{\frac{1}{1+\theta_w}} dh \right]^{1+\theta_w}$$

Where  $\theta_w > 0$ . The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate  $W_t(h)$  as given, and then sells units of the labor index to the production sector at their unit cost  $W_t$ :

$$(11) \quad W_t = \left[ \int_0^1 W_t(h)^{-1/\theta_w} dh \right]^{-\theta_w}.$$

It is natural to interpret  $W_t$  as the aggregate wage index. The aggregator's demand for the labor hours of household  $h$  – or equivalently, the total demand for this household's labor by all goods-producing firms – is given by

$$(12) \quad N_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-(1+\theta_w)/\theta_w} L_t.$$

The utility functional of household  $h$  is

$$(13) \quad \begin{aligned} \varepsilon_t \sum_{j=0}^{\infty} \beta^j & \left( \mathbb{U}(C_{t+j}(h), Q_{t+j}) + \mathbb{V}(N_{t+j}(h), Z_{t+j}) + \frac{\mu_0}{1-\mu} \left( \frac{M_{t+j}(h)}{P_{t+j}} \right)^{1-\mu} \right), \\ \mathbb{U}(C_t(h), Q_t) &= \frac{1}{1-\sigma} (C_t(h) - Q_t)^{1-\sigma}, \\ \mathbb{V}(N_t(h), Z_t) &= \frac{1}{1-\chi} (1 - N_t(h) - Z_t)^{1-\chi}, \end{aligned}$$

where the operator  $\varepsilon_t$  here represents the conditional expectation over all states of nature, and the discount factor  $\beta$  satisfies  $0 < \beta < 1$ . The period utility function is separable in three arguments: net consumption, net leisure, and real money balances. Net consumption is defined by subtracting the consumption shock  $Q_t$  from the household's consumption index  $C_t(h)$ . Net leisure is defined by subtracting hours worked  $N_t(h)$  and the leisure shock  $Z_t$  from the household's time endowment (normalized to unity). The consumption and leisure shocks are common to all households. Real money balances are nominal money holdings  $M_t(h)$  deflated by the aggregate price index  $P_t$ .

Household  $h$ 's budget constraint in period  $t$  states that consumption expenditures plus asset accumulation must equal disposable income:

$$(14) \quad \begin{aligned} P_t C_t(h) + M_t(h) + M_{t-1}(h) + \delta_{t+1,t} B_t(h) - B_{t-1}(h) \\ = (1 + \tau_w) W_t(h) N_t(h) + \Gamma_t(h) + T_t(h). \end{aligned}$$

Asset accumulation consists of increases in money holdings and the net acquisition of state-contingent claims. Each element of the vector  $\delta_{t+1,t}$  represents the price of an asset that will pay one unit of currency in a particular state of nature in the subsequent period, while the corresponding element of the vector  $B_t(h)$  represents the quantity of such claims purchased by the household.  $B_{t-1}(h)$  indicates the value of the household's claims given the current realization of the



state of nature. Labor income  $W_t(h)N_t(h)$  is subsidized at a fixed rate  $\tau_w$ . Each household owns an equal share of all firms and of the aggregate capital stock, and receives an aliquot share  $\Gamma_t(h)$  of aggregate profits and rental income. Finally, each household receives a lump-sum government transfer  $T_t(h)$ . The government's budget is balanced every period, so that total lump-sum transfers are equal to seignorage revenue less output and labor subsidies.

In every period  $t$ , each household  $h$  maximizes the utility functional (13) with respect to consumption, money balances, and holdings of contingent claims, subject to its labor demand function (12) and its budget constraint (14). The first-order conditions for consumption and holdings of state-contingent claims imply the familiar 'consumption Euler equation' linking the marginal cost of foregoing a unit of consumption in the current period to the expected marginal benefit in the following period:

$$(15) \quad \mathbb{U}_{C,t} = \varepsilon_t [\beta(1+R_t)\mathbb{U}_{C,t+1}] = \xi_t \left[ \beta(1+I_t) \frac{P_t}{P_{t+1}} \mathbb{U}_{C,t+1} \right]$$

where the risk-free real interest rate  $R_t$  is the rate of return on an asset that pays one unit of consumption under every state of nature at time  $t+1$ , and the nominal interest rate  $I_t$  is the rate of return on an asset that pays one unit of currency under every state of nature at time  $t+1$ . Note that the omission of the household-specific index  $h$  in Eq. (15) reflects our assumption of complete contingent claims markets for consumption (but not for leisure), which implies that consumption is identical across households in every period ( $C_t(h) = C_t$ ).

Households set nominal wages in staggered contracts that are analogous to the price contracts described above. In particular, a constant fraction  $(1 - \xi_w)$  of households renegotiate their wage contracts in each period. In any period  $t$  in which household  $h$  is able to reset its contract wage, the household maximizes its utility functional (13) with respect to the wage rate  $W_t(h)$ , yielding the following first-order condition:

$$(16) \quad \varepsilon_t \sum_{j=0}^{\infty} \beta^j \xi_w^j \left( \frac{(1+\tau_w)}{(1+\theta_w)} \frac{\Pi^j W_t(h)}{P_{t+j}} \mathbb{U}_{C,t+j} \mathbb{V}_{N(h),t+j} \right) N_{t+j}(h) = 0.$$

where  $\varepsilon_t$  here indicates the conditional expectation taken only over states of nature in which the household is unable to reset its wage. Whenever the household is not allowed to renegotiate its contract, its wage rate is automatically increased at the unconditional mean rate of gross inflation,  $\Pi$ . Thus, if household  $h$  has not reset its contract wage since period  $t$ , then its wage rate in period  $t+j$  is  $W_{t+j}(h) = \Pi^j W_t(h)$ .

According to Eq. (16), the household sets its wage so that the discounted marginal utility of the income (inclusive of subsidies) from an additional unit of labor is equal to its discounted marginal disutility, in expected value terms. We assume that employment is subsidized to eliminate the monopolistic distortion associated with a positive markup; that is,  $\tau_w = \theta_w$ . Thus, in the limiting case in which all households are allowed to set their wages every period ( $\xi_w \rightarrow 0$ ), Eq. (16) reduces to the condition that the real wage equals the marginal rate of substitution of consumption for leisure ( $MRS_t$ ):

$$(17) \quad W_t / P_t = MRS_t,$$

$$(18) \quad MRS_t = -\frac{V_{N,t}}{U_{C,t}} = \frac{(C_t - Q_t)^{-\sigma}}{(1 - N_t - Z_t)^{-\chi}}.$$

### 3. The Steady State

The non-stochastic steady state of our model is derived by setting the three shocks to their mean values ( $\bar{X}$ ,  $\bar{Q}$  and  $\bar{Z}$ ). Given that both wage and price contracts are indexed to the steady-state inflation rate  $\bar{\pi}$ , the steady state is the same as if wages and prices were fully flexible. Thus, given our assumptions about production and employment subsidies, the steady state is Pareto optimal. All firms produce the same amount of output ( $\bar{Y}(f) = \bar{Y}$ ), using the same amount of labor, and all households supply the same quantity of labor ( $\bar{L}(f) = \bar{L} = \bar{N} = \bar{N}(h)$ ), where variables with bars represent steady-state values. Using the production function (4) to solve for labor hours in terms of output, equilibrium values of the real wage and output are determined by the condition that  $MPL_t = MRS_t$ , using Eqs. (6) and (18). The real interest rate  $\bar{R}$  is determined by the consumption Euler equation (15).<sup>8</sup>

### 4. The Pareto Optimum

For comparative purposes, we consider the equilibrium of our model under flexible prices and wages, henceforth referred to as the Pareto optimum. We follow the standard approach of log-linearizing around the steady state of the model. Small letters denote the deviations of logarithms of the corresponding variables from their steady-state levels, and letters with asterisks represent Pareto-op-

<sup>8</sup> We set  $\bar{Q} = 0.3163$ ,  $\bar{X} = 4.0266$ ,  $\bar{Z} = 0.03$ , and  $\bar{K} = 30 \bar{Q}$ . Using the baseline calibration described in Section 5.1 (namely,  $\alpha = 0.3$ , and  $\sigma = \chi = 1.5$ ), we obtain  $\bar{L} = 0.27$  and  $\bar{Y} = 10 \bar{Q} = 3.163$ . Thus,  $\bar{L}$  and  $\bar{Z}$  together account for about one-third of the household's time endowment, and the steady-state capital/output ratio is equal to 3.

timal values of the corresponding variables. We solve for values of Pareto-optimal output ( $y_t^*$ ), the real wage ( $\zeta_t^*$ ), and real interest rate ( $r_t^*$ ) using the same equations that were used above to obtain the steady state:

$$(19) \quad \begin{aligned} y_t^* &= \frac{1 + \chi_N^{\ell}}{\Lambda} x_t + \frac{(1 - \alpha)\sigma^{\ell_Q}}{\Lambda} q_t - \frac{(1 - \alpha)\chi_Z^{\ell}}{\Lambda} z_t, \\ \zeta_t^* &= \frac{\chi_N^{\ell} + \alpha\ell_C}{\Lambda} x_t - \frac{\alpha\sigma^{\ell_Q}}{\Lambda} q_t + \frac{\alpha\chi_N^{\ell}}{\Lambda} z_t, \\ r_t^* &= \alpha\ell_C (y_{t+1|t}^* - y_t^*) + \sigma^{\ell_Q} (q_{t+1|t} - q_t), \\ \Lambda &= \alpha + \chi_N^{\ell} + (1 - \alpha)\sigma^{\ell_C}, \end{aligned}$$

where

$$(20) \quad \ell_C = \frac{\bar{C}}{(\bar{C} - \bar{Q})}, \ell_Q = \frac{\bar{Q}}{(\bar{C} - \bar{Q})}, \ell_N = \frac{\bar{N}}{(1 - \bar{N} - \bar{Z})}, \ell_Z = \frac{\bar{Z}}{(1 - \bar{N} - \bar{Z})}.$$

The subscript  $t+1|t$  indicates a one-step-ahead forecast of the variable based on information available through period  $t$ .

As usual, a positive productivity shock  $x_t$  raises both  $y_t^*$  and  $\zeta_t^*$ . A positive consumption shock  $q_t$  raises the marginal utility of a given level of the consumption index, inducing an increase in labor supply that raises  $y_t^*$  and reduces  $\zeta_t^*$ . A positive leisure shock  $z_t$  directly reduces labor supply by raising the marginal disutility of a given amount of labor hours, thereby decreasing  $y_t^*$  and increasing  $\zeta_t^*$ .

### 5. The Dynamic Equilibrium

With staggered wage and price setting, the key equations of the model are listed in Table 1. Given that output can deviate from its Pareto-optimal level, we define the output gap  $g_t = y_t - y_t^*$ .

In the consumption Euler equation (T1.1) (see Table 1), the expected change in the output gap depends on the deviation of the short-term real interest rate (that is, the nominal interest rate  $i_t$  less expected output price inflation  $\pi_{t+1|t}$ ) from the equilibrium real interest rate  $r_t^*$ . Solving the equation forward, the current output gap depends negatively on an unweighted sum of current and future short-term real interest rates, naturally interpreted as the ‘long-term’ real interest rate. This equation resembles a Keynesian IS curve.<sup>9</sup>

Equations (T1.2) and (T1.3) are simple transformations of Eqs. (6) and (18), respectively. The marginal product of labor,  $mpl_t$ , is negatively related to the

<sup>9</sup> See, for example, Woodford (1996) and Kerr/King (1996).

output gap, while the marginal rate of substitution,  $mrs_t$ , is positively related to the output gap. The output gap is zero at the intersection of the  $mpl_t$  and  $mrs_t$  schedules, namely, at the Pareto-optimal real wage rate,  $\zeta_t^*$ .

Using the first-order condition of each price-setting firm (8), it is straightforward to obtain an aggregate equation that has been derived in earlier work: namely, price inflation depends on the percentage deviation of real marginal cost from its constant desired level of unity.<sup>10</sup> Our price-setting equation (T1.4) follows from the equality between real marginal cost and the ratio of the real wage to the marginal product of labor.

Current price inflation (as a deviation from steady state) depends positively both on expected price inflation and on the percentage by which the real wage,  $\zeta_t^*$ , exceeds the marginal product of labor,  $mpl_t$ . Solving the equation forward reveals that price inflation depends on current and expected future gaps between real wages and marginal products of labor. Thus, price inflation is at its steady-state value only when the real wage and the marginal product of labor are equal and are expected to remain so. Otherwise, there is a non-degenerate distribution of output prices across firms, and  $mpl_t - \zeta_t$  should be interpreted as the average across firms.

Table 1  
Key Equations

$g_t = g_{(t+1 t)} - \frac{1}{\sigma \ell_C} (i_t - \pi_{(t+1 t)} - r_t^*)$	(goods demand)	(T1.1)
$mpl_t = \zeta_t^* - \lambda_{mpl} g_t$ where $\lambda_{mpl} = \alpha / (1 - \alpha)$	(marginal product of labor)	(T1.2)
$mrs_t = \zeta_t^* + \lambda_{mrs} g_t$ where $\lambda_{mrs} = \frac{\chi^{\ell_N}}{1 - \alpha} + \sigma \ell_C$	(marginal rate of substitution)	(T1.3)
$\pi_t = \beta \pi_{t+1 t} + \kappa_p (\zeta_t - mpl_t)$ where $\kappa_p = (1 - \xi_p \beta) (1 - \xi_p) / \xi_p$	(price setting)	(T1.4)
$\omega_t = \beta \omega_{t+1 t} + \kappa_w (mrs_t - \zeta_t)$ where $\kappa_w = \frac{(1 - \xi_w \beta) (1 - \xi_w)}{\xi_w \left( 1 + \chi^{\ell_N} \left( \frac{1 + \theta_w}{\theta_w} \right) \right)}$	(wage setting)	(T1.5)
$\zeta_t = \zeta_{t-1} + \omega_t - \pi_t$	(real wage change)	(T1.6)

<sup>10</sup> See Yun (1996) and the papers cited in Footnote 2. A very similar price-setting equation is implied by the assumption of quadratic menu costs of adjusting nominal prices, as in Rotemberg (1996).

As shown in Appendix A, the wage-setting equation (T1.5) is derived using the household's first-order condition for setting its contract wage rate (16). This equation states that the amount by which current wage inflation exceeds its steady-state value ( $\Pi$ ) depends on the percentage by which households' average marginal rate of substitution,  $mrs_t$ , exceeds the real wage  $\zeta_t$ , taking expected wage inflation next period as given. Wage inflation is at its steady-state value only when the real wage and the marginal rate of substitution are equal and are expected to remain so. Otherwise, there is a non-degenerate distribution of wage rates and labor hours across households, and  $mrs_t - \zeta_t$  should be interpreted as the average across all households. The wage-setting equation, like the pricesetting equation, can be expressed equivalently in terms of the wage markup, namely, the percentage deviation of the real wage from the  $mrs_t$  of households.

The identity (T1.6) expresses the change in the real wage as the difference between wage inflation and price inflation. Finally, a monetary policy rule is required to close the model. Such a rule is not listed in Table 1, but in Section 5 we will consider feedback rules in which the short-term nominal interest rate  $i_t$  (expressed as a deviation from its steady-state value) responds linearly to one or more of the endogenous variables and exogenous disturbances. It is interesting to note that the model in Table 1 has some formal similarity to the earlier work on 'disequilibrium' models.<sup>11</sup> In particular, wages and prices are subject to nominal inertia and exhibit partial adjustment toward the Pareto-optimal equilibrium. However, the wage and price equations in Table 1 are derived from optimizing behavior, and thus depend on the underlying structure of preferences and technology as well as exogenously specified mean contract duration parameters.

### III. The Welfare Function

The monetary policymaker maximizes the unconditional expectation of the unweighted average of household utility functionals. This problem is equivalent to maximizing the unconditional expectation of the average of household period utility functions, referred to hereafter as the policymaker's period welfare function.<sup>12</sup>

$$(21) \quad \mathbb{W}_t = \mathbb{U}(C_t, Q_t) + \int_0^1 \mathbb{V}(N_t(h), Z_t) dh.$$

<sup>11</sup> Earlier work on disequilibrium models by Clower, Patinkin, Barro and Grossman, Benassy, Grandmont, Malinvaud, Negishi, and others is cited in *Cuddington et al.* (1984).

<sup>12</sup> This equivalence follows from  $\mathcal{E} \sum_{j=0}^{\infty} \beta^j \mathbb{W}_{t+j} = (1/(1-\beta)) \mathcal{E}(\mathbb{W}_t)$ .

In this expression, consumption is identical across households whereas labor hours may vary across households (due to complete contingent claims for consumption but not for leisure). In addition, this expression for  $\mathbb{W}_t$  reflects our assumption that the welfare losses related to fluctuations in real balances are sufficiently small to be ignored.<sup>13</sup> The Pareto-optimal period welfare function is given by  $\mathbb{W}_t^* = \mathbb{U}(C_t^*, Q_t) + \mathbb{V}(N_t^*, Z_t)$ .

### 1. Approximation of the Period Welfare Function

To analyze the deviation of the policymaker's period welfare from the Pareto optimum, we derive the second-order Taylor approximations of  $\mathbb{W}_t$  and  $\mathbb{W}_t^*$  around the steady-state period welfare level  $\bar{\mathbb{W}}$ , and take their difference.<sup>14</sup> As shown in Appendix B,

$$(22) \quad \begin{aligned} \mathbb{W}_t - \mathbb{W}_t^* = & -\frac{1}{2} \mathbb{U}_C(\bar{C}, \bar{Q}) \bar{C} (\lambda_{mrs} + \lambda_{mpl}) g_t^2 + \frac{1}{2} \mathbb{V}_{NN}(\bar{N}, \bar{Z}) \bar{N}^2 \text{var}_h n_t(h) \\ & + \frac{1}{2} \mathbb{V}_N(\bar{N}, \bar{Z}) \bar{N} \left( \frac{\theta_w}{1 + \theta_w} \text{var}_h n_t(h) + \frac{1}{1 - \alpha} \frac{\theta_p}{1 + \theta_p} \text{var}_f y_t(f) \right) \end{aligned}$$

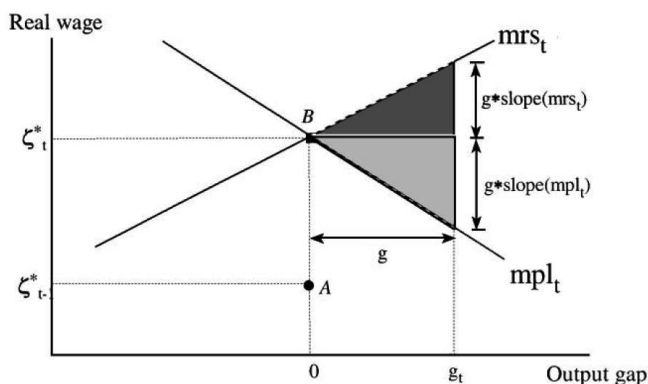


Figure 1: Welfare Costs of Output Gap Deviations

where  $n_t(h)$  indicates the percent deviation from steady state of the labor hours  $N_t(h)$  of household  $h$ , and  $\text{var}_h n_t(h)$  indicates the cross-sectional dispersion of  $n_t(h)$  around the cross-sectional average  $\mathcal{E}_h n_t(h)$ . Similarly,  $y_t(f)$  indicates the

<sup>13</sup> That is, we assume that the weight  $\mu_0$  in Eq. (13) is arbitrarily close to zero.

<sup>14</sup> The economy with staggered contracts has the same steady state as the Pareto-optimal economy, because both wage and price contracts are indexed to steady-state inflation.

percent deviation from steady state of the output  $Y_t(f)$  of firm  $f$ , and  $\text{var}_f y_t(f)$  indicates the cross-sectional dispersion of  $y_t(f)$ .<sup>15</sup> Note that the marginal disutility of labor is positive and increasing (i.e.,  $-\mathbb{V}_N(\bar{N}, \bar{Z}) > 0$  and  $-\mathbb{V}_{NN}(\bar{N}, \bar{Z}) > 0$ ), so that all three terms on the right-hand side of Eq. (22) are negative.

The first term captures the period welfare cost of variation in the output gap. For a given output gap  $g_t$ , Fig. 1 depicts this welfare cost (in percentage terms) as the shaded area between the upward-sloping  $\text{mrs}_t$  schedule (Eq. (T1.3)) and the downward-sloping  $\text{mpl}_t$  schedule (Eq. (T1.5)).<sup>16</sup> Multiplying the shaded area by  $\mathbb{U}_C(\bar{C}, \bar{Q})\bar{C}$  gives the period welfare cost in terms of utility.

The remaining two terms capture the period welfare costs of cross-sectional dispersion that arise because of staggered wage and price contracts. Even when the output gap is zero, staggered wage setting can lead to dispersion in hours worked across households, while staggered price setting can lead to dispersion in differentiated goods production across firms. Cross-sectional dispersion in hours imposes a welfare cost (captured by the second term) because households dislike variation in their labor supply, i.e., because households have increasing marginal disutility of labor ( $-\mathbb{V}_{NN}(\bar{N}, \bar{Z}) > 0$ ).

In addition, cross-sectional dispersion in employment and in production each impose inefficiencies (captured by the third term) by raising the aggregate labor hours,  $N_t = \int_0^1 N_t(h) dh$ , needed to produce a given level of the output index.

These inefficiencies would arise even if the marginal disutility of labor were constant ( $\mathbb{V}_{NN}(\bar{N}, \bar{Z}) = 0$ ).<sup>17</sup> Labor services of households are imperfect substitutes in production, and differentiated goods are imperfect substitutes in consumption; thus, the magnitudes of the inefficiencies increase with the degrees of concavity of the labor index and of the output index (as determined by the values of the wage markup rate  $\theta_w$  and the price markup rate  $\theta_p$ ). Since each household's labor hours enter symmetrically into the aggregate labor index, and every household has equal weight in the social welfare function, the Pareto-

<sup>15</sup> That is,  $\text{var}_h n_t(h) = \int_0^1 (n_t(h) - E_h n_t(h))^2 dh$  and  $\text{var}_f y_t(f) = \int_0^1 (y_t(f) - E_f y_t(f))^2 df$ , where  $E_h n_t(h) = \int_0^1 n_t(h) dh$  and  $E_f y_t(f) = \int_0^1 y_t(f) df$ . Note that  $\text{var}_h n_t(h) = \text{var}_h \ln N_t(h)$ .

<sup>16</sup> This graphical representation is used in Aizenman/Frenkel (1986) to show that the welfare cost of variations in the output gap can be represented by a loss in economic surplus.

<sup>17</sup> As a second-order approximation, Eq. (22) omits higher-order terms that involve the interaction between productive inefficiencies and aggregate output fluctuations, as well as higher-order terms associated with the response of employment dispersion to fluctuations in the labor hours of the average household.

optimal equilibrium has the property that the number of labor hours is identical across households. Under staggered wage setting, however, the economy no longer exhibits this optimality property. The inefficiency associated with cross-sectional dispersion of labor hours can be expressed in terms of the percentage increase in aggregate labor hours required to produce a given level of the labor index:

$$(23) \quad n_t = l_t + \frac{1}{2} \left( \frac{\theta_w}{1 + \theta_w} \right) \text{var}_h n_t(h)$$

where  $n_t$  is the percent deviation from steady state of average labor hours,  $N_t$ , and  $l_t$  is the percent deviation from steady state of the labor index,  $L_t$ .

Similarly, staggered price setting induces cross-sectional dispersion in production, and thereby increases the average level of differentiated goods output required to produce a given level of the output index. Since all firms face the same Cobb-Douglas production function (Eq. (4)), this inefficiency can be expressed in terms of the percentage increase in the labor index required to produce a given level of the output index:

$$(24) \quad l_t = \frac{1}{(1 - \alpha)} (y_t - x_t) + \frac{1}{2} \frac{1}{(1 - \alpha)} \left( \frac{\theta_p}{1 + \theta_p} \right) \text{var}_f y_t(f)$$

where  $y_t$  and  $x_t$  are the percent deviations from steady state of the output index  $Y_t$  and of total factor productivity  $X_t$ , respectively. The second term on the right-hand side of Eq. (24) represents the increased amount of labor hours required because of cross-sectional dispersion in production.

The final term in the welfare approximation (Eq. (22)) is obtained by multiplying the inefficiency terms in Eqs. (23) and (24) by the factor  $\mathbb{V}_N(\bar{N}, \bar{Z}) \bar{N}$ .

The policymaker's objective is to maximize the unconditional expectation of Eq. (22). The resulting equation can be expressed (to second order) in terms of the unconditional variances of the output gap, price inflation, and wage inflation. First, Appendix B demonstrates that  $\varepsilon g_t^2 = \text{var}(g_t)$  where  $\text{var}(gt)$  indicates unconditional variance. Next, the labor demand function (12) of each household directly implies that

$$(25) \quad \begin{aligned} \text{var}_h n_t(h) &= \left( \frac{1 + \theta_w}{\theta_w} \right)^2 \text{var}_n \ln W_t(h) \\ &\equiv \left( \frac{1 + \theta_w}{\theta_w} \right)^2 \varepsilon_h [\ln W_t(h) - \varepsilon_h \ln W_t(h)]^2. \end{aligned}$$

Thus, cross-sectional employment dispersion varies directly with wage dispersion, with the former tending toward infinity as labor services become closer to



perfect substitutes (i.e., as the wage markup rate  $\theta_w$  approaches zero). Moreover, Appendix B demonstrates that

$$(26) \quad \varepsilon[\text{var}_h \ln W_t(h)] = \frac{\xi_w}{(1 - \xi_w)^2} \text{var}(\omega_t).$$

Thus, cross-sectional wage dispersion associated with a given variance of wage inflation increases with the average duration of wage contracts. Combining Eqs. (25) and (26) yields an expression for  $\varepsilon[\text{var}_h \pi_t(h)]$  in terms of  $\text{var}(\omega_t)$ . The analogous relation

$$(26a) \quad \varepsilon[\text{var}_f y_t(f)] = \left( \frac{1 + \theta_p}{\theta_p} \right)^2 \left( \frac{\xi_p}{(1 - \xi_p)^2} \right) \text{var}(\pi_t)$$

is derived in *Rotemberg and Woodford* (1999). Thus, the expected deviation of social welfare from its Pareto-optimal level can be expressed as

$$(27) \quad \begin{aligned} \frac{\varepsilon[\mathbb{W} - \mathbb{W}^*]}{\mathbb{U}_C(\bar{C}, \bar{Q})\bar{C}} = & -\frac{1}{2}(\lambda_{mrs} + \lambda_{mpl}) \text{var}(g_t) \\ & - \frac{1}{2} \left( \frac{1 + \theta_p}{\theta_p} \right) \left( \frac{1 - \beta \xi_p}{1 - \xi_p} \right) \frac{1}{\kappa_p} \text{var}(\pi_t) \\ & - \frac{1}{2} \left( \frac{1 + \theta_w}{\theta_w} \right) \left( \frac{1 - \beta \xi_w}{1 - \xi_w} \right) \frac{1 - \alpha}{\kappa_w} \text{var}(\omega_t) \end{aligned}$$

where the price and wage adjustment coefficients  $\kappa_p$  and  $\kappa_w$  are defined in Table 1 above. The welfare deviation from the Pareto-optimal level is scaled by  $\mathbb{U}_C(\bar{C}, \bar{Q})\bar{C}$ , so that the right-hand side of Eq. (27) expresses these welfare losses as a fraction of Pareto-optimal consumption.

Several important qualitative features of the social welfare function are evident from Eq. (27). The welfare cost of price inflation volatility increases with the degree of substitutability across differentiated goods (which is inversely related to the price markup rate  $\theta_p$ ) and with the mean duration of price contracts (which varies positively with  $\xi_p$  and negatively with  $\kappa_p$ ).<sup>18</sup> Welfare is independent of the variance of price inflation only in the special case of completely flexible prices (i.e.,  $\xi_p = 0$  and hence  $\kappa_p = \infty$ ). Similarly, the welfare cost of wage inflation volatility increases with the degree of substitutability across differentiated labor inputs (which is inversely related to the wage markup rate  $\theta_w$ ) and

<sup>18</sup> It is important to note that our log-linear approximation becomes relatively inaccurate as the degree of substitutability of differentiated goods or labor services approaches infinity, that is, as either  $\theta_p$  or  $\theta_w$  approach zero.

with the mean duration of wage contracts (which varies positively with  $\xi_w$  and negatively with  $\kappa_w$ ). Finally, it should be noted that the welfare cost of output gap volatility does not depend on either  $\xi_p$  or  $\xi_w$ . Thus, the *relative* weight on output gap volatility declines with the mean duration of price contracts and the mean duration of wage contracts.

#### IV. The Policy Frontier

We have shown that the policymaker's welfare function can be expressed in terms of the variances of three aggregate variables: the output gap, price inflation, and wage inflation. We demonstrate the impossibility of completely stabilizing more than one of these three variables. This result depends only on the aggregate supply relations (T1.2)–(T1.5), and not on the particular specification of goods demand or the monetary policy rule. It follows that monetary policy cannot achieve the Pareto-optimal level of social welfare. Instead, the policymaker faces tradeoffs in stabilizing the three variables; these tradeoffs are summarized by the policy frontier. The Pareto optimum can be achieved only in the two special cases in which either prices or wages are completely flexible.

##### 1. The General Case

Our result for the general case of staggered wage and price setting is stated in the following proposition:

*Proposition 1. With staggered wage and price setting ( $\kappa_p = 0$  and  $\xi_p > 0$ ), there exists a tradeoff in stabilizing the output gap, the price inflation rate, and the wage inflation rate: it is impossible for more than one of the three variables  $g_t$ ,  $\pi_t$ , and  $\omega_t$  to have zero variance. Therefore, monetary policy cannot attain the Pareto-optimal social welfare level.*

*Proof.* From Eq. (T1.4), price inflation remains at steady state if and only if the real wage and the marginal product of labor are equal; i.e.,  $\pi_t = \pi_{t+1|t} = 0$  if and only if  $\zeta_t = mpl_t$ , where the  $mpl_t$  schedule is given by Eq. (T1.2). Similarly, from Eq. (T1.5), wage inflation remains at steady state if and only if the real wage and the marginal rate of substitution are equal; i.e.,  $\omega_t = \omega_{t+1|t} = 0$  if and only if  $\zeta_t = mrs_t$ , where the  $mrs_t$  schedule is given by Eq. (T1.3). The  $mpl_t$  and  $mrs_t$  schedules intersect at the Pareto optimum; i.e.,  $g_t = 0$  and  $\zeta_t = \zeta_t^*$  if and only if  $\zeta_t = mpl_t = mrs_t$ . Thus, any two of the three conditions ( $g_t = 0$ ,  $\zeta_t = mpl_t$ , and  $\zeta_t = mrs_t$ ) imply the third condition, so that the output gap would remain at zero and nominal wage inflation and price inflation would remain constant only if the aggregate real wage rate were continuously at its Pareto-optimal level. However, it is evident from Eq. (19) that the Pareto-optimal

real wage  $\zeta_t^*$  moves in response to each of the exogenous shocks, whereas the actual real wage  $\zeta_t$  would always remain at its steady-state value if neither prices nor wages ever adjusted. Given this contradiction, it follows that no more than one of the three variables  $g_t$ ,  $\pi_t$ , and  $\omega_t$  can have zero variance when the exogenous shocks have non-zero variance. Therefore, the Pareto-optimal social welfare level is infeasible, because the variances of all three variables enter the policymaker's welfare function with weights that are strictly negative.

The proof is illustrated in Fig. 1. Point A represents an initial Pareto-optimal equilibrium. Point B represents the Pareto-optimal equilibrium following a positive productivity shock.<sup>19</sup> Completely stabilizing any two of the three variables  $g_t$ ,  $\pi_t$ , and  $\omega_t$  is impossible, because doing so would imply that the economy was simultaneously at both points A and B.

## 2. Two Special Cases

As noted in the introduction, recent studies have characterized optimal monetary policy in optimizing-agent models in which staggered price setting is the sole form of nominal rigidity. The salient finding of these studies is that monetary policy faces no tradeoff between minimizing variability in the output gap and minimizing variability in price inflation, so the Pareto optimal welfare level can be attained. This finding is consistent with the implication of the special case of our model in which wages are completely flexible. We prove the following proposition:

*Proposition 2. (A) With staggered price contracts and completely flexible wages ( $\xi_p > 0$  and  $\xi_w = 0$ ), monetary policy can completely stabilize price inflation and the output gap, thereby attaining the Pareto-optimal social welfare level.*

*(B) With staggered wage contracts and completely flexible prices ( $\xi_w > 0$  and  $\xi_p = 0$ ), monetary policy can completely stabilize wage inflation and the output gap, thereby attaining the Pareto-optimal social welfare level.*

To prove Proposition 2(A), note that in this case, nominal wages can adjust freely to ensure that the real wage is equal to the marginal rate of substitution ( $\zeta_t = mrs_t$ ). Combining this condition with Eqs. (T1.2), (T1.3), and (T1.4) yields an expectational Phillips curve that looks reasonably familiar except for the absence of an error term:

$$(28) \quad \pi_t = \beta\pi_{t+1|t} + \left( \frac{\kappa_p \Lambda}{1 - \alpha} \right) g_t.$$

<sup>19</sup> According to Eqs. (T1.3) and (T1.5) any shock shifts both schedules vertically by the amount of the change in the equilibrium real wage. We consider a positive productivity shock as an example.

This equation implies that the output gap has zero variance if price inflation is completely stabilized (i. e.,  $g_t = 0$  if  $\pi_t = \pi_{t+1|t} = 0$ ). Solving Eq. (28) forward shows that stabilizing the output gap also stabilizes price inflation. Given that wages are completely flexible, the variance of wage inflation receives zero weight in the social welfare function (27). Thus, it is possible to attain the Pareto-optimal social welfare level by strictly targeting either price inflation (as recommended by Goodfriend and King (1997) and King and Wolman (1999)), or equivalently, the output gap.

Proposition 2(A) indicates that staggered price setting by itself does not imply a price inflation-output gap variance tradeoff. To obtain such a tradeoff, one approach taken in the literature has been to add an exogenous shock to Eq. (28).<sup>20</sup> Such a shock has been interpreted as representing aggregate pricing mistakes or other unexplained deviations from the optimality condition (28). In contrast, a price inflation-output gap variance tradeoff arises endogenously in our model with staggered wage and price setting. By substituting the  $mpl_t$  schedule (T1.2) into the price-setting equation (T1.4), we obtain the following relationship:

$$(29) \quad \pi_t = \beta \pi_{t+1|t} + \kappa_p \lambda_{mpl} g_t + \kappa_p (\zeta_t - \zeta_t^*).$$

A stabilization tradeoff arises because of the final term  $\kappa_p (\zeta_t - \zeta_t^*)$  in Eq. (29), rather than from an ad hoc shock. Furthermore, this tradeoff depends on the preference and technology parameters of the model as well as the exogenous disturbances  $x_t$ ,  $q_t$ , and  $z_t$ .

To prove Proposition 2(B), note that prices can adjust freely to ensure that the real wage is equal to the marginal product of labor ( $\zeta_t = mpl_t$ ). Combining this condition with Eqs. (T1.2), (T1.3), and (T1.5) yields the following relationship:

$$(30) \quad \omega_t = \beta \omega_{t+1|t} + \left( \frac{\kappa_w \Lambda}{1 - \alpha} \right) g_t.$$

Comparison of Eqs. (28) and (30) reveals the formal symmetry between the two special cases. Eq. (30) implies that complete wage inflation stabilization ( $\omega_t = \omega_{t+1|t} = 0$ ) stabilizes the output gap, while prices adjust freely to keep the real wage at its Pareto-optimal value. Given that the variance of price inflation does not enter the social welfare function (27), it is possible to attain the Pareto-optimal social welfare level.

Evidently, in this special case with staggered wage setting and completely flexible prices, complete output gap stabilization generates positive variance of price

<sup>20</sup> See Kiley (1998) and McCallum/Nelson (1999). In the resulting equation, if price inflation is completely stabilized, then the variance of the output gap is proportional to the variance of the exogenous shock.

inflation. Such a variance tradeoff has been derived previously in other models with sticky wages and flexible prices.<sup>21</sup> Nevertheless, as is evident from Proposition 2(B), this tradeoff does not necessarily have any consequences for social welfare.

## V. Optimal Monetary Policy

In this section we use numerical methods to characterize optimal monetary policy. In particular, for specified values of the structural parameters, we find the interest rate rule that maximizes the welfare function (27) subject to the loglinearized behavioral equations given in Table 1. For the sake of brevity and clarity, we focus exclusively on volatility induced by exogenous productivity shocks; consumption and leisure shocks imply qualitatively similar properties of the monetary policy frontier.<sup>22</sup>

### 1. Parameterization and Computation

Throughout this section, we use a discount factor  $\beta$  of 0.99 (corresponding to a quarterly periodicity of the model), and we use household utility parameters  $\sigma = \chi = 1.5$  (so that utility is nearly logarithmic in consumption and leisure). Unless otherwise specified, the Cobb-Douglas capital share parameter  $\alpha = 0.3$  (implying that output has a labor elasticity of 0.7); the wage and price markup rates  $\theta_w = \theta_p = \frac{1}{3}$ ; and the wage and price contract duration parameters  $\xi_w = \xi_p = 0.75$  (implying an average contract duration of  $1/(1-0.75) = 4$  quarters). We assume that the productivity shock  $x_t$  follows an AR(1) process with first-order autocorrelation of 0.95, where the innovation  $\varepsilon_{x,t}$  is i.i.d. with mean zero and variance  $\sigma_{\varepsilon_x}^2$ .

The full model consists of the equations in Table 1 and the optimal interest rate rule. Given our assumptions about the exogenous shocks, the optimal interest rate rule has the following general form:<sup>23</sup>

$$(31) \quad i_t = \gamma_0 \pi_t + \gamma_1 i_{t-1} + \gamma_2 \zeta_{t-1} + \gamma_3 x_{t-1} + \gamma_4 \varepsilon_{x,t} + \gamma_5 \zeta_{t-2} + \gamma_6 x_{t-2} + \gamma_7 \varepsilon_{x,t-1}.$$

<sup>21</sup> Examples include Levin (1989), Bryant *et al.* (1993), Henderson/McKibbin (1993), Blanchard (1997), and Friedman (1999).

<sup>22</sup> See Erceg *et al.* (1998). By taking this approach, we also avoid the need to calibrate a contemporaneous covariance matrix for the three disturbances.

<sup>23</sup> A detailed description of the solution algorithm and recent enhancements may be found in Anderson (1997). Using Matlab version 5.2 on a 400 Mhz Pentium II, this algorithm generates the rational expectations solution within a few seconds for every case considered here.

It is important to note that this rule includes a fixed parameter  $\gamma_0$  on current price inflation  $\pi_t$ ; this parameter must be large enough to ensure determinacy (i.e., the existence of a unique stationary rational expectations equilibrium). The particular value chosen for  $\gamma_0$  does not affect the reduced-form solution when the other parameters in Eq. (31) are chosen optimally.

To confirm that the determinacy conditions are satisfied and to compute the reduced-form solution of the model for a given set of parameters, we use the numerical algorithm of *Anderson/Moore* (1985), which provides an efficient implementation of the method proposed by *Blanchard/Kahn* (1980). Having obtained the reduced-form solution, it is straightforward to compute the variances of the output gap, price inflation, and wage inflation. These variances in turn are used to evaluate the welfare function (27). For a given set of structural parameters, we use a hill-climbing algorithm to determine the values of the monetary policy parameters that maximize social welfare.<sup>24</sup> Throughout the remainder of the paper, the variances of the endogenous variables and the corresponding welfare loss are all scaled by the productivity innovation variance.

## 2. Geometric Representation

Using the numerical methods described above, we can depict the policymaker's optimization problem geometrically, thereby highlighting the parallels with the standard social planner's problem. In particular, each level set of the Figure 2 welfare function (27) corresponds to an indifference locus and forms a plane in 3-dimensional  $\{\text{Var}(g_t), \text{Var}(\pi_t), \text{Var}(w_t)\}$  space. Since the weight on each variable is negative, indifference loci further from the origin are associated with lower social welfare. The behavioral equations in Table 1 determine the policy frontier, i.e., the boundaries of the policymaker's opportunity set in  $\{\text{Var}(g_t), \text{Var}(\pi_t), \text{Var}(w_t)\}$  space. The policy frontier has the property that the variance of any single variable cannot be reduced without increasing the variance of one or both of the other variables. The optimal monetary policy outcome occurs at the point of tangency between the policy frontier and one of the indifference loci.

Fig. 2 portrays 2-dimensional slices of the 3-dimensional variance space for the calibration described above. For example, the upper-left panel depicts the policy frontier and the relevant indifference locus in terms of the variances of the output gap and price inflation, holding the variance of wage inflation con-

<sup>24</sup> A detailed description of the solution algorithm and recent enhancements may be found in *Anderson* (1997). Using Matlab version 5.2 on a 400 Mhz Pentium II, this algorithm generates the rational expectations solution within a few seconds for every case considered here.

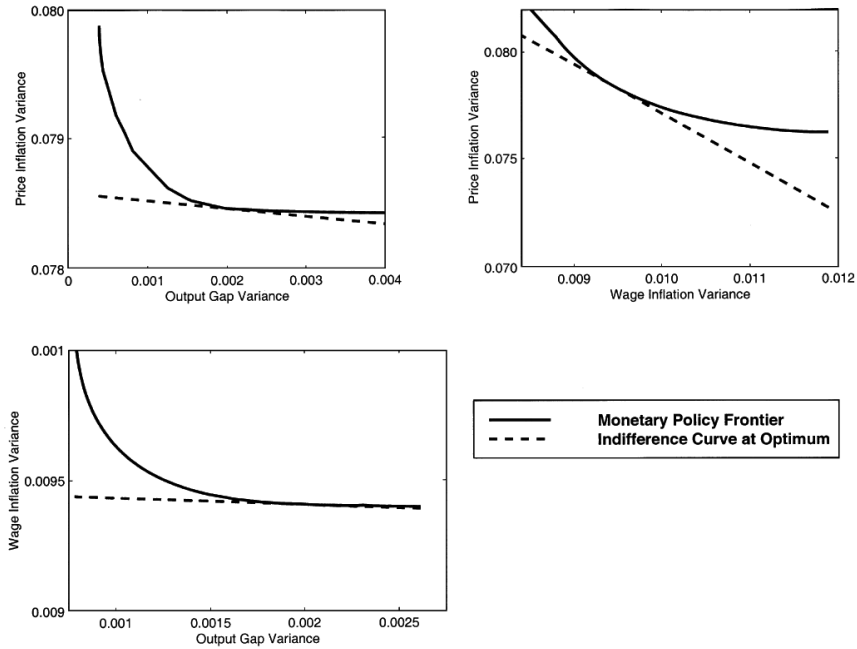


Figure 2: Three Views of the Monetary Policy Frontier

stant at its optimal value. Each slice of the policy frontier is downward sloping, and the origin does not lie on the policy frontier. Thus, as indicated by Proposition 1, there is a non-trivial tradeoff in stabilizing the corresponding pair of variables, and the Pareto-optimal welfare level is infeasible.

In contrast, in the special case of staggered price contracts and completely flexible wages (not shown), the policy frontier contains a point at which  $var(\pi_t) = var(g_t) = 0$ ; i.e., the policy frontier intersects the  $var(\omega_t)$  axis. In this special case,  $var(\omega_t)$  has zero weight in the welfare function (27), implying that the indifference loci are parallel to  $var(\omega_t)$  axis. Thus, the optimum is found where the policy frontier intersects the  $var(\omega_t)$  axis, and the optimal policy rule yields the Pareto-optimal welfare level.

### 3. Implications of Wage and Price Contract Duration

As we have seen, the policymaker faces a non-trivial stabilization problem when both wages and prices are determined by staggered contracts. Now we use numerical methods to consider a grid of values from 0 to 0.9 for the contract duration parameters  $\xi_w$  and  $\xi_p$ ; i.e., for each type of contract, the average dura-

tion ranges from one quarter (complete flexibility) up to ten quarters. For each combination of  $\xi_w$  and  $\xi_p$ , we determine the optimal monetary policy rule and the corresponding implications for aggregate volatility and social welfare.

In Fig. 3, panels A, B and C depict the optimal variances of price inflation, wage inflation, and the output gap, respectively, while panel D depicts the welfare loss incurred under the optimal policy relative to the Pareto optimum. Each contour line indicates the combinations of mean wage and price contract durations under which the optimal policy yields the specified variance or welfare loss. For example, the contour lines in panel A represent a circular staircase starting on the vertical axis and going upward in a clockwise direction, while the contour lines in panel B represent a circular staircase starting on the horizontal axis and going upward in a counterclockwise direction.

Fig. 3 illustrates the implications of Proposition 2, namely, that monetary policy can attain the Pareto optimum if either wages or prices are completely flexible. The vertical axis of each panel corresponds to the special case of staggered price contracts and completely flexible wages; i.e., wage contracts have mean duration of one quarter (implying that all households revise their wage contracts every period), and social welfare does not depend on the variance of wage inflation. Along this axis, the optimal policy rule attains the Pareto-optimal welfare

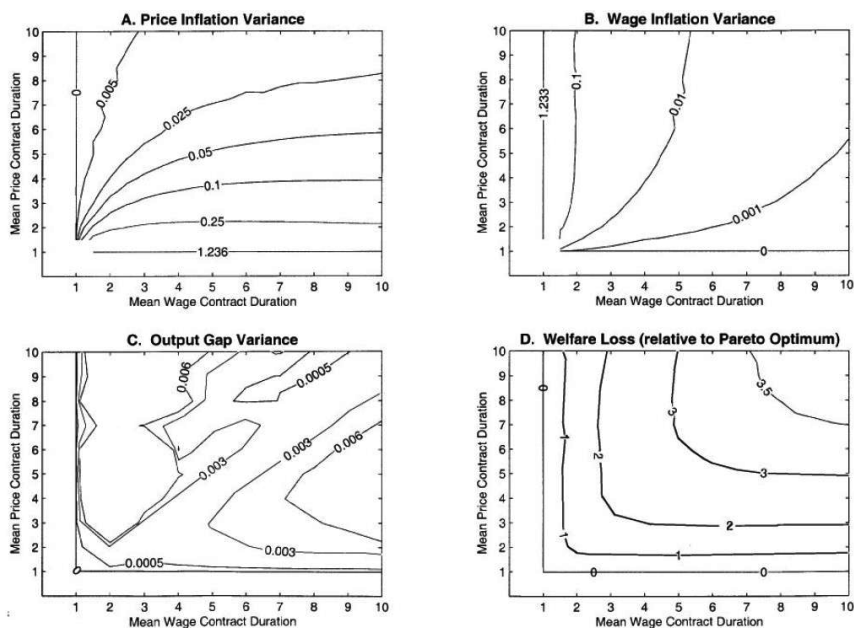


Figure 3: Optimal Monetary Policy for Alternative Contract Durations



level by completely stabilizing both price inflation and the output gap, while the variance of wage inflation reaches its maximum value. The horizontal axis corresponds to the symmetric special case of staggered wage contracts and completely flexible prices; in this case, the Pareto-optimal welfare level is attained by completely stabilizing wage inflation and the output gap, while the variance of price inflation reaches its maximum value.

More generally, Fig. 3 highlights an important feature of the optimal policy rule: movement in the more flexible nominal variable accounts for a relatively larger share of the optimal real wage adjustment. When price contracts have longer mean duration than wage contracts (the northwest quadrant of each panel), the optimal variance of price inflation is relatively low and the optimal variance of wage inflation is relatively high. In contrast, when price contracts have shorter mean duration than wage contracts (the southeast quadrant), the optimal policy rule is associated with relatively high price inflation variance and relatively low wage inflation variance.

Finally, the optimal variance of the output gap is quite low for every combination of wage and price contract durations, even though each of these calibrations implies a relatively low weight on the output gap variance in the social welfare function (27). This result suggests that the optimal policy might be well approximated by strict output gap targeting; we explicitly investigate this rule (and others) in the following section.

## VI. Alternative Monetary Policy Rules

Using the performance of the optimal monetary policy rule as a benchmark, we analyze the welfare costs of five alternative monetary policy rules. Each of the first three rules focuses exclusively on stabilizing a single variable: price inflation, wage inflation, or the output gap. Next, we consider a hybrid rule in which the table 2 interest rate responds to both price inflation and the output gap; this rule is similar in form to those considered recently by several analysts.<sup>25</sup> Finally, we consider a hybrid rule in which the interest rate responds to both price inflation and wage inflation. This rule is a natural one to consider in a model with two forms of nominal inertia, and has the advantage that the policymaker need not know the Pareto-optimal level of output.

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<sup>25</sup> Early analysis of such rules may be found in *Bryant et al. (1993)*, *Henderson/McKibbin (1993)*, and *Taylor (1993)*, whose name is frequently associated with such rules.

Table 2

Welfare Costs of Alternative Policy Rules<sup>a</sup>

	Optimal policy	Strict targeting			Hybrid targeting	
		Price inflation	Wage inflation	Output gap	Price inflation output gap	Price inflation wage inflation
Baseline parameter	2.4	18.6	3.5	2.4	2.4	2.4
Wage contract mean duration $(1 - \xi_w)^{-1}$						
1	0	0	3.5	0	0	0
2	1.6	6.5	3.5	0	0	0
4	2.4	18.6	3.5	1.6	1.6	1.6
8	2.7	36.8	3.5	2.7	2.7	2.7
Labor elasticity of output ( $\alpha$ )						
0	2.2	702.5	2.6	2.2	2.2	2.2
0.1	2.3	77.8	2.8	2.3	2.3	2.3
0.2	2.4	33.5	3.1	2.4	2.4	2.4
0.3	2.4	18.6	3.5	2.4	2.4	2.4
0.4	2.4	11.3	4.0	2.5	2.5	2.4
0.5	2.3	7.22	4.7	2.5	2.5	2.3
Price markup rate ( $\theta_p$ )						
0.05	7.3	18.6	16.9	10.9	8.3	7.6
0.10	5.0	18.6	9.0	5.9	5.6	5.1
0.25	2.9	18.6	4.3	2.9	2.9	2.9
0.50	1.9	18.6	2.7	1.9	1.9	1.9

<sup>a</sup> Each welfare loss is expressed as a fraction of Pareto-optimal consumption, divided by the productivity innovation variance.

The top row of Table 2 indicates the welfare losses of these rules under the baseline calibration given in Section 5.1. The results of some sensitivity analysis are reported in the remainder of the table. In particular, we evaluate the performance of each rule for a range of values of three structural parameters:  $\alpha$ ,  $\xi_w$ , and  $\theta_p$ . Each parameter is varied in turn, while keeping all other structural parameters at their baseline values. For a given set of structural parameters,

a hill-climbing algorithm is used to determine the coefficients of each hybrid rule that maximize the social welfare function (27).

The welfare costs of strict price inflation targeting are high under the baseline calibration, and increase further when the mean wage contract duration is very long (i.e., when  $\xi_w$  is large) or when the  $mpl_t$  schedule is nearly flat (i.e., when  $\alpha$  is small). When the  $mpl_t$  schedule is relatively flat, a given shock to total factor productivity has a larger impact on the equilibrium real wage, and thereby requires a larger adjustment of nominal wage rates (given that prices remain constant). When mean wage contract duration is long, only a small fraction of households adjust their wage contracts in any given period, so that a given movement in the aggregate nominal wage rate is associated with a relatively high level of cross-sectional employment dispersion.

Strict wage inflation targeting performs much better than strict price inflation targeting for every combination of structural parameters considered in Table 2. The performance of strict wage inflation targeting would deteriorate if the model were modified to eliminate the wealth effect on labor supply and to incorporate firm-specific costs of adjusting capital or labor (thereby flattening the  $mrs_t$  schedule and increasing the cost of price inflation volatility, respectively).

Strict output gap targeting does nearly as well as the optimal rule regardless of the relative duration of wage and price contracts or of the value of  $\alpha$  in the range considered. This policy is generally consistent with a key feature of the optimal rule: both nominal wages and prices adjust in response to the real wage deviation from its equilibrium value, with the more flexible variable automatically accounting for a larger share of the adjustment process. However, this policy generates noticeable welfare costs when the price markup rate  $\theta_p$  is very small: in this case, the social welfare function (27) assigns very high weight to the variance of price inflation, while complete output gap stabilization induces slightly more price inflation volatility than the optimal policy rule.

Finally, both constrained-optimal hybrid rules perform nearly as well as the optimal rule in all cases.

## VII. Conclusions

When both wages and prices are determined by staggered nominal contracts, monetary policy cannot achieve the Pareto-optimal welfare level, and the optimal policy rule depends on the underlying structure and parameter values of the model. The Pareto optimum is only feasible if either wages or prices are completely flexible. Thus, while considerations of parsimony alone might suggest an exclusive focus on either staggered price setting or staggered wage setting, the inclusion of both types of nominal inertia makes a critical difference in the monetary policy problem.

More generally, our analysis suggests that the existence of a monetary policy tradeoff is not contingent on the particular specification of staggered wage and price setting considered here. For example, our model is isomorphic to a model with differentiated goods at two stages of production, in which the prices of both intermediate inputs and final goods are determined by staggered nominal contracts. We conjecture that a monetary policy tradeoff would also exist under alternative formulations, such as (1) one-period input price contracts and staggered output price contracts, and (2) flexible input prices and staggered output price contracts, where differentiated goods producers face idiosyncratic productivity shocks or shifts in relative demand. In each of these cases, the prices of several inputs and/or outputs are determined by nominal contracts that are not completely synchronized, and some of the relative prices of these items vary in response to exogenous shocks in the Pareto-optimal equilibrium.

Although it is worthwhile to consider alternative formulations of nominal inertia, we believe that both wages and prices are sticky in actual economies, and that the relative price of labor plays an important role in generating a non-trivial policy tradeoff. Our position is consistent with the long history of analyses (dating back at least to *Keynes* (1935)) in which nominal wage inertia plays a significant role in generating aggregate fluctuations. In contrast, recent contributions have emphasized sticky prices rather than sticky wages, at least in part because state-contingent employment contracts can, in principle, prevent any misallocation of labor due to nominal wage contracts.<sup>26</sup> However, one can also imagine state-contingent output contracts which ensure that sticky prices have no allocative effects; such state-contingent contracts are neither more nor less plausible than the analogous employment contracts. Hence, it seems reasonable to assume that both wage and price contracts have significant allocative effects, at least until further guidance is provided by empirical research.

We have used numerical methods to analyze the properties of optimal monetary policy and to quantify the welfare losses of alternative policy rules. For the specifications considered here, we find that strict price inflation targeting generates relatively large welfare losses, whereas several other simple policy rules perform nearly as well as the optimal rule. These findings should be investigated further in models that relax some of our key simplifying assumptions such as complete consumption risk sharing and complete information of private agents and policymakers.<sup>27</sup>

<sup>26</sup> *Barro* (1977) is sometimes cited to support this view. However, Barro himself applies this argument to prices as well as wages.

<sup>27</sup> Other key simplifying assumptions include time-separable preferences, no capital accumulation, no adjustment costs, and exogenous duration of wage and price contracts. The sensitivity of contract duration to monetary policy has been studied previously by *Canzoneri* (1980), *Gray* (1978), and *Dotsey et al.* (1997), among others.

## Appendix A

In this appendix, we derive the aggregate wage-setting equation (T1.5) using first-order Taylor approximations where appropriate. For every household  $\tilde{h}_t$  that resets its contract wage in period  $t$ , define  $d_t(\tilde{h}_t) = \ln W_t(\tilde{h}_t) - \ln W_t$ . Then the log differential of the first-order condition (16) around the steady state is

$$(A.1) \quad \frac{d_t(\tilde{h}_t)}{1 - \xi_w \beta} + \varepsilon_t \sum_{j=0}^{\infty} \xi_w^j \beta^j (\zeta_{t+j} + \hat{U}_{C,t+j} + \hat{V}_{N,t+j}(\tilde{h}_t) - \sum_{k=1}^j \omega_{t+k}) = 0.$$

Using the labor demand Eq. (12), we can rewrite  $\hat{V}_{N,t+j}(\tilde{h}_t)$  as

$$(A.2) \quad \hat{V}_{N,t+j}(\tilde{h}_t) = \hat{V}_{N,t+j} + \chi^\ell_N \left( \frac{1 + \theta_w}{\theta_w} \right) \left( d_t(\tilde{h}_t) - \sum_{k=1}^j \omega_{t+k} \right)$$

where  $-\hat{V}_{N,t+j} = (\ell_N l_t + \ell_Z z_t)$  is the average of marginal disutilities of labor across households. Eq. (18) implies that  $mrs_t = -(\hat{U}_{C,t} + \hat{V}_{N,t})$ , while the aggregate wage definition (11) implies that  $d_t(\tilde{h}_t) = (\xi_w / (1 - \xi_w)) \omega_t$ . Thus, substituting (A.2) into (A.1) yields

$$(A.3) \quad \frac{\xi_w}{(1 - \xi_w)(1 - \xi_w \beta)} \omega_t = \varepsilon_t \sum_{j=1}^{\infty} \xi_w^j \beta^j \sum_{k=1}^j \omega_{t+k} + \varepsilon_t \sum_{j=0}^{\infty} \xi_w^j \beta^j \left[ \frac{mrs_{t+j} - \zeta_{t+j}}{1 + \left( \frac{1 + \theta_w}{\theta_w} \right) \chi^\ell_N} \right].$$

Forwarding (A.3) by one period, multiplying the result by  $\xi_w \beta$ , subtracting the outcome from (A.3), and rearranging yields (T1.5).

## Appendix B

In this appendix, we derive the approximation of  $W_t - W_t^*$  given in Eq. (22). We also show that  $\varepsilon g_t^2 = \text{var}(g_t)$  and that  $\varepsilon[\text{var}_{h_t} \{\ln W_t(h)\}] = (\xi_w / (1 - \xi_w)^2) \text{var}(\omega_t)$  (see equation (26)). Throughout this appendix, we use second-order Taylor approximations.

We use two approximations repeatedly. If  $A$  is a generic variable, the relationship between its arithmetic and logarithmic percentage changes is

$$(B.1) \quad \frac{A - \bar{A}}{\bar{A}} = \frac{dA}{A} \approx a + \frac{1}{2} a^2, \quad a \equiv \ln A - \ln \bar{A}$$

If  $A = \left[ \int_0^1 A(j)^\phi dj \right]^{1/\phi}$ , the logarithmic approximation of  $A$  is

$$(B.2) \quad a \approx \varepsilon_j a(j) + \frac{1}{2} \phi \left( \varepsilon_j a(j)^2 - (\varepsilon_j a(j))^2 \right) = \varepsilon_j a(j) + \frac{1}{2} \phi \text{var}_j a(j).$$

### B.1. The Derivation of the Approximation of $\mathbb{W} - \mathbb{W}^*$

Eq. (21) without time subscripts is repeated here for convenience:

$$(B.3) \quad \mathbb{W} = \mathbb{U}(C, Q) + \int_0^1 \mathbb{V}(N(h), Z) dh = \mathbb{U}(C, Q) + \varepsilon_h \mathbb{V}(N(h), Z).$$

First, we approximate  $\mathbb{U}(C, Q)$ :

$$(B.4) \quad \begin{aligned} \mathbb{U}(C, Q) \approx & \bar{\mathbb{U}} + \mathbb{U}_C \bar{C} \frac{dC}{C} + \mathbb{U}_Q \bar{Q} \frac{dQ}{Q} \\ & + \frac{1}{2} \left( \mathbb{U}_{CC} \bar{C}^2 \left( \frac{dC}{C} \right)^2 + 2 \mathbb{U}_{CQ} \bar{C} \bar{Q} \frac{dC}{C} \frac{dQ}{Q} + \mathbb{U}_{QQ} \bar{Q}^2 \left( \frac{dQ}{Q} \right)^2 \right) \end{aligned}$$

Making use of result (B.1) yields

$$(B.5) \quad \begin{aligned} \mathbb{U}(C, Q) \approx & \bar{\mathbb{U}} + \mathbb{U}_C \bar{C} \left( y + \frac{1}{2} y^2 \right) + \mathbb{U}_Q \bar{Q} \left( q + \frac{1}{2} q^2 \right) \\ & + \frac{1}{2} \left( \mathbb{U}_{CC} \bar{C}^2 y^2 + 2 \mathbb{U}_{CQ} \bar{C} \bar{Q} y q + \mathbb{U}_{QQ} \bar{Q}^2 q^2 \right). \end{aligned}$$

Next we appropriate  $\varepsilon_h \mathbb{V}(N(h), Z)$

$$(B.6) \quad \begin{aligned} \varepsilon_h \mathbb{V}(N(h), Z) \approx & \bar{\mathbb{V}} + \varepsilon_h \mathbb{V}_N \bar{N} \frac{dN(h)}{\bar{N}} + \mathbb{V}_Z \bar{Z} \frac{dZ}{\bar{Z}} \\ & + \frac{1}{2} \left( \varepsilon_h \mathbb{V}_{NN} \bar{N}^2 \left( \frac{dN(h)}{\bar{N}} \right)^2 + \varepsilon_h \mathbb{V}_{NZ} \bar{N} \bar{Z} \frac{dN(h)}{\bar{N}} \frac{dZ}{\bar{Z}} \right. \\ & \left. + \mathbb{V}_{ZZ} \bar{Z}^2 \left( \frac{dZ}{\bar{Z}} \right)^2 \right) \end{aligned}$$

Making use of the result (B.1) yields

$$(B.7) \quad \begin{aligned} \varepsilon_h \mathbb{V}(N(h), Z) \approx & \bar{\mathbb{V}} + \mathbb{V}_N \bar{N} \left( \varepsilon_h n(h) + \frac{1}{2} \varepsilon_h n(h)^2 \right) + \mathbb{V}_Z \bar{Z} \left( z + \frac{1}{2} z^2 \right) \\ & + \frac{1}{2} \left( \mathbb{V}_{NN} \bar{N}^2 \varepsilon_h n(h)^2 + 2 \mathbb{V}_{NZ} \bar{N} \bar{Z} z \varepsilon_h n(h) + \mathbb{V}_{ZZ} \bar{Z}^2 z^2 \right). \end{aligned}$$

The aggregate supply of labor by households is  $L = \left[ \int_0^1 N(h)^{1/(1+\theta_w)} dh \right]^{1+\theta_w}$ . Thus,

$$(B.8) \quad \begin{aligned} l = \ln \left[ \int_0^1 N(h)^{1/(1+\theta_w)} dh \right]^{1+\theta_w} - \ln \bar{L} \approx & \varepsilon_h n(h) \\ & + \frac{1}{2} \left( \frac{1}{1+\theta_w} \right) \text{var}_h n(h). \end{aligned}$$

The aggregate demand for labor by firms is  $L = \int_0^1 L(f) df = \varepsilon_f L(f)$ . Thus,

$$(B.9) \quad l = \ln \varepsilon_f L(f) - \ln \bar{L} \approx \varepsilon_f l(f) + \frac{1}{2} \text{var}_f l(f).$$

All firms choose identical capital labor ratios ( $K(f)/L(f)$ ) equal to the aggregate ratio ( $K/L$ ) because they face the same factor prices, so

$$(B.10) \quad Y(f) = \left( \frac{K(f)}{L(f)} \right)^\alpha L(f) X = \left( \frac{K}{L} \right)^\alpha L(f) X.$$

Since the total amount of capital is fixed, Eq. (B.10) in turn implies

$$(B.11) \quad y(f) = x - \alpha l + l(f), \varepsilon_f y(f) = x - \alpha l + \varepsilon_f l(f),$$

$$\text{var}_f y(f) = \text{var}_f l(f).$$

Substituting the relationships in Eq. (B.11) into Eq. (B.9), and eliminating  $\varepsilon_f y(f)$  using Eq. (B.2) yields Eq. (24), repeated here for convenience:

$$(B.12) \quad l \approx \frac{1}{1-\alpha} (y-x) + \frac{1}{2} \left( \frac{1}{1-\alpha} \right) \left( \frac{\theta_p}{1+\theta_p} \right) \text{var}_f y(f).$$

Solving Eq. (B.8) for  $\varepsilon_h n(h)$  and eliminating  $l$  using Eq. (B.12) yields

$$(B.13) \quad \varepsilon_h n(h) \approx \frac{1}{1-\alpha} (y-x) + \frac{1}{2} \left( \frac{1}{1-\alpha} \right) \left( \frac{\theta_p}{1+\theta_p} \right) \text{var}_f y(f)$$

$$- \frac{1}{2} \left( \frac{1}{1+\theta_w} \right) \text{var}_h n(h).$$

Using the relationship  $\varepsilon_h n(h)^2 = \text{var}_h n(h) + [\varepsilon_h n(h)]^2$  to eliminate  $\varepsilon_h n(h)^2$ , and Eq. (B.13) to eliminate  $\varepsilon_h n(h)$ , Eq. (B.7) can be rewritten as

$$(B.14) \quad \varepsilon_h \nabla(N(h), Z) dh \approx \bar{\nabla} + \nabla_N \bar{N} \left( \frac{y-x}{1-\alpha} \right)$$

$$+ \frac{1}{2} [(\nabla_Z \bar{Z} + \nabla_{ZZ} \bar{Z}^2)] z^2 + 2 \nabla_{NZ} \bar{N} \bar{Z} \left( \frac{(y-x)z}{1-\alpha} \right)$$

$$+ (\nabla_N \bar{N} + 2 \nabla_{NN} \bar{N}^2) \left( \frac{y-x}{1-\alpha} \right)^2$$

$$+ \left( \nabla_N \bar{N} \left( \frac{\theta_w}{1+\theta_w} \right) + \nabla_{NN} \bar{N}^2 \right) \text{var}_h n(h)$$

$$+ \left( \frac{\nabla_N \bar{N}}{1-\alpha} \right) \left( \frac{\theta_p}{1+\theta_p} \right) \text{var}_f y(f)].$$

Approximating the utility associated with consumption and labor at the Pareto optimum,  $\mathbb{U}(C^*, Q)$  and  $\varepsilon_h \mathbb{V}(N^*(h), Z)$ , respectively, in analogous ways and subtracting the sum of the results from the sum of Eqs. (B.5) and (B.14) yields

$$\begin{aligned} \mathbb{W} - \mathbb{W}^* = & \left( \frac{-(\mathbb{V}_N \bar{N} + \mathbb{V}_{NN} \bar{N}^2)}{(1-\alpha)^2} x + \mathbb{U}_{CQ} \bar{C} \bar{Q} q + \frac{\mathbb{V}_{NZ} \bar{N} \bar{Z}}{1-\alpha} z \right) (y - y^*) \\ & + \frac{1}{2} \left( \mathbb{U}_C \bar{C} + \mathbb{U}_{CC} \bar{C}^2 + \frac{\mathbb{V}_N \bar{N} + \mathbb{V}_{NN} \bar{N}^2}{(1-\alpha)^2} \right) (y^2 - y^{*2}) \\ & + \frac{1}{2} \left( \mathbb{V}_N \bar{N} \left( \frac{\theta_w}{1+\theta_w} \right) + \mathbb{V}_{NN} \bar{N}^2 \right) \text{var}_h n(h) \\ & + \frac{1}{21-\alpha} \frac{\mathbb{V}_N \bar{N}}{(1+\theta_p)} \text{var}_f y(f) \end{aligned} \quad (\text{B.15})$$

since the first-order condition (17) implies that  $\mathbb{U}_C \bar{C} + \frac{\mathbb{V}_N \bar{N}}{1-\alpha} = 0$ . Our model implies that  $\mathbb{U}_C = (C - Q)^{-\sigma}$ ,  $\mathbb{U}_{CC} = -\mathbb{U}_{CQ} = -\sigma(C - Q)^{-\sigma-1}$ ,  $\mathbb{V}_N = -(1 - N - Z)^{-\chi}$ , and  $\mathbb{V}_{NN} = V_{NZ} = -\chi(1 - N - Z)^{-\chi-1}$ . Thus,  $\mathbb{U}_{CC} / \mathbb{U}_C = -\mathbb{U}_{CQ} / \mathbb{U}_C = -\sigma / (C - Q)$  and  $\mathbb{V}_{NN} = V_{NZ} / \mathbb{V}_N = \chi / (1 - N - Z)$ . Using these relationships together with the solution for Pareto-optimal output (19), the first and second lines can be expressed as  $(\mathbb{U}_C \bar{C} / (1-\alpha)) y^* (y - y^*)$  and  $(\mathbb{U}_C \bar{C} / 2(1-\alpha)) (y^2 - y^{*2})$ , respectively. Combining terms we arrive at Eq. (22).

### B.2. Proof that $\varepsilon g_t^2 = \text{var}(g_t)$

Now we show that  $\varepsilon g_t$  is of second order, so that  $(\varepsilon g_t)^2$  can be neglected in the second-order approximation. We assume that the model has a unique stationary solution, so that the deviation of aggregate output  $Y_t$  from the Pareto optimum  $Y_t^*$  can be expressed as  $Y_t - Y_t^* = Y(\eta_t, \eta_{t-1}, \eta_{t-2}, \dots)$ , where  $\eta_t$  is the vector of mean-zero i.i.d. innovations at time  $t$ . Because the economy with staggered contracts has the same steady state as the Pareto-optimal economy,  $Y(0, 0, 0, \dots) = 0$ . Thus, taking unconditional expectations of the second-order Taylor approximation yields

$$\mathbb{E} \left( \frac{Y_t - Y_t^*}{Y} \right) = \frac{1}{2Y} \sum_{j=0}^{\infty} Y_{\eta_{t-j}, \eta_{t-j}} \text{var}(\eta_t). \quad (\text{B.16})$$

Applying expression (B.1) to  $Y_t - Y_t^*$ , taking unconditional expectations, and rearranging terms yields

$$\varepsilon g_t = \varepsilon \left( \frac{Y_t - Y_t^*}{Y} \right) - \frac{1}{2} [\text{var}(g_t) + (\varepsilon g_t)^2]. \quad (\text{B.17})$$

Taken together, Eqs. (B.16) and (B.17) imply that  $\varepsilon g_t$  is of second order.



### B.3. The Approximation of $\varepsilon \text{var}_h \{\ln W_t(h)\}$

As in Appendix A, let  $\tilde{W}_t(\tilde{h}_t)$  indicate the wage of every household that resets its contract wage in period  $t$ , and let  $d_t(\tilde{h}_t) = \ln W_t$ . Note that  $\ln W_t(h) = \ln W_{t-1}(h) + \ln \Pi$  for each of the remaining  $\xi_w$  households that cannot reset their wages. Thus, cross-sectional wage dispersion is

$$(B.18) \quad \text{var}_h \ln W_t(h) = \xi_w \varepsilon_h (\ln W_{t-1}(h) + \ln \Pi - \varepsilon_h \ln W_t(h))^2 + (1 - \xi_w) (\ln W_t(\tilde{h}_t) - \varepsilon_h \ln W_t(h))^2.$$

For those households that cannot reset their wages, the wage dispersion around the current aggregate wage is

$$(B.19) \quad \varepsilon_h (\ln W_{t-1}(h) + \ln \Pi - \varepsilon_h \ln W_t(h))^2 = \text{var}_h \ln W_{t-1}(h) + \omega_t^2.$$

because  $\omega_t = \ln W_t - \ln W_{t-1} - \ln \Pi$  and because  $(\varepsilon_h \ln W_t(h) - \ln W_t)$  and  $(\varepsilon_h \ln W_t(h) - \ln W_t)$  are of second order from (B.2) so their squares and cross products can be ignored in the second-order approximation.

Similarly,  $(\ln W_t(\tilde{h}_t) - \varepsilon_h \ln W_t(h))^2 = (d_t(\tilde{h}_t))^2$ . Thus, the squared logarithmic deviation of the contract wage from the current aggregate wage is

$$(B.20) \quad (\ln W_t(\tilde{h}_t) - \varepsilon_h \ln W_t(h))^2 = \left( \frac{\xi_w}{1 - \xi_w} \right)^2 \omega_t^2.$$

Substituting Eqs. (B.19) and (B.20) into Eq. (B.18) and rearranging terms, we obtain

$$(B.21) \quad \text{var}_h \ln W_t(h) = \xi_w \text{var}_h \ln W_{t-1}(h) + \frac{\xi_w}{1 - \xi_w} \omega_t^2.$$

Finally, taking unconditional expectations and rearranging terms yields

$$(B.22) \quad \varepsilon \text{var}_h \ln W_t(h) = \frac{\xi_w}{(1 - \xi_w)^2} \varepsilon \omega_t^2 = \frac{\xi_w}{(1 - \xi_w)^2} \text{var}(\omega_t).$$

Note that  $\varepsilon \omega_t^2 = \text{var}(\omega_t)$  because  $\varepsilon \omega_t$  (like  $\varepsilon g_t$ ) is of second order.

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