

Sannino, Francesco

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# The Equilibrium Size and Value-Added of Venture Capital

FRANCESCO SANNINO\*

## ABSTRACT

I model positive sorting of entrepreneurs across the high and low value-added segments of the venture capital market. Aiming to attract high-quality entrepreneurs, inefficiently many venture capitalists (VCs) commit to provide high value-added by forming small portfolios. This draws the marginal entrepreneur away from the low value-added segment, reducing match quality in the high value-added segment too. There is underinvestment. Multiple equilibria may emerge, and they differ in aggregate investment. The model rationalizes evidence on VC returns and value-added along fundraising “waves” and when the cost of entrepreneurship falls, and generates untested predictions on the size and value-added of venture capital.

VENTURE CAPITALISTS (VCs) ADD VALUE TO the companies they finance (Hellmann and Puri (2002), Gompers et al. (2020)).<sup>1</sup> How much they can do so is determined largely by the resources, such as human capital, that they choose to employ and devote to each firm in their portfolio (Ewens and Rhodes-Kropf (2015)). As these resources are scarce, there is a trade-off between the number of firms in a VC’s portfolio and the value-added each firm will receive. In light of VCs’ championed role in boosting growth, and the evidence that

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Correspondence: Francesco Sannino, Department of Finance, Adickesallee 32-34, Frankfurt am Main, 60322, Germany. e-mail: [f.sannino@fs.de](mailto:f.sannino@fs.de).

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<sup>1</sup> For example, they help select top management, connect firms to investors and customers, and experiment with innovative business strategies. See Da Rin, Hellmann, and Puri (2013) for a survey on venture capital.

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venture capital is often undersupplied (Samila and Sorenson (2011)), it is only natural to ask what determines VCs' portfolio size, and whether VCs' value-adding capacity is employed in its full potential.

Theories have been proposed to explain why VCs choose to limit the scale at which they operate (Kaplan and Schoar (2005), Metrick and Yasuda (2010)). The main contribution of this paper is to move beyond the individual VC's problem to study portfolio size and value-added as market equilibrium phenomena. It is shown that, in equilibrium, VCs commit excessive resources to their portfolio firms, therefore too few firms receive financing, compared to a second-best benchmark. The result suggests that the limited supply of venture capital can be the manifestation of an inefficiency (Lerner and Nanda (2020)). Moreover, the model supports the coexistence of equilibria that differ in the extent to which VCs choose to spread their resources across multiple firms, offering an interpretation of the variation observed between industries in VCs' involvement in the firms they back (Gompers et al. (2020)).

The analysis builds on the fundamental observation that, as much as entrepreneurs want to match with those VCs who can add the most value, VCs want to attract the best entrepreneurs (Sørensen (2007), Korteweg and Sørensen (2017), Nanda, Samila, and Sorenson (2020)).<sup>2</sup> The model endogenizes positive sorting of entrepreneurs across the high and low value-added segments of the venture capital market. Self-selection of high-quality entrepreneurs into the high value-added segment motivates VCs to operate at small scale. VCs, however, fail to internalize the adverse effect that a large supply of high value-added funds has on equilibrium sorting: It discourages entrepreneurs of mediocre quality from seeking financing in the low value-added segment of the market, where they would improve average match quality. Moreover, adverse selection in the low value-added segment may worsen when more VCs choose to provide high value-added, reinforcing their motive to do so, thus representing a source of equilibrium multiplicity.

The model offers untested predictions on the determinants of VCs' value-adding input and portfolio size, and provides market-equilibrium explanations for patterns in the industry that have been attributed to changes in individual VCs' risk appetite and investment strategy.

Finally, despite widespread skepticism toward subsidies that encourage private VC activity (Lerner (2009)), this paper offers one reason why subsidizing entry of less sophisticated VCs can be beneficial.<sup>3</sup>

In the model, there are VCs and entrepreneurs. To capture scarcity in VCs' input, I assume that the *attention* they devote to each investment decreases as the number of projects they finance increases. In the baseline model, attention is either high or low. Each entrepreneur owns one project—differing in *quality*,

<sup>2</sup> Interestingly, Nanda, Samila, and Sorenson (2020) quote Chris Dixon, partner at Andreessen Horowitz: “Success in VC is probably 10% about picking, and 90% about sourcing the right deals and having entrepreneurs choose your firm as a partner.”

<sup>3</sup> One of the arguments in Lerner (2009) is that programs such as the Canadian Labor Fund Program disproportionately favor less skilled VCs.

which lies in a continuum—and needs the input of a VC to become profitable. A project's return is a function of project quality and VC attention. Importantly, the two inputs are complements.

First, VCs choose a size—and therefore a fund capacity and level of attention—to which they commit. VCs' size is observable by entrepreneurs, who also privately observe their projects' quality. Entrepreneurs then search for a VC: Each entrepreneur directs their search at a submarket, defined by the VC's attention. As a reflection of asymmetric information, when entrepreneurs are in excess of VCs in a given submarket, they are matched at random to VCs up to total capacity. Returns are then produced and shared between the VC and the entrepreneur.

In equilibrium, high-attention VCs must be in greater demand, and thus by searching for them, entrepreneurs face a higher probability of remaining unmatched. Due to complementarities in the returns function, better entrepreneurs are more willing to take this risk. Positive sorting follows: Only entrepreneurs above a quality threshold search for high-attention VCs.

Compared to a benchmark (second-best) in which VCs' attention is chosen to maximize aggregate returns subject to entrepreneurs' self-selection, too many VCs choose to provide high attention. The reason is as follows. When the supply of funds in the high-attention market increases (relatively to the supply of funds in the alternative market), the quality of the marginal, indifferent entrepreneur decreases in equilibrium. Given the matching protocol, when the marginal entrepreneur is of lower quality, the average quality of all inframarginal entrepreneurs in the VCs' portfolios is lower.<sup>4</sup> A second-best allocation internalizes the negative externality that high-attention VCs impose through sorting: In the second-best solution, low-attention VCs are in larger measure than in equilibrium to ensure the marginal entrepreneur is of higher quality and all VCs finance higher quality entrepreneurs.

The gains from attracting high-quality entrepreneurs with high attention depend, in equilibrium, on the quality of the marginal entrepreneur, which depends in turn on the relative supply of funds in the two submarkets: Equilibrium multiplicity may emerge. When not unique, equilibria are Pareto-ranked, and the dominated equilibria feature higher VC attention, and thus fewer firms being financed.

To endogenize the supply of venture capital, and study how equilibria change along fundraising waves, I also explore a version of the model in which VCs differ in skill: More skilled VCs provide higher attention for any given portfolio

<sup>4</sup> It is implicit in the matching protocol that asymmetric information persists at the matching stage. I assume random matching within submarkets for tractability, but show in dedicated extensions that the mechanism operates when VCs receive imperfect signals about project quality ex post, and use them to select the best projects or reallocate their attention toward the most promising ones. I argue that an alternative to imperfect learning is to assume that it is costly for VCs to produce fully informative signals.

size.<sup>5</sup> The results described above continue to hold. Under this specification, I further show that entry of low-skilled VCs is a positive externality to incumbent VCs. The reason is that low-skilled VCs choose to devote low attention to their firms; the larger supply of low-attention funds increases the quality of the marginal entrepreneur, improving matching opportunities for all incumbent VCs.

*Positive Results.* In the model, entry of less skilled VCs implies that the quality of the marginal entrepreneur increases, which changes the composition of firms in VCs' portfolios; this causes more funded projects to end up on the lower side of the returns distribution, and higher returns for those projects that perform best. The result gives a unified explanation for empirical findings concerning fundraising booms (Kaplan and Schoar (2005), Nanda and Rhodes-Kropf (2013), and Yung (2017)). The sorting channel is novel and has not been tested empirically.

Consistent with the findings in Ewens, Nanda, and Rhodes-Kropf (2017), the model predicts that lower costs of entrepreneurship imply larger portfolios, lower equilibrium VC value-adding input, and a more dispersed distribution of returns in the industry. When the cost of entrepreneurship is low, venture capital is in higher demand and VCs, who can fill up more of their capacity, tilt their strategy in favor of forming large, low-attention funds. Once again, the change in the relative supply of funds across the two segments of the market triggers entrepreneurs' response, and the improved matching opportunities of high value-adding VCs imply more dispersed returns.

I derive two additional untested predictions: by amplifying the motive to attract high-quality entrepreneurs, both (i) markets in which the distribution of project quality exhibits more upside and downside risk and (ii) markets in which deal flow is in smaller part proprietary imply higher VC value-adding input and thus smaller portfolios.

*Realism of the Main Assumptions.* There are three key assumptions in the model: (i) scarcity in VCs' input, (ii) entrepreneurs self-selection, and (iii) complementarities in the returns function. To support the first, diseconomies of scale at the fund level are found in Kaplan and Schoar (2005), Robinson and Sensoy (2013, 2016), and Rossi (2019).<sup>6</sup> Importantly, Kaplan and Strömberg (2004), Humphery-Jenner (2011), Cumming and Dai (2011), Lopez-de Silanes, Phalippou, and Gottschalg (2015), and Abuzov (2019) document that scarcity of VCs' human capital drives diseconomies of scale in the industry. Ewens and Rhodes-Kropf (2015) show that there exists significant variation in the quality of partners' human capital within a VC firm, and that better partners

<sup>5</sup> Abundant evidence shows that VCs differ in their ability to generate returns and to help companies reach the initial public offering stage (Sørensen (2007), Ewens and Rhodes-Kropf (2015), Korteweg and Sørensen (2017)).

<sup>6</sup> The assumption that a VC's input is scarce is explicit in some works (Fulghieri and Sevilir (2009)) and implicit in others (Michelacci and Suarez (2004), Jovanovic and Szentes (2013), and Silveira and Wright (2015)) where VCs—constrained to monitor one project at a time—have to exit the investment to free up their human capital for a new project.

contribute more to VC value-added, confirming that human capital is scarce not only in quantity (e.g., “time”), but also in quality.<sup>7</sup>

Concerning the second assumption, note that in contrast to other delegated asset managers (e.g., general partners in buyouts, mutual fund managers), VCs invest in targets that are interested in their ability to add value. Hsu (2004) finds that entrepreneurs accept worse terms in order to affiliate with VCs who can provide greater value-added. Nanda, Samila, and Sorenson (2020) find that a VC’s initial success generates superior future performance through access to better deal flow, consistent with the idea that the best entrepreneurs self-select into the VC funds that they perceive can add more value.

Finally, the positive assortative matching (PAM) between VCs and entrepreneurs is indirect evidence of complementarities in the inputs that they provide.<sup>8</sup> Based on a survey, Gompers et al. (2020) find that according to VCs, the three primary ways they add value are through “strategic guidance,” “connecting investors,” and “connecting customers.” These activities are likely to be particularly valuable for high-quality firms, as they complement (and do not substitute for) a good business model.<sup>9,10</sup>

*Literature.* In the context of mutual funds, Berk and Green (2004) study equilibrium fund size. They model decreasing returns in reduced form. In my model, entrepreneurs’ search behavior—an important and distinctive feature of the venture capital market—endogenizes the returns that VCs generate at any given scale. Accounting for this behavior is the key to derive all of the results presented in the paper.

Prior literature studies optimal portfolio size (Fulghieri and Sevilir (2009)) or fund size (Inderst, Mueller, and Münnich (2006), Marquez, Nanda, and Yavuz (2014)) in venture capital. These works study the VC’s problem in isolation and therefore cannot address the question of efficiency in the market equilibrium. Marquez, Nanda, and Yavuz (2014) also assume that a VC’s deal flow is endogenous and posit that VCs seek to improve their reputation to attract

<sup>7</sup> By exploiting partners’ movements between VC firms, they find that the explanatory power of partner human capital for variation in performance is two to five times greater than that of VC firm fixed effects; to the extent that skilled partners are in scarce supply, the implication is that scaling up a VC firm by hiring additional partners reduces the firm’s average human capital.

<sup>8</sup> Indeed, complementarities are at the heart of the seminal matching model proposed by Sørensen (2007).

<sup>9</sup> Quoting Fred Wilson (managing partner at Union Square Ventures): “When it’s clear the founder only wants your money and has no interest in your advice, it is hard to get excited about the investment. When it seems that all the founder wants is your advice and isn’t worried about getting money, it makes you want to work with that founder” (full text at <https://avc.com/2015/12/advice-and-money/>).

<sup>10</sup> Another role of VCs is to solve agency problems, which I abstract from. While actions such as retaining control or replacing management (Kaplan, Martel, and Strömberg (2007)) are not necessarily advantageous for the founders, the ability to enforce such contracts is ex ante value enhancing, and arguably a function of the number and quality of partners in a venture capital firm.

the best firms.<sup>11</sup> While they model a fund's portfolio quality in reduced form, I characterize sorting explicitly; this matters because different sorting outcomes are the reason why multiple equilibria emerge (and are Pareto-ranked), and the second-best solution differs from the equilibrium.

By endogenizing the distribution of VCs offering different value-added, the analysis differs from the seminal matching model of Sørensen (2007). Other concurrent theoretical research emphasizes matching in the venture capital market: Both Cong and Xiao (2022) and Hong, Serfes, and Thiele (2020) assume perfect information and that every VC is matched with one entrepreneur only.

In the dynamic models of Michelacci and Suarez (2004), Jovanovic and Szentes (2013), and Silveira and Wright (2015), due to scarcity in human capital, VCs have incentives to terminate projects early. These models do not study how heterogeneous entrepreneurs and VCs match.

To focus the discussion on its novel part, the model assumes away all agency problems associated with venture capital financing, which have received comparatively more space in the literature.<sup>12</sup>

I introduce adverse selection to a matching environment following Guerrieri, Shimer, and Wright (2010).<sup>13</sup> One important difference is that, in my model, the uninformed side of the market (the VCs) is scarce and therefore earns rents in expectation. Moreover, I provide conditions for sorting in a matching environment with nontransferable utilities and search frictions, contributing to a literature that includes Eeckhout and Kircher (2010, 2018). Finally, the result that VCs who want to attract entrepreneurs of unobservable quality overcommit their resources resonates overinvestment results in models in which sunk investments precede a matching stage that, contrary to my setting, is frictionless (e.g., de Meza and Lockwood (2010) and Mailath, Postlewaite, and Samuelson (2017)).

*Roadmap.* In Section I, I introduce the model and characterize its equilibria. In Section II, I discuss equilibrium efficiency and multiplicity. In Section III, I endogenize VCs' entry and study related comparative statics. In Section IV, I analyze the effects of changes in the cost of entering entrepreneurship. In Section V, I discuss the model's predictions and policy implications. In Section VI, I discuss extensions and robustness.

<sup>11</sup> VCs' reputation-building motive is also at the heart of Piacentino (2019). She shows that the conservatism of reputation-motivated unskilled VCs can be beneficial because it generates a certification effect, making VC-backed firms more likely to raise capital in an IPO.

<sup>12</sup> Notable examples are Cornelli and Yosha (2003) and Repullo and Suarez (2004), who focus on security design with information production. In Schmidt (2003), the double moral hazard problem between the two parties justifies the use of convertible preferred equity. Hellmann (2006) extends this analysis to distinguish between exit via IPO and exit via private acquisition. Casamatta (2003) studies the emergence of external financing from VCs who also provide value-added, and the optimality of common stocks versus preferred equity.

<sup>13</sup> Using their terminology, the VCs in my model are "uninformed principals" that post contracts (summarized by the level of attention) and "informed agents" (entrepreneurs) who direct their search based on their type.



# I. Model

*Agents.* There are identical VCs in measure normalized to one, and ex ante identical entrepreneurs in arbitrarily large measure. Entrepreneurs can enter the market upon paying a startup cost  $c > 0$ . If they do so, they draw their type, denoted  $\lambda$ , which represents the quality of their project and is distributed according to the measure  $F$ , admitting a continuous density  $f$  with full support  $[\underline{\lambda}, \bar{\lambda}] \subset \mathbb{R}$ . A higher  $\lambda$  indicates a better project in the sense specified in the next paragraph. Entrepreneurs need money and the VC's input to turn their projects into profitable firms.

*Projects.* All projects require only one unit of funding to become a firm and are not scalable. Each project's return  $R$  is a function of both the VC's input—which I call attention, denoted  $a$ —and the project's quality  $\lambda$ . Denote this function  $R(a, \lambda)$ . The partial derivatives  $R_a(a, \lambda)$  and  $R_\lambda(a, \lambda)$  are strictly positive for all  $a, \lambda \in \mathbb{R}$ , and  $R(a, \lambda)$  is twice continuously differentiable in both arguments.

*The Attention Function.* Denote by  $m$  the measure of projects that a given VC finances at a given time. The attention each VC devotes to each project is a function of the measure of firms to which it is matched,  $a := a(m)$ . Specifically,  $a(m)$  is the step function

$$a(m) = \begin{cases} a_1 & \forall m \in [0, m_1] \\ a_0 & \forall m \in (m_1, m_0] \end{cases}, \quad (1)$$

with  $a_1 > a_0$  and  $m_0 > m_1$ . In words, a VC's input is diluted by financing more projects at a given time. The parameter  $m_0$  is the VC's maximum portfolio size. I choose this reduced-form specification to describe all manifestations of scarcity in the resources that VCs employ to operate their business.<sup>14</sup>

*Matching and Information.* While VCs have no private information, entrepreneurs are privately informed about their type,  $\lambda$ . I therefore study directed search from the entrepreneurs' side. Since VCs differ in the (observable) size of their funds  $w$ —the choice of which is clarified in the next paragraph—each fund size  $w$  chosen in equilibrium will define a *submarket* to which entrepreneurs can direct their search; different entrepreneurs who search in a given submarket are randomly assigned to VCs. The matching technology is Leontief: as many matches are formed in each submarket as possible.

*Strategies, Payoffs, and Timing.* In the first stage of the game, each VC raises an amount of capital  $w$ , at a constant marginal cost of  $R_0$  per unit of capital. As all projects require one unit of capital, I refer to fund size  $w$  as the fund's capacity. In the second stage, entrepreneurs observe the distribution of fund capacities  $w$  implied by the first stage and choose whether or not to pay the startup

<sup>14</sup> One could also posit moral hazard (outside the current model) as a channel generating a similar tradeoff. In a setting in which VCs are subject to a moral hazard problem (Holmstrom and Tirole (1997)), sufficient correlation across portfolio projects generates an endogenous limit on the number of firms to which VCs can provide high value-added while preserving their incentives not to shirk (see Dessi (2005)).



cost. Those who do can direct their search toward different VCs. Then, matching occurs. VCs receive a fixed share  $\alpha \in (0, 1)$  of the returns generated from every investment. Entrepreneurs, conditional on matching to a VC, receive the residual share of the returns from their own projects,  $1 - \alpha$ . All agents are risk neutral and maximize expected profits.

Below I discuss the role of some of the assumptions.

*Exogenous Sharing Rule and Contracts.* As often occurs in the matching literature and following Sørensen (2007), who also studies venture capital, I assume for tractability that VCs and entrepreneurs receive a fixed fraction of the surplus they create. While one interpretation is that each party receives a fixed equity share, the reduced-form model can describe, with minor adjustments, different and more complex forms of financing (one example is described in the Internet Appendix, Section VIII).<sup>15</sup> A detailed discussion is in Section VI.B, where I also consider the case in which VCs can commit to different contracts and do so to screen the best entrepreneurs.

*VC Compensation.* In reality, VCs receive a fixed fee proportional to assets under management and a variable payment typically consisting of a fraction of the fund's returns above a certain benchmark.<sup>16</sup> This compensation structure finds theoretical justification in agency problems inherent in the VC-investor relationship. In my model, VCs act in the best interest of the fund, an assumption that isolates the novel mechanism. Adding performance-based compensation would tilt VCs' trade-off in favor of generating higher returns per assets under management. Since the model generates excessively small funds and underinvestment, this assumption would exacerbate the forces that are responsible for the main results.

*Entrepreneur Effort Provision.* Entrepreneurial effort is essential in venture capital. Arguably, the VC's involvement in the project includes monitoring, which improves entrepreneurs' incentive to exert effort. This potential benefit from higher attention is embedded in the reduced form I propose. To the extent that VCs internalize their effect on returns, modeling effort explicitly would not interact with the paper's core mechanism.

*Alternative Specifications: Continuous Attention and Coarse Search.* I assume that attention is a step function of  $m$  to discipline the characterization of sorting patterns, and second-best assignment, by inducing a discrete number of relevant submarkets. In the Internet Appendix (Section III), I propose an alternative model in which  $\lambda$  is discrete and  $a(m)$  is continuous in  $m$ . Another way to preserve tractability is to assume that  $\lambda$  and  $a$  are continuous, but entrepreneurs' search strategy is based on a coarse classification of VCs (see Section IV in the Internet Appendix).

<sup>15</sup> The Internet Appendix is available in the online version of this article on The Journal of Finance website.

<sup>16</sup> For an empirical study of VCs' compensation structure, its determinants, and variation over fundraising cycles, see Robinson and Sensoy (2013).

### A. Entry and Sorting Subgame

Consider the subgame in which entrepreneurs, after observing the distribution of VCs' fund capacities, make the entry and directed search decision. While every fund capacity  $w$  chosen by VCs defines a unique submarket, I simplify the subgame and reduce entrepreneurs' strategy to one in which their search is directed to at most two relevant submarkets: one that aggregates all VCs who provide high attention and one that aggregates all VCs who provide low attention.

The attention that an entrepreneur receives depends on how many projects the VC finances in equilibrium, a quantity that depends in turn on all the entrepreneurs' strategy profile. To circumvent this complication, I exploit the fact that VCs are in scarce supply. With the additional restriction that low-attention VCs are sufficiently valuable to the average entrepreneur, the assumption guarantees that all equilibria feature sufficiently large entry of entrepreneurs to cause excess demand for VCs in every submarket. The implication is that all VCs must operate at full capacity, and their attention is fully determined by their decision in the fundraising stage. The additional restriction reads as follows.

ASSUMPTION 1:  $(1 - \alpha)E_{\lambda}R(a_0, \lambda) > c$ .

In words: an entrepreneur strictly benefits from paying the startup cost, discovering the project's quality, and matching with probability one to a VC that provides low attention.<sup>17</sup> The result below specifies the sense in which the model can be simplified.

LEMMA 1 (The reduced model): *For any equilibrium of the entry and sorting subgame, there is one in a reduced game such that: (i) upon entry, entrepreneurs can only direct their search toward different attention levels, that is, their strategy maps  $[\lambda, \bar{\lambda}]$  to a distribution over the two submarkets  $\{a_0, a_1\}$ ; (ii) in every submarket, as many entrepreneurs are matched to VCs as possible; (iii) within submarkets, VCs are matched randomly to entrepreneurs, and the measure of entrepreneurs who each VC is matched to is proportional to their capacity; and (iv) entrepreneurs receive the same payoff as in the original equilibrium.*

In light of Lemma 1, I assume that entrepreneurs can only choose between the two relevant submarkets,  $\{a_0, a_1\}$ . Conditional on entry, entrepreneurs choose a search strategy—a distribution  $s$  over  $\{a_0, a_1\}$ . While entrepreneurs can follow a mix strategy, the restrictions that I impose next permit a focus on pure-strategy equilibria without loss of generality. Denoting by  $\mathbf{s}$  the vector of all entrepreneurs' search strategies, and by  $\mathcal{E}$  the measure of entrepreneurs who choose to enter, a strategy profile is a vector  $(\mathcal{E}, \mathbf{s})$ .

An entrepreneur who searches in submarket  $a_i$ —with  $i \in \{0, 1\}$ —succeeds in matching with probability  $Q_i$ , which depends on the ratio of total fund capacity (the supply of funds) to entrepreneurs searching in the submarket (demand).

<sup>17</sup> I relax this assumption in Section IV.

This ratio is denoted by  $\theta_i$ . The demand for venture capital is satisfied up to capacity, and thus:<sup>18</sup>

$$Q_i := \min \{\theta_i, 1\}. \quad (2)$$

I refer to a submarket in which  $Q_i$  is lower as a more congested submarket.<sup>19</sup> Given VCs' fundraising, congestion is determined by entrepreneurs' strategy profile, and thus is denoted by  $Q_i(\mathcal{E}, \mathbf{s})$ . Entrepreneurs are atomistic and therefore take the functions  $Q_i(\mathcal{E}, \mathbf{s})$  as given. The conditions characterizing an equilibrium in the subgame are given below.

**DEFINITION 1** (Equilibrium in the subgame): A strategy profile  $(\mathcal{E}, \mathbf{s})$  constitutes an equilibrium of the entry and sorting subgame if:

- (i) a type- $\lambda$  entrepreneur searches in submarket  $a_i$  only if  $i$  maximizes  $(1 - \alpha)Q_i(\mathcal{E}, \mathbf{s})R(a_i, \lambda)$ ;
- (ii) if  $\mathcal{E} > 0$ , entrepreneurs are indifferent between entering and staying out of the market:

$$\int_{\lambda} \max_{i \in [0,1]} (1 - \alpha)Q_i(\mathcal{E}, \mathbf{s})R(a_i, \lambda) dF(\lambda) = c. \quad (3)$$

Part (i) imposes optimality. Part (ii) follows from the assumption that entrepreneurs are in large supply.

I characterize the subgame equilibria under the assumption that the function  $R(a, \lambda)$  exhibits a strong form of complementarity in its two arguments, as stated below.

**ASSUMPTION 2:**  $R(a, \lambda)$  is everywhere log-supermodular. (Equivalently, the ratio  $R(a, \lambda)/R(a', \lambda)$  is increasing in  $\lambda$  for any  $a > a'$ ).

This form of complementarity implies the weaker concept of supermodularity. Under log-supermodularity, equilibria are necessarily characterized by an assortative matching pattern. Formally, letting  $\Lambda^s(a)$  be the set of entrepreneurs searching in market  $a$  under strategy  $s$ , the following holds.

**PROPOSITION 1** (Sorting): Define PAM as an allocation of entrepreneurs into attention levels such that  $\inf \Lambda^s(a_1) \geq \sup \Lambda^s(a_0)$ . Then, Assumption 2 is necessary and sufficient for all equilibria to exhibit PAM.

Under PAM, any entrepreneur searching in a low-attention market has to be of lower quality than any of those seeking higher attention. However, the pooling of different entrepreneurs at a given attention level is compatible with the definition.<sup>20</sup> The intuition for why log-supermodularity is sufficient to generate PAM is straightforward. Given probabilities  $Q_0$  and  $Q_1$ , an entrepreneur

<sup>18</sup> By assuming that as many matches are formed in every submarket as possible, I abstract from inefficiencies that might arise from matching frictions within the submarket.

<sup>19</sup> The direct consequence of Assumption 1 is that, in all equilibria,  $\theta_i < 1$  and thus  $Q_i < 1$ .

<sup>20</sup> A key to relate the result in Proposition 1 to existing literature is that utilities are assumed to be nontransferable. In the directed search framework of Eeckhout and Kircher (2010), sellers can commit to posted prices, and although supermodularity per se is not sufficient, the

of quality  $\tilde{\lambda}$  prefers to seek VCs in the high-attention market if and only if  $(1 - \alpha)Q_1R(a_1, \tilde{\lambda}) > (1 - \alpha)Q_0R(a_0, \tilde{\lambda})$ . Rearranged, the condition is

$$\underbrace{\frac{R(a_1, \tilde{\lambda})}{R(a_0, \tilde{\lambda})}}_{\text{increasing in } \tilde{\lambda}} > \frac{Q_0}{Q_1}. \quad (4)$$

The left-hand side is increasing in  $\tilde{\lambda}$  by log-supermodularity. Therefore, if an entrepreneur of quality  $\tilde{\lambda}$  chooses the high-attention market, so will all entrepreneurs of higher quality.

### B. The Market Equilibrium

In this section, I study the VC's choice at the initial stage, when choosing capacity  $w$ . I therefore endogenize the measure of high- and low-attention funds in the market, and characterize equilibria of the entire game. VCs take into account that committing different attention levels attracts different sets of entrepreneurs. For a VC, the optimal size must solve

$$w \in \arg \max_{\tilde{w}} \underbrace{\tilde{w} \{ \alpha \mathbb{E} [R(a(\tilde{w}), \lambda) | \lambda \in \Lambda^s(a(\tilde{w}))] - R_0 \}}_{\text{total excess returns}}. \quad (5)$$

Their optimal decision is further simplified below.

*VC Strategy.* When capacity is set within the intervals  $[0, m_1]$  and  $[m_1, m_0]$ , marginal returns are constant—attention is constant, and so is the set of entrepreneurs the VC expects to attract. It follows that the relevant strategic choice of a VC is which level of attention  $a_i$  to offer. The VC chooses the maximum fund size conditional on the chosen attention  $a_i$ , that is,  $m_i$ . VCs' strategy profile, can then be fully described by the variable  $\chi \in [0, 1]$ , which represents the measure of VCs who choose to provide high attention (recall that the total measure of VCs is normalized to one).

*Equilibrium Concept.* The notion of Weak Perfect Bayesian Equilibrium only disciplines beliefs on the equilibrium path.<sup>21</sup> Formally, VC beliefs about the composition of entrepreneurs in a given market can be represented by a mapping  $\beta : \{a_0, a_1\} \rightarrow \Delta([\underline{\lambda}, \bar{\lambda}])$ . When some VCs do offer funds in market  $a_i$  in equilibrium, the distribution  $\beta_{a_i}$  is derived using Bayes rule. For submarkets in which no VC offers funds, I impose a restriction on beliefs that will help characterize noninterior equilibria. Nonetheless, I emphasize that the vast majority of the results concern interior equilibria. The approach that I follow is based on

requirements for PAM to emerge are milder: the degree of supermodularity depends on the elasticity of substitution in the matching function. Notably here, with nontransferable utilities, the result that  $R$  must be log-supermodular holds true under any specification of the matching function. Log-supermodularity is also necessary for PAM to emerge in dynamic search models with anonymous search and nontransferable utilities (Smith (2011)).

<sup>21</sup> For a formal definition of Weak Perfect Bayesian Equilibrium, see Definition 9.C.3 in Mas-Colell, Whinston, and Green (1995).

Guerrieri, Shimer, and Wright (2010), who consider a similar setting. I state the restriction first, and then explain the intuition behind it.

**Requirement 1:** Take an off-equilibrium  $a_j$ . Given a subgame equilibrium  $(\mathcal{E}, s)$  and associated entrepreneur expected payoff  $\pi_\lambda$ , the type  $\lambda$  is in the support of  $\beta_{a_j}$  if and only if the set

$$\mathcal{Q}(\lambda; a_j) := \{Q \in [0, 1] \mid Q(1 - \alpha)R(a_j, \lambda) \geq \pi_\lambda\} \quad (6)$$

is maximal. If  $\mathcal{Q}(\lambda; a_j)$  is empty for all  $\lambda$ , VCs expect to find no entrepreneur in market  $a_j$ .<sup>22</sup>

The restriction has a simple interpretation: A VC that is contemplating offering attention  $a_j$  must believe that this offer would attract the type that is willing to face the highest congestion, that is, to deviate at the lowest matching probability.<sup>23</sup>

**DEFINITION 2 (Equilibrium):** An equilibrium is a vector  $(\mathcal{E}, s, \chi, \beta)$  constituting a Weak Perfect Bayesian Equilibrium, with the restriction that  $\beta$  satisfies Requirement 1 off the equilibrium path.

Before I proceed with the equilibrium characterization, I make one further restriction to avoid the emergence of a trivial degenerate equilibrium.

**ASSUMPTION 3:**  $R(a_1, \underline{\lambda})/R(a_0, \underline{\lambda}) < (1 - \alpha)\mathbb{E}_\lambda R(a_1, \lambda)/c$ .

Under this assumption, an equilibrium in which every VC raises high-attention funds can only exist because of VCs' motive to attract the best entrepreneurs.<sup>24</sup> Assumption 3 is relaxed in Section IV, and does not drive any of the results of interest. In what follows, I characterize all equilibria of the game.

**PROPOSITION 2 (Equilibrium characterization):**

- (i) *An equilibrium of the game always exists.*
- (ii) *(Interior Equilibria). Any interior equilibrium is characterized by a vector  $(\chi, \lambda_0)$  such that given a strategy profile implying matching probabilities  $Q_0$  and  $Q_1$ , all entrepreneurs below (above)  $\lambda_0$  search for low (high)*

<sup>22</sup> For a given collection of sets  $\mathcal{Q}(\lambda; a_j)$ ,  $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ ,  $\mathcal{Q}(\hat{\lambda}; a_j)$  is said to be maximal if it is not a subset of any other  $\mathcal{Q}(\lambda; a_j)$ .

<sup>23</sup> The elements in  $\mathcal{Q}(\lambda; a_j)$  can be thought of as the match probabilities resulting from VCs off-equilibrium behavior, the vacancies posted at attention  $a_j$ . Requirement 1 can be interpreted as follows: "the type that is expected to search in  $a_j$  is the one for which there is a larger set of VCs off-equilibrium actions that would make this deviation profitable." In this sense, Requirement 1 is an adaptation of condition D1 for signaling games (Cho and Kreps (1987)). In the Internet Appendix (Section X), it is shown that the same equilibria are selected under an alternative refinement. Specifically, I focus on perturbed games where a fraction of VCs chooses fund size nonstrategically—so that no submarket is empty—and take the limit as this fraction approaches zero.

<sup>24</sup> If the assumption would not hold, such equilibrium would always exist because no entrepreneur would search in the alternative market, no matter how large the difference in congestion would be across markets.

attention. Moreover, given  $\lambda_0$ , VCs are indifferent between offering high- or low-attention funds. The vector  $(\chi, \lambda_0)$  satisfies

$$m_0\{\alpha E[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0\} = m_1\{\alpha E[R(a_1, \lambda) \mid \lambda \geq \lambda_0] - R_0\}, \quad (7)$$

$$Q_0(1 - \alpha)R(a_0, \lambda_0) = Q_1(1 - \alpha)R(a_1, \lambda_0). \quad (8)$$

(iii) (Corner Equilibria).

- There exists an equilibrium in which every VC provides low attention if and only if

$$m_0\{\alpha E[R(a_0, \lambda)] - R_0\} \geq m_1\{\alpha R(a_1, \bar{\lambda}) - R_0\}. \quad (9)$$

- There exists an equilibrium in which every VC provides high attention if and only if

$$m_1\{\alpha E[R(a_1, \lambda)] - R_0\} \geq m_0\{\alpha R(a_0, \underline{\lambda}) - R_0\}. \quad (10)$$

Note that noninterior equilibria, described in part (iii), represent the only case in which the requirement on off-equilibrium beliefs kicks in. In a candidate equilibrium in which every VC raises a large fund, deviating to offer high attention would attract the highest quality entrepreneur, the one most willing to sacrifice matching probability to receive higher attention. Similarly, in a candidate equilibrium in which every VC forms a small portfolio, deviating to offer low attention would attract the lowest quality entrepreneur, the one most willing to sacrifice attention to match with a higher probability. The conditions in (iii) ensure that such deviations are not profitable.

## II. Efficiency and Equilibrium Multiplicity

In this section, I study efficiency of the market equilibrium, describe when multiple equilibria can emerge, and characterize their properties. The two main propositions are connected by a common theme, namely, a sorting externality that VCs impose on each other. To visualize it, denote by  $M_i$  the aggregate supply of VC funds (total funds capacity) in a given market  $i$ , in a given equilibrium and note that each  $M_i$  is a function of  $\chi$ ,

$$M_0(\chi) = (1 - \chi)m_0; \quad M_1(\chi) = \chi m_1. \quad (11)$$

LEMMA 2 (The sorting externality): *Given  $M_0$  and  $M_1$ , the threshold  $\lambda_0$  identifying the marginal entrepreneur is unique. The threshold is monotonically increasing in the ratio  $M_0/M_1$ . The quality of projects financed by each VC conditional on attention (and size) is higher the higher is  $M_0/M_1$ .*

The larger the ratio  $M_0/M_1$ , the more congested is the high-attention market, holding entrepreneurs' sorting constant. By log-supermodularity, the marginal, indifferent entrepreneur's quality must be higher to make condition (8) hold. Because of excess demand for VCs and uniform random match-

ing within submarkets, the fact that the marginal entrepreneur is of higher quality implies that, within each segment, all VCs benefit from higher quality matches. In particular, VCs in the low-attention segment face a distribution that has higher quality at the top, and those in the high-attention segment face a distribution that has higher quality at the bottom.<sup>25</sup> It is crucial that different entrepreneurs “compete” for the scarce supply of venture capital in each submarket. This is highlighted in the following example, where the economy is modified to better illustrate the effect.

**EXAMPLE 1:** There are two VCs and three entrepreneurs, with qualities  $\lambda_h > \lambda_m > \lambda_l$ . The first VC provides high attention, while the second provides low attention—attention is assumed to be exogenous and each VC matches to one entrepreneur only. Two arrangements are compatible with PAM: either (i)  $\lambda_l$  and  $\lambda_m$  search for low attention, while  $\lambda_h$  searches for high attention, or (ii)  $\lambda_l$  searches for low attention, while  $\lambda_m$  and  $\lambda_h$  search for high attention. It is easy to see that each VC is better off when sorting is as in case (i).<sup>26</sup> Notably, the addition of a third VC providing either low or high attention would imply only one sorting outcome, muting the sorting effect on match quality.

*Efficiency.* Consider the next example, which illustrates one extreme manifestation of VCs’ failure to internalize their impact on equilibrium sorting and thus on the equilibrium value of committing high or low attention.

**EXAMPLE 2:** In the original economy, set  $R_0 = 0$  for simplicity and assume that for all  $\lambda$ ,  $m_0 R(a_0, \lambda) > m_1 R(a_1, \lambda)$ : Absent selection considerations, a large portfolio is optimal. From Proposition 2, an equilibrium in which every VC commits high attention exists as long as  $m_1 \mathbb{E}[R(a_1, \lambda)] > m_0 R(a_0, \underline{\lambda})$ , that is, no VC wishes to deviate and attract the worst entrepreneur. In such an equilibrium, all VCs are worse off than they would be if they all raised a large fund:  $m_1 \mathbb{E}[R(a_1, \lambda)] < m_0 \mathbb{E}[R(a_0, \lambda)]$ . In this example, the only reason VCs commit high attention is that, if they did not, they would attract low-quality entrepreneurs. However, in a corner equilibrium, this strategy confers no benefits, as all VCs are in the same submarket and face the same distribution of entrepreneurs.

Ex ante, total welfare in the economy amounts to the expected aggregate returns to VCs net of funding costs (entrepreneurs’ expected payoff is zero due

<sup>25</sup> The externality is reminiscent of the cream-skimming effect in Bolton, Santos, and Scheinkman (2016), where the presence of informed investors trading over-the-counter reduces the quality of the assets sold in the organized exchange. In my model, high-attention VCs draw the best entrepreneurs away from the low-attention market; in addition, by doing so, they also reduce average project quality in their submarket.

<sup>26</sup> The low-attention VC generates expected returns  $1/2 R(a_0, \lambda_l) + 1/2 R(a_0, \lambda_m)$  in case (i), and  $R(a_0, \lambda_l)$  in case (ii); the high-attention VC generates  $R(a_1, \lambda_h)$  in case (i) and  $1/2 R(a_1, \lambda_m) + 1/2 R(a_1, \lambda_h)$  in case (ii). A similar effect, but limited to low-attention VCs, would arise from a version of the model in which VCs can select the best entrepreneurs in their respective submarket. However, this would not be compatible with entrepreneurs’ optimal search. I relax uniform random matching and study an extension with imperfect screening in the Internet Appendix (Section V).



to free entry). Therefore,

$$\text{expected welfare} = \sum_{i \in [0,1]} M_i(\chi) \{ \alpha \mathbb{E}[R(a_i, \lambda) | \lambda \in \Lambda_i^s] - R_0 \}. \quad (12)$$

I now study the welfare-maximizing profile of VCs' size when its induced effect on entrepreneurs' sorting is taken into account. I refer to such a solution as the *second-best allocation*.

**DEFINITION 3: A second-best allocation** is an allocation of VCs into the high- and low-attention markets,  $(\chi^{sb}, 1 - \chi^{sb})$ , that solves:

$$\max_{\chi^{sb} \in [0,1]} \sum_{i \in [0,1]} M_i(\chi^{sb}) \{ \alpha \mathbb{E}[R(a_i, \lambda) | \lambda \in \Lambda_i^s] - R_0 \} \quad (13)$$

subject to strategy **s** constituting an equilibrium of the sorting subgame.

I highlight two aspects of this exercise. First, the second-best allocation is constrained by entrepreneurs' optimal response. Thus, the benchmark that I compare the model's equilibrium against is subject to the congestion externalities that entrepreneurs impose on each other, a friction that is embedded in random matching: Any departure of the equilibrium from the second-best allocation will come from VCs' choice prior to matching. Second, entrepreneurs break even in every equilibrium and in every candidate second-best solution: Entrepreneurs' payoff cannot be affected by  $\chi^{sb}$ . This feature mutes any inefficiency that may result from the way VCs and entrepreneurs share surplus ex post.

By varying the measure  $\chi$ , one can only affect the sum of aggregate returns across VCs. Given the above, if equilibria are not second-best efficient, it must be the case that VCs are imposing an externality onto each other. While VCs optimally choose attention to maximize their portfolio's returns given the quality of the projects they finance, a second-best allocation maximizes aggregate returns across VCs, accounting for the effect of relative fund supply on sorting. That is, the sorting externality can fully explain the result below.

**PROPOSITION 3 (Inefficiently small VCs):** *In every interior equilibrium, too many VCs choose high attention compared to the second-best solution, that is,  $\chi^{sb} < \chi$ . Consequently, equilibrium aggregate investment is lower than in the second-best allocation. If a corner equilibrium in which every VC raises a large fund ( $\chi = 0$ ) exists, it is second-best efficient.*

By Lemma 2, the quality of the marginal entrepreneur,  $\lambda_0$ , decreases in  $\chi$ . VCs take  $\lambda_0$  as given. Collectively, however, by contributing to a larger supply of high-attention funds, VCs who choose to remain small reduce  $\lambda_0$  and therefore impose a negative externality on all other VCs; conversely, VCs who form large portfolios impose a positive externality on all other VCs. Because in interior equilibria VCs produce the same aggregate returns from either strategy (i.e., they are indifferent by Proposition 2), it is always welfare improving, at the margin, to decrease  $\chi$ . In equilibrium, the market for high-attention funds

is not sufficiently congested, that is,  $\chi$  is too large, and therefore  $\lambda_0$  is too small and the quality of firms financed in each submarket is too low, compared to the second-best solution.<sup>27</sup>

The fact that VCs must raise small funds to commit high-attention implies that aggregate investment would be higher under second-best. Hence, another manifestation of the inefficiency is underinvestment in the venture capital market.

**Multiple Equilibria.** I derive a necessary condition for multiple equilibria to emerge, to illustrate whether and how a form of *strategic complementarity* among VCs exists. To this end, I define the function

$$\phi(\tilde{\lambda}) := \frac{\alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \tilde{\lambda}] - R_0}{\alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \tilde{\lambda}] - R_0}, \quad (14)$$

which is the expected excess return per unit of funds from choosing attention  $a_1$  and attracting entrepreneurs above or equal to some  $\tilde{\lambda}$ , relative to the excess return from choosing attention  $a_0$  and attracting entrepreneurs weakly below the threshold  $\tilde{\lambda}$ . In addition, I compare multiple equilibria in terms of welfare, VC attention, and size. All results are summarized in Proposition 4.

PROPOSITION 4 (Multiple Equilibria):

- (i) *Multiple equilibria can emerge only if the function  $\phi(\tilde{\lambda})$  is somewhere decreasing.*
- (ii) *All equilibria can be Pareto-ranked. An equilibrium Pareto-dominates another equilibrium if and only if  $\lambda_0$  is larger.*
- (iii) *Furthermore,  $\chi$  is lower, and  $Q_0/Q_1$  and  $M_0/M_1$  are larger, in the Pareto-dominant equilibrium.<sup>28</sup>*
- (iv) *In a Pareto-dominant equilibrium, average attention by VCs is lower, the average size of VCs' funds is larger, and the measure of firms receiving financing is larger.*

COROLLARY 1 (Ranking interior and corner equilibria): *An equilibrium in which every VC forms a low-attention fund—if it exists—Pareto-dominates every other equilibrium. An equilibrium in which every VC forms a high-attention fund—if it exists—is Pareto-dominated by every other equilibrium.*

Equilibrium multiplicity emerges due to the following feedback effect: By the sorting externality, VCs determine the endogenous  $\lambda_0$  and thus the quality of

<sup>27</sup> That entrepreneurs cannot fully separate in equilibrium may appear central to the result, but it is not. As I discuss in Section VI.A and in the Internet Appendix (Section III), in a setting in which there are two types of entrepreneurs and each type searches in a different submarket in equilibrium, I prove that it is welfare improving to increase the supply of funds in the low-attention submarket to make some high-quality entrepreneur search in it with positive probability.

<sup>28</sup> In comparing welfare in different equilibria, as some may be noninterior, I will need to define the matching probability in empty markets. To do so, I will use the lowest  $Q$  such that the type(s) selected by Requirement 1 will (weakly) benefit from deviating.

projects that they attract in each submarket. In turn, the quality of projects in the two submarkets determines the trade-off between forming high- or low-attention funds. When the function  $\phi(\tilde{\lambda})$  is decreasing, it is particularly important for VCs not to attract entrepreneurs at the lower end of the distribution (and less important to attract those at the higher end). If many VCs choose to form small, high-attention portfolios, the quality of the marginal entrepreneur  $\lambda_0$  is low. When  $\phi(\tilde{\lambda})$  is decreasing, a low  $\lambda_0$  means that forming a large fund is particularly unprofitable, making it consistent for many VCs to supply high-attention funds in equilibrium. That is, when  $\phi(\tilde{\lambda})$  is decreasing, VCs' strategies are complements.<sup>29</sup>

Note that both the numerator and the denominator of  $\phi(\tilde{\lambda})$  increase in  $\tilde{\lambda}$ , and the function need not be monotone. Its behavior depends jointly on  $R$  and on the distribution of projects' quality,  $F$ . In the example below, these primitives imply that  $\phi(\tilde{\lambda})$  is decreasing.

**EXAMPLE 3:** Quality  $\lambda$  is uniformly distributed over the support  $[0, 1]$ . Returns are given by  $R(a, \lambda) = a + (a - \delta)\rho(\lambda)$  with  $a > \delta > 0$ .<sup>30</sup> If  $\rho(\cdot)$  is an increasing linear function, the ratio  $\phi(\tilde{\lambda})$  is decreasing in  $\tilde{\lambda}$  for any  $a_1 > a_0$  and any  $R_0 > 0$  so long as  $\delta$  is sufficiently small.

Proposition 4 reveals that endogenous sorting can be a source of strategic complementarity among VCs, who might remain stuck on an equilibrium with high-attention small portfolios, anticipating that a deviation to the alternative style would worsen portfolio quality substantially. To the extent that VCs tend to specialize in certain industries, the result provides an interpretation for the variation in VCs' investment style observed, for example, between sectors such as healthcare and IT (Gompers et al. (2020)).<sup>31</sup>

### III. Fundraising Waves and the Role of VC Entry

#### A. Analysis

In this section, I model the VCs' entry decision. To produce explicit results concerning the effect of VCs' entry on the market equilibrium, and to discipline the comparative statics discussed hereafter, I introduce a dimension along which VCs differ exogenously. I interpret my findings in light of recent

<sup>29</sup> In models of competitive directed search, Guerrieri and Shimer (2018) and Williams (2021) show that, under multidimensional private information, multiple equilibria can exist, despite a restriction on out-of-equilibrium beliefs similar to Requirement 1. Contrary to my setting, multiplicity is due to out-of-equilibrium beliefs, which, because of multidimensional private information, cannot be uniquely pinned down despite the restriction.

<sup>30</sup> This function is log-supermodular whenever  $\rho' > 0$ .

<sup>31</sup> Specifically, the survey in Gompers et al. (2016) and Gompers et al. (2020) documents that VCs in the IT sector close significantly more deals, hire fewer board members, and employ fewer venture partners in the firms they back, compared to their peers (in line with Ewens, Nanda, and Rhodes-Kropf (2017) and Ewens and Malenko (2020), who document that VCs in the IT sector place less emphasis on active governance, and back more firms). By contrast, the survey reveals that the exact opposite holds for VCs in the healthcare sector.

evidence about VCs' returns over fundraising "waves" and the debate around policies that encourage VCs' investment.

*Heterogeneous VCs.* VCs are endowed with publicly observable skill,  $x$ . As will become clear below, skill  $x$  can be assumed to be distributed uniformly over  $[0, 1]$  without loss of generality. For simplicity, skill does not affect returns per se, but implies that VCs differ in their ability to scale up their funds without sacrificing attention. Specifically, the functions  $m_0^x$  and  $m_1^x$ —defining the maximum capacities at the two attention levels for a VC with skill  $x$ —satisfy

- (i)  $m_i^x$  is increasing in  $x$ , for  $i = 0, 1$  and  $\forall x$
- (ii)  $\frac{m_1^x}{m_0^x}$  is increasing in  $x$

Better VCs can run more projects at a given level of attention and are relatively more efficient at providing high attention. This is consistent with the empirical evidence: There are decreasing returns to scale at the fund level and more skilled VCs manage larger funds and provide greater value-added to their companies.<sup>32</sup> The crucial consequence of this assumption is that, in interior equilibria, lower skilled VCs will choose to provide low attention.

REMARK: All results of Sections I and II extend naturally to the model with heterogeneous VCs. Interior equilibria are characterized by two cutoffs,  $(x_0, \lambda_0)$ . VCs choose to provide high attention if and only if their skill is above  $x_0$ , and  $x_0$  solves

$$m_0^{x_0} \{\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0\} = m_1^{x_0} \{\alpha \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0] - R_0\}, \quad (15)$$

where equation (15) replaces equation (7) in Proposition 2.<sup>33</sup>

*Entry Decision and Equilibrium Characterization.* To make the entry decision nontrivial, assume that raising a fund requires an initial fixed investment of  $\kappa > 0$ . With endogenous entry of VCs, an interior equilibrium is a vector  $(\underline{x}, x_0, \lambda_0)$  such that the cutoffs  $(x_0, \lambda_0)$  satisfy conditions (8) and (15), and the

<sup>32</sup> In fact, a positive relation between fund size and returns exists in the cross section that reverts to negative when VC fixed effects are accounted for. Evidence about the size-return relationship can be found, among others, in Kaplan and Schoar (2005), Harris, Jenkinson, and Kaplan (2014), Robinson and Sensoy (2013, 2016), and Rossi (2019), while greater value-added by more reputable VCs is documented in Sørensen (2007).

<sup>33</sup> In the Appendix, I provide details on how the previous results extend to this more general framework. Equation (15) will be central to study various comparative statics. When the relative return of providing high attention increases (decreases) exogenously, the marginal VC,  $x_0$ , decreases (increases), generating predictions on VC's attention and fund size in equilibrium. This contrasts with the baseline model, where VCs' indifference condition has to continue to hold after exogenous parameters vary marginally, that is, the value of forming a high relative to a low-attention fund must remain constant. Depending on primitives,  $\lambda_0$  has to either increase or decrease, and since  $\lambda_0$  is a unique function of  $\chi$ , this would dictate whether the measure of VCs offering high-attention funds must increase or decrease. The predictions on VC's attention and size could, therefore, run against economic intuition.

marginal VC  $\underline{x}$  is indifferent between entering and staying out. Formally:

$$m_0^{\underline{x}}\{\alpha \mathbb{E}[R(\alpha_0, \lambda) \mid \lambda \leq \lambda_0] - R_0\} = \kappa. \quad (16)$$

Below I state the equivalents of the welfare results in Propositions 3 and 4 in case VC entry is endogenous. I adapt the definition of second-best (Definition 3) to the case in which the allocation of VCs across high- and low-attention funds as well as the measure of VCs who enter the market (hence the cutoff  $\underline{x}$ ) are jointly chosen to maximize aggregate welfare.

PROPOSITION 5 (Welfare with endogenous entry):

- (i) *In every interior equilibrium, the measure of VCs who enter is lower ( $\underline{x}^{sb} < \underline{x}$ ), the supply of low-attention funds is smaller (in absolute terms as well as relative to the supply of high-attention funds), and, consequently, equilibrium VC investments are lower than in the second-best allocation (thus, the marginal entrepreneur is lower:  $\lambda_0 < \lambda_0^{sb}$ ).*
- (ii) *In the second-best, the excess returns to the active VC with the lowest skill are strictly below entry costs.*
- (iii) *All interior equilibria are Pareto-ranked and the Pareto-dominant equilibria feature a larger cutoff  $\lambda_0$ , a larger supply of low-attention funds (in absolute terms as well as relative to the supply of high-attention funds), and more VC investments.*

In equilibrium, the measure of active VCs in equilibrium is lower than under second-best. Entry of low-skilled VCs represents a positive externality on all other VCs. The reason is that the lower skilled VCs choose to provide low attention. In line with the intuition behind Proposition 3, the larger supply reduces congestion in the low-attention market, attracting demand by the marginal entrepreneur. The cutoff  $\lambda_0$  increases, improving matching opportunities for all incumbent VCs. Once again, individual VCs do not internalize this externality: as part (ii) states, for the lowest skilled VC that enters the market in a second-best allocation, the private returns from entry are negative.

*Hot Markets and Comparative Statics.* Determining why capital committed to venture funds is volatile and exhibits cyclical behavior is beyond the scope of this paper.<sup>34</sup> I thus interpret “hot” markets as times in which, for reasons exogenous to the model, VCs’ fundraising is cheaper. In the model, this means that the initial investment,  $\kappa$ , or, similarly, the marginal cost of raising capital,  $R_0$ , is lower.<sup>35</sup> To identify the comparative statics, I restrict attention to stable equilibria (formally defined in the Appendix).<sup>36</sup>

PROPOSITION 6 (Hot markets): *For every interior stable equilibrium  $(x_0, \lambda_0)$ :*

<sup>34</sup> For a theoretical analysis of venture capital cyclicity and risk, see the general equilibrium model with endogenous growth developed by Opp (2019).

<sup>35</sup> There is no substantial difference in the comparative statics taken with respect to these two variables, and thus either interpretation is valid.

<sup>36</sup> In words, an equilibrium is stable if, after any small perturbation forces some agents’ strategies away from it, behavior converges back to the original equilibrium.

- (i) as  $\kappa$  decreases, more VCs enter and better entrepreneurs search in the low-attention market, that is,  $\frac{\partial x}{\partial \kappa} > 0$  and  $\frac{\partial \lambda_0}{\partial \kappa} < 0$ .
- (ii) As  $\kappa$  decreases, all VCs generate higher total returns. Total VC profits increase by strictly more than the reduction in entry costs.
- (iii) As  $\kappa$  decreases, more VCs raise large funds, that is,  $\frac{\partial x_0}{\partial \kappa} < 0$ , if and only if  $\phi_{\tilde{\lambda}}(\lambda_0) < 0$ .
- (iv) The statements in (i) and (ii) hold when the comparative statics is taken with respect to  $R_0$ . The condition in (iii) is sufficient for  $\frac{\partial x_0}{\partial R_0} < 0$ .

The intuition for the results is as follows. The lower entry or lower funding costs make it more profitable for all VCs to operate. Thus, more relatively unskilled VCs choose to enter the market and offer low-attention funds. The larger supply of funds in the market for low attention results in the positive externality highlighted in Proposition 5. Moreover, when the function  $\phi(\tilde{\lambda})$  is decreasing in  $\tilde{\lambda}$ , hot markets are characterized by lower returns to high-attention funds, relative to low-attention funds, and as a result by higher average fund size and aggregate investment.

*Consistent Empirical Evidence and Policy Implications.* The results in Proposition 5 (parts (iii) and (iv)) offer a novel rationale for the findings in Brander, Du, and Hellmann (2014) that the presence of government-sponsored VCs—which are considered by many to be less sophisticated than private VCs—does not crowd out but instead increases investments from private VCs at the market level. Moreover, Proposition 6 suggests that a policy that induces, via subsidies, entry of unskilled VCs generates benefits that, at the margin, exceed its direct costs. I discuss this and other policy implications in Section V. More empirical predictions related to entry and fundraising waves, and their relationship with existing evidence, are discussed in the next subsection.

## B. Distribution of Returns and Project Quality

### B.1. Distribution of Returns in Hot Times

Consider the distribution of returns of funded projects. The shape of this distribution is determined jointly by the ex ante distribution of project quality and the equilibrium allocation of VCs and entrepreneurs across the two submarkets. By Proposition 6, entry of VCs means that the quality of the marginal entrepreneur increases, which improves matching opportunities for incumbents VCs. This affects the distribution of returns as formalized in Corollary 2.

**COROLLARY 2** (Hot markets and the distribution of returns): *There exists a point  $\tilde{R}$  in the distribution of returns such that, as  $\kappa$  ( $R_0$ ) decreases:*

- (i) *expected returns are (weakly) higher conditional on being above  $R$ , for any  $R > \tilde{R}$ , and are more likely to be below  $\tilde{R}$ . The first effect is strict for some  $R$  sufficiently close to  $\tilde{R}$ .*

- (ii) *When the supply of low-attention funds grows sufficiently (relative to entrepreneurs' demand for such funds), returns are also more likely to be below any  $R$  that satisfies  $R < \tilde{R}$  (larger downside risk).*

Due to entry, a larger proportion of aggregate investment is made by low-attention VCs, which produce relatively low returns. This is the direct effect of entry. Its indirect effect, operating through the entrepreneurs' optimal response, implies higher expected returns at the top of the distribution (part (i)). Within the region of returns generated by low-attention VCs, the larger supply of funds increases the density of the returns distribution at any point, but the additional demand by higher quality entrepreneurs implies, due to random matching, that lower quality projects are less likely to be financed, decreasing the density conditional on a given level of supply. Only when the increase in the supply of low-attention funds is sufficiently strong does the distribution of returns exhibit more mass in the left tail, and hence larger downside risk (part (ii)). In the [Appendix](#), I construct examples in which this condition holds, and examples in which it is violated.<sup>37</sup>

*Consistent Empirical Evidence.* Corollary 2 offers an unified interpretation of some empirical findings on venture capital fundraising “waves.” Kaplan and Schoar (2005) document that the largest proportion of the capital raised in times when aggregate fundraising is high goes to poorly performing funds. Nanda and Rhodes-Kropf (2013) find that those startups that are financed in hot seasons are more likely to fail, but, conditional on not failing, they receive higher IPO valuations. The authors rationalize this finding with increased risk-appetite of VCs in hot times.<sup>38</sup> Yung (2017) looks instead at fund returns and finds larger dispersion in returns across funds when they are raised during hot seasons; in particular, returns on the left tail are lower and more sensitive to market heat within the subset of new entrant VCs, whereas returns on the right tail are higher and more sensitive to market heat within the subset of incumbent VCs, consistent with my explanation. The author posits that the right-tail effect is caused by the larger rents that more skilled VCs earn when the demand for venture capital is higher. This theory, however, does not explain why Nanda and Rhodes-Kropf (2013) find that in hot times, portfolio firms, not just the VCs, perform better conditional on being on the right end of the distribution. A widespread interpretation of hot markets associates them with times in which lower skilled VCs raise funds, and the VCs' marginal investment is devoted to lower quality entrepreneurs. Any model that features these

<sup>37</sup> The condition is (A72) in the [Appendix](#). Intuitively, when the distribution of project quality places sufficiently large mass on its extremes, the sorting effect cannot dominate the direct increase in the supply of capital due to entry; the opposite is true when the ratio  $R(a_1, \lambda)/R(a_0, \lambda)$  is sufficiently uniform across  $\lambda$ s, so that the marginal entrepreneur has to increase substantially to make entrepreneurs' indifference condition hold when more capital is supplied in the low-attention market.

<sup>38</sup> Their explanation applies to incumbent and newly entered VCs, although the effect appears to be more pronounced for more experienced VCs. In my model, the effects on returns to investments by incumbent VCs are essential for the results to hold.



relations can match the evidence of higher failure rates.<sup>39</sup> In contrast, the evidence at the top can only result from more indirect channels. In my model, the effect is driven by how entrepreneurs, depending on fund availability, choose to sort across the different segments of the market.

### B.2. Distribution of Project Quality in Hot Times

Consider now the equilibrium distribution of the quality of projects financed, and how it is affected by VCs' entry.

**COROLLARY 3** (Hot markets and the distribution of project quality): *As  $\kappa$  ( $R_0$ ) decreases, average project quality conditional on VC attention increases. Under the condition in Corollary 2, part (ii), the unconditional distribution of VC-financed project quality worsens in the sense of first-order stochastic dominance.*

The result in Corollary 3, together with Proposition 6, provides an alternative and more nuanced interpretation of the widespread view that worse investments are made by VCs in boom times (Gompers and Lerner (2003)). Consider again the narrative according to which, in hot times, VCs “move down the supply curve” and finance projects of lower quality. The result would be a mechanical decrease in the average quality of projects financed by all VCs. In my model, unconditional average project quality may also decrease in hot times, since more VCs offer low-attention funds and thus entrepreneurs' demand in the low-attention segment is disproportionately satisfied. However, conditional on VCs' attention, average quality increases in hot times, and all VCs make strictly higher total gross and excess returns. This occurs because the additional supply of low-attention funds generates the sorting externality.

## IV. The Cost of Entrepreneurship

In the sections above, the cost of entering entrepreneurship  $c$  is assumed to be sufficiently small that entrepreneurs enter the market until congestion kicks in at every submarket (Assumption 1). This assumption is convenient for a tractable exposition of the paper's main message, and it implies that VCs are in excess demand, a feature that appears realistic given the small number of firms that successfully receive venture capital. The assumption, however, hides interesting effects of the parameter  $c$  that only operate when this takes higher values. In the Appendix, I characterize three thresholds for  $c$ , denoted respectively,  $c_L$ ,  $c_M$ , and  $c_H$ . Assumptions A1 and A3 correspond to the case in which  $c < c_L$ . Below I analyze the remaining cases. For simplicity, I switch back to the baseline model with homogeneous VCs. All results can be extended to the more general case.

<sup>39</sup> I model expected returns in reduced form, that is without explicitly describing their risk profile, and VCs' skill has no direct impact on returns. These two simplifying assumptions could be relaxed to deliver the result more directly.

It is possible to find equilibria in which low-attention VCs are in excess supply. I assume that, when this is the case, such VCs use up an equal proportion, denoted by  $\gamma$ , of their capacity, and this proportion ensures market clearing.

*Intermediate costs.* Assume that costs are intermediate:  $c_L \leq c \leq c_M$ . In this region, for sufficiently high costs, there exist equilibria in which every VC raises a small fund. The reason is that few entrepreneurs choose to pay the entry cost  $c$ , and thus congestion is so small that a deviating VC would attract no entrepreneur, even if such an entrepreneur would match with probability one (see the discussion about Assumption 3).

More interestingly, such equilibria can coexist with interior equilibria in which funds are offered in both submarkets but entrepreneurs are not rationed in the low-attention market ( $Q_0 = 1$ ). In such a situation, VCs in the low-attention market are in excess supply and cannot fill up all of their capacity. An equilibrium of this type is therefore pinned down by jointly imposing the following conditions.

- (i) Entrepreneurs' entry and search strategies are such that entrepreneurs are ex ante indifferent and ex post optimize their search decision, that is,

$$\mathbb{E}_\lambda \max \{ (1 - \alpha)R(a_0, \lambda), Q_1(1 - \alpha)R(a_1, \lambda) \} = c, \quad (17)$$

where  $Q_1 = M_1 / \mathcal{E}(1 - F(\lambda_0))$  and

$$(1 - \alpha)R(a_0, \lambda_0) = Q_1(1 - \alpha)R(a_1, \lambda_0). \quad (18)$$

- (ii) VCs optimally choose their capacity, anticipating that the demand for low-attention funds and the market-clearing condition determine the size of their investment. In an interior equilibrium,

$$\gamma m_0 \{ \alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0 \} = m_1 \{ \alpha \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0] - R_0 \}, \quad (19)$$

with  $\gamma m_0 > m_1$  and ensuring market clearing:

$$(1 - \chi) \gamma m_0 = \mathcal{E} F(\lambda_0). \quad (20)$$

The market-clearing equation for the low-attention market reveals how the measure of entrepreneurs who enter,  $\mathcal{E}$ , which is pinned down by imposing the requirement that entrepreneurs' expected payoff equals the cost of entrepreneurship, affects VCs' decision: The measure  $\mathcal{E}$  determines the extent to which VCs can scale up their funds, that is,  $\gamma$ . This effect is muted when the endogenous demand for venture capital ( $\mathcal{E}$ ) is so high that low-attention funds operate in equilibrium at their maximum capacity.

PROPOSITION 7 (Marginal effect of the cost of entrepreneurship): *For every interior stable equilibrium  $(\chi, \lambda_0)$ , as  $c$  decreases:*

- (i) *relatively more low-attention investments are made, low-attention funds get bigger, and better entrepreneurs search in the low-attention market, that is,  $\frac{\partial \gamma M_0/M_1}{\partial c} < 0$ ,  $\frac{\partial \gamma}{\partial c} < 0$ , and  $\frac{\partial \lambda_0}{\partial c} < 0$ ;*
- (ii) *the distributions of returns and of project quality change as in Corollaries 2 and 3.*

Intuitively, lower costs of entrepreneurship are associated with an inflow of entrepreneurs in the market (the measure  $\mathcal{E}$  increases). The larger demand for venture capital has two implications. First, those VCs who offer low-attention funds can finance more projects. Second, congestion in the high-attention market increases due to reduced supply, leading some entrepreneurs to switch to the low-attention market. While it may seem mechanical that more demand for venture capital implies that VCs raise larger funds and hence add less value in equilibrium, it is only because of sorting that it also results in higher project quality conditional on VC attention and a more “dispersed” distribution of returns in the sense specified in Corollary 2.

*High Costs.* If costs are higher than  $c_M$ , yet sufficiently small to support equilibria with entry of a positive measure of entrepreneurs, that is, if  $c_M < c \leq c_H$ , it can be proven that the unique equilibrium of the game involves all VCs raising small, high-attention funds.

*Consistent Empirical Evidence.* Taken together, the results are broadly consistent with the patterns documented in Ewens, Nanda, and Rhodes-Kropf (2017). The authors identify a particular shock to the cost of starting new businesses and posit that VCs responded to it by shifting to a “spray and pray” investment approach. Their findings that VCs form larger portfolios and reduce their active governance, and that investments in the treated sector exhibit more disperse outcomes, corroborate their interpretation. My model indicates that the dispersion effect could emerge at the industry rather than the fund level, as a result of changes in the relative demand and supply of venture capital across different segments of the market and in their implications for the way different entrepreneurs match with different VCs.

## V. Novel Predictions and Policy Implications

### A. Novel Predictions

In this section, I discuss the novel predictions that emerge from the analysis. All predictions are related to VCs’ input in the firms they finance, which in the model is framed as VC attention. To proxy for the VC input, one could use the number of individual VC partners divided by the number of portfolio investments, possibly adjusting for the quality of the individual VC partner or VC firm (as measured by historical performance). In addition, the presence of a VC in a portfolio firm’s board can proxy for active governance, which

arguably employs scarce resources to improve firm performance. These proxies have precedent in the literature (Ewens and Rhodes-Kropf (2015) and Ewens, Nanda, and Rhodes-Kropf (2017)).

First, the model identifies endogenous deal flow as one source of decreasing returns to portfolio size, informing the literature on the scalability of venture capital.

**PREDICTION 1** (Sorting and returns to portfolio size): The quality of all firms in a VC's portfolio is increasing in the VC's input. Fixing the quality of firms in the VC's portfolio, the return to financing the marginal firm is strictly positive.

I stress that Prediction 1 pertains all firms in the VC's portfolio, not just the marginal, lowest quality firm. The model's central idea is that the quality of firms demanding capital is determined by the resources that VCs commit to employ (input). Ignoring this effect would lead the researcher to underestimate the limits to scalability in venture capital. The strength of the effect of VC input on the quality of deal flow depends on the distribution of VCs and firms across submarkets at a given point in time. Therefore, the fact that the historical analysis in Rossi (2019) fails to uncover decreasing returns to scale need not imply that VCs who do not increase their funds' size when they have the opportunity to do so (Metrick and Yasuda (2010)) are sacrificing profits. The pattern may reflect VCs' optimal response to endogenous sorting.

Second, Proposition 6 and Corollary 3 have implications that I restate below.

**PREDICTION 2** (Hot markets and the effect of entry on project quality): An exogenous increase in the supply of capital by low-skilled VCs (i) increases total excess returns to all VCs; (ii) increases the quality of firms financed by incumbent VCs, conditional on the VC's input; and (iii) may decrease the unconditional average quality of firms financed.

Importantly, the positive effect on financed firms' quality conditional on the VC's input is novel and crucial to distinguish this model's implications from more standard narratives of boom times, which predict that additional, lower quality firms receive financing when more capital flows in this asset class (as a reflection of irrational behavior or a rational response to changing opportunities).<sup>40</sup>

Third, the central economic force in the model is that VCs commit high attention to attract higher quality entrepreneurs. The effect is stronger the higher the downside risk—defined as the probability of financing extremely low-quality entrepreneurs by matching in the low-attention submarket—and the higher the upside risk—the probability of financing extremely high-quality entrepreneurs by matching in the high-attention submarket. In the [Internet Appendix](#) (Section I), I formally confirm this intuition by parameterizing upside and downside risk in the distribution of project quality and performing comparative statics around interior equilibria. The empirical prediction is stated below.

<sup>40</sup> See, for example, Yung (2017).

**PREDICTION 3** (Effect of upside and downside risk in project quality on VC input and size): If the distribution of project quality exhibits more downside and upside risk (defined in sufficiently extreme regions of the domain), average VC input is higher, and thus average portfolio size and total investment are lower.

Fourth, the model can be extended as follows. Some entrepreneurs actively approach VCs, and in doing so trade off VCs attention and the higher likelihood of being rationed. This is what my stylized model of directed search describes. Other entrepreneurs are referred to VCs by investors and portfolio companies: The VCs, through their networks, can access a “proprietary deal flow” that is not subject to entrepreneurs’ self-selection. While I detail this extension in the [Internet Appendix](#) (Section II), I present the main insight and empirical prediction below.

**PREDICTION 4** (Effect of source of deal flow on VC input and size): An increase in the share of matches generated from directed search compared to those that come from proprietary deal flow leads to an increase in average VC input—and thus a decrease in average portfolio size and total investment—if the average quality of projects from proprietary deal flow is sufficiently low.

Intuitively, if deal flow is in larger part proprietary, the motive to attract high-quality entrepreneurs is weaker, inducing fewer VCs to commit high attention. The caveat is that if the quality of entrepreneurs who a VC can directly source is sufficiently high, this effect may be reverted: Due to complementarities in the return function, VCs may find it optimal to concentrate their resources on only a few such projects.

### *B. Policy Implications and Optimal Taxation*

*Subsidizing Entry.* One implication of Proposition 5 is that, by favoring entry of unskilled VCs, subsidizing VCs’ activity is a policy that, at the margin, generates benefits beyond its direct costs. More generally, the analysis suggests that alternatives to VCs known to add less value, such as angel investors, may represent a positive externality. This result complements the analysis in Hellmann and Thiele (2015), who focus on the interaction between angels and VCs. Naturally, the merits of such a policy should be evaluated against the opportunity cost of public funds and distortionary taxation. For such an evaluation, one would need an estimate of the benefits of the sorting externality induced by VC entry.

*Banks in Venture Capital.* It has been documented that the availability of banks investing in venture capital funds as limited partners determines VCs’ fundraising capacity (Chen and Ewens (2021)). While in my model VCs are not financially constrained, a larger investor base would arguably reduce VCs’ cost of funding ( $R_0$ ), causing entry and inducing more VCs to raise large funds as discussed in Section III. Banks also invest in young firms directly, acting as general partners (Hellmann, Lindsey, and Puri (2008)). Whether they compete with high value-adding VCs for the best entrepreneurs or whether they choose

to spread their resources across large portfolios is an empirical question. This paper provides one channel through which, depending on their value-adding strategy, new financiers entering the market may induce a positive or a negative externality on the quality of the firms that receive venture capital.

*Optimal Tax.* I show, and formally state below, that there always exists a tax on VC profits that implements the second-best allocation. The optimal tax imposed on VCs equals a positive share of total profits  $\tau \in (0, 1)$  when the returns per unit of investment exceed a certain level, and zero otherwise. Intuitively, the tax tilts VCs' trade-off in favor of forming large, low-attention funds that generate lower returns per assets under management. Note that, at the sorting outcome implied by the second-best allocation, VCs would strictly prefer the high-attention fund in the absence of the tax.<sup>41</sup> As this is the case when  $\tau = 0$ , there must always exist a sufficiently large  $\tau$  to ensure that the relative return of a high- versus a low-attention fund induces the optimal measure of VCs to raise a large fund.

**PROPOSITION 8 (Optimal tax):** *Assume that a share  $\tau$  of the total profits made by VCs can be taxed, and  $\tau$  can be a function of the realized gross returns to VCs per assets under management, denoted by  $R^{vc}$ . Then, there exists a unique  $\tau^*$  such that the tax profile*

$$\tau(R) = \begin{cases} 0 & \text{if } R^{vc} < \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0^{sb}] \\ \tau^* & \text{otherwise} \end{cases} \quad (21)$$

*implements the second-best allocation.*

Notably, “overvalued” startups can represent an indirect way of improving the market outcome that would similarly operate on the trade-off between large, low-return funds and small, high-return funds.<sup>42</sup> The argument echoes the result in Miao and Wang (2018) that stock price bubbles alleviate inefficient underinvestment.<sup>43</sup>

## VI. Robustness and Extensions

I conclude the paper with some considerations on the main mechanism's robustness. I also discuss extensions that incorporate several features of the venture capital market.

<sup>41</sup> Equivalently, in the variant with heterogeneous VCs, it can be proven that the marginal VC would prefer high-attention funds at the induced second-best  $\lambda_0^{sb}$ .

<sup>42</sup> One way to model overvaluation would be to assume that the return an investment generates is  $R(a, \lambda) + b$ , where  $b$  represents a “bubble” component. The relative return of attracting high- versus low-quality entrepreneurs would decrease in  $b$ , motivating more VCs to form large portfolios. Different from the tax case,  $b$  would also affect entrepreneurs' optimal search. By reducing the motive to match with high-attention VCs, a larger  $b$  would also improve sorting. The formal analysis is omitted for brevity.

<sup>43</sup> In their model, bubbles relax credit constraints.

### A. Alternative Matching Models

Consider an alternative model in which the probability of matching is independent of total demand and supply. Assume that VCs and entrepreneurs negotiate, upon matching, a transfer that reflects competition among VCs within a submarket: The more VCs are in a submarket, the higher the transfer,  $p$ . Due to supermodularity, PAM would hold and the indifferent entrepreneur would satisfy  $(1 - \alpha)R(a_0, \lambda_0) + p_0 = (1 - \alpha)R(a_1, \lambda_0) + p_1$ . Clearly,  $p_0 > p_1$  and the threshold  $\lambda_0$  is increasing in the difference  $p_0 - p_1$ , indicating that a larger supply of high-attention VCs would imply a lower  $\lambda_0$ , an effect similar to the sorting externality that I discuss above.

Moreover, the inefficiency that I uncover exists in a version of the model in which perfect separation occurs in equilibrium. In the [Internet Appendix](#) (Section III), a model with continuous attention is introduced in which there are two types of VCs and two types of entrepreneurs.<sup>44</sup> In equilibrium, different VCs offer different levels of attention and entrepreneurs separate perfectly. I find that when low attention is not too detrimental, the equilibrium can be Pareto-improved by letting more VCs raise large funds, making the low-attention market attractive to some high-quality entrepreneurs, who then search in both markets with positive probability.

Finally, the static directed search framework approximates a market in which the cost of approaching multiple VCs simultaneously is large. In a dynamic search setup, such a cost can be represented by a discount factor applied between successive search attempts.<sup>45</sup> Modeling the dynamics of a market with directed search and private information and in which such a discount factor is not prohibitively high is beyond the scope of this paper. Using insights from this paper and existing literature (Wright et al. (2021)), one could envision the existence of equilibria in which some VCs fill up their capacity more slowly, that is, at later stages, provided they expect better entrepreneurs to demand financing in later stages (if two successive stages  $t, t + 1$  are characterized by cutoff strategies  $\lambda_0^t$  and  $\lambda_0^{t+1}$ , this means  $\lambda_0^{t+1} > \lambda_0^t$ ).<sup>46</sup> Interestingly, those VCs who supply funds in future stages would exercise a negative externality on those that supply funds in earlier stages, since lower congestion at time  $t + 1$  would encourage entrepreneurs to attempt matching in (and therefore overcrowd) the high-value-added segment of the market at time  $t$ . The extension

<sup>44</sup> While the extension is mainly introduced to allow for a more realistic continuous size-attention trade-off, it provides a convincing case for why imperfect separation is driving the inefficiency.

<sup>45</sup> This could reflect preferences as well as, realistically, technological frictions in the search process.

<sup>46</sup> One can verify that, with sufficiently strong complementarities in  $R$ , such an outcome can be compatible with optimal search behavior: Given congestion  $Q_0^t > Q_1^t$  and  $Q_0^{t+1} > Q_1^{t+1}$ , low-quality entrepreneurs would prefer to always search in the low-attention market, high-quality entrepreneurs would prefer to always search in the high-attention market, and entrepreneurs in  $[\lambda_0^t, \lambda_0^{t+1}]$  would first attempt matching to high-attention VCs, and if unsuccessful, search for low-attention VCs at  $t + 1$ .



could offer a novel angle to think about the life-cycle of venture capital funds. I leave such analysis for future work.

### *B. The Financing Contract*

The exogenous surplus share assumption makes the model tractable, and has been used previously (Sørensen (2007)). While its most immediate interpretation is that each party receives an exogenous equity share, I show in the [Internet Appendix](#) (Section VIII) that the model can accommodate an environment in which the financing contract is a mix of equity and debt.<sup>47</sup> Crucially, any specification must assume that (i) VCs' payoff is positively affected by project quality and (ii) high-quality entrepreneurs gain more from receiving high attention. Regardless of the full contractual arrangement, condition (i) would be satisfied if VCs must hold a minimum equity stake—which is realistic and typically necessary to induce optimal effort by VCs.

VCs may use contracts to screen entrepreneurs or to attract entrepreneurs of higher quality. In the [Internet Appendix](#) (Section IX), I present a version of the model in which VCs can “post” contracts: An equity share and a transfer paid to entrepreneurs upon matching. By offering them a higher equity share, VCs can form large portfolios while remaining attractive to high-quality entrepreneurs, that is, the adverse selection concern associated with providing low attention is reduced. However, the motive to form small portfolios to attract the best entrepreneurs remains, implying equilibria in which VCs efficiently operate at large scale remain fragile to such deviations. I also show that the presence of a minimum equity share to be given to VCs effectively limits their ability to use contracts as screening devices. As discussed in Sørensen (2007), it is also difficult for VCs to commit contract terms *ex ante*, given successive investment rounds naturally involve new investors.

Finally, I assume that VCs and entrepreneurs cannot negotiate different surplus shares (and contracts) *ex post*, which is natural due to entrepreneurs' private information. Ewens, Gorbenko, and Korteweg (2022) estimate a model of venture capital in which negotiations occur after VCs and entrepreneurs learn about each others' quality and find that matching patterns do not necessarily exhibit positive assortativity in the entire quality domain. This is not to be seen in contrast with my model, where the focus is on initial selection of entrepreneurs—below, I argue that self-selection and its consequences remain a robust feature even when some form of learning is allowed.

### *C. Ex Post Screening and Reallocating Attention*

For tractability and transparency, VCs are assumed not to observe any signal of project quality *ex post*. Realistically, VCs engage in due diligence and gradually learn about the quality of the projects they finance. The nature of

<sup>47</sup> For an environment with pure debt contracts to generate the same mechanism, the probability of debt not being repaid must be an increasing function of project quality.

the projects receiving venture capital suggests that this learning is imperfect and substantial private information persists.<sup>48</sup> I highlight two ways in which learning can interact with the model's mechanism. In both cases, VCs still have a motive not to raise large funds in order to avoid adverse selection, provided learning is not perfect; such a mechanism fully unravels only in the (unrealistic) limit case that learning is perfect.

First, VCs who learn project quality upon matching can choose to fill their capacity with the best projects that apply, and reject the others. In turn, this affects entrepreneurs' choice *ex ante*. Intuitively, with perfect (or costless) learning, the sorting equilibrium will completely unravel. In the [Internet Appendix](#) (Section V), I assume that with an exogenous probability  $p < 1$  that is independent across VCs, a VC can observe project quality (one interpretation is that VCs receive imprecise signals about project quality). I show that the negative externality that high-attention funds impose continues to exist.

Second, when learning about projects' quality, VCs may optimally shift their attention toward the most promising projects. Note that by raising a smaller fund, and thus limiting their capacity, VCs can commit their scarce attention to fewer projects: In the model, a small fund means entrepreneurs will receive the highest attention.<sup>49</sup> In the [Internet Appendix](#) (Section VI), I formally illustrate how the model's core mechanism is robust to attention reallocation provided some attention cannot be shifted away from the original project—a friction that can be thought of as a manifestation of gradual learning over time.

#### D. Endogenous Entrepreneurs Quality

Assume that, after learning their quality, agents who become entrepreneurs can choose to receive an outside option  $u$  instead of venture capital financing. I distinguish two alternative scenarios. I provide details in the [Internet Appendix](#) (Section VII).

*Unmatched Entrepreneurs Receive Outside Option.* In a setting in which an entrepreneur who fails to match can receive their outside option  $u$ , the only addition to the original model is the condition that all entrepreneurs who receive low attention do not prefer their outside option, that is,  $(1 - \alpha)R(a_0, \lambda) \geq u$  and the same holds for those  $\lambda$ s receiving high attention, or  $(1 - \alpha)R(a_1, \lambda) \geq u$ . The analysis follows the original model. While the main results continue to hold, one intriguing new effect is that equilibria in which all VCs offer low attention are more likely to emerge the larger the outside option  $u$ .<sup>50</sup>

<sup>48</sup> Examples such as the events leading to the liquidation of the startup *Theranos* demonstrate how investors' learning can be very limited.

<sup>49</sup> The way venture capital funds are structured allows VCs to commit not to dilute their resources across too many portfolio companies. Contractual restrictions prevent VCs from forming successor funds before the existing one is substantially invested (Gompers and Lerner (1996) and Ramsinghani (2014)). Typically, such covenants find economic justification in agency problems between VCs and their investors (Cumming and Johan (2013)).

<sup>50</sup> The reason is that a subset of low-quality entrepreneurs would remain inactive and prefer their outside option, making the VCs finance, on average, better entrepreneurs, and therefore

*Unmatched Entrepreneurs Lose the Outside Option (Costly Search).* If one introduces the additional friction that searching for the first VC is costly, in the sense that entrepreneurs who remain unmatched cannot receive their outside option, the two restrictions above would be replaced by  $Q_0(1 - \alpha)R(a_0, \lambda) \geq u$  and  $Q_1(1 - \alpha)R(a_1, \lambda) \geq u$ . For sufficiently large  $u$ , only entrepreneurs above a threshold would prefer to search in the market, with the threshold  $\lambda_l$  satisfying  $Q_0(1 - \alpha)R(a_0, \lambda_l) = u$ . This addition to the model comes at a considerable cost in terms of tractability.<sup>51</sup> Plausibly, in the modified model, a direct effect of increasing the supply of low-attention VCs would be to induce more entrepreneurs to search, reducing  $\lambda_l$ . This effect would work in parallel to and against the sorting externality in the low-attention market. In the [Internet Appendix](#), I analyze a specification of the model and offer a numerical analysis that confirms the intuition above and I provide examples in which the sorting externality prevails.

#### E. Entrepreneurial Rents and the Hold-Up Problem

Consider an economy in which entrepreneurs are not in large supply and can make rents in expectation. VCs' choice to commit high attention is, in effect, a costly investment in the return to every match with an entrepreneur. Should the good entrepreneurs who select in the high-attention market capture a larger surplus share, VCs would have a lower incentive to "invest" in attention ex ante. This is the classic hold-up problem, a version of which has been analyzed by Bester (2013) in a matching environment: Intuitively, the long side of the market does not invest. That model features perfect information and random search. Due to the mechanism I uncover, effects would be more nuanced in my environment. Empirically, there is no evidence that high-quality entrepreneurs obtain more favorable terms from the highly skilled VCs they match with, compared to what low-quality entrepreneurs would obtain. Hsu (2004) and Ewens, Gorbenco, and Korteweg (2022) support the opposite case: Skilled VCs are scarce and those that generate more value-added extract a larger surplus share. In my model, this would represent an additional motive to commit high attention.

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be less willing to deviate to offer high attention. I show that the set of parameters and family of functions such that this corner equilibrium exists given outside option  $u$  contains the set of parameters and family of functions such that it exists when this outside option is lower.

<sup>51</sup> In particular, the sorting subgame equilibrium is not unique given VC funds supply, and an interior solution  $(\lambda_l, \lambda_0)$  need not exist. Moreover, the threshold  $\lambda_l$  does not depend on the relative congestion  $Q_0/Q_1$ , but on the absolute  $Q_0$ , implying the measure of entrepreneurs who enter in the first stage,  $\mathcal{E}$ , has an allocative effect in equilibrium and therefore represents an additional source of externality.

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## Appendix

*The Sorting Subgame.* It is immediate to generalize the arguments of Section I to the case in which VCs are heterogeneous in skill (introduced in Section III). I therefore prove the statements in the more general setup. Note that skill does not enter the function  $R$  directly, but only indirectly through the attention function  $a(m, x)$ . Every combination of fund size and VC skill,  $(w, x)$ , is a submarket toward which entrepreneurs can direct search. Denote by  $\mathcal{M}$  the set of all available submarkets, so that  $\mathcal{M} \subseteq [\underline{w}, \bar{w}] \times [\underline{x}, \bar{x}]$ . Formally, the entrepreneur strategy is a mapping  $s : [\underline{\lambda}, \bar{\lambda}] \rightarrow \Delta(\mathcal{M})$ . For every  $\lambda$ , the strategy generates a cumulative density function  $S(w, x; \lambda)$ . Define  $\tilde{S}(w, x, \mathcal{E})$  as the measure of entrepreneurs searching in markets with size below  $w$  and VC skill below  $x$ , given the measure of entrepreneurs who enter  $\mathcal{E}$ . This is given by summing the search strategy over all entrepreneurs, so

$$\tilde{S}(w, x; \mathcal{E}) = \int_{\lambda} \mathcal{E} S(w, x; \lambda) dF(\lambda). \quad (\text{A1})$$

Define  $H(w, x)$  as the equilibrium measure of VCs with skill below  $x$  that raise funds of size below  $w$ . The aggregate supply of funds in submarkets below  $(w, x)$  is given by  $\int_{-\infty}^x \int_{-\infty}^w \hat{w} dH(\hat{w}, \hat{x})$ . Therefore, market tightness, denoted by  $\theta_{w,x}(\mathcal{E})$ —where I make explicit that it depends on  $\mathcal{E}$ —will solve:

$$\int_{-\infty}^x \int_{-\infty}^w \hat{w} dH(\hat{w}, \hat{x}) = \int_{-\infty}^x \int_{-\infty}^w \theta_{\hat{w}, \hat{x}}(\mathcal{E}) d\tilde{S}(\hat{w}, \hat{x}; \mathcal{E}). \quad (\text{A2})$$

Note that since no VC will offer funds in submarkets in which no entrepreneur is searching, the measure  $\tilde{H}(w, x) := \int_{-\infty}^x \int_{-\infty}^w \hat{w} dH(\hat{w}, \hat{x})$  is absolutely continuous with respect to  $\tilde{S}$ . Therefore, market tightness in every submarket available in equilibrium is well defined and given by the Radon-Nikodym derivative  $d\tilde{H}/d\tilde{S}$ .

PROOF OF LEMMA 1: I first prove that, under Assumption 1, all VCs operate at full capacity (formally, denoting by  $n_{w,x}$  the measure of entrepreneurs who match with a VC in submarket  $(w, x)$ , I prove that  $Q_{\tilde{w}, \tilde{x}}(\mathcal{E}) < 1$  and thus  $n_{w,x} = w$ ). Consider first those submarkets  $(w, x)$  for which  $\tilde{S}(w, x; \mathcal{E}) > 0$ . Take one submarket  $(\tilde{w}, \tilde{x})$  in which there is no congestion, that is  $Q_{\tilde{w}, \tilde{x}}(\mathcal{E}) = 1$ . By entering and searching in this submarket, an entrepreneur that is outside the market gets in expectation

$$(1 - \alpha) \mathbb{E} R(a(n_{\tilde{w}, \tilde{x}}(\mathcal{E}), \tilde{x}), \lambda) - c \geq (1 - \alpha) \mathbb{E} R(a_0, \lambda) - c > 0, \quad (\text{A3})$$

where the last inequality is Assumption 1. Hence, when  $\tilde{S}(w, x; \mathcal{E}) > 0$ , it must be the case that  $Q_{\tilde{w}, \tilde{x}}(\mathcal{E}) < 1$ . To show that  $\tilde{S}(w, x; \mathcal{E}) > 0$  for all  $(w, x)$ , assume

not and denote  $(\hat{w}, \hat{x})$  the submarket in which no entrepreneur searches. Then any type  $\lambda$  that entered the market could search in  $(\hat{w}, \hat{x})$ , as this would give:

$$(1 - \alpha)R(a_1, \lambda) > Q_{w,x}(\mathcal{E})(1 - \alpha)R(a(n_{w,x}(\mathcal{E}), x), \lambda) \quad \forall (w, x), \quad (\text{A4})$$

and hence the deviation would be profitable.

Given VCs operate at full capacity, the payoff to type  $\lambda$  in market  $(w, x)$  conditional on matching is given by  $(1 - \alpha)R(a(w, x), \lambda)$ .

I can now prove the statement of Lemma 1. Take two markets  $(w, x)$  and  $(w', x')$ , with associated attention levels  $a$  and  $a'$ , with  $a = a'$ . Assume that  $Q_{w,x}(\mathcal{E}) > Q_{w',x'}(\mathcal{E})$ . Then, any entrepreneur searching in  $(w', x')$  could deviate to  $(w, x)$  and get:

$$Q_{w,x}(\mathcal{E})(1 - \alpha)R(a, \lambda) > Q_{w',x'}(\mathcal{E})(1 - \alpha)R(a', \lambda). \quad (\text{A5})$$

Therefore, for any two markets in which attention is constant, congestion is constant. Formally,  $Q_{w,x}(\mathcal{E}) = Q_{w',x'}(\mathcal{E})$  if  $a(w, x) = a(w', x')$ .

Entrepreneurs maximize  $Q_{w,x}(\mathcal{E})(1 - \alpha)R(a(w, x), \lambda)$ , and, by the results above,  $Q_{w,x}(\mathcal{E}) = \theta_{w,x}(\mathcal{E})$ . Since in the reduced model described in Lemma 1 entrepreneurs are matched to VCs within submarkets proportionally to each VC size, it has to be the case that if one VC operates at full capacity, all of them do. Thus, the results above apply to the transformed model as well. Because  $R(a(w, x), \lambda)$  is constant across an iso-attention region, all that remains to show is that the ratio of total supply to total demand is the same in submarket  $a$  as it is at any point in the iso-attention region in the original model. That is, formally,  $\theta(a) = \theta_{w,x}(\mathcal{E})\forall (w, x) : a(w, x) = a$ . Denote by  $\Gamma(a)$  the measure of funds across all VCs at a given iso-attention region,  $a$ , and  $S(a)$  the measure of entrepreneurs searching in the same region. Then,

$$S(a) := \int_{\{(w,x):a(w,x)=a\}} \int_{\lambda} \mathcal{E} dS(w, x; \lambda) \quad (\text{A6})$$

and

$$\Gamma(a) := \int_{\{(w,x):a(w,x)=a\}} wdH(w, x). \quad (\text{A7})$$

Note that, by the definition of  $\theta_{w,x}(\mathcal{E})$ ,

$$d\tilde{S}(w, x; \mathcal{E})\theta_{w,x}(\mathcal{E}) = wdH(w, x). \quad (\text{A8})$$

Integrating on both sides over a given iso-attention region, and taking  $\theta_{w,x}(\mathcal{E})$  outside of the integral (since it is constant), it follows that

$$\theta_{w,x}(\mathcal{E}) \int_{\{(w,x):a(w,x)=a\}} d\tilde{S}(w, x, \mathcal{E}) = \int_{\{(w,x):a(w,x)=a\}} wdH(w, x). \quad (\text{A9})$$

Therefore,

$$\theta_{w,x}(\mathcal{E}) = \left( \int_{\{(w,x):a(w,x)=a\}} wdH(w,x) / \int_{\{(w,x):a(w,x)=a\}} d\tilde{S}(w,x,\mathcal{E}) \right) = \Gamma(a)/S(a) = \theta(a), \quad (\text{A10})$$

since, by definition,  $\theta(a)$  is the ratio of total funds over the measure of entrepreneurs in the submarket defined by  $a$ .  $\square$

**PROOF OF PROPOSITION 1: Sufficiency.** Assume that  $R(a, \lambda)$  is log-supermodular everywhere. If there is an equilibrium that does not exhibit PAM everywhere, then there must exist at least two markets  $a, a'$  with  $a > a'$ , and two types  $\lambda', \lambda$  with  $\lambda' > \lambda$  such that  $\lambda$  searches in  $a$  and  $\lambda'$  searches in  $a'$ .<sup>52</sup> Optimality of the search strategy requires that type  $\lambda$  is at least as well off searching in  $a$  as in  $a'$ , and, similarly,  $\lambda'$  (weakly) prefers  $a'$  to  $a$ , that is,

$$Q(a)R(a, \lambda) \geq Q(a')R(a', \lambda), \quad (\text{A11})$$

$$Q(a')R(a', \lambda') \geq Q(a)R(a, \lambda'). \quad (\text{A12})$$

The two inequalities imply that

$$\frac{R(a, \lambda)}{R(a', \lambda)} \geq \frac{R(a, \lambda')}{R(a', \lambda')}, \quad (\text{A13})$$

which contradicts the fact that  $R(a, \lambda)$  is log-supermodular.

**Necessity.** Assume that  $R(a, \lambda)$  is not log-supermodular at some point  $(\hat{a}, \hat{\lambda})$ . The continuity properties of  $R(a, \lambda)$  (see Section I) imply that there exists a number  $\varepsilon > 0$  such that the function is not log-supermodular anywhere in  $[\hat{a} - \varepsilon, \hat{a} + \varepsilon] \times [\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ . I construct an economy in which negative assortative matching (NAM) could be supported, and hence a contradiction arises. Let the support of  $f$  be exactly  $[\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ , and  $a_i \in [\hat{a} - \varepsilon, \hat{a} + \varepsilon]$ , for all  $i$ . By construction, all matches will be in  $[\hat{a} - \varepsilon, \hat{a} + \varepsilon] \times [\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ . For only PAM sorting patterns to emerge, a necessary condition is that for at least two  $(\lambda, \lambda')$  with  $\lambda > \lambda'$  and two  $(a, a')$  with  $a > a'$ ,

$$Q(a)R(a, \lambda) \geq Q(a')R(a', \lambda), \quad (\text{A14})$$

$$Q(a')R(a', \lambda') \geq Q(a)R(a, \lambda'), \quad (\text{A15})$$

and, crucially, at least one of the two inequalities—for at least one such pair—is strict.<sup>53</sup> When (A14), (A15), or both are satisfied with strict inequality, it holds that

$$\frac{R(a, \lambda)}{R(a', \lambda)} > \frac{R(a, \lambda')}{R(a', \lambda')}, \quad (\text{A16})$$

<sup>52</sup> This proof follows through when  $a$  can take any finite number of values. In fact, the result holds in the continuum too, but must be proven via different techniques. In other words, Proposition 1 is valid when the domain of the attention function is any subset of  $\mathbb{R}$ .

<sup>53</sup> Otherwise, it would be possible to support an equilibrium with NAM, and the contradiction would immediately arise.

which means that  $R(a, \lambda)$  is log-supermodular somewhere in  $[\hat{a} - \varepsilon, \hat{a} + \varepsilon] \times [\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon]$ , a contradiction.  $\square$

**PROOF PROPOSITION 2: Interior Equilibria.** By Proposition 1, sorting in the subgame must exhibit PAM: All entrepreneurs below  $\lambda_0$  search in market  $a_0$ , whereas those above search in  $a_1$ . Type  $\lambda_0$  must therefore satisfy the indifference condition (8). Clearly, for an equilibrium to be interior, VCs must be indifferent between either strategy, giving condition (7).

[In the general case with heterogeneous VCs, a VC with skill  $x$  will prefer the large fund if and only if

$$m_0^x \{\alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0] - R_0\} > m_1^x \{\alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0] - R_0\}, \quad (\text{A17})$$

which can be rewritten as

$$\frac{m_1^x}{m_0^x} < \frac{\alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0] - R_0}{\alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0] - R_0}. \quad (\text{A18})$$

The right-hand side of (A18) is independent of  $x$ . The left-hand side is continuous and increasing in  $x$ . Therefore, if (A18) holds for some  $x$ , it will hold for any VC with ability below  $x$ . Moreover, if the same inequality is reverted for some  $x' > x$ , then, by the Intermediate Value Theorem, there exists a level of skill  $\tilde{x} \in [x, x']$  such that the payoff from  $a_0$  and  $a_1$  is the same.]

**Corner Equilibria.** Consider first a candidate equilibrium in which every VC raises a large fund (i.e.,  $\chi = 0$ , or, if VCs differ in skill,  $x_0 = \bar{x}$ ). The belief  $\beta_{a_1}$  is off-equilibrium and therefore disciplined by Requirement 1. To identify it, note that the set  $\mathcal{Q}(\lambda; a_1)$  of matching probabilities at which type  $\lambda$  is willing to search in  $a_1$  consists of all  $Q$ s such that

$$Q(1 - \alpha)R(a_1, \lambda) \geq Q_0(1 - \alpha)R(a_0, \lambda), \quad (\text{A19})$$

and therefore, the sets  $\mathcal{Q}(\lambda; a_1)$  are intervals  $[\underline{Q}(\lambda), 1]$  where  $\underline{Q}(\lambda)$  satisfies (A20) with equality for type  $\lambda$ . Since

$$\underline{Q}(\lambda) = Q_0 \frac{R(a_0, \lambda)}{R(a_1, \lambda)} \quad (\text{A20})$$

is decreasing in  $\lambda$  by log-supermodularity of  $R$ , it follows that  $\mathcal{Q}(\lambda; a_1)$  is maximal for type  $\bar{\lambda}$ . The belief  $\beta(a_1)$  is degenerate at  $\bar{\lambda}$  and therefore the condition in Proposition 2 ensures that no VC is willing to deviate.

The same logic applies to the case in which every VC raises a small fund (i.e.,  $\chi = 1$ , or, if VCs differ in skill,  $x_0 = \underline{x}$ ), to show that  $\mathcal{Q}(\lambda; a_0)$  is maximal for type  $\underline{\lambda}$ .

It remains to show that both  $\mathcal{Q}(\lambda; a_1)$  and  $\mathcal{Q}(\lambda; a_0)$  are not empty—that is, that such  $\underline{Q}(\lambda)$  exist—so that the two conditions are indeed necessary. When every VC is raising a large fund, the free-entry condition—condition (ii) in



Definition 1—gives:

$$Q_0(1 - \alpha)\mathbb{E}_\lambda R(a_0, \lambda) = c, \quad (\text{A21})$$

and rearranging:

$$Q_0 = \frac{c}{(1 - \alpha)\mathbb{E}_\lambda R(a_0, \lambda)}, \quad (\text{A22})$$

which is smaller than one by Assumption 1. For  $\mathcal{Q}(\lambda; a_1)$  to be nonempty, it must be the case that

$$Q(1 - \alpha)R(a_1, \lambda) \geq Q_0(1 - \alpha)R(a_0, \lambda) \quad (\text{A23})$$

for some  $Q \in [0, 1]$  and some  $\lambda$ . Clearly, this is true for any  $Q \geq Q_0$  and any  $\lambda$ .

When every VC is raising a small fund, the free-entry condition gives

$$Q_1 = \frac{c}{(1 - \alpha)\mathbb{E}_\lambda R(a_1, \lambda)}. \quad (\text{A24})$$

To ensure that there exists a  $Q$  and a  $\lambda$  that would search in  $a_0$  if the probability of finding a match is  $Q$ , set  $Q = 1$  and consider type  $\underline{\lambda}$ . Then, searching in the off-equilibrium market is worthwhile if

$$(1 - \alpha)R(a_0, \underline{\lambda}) > \frac{c}{(1 - \alpha)\mathbb{E}_\lambda R(a_1, \lambda)}(1 - \alpha)R(a_1, \underline{\lambda}). \quad (\text{A25})$$

Rearranged, this gives

$$\frac{(1 - \alpha)\mathbb{E}_\lambda R(a_1, \lambda)}{c} > \frac{R(a_1, \underline{\lambda})}{R(a_0, \underline{\lambda})}, \quad (\text{A26})$$

which is exactly Assumption 3.

*Existence.* The existence proof proceeds in four steps.

Step 1. I first show that, given any equilibrium strategy vector  $(\chi, \lambda_0)$ , a measure  $\mathcal{E}$  such that equilibrium  $Q_i$  satisfies free entry exists. Note that entrepreneurs' free entry can be rewritten as

$$\mathbb{E}_\lambda \max \{Q_0(1 - \alpha)R(a_0, \lambda), Q_1(1 - \alpha)R(a_1, \lambda)\} = c. \quad (\text{A27})$$

Substituting the definitions of  $Q_0$  and  $Q_1$ , where  $M_0$  and  $M_1$  denote total fund supply in the two submarkets, gives

$$\mathbb{E}_\lambda \max \left\{ \frac{M_0}{\mathcal{E}(F(\lambda_0))}(1 - \alpha)R(a_0, \lambda), \frac{M_1}{\mathcal{E}(1 - F(\lambda_0))}(1 - \alpha)R(a_1, \lambda) \right\} = c. \quad (\text{A28})$$

Thus,

$$\frac{1}{\mathcal{E}} \mathbb{E}_\lambda \max \left\{ \frac{M_0}{F(\lambda_0)}(1 - \alpha)R(a_0, \lambda), \frac{M_1}{(1 - F(\lambda_0))}(1 - \alpha)R(a_1, \lambda) \right\} = c. \quad (\text{A29})$$

Since by Assumption 1,  $(1 - \alpha)\mathbb{E}_\lambda R(a_0, \lambda) > c$ , there must exist a sufficiently small  $\varepsilon \in \mathbb{R}$  such that the left-hand side of (A29) is larger than the right-hand side. As  $\varepsilon$  grows and approaches infinity, the opposite inequality holds. Therefore, by the Intermediate Value Theorem, an  $\varepsilon$  that satisfies (A29) exists.

Step 2. I show that any equilibrium VC strategy profile induces one and only one cutoff in the entrepreneurs' search strategy. For any measure of VCs in the high-attention market,  $\chi$ , the indifferent entrepreneur,  $\lambda_0$ , solves equation (8). It is useful to rewrite it as

$$\frac{R(a_1, \lambda_0)}{R(a_0, \lambda_0)} = \frac{M_0(\chi)}{M_1(\chi)} \frac{(1 - F(\lambda_0))}{F(\lambda_0)}. \quad (\text{A30})$$

The left-hand side of (A30) is continuous and strictly increasing in  $\lambda_0$  by assumption (as  $R$  is continuous in both arguments and is log-supermodular). The right-hand side is continuous and strictly decreasing in  $\lambda_0$ . In particular, note that the left-hand side is positive and finite for all  $\lambda_0 \in [\underline{\lambda}, \bar{\lambda}]$ . The right-hand side is zero as  $\lambda_0$  tends to  $\bar{\lambda}$  and tends to infinity as  $\lambda_0$  tends to  $\underline{\lambda}$ . Therefore, by the Intermediate Value Theorem, for any given  $\chi$ , there exists a cutoff  $\lambda_0$ , and this cutoff is unique by monotonicity. Recall that  $M_0 = (1 - \chi)m_0$  and  $M_1 = \chi m_1$ . Therefore, the ratio  $M_0(\chi)/M_1(\chi)$  is continuous and decreasing in  $\chi$ . Thus, the (unique) function  $\lambda_0(\chi)$  is continuous and decreasing. Moreover, when  $\chi$  tends to one,  $M_0(\chi)/M_1(\chi)$  tends to zero, and hence  $\lim_{\chi \rightarrow 1} \lambda_0(\chi) = \underline{\lambda}$ . Similarly, if  $\chi$  tends to zero,  $M_0(\chi)/M_1(\chi)$  tends to infinity, and hence  $\lim_{\chi \rightarrow 0} \lambda_0(\chi) = \bar{\lambda}$ . The limits of  $\lambda_0(\chi)$  in the two corner equilibria are, therefore, the types selected by Requirement 1.

Step 3. Substitute  $\lambda_0 = \lambda_0(\chi)$  into the VC's indifference condition and observe that, if a solution to the equation below exists, an interior equilibrium  $(\chi, \lambda_0)$  exists.

$$m_1\{\alpha\mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0(\chi)]\} - R_0 = m_0\{\alpha\mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0(\chi)] - R_0\}. \quad (\text{A31})$$

Step 4. I now cover the case in which an interior equilibrium does not exist, and therefore the two functions  $m_0/m_1$  and

$$\phi(\lambda_0(\chi)) := \frac{\alpha\mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0(\chi)] - R_0}{\alpha\mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0(\chi)] - R_0} \quad (\text{A32})$$

do not intersect at any  $\chi \in [0, 1]$ . Start from the case  $m_0/m_1 > \phi(\lambda_0(\chi))$  for all  $\chi \in [0, 1]$ . Note that the inequality also applies at  $\chi = 0$ , that is,  $m_0/m_1 > \phi(\lambda_0(0)) = \phi(\bar{\lambda})$ , and note further that  $\phi(\bar{\lambda}) = (\alpha R(a_1, \bar{\lambda}) - R_0)/(\alpha\mathbb{E}[R(a_0, \lambda)] - R_0)$ . Consider a candidate equilibrium in which every VC offers low attention. Then, from Proposition 2 part (iii), by deviating to high attention a VC will attract the highest quality entrepreneur,  $\bar{\lambda}$ . No VC finds this profitable as long as:

$$m_0\{\alpha\mathbb{E}[R(a_0, \lambda)] - R_0\} > m_1\{\alpha R(a_1, \bar{\lambda}) - R_0\}, \quad (\text{A33})$$

which is satisfied because of the case I am restricting attention to. Therefore, a (corner) equilibrium where every VC offers  $a_0$  exists.

Assume instead that  $m_0/m_1 < \phi(\lambda_0(\chi))$  for all  $\chi \in [0, 1]$ . Note that the inequality also applies at  $\chi = 1$ , that is,  $m_0/m_1 < \phi(\lambda_0(1)) = \phi(\underline{\lambda})$ , and note further that  $\phi(\underline{\lambda}) = (\alpha \mathbb{E}[R(a_1, \lambda)] - R_0)/(\alpha R(a_0, \underline{\lambda}) - R_0)$ . Consider a candidate equilibrium in which every VC offers high attention. Then from Proposition 2, part (iii), by deviating to low attention a VC will attract the lowest quality entrepreneur,  $\underline{\lambda}$ . No VC finds this profitable as long as

$$m_0\{\alpha R(a_0, \underline{\lambda}) - R_0\} < m_1\{\alpha \mathbb{E}[R(a_1, \lambda)] - R_0\}, \quad (\text{A34})$$

which is satisfied because of the case I am restricting attention to. Therefore, a (corner) equilibrium in which every VC offers  $a_1$  exists.

[It is immediate to extend these arguments to the case with heterogeneous VCs, after noting that in such a case,  $M_0 = \int_0^{x_0} m_0^x dx$ , and  $M_1 = \int_{x_0}^1 m_1^x dx$ . Therefore, the ratio  $M_0(x_0)/M_1(x_0)$  is continuous and increasing in  $x_0$ , as  $\partial m_i^x / \partial x$  is positive,  $m_0^x$  is continuous, and the skill distribution is continuous. Therefore, the (unique) function  $\lambda_0(x_0)$  is continuous and increasing. Moreover,  $\lim_{x_0 \rightarrow \underline{x}} \lambda_0(x_0) = \underline{\lambda}$  and  $\lim_{x_0 \rightarrow \bar{x}} \lambda_0(x_0) = \bar{\lambda}$ .]  $\square$

PROOF LEMMA 2: The previous proof shows that  $\lambda_0$  is unique given  $\chi$ , and that the function  $\lambda_0(\chi)$  is monotone and strictly decreasing everywhere in  $(0, 1)$ . Noting that the ratio  $M_0/M_1$  is given by

$$\frac{(1 - \chi)m_0}{\chi m_1}, \quad (\text{A35})$$

the statement follows from observing that  $M_0/M_1$  is uniquely pinned down by  $\chi$ , and the function  $\frac{M_0}{M_1}(\chi)$  is monotone and decreasing everywhere in  $(0, 1)$ .

[The argument applies to the case with heterogeneous VCs:  $\frac{M_0}{M_1}(x_0)$  is monotone and increasing in the cutoff  $x_0 \in (0, 1)$ .]  $\square$

PROOF PROPOSITION 3: Denote by  $\chi^*$  the lowest interior equilibrium measure  $\chi$ . This corresponds, by Proposition 4 (proven below), to the Pareto-superior interior equilibrium. In the remainder of the proof, I set investors' outside option,  $R_0$ , to zero. This serves to better visualize each equation; a strictly positive  $R_0$  would make no difference. The proof proceeds in two steps.

Step 1. First, I show that any allocation  $\tilde{\chi} > \chi^*$  delivers lower welfare than  $\chi^*$ . Note that welfare induced by an allocation  $\tilde{\chi}$  is bounded above by what total returns would be if, given the sorting subgame, VCs could optimally select fund size—as they do in equilibrium. Since VCs are indifferent in interior equilibria, this means that welfare  $W$  satisfies

$$W(\tilde{\chi}) \leq \max \{m_0 \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(\tilde{\chi})], m_1 \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(\tilde{\chi})]\}. \quad (\text{A36})$$

Hence, for all  $\tilde{\chi} > \chi^*$ ,

$$W(\chi^*) = m_0 \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(\chi^*)] = m_1 \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(\chi^*)] > W(\tilde{\chi}), \quad (\text{A37})$$

where the inequality is strict because  $\tilde{\chi} > \chi^*$  implies  $\lambda_0(\tilde{\chi}) < \lambda_0(\chi^*)$ .

Step 2. Second, I show that the second-best problem can be improved by a marginal decrease in  $\chi$ , starting from  $\chi^*$ . To see why, write the objective function

$$W(\chi) = M_0(\chi) \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(\chi)] + M_1(\chi) \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(\chi)]. \quad (\text{A38})$$

So

$$\begin{aligned} \frac{\partial W(\chi)}{\partial \chi} \Big|_{\chi=\chi^*} &= \underbrace{\{-m_0 \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(\chi^*)] + m_1 \alpha \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(\chi^*)]\}}_{=0} \\ &\quad + \alpha \left\{ \left[ M_0(\chi^*) \frac{\partial \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(\chi^*)]}{\partial \lambda_0} + M_1(\chi^*) \frac{\partial \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(\chi^*)]}{\partial \lambda_0} \right] \frac{\partial \lambda_0(\chi^*)}{\partial \chi} \right\} < 0, \end{aligned} \quad (\text{A39})$$

where the term in the first bracket is zero because VCs are indifferent in interior equilibria. Evidently, no such marginal deviation is possible if one starts from a corner equilibrium in which every VC raises a low-attention fund. Hence, from the results in steps 1 and 2, if such an equilibrium exists, it corresponds to the second-best allocation.

[Noting that any equilibrium in which the cutoff VC  $\tilde{x}_0$  satisfies  $\tilde{x}_0 < x_0^*$  delivers lower welfare than  $x_0^*$ , where  $x_0^*$  is the highest cutoff VC in the model with heterogeneous VCs, that the functions  $m_0^x$  and  $m_1^x$  are increasing and continuous, and that VCs' skill distribution is continuous, the same logic can be used to show that the second-best allocation is a cutoff  $x_0^{sb}$ , which is higher than any equilibrium cutoff  $x_0$ .]  $\square$

PROOF PROPOSITION 4: Part (i). Recall that the (unique) function  $\lambda_0(\chi)$  is continuous and increasing. Therefore, when the function  $\phi(\bar{\lambda})$  is increasing everywhere, the functions  $m_0/m_1$  (which is constant) and  $\phi(\lambda_0(\chi))$  can intersect only at one interior  $\chi$ . Assume such an interior solution exists. As  $\chi$  decreases and approaches zero,  $\lambda_0(\chi)$  increases and approaches  $\bar{\lambda}$ , and  $\phi$  increases and approaches

$$\frac{\alpha R(a_1, \bar{\lambda}) - R_0}{\alpha \mathbb{E}[R(a_0, \bar{\lambda})] - R_0} > \frac{m_0}{m_1}, \quad (\text{A40})$$

which, rearranged, implies that in a candidate corner equilibrium in which every VC offers low attention, it would be profitable to deviate to high attention and attract the best entrepreneur. Similarly, as  $\chi$  increases and approaches one,  $\lambda_0(\chi)$  decreases and approaches  $\underline{\lambda}$ , and  $\phi$  decreases and approaches

$$\frac{\alpha \mathbb{E}[R(a_1, \lambda)] - R_0}{\alpha R(a_0, \underline{\lambda}) - R_0} < \frac{m_0}{m_1}, \quad (\text{A41})$$

which, rearranged, would imply that VCs have an incentive to deviate to low attention from a candidate equilibrium where every VC offers high attention.

If instead an interior  $\chi$  does not exist, the fact that  $\phi(\tilde{\lambda})$  is increasing everywhere would similarly imply that the unique equilibrium is either  $\chi = 0$  or  $\chi = 1$ . Therefore, the equilibrium must be unique.

[The same argument can be made with heterogeneous VCs, noting that  $m_0^{x_0}/m_1^{x_0}$  is monotone and decreasing in  $x_0$  by assumption.]

Part (ii). For part (ii), it is sufficient to observe that the payoff to a VC that chooses high attention in any interior equilibrium is increasing in  $\lambda_0$ . Since it is an interior equilibrium, this is also the payoff to a VC that chooses low attention.

[In the model with heterogeneous VCs, it also holds that, in an interior equilibrium, the payoff to any VC in the economy is strictly increasing in  $\lambda_0$ . Since VCs can optimally choose fund size given  $\lambda_0$ , all VCs must be better off in an equilibrium where  $\lambda_0$  is larger.]

Part (iii). This part contains three separate statements.

First, denote by *I* the equilibrium exhibiting lower ratio  $Q_1/Q_0$ , and by *II* the other equilibrium. Use the superscripts *I* and *II* to denote the cutoffs  $\lambda_0$  under equilibria *I* and *II*. It can be shown that  $\lambda_0^I > \lambda_0^{II}$ . This is easily seen by rewriting the entrepreneur's indifference condition as

$$\frac{R(a_1, \lambda_0)}{R(a_0, \lambda_0)} = \frac{Q_0}{Q_1}. \quad (\text{A42})$$

Condition (A42) has to hold under both equilibrium values  $\lambda_0^I$  and  $\lambda_0^{II}$ . If, under equilibrium *I*, the right-hand side of (A42) is larger than under *II*, since  $R$  is log-supermodular, it must be the case that  $\lambda_0^I > \lambda_0^{II}$ . Welfare is then higher in equilibrium *I* by part (i).

Similarly, let *I* be the equilibrium exhibiting a lower ratio  $M_1/M_0$ , and *II* be the other equilibrium. Use the superscripts *I* and *II* to denote the cutoffs  $\lambda_0$  under equilibria *I* and *II*. It can be shown that  $\lambda_0^I > \lambda_0^{II}$ . To prove this, assume this is not the case. That is, assume  $\lambda_0^I \leq \lambda_0^{II}$ . First focus on the case where the inequality is strict. Rewrite the indifference condition for type  $\lambda_0$ , as

$$\frac{R(a_1, \lambda_0)}{R(a_0, \lambda_0)} = \frac{M_0}{M_1} \frac{1 - F(\lambda_0)}{F(\lambda_0)}. \quad (\text{A43})$$

When  $\lambda_0$  is lower, the left-hand side of (A43) is lower (due to log-supermodularity). The ratio  $M_0/M_1$  is higher in equilibrium *I* by assumption. Moreover,  $(1 - F(\lambda_0))/F(\lambda_0)$  is higher under *I*. Hence, (A43) cannot hold. It remains to show that it cannot be the case that  $\lambda_0^I = \lambda_0^{II}$ . Assume to the contrary that it is the case. This would imply that under the two equilibria, the left-hand side of (A43) stays constant, as well as the ratio  $(1 - F(\lambda_0))/F(\lambda_0)$ . Because  $M_0/M_1$  is not the same under the two equilibria, (A43) cannot hold and the desired contradiction arises. This statement does not apply to corner equilibria since, in those cases,  $M_0/M_1$  is not well defined.

Finally, it is immediate to observe that  $M_0/M_1$  is higher if and only if  $\chi$  is lower, completing the proof.

[In the model with heterogeneous VCs, the same arguments imply that the indifferent VC,  $x_0$ , is higher in a Pareto-superior equilibrium.]

Part (iv). The statements in part (iv) are immediate consequences of part (iii). In particular, note that the total measure of firms receiving financing is in equilibrium given by the sum  $M_0 + M_1$ , which is clearly decreasing in  $\chi$  since  $m_0 > m_1$ .

[In the model with heterogeneous VCs, given  $x_0^I > x_0^{II}$ , average fund size in the low-attention market is given by  $\int_0^{x_0^I} m_0^x dx / x_0^I$  and thus is larger in equilibrium I, since  $\partial m_0^x / \partial x > 0$ . Similarly, average fund size in the high-attention market, which is given by  $\int_{x_0^I}^1 m_1^x dx / (1 - x_0^I)$ , is larger in equilibrium I, since  $\partial m_1^x / \partial x > 0$ . The fact that total fundraising is larger in equilibrium I simply comes from the fact that, comparing the two economies, there is a subset of VCs who make the same fundraising decision; those that do not are all VCs in  $x \in [x_0^{II}, x_0^I]$ , which raise small funds in equilibrium II and large funds in equilibrium I.]

*Corner Equilibria.* The statements in (ii), (iii), and (iv) also apply to corner equilibria. In such equilibria, the ratio  $Q_0/Q_1$  is pinned down by the indifference condition of the type that is selected by Requirement 1. In the equilibrium in which every VC offers  $a_0$ , this type is  $\bar{\lambda}$  and therefore the ratio is given by  $R(a_1, \bar{\lambda})/R(a_0, \bar{\lambda})$ . Hence, it is higher than in any other equilibrium. Conversely, in the equilibrium in which every VC offers  $a_1$ , this type is  $\underline{\lambda}$  and therefore the ratio  $Q_0/Q_1$  is given by  $R(a_1, \underline{\lambda})/R(a_0, \underline{\lambda})$ . Hence, it is lower than in any other equilibrium. Moreover, the ratio  $M_0/M_1$  tends to infinity as we approach an equilibrium in which all VCs offer  $a_0$  and is zero as we approach an equilibrium in which all VCs offer  $a_1$ . Corollary 1 proves that these are, respectively, the highest welfare and lowest welfare equilibria.  $\square$

**PROOF COROLLARY 1:** Take the equilibrium in which every VC raises a large fund. By part (iii) of Proposition 2, every VC generates larger aggregate excess returns than they would by offering  $a_1$  in any other equilibrium. Since, in all equilibria, returns from offering  $a_0$  are bounded above by  $m_0\{\alpha \mathbb{E}[R(a_0, \lambda)] - R_0\}$ —due to negative selection in market  $a_0$ —the result follows.

Take the equilibrium in which every VC raises a small fund. In this equilibrium, every VC's excess returns are  $\alpha \mathbb{E}[R(a_1, \lambda)] - R_0$ . Due to positive selection in market  $a_1$ , this is the lower bound on what a VC generates in any other equilibrium by offering  $a_1$ .

[In the model with heterogeneous VCs, the result is proven by noting that aggregate welfare in a corner equilibrium in which all VCs offer  $a_0$ , and thus  $x_0 = 0$  equals aggregate welfare in an interior equilibrium in the limit when  $x_0$  approaches zero, and aggregate welfare in a corner equilibrium in which all VCs offer  $a_1$ , thus  $x_0 = 1$  equals aggregate welfare in an interior equilibrium in the limit when  $x_0$  approaches one (which can be proven using the limit of  $\lambda_0(x_0)$  as proven in the existence proof, and the continuity of  $R$  and  $f$ ). Since all interior equilibria are ranked, the result follows.]  $\square$

*Stable Equilibria.* An interior equilibrium satisfies conditions (16), (15), and (8), which are rearranged below as

$$\kappa - m_0^x \{\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0\} = 0, \quad (\text{A44})$$

$$\frac{\alpha \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0] - R_0}{\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0} - \frac{m_0^{x_0}}{m_1^{x_0}} = 0, \quad (\text{A45})$$

$$\frac{R(a_1, \lambda_0)}{R(a_0, \lambda_0)} - \frac{M_0(\underline{x}, x_0)}{M_1(x_0)} \frac{1 - F(\lambda_0)}{F(\lambda_0)} = 0. \quad (\text{A46})$$

Note that I have made explicit in (A46) the dependence of  $M_0$  on  $\underline{x}$ .<sup>54</sup> Let  $\mu^1(\underline{x}, x_0, \lambda_0)$ ,  $\mu^2(\underline{x}, x_0, \lambda_0)$ , and  $\mu^3(\underline{x}, x_0, \lambda_0)$  be the left-hand sides of (A44), (A45), and (A46), respectively.  $\square$

**DEFINITION A1: (Stable Equilibria).** An equilibrium  $(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0)$  is stable if it is an asymptotically stable fixed point of the function  $\Theta: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  defined by

$$\Theta(\underline{x}, x_0, \lambda_0) = \begin{bmatrix} \mu^1(\underline{x}, x_0, \lambda_0) + \underline{x} \\ \mu^2(\underline{x}, x_0, \lambda_0) + x_0 \\ \mu^3(\underline{x}, x_0, \lambda_0) + \lambda_0 \end{bmatrix}. \quad (\text{A47})$$

A fixed point is asymptotically stable if and only if all eigenvalues of the Jacobian of  $\Theta$ —denoted  $J(\Theta)$ —are smaller than one in absolute value. Note that the matrix  $J(\Theta)$  is equivalent to  $J'(\Theta) + I$ , where  $I$  is the  $3 \times 3$  identity matrix and  $J'(\Theta)$  is

$$\begin{bmatrix} \mu_{\underline{x}}^1(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) & \mu_{x_0}^1(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) & \mu_{\lambda_0}^1(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) \\ \mu_{\underline{x}}^2(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) & \mu_{x_0}^2(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) & \mu_{\lambda_0}^2(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) \\ \mu_{\underline{x}}^3(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) & \mu_{x_0}^3(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) & \mu_{\lambda_0}^3(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0) \end{bmatrix}. \quad (\text{A48})$$

Denoting by  $[\varepsilon_1, \varepsilon_2, \varepsilon_3]$  the eigenvalues of  $J'(\Theta)$ , it thus holds that the eigenvalues of  $J(\Theta)$  are  $[\varepsilon_1 + 1, \varepsilon_2 + 1, \varepsilon_3 + 1]$ . It must therefore be the case that  $\varepsilon_1, \varepsilon_2$ , and  $\varepsilon_3$  are negative. This implies that a necessary condition for a fixed point  $(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0)$  to be asymptotically stable is that the determinant of the corresponding  $J'(\Theta)$  is negative. Noting that  $\mu_{x_0}^1 = \mu_{\underline{x}}^2 = 0$ , this is the case if and only if, at  $(\tilde{x}, \tilde{x}_0, \tilde{\lambda}_0)$  the following holds:

$$\mu_{\underline{x}}^1 \mu_{x_0}^2 \mu_{\lambda_0}^3 - \mu_{\underline{x}}^1 \mu_{\lambda_0}^2 \mu_{x_0}^3 < \mu_{\lambda_0}^1 \mu_{x_0}^2 \mu_{\underline{x}}^3. \quad (\text{A49})$$

**PROOF OF PROPOSITION 5 (Ranking multiple equilibria):** Analogous to the proof of Proposition 4, note that given any two equilibria I and II, the first equilibrium generates higher welfare than (and Pareto dominates) the second if and only if  $\lambda_0^I > \lambda_0^{II}$ . And it is also analogous to prove that this implies

$$\frac{Q_0^I}{Q_1^I} > \frac{Q_0^{II}}{Q_1^{II}} \quad (\text{A50})$$

<sup>54</sup> One can verify from equation (A46) that, given a couple  $(\underline{x}, x_0)$ , the induced cutoff  $\lambda_0$  is unique.



and

$$\frac{M_0^I}{M_1^I} > \frac{M_0^{II}}{M_1^{II}}. \quad (\text{A51})$$

Moreover, since  $\lambda_0^I > \lambda_0^{II}$ , excess returns are higher in the low-attention market in equilibrium I, implying that more VCs must be willing to establish funds:  $\underline{x}^I < \underline{x}^{II}$ .

To prove that this implies fewer investments in the dominated equilibria, distinguish two cases: if  $M_1^I > M_1^{II}$ , then (A51) implies  $M_0^I > M_0^{II}$ , and therefore  $M_0^I + M_1^I > M_0^{II} + M_1^{II}$ , which means aggregate investment is higher in the dominant equilibrium; if instead  $M_1^I < M_1^{II}$ , recalling the definition of  $M_1$ , it must be the case that  $x_0^I > x_0^{II}$ . In this case, more VCs operate in the market and all VCs in  $x \in [x_0^{II}, x_0^I]$  raise small funds in equilibrium II and large funds in equilibrium I, implying once again that  $M_0^I > M_0^{II}$  and that  $M_0^I + M_1^I > M_0^{II} + M_1^{II}$ .

(Second-Best). First, the proof uses the fact that the indifferent entrepreneur  $\lambda_0$  is a monotone function of the thresholds  $\underline{x}$  and  $x_0$  and, in particular

$$\frac{\partial \lambda_0(\underline{x}, x_0)}{\partial \underline{x}} < 0, \quad \frac{\partial \lambda_0(\underline{x}, x_0)}{\partial x_0} > 0. \quad (\text{A52})$$

Both can be seen by direct calculations, using the indifference condition:

$$\frac{R(a_1, \lambda_0)}{R(a_0, \lambda_0)} - \frac{M_0(\underline{x}, x_0)}{M_1(x_0)} \frac{1 - F(\lambda_0)}{F(\lambda_0)} = 0, \quad (\text{A53})$$

noting that  $M_0(\underline{x}, x_0)$  is strictly decreasing in  $\underline{x}$  and strictly increasing in  $x_0$ , and that  $M_1(x_0)$  is strictly decreasing in  $x_0$ .

Again, to better visualize each equation, I set the fundraising costs,  $R_0$ , to zero. Denote by  $\underline{x}^*$  the lowest equilibrium marginal VC that enters the market and  $\lambda_0^*$  the highest equilibrium marginal entrepreneur that is indifferent between the two submarkets. For reasons analogous to those underlying the proof of Proposition 3, a marginal decrease in  $\underline{x}$  or increase  $x_0$  away from  $\underline{x}^*$  and  $x_0^*$  improves welfare by increasing the threshold  $\lambda_0^*$ . Therefore, the second-best allocation must necessarily induce  $\lambda_0 > \lambda_0^*$  (inducing  $\lambda_0 \leq \lambda_0^*$  would not be optimal). Note that this means, for all  $x$ ,

$$m_0^x \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0] > m_0^x \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0^*]. \quad (\text{A54})$$

It is possible to prove that for the second-best marginal VC,  $\underline{x}^{sb}$ , the payoff from entry is strictly lower than the cost  $\kappa$ . Indeed, if it was (weakly) above  $\kappa$ ,

a marginal decrease in  $\underline{x}$  would improve welfare:

$$\begin{aligned} \frac{\partial W(\underline{x})}{\partial \underline{x}} &= \underbrace{-\{m_0^x \alpha \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0] - \kappa\}}_{\leq 0} \\ &+ \alpha \left\{ \left[ M_0(\underline{x}) \frac{\partial \mathbb{E}[R(a_0, \lambda) | \lambda \leq \lambda_0(\underline{x})]}{\partial \lambda_0} + M_1(\underline{x}) \frac{\partial \mathbb{E}[R(a_1, \lambda) | \lambda \geq \lambda_0(\underline{x})]}{\partial \lambda_0} \right] \frac{\partial \lambda_0(\underline{x})}{\partial \underline{x}} \right\} < 0. \end{aligned} \quad (\text{A55})$$

The only way to support both equation (A54) and the fact that the marginal VC generates profits below  $\kappa$  is if  $\underline{x}^{sb} < \underline{x}^*$ . By a logic analogous to that used to rank equilibria, this in turn means that  $M_0^{sb} > M_0^*$ , and  $M_0^{sb} + M_1^{sb} > M_0^* + M_1^*$ , completing the proof.  $\square$

**PROOF OF PROPOSITION 6:** Part (i). Consider the indifference conditions (A44), (A45), and (A46). Denote by  $\mu_\kappa^1(\cdot)$ ,  $\mu_\kappa^2(\cdot)$ , and  $\mu_\kappa^3(\cdot)$  the partial derivatives of  $\mu^1$ ,  $\mu^2$ , and  $\mu^3$  with respect to  $\kappa$ . Note that  $\mu_{x_0}^1 = \mu_{\underline{x}}^2 = \mu_\kappa^2 = \mu_\kappa^3 = 0$ . Using the Implicit Function Theorem, evaluated at an equilibrium, the following must hold:

$$\frac{\partial \lambda_0}{\partial \kappa} \underbrace{\left[ \mu_{\lambda_0}^3 - \mu_{x_0}^3 \frac{\mu_{\lambda_0}^2}{\mu_{x_0}^2} - \mu_{\underline{x}}^3 \frac{\mu_{\lambda_0}^1}{\mu_{\underline{x}}^1} \right]}_{:=B} = \mu_{\underline{x}}^3 \frac{\mu_\kappa^1}{\mu_{\underline{x}}^1}, \quad (\text{A56})$$

where I define  $B$  as the number multiplying  $\frac{\partial \lambda_0}{\partial \kappa}$  on the left-hand side of (A56). Note by direct calculation that  $\mu_\kappa^1 > 0$ ,  $\mu_{\underline{x}}^1 < 0$ ,  $\mu_{x_0}^2 > 0$ ,  $\mu_{x_0}^3 > 0$ , and  $\mu_{\underline{x}}^3 < 0$ . It follows that the right-hand side of (A56) is strictly positive, and therefore  $\text{sign}(\frac{\partial \lambda_0}{\partial \kappa}) = \text{sign}(B)$ . Thus,

$$\frac{\partial \lambda_0}{\partial \kappa} < 0 \iff \mu_{\underline{x}}^1 \mu_{x_0}^2 \mu_{\lambda_0}^3 - \mu_{\underline{x}}^1 \mu_{\lambda_0}^2 \mu_{x_0}^3 < \mu_{\lambda_0}^1 \mu_{x_0}^2 \mu_{\underline{x}}^3, \quad (\text{A57})$$

which is the same as condition (A49) and therefore implied by stability. Similarly, for  $\underline{x}$  one gets that

$$\frac{\partial \underline{x}}{\partial \kappa} = -\frac{\mu_{\lambda_0}^1}{\mu_{\underline{x}}^1} \frac{\partial \lambda_0}{\partial \kappa} - \frac{\mu_\kappa^1}{\mu_{\underline{x}}^1} > 0, \quad (\text{A58})$$

which holds for all stable equilibria, given the calculations above.

Part (ii). To prove part (ii), note that due to the increase in  $\lambda_0$ , expected excess returns to VCs are higher in every submarket. This compositional effect operates on top of the direct effect of lower entry costs, and hence the result follows.

Part (iii). To prove part (iii), note that

$$\frac{\partial x_0}{\partial \kappa} = -\frac{\partial \lambda_0}{\partial \kappa} \frac{\mu_{\lambda_0}^2}{\mu_{x_0}^2} < 0 \iff \mu_{\lambda_0}^2 < 0 \quad (\text{A59})$$

given I am considering stable equilibria. By direct calculation,  $\mu_{\lambda_0}^2 < 0 \iff \phi_{\lambda}(\lambda_0) < 0$ .

Part (iv). For part (iv), similar arguments apply. Consider the indifference conditions (A44), (A45), and (A46). Denote by  $\mu_{R_0}^1(\cdot)$ ,  $\mu_{R_0}^2(\cdot)$ , and  $\mu_{R_0}^3(\cdot)$  the partial derivative functions of  $\mu^1$ ,  $\mu^2$ , and  $\mu^3$  with respect to  $R_0$ . Using the Implicit Function Theorem, evaluated at an equilibrium, the following must hold:

$$\frac{\partial \lambda_0}{\partial R_0} \underbrace{\left[ \mu_{\lambda_0}^3 - \mu_{x_0}^3 \frac{\mu_{\lambda_0}^2}{\mu_{x_0}^2} - \mu_{\underline{x}}^3 \frac{\mu_{\lambda_0}^1}{\mu_{\underline{x}}^1} \right]}_{:=C} = \mu_{x_0}^3 \frac{\mu_{R_0}^2}{\mu_{x_0}^2} + \mu_{\underline{x}}^3 \frac{\mu_{R_0}^1}{\mu_{\underline{x}}^1}, \quad (\text{A60})$$

where I define  $C$  as the number multiplying  $\frac{\partial \lambda_0}{\partial R_0}$  on the left-hand side of (A60). Note by direct calculation that  $\mu_{R_0}^1 > 0$ ,  $\mu_{\underline{x}}^1 < 0$ ,  $\mu_{R_0}^2 > 0$ ,  $\mu_{x_0}^2 > 0$ ,  $\mu_{x_0}^3 > 0$ , and  $\mu_{\underline{x}}^3 < 0$ . It follows that the right-hand side of (A60) is strictly positive, therefore  $\text{sign}(\frac{\partial \lambda_0}{\partial R_0}) = \text{sign}(C)$ . Thus,

$$\frac{\partial \lambda_0}{\partial R_0} < 0 \iff \mu_{\underline{x}}^1 \mu_{x_0}^2 \mu_{\lambda_0}^3 - \mu_{\underline{x}}^1 \mu_{\lambda_0}^2 \mu_{x_0}^3 < \mu_{\lambda_0}^1 \mu_{x_0}^2 \mu_{\underline{x}}^3, \quad (\text{A61})$$

which is the same as condition (A49) and therefore implied by stability. Similarly, for  $\underline{x}$  one gets that

$$\frac{\partial \underline{x}}{\partial R_0} > 0 \iff -\frac{\mu_{\lambda_0}^1}{\mu_{\underline{x}}^1} \frac{\partial \lambda_0}{\partial R_0} - \frac{\mu_{R_0}^1}{\mu_{\underline{x}}^1} > 0, \quad (\text{A62})$$

which holds for all stable equilibria, given the calculations above.

The sufficient condition for  $x_0$  to be decreasing in  $R_0$  can be derived from differentiating  $\mu^2$ , to get

$$\frac{\partial x_0}{\partial R_0} = -\frac{\partial \lambda_0}{\partial R_0} \frac{\mu_{\lambda_0}^2}{\mu_{x_0}^2} - \frac{\mu_{R_0}^2}{\mu_{x_0}^2}, \quad (\text{A63})$$

which is negative if  $\mu_{\lambda_0}^2$ . Substituting  $\frac{\partial \lambda_0}{\partial R_0}$  into the above reveals, however, that the same condition is not necessary for the derivative to be negative.  $\square$

PROOF OF COROLLARY 2: The result above establishes the marginal effects of  $\kappa$  and  $R_0$  on the equilibrium cutoffs  $\underline{x}$ ,  $x_0$ , and  $\lambda_0$ . Because of their symmetric effects, I focus on a marginal change in  $\kappa$ . It is useful to note that since the function  $\lambda_0(\kappa)$  is strictly decreasing, the ratio

$$\omega_0(\kappa) := \frac{M_0(\kappa)}{M_0(\kappa) + M_1(\kappa)} \quad (\text{A64})$$

must be strictly decreasing in  $\kappa$ , a direct consequence of the fact that the cutoff  $\lambda_0$  is a strictly increasing function of  $M_0/M_1$ .

To better visualize the equations, I ignore the exogenous share  $\alpha$  and I compute returns gross of funding costs  $R_0$ . The distribution of net returns to VCs will exhibit the same properties.

Part (i). Denote by  $r$  the equilibrium returns and note that  $r \in [R(a_0, \underline{\lambda}), R(a_0, \lambda_0)] \cup [R(a_1, \lambda_0), R(a_1, \bar{\lambda})]$ . Since, by assumption, the function  $R$  is monotonically increasing in  $\lambda$ , fix attention at  $a_0$  and  $a_1$  and, respectively, define the functions  $R_0(\lambda) := R(\lambda, a_0)$  and  $R_1(\lambda) := R(\lambda, a_1)$ , as well as their inverse  $R_0^{-1}(r)$  and  $R_1^{-1}(r)$ , mapping values of  $\lambda$  into return,  $r$ , and vice versa. In the model, because of uniform random matching within submarkets, the distribution of returns is shaped by the underlying distribution of project's quality: This is the truncation of  $F$  for  $\lambda \in [\underline{\lambda}, \lambda_0(\kappa)]$  in market  $a_0$  and the truncation of  $F$  for  $\lambda \in [\lambda_0(\kappa), \bar{\lambda}]$  in market  $a_1$ . Fix an equilibrium and consider the distribution of returns. The cumulative density function of the returns is a step function, denoted by  $Y$ , that takes the form

$$Y(r; \kappa) = \begin{cases} \omega_0(\kappa) \frac{F(R_0^{-1}(r))}{F(\lambda_0(\kappa))} & \text{if } r \leq R(a_0, \lambda_0(\kappa)) \\ \omega_0(\kappa) & \text{if } r \in [R(a_0, \lambda_0(\kappa)), R(a_1, \lambda_0(\kappa))] \\ \omega_0(\kappa) + (1 - \omega_0(\kappa)) \frac{F(R_1^{-1}(r)) - F(\lambda_0(\kappa))}{1 - F(\lambda_0(\kappa))} & \text{otherwise.} \end{cases} \quad (\text{A65})$$

The associated probability density function,  $y$ , is given by

$$y(r; \kappa) = \begin{cases} \omega_0(\kappa) \frac{f(R_0^{-1}(r))/R'_0(R_0^{-1}(r))}{F(\lambda_0(\kappa))} & \text{if } r \leq R(a_0, \lambda_0(\kappa)) \\ 0 & \text{if } r \in [R(a_0, \lambda_0(\kappa)), R(a_1, \lambda_0(\kappa))] \\ (1 - \omega_0(\kappa)) \frac{f(R_1^{-1}(r))/R'_1(R_1^{-1}(r))}{1 - F(\lambda_0(\kappa))} & \text{otherwise.} \end{cases} \quad (\text{A66})$$

Consider a marginal negative change in  $\kappa$ . Since  $\omega'_0(\kappa) < 0$ , we have that at  $r = R(a_0, \lambda_0(\kappa))$ ,  $Y(r; \kappa)$  increases. That is, returns are more likely to fall below  $R(a_0, \lambda_0(\kappa))$ .

Consider now the expectation of  $r$  conditional on being larger than  $R(a_0, \lambda_0(\kappa))$ . This is

$$\int_{R(a_0, \lambda_0(\kappa))}^{R(a_1, \bar{\lambda})} r dY(r; \kappa) / (1 - \omega_0(\kappa)). \quad (\text{A67})$$

When  $\kappa$  decreases,  $\lambda_0(\kappa)$  and  $\omega_0(\kappa)$  increase and therefore the expectation increases. The same is true when conditioning on  $r$  being above any number in the set  $(R(a_0, \lambda_0(\kappa)), R(a_1, \lambda_0(\kappa))]$ . The conditional expectation given that  $r$  is above some  $\hat{r} > R(a_1, \lambda_0(\kappa))$  is instead given by

$$\int_{\hat{r}}^{R(a_1, \bar{\lambda})} r dY(r; \kappa) / (1 - Y(\hat{r}; \kappa)). \quad (\text{A68})$$

I show that this is unaffected by the change in  $\kappa$ . First,  $(1 - Y(\hat{r}; \kappa))$  is equivalent to

$$\int_{\hat{r}}^{R(a_1, \bar{\lambda})} dY(r; \kappa) = (1 - \omega_0(\kappa)) \int_{\hat{r}}^{R(a_1, \bar{\lambda})} \frac{f(R_1^{-1}(r))/R_1'(R_1^{-1}(r))}{1 - F(\lambda_0(\kappa))}. \quad (\text{A69})$$

Therefore, using the form of  $y(r; \kappa)$  derived above, the conditional expectation can be written as

$$\frac{\int_{\hat{r}}^{R(a_1, \bar{\lambda})} r dY(r; \kappa)}{(1 - Y(\hat{r}; \kappa))} = \frac{\int_{\hat{r}}^{R(a_1, \bar{\lambda})} r f(R_1^{-1}(r))/R_1'(R_1^{-1}(r)) dr}{1 - F(R_1^{-1}(\hat{r}))}, \quad (\text{A70})$$

which does not depend on  $\kappa$ . Hence, returns conditional on being above any number in  $[(R(a_0, \lambda_0(\kappa)), R(a_1, \lambda_0(\kappa)))]$  are strictly higher when  $\kappa$  decreases, whereas those conditional on being higher than some  $\hat{r} > R(a_1, \lambda_0(\kappa))$  stay constant.

Part (ii). Inspecting the function  $Y(\hat{r}; \kappa)$ , it is evident that should the ratio

$$\tilde{\omega}_0(\kappa) := \frac{\omega_0(\kappa)}{F(\lambda_0(\kappa))} \quad (\text{A71})$$

be (weakly) decreasing in  $\kappa$ , the cumulative distribution would be (weakly) higher for any  $r < R(a_0, \lambda_0(\kappa))$  as  $\kappa$  decreases. Using the definition of  $\omega_0(\kappa)$ , the required condition can be written as

$$F(\lambda_0(\kappa))[M_0'(\kappa)M_1(\kappa) - M_0(\kappa)M_1'(\kappa)] \leq M_0(\kappa)[M_0(\kappa) + M_1(\kappa)]f(\lambda_0(\kappa))\lambda_0'(\kappa). \quad (\text{A72})$$

An example in which (A72) holds. Without additional restrictions, it is not possible to assess whether, in general, (A72) holds in a given equilibrium. Here I describe an environment where it holds in all interior equilibria. Consider the case in which entrepreneurs' quality  $\lambda$  is drawn from a distribution  $f$  that places arbitrarily large mass to its extremes,  $\underline{\lambda}$  and  $\bar{\lambda}$ . Formally, I study the limit in which  $f(\lambda) = 0$  for all  $\lambda \in (\underline{\lambda}, \bar{\lambda})$ . So long as  $f$  is positive in the support, no assumption is violated. In particular, note that the comparative statics derived above follow through. Inspecting the equilibrium conditions, the function  $\mu_{\lambda_0}^3$  can be derived analytically to get at any interior equilibrium

$$\mu_{\lambda_0}^3 = \frac{R_{\lambda}(a_1, \lambda_0)R(a_0, \lambda_0) - R_{\lambda}(a_0, \lambda_0)R(a_1, \lambda_0)}{R(a_0, \lambda_0)^2} + \frac{M_0}{M_1} \frac{f(\lambda_0)}{F(\lambda_0)^2}. \quad (\text{A73})$$

The first term is positive and finite by assumption (log-supermodularity and continuous second derivatives), while the first term converges to zero in the limit in which the distribution places all mass on its extremes. Similarly, one can compute  $\mu_{\lambda_0}^1$  and  $\mu_{\lambda_0}^2$  to show that they converge to zero as  $f$  converges to zero. Inspecting the formula for  $\lambda_0'(\kappa)$  derived in the proof of Proposition 6, one sees that  $\lambda_0'(\kappa)$  remains strictly negative and finite as  $f$  converges to zero. To

ensure all equilibria are indeed interior, assume

$$m_0^x \{\alpha \mathbb{E}[R(a_0, \lambda)] - R_0\} < m_1^x \{\alpha R(a_1, \bar{\lambda}) - R_0\} \quad (\text{A74})$$

and

$$m_0^x \{\alpha R(a_0, \underline{\lambda}) - R_0\} > m_1^x \{\alpha \mathbb{E}[R(a_1, \lambda)] - R_0\} \quad (\text{A75})$$

for all skill levels  $x$ , and

$$m_0^x \{\alpha R(a_0, \bar{\lambda}) - R_0\} > \kappa \quad (\text{A76})$$

for some sufficiently small  $x$ . Inspecting condition (A72), note that an upper bound to  $M'_0(\kappa)M_1(\kappa) - M_0(\kappa)M'_1(\kappa)$  is given by  $M'_0(\kappa)M_1(\kappa)$ . Moreover,  $M'_0 = m_0^{x_0}x'_0(\kappa) - m_0^x x'(\kappa)$ . Noting that  $x'_0(\kappa) < 0$  and  $x'(\kappa) = -\mu_{\lambda_0}^1/\mu_{\underline{x}}^1 \lambda'_0(\kappa) - \mu_{\underline{x}}^1/\mu_{\underline{x}}^1 < -\mu_{\underline{x}}^1/\mu_{\underline{x}}^1$ , one can replace the left-hand side of (A72) with its upper bound to get the sufficient condition

$$-F(\lambda_0(\kappa)) \frac{\mu_{\kappa}^1}{\mu_{\underline{x}}^1} \leq M_0(\kappa)[M_0(\kappa) + M_1(\kappa)]f(\lambda_0(\kappa))\lambda'_0(\kappa). \quad (\text{A77})$$

By direct calculation, one can see that while the left-hand side is always strictly negative, the right-hand side is strictly negative but converges to zero as  $f(\lambda_0(\kappa))$  converges to zero, proving that condition (A72) holds (with strictly inequality) for sufficiently small  $f$ .

An example in which (A72) does not hold. To construct an economy in which (A72) does not hold, consider a specification in which the derivative of  $R(a_1, \lambda)/R(a_0, \lambda)$  is taken arbitrarily small. Log-supermodularity implies that this ratio is strictly increasing. Consider the linear specification  $R(a, \lambda) = a + (a - \delta)\lambda$ . As  $\delta$  converges to zero, the ratio  $R(a_1, \lambda)/R(a_0, \lambda)$  converges to the constant  $a_1/a_0$ . Note that all comparative statics continue to hold and the partial derivatives of the functions  $\mu^1$ ,  $\mu^2$ , and  $\mu^3$  are continuous in  $\delta$ . Set  $\delta = 0$ , and note the entrepreneur's indifference condition is

$$\frac{a_1}{a_0} - \frac{M_0(\underline{x}, x_0)}{M_1(x_0)} \frac{1 - F(\lambda_0)}{F(\lambda_0)} = 0. \quad (\text{A78})$$

The second term must therefore be constant when comparative statics are taken with respect to  $\kappa$  (or  $R_0$ ) around an interior equilibrium. This implies

$$[1 - F(\lambda_0(\kappa))]F(\lambda_0(\kappa))[M'_0(\kappa)M_1(\kappa) - M_0(\kappa)M'_1(\kappa)] = M_0(\kappa)M_1(\kappa)f(\lambda_0(\kappa))\lambda'_0(\kappa). \quad (\text{A79})$$

Replacing this condition in condition (A72) gives

$$\frac{M_1(\kappa)}{M_0(\kappa) + M_1(\kappa)} \frac{1}{1 - F(\lambda_0(\kappa))} \geq 1, \quad (\text{A80})$$

leading to

$$\frac{M_1(\kappa)}{M_0(\kappa)} \frac{F(\lambda_0(\kappa))}{1 - F(\lambda_0(\kappa))} \geq 1. \quad (\text{A81})$$

Condition (A81) cannot be satisfied in an interior equilibrium, because it would imply that the relative congestion in the high-attention market is lower than in the low-attention market (in this case, it must indeed be equal to  $a_0/a_1 < 1$ ). By continuity, condition (A72) is thus also violated when the parameter  $\delta$  takes positive but sufficiently small values.  $\square$

PROOF OF COROLLARY 3: The condition (A72) in the proof of Corollary 2 ensures that the ratio

$$\tilde{\omega}_0(\kappa) := \frac{M_0(\kappa)}{M_0(\kappa) + M_1(\kappa)} \frac{1}{F(\lambda_0(\kappa))} \quad (\text{A82})$$

is decreasing in  $\kappa$ . To prove the statement, also note that

$$\tilde{\omega}_1(\kappa) := \frac{M_1(\kappa)}{M_0(\kappa) + M_1(\kappa)} \frac{1}{1 - F(\lambda_0(\kappa))} \quad (\text{A83})$$

is increasing in  $\kappa$ . To prove  $\tilde{\omega}_1(\kappa)$  is always increasing, note that this is true if and only if

$$[1 - F(\lambda_0(\kappa))] [M'_0(\kappa)M_1(\kappa) - M_0(\kappa)M'_1(\kappa)] < M_1(\kappa)[M_0(\kappa) + M_1(\kappa)]f(\lambda_0(\kappa))\lambda'_0(\kappa), \quad (\text{A84})$$

which is implied by (A72) if

$$\frac{M_0(\kappa)}{F(\lambda_0(\kappa))} > \frac{M_1(\kappa)}{1 - F(\lambda_0(\kappa))}, \quad (\text{A85})$$

which holds in every interior equilibrium because congestion is lower in the low-attention market in every equilibrium.

Consider the distribution of project quality in the market and denote by  $J$  its cumulative density function:

$$J(\lambda; \kappa) = \begin{cases} \omega_0(\kappa) \frac{F(\lambda)}{F(\lambda_0(\kappa))} & \text{if } \lambda \leq \lambda_0(\kappa) \\ \omega_0(\kappa) + (1 - \omega_0(\kappa)) \frac{F(\lambda) - F(\lambda_0(\kappa))}{1 - F(\lambda_0(\kappa))} & \text{otherwise.} \end{cases} \quad (\text{A86})$$

The function  $J$  is continuous at  $\lambda_0(\kappa)$ . In the region  $\lambda \leq \lambda_0(\kappa)$ , it is decreasing in  $\kappa$  because  $\tilde{\omega}_0(\kappa)$  is decreasing in  $\kappa$ . In the region  $\lambda > \lambda_0(\kappa)$ ,  $J$  can also be written as

$$J(\lambda; \kappa) = 1 - \omega_1(\kappa) \frac{1 - F(\lambda)}{1 - F(\lambda_0(\kappa))} = 1 - \tilde{\omega}_1(\kappa)(1 - F(\lambda)), \quad (\text{A87})$$

which is decreasing in  $\kappa$  because  $\tilde{\omega}_1(\kappa)$  is increasing in  $\kappa$ . Therefore, as  $\kappa$  decreases, the cumulative density function is higher at any point in its support,



proving it is first-order stochastically dominated by the distribution implied by a higher  $\kappa$ .  $\square$

*Equilibrium Characterization for All Values of  $c$ .* To define the lowest threshold for  $c$ , observe that Assumptions A1 and A3 imply the restriction

$$c < (1 - \alpha) \min \left\{ \mathbb{E}_\lambda R(a_0, \lambda), \mathbb{E}_\lambda R(a_1, \lambda) \frac{R(a_1, \underline{\lambda})}{R(a_0, \underline{\lambda})} \right\} := c_L. \quad (\text{A88})$$

The objective of Section IV is to characterize all cases in which  $c > c_L$ . Define  $c_H$  as the highest cost to ensure that an equilibrium with positive entry of entrepreneurs exists. This is given by

$$(1 - \alpha) \mathbb{E}_\lambda R(a_1, \lambda) := c_H. \quad (\text{A89})$$

If  $c > c_H$ , no entrepreneur finds it ex ante optimal to enter even when all VCs offer high attention and no entrepreneur must endure congestion in equilibrium.

I first prove that an equilibrium in which all VCs raise a small fund exists whenever  $c > (1 - \alpha) \mathbb{E}_\lambda R(a_1, \lambda) R(a_1, \underline{\lambda}) / R(a_0, \underline{\lambda})$ . To do so, note that in such a conjectured equilibrium the matching probability is

$$Q_1 = \frac{c}{(1 - \alpha) \mathbb{E}_\lambda R(a_1, \lambda)}, \quad (\text{A90})$$

from which one can show, as in the proof of Proposition 2, that the set  $\mathcal{Q}(\lambda; a_0)$  is empty for all  $\lambda$ s, and therefore no VC wishes to deviate to the large fund. Since  $(1 - \alpha) \mathbb{E}_\lambda R(a_1, \lambda) R(a_1, \underline{\lambda}) / R(a_0, \underline{\lambda}) \geq c_L$ , it follows that the equilibrium exists when  $c$  is sufficiently larger than  $c_L$ .

Second, I prove an interior equilibrium where  $Q_0 = 1$ , if it exists, can only be supported in the region  $c_L \leq c \leq c_M$ , where  $c_M$  is a value that satisfies  $c_M < c_H$ . The first inequality is due to the fact that when  $c < c_L$  (implied by Assumptions 1 and 3), it must be the case that  $Q_0 < 1$ , by Lemma 1. To prove the second inequality, it must be proven that such equilibrium cannot exist at any  $c \geq \tilde{c}$  for some  $\tilde{c}$  satisfying  $c_L \leq \tilde{c} < c_H$ .

To do so, note that an equilibrium with  $Q_0 = 1$  is interior if and only if  $Q_1 < 1$ , as otherwise no entrepreneur would search in the low-attention market. The free-entry condition reads

$$\mathbb{E}_\lambda (1 - \alpha) \max \{ R(a_0, \lambda), Q_1 R(a_1, \lambda) \} = c. \quad (\text{A91})$$

When  $c = c_H$ , it has no solution since

$$\begin{aligned} \mathbb{E}_\lambda (1 - \alpha) \max \{ R(a_0, \lambda), Q_1 R(a_1, \lambda) \} &< \mathbb{E}_\lambda (1 - \alpha) \max \{ R(a_0, \lambda), R(a_1, \lambda) \} \\ &= \mathbb{E}_\lambda (1 - \alpha) R(a_1, \lambda) \\ &= c_H. \end{aligned} \quad (\text{A92})$$

By continuity, there exists a  $\tilde{c}$  such that the inequality continues to hold for all  $c \geq \tilde{c}$  with  $(c_H - \tilde{c}) > 0$  and sufficiently small. I define  $c_M$  as

$$c_M := \max\{c_L, \tilde{c}\}. \quad (\text{A93})$$

Since  $c_L$  can be made arbitrarily small by setting  $\mathbb{E}_\lambda R(a_0, \lambda)$  arbitrarily small, cases can be found such that  $\tilde{c} > c_L$ . To summarize, (i) when  $c_L \leq c \leq c_M$ , there always exist an equilibrium in which all VCs raise a small, high-attention fund and there may exist interior equilibria in which both submarkets are nonempty and  $Q_0 = 1$ ; (ii) when  $c_M \leq c \leq c_H$ , the equilibrium in which all VCs raise a small, high-attention fund is unique; and (iii) when  $c > c_H$ , there exists no equilibrium in which a positive measure of entrepreneurs enter the market.

PROOF OF PROPOSITION 7: The interior equilibrium, when it exists, is a solution  $(\mathcal{E}, \chi, \lambda_0, \gamma)$  to the system

$$\mathbb{E}_\lambda \max\{(1 - \alpha)R(a_0, \lambda), Q_1(1 - \alpha)R(a_1, \lambda)\} - c = 0, \quad (\text{A94})$$

$$\frac{\alpha \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0] - R_0}{\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0} - \frac{\gamma m_0}{m_1} = 0, \quad (\text{A95})$$

and

$$\frac{\chi m_1}{\mathcal{E}(1 - F(\lambda_0))} \frac{R(a_1, \lambda_0)}{R(a_0, \lambda_0)} - 1 = 0, \quad (\text{A96})$$

and the market-clearing equation

$$(1 - \chi)\gamma m_0 = \mathcal{E}F(\lambda_0). \quad (\text{A97})$$

Using the definition of  $Q_1$  and substituting  $\gamma$  from the market-clearing equation, one can rewrite equation (A94) as

$$(1 - \alpha)\mathcal{E}F(\lambda_0)\mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] + (1 - \alpha)\chi m_1 \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0] - \mathcal{E}c = 0 \quad (\text{A98})$$

and equation (A95) as

$$\frac{\alpha \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0] - R_0}{\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0] - R_0} - \frac{\frac{\mathcal{E}F(\lambda_0)}{(1 - \chi)}}{m_1} = 0. \quad (\text{A99})$$

There are, therefore, three endogenous variables to fully describe an equilibrium:  $(\mathcal{E}, \chi, \lambda_0)$ . Define  $\eta^1(\mathcal{E}, \chi, \lambda_0)$ ,  $\eta^2(\mathcal{E}, \chi, \lambda_0)$ , and  $\eta^3(\mathcal{E}, \chi, \lambda_0)$  as the left-hand side of equations (A98), (A99), and (A96), respectively. By direct calculations, one can verify the signs of the partial derivatives:  $\eta^1_{\mathcal{E}} < 0$ ,  $\eta^1_{\lambda_0} > 0$ ,  $\eta^1_{\chi} > 0$ ,  $\eta^2_{\mathcal{E}} < 0$ ,  $\eta^2_{\chi} < 0$ ,  $\eta^2_{\lambda_0} < 0$ ,  $\eta^3_{\lambda_0} > 0$ , and  $\eta^3_{\chi} > 0$ . Since the comparative statics is

performed around stable equilibria, define the function

$$\Omega(\mathcal{E}, \chi, \lambda_0) = \begin{bmatrix} \eta^1(\mathcal{E}, \chi, \lambda_0) + \mathcal{E} \\ \eta^2(\mathcal{E}, \chi, \lambda_0) + \chi \\ \eta^3(\mathcal{E}, \chi, \lambda_0) + \lambda_0 \end{bmatrix}. \quad (\text{A100})$$

Using the same argument that leads to stability condition (A49), a necessary condition for an equilibrium  $(\mathcal{E}, \chi, \lambda_0)$  to be stable is that the eigenvalues of the Jacobian of  $\Omega$  are smaller than one in absolute value. As before, a necessary condition is that the determinant of the Jacobian minus the identity matrix is negative, and thus

$$\begin{aligned} & \eta^1_{\mathcal{E}} \left( \eta^2_{\chi} \eta^3_{\lambda_0} - \eta^2_{\lambda_0} \eta^3_{\chi} \right) - \eta^1_{\chi} \left( \eta^2_{\mathcal{E}} \eta^3_{\lambda_0} - \eta^2_{\lambda_0} \eta^3_{\mathcal{E}} \right) \\ & \quad + \eta^1_{\lambda_0} \left( \eta^2_{\mathcal{E}} \eta^3_{\chi} - \eta^2_{\chi} \eta^3_{\mathcal{E}} \right) < 0. \end{aligned} \quad (\text{A101})$$

It is now possible to prove the statement.

Part (i). Consider the indifference conditions (A98), (A99), and (A96). Denote by  $\eta^1_c(\cdot)$ ,  $\eta^2_c(\cdot)$ , and  $\eta^3_c(\cdot)$  the partial derivative functions of  $\eta^1$ ,  $\eta^2$ , and  $\eta^3$  with respect to  $c$ . Note that  $\eta^2_c = \eta^3_c = 0$ . Using the Implicit Function Theorem, evaluated at an equilibrium  $(\mathcal{E}, \chi, \lambda_0)$ , the following must hold:

$$\frac{\partial \lambda_0}{\partial c} \underbrace{\left[ 1 - \frac{\eta^1_{\chi} \eta^2_{\lambda_0}}{\eta^1_{\lambda_0} \eta^2_{\chi}} - \frac{(\eta^1_{\chi} \eta^2_{\mathcal{E}} - \eta^1_{\mathcal{E}} \eta^2_{\chi})}{\eta^1_{\lambda_0} \eta^2_{\chi}} \frac{(\eta^3_{\chi} \eta^2_{\lambda_0} - \eta^3_{\lambda_0} \eta^2_{\chi})}{(\eta^3_{\mathcal{E}} \eta^2_{\chi} - \eta^3_{\chi} \eta^2_{\mathcal{E}})} \right]}_{:=D} = -\eta^1_c. \quad (\text{A102})$$

Since  $\eta^1_c < 0$ , the necessary and sufficient condition for  $\frac{\partial \lambda_0}{\partial c} < 0$  is that  $D < 0$ . This holds for all stable equilibria, as it is implied by (A101).

Given that  $Q_0 = 1$  in equilibrium, it has to be the case that  $\lambda_0$  increases (decreases) if and only if  $Q_1$  decreases (increases), and hence if and only if  $\frac{\gamma M_0}{M_1}$  increases (decreases). It follows that  $\partial(\frac{\gamma M_0}{M_1})/\partial c < 0$ .

To study the effect of  $c$  on  $\gamma$ , note first that the market-clearing equation can be used to substitute  $\mathcal{E}$  in the system defined by  $\eta^1(\mathcal{E}, \chi, \lambda_0)$ ,  $\eta^2(\mathcal{E}, \chi, \lambda_0)$ , and  $\eta^3(\mathcal{E}, \chi, \lambda_0)$ ; the resulting system pins down the equilibrium  $(\gamma, \chi, \lambda_0)$ , and its partial derivatives with respect to  $(\gamma, \chi, \lambda_0, c)$  have the same signs as the partial derivatives of the original system with respect to  $(\mathcal{E}, \chi, \lambda_0, c)$ . It follows that  $\text{sign}(\frac{\partial \gamma}{\partial c}) = \text{sign}(\frac{\partial \mathcal{E}}{\partial c})$ . To study  $\frac{\partial \mathcal{E}}{\partial c}$ , consider the following three cases:

Case I. Assume  $\frac{\partial \chi}{\partial c} < 0$ . For  $\partial(\frac{\gamma M_0}{M_1})/\partial c < 0$  to hold, it has to be the case that  $\frac{\partial \gamma}{\partial c} < 0$ , and thus  $\frac{\partial \mathcal{E}}{\partial c} < 0$ .

Case II. Assume  $\frac{\partial \chi}{\partial c} > 0$  and  $\eta^2_{\lambda_0} > 0$ . Differentiating  $\eta^2$ , it must hold that

$$\eta^2_{\mathcal{E}} d\mathcal{E} + \eta^2_{\chi} d\chi + \eta^2_{\lambda_0} d\lambda_0 + \eta^2_c dc = 0, \quad (\text{A103})$$

and thus

$$\frac{d\mathcal{E}}{dc} = -\frac{\eta_c^2}{\eta_\varepsilon^2} - \frac{\eta_\chi^2}{\eta_\varepsilon^2} \frac{d\chi}{dc} - \frac{\eta_{\lambda_0}^2}{\eta_\varepsilon^2} \frac{d\lambda_0}{dc} < 0. \quad (\text{A104})$$

Case III. Assume  $\eta_{\lambda_0}^2 < 0$ . In this case, the stability condition (A101) can hold only if  $\eta_\chi^2 \eta_{\lambda_0}^3 > \eta_{\lambda_0}^2 \eta_\chi^3$ . Using again total differentiation,

$$\frac{d\mathcal{E}}{dc} = \frac{d\lambda_0}{dc} \left( \frac{\eta_{\lambda_0}^2 \eta_\chi^3 - \eta_\chi^2 \eta_{\lambda_0}^3}{\eta_\varepsilon^3 \eta_\chi^2 - \eta_\chi^3 \eta_\varepsilon^2} \right), \quad (\text{A105})$$

which implies that  $\frac{d\mathcal{E}}{dc} < 0$ . This case completes the proof that around a stable equilibrium,  $\frac{\partial \mathcal{E}}{\partial c} < 0$  and  $\frac{\partial \gamma}{\partial c} < 0$ .

Part (ii). To prove the second part of the statement, similar to the proof of Corollary 2, note that  $\frac{\partial \lambda_0}{\partial c} < 0$  and  $\partial(\frac{\gamma M_0}{M_1})/\partial c < 0$  imply a change in the distribution of returns that mirrors the marginal effect of  $\kappa$  and  $R_0$  described in Corollary 2, part (i). For the effects described in Corollary 2, part (ii), and Corollary 3 to hold after a marginal change in  $c$  around interior equilibria with  $Q_0 = 1$ , it must be the case that the ratio

$$\omega_0(c) := \frac{\tilde{M}_0(c)}{\tilde{M}_0(c) + M_1(c)} \frac{1}{F(\lambda_0(c))} \quad (\text{A106})$$

is decreasing in  $c$ , where  $\tilde{M}_0(c) := \gamma(c)M_0(c)$ . This can be rewritten as

$$F(\lambda_0(c)) [\tilde{M}_0'(c)M_1(c) - \tilde{M}_0(c)M_1'(c)] < \tilde{M}_0(c) [\tilde{M}_0(c) + M_1(c)] f(\lambda_0(c)) \lambda_0'(c). \quad (\text{A107})$$

This is the equivalent of equation (A72), with the function  $\tilde{M}_0(c)$  replacing  $M_0(c)$ . Analogous arguments can be used to find examples in which the condition holds, as well as examples in which it is violated.  $\square$

PROOF OF PROPOSITION 8: If  $\chi^{sb} = \chi = 0$ , or if  $\chi^{sb} = \chi = 1$ , then a zero tax implements second-best. When  $\chi$  is interior,  $\chi^{sb} < \chi$ . Note that in this case, for every cutoff  $\lambda_0^{sb}$ , induced by  $\chi^{sb}$  it holds that

$$\frac{\alpha \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0^{sb}] - R_0}{\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0^{sb}] - R_0} > \frac{m_0}{m_1}. \quad (\text{A108})$$

Indeed, if (A108) would not hold, a marginal decrease in  $\chi^{sb}$  would cause an increase in the induced cutoff  $\lambda_0$ . Every VC would be strictly better off, implying the original  $\chi^{sb}$  is not the second-best solution.

The tax profile in Proposition 8 implies that for a given choice of  $\tau^*$ , a VC prefers to raise a large, low-attention fund if and only if

$$\frac{m_0}{m_1(1 - \tau^*)} > \frac{\alpha \mathbb{E}[R(a_1, \lambda) \mid \lambda \geq \lambda_0^{sb}] - R_0}{\alpha \mathbb{E}[R(a_0, \lambda) \mid \lambda \leq \lambda_0^{sb}] - R_0}. \quad (\text{A109})$$

Condition (A108) implies that, at  $\tau^* = 0$ , VCs strictly prefers the small, high-attention fund. The opposite must clearly hold for  $\tau^*$  sufficiently close to one, and thus a unique  $\tau^*$  exists such that all VCs are indifferent, and the second-best measure  $\chi^{sb}$ , inducing sorting  $\lambda_0^{sb}$ , can be implemented in equilibrium.

[The result extends naturally to heterogeneous VCs, by similarly observing that (A108) must hold for the second-best cutoff VC,  $x_0^{sb}$ .]  $\square$

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## Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

**Appendix S1:** Internet Appendix.  
**Replication Code.**