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Cross-border DCF valuation: discounting cash flows in foreign currency

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Abstract

The paper seeks to develop a comprehensive framework to cross-border discounted cash flow valuation. Although the literature on company valuation and on international financial management is vast, such a framework has not yet been proposed. We build upon well-known fundamentals and relevant contributions, e.g. on the derivation of the risk-adjusted rate of return. Relevant risks are exchange rate risk, business risk, financial risk, the risk of the tax effects induced by debt financing, and the risk of default. Additional tax effects beyond the well-known tax shield on interest expenses must be considered. Risk discounts from cash flows and risk premia to be added to risk-free interest rates are derived according to the global capital asset pricing model. A conceptual choice occurs not only between the foreign currency and the home currency approach, but also regarding the estimation of future exchange rates. The paper shows how a valuation can be implemented with or without consideration of covariances between cash flows and rate of returns with exchange rates. It also derives the discount rates if forward exchange rates are applied. We discuss the consequences of assuming the uncovered interest parity to hold. We assume deterministic debt and apply the adjusted present value approach. In addition, we derive the RADR to be used in the flow to equity and weighted average cost of capital approach. The paper addresses not only the valuation of a foreign company, but also the valuation of a domestic company that generates cash flows in foreign currency and/or uses debt in foreign currency.

Keywords Exchange risk · Foreign currency · Cross-border valuation · Valuation · International finance

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1 Introduction

The literature on company valuation is vast. Key contributions to discounted cash flow (DCF) valuation were made by Modigliani and Miller (1958, 1963), Miles and Ezzell (1980), Harris and Pringle (1985) and Inselbag and Kaufold (1997). Of course, this is also true for the literature on international financial theory. For example, forecasting and hedging flexible exchange rates, international parity theories, the properties of international capital markets, or the pricing of assets in these markets has been of interest to researchers over decades. The textbooks of Sercu (2009) and Bekaert and Hodrick (2018) provide a good overview and a thorough analysis of this field.

The interface between these two streams of the literature, the cross-border valuation of companies, has been analyzed extensively when it comes to the expected rate of returns for shareholders (cost of equity). Numerous papers have analyzed the risk-return relationship based on the capital asset pricing model (CAPM). Mehra (1978), Solnik (1974), Sercu (1980) and Stulz (1995) suggested different models, which have been discussed in the following from a conceptual perspective (see for example Ruiz de Vargas and Breuer 2018, 2019) while e.g. Ejara et al. (2019) provide an empirical analysis. Several papers and chapters in textbooks address the valuation of cash flows in foreign currency (FC) using the home currency (HC) and the foreign currency (FC) approach (e.g. Bekaert and Hodrick 2018; Breuer 2001; Butler et al. 2013; Ruiz de Vargas 2019). Leading textbooks on corporate finance such as Berk and DeMarzo (2020) or Brealey et al. (2019) cover cross-border valuations only briefly. This is true also for Koller et al. (2015), although these authors cover the use of the CAPM for cross-border valuation more closely than Berk and DeMarzo (2020) or Brealey et al. (2019). Holthausen and Zmijewski (2020) provide a reconciliation between the FC and the HC approach, and cover additional aspects, e.g. tax effects.

However, there is a gap in the literature when it comes to a comprehensive analysis of a cross-border DCF valuation of unlevered and levered companies. Topics to be addressed are, for instance, the identification of the relevant risks and the pricing of these risks, or a consistent integration of the effects of capital structure on cash flows and risks. A thorough understanding of these topics is necessary for a consistent implementation of the established DCF approaches to the FC approach and the HC approach. This includes the definition of risk adjusted discount rates (RADR). Relevant risks are the exchange rate risk, business risk, financial risk, the risk of the tax effects caused by debt financing, and the risk of default. The application of the CAPM for the pricing of risk and the derivation of the RADR, as well as the proper definition of the terminal value impose additional challenges for the valuation of a foreign company or a domestic company that generates cash flows in FC and/or uses debt denominated in a FC. Basically, the established tool kit to DCF valuation has to be extended in order to cope with different currencies by linking fundamental contributions of macroeconomics and

financial theory. This leads to conceptual choices not only between the FC and the HC approach, but also regarding the estimation of exchange rates to be used over the forecast horizon. To the best of my knowledge, a paper that tries to develop a comprehensive framework is still missing. This paper aims at filling this gap.

For achieving this objective, Sect. 2 presents assumptions. It discusses basic principles by applying Cox et al. (1979). It also contains a parsimonious application of the value additivity principle (Schall 1972; Haley and Schall 1979) to cross-border valuation. This second part of Sect. 2 shows in principle how a cross-border valuation can be implemented consistently. This will prove helpful for the valuation of an unlevered foreign company (Sect. 3) and a levered foreign company (Sect. 4). In Sects. 2, 3, and 4, we introduce a series of risk categories beginning with exchange rate risk (Sect. 2), followed by business risk (Sect. 3), financial risk (Sect. 4), the risk of default (Sect. 4), and the risk attributable to tax effects (Sect. 5). For the pricing of risk, we will refer to the global CAPM. We discuss the link between the inputs to the CAPM for the HC perspective and for the FC perspective. We derive RADR definitions and value equations, if expected spot exchange rates or forward exchange rates are used, for both unlevered (Sect. 3) and levered companies (Sect. 4). If forward exchange rates are used, we discuss the consequences of assuming the uncovered interest parity to hold. Section 5 deals with the valuation of a domestic company that uses debt denominated both in domestic and in foreign currency. This requires the consideration of a tax effect beyond the well-known tax shield on interest expenses, because repayments are also subject to exchange rate risk. In Sect. 6, we compare our findings with selected literature contributions. Section 7 provides conclusions.

2 Assumptions and fundamentals

2.1 Assumptions

We establish the following assumptions for our analysis:

- We consider two countries. Relevant currencies are the home (domestic) currency (HC) and the foreign currency (FC).
- The foreign company to be valued in a discrete-time setting generates only cash flows in that FC. We assume the payout of free cash flows (residual dividend policy).
- If the foreign company uses debt financing (Sect. 4), it employs debt in that FC. In Sect. 5 we assume a domestic company that generates cash flows in both domestic and foreign currency and carries debt denominated in both currencies.
- Both foreign and domestic companies are valued from the perspective of a domestic investor. The valuation result, the value of equity, is to be denominated in HC. We use the direct quotation, i.e. the price for one unit of FC quoted in HC, throughout the paper. We do not discuss the implications of the Siegel's paradox (see Siegel 1972, Breuer 2015, pp. 43–48, Solnik 1993, p. 184, Hull 2018, p. 697).

- The capital markets of the home and foreign country are perfect, complete, and fully integrated. This implies that they are free of arbitrage opportunities. Risk-averse capital market participants process the information into homogeneous expectations.
- Following the previous assumptions, the covered interest parity (CIP) holds. We do not require the uncovered interest parity (UIP) to hold, but refer to it on several occasions to illustrate its consequences.¹
- Risk is priced according to the global CAPM. This requires the relative purchase price parity to hold (see Koller et al. 2015, p. 496, Bekaert and Hodrick 2018, p. 569, Stulz 1995, p. 12). Alternative CAPM settings as suggested by Solnik and Sercu (see Sercu 2009, Chapter 19) or Mehra (1978) are not considered here. This choice could be justified by empirical evidence provided by Krapl and O'Brien (2016) and Ejara et al. (2019), supported by the cautious recommendation of Sercu (2009), pp. 687–693, and of others as cited in Ruiz de Vargas (2019), p. 1668.
- Domestic and foreign corporate income is subject to a constant and identical corporate tax rate (for differential international taxes see Senbet 1979). Personal income taxes, barriers to repatriation of cash flows, and transaction costs do not exist.²
- The nominal risk-free rates are deterministic for both countries. There is no money illusion, and the international Fisher hypothesis holds (see Ruiz de Vargas and Breuer 2018, p. 15). This allows us to exclude a discussion of state-contingent inflation rates and inflation risk.
- We consider a binomial one-period case first. A multi-period model can be derived by analyzing a chain of single period models in which the intertemporal connection of states (stochastic independence vs. stochastic dependence) becomes relevant, and a recursive valuation (roll-back-procedure) is to be applied. We use a multi-period setting later, but simplify the model by using expected values without addressing the intertemporal connections, and by applying the global CAPM in a multi-period setting. The latter assumption implies that risk-free rates are deterministic over an infinite time span, and that the distribution of the return on the market portfolio is time invariant.

2.2 Exchange rates and the present value of one unit in foreign currency

First, we summarize the fundamental links between current spot exchange rates, expected spot exchange rates and forward exchange rates. We then prepare the analysis of cross-border valuations by valuing an investment in one unit of FC.

¹ For a discussion of international parity theories, see, for example, Bekaert and Hodrick (2018), Chapters 7, 8, 10, and Breuer (2015), Chapter 2.

² For overviews on how provisions of the tax code on e.g. double taxation agreements, or the remittance of foreign earnings might become relevant, see e.g. Eiteman et al. (2016), Chapter 15, Holthausen and Zmijewski (2020), Chapter 17.4, Sercu (2009), Chapter 20, Senbet (1979), Hodder and Senbet (1990), Chowdhry and Coval (1998).

The covered interest parity establishes the well-known link between (current) spot exchange rate S and forward exchange rate F that is the ratio of one plus the risk-free interest rate i in both countries (HC and FC):

$$F_1 = S_0 \frac{1 + i_{HC,1}}{1 + i_{FC,1}} \tag{1}$$

Our discussion will not require the uncovered interest parity or the unbiasedness hypothesis (foreign exchange expectation) to hold. The UIP states that the expected exchange rate $E[S]$ is equal to the (current) spot exchange rate S multiplied with the ratio of one plus the risk-free interest rate i in both countries. The unbiasedness hypothesis (UH) links CIP and UIP, by assuming that the forward exchange rate is an unbiased estimator of the expected spot exchange rate (see, for example, Breuer 2015, pp. 41–51, Bekaert and Hodrick 2018, pp. 269–272, Sercu 2009, p. 430).

$$E[S_1] = S_0 \frac{1 + i_{HC,1}}{1 + i_{FC,1}} = F_1 \tag{2}$$

Instead, we will model the link between expected spot exchange rate and forward exchange rate generally without an assumption regarding the magnitude or the sign of the difference between both rates.

For a binomial setting, the assumption of a complete capital market requires the prices for two Arrow Debreu Securities, one of them being a risk-free security. These prices determine the risk-neutral probability q for the up-state and $(1 - q)$ for the down-state (Cox et al. 1979). The relationship between the return in the down state d , the risk-free return, and the return in the up-state u is $d < 1 + i < u$. We use this setting for the domestic capital market first. The risk-neutral probability q is defined by (Cox et al. 1979, pp. 234–235):

$$q = \frac{(1 + i_{HC}) - d}{u - d} \tag{3}$$

The present value (V_S) of one monetary unit in FC received in one year can be written as the present value of the certainty equivalent (CE), the forward exchange rate. Alternatively, using the RADR (risk adjusted discount rates) approach, V_S is defined as the expected spot exchange rate discounted with the risk equivalent expected rate of return, $E[r_{V_S}]$, using the probability p for the up-state and $(1 - p)$ for the down-state respectively (Sercu 2009, pp. 134–137, 432, Hull 2018, p. 129):

$$V_{S,0} = \frac{1}{1 + i_{HC}} \underbrace{[qS_{u,1} + (1 - q)S_{d,1}]}_{F_1} = \frac{1}{1 + E[r_{V_S}]} \underbrace{[pS_{u,1} + (1 - p)S_{d,1}]}_{E[S_1]} \tag{4}$$

Using Eq. (1), we can rewrite the present value using the current spot exchange rate:

$$V_{S,0} = S_0(1 + i_{FC})^{-1} \tag{5}$$

The risk equivalent rate r_{VS} is the sum of the risk-free home interest rate i_{HC} and the premium for the exchange rate risk, z_{VS} . It is worth noting that this rate is not equal to the expected change in exchange rates (δ_S), which is defined as the ratio of the expected spot exchange rate in $t=1$ divided by the spot exchange rate in $t=0$ (minus one). The risk equivalent rate r_{VS} rather relates the expected HC equivalent of a state-contingent cash flow of one unit in FC to its present value. Stated alternatively, it relates the expected value of the state-contingent future exchange rates multiplied by the i_{FC} (plus one) to the current spot exchange rate, thus implying an investment of one unit FC in $t=0$ at i_{FC} for 1 year. The rates are linked³:

$$E[r_{VS}] = \frac{E[S_1]}{V_{S,0}} - 1 = (1 + i_{FC}) \frac{E[S_1]}{S_0} - 1 = (1 + i_{FC})(1 + E[\delta_S]) - 1 \quad (6)$$

The right-hand side (RHS) of this equation can be interpreted from an economic perspective in an alternative manner: if an investor has one unit of FC now and will exchange it one period later, he will reinvest it for one year at the rate i_{FC} . Therefore, he is not only experiencing a change in exchange rates, when he converts this one unit of FC into HC in $t=1$, but he earns the risk-free return, i_{FC} , too. We will return to the distinction between r_{VS} and δ_S later.

The expected spot exchange rate is of interest for a cross-border valuation. While the current spot exchange rate and the forward exchange rate are observable, the expected spot exchange rate is not directly observable. However, we can establish a link between different exchange rates, and can come up with a definition of the expected spot exchange rate. For a binomial distribution of the exchange rates, the expected spot exchange rate can be defined with reference to F and one of the state-contingent exchange rates. With the risk-neutral definition of F_I in Eq. (4), the spot exchange rate in the up-state follows the spot exchange rate in the down-state:

$$S_{u,1} = \frac{F_1 - S_{d,1}}{q} + S_{d,1} \quad (7)$$

The observable forward exchange rate together with the (assumed) spot exchange rate for the down state are sufficient to determine the spot exchange rate for the up state, thus completing the binomial exchange rate distribution. Alternatively, we could complete the binomial distribution beginning with S_u . Given S_u and S_d , the expected spot exchange rate follows from using the probability p for the up-state and $(1 - p)$ for the down-state respectively, as shown by the RHS of Eq. (4).

We will revisit the pricing of risk in Sect. 3 in more detail, but further illustrate the link between the expected spot exchange rate and the forward exchange rate here. For this reason, we introduce RD_S denoting the discount from the expected spot exchange rate due to the exchange rate risk. This cash flow equivalent to z_{VS} is the difference between the expected spot exchange rate and the forward exchange

³ Equation (6) could also be derived by using the equations in Sercu (2009), p. 136, and Bekaert and Hodrick (2018), pp. 263–263.

rate. Given the binomial framework used in this section, the risk discount equals the difference between the spot exchange rate in the up-state and the spot exchange rate in the down-state (Δ_S) multiplied by the difference between the ‘regular’ probability p and the risk-neutral probability q , each referring to the up-state (see [Appendix](#)):

$$RD_S = E[S_1] - F_1 = (p - q)\Delta_S \quad (8)$$

Since we assumed risk-aversion, p exceeds q . Thus, the sign of the risk discount is positive, if the exchange rate in the up-state is higher than the rate in the down-state, i.e. $\Delta_S > 0$. The UIP in combination with the unbiasedness hypothesis would assume a risk premium of zero. In a binomial setting, this implies either Δ_S to be zero, which would imply a state-independent exchange rate, or $p = q$, which implies risk neutrality.

The expected spot exchange rate can be expressed as:

$$E[S_1] = F_1 + (p - q)\Delta_S \quad (9)$$

Using Eqs. (4) and (5), the expected spot exchange rate can be expressed more generally:

$$E[S_1] = \left(1 + E[r_{V_S}]\right)V_{S,0} = \left(1 + i_{HC} + z_{V_S}\right)S_0(1 + i_{FC})^{-1} \quad (10)$$

The expected spot exchange rate depends upon the premium for the exchange rate risk, z_{V_S} , which may be estimated empirically, and the risk-free interest rates and the spot exchange rate, which are observable at the valuation date.

2.3 Value additivity and general valuation framework

Based upon these fundamental links, we develop a general understanding of how to discount an expected cash flow in FC (A) occurring one year from now to its present value in the HC (V_{HC}). We will address the definition of the cash flow to be discounted and the derivation of the RADR in later chapters. In this section, we apply the value additivity principle (Schall 1972; Haley and Schall 1979) to DCF valuation (see Schüller 2018a).

d_A stands for the risk-equivalent factor to discount the expected cash flow in FC (A) and is defined as $1 + \text{RADR}$. A could be converted into the HC by multiplying it by the expected spot exchange rate, which requires the discount factor to be converted into the HC as well. This is done by multiplying d_A with d_S , which is defined as $1 + \delta_S$.

Since state contingent cash flows in FC as well as RADR in FC might depend upon state contingent exchange rates, the respective covariances must be considered in both the numerator and the denominator (see, for example, Sercu 2009, pp. 668–669 or O’Brien 2017, pp. 58, 124). However, the covariances in the numerator and denominator cancel out, and the valuation can be done without them (see [Appendix](#)):

$$V_{HC,0} = \frac{\overbrace{E[A_1] \cdot E[S_1]}^{E[A_1 \cdot S_1]} + \text{cov}(A_1, S_1)}{d_A d_S + \text{cov}(d_A, d_S)} = \frac{E[A_1] \cdot E[S_1]}{d_A d_S} \quad (11)$$

In addition to being helpful for valuation practitioners, this result enables us to link two approaches to value cash flows in foreign currencies. Equation (11) represents the HC approach that is characterized by converting the FC cash flows first into HC, and then discount the cash flows in HC to their present value in HC.

Alternatively, the cash flows in FC can be discounted to their present value in FC first, and then can be converted into HC using the current spot exchange rate (FC approach). It can be expressed by starting with the RHS of Eq. (11) and using the definition of d_S , i.e. the expected spot exchange rate divided by the current spot exchange rate:

$$V_{HC,0} = \frac{E[A_1] \cdot E[S_1]}{d_A \frac{E[S_1]}{S_0}} = \frac{E[A_1]}{\underbrace{d_A}_{V_{FC,0}}} S_0 \quad (12)$$

The covariances are irrelevant for the FC approach.

This basic valuation principle can also be used to illustrate how the discount factor d_A in the denominator on the RHS of Eq. (11) must be modified, if the forward exchange rate is used in the numerator instead of the expected spot exchange rate. It can be shown that d_A must be multiplied by the ratio of the risk-free interest rates each increased by 1 (see [Appendix](#)):

$$V_{HC,0} = \frac{E[A_1] \cdot F_1}{d_A \frac{1+i_{HC}}{1+i_{FC}}} \quad (13)$$

Thus, other than in Eq. (11) the rate d_A is not multiplied by d_S , i.e. the ratio between expected spot exchange rate and current spot exchange rate, for discounting cash flows converted with the expected spot exchange rate, but has to be multiplied by the ratio of the forward exchange rate to the current spot exchange rate, i.e. the ratio based upon the risk-free interest rates, instead. This result does not depend upon the UIP to hold. At first sight, this result might look familiar considering the approaches suggested by textbooks as Bekaert and Hodrick (2018), pp. 726–734, Berk and DeMarzo (2020), pp. 1099–1103, Brealey et al. (2019), pp. 731–734, Holthausen and Zmijewski (2020), pp. 857–868, and Koller et al. (2015), pp. 490–493. However, our discussion in the next chapters will extend the basic principle shown in Eq. (13), and will enable us to reveal the underlying assumptions, which in turn makes it possible to identify some gaps and inconsistencies of these textbook-approaches in Sect. 6.

In summary, the general valuation framework showed (a) that the valuation can be conducted in a consistent manner with and without covariances, (b) that the valuation can be done with the HC or the FC approach, and (c) that using forward

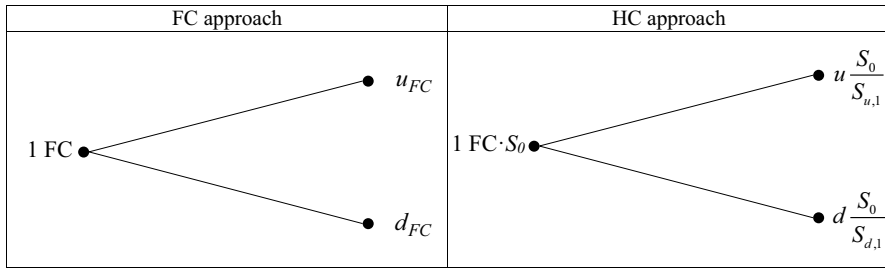


Fig. 1 FC approach and HC approach for a risky investment of one unit of foreign currency

exchange rates instead of expected spot exchange rates requires a modification of the discount rate.

3 Valuing an unlevered foreign company

3.1 Risk-neutral valuation

We now will extend the discussion by applying the previous results to the valuation of a foreign company that is financed by equity only (unlevered foreign company). Different from Sect. 2.2, when we valued one unit FC while exchange rate risk was the only relevant risk, we now consider risky cash flows in FC as well. Thus, previously we discussed price risk for one currency unit, now we introduce volume risk, i.e. the risk caused by state-contingent volumes of currency units (FCF in FC), additionally. We will denote the volume risk as business risk in the following.

An unlevered free cash flow after corporate taxes (FCF_U) occurring in $t=1$ can be valued according to the risk-neutral version of the HC approach⁴:

$$V_{U,HC,0} = [qFCF_{Uu,FC,1}S_{u,1} + (1 - q)FCF_{Ud,FC,1}S_{d,1}] (1 + i_{HC})^{-1} \quad (14)$$

Applying the risk-neutral probabilities to state-contingent cash flows leads to the certainty equivalent of the cash flows, which is discounted by the risk-free rate of return.

For deriving a risk-neutral version of the FC approach, we first need to understand the return on the risky security, which serves as a point of reference for the pricing on the capital market. Figure 1 illustrates the returns on an investment of one unit FC into this risky security at the valuation date ($t=0$) for the FC and the HC approach.

Thus, the FC-returns are defined by:

⁴ Needless to say, that risk-neutral valuation does not imply risk-neutral investors, but risk-averse ones. The method is labelled to be risk-neutral, because risk-neutral probabilities, which lead to certainty equivalents, are used.

$$u_{FC} = u \frac{S_0}{S_{u,1}}; d_{FC} = d \frac{S_0}{S_{d,1}} \tag{15}$$

And the FC-risk-neutral probability follows from:

$$q_{FC} = \frac{(1 + i_{FC}) - d_{FC}}{u_{FC} - d_{FC}} = \frac{(1 + i_{HC}) \frac{S_0}{F_1} - d \frac{S_0}{S_{d,1}}}{u \frac{S_0}{S_{u,1}} - d \frac{S_0}{S_{d,1}}} = \frac{(1 + i_{HC}) F_1^{-1} - d S_{d,1}^{-1}}{u S_{u,1}^{-1} - d S_{d,1}^{-1}} \tag{16}$$

Equation (16) begins with Cox et al. (1979, pp. 234–235), and is applied to cross-border valuation. To the best of my knowledge, a similar equation has not yet been presented in the literature. It shows the transition from the HC perspective to the FC perspective for the relevant parameters. It is worth mentioning, that if the UIP is assumed to hold, and the forward exchange rate is set equal to the expected spot exchange rate according to the UH, the risk-neutral probability would be the same for the FC and the HC perspective:

$$q_{FC} = \frac{(1 + i_{FC}) - d_{FC}}{u_{FC} - d_{FC}} = \frac{(1 + i_{HC}) \frac{S_0}{F_1} - d \frac{S_0}{F_1}}{u \frac{S_0}{F_1} - d \frac{S_0}{F_1}} = \frac{(1 + i_{HC}) - d}{u - d} = q \tag{17}$$

Under these assumptions, the state-contingent exchange rates as used in Eq. (16) can be replaced by the forward exchange rate. By doing so, we refer to the reasoning behind Eq. (9): Since we assume risk aversion ($p \neq q$), the difference between the exchange rates in states u and d must be zero.

Coming back to the general case and using the FC-version of the risk-neutral probability according to Eq. (16) allows us to value the company in two equivalent ways:

$$\begin{aligned} V_{U,HC,0} &= S_0 [q_{FC} FCF_{Uu,FC,1} + (1 - q_{FC}) FCF_{Ud,FC,1}] (1 + i_{FC})^{-1} \\ &= F_1 \underbrace{[q_{FC} FCF_{Uu,FC,1} + (1 - q_{FC}) FCF_{Ud,FC,1}]}_{CE_{FC,1}} (1 + i_{HC})^{-1} \end{aligned} \tag{18}$$

The first line of this equation is a straight-forward implementation of the FC approach, while the second line is the HC approach. It follows from substituting S using Eq. (1). Based upon certainty equivalents, using the forward exchange rate to convert cash flows from FC into HC is equivalent to using state contingent exchange rates.

We can use this relation to separate the risk due to state contingent exchange rates (price risk or exchange rate risk) from the risk of state contingent FCF_U (volume risk or business risk). Using CE_U for the certainty equivalent in units of FCF_U , and RD_{FCFu} for the risk discount to be subtracted from the expected FCF_U , as well as RD_S for the risk discount to be subtracted from the expected spot exchange rate, we get (see Appendix):

$$\begin{aligned}
 V_{U,HC,0} &= F_1 \underbrace{\left[q_{FC} FCF_{Uu,FC,1} + (1 - q_{FC}) FCF_{Ud,FC,1} \right]}_{CE_{U,FC,1}} (1 + i_{HC})^{-1} \\
 &\quad \underbrace{\hspace{10em}}_{CE_{U,HC,1}} \\
 &= (E[S_1] - RD_S) \left(E[FCF_{U,FC,1}] - RD_{FCF_{U,FC}} \right) (1 + i_{HC})^{-1}
 \end{aligned}
 \tag{19}$$

After expanding, we get a stepwise definition of the unlevered certainty equivalent in HC:

$$V_{U,HC,0} = \left(\underbrace{E[FCF_{U,FC,1}] E[S_1]}_{(a)} - \underbrace{RD_S E[FCF_{U,FC,1}]}_{(b)} - \underbrace{RD_{FCF_{U,FC}} E[S_1]}_{(c)} + \underbrace{RD_S RD_{FCF_{U,FC}}}_{(d)} \right) (1 + i_{HC})^{-1}
 \tag{20}$$

The starting point is the expected *FCF* multiplied by the expected spot exchange rate (a). Then, the discount for the exchange risk per monetary unit is multiplied by the relevant number of units, i.e. the expected FCF_U (b), and subtracted from (a). In analogy to r_{VS} , RD_S refers to the risk considered in the present value of one unit FC. (c) is also to be subtracted from (a), and represents the business risk in FC, translated into domestic currency. (d) is the cross-product of the risk discounts.

In summary, introducing risky unlevered *FCF* implies the introduction of a second category of risk besides the exchange rate risk, which is the business risk. The risk-neutral valuation framework can be applied also for the FC approach, but requires a definition of the exogenous valuation parameters u , d , and q . If the UIP and the UH are assumed to hold, these parameters are the same for both the HC and the FC approach. Finally, the transition from FC approach to HC approach reveals that an application of the HC approach requires a bottom up understanding of the business risk in FC and the exchange rate risk. It also reveals that a simple addition of risk discounts is not sufficient, but crosslinks between the components have to be accounted for as well.

3.2 CAPM-based pricing of exchange rate risk

So far, we have not addressed the pricing of risk. We will refer to the global CAPM for pricing both exchange rate risk and the business risk. We start with the pricing of exchange rate risk in this section, and deal with business risk in Sect. 3.3. It is possible to conduct a risk-neutral valuation based upon the CAPM, if one interprets the market portfolio M as the risky Arrow-Debreu security with known state-contingent prices. An investment in M in $t=0$ yields a return in the up-state of $u = 1 + r_{M,u}$ and in the down-state of $d = 1 + r_{M,d}$. The equations shown in the previous section still hold. The FC approach and the HC approach can be applied, if the CAPM parameters are defined accordingly. We will return to that requirement later.

Now, we will price exchange rate risk. Exchange rate risk does not affect the FC approach directly. It does affect it indirectly, if the parameters u , d , and q are derived

from their respective HC-values, as illustrated in Sect. 3.1. Thus, we focus on the HC approach. Applying Rubinstein (1973) or Fama (1977) to the HC approach, one unit FC in $t=1$ can be valued referring to either certainty equivalents or RADR, where MRP stands for market risk premium (difference between the expected rate of return on the market portfolio and the risk-free rate). The following equation also includes λ_{HC} , the market price for risk, and the beta value representing the systematic risk of investing one unit FC in $t=0$ (or, equivalently, the systematic risk inherent in the present value of one unit FC received in $t=1$).

$$V_{S,0} = \frac{E[S_1] - \overbrace{\lambda_{HC} \text{cov}(S_1, r_{M,HC})}^{\frac{E[MRP_{HC}]}{\sigma_{M,HC}^2}}}{1 + i_{HC}} = \frac{E[S_1]}{1 + i_{HC} + \underbrace{\beta_{V_S} \frac{E[MRP_{HC}]}{\text{cov}(r_{V_S}, r_{M,HC})} E[r_{M,HC}]^{-i_{HC}}}_{E[r_{V_S}]}} \tag{21}$$

The expected rate of return on the present value of one unit in FC, defined in the denominator on the RHS of this equation, is linked to the expected rate of change in the exchange rate as shown in Eq. (6). This definition in the denominator on the RHS is the CAPM-equivalent to the RHS of Eq. (4). Other authors like Zwirner (1989), pp. 96–102, and Ruiz de Vargas and Breuer (2018), refer to a coefficient that relates to the expected change in the exchange rate. The link between both betas is a simple one:

$$\begin{aligned} \text{cov}(r_{V_S}, r_{M,HC}) &= \text{cov}(\delta_S, r_{M,HC})(1 + i_{FC}) \\ \beta_{V_S} &= \beta_S(1 + i_{FC}) \end{aligned} \tag{22}$$

When it comes to valuation, β_{V_S} must be used. If one would try to argue that δ_S is a rate of return, the counterargument would be that this rate misses the reinvestment at i_{FC} . This is exactly covered by the transition to β_{V_S} , as shown by Eq. (22). Bekaert and Hodrick derive this link form comparing—in their words—the excess return and the forward market return on a FC investment (Bekaert and Hodrick 2018, p. 277).

The usual definition of a beta value suffers from a circular reference: the covariance refers to the rate of return, which in turn depends upon the valuation result, because the rate of return relates a surplus to its present value. However, we can use a binomial distribution to circumvent this problem. A binomial distribution can be rearranged by using the surplus in state d as a certain surplus in both states, d and u, and the difference (Δ) between the surplus in state u and state d as a risky distribution (state u: Δ ; state d: 0). As shown in Schöler (2018b), the beta value for this risky distribution can be defined without a circular reference:

$$\beta_{\Delta|0} = \frac{1 + i}{i - r_{M,d}} \tag{23}$$

Because β_{VS} is the value-weighted average consisting of this beta value and a beta value of zero for the certain spot exchange rate S_d , we can conclude that

$$\beta_{VS} = \beta_{\Delta} \frac{V_{\Delta}}{V_S} + 0 \frac{S_{d,1}(1+i_{HC})^{-1}}{V_S} = \beta_{\Delta} \left[1 - \frac{S_{d,1}(1+i_{HC})^{-1}}{S_0(1+i_{FC})^{-1}} \right] = \beta_{\Delta} \left(1 - \frac{S_{d,1}}{F_1} \right) \tag{24}$$

This equation is not only useful when it comes to the derivation of the RADR, but also with regard to the link between observable parameters and the expected spot exchange rate. In analogy to our analysis in Sect. 2.2, the expected spot rate is defined as the sum of the forward exchange rate and the risk discount for the exchange rate risk. It follows for the binomial case (see [Appendix](#)):

$$E[S_1] = F_1 + RD_S = F_1 + (F_1 - S_{d,1}) \frac{E[MRP_{HC}]}{i_{HC} - r_{M,d}} \tag{25}$$

Departing the binomial case, we could write more generally for the expected spot exchange rate (see "[Appendix](#)", and also Ruiz de Vargas 2019, p. 1663, who uses β_S):

$$E[S_1] = F_1 + RD_S = F_1 + F_1(1+i_{HC})^{-1} \beta_{VS} E[MRP_{HC}] = F_1 + S_0(1+i_{FC})^{-1} \beta_{VS} E[MRP_{HC}] \tag{26}$$

3.3 CAPM-based pricing of business risk and exchange rate risk

Introducing a risky unlevered free cash flow in $t=1$, again referring to Rubinstein (1973) or Fama (1977), and using Eq. (20) for formulating the HC-approach, leads to the following equations to value the unlevered company:

Based upon certainty equivalents:

3.3.1 FC approach

$$V_{U,HC,0} = S_0 \left[E[FCF_{U,FC,1}] - \lambda_{FC} \text{cov}(FCF_{U,FC,1}, r_{M,FC}) \right] (1+i_{FC})^{-1}$$

with $\lambda_{FC} = \frac{E[MRP_{FC}]}{\sigma_{M,FC}^2}$ (27)

3.3.2 HC approach (see "[Appendix](#)")

$$V_{U,HC,0} = \left[E[FCF_{U,HC,1}] - \lambda_{HC} \text{cov}(FCF_{U,HC,1}, r_{M,HC}) \right] (1+i_{HC})^{-1}$$

$$= \left[\underbrace{E[FCF_{U,FC,1}] E[S_1]}_{(a)} - \underbrace{\lambda_{HC} \text{cov}(S_1, r_{M,HC}) E[FCF_{U,FC,1}]}_{(b)} \right.$$

$$\left. - \underbrace{\lambda_{FC} \text{cov}(FCF_{U,FC,1}, r_{M,FC}) E[S_1]}_{(c)} + \underbrace{\lambda_{HC} \text{cov}(S_1, r_{M,HC}) \lambda_{FC} \text{cov}(FCF_{U,FC,1}, r_{M,FC})}_{(d)} \right] (1+i_{HC})^{-1} \tag{28}$$

Equation (28) is the CAPM-based equivalent to Eqs. (19) and (20). (b) represents the discount for the exchange rate risk, (c) is the discount compensating the business risk, and (d) is the cross-product of the risk discounts.

Based upon RADR:

3.3.3 FC approach

$$V_{U,HC,0} = S_0 E[FCF_{U,FC,1}] \underbrace{(1 + i + \beta_{U,FC} MRP_{FC})^{-1}}_{1+E[r_{U,FC}]} \tag{29}$$

3.3.4 HC approach

$$V_{U,HC,0} = \underbrace{[E[FCF_{U,FC,1}]E[S_1] + \text{cov}(FCF_{U,FC,1}, S_1)]}_{E[FCF_{U,HC,1}]} \underbrace{(1 + i + \beta_{U,HC} MRP_{HC})^{-1}}_{1+E[r_{U,HC}]} \tag{30}$$

In order to develop a better understanding of the risk premia relevant for the derivation of $r_{U,HC}$ in Eq. (30), and for transitioning from the FC approach to the HC approach, we need to address the inputs for defining the discount rates for the FC and the HC approach.

Table 2 in the Appendix contains the parameters determining the market price of risk (λ) and the amount of relevant risk for both approaches. The relevant risk is measured by the covariance of the unlevered free cash flows with the market return for the CE-based valuation, and by the covariance of the unlevered rates of return with the market return for the RADR-based valuation.

As Table 2 in the Appendix illustrates, there is no simple transition between FC and HC approach for the market price of risk ($\lambda_{FC} \rightleftharpoons \lambda_{HC}$) and the volume of risk ($\text{cov}_{FC} \rightleftharpoons \text{cov}_{HC}$). Goodman (1960), p. 712, proves the transition for the variance. Ruiz de Vargas and Breuer (2019), p. 370, show the simplifications required for the covariance of the rate of returns of the company with the market rate of return divided by the variance of the market return, i.e. the beta value, being identical for the HC and the FC approach.

The transition between the resulting rate of returns, i.e. the unlevered cost of equity, from FC approach to HC approach is (see O’Brien 2017, pp. 58, 124):

$$E[r_{U,HC}] = (1 + E[r_{U,FC}])(1 + E[\delta_S]) + \text{cov}(r_{U,FC}, \delta_S) - 1 \tag{31}$$

This is the rate for discounting the expected unlevered FCF in HC. It may be rearranged to solve for $r_{U,FC}$.

With a little help of Eq. (6), we can divide the unlevered cost of equity into the risk-free rate of return, the premia for the exchange rate risk (b) and for the business risk (c) and the covariance-term (d):

$$\begin{aligned}
 E[r_{U,HC}] &= i_{FC}(1 + E[\delta_S]) + E[\delta_S] + z_{U,FC}(1 + E[\delta_S]) + \text{cov}(r_{U,FC}, \delta_S) \\
 &= E[r_{V_S}] + z_{U,FC}(1 + E[\delta_S]) + \text{cov}(r_{U,FC}, \delta_S) \\
 &= i_{HC} + \underbrace{z_{V_S}}_{(b)} + \underbrace{z_{U,FC}(1 + E[\delta_S])}_{(c)} + \underbrace{\text{cov}(r_{U,FC}, \delta_S)}_{(d)} \tag{32}
 \end{aligned}$$

$z_{U,HC}$

This definition also illustrates how the risk premium to be added to the risk-free rate is linked to the risk premium used in the FC approach. It also serves as the RADR-counterpart to the CE-based decomposition of risk discounts used in Eq. (20). Coming back to the CAPM for pricing the risk, we can derive the RADR-counterpart to the CE-based Eq. (28):

$$\begin{aligned}
 E[r_{U,HC}] &= i_{HC} + \underbrace{\beta_{V_S} E[MRP_{HC}]}_{(b)} + \underbrace{\beta_{U,FC} E[MRP_{FC}]}_{(c)} (1 + E[\delta_S]) + \underbrace{\text{cov}(r_{U,FC}, \delta_S)}_{(d)} \\
 &\qquad\qquad\qquad \underbrace{\hspace{15em}}_{\beta_{U,HC} E[MRP_{HC}]} \tag{33}
 \end{aligned}$$

If the forward exchange rate is used instead of the expected spot exchange rate, the discount rate must be adjusted as Eq. (13) has shown for the basic framework. Applied to the definition of the RADR in Eq. (33), we get:

$$\begin{aligned}
 E[r_{U,HC}]' &= (1 + E[r_{U,FC}]) \frac{1 + i_{HC}}{1 + i_{FC}} - 1 = i_{HC} + z_{U,FC} \frac{1 + i_{HC}}{1 + i_{FC}} \\
 &\qquad\qquad\qquad \underbrace{\hspace{10em}}_{z_{U,FC}} \\
 &= i_{HC} + \underbrace{\beta_{U,FC} E[MRP_{FC}]}_{(c)'} \frac{1 + i_{HC}}{1 + i_{FC}} \tag{34}
 \end{aligned}$$

Thus, if the state-independent forward exchange rate is used for the valuation, there is neither an exchange rate risk premium nor an exchange rate related covariance to be considered in the discount rate. The forward exchange rate is the certainty equivalent of the state-contingent exchange rates. Terms (b) and (d) equal zero. Put differently, if the numerator is free of exchange rate risk, since the forward exchange rate is used and the covariance between the FCF FC and the exchange rate is not relevant, the denominator must not consider either a premium for exchange rate risk or the covariance of the RADR FC with the exchange rate. It should be noted that the business risk premium (c)' must be calculated from a FC perspective, and is not adjusted by the expected change in exchange rates as in Eq. (32), but by the ratio of (one plus) the risk-free interest rates.

Unlevered company value follows from:

$$V_{U,HC,0} = E[FCF_{U,FC,1}]F_1\left(1 + E[r_{U,HC}]\right)^{-1} \tag{35}$$

With the UIP and the UH, the expected spot exchange rate is equal to the forward exchange rate. Under this assumption, the change between HC and FC approach becomes more straightforward in terms of the risk parameters (Table 3 in the Appendix). It can be shown that the beta values are the same for the HC and the FC approach ($\beta_{U,HC} = \beta_{U,FC}$), and the market risk premia are linked by the ratio of the interest rates (see Appendix):

$$E[MRP_{HC}] = E[MRP_{FC}] \frac{1 + i_{HC}}{1 + i_{FC}} \tag{36}$$

Unlike for the framework set for Eq. (34), the UIP in combination with the UH implies that all market participants are setting the expected spot exchange rate equal to the forward exchange rate. The expected rate of return would not require a derivation of the beta value in FC, but could be based upon the beta value in HC:

$$E[r_{U,HC}]'' = i_{HC} + \underbrace{\beta_{U,FC}}_{= \beta_{U,HC}} \underbrace{E[MRP_{FC}] \frac{1 + i_{HC}}{1 + i_{FC}}}_{= E[MRP_{HC}]} = i_{HC} + z_{U,FC} \frac{1 + i_{HC}}{1 + i_{FC}} \tag{37}$$

(c)''

Finally, we can compare the discount rates used, if forward exchange rates are used, i.e. we can compare Eqs. (34) and (37): If the forward exchange rate is applied in the general valuation setting by referring to Eq. (34), the FC beta value is used. If the use of forward exchange rates is justified by the UIP and the UH to hold, the (identical) HC beta value could be used as well. It should be noted that beta and, if e.g. the market return in HC is used as a starting point, the market returns in FC depend upon the UIP and UH assumption.

For the remainder of the paper, we will show the definitions of the RADR and the value equations for the HC approach for the use of both expected spot exchange rates and forward exchange rates. It is not necessary to show all of that additionally for the UIP assumption, because the difference to using the forward exchange rates in general is the link implied by the UIP between the risk premium from a FC and HC perspective.

3.4 Multi-period case

An extension of the forecast horizon to an infinite number of periods leads, inter alia, to an introduction of a perpetuity that must be addressed consistently. It is beyond the scope of the paper to deal with all the challenges related to model the terminal value, i.e. the value of a growing perpetuity. In this section, we assume a steady state of all inputs to a RADR valuation within the FC approach at the beginning of the

growing perpetuity, e.g. free cash flows, financing cash flows, and tax effects, and do not differentiate between inflation and real growth. Hence, this is a simplified analysis.

We only point out the following specifics for a multi-period valuation with the HC approach:

- *Forward interest rates:* Even if a domestic company, which generates only free cash flows in HC and uses only domestic sources of finance, is to be valued, varying capital structures and risk premia require the use of forward interest rates instead of spot interest rates. A recursive valuation (roll-back procedure) must be applied (see, for example, Drukarczyk and Schüler 2016, pp. 242–244). For a cross-border valuation, an additional reason for using forward interest rates is the relation between current spot exchange rate and forward exchange rates, because the change of the forward exchange rate over time depends upon the development of the forward interest rates:

$$F_t = S_0 \frac{\prod_{\tau=1}^t (1 + i_{HC|\tau-1,\tau})}{\prod_{\tau=1}^t (1 + i_{FC|\tau-1,\tau})} \tag{38}$$

Forward interest rates are only observable up to the longest available maturity of risk-free bonds. One could assume in a pragmatic manner that the last period T before the beginning of the perpetuity coincides with the last period for which forward interest rates are observable, and forward interest rates remain constant thereafter.

An implication arises with regard to the link between RADR in HC and FC as shown, for example, by Eq. (34): as Bekaert and Hodrick (2018), p. 723, point out, the RADR in one currency cannot be constant, if the RADR in the other currency is constant, unless the term structure of the interest rates is flat.

- *Change in exchange rates:* The change of forward exchange rates (δ_F) is relevant, if the valuation is referring to forward exchange rates as illustrated by Eq. (13), or Eq. (35). If expected spot exchange rates are used directly as shown, for example, by Eqs. (11) or (30), the change of expected spot exchange rates over time (δ_S) is needed. Combining the respective change in exchange rates with the growth of the free cash flows in FC ($g_{FC,FC}$) leads to adjusted growth rates that are necessary to calculate the terminal value within the HC approach. The expected values of these growth rates that are state-dependent, but time-invariant, are:

$$\begin{aligned} E[g_{FC,HC,F}] &= (1 + E[g_{FC,FC}]) (1 + \delta_F) - 1 = (1 + E[g_{FC,FC}]) \frac{1 + i_{HC|T-1,T}}{1 + i_{FC|T-1,T}} - 1 \\ E[g_{FC,HC,S}] &= (1 + E[g_{FC,FC}]) (1 + E[\delta_S]) + \text{cov}(g_{FC,FC}, \delta_S) - 1 \end{aligned} \tag{39}$$

One implication of these definitions is that the growth of the FCF denominated in HC should be calculated starting with the rate of growth of the FCF in FC. Then, the

estimated rate of growth in the exchange rates should be considered. Put differently, one should be aware that setting a growth rate for the FCF in HC pragmatically not only implies an assumption about the growth rate of the FCF in FC, but also about the change in exchange rates.

Whether the growth in FCF is higher in HC or in FC, depends upon the relation between the risk-free forward rates in HC and FC if forward exchange rates are used: $g_{FCF,HC,F} > g_{FCF,FC,F}$ if $i_{HC|T-1,T} > i_{FC|T-1,T}$, and $g_{FCF,HC,F} < g_{FCF,FC,F}$ if $i_{HC|T-1,T} < i_{FC|T-1,T}$. If expected spot exchange rates are used: $g_{FCF,HC,s} > g_{FCF,FC,s}$ if the expected change in exchange rates is positive, and $g_{FCF,HC,s} < g_{FCF,FC,s}$ if the expected change in exchange rates is negative.

If a foreign company is supposed to be valued assuming a constant growth rate of the FCF in HC and a constant exchange rate, and referring to the first line in Eq. (39) and δ_F respectively, the FCF in FC would be underestimated if $i_{HC|T-1,T} < i_{FC|T-1,T}$ and overestimated if $i_{HC|T-1,T} > i_{FC|T-1,T}$; with reference to the second part of Eq. (39), the FCF in FC would be underestimated if the expected change in exchange rates is negative and overestimated if the expected change in exchange rates is positive, and one erroneously applies a constant exchange rate.

Assuming a constant premium for business risk from the FC perspective and using Eq. (39), the terminal value of growing free cash flows using, for example, forward exchange rates is:

$$\begin{aligned}
 V_{U,HC,T} &= \frac{E[FCF_{U,FC,T}]F_T(1 + E[g_{FCF,HC,F}])}{\left[(1 + E[r_{U,FC,T}]) \frac{1+i_{HC|T-1,T}}{1+i_{FC|T-1,T}} - 1 \right] - \left[(1 + E[g_{FCF,FC}]) \frac{1+i_{HC|T-1,T}}{1+i_{FC|T-1,T}} - 1 \right]} \\
 &= F_T \underbrace{\frac{E[FCF_{U,FC,T}](1 + E[g_{FCF,FC}])}{E[r_{U,FC,T}] - E[g_{FCF,FC}]}_{V_{U,FC,T}}
 \end{aligned}
 \tag{40}$$

The last term in this equation is the FC approach based upon the forward exchange rate in period T . Of course, we could write the terminal value also by using the expected spot exchange rate and the growth rate based upon the spot exchange rates as shown by the second line of Eq. (39).

After a recursive valuation process (roll-back procedure), the company’s value at the valuation date $t=0$ is derived by:

$$\begin{aligned}
 V_{U,FC,0} &= \sum_{t=1}^T E[FCF_{U,FC,t}] \prod_{\tau=1}^t (1 + E[r_{U,FC|t-1,\tau}])^{-1} + E[V_{U,FC,T}] \prod_{\tau=1}^T (1 + E[r_{U,FC|t-1,\tau}])^{-1} \\
 &= (E[FCF_{U,FC,1}] + E[V_{U,FC,1}]) (1 + E[r_{U,FC|0,1}])^{-1}
 \end{aligned}
 \tag{41}$$

For completing the FC approach, this value has to be multiplied by the spot exchange rate at the valuation date, to get the unlevered company value in HC.

For the HC approach, we can write by referring to the last step of the recursive valuation with noting that the expected company value at $t=T$ (terminal value) is to be derived according to Eq. (40):

$$\begin{aligned}
 V_{U,HC,0} &= (E[FCF_{U,FC,1}]E[S_1] + \text{cov}(FCF_{U,FC,1}, S_1) + E[V_{U,HC,1}]) (1 + E[r_{U,HC|0,1}])^{-1} \\
 &= (E[FCF_{U,FC,1}]F + E[V_{U,HC,1}]) \left(1 + i_{HC|0,1} + \beta_{U,FC} E[MRP_{FC}] \frac{1 + i_{HC|0,1}}{1 + i_{FC|0,1}} \right)^{-1}
 \end{aligned}
 \tag{42}$$

4 Valuing a levered foreign company

4.1 Assumptions

This chapter introduces debt financing. Debt financing alters the amount of cash flow to be valued because of tax shields, but also the distribution and the riskiness of cash flow components. Thus, we are adding financial risk and risk of default to our analysis that dealt with exchange rate risk and business risk so far. For the sake of efficiency, we focus on the changes of the valuation framework induced by debt.

If a foreign company uses debt in foreign currency (D_{FC}), interest expenses lead to a corporate tax shield. We follow Modigliani and Miller (1958, 1963), while adding the following assumptions:

- We consider companies with limited liability. In the case of alternative private debt financing, the liability of the private shareholders is limited to the dividends paid out by the unlevered company. Thus, debt financing is not value increasing due to the limited liability of the company, because the liability of the private shareholders is limited also (Stiglitz 1969, p. 788).
- We assume deterministic levels of debt employed. Therefore, we do not assume a financial policy that defines debt in % of total company value (target leverage ratio). As a result, our valuation framework begins with the APV (adjusted present value) approach, and not the weighted average cost of capital (WACC) approach (see, for example, Inselbag and Kaufold 1997). Nevertheless, we develop the WACC and FTE (flow to equity) framework which reconciles with the APV results.
- Lenders demand a contractual interest rate (r_D^*) that compensates them ex ante for the risk of default. As a consequence, the face value and the market value of debt are identical.
- For modelling the tax effects of debt financing, we assume a tax-regime as used by Sick (1990): A state-contingent tax shield is defined as the state-contingent rate of return for the lenders (on the face value of debt) multiplied by the corporate tax rate and the face value of debt. This tax-regime implies that accounting income resulting from an incomplete repayment of debt leads to tax payments.⁵
- We do not discuss whether a bankruptcy proceeding is started if a default occurs nor which consequences this might have in terms of risk and wealth transfers between owners and lenders, or the valuation framework in general. We do not consider cost of financial distress.

⁵ This assumption is also used by Kruschwitz et al. (2005). For other assumptions regarding the tax regime see, for example, Cooper and Nyborg (2008) or Kraus and Lahmann (2016).

4.2 APV approach

This approach is an immediate implementation of the value additivity principle. The value of equity (E) follows from adding the present value of the tax shields (V_{TS}) to the unlevered company value (already discussed in Sect. 3), and subtracting the value of debt from the resulting levered company value (V_L). What is left to explain is the valuation of the tax shields and the valuation of debt.

For the FC approach, we apply Sick (1990) to the valuation of the tax shields from debt denominated in FC. Sick (1990) pointed out that the present value of state-contingent tax shields must equal the certainty-equivalent of these tax shields discounted by the risk-free rate. For the tax-regime assumed by Sick (1990), a state-contingent tax shield equals the state-contingent rate of return for the lenders multiplied by the corporate tax rate and the face value of debt, as mentioned in the previous section. Then, the certainty-equivalent of these tax shields is defined as the tax rate multiplied by the risk-free rate and the face value of debt. This is a straightforward application of a certainty-equivalent. It is relevant for two reasons: First, an increase in the risk of default for a given level of debt does not change the levered company value (see also Kruschwitz et al. 2005). Second, the valuation of the tax shields becomes simple. Applying this idea to a multi-period context and to deterministic levels of debt, the present value of the tax shields on interest expenses on debt denominated in FC, to be derived in a recursive manner using forward interest rates, is:

$$V_{TS,FC,0} = \left(\tau_C i_{FC|0,1} D_{FC,0} + V_{TS,FC,1} \right) (1 + i_{FC|0,1})^{-1} \quad (43)$$

The value of debt is the present value of the expected payments to the lenders (CF_D) until maturity. They are based on the contractual interest payments and depend upon the assumptions regarding default. The RADR is the expected rate of return to the lenders (r_D). That rate can be defined by referring to the CAPM and therefore to debt beta (β_D). Debt can be valued recursively by:

$$D_{FC,0} = \left(E[CF_{D,FC,1}] + E[D_{FC,1}] \right) \left(1 + \underbrace{E[r_{D,FC|0,1}]}_{i_{FC|0,1} + \beta_{D,FC,1} E[MRP_{FC}]} \right)^{-1} \quad (44)$$

Since we assume that the lenders are able to charge the interest rate that compensates them for the risk of default in each future period, the present value of the expected payments to the lenders (market value of debt) equals the face value of debt.

The value of equity follows from:

$$E_{HC,0} = \left(\underbrace{V_{U,FC,0} + V_{TS,FC,0} - D_{FC,0}}_{V_{L,FC,0}} \right) S_0 = \underbrace{V_{U,HC,0} + V_{TS,HC,0} - D_{HC,0}}_{V_{L,HC,0}} \quad (45)$$

If one would, contrary to our assumption, consider cost of financial distress, these cost would lower the value of equity: Since we assumed homogeneous expectations and the ability of lenders to demand a risk-equivalent interest rate, lenders will consider any of these costs that could decrease their cash flows while deriving the risk equivalent cost of debt. Thus, cost of financial distress would decrease the cash flows paid out to the owners.

For the HC approach, a foreign company that uses debt in the same FC (and is taxed abroad) delivers tax shields which are subject to exchange rate risk. Then, tax shields on FC debt that can be valued by transforming them into certainty equivalents and discounting them with the risk-free rate for the FC approach become risky from the HC perspective. Due to the exchange rate risk, the risk equivalent discount rate r_{VS} must be used. With the covariance between the certainty equivalent of the tax shield in FC and the state-contingent exchange rates being equal to zero, the value of the tax shields is:

$$V_{TS,HC,0} = \left(\tau_C i_{FC|0,1} D_{FC,0} E[S_1] + E[V_{TS,HC,1}] \right) \left(1 + r_{VS|0,1} \right)^{-1} \quad (46)$$

The rate for discounting the expected payments to lenders, converted by the expected spot exchange rate into domestic currency, can be derived by applying Eq. (33) and skipping the time index for the ease of presentation:

$$\begin{aligned} E[r_{D,HC}] &= (1 + E[r_{D,FC}]) (1 + E[\delta_S]) + \text{cov}(r_{D,FC,1}, \delta_S) - 1 \\ &= i_{HC} + \beta_{VS} E[MRP_{HC}] + \beta_{D,FC} E[MRP_{FC}] (1 + E[\delta_S]) + \text{cov}(r_{D,FC}, \delta_S) \end{aligned} \quad (47)$$

If forward exchange rates are used, the discount rate analogous to Eq. (34) is:

$$E[r_{D,HC}]' = (1 + E[r_{D,FC}]) \frac{1 + i_{HC}}{1 + i_{FC}} - 1 = i_{HC} + \beta_{D,FC} E[MRP_{FC}] \frac{1 + i_{HC}}{1 + i_{FC}} \quad (48)$$

Analogous to the derivation of the unlevered company value, assuming the UIP to hold would lead to the identity of the beta value for the FC and the HC perspective.

Using the rates as shown by Eqs. (47) and (48) in a multi-period context, and accounting for the covariance of the payments to lenders with the state-contingent exchange rates if expected spot exchange rates are used, we can value the expected payments to lenders with the HC approach by:

$$\begin{aligned}
 D_{HC,0} &= \left[(E[CF_{D,FC,1}]E[S_1] + \text{cov}(CF_{D,FC,1}, S_1) + E[D_{HC,1}]) \right] (1 + E[r_{D,HC|0,1}])^{-1} \\
 &= (E[CF_{D,FC,1}]F_1 + E[D_{HC,1}]) \left(1 + E[r_{D,HC|0,1}] \right)^{-1}
 \end{aligned} \tag{49}$$

Obviously, the HC market value of debt denominated in FC depends on the exchange rate. As usual, the value of equity according to the HC approach follows from subtracting the value of debt from the sum of the unlevered company value and the value of the tax shields.

4.3 FTE and WACC approach

The FTE approach requires the expected levered free cash flows (FCF_L) to be discounted by the levered cost of equity, i.e. the rate of return expected from the owners of a levered company. Using the value additivity principle, this rate of return can be defined as the weighted average of the relevant risk premia (see Schüler 2018a). Disregarding the time index for the ease of presentation again and relaxing the assumption of risk-free tax shields on interest expenses, we can write for the FC approach with reference to Inselbag and Kaufold (1997) and Schüler (2018a):

$$\begin{aligned}
 E[r_{L,FC}] &= i_{FC} + z_{U,FC} \frac{V_{U,FC}}{E_{FC}} - z_{D,FC} \frac{D_{FC}}{E_{FC}} + z_{V_{TS,FC}} \frac{V_{TS,FC}}{E_{FC}} \\
 &= E[r_{U,FC}] + (E[r_{U,FC}] - E[r_{D,FC}]) \frac{D_{FC}}{E_{FC}} - (E[r_{U,FC}] - E[r_{V_{TS,FC}}]) \frac{V_{TS,FC}}{E_{FC}}
 \end{aligned} \tag{50}$$

If the CAPM is to be used, the levered beta equals the weighted average of the betas attributable to the components of the unlevered free cash flow (see Drukarczyk and Schüler 2016, p. 371):

$$\beta_{L,FC} = \beta_{U,FC} + \beta_{U,FC} \frac{D_{FC} - V_{TS,FC}}{E_{FC}} - \beta_{D,FC} \frac{D_{FC}}{E_{FC}} + \beta_{V_{TS,FC}} \frac{V_{TS,FC}}{E_{FC}} \tag{51}$$

The value of equity is the present value of the levered free cash flows, i.e. the unlevered free cash flows after interest payments plus the change in debt employed and the tax shields, all in FC. The value of equity in HC equals the value of equity in FC multiplied by the current spot exchange rate.

For the WACC approach, the unlevered free cash flows are discounted by the WACC:

$$WACC_{FC} = r_{L,FC} \frac{E_{FC}}{V_{L,FC}} + r_{D,FC} \frac{D_{FC}}{V_{L,FC}} - \frac{TS_{FC}}{V_{L,FC}} \tag{52}$$

The valuation results need to be known before the levered cost of equity or the WACC can be derived. This circular reference does not impose a problem for the WACC approach, if the value ratios used in the definitions are known (see Inselbag

and Kaufold 1997; Miles and Ezzell 1980). For the FTE approach, a circular reference remains even for known value ratios since the payments to creditors need to be known for deriving the levered FCF (see Inselbag and Kaufold 1997, p. 122).

For the HC approach, the exchange rate risk and the choice to deal with it by applying expected spot exchange rates or forward exchange rates is relevant for the FTE and the WACC approach, too. Discount rates and value equations are presented in Table 1. For deriving the RADR, we start with Eq. (33), if expected exchange rates are used, and Eq. (34), if forward exchange rates are used.

Besides the risk-free rate of return, the discount rates consist of a premium for the exchange rate risk (b), if the expected spot exchange rates are used. This premium does not apply, if forward exchange rates are used in the numerator of the value equation and the adjusted discount rate, $r_{L,HC}'$, is used in the denominator. Risk premia for business risk (c), and financial risk (d), which is the business risk borne by the owners instead of the lenders, must be accounted for in both alternatives. The risk of default (e) is borne by the lenders, and thus reduces the risk borne by the owners. We also include a beta value for the tax shields (f) to generalize the definition, although our previous assumptions required the RADR from a FC perspective to be the risk-free rate (FC). In this case, the beta value is zero. The covariance related to the foreign exchange rate (g) is to be considered, if expected spot exchange rates are used.

It should be noted that levered beta values derived empirically by the index model imply all these risk premia. This should also be kept in mind when peer group betas are used. Another practical implication is that because forward interest rates need to be used for a multi-period valuation, discount rates cannot be constant. In addition, changing exchange rates lead to varying discount rates even if they are constant in FC. If one refers to the WACC approach, for example, hoping that it is easier because the WACC is constant, the valuation would produce correct results only by coincidence. Since we assumed debt employed to be deterministic, the APV approach was used in line with the recommendation in the literature (see, for example, Inselbag and Kaufold 1997). In the literature on cross-border valuation, recommendations to use the APV approach can be found also (see also Bekaert and Hodrick 2018, p. 665, Lessard 1979, Sercu 2009, Chapter 21). If a target leverage ratio would be assumed, the WACC approach would be recommended for the FC approach without considering exchange rate risk. For the HC approach, it would have to be checked how the exchange rate risk that must be considered for each planning period can be addressed within the WACC approach. One could conclude that the FC approach must be applied first, in order to transfer the target leverage ratio from the FC perspective into the HC perspective.

Table 1 FTE and WACC approach

| | Value equation |
|----------------------|---|
| RADR | |
| FTE approach | |
| ...based upon $E[S]$ | $E[V_{L,HC}] = i_{HC} + \beta_{L,HC} E[MRP_{HC}]$ $= i_{HC} + \beta_{L,HC} \underbrace{E[MRP_{HC}]}_{(a)} + \underbrace{\left(\beta_{U,FC} + \beta_{L,FC} \frac{D_{FC} - V_{TS,FC}}{E_{FC}} - \beta_{D,FC} \frac{D_{FC}}{E_{FC}} + \beta_{TS,FC} \frac{V_{TS,FC}}{E_{FC}} \right)}_{(b)} \underbrace{E[MRP_{FC}]}_{(c)} (1 + E[\delta_S])$ $+ \underbrace{\text{cov}(r_{L,FC}, \delta_S)}_{(d)}$ $E_{HC,0} = ((E[FCF_{L,FC}]E[S] + \text{cov}(FCF_{L,FC}, \delta_S) + E[E_{HC,1}])(1 + E[r_{L,HC(0,1)}])^{-1}$ |
| ...based upon F | $E[r_{L,HC}]' = (1 + E[r_{L,FC}]) \frac{1 + i_{HC}}{1 + i_{FC}}$ $= i_{HC} + \underbrace{\left(\beta_{U,FC} + \beta_{L,FC} \frac{D_{FC} - V_{TS,FC}}{E_{FC}} - \beta_{D,FC} \frac{D_{FC}}{E_{FC}} + \beta_{TS,FC} \frac{V_{TS,FC}}{E_{FC}} \right)}_{(d)} \underbrace{E[MRP_{FC}]}_{(e)} \frac{1 + i_{HC}}{1 + i_{FC}}$ $E_{HC,0} = (E[FCF_{L,FC}]F_1 + E[E_{HC,1}])(1 + E[r_{L,HC(0,1)}])^{-1}$ |
| WACC approach | |
| ...based upon $E[S]$ | $WACC_{HC} = (1 + WACC_{FC})(1 + E[\delta_S]) + \text{cov}(WACC_{FC}, \delta_S) - 1$ $V_{L,HC0} = ((E[FCF_{U,FC}]E[S] + \text{cov}(FCF_{U,FC}, \delta_S) + E[V_{L,HC,1}])(1 + WACC_{HC(0,1)}^{-1})^{-1}$ |
| ...based upon F | $WACC'_{HC} = (1 + WACC_{FC}) \frac{1 + i_{HC}}{1 + i_{FC}} - 1$ $V_{L,HC0} = (E[FCF_{U,FC}]F_1 + E[V_{L,HC,1}])(1 + WACC'_{HC(0,1)}^{-1})^{-1}$ |

5 Valuing a domestic company that generates cash flows in foreign currency or employs foreign currency debt

First, we assume that a domestic company generates cash flows in the home and the FC, but is financed domestically. Cash flows in FC can be valued separately by either the HC or the FC approach as shown above. Alternatively, the cash flows in FC can be integrated in the overall cash flow forecast of the domestic company. It does not seem to be efficient to convert its domestic cash flows into FC, add them to the cash flows in FC, and apply the FC approach. Rather, one would convert the cash flows in FC into the HC, add them to the domestic cash flows, and apply the HC approach.

Next, we discuss the tax effects for a domestic company that is financed partially by debt in FC. According to the HC approach, the tax shield on interest expenses (TS) on foreign debt must be converted into the HC and must be added to the tax shields on domestic debt. Equation (46) can be applied accordingly.

Interestingly, tax effects do not only consist of the well-known tax shields on interest expenses, but also tax effects resulting from currency effects related to changes in debt employed. These effects (TFx) could occur due to changes in the exchange rate within the time period between obtaining and repaying debt. They are a financial manifestation of the transaction risk. They will occur if the tax regime considers them to be part of the taxable income. This might be the case, if a repayment of FC-debt converted into HC exceeds the amount of debt received in HC originally and therefore reduces taxable income, or if a repayment converted into HC is lower than the amount of debt received in HC and therefore increases taxable income. The period in which the debt financing was obtained is labelled s , and the repayment occurs in period t (with $s < t$). In order to keep the focus on repayments only, a more general formulation referring to the change in debt should be avoided. This prevents increases in debt from mistakenly influencing the tax effects. In order to avoid compensating effects between different debt instruments, each debt instrument should be addressed separately. For the HC approach, we can write repayment defined as RP :

$$E[TFx_{HC,t}] = \tau_C E[RP_{FC,t}] (E[S_t] - S_s) + \text{cov}(TFx_{FC,t}, S_t) \quad (53)$$

It is worth noting that this effect is one example when the tax base differs from the usual simplified definition of tax shields as being the product of tax rate times interest expenses. Specific provisions of the tax code or tax loss carryforwards, for instance, lead to a more complicated framework (see, for example, Baetge et al. 2019, Friedrich 2016 and Lübbehüsen 2000).

The change in exchange rates between the two relevant periods is to be considered, and not only an exchange rate at a specific point in time. The covariance between the effect and the spot exchange rate is also relevant. Zwirner (1989), pp. 232–234, Click and Coval (2002), p. 138, Eiteman et al. (2016), p. 537, Bekaert and Hodrick (2018), p. 742 and Ruiz de Vargas (2019), p. 1672, also point out the effect, although at differing levels of precision. Eiteman et al. (2016), for example, consider tax effects when deriving free cash flows in FC, but they neither value them nor

integrate them into the discount rates, and mistakenly apply them not only to repayments, but to interest payments as well.⁶

For the FC approach, we can write:

$$E[TFx_{FC,t}] = E[TFx_{HC,t}]E[S_t^{-1}] + cov(TFx_{HC,t}, S_t^{-1}) \tag{54}$$

One could argue that the FC approach is only a second-best solution to address these tax effects, because TFx is defined in HC and needs to be converted back into FC.

The total tax effect induced by debt financing (TE) is the sum of TS and TFx . It can also be derived by subtracting levered taxes from unlevered taxes. The value of the tax effects (V_{TE}) at the valuation date equals the present value of all expected future tax effects. Although we transformed state-contingent tax shields on interest expenses (TS) from an FC perspective into certainty equivalents following Sick (1990), they are risky from an HC perspective because of the exchange rate risk (see Sect. 4.2 and Eq. 46 specifically). In addition, TFx cannot be expected to be risk-free due to exchange rate risk and additionally due to the risk of default. Generally speaking, and with reference to the value additivity principle, the rate for discounting the tax effects would need to include risk premia for both TS and TFx :

$$E[r_{TE,HC}] = i_{HC} + z_{TS,HC} \frac{V_{TS,HC}}{V_{TE,HC}} + z_{TFx,HC} \frac{V_{TFx,HC}}{V_{TE,HC}} \tag{55}$$

Within the APV approach, levered company value is the sum of the present value of the tax effects TE and the unlevered company value. Before obtaining the value of equity, debt must be valued and subtracted from the total levered company value. For applying the FTE or WACC approach, we can refer to Sect. 4.3. The equations presented there are also applicable in general for the valuation of a domestic company that uses debt in FC, but need to consider the tax effects (TE) instead of the tax shields on interest expenses (TS). The recommendation to use APV if deterministic debt levels are assumed, and WACC if debt is planned in % of total levered company value was established within a single currency framework. Before the WACC approach can be used from a HC perspective, it has to be made sure that target leverage ratios in % of total levered company value can be applied in HC although debt is denominated in FC and is subject to exchange rate risk, and although TE is risky. That TE can be planned with a fixed link to the target leverage ratio should not be taken for granted.

⁶ Interest payments in foreign currency do not lead to this tax effect, because other than for repayments, there is no corresponding prior cash inflow, which for repayments is the initial inflow of debt into the company converted into HC at the exchange rate in period s . Thus, there is no change in exchange rates between cash inflow (initial payout) and cash outflow (repayment) to be considered. Only the exchange rate at the time of the interest payment is relevant.

6 Assessment of textbook-approaches

The results of the assessment of Bekaert and Hodrick (2018), Chapters 15 and 16, Berk and DeMarzo (2020), Chapter 31, Brealey et al. (2019), Chapter 27, Holthausen and Zmijewski (2020), Chapter 17 and Koller et al. (2015), Chapter 23, with regard to the analysis of the paper are as follows:

1. As shown by Eq. (11), the valuation can be conducted either with covariances between cash flow or rate of returns with exchange rates considered in both numerator and denominator, or without considering them. Bekaert and Hodrick (2018), Berk and DeMarzo (2020) and Holthausen and Zmijewski (2020) mention the covariance of the FCF in FC with the exchange rate, but assume it to be zero. The covariance in the discount rate is not addressed in these three textbooks. The other two textbooks do not mention these covariances at all. Although the textbooks mostly use forward exchange rates to explain cross-border valuation, it remains unclear how covariances would have to be dealt with, if expected exchange rates would be used.
2. We have discussed the conceptual consequences of using forward exchange rates instead of expected spot exchange rates. Equation (13) contains the rate to discount cash flows that have been converted by the forward exchange rates instead of the expected spot exchange rates. Equation (34) applies this general principle to the unlevered case. The formulae for the levered case are shown in Table 1. Equation (37) and Table 3 show the RADR and its components assuming the UIP and the UH to hold. In any case, our analysis started with r_{FC} . Koller et al. (2015) do not use RADR for their demonstration of the HC and the FC approach, but risk-free rates. Brealey et al. (2019) begin the analysis with r_{FC} , use forward exchange rates and adjust r_{FC} according to Eq. (34). Besides neglecting debt and taxes, and not clearly motivating the use of forward exchange rates, their approach reconciles with our analysis for the unlevered case without growth. The other three textbooks begin their discussion of the RADR to be used for the HC approach and the FC approach with the HC-RADR, and combine it with using forward exchange rates to convert FC cash flows (Bekaert and Hodrick 2018; Berk and DeMarzo 2020; Holthausen and Zmijewski 2020). They use Eq. (34) with r_{FC} as the dependent variable. If one derives r_{FC} from r_{HC} while using forward exchange rates, one assumes implicitly that r_{HC} equals r_{HC}' . Thus, the authors neglect the premium for exchange rate risks (b) and the exchange rate related covariance [unlevered case: (d); levered case: (g)]. They imply that these risk effects are zero. Otherwise, they would have to adjust r_{HC} , a rate which fits to the discounting of expected values, for (b), and (d) or (g). Their discussion is also not sufficiently detailed to judge whether they also imply that the beta values are identical in HC and in FC.
3. The use of forward interest rates is necessary in order to deliver consistent valuation results. Bekaert and Hodrick (2018) and Koller et al. (2015) consider the term structure of interest rates. However, Koller et al. (2015) do not address business risk, and work with risk-free forward interest rates only. Holthausen and Zmijewski (2020) consider the term structure of interest rates implicitly by using the change in quotations of forward exchange rates as a proxy for linking

- the RADR in HC to the RADR in FC. The other textbooks do not consider the term structure of interest rates.
4. We have shown some principles regarding the growth rate to be used for deriving the terminal value. None of the textbooks discusses the derivation of the terminal value in cross-border valuation.
 5. Bekaert and Hodrick (2018) use the APV in Chapter 15 and refer to WACC and FTE in Chapter 16, too. They review the use of the different approaches in line with the established reasoning, shown e.g. in Inselbag and Kaufold (1997), but they do so without a specific reference to cross-border valuation. Berk and DeMarzo (2020) apply a constant WACC and assume only debt in HC. Neither of the two textbooks addresses the risk of default. The other textbooks do not cover debt financing at all, thus assuming unlevered companies.
 6. There are more tax effects than just the tax shields on interest expenses, if debt denominated in FC is used. As mentioned before, Bekaert and Hodrick (2018) cover TFx besides the usual tax shields. However, they do not integrate tax effects in the DCF valuation. Holthausen and Zmijewski (2020) discuss a number of tax effects; they do not include them in their valuation example, for which they assume an unlevered company. The other textbooks do not mention the tax effects on debt denominated in FC.

As always, there is room for improvement. The review confirms the gap in the literature mentioned in Sect. 1.

7 Conclusions

As well established in the literature, FC and HC approach applied consistently are equivalent. By extending the existing literature, the paper shows the requirements for both approaches to be defined consistently for both a risk-neutral and a CAPM-based valuation framework. Furthermore, the analysis addresses the valuation of unlevered companies as well as levered companies. Risk discounts from expected cash flows (certainty equivalents) and risk premia as part of the respective RADRs (unlevered and levered cost of equity, WACC, cost of debt) are derived for a number of risks including: exchange rate risk, business risk, financial risk, and the risk of default.

The conclusions are:

1. The covariance of the free cash flows with the exchange rates is the numerator-equivalent of the covariance of the rate of returns with the exchange rates in the denominator of the value equation. The paper contributes to the literature by showing that a valuation can consider either the covariances in both the numerator and the denominator, or none of them. Both alternatives deliver identical company values. This conclusion holds for the valuation of an unlevered company as well as for the valuation of a levered company, because the covariance of the levered FCF can be written as the sum of the covariances of its components starting with the covariance of the unlevered FCF . If a discount rate r_{HC} is determined not indirectly by starting with a rate r_{FC} and adjusted for exchange rate effects as shown by Eq. (32) for example,

- but is calculated directly by adding the domestic risk-free rate to the risk premium in HC that is estimated empirically with the index model, the resulting discount rate is to be interpreted as including all covariances (based on rate of returns). Thus, the covariance between FCF in FC and exchange rates need to be accounted for in the numerator. Alternatively, FCF in FC net of their covariance with the exchange rate are to be discounted by the rate shown, for example, by Eq. (11).
2. For the HC approach, (1) expected spot exchange rates or (2) forward exchange rates can be used for the conversion of free cash flows in FC into HC. If the forward exchange rates are used, one needs to differentiate between their application with (2a) assuming the UIP and the UH to hold and without this assumption, i.e. if forward exchange rates are interpreted as certainty equivalents (2b). The paper develops the valuation framework for all three cases. If expected spot rates are used, all risk premia are relevant and the RADR in HC has to include a premium for exchange rate risk. This implies that if the beta value in HC is estimated empirically without any risk adjustments, it is assumed to contain premia for the risks mentioned above including exchange rate risk. It can be used to discount cash flows converted by expected spot exchange rates. It can only be applied to free cash flows converted by forward exchange rates if the UIP and the UH are assumed to hold (2a). If the forward exchange rates are used to convert cash flows into HC without assuming the UIP and the UH to hold, and treated like certainty equivalents (2b), the discount rate must not contain exchange rate related risk effects. The appropriate RADR (for a direct quotation) would be the RADR in FC adjusted by the ratio of the risk-free interest rates in HC and FC (plus one). These results hold for valuing both unlevered and levered companies. An analysis covering all these cases has not been presented in the literature before.
 3. For company valuation, exchange rate risk is measured with reference to the present value of one future unit of FC (price risk), or, equivalently, by considering the investment of one unit of FC at the risk-free interest rate in combination with the expected change in exchange rates. This is a technical clarification with regard to papers that measure exchange rate risk by comparing the change in exchange rates with the returns on the market portfolio. Another technical clarification of our analysis is that a multi-period valuation requires the use of forward interest rates also when it comes to cross-border valuation.
 4. For calculating growth rates to derive the terminal value within the HC approach, the rate of change in exchange rates has to be considered besides the growth in FCF in FC. Using a constant growth rate in HC might lead to inconsistencies. Therefore, the rate of growth of FCF in FC should be the starting point. It needs to be combined with the ratio of the forward interest rates in HC and FC (plus one), if forward exchange rates are used, or with the expected change in exchange rates, if expected spot exchange rates are used.
 5. In line with the assumption of deterministic debt employed, our analysis starts with the APV approach, although the RADRs for the FTE and the WACC approach were derived as well. With regard to using target leverage ratios for planning future debt financing, which would justify the use of the WACC approach, we pointed out that the leverage ratios needed to be based upon FC

values. Besides varying forward interest rates, exchange rate risk would lead to time-dependent RADR.

6. The repayment of debt in FC used by a domestic company triggers a tax effect in addition to the well-known tax shields on interest expenses. This additional tax effect can be found in the literature, but has not been integrated into cross-border DCF valuation in a consistent manner before.

As always, the results depend upon the assumptions made, thus limiting the applicability of the conclusions, if the assumptions were different. One of the two most notable limitations can be attributed to result No. 4, because the terminal value is subject to quite a few challenges as mentioned above. Secondly, result No. 5 is limited by assuming deterministic levels of debt employed only and not using alternative financing policies like a target debt ratio based upon market values or book values, and by not modeling the risk of default in a multi-period setting extensively (including the preconditions for market values of debt equaling book values and the costs of financial distress).

Future normative research could overcome these limitations, for example by analyzing different financial policies and modelling the consequences of the risk of default in greater detail. The paper might also provide a starting point for empirical research. For instance, existing empirical evidence could be analyzed or genuine empirical research could be conducted with regard to the empirical relevance of the premium for exchange rate risks (b) and the exchange rate related covariance [unlevered case: (d); levered case: (g)], and also with regard to the difference between the beta values and market risk premia from an HC and a FC perspective. This could help to analyze the validity of the UIP and the UH, for which beta FC would equal beta HC, and to choose between HC and FC approach, and to choose between using forward exchange rates or expected spot exchange rates.

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Appendix

See Tables 2 and 3.

Equation (8):

Using Δ_S as the difference $S_u - S_d$, we can write:

$$E[S_1] = pS_{u,1} + (1 - p)S_{d,1} = p\Delta_S + S_{d,1}$$

And:

$$F_1 = qS_{u,1} + (1 - q)S_{d,1} = q\Delta_S + S_{d,1}$$

Since S_d is part of both equations, Eq. (8) is the difference between them:

$$RD_S = E[S_1] - F_1 = p\Delta_S - q\Delta_S = (p - q)\Delta_S$$

Equation (11):

We begin with:

$$V_{HC,0} = \frac{E[A_1] \cdot E[S_1] + \text{cov}(d_A, d_S) V_{HC,0}}{d_A d_S + \text{cov}(d_A, d_S)}$$

With $\text{cov}(d_A, d_S) = \text{cov}(A_1, S_1) \frac{1}{V_{FC,0}} \frac{1}{S_0} = \text{cov}(A_1, S_1) V_{HC,0}^{-1}$.

After rearranging, we get:

$$V_{HC,0}(d_A d_S + \text{cov}(d_A, d_S)) - \text{cov}(d_A, d_S) V_{HC,0} = E[A_1] \cdot E[S_1]$$

$$V_{HC,0} = \frac{E[A_1] \cdot E[S_1]}{d_A d_S}$$

Equation (13):

With x for the modification of the discount factor d_A , we begin with:

$$V_{HC,0} = \frac{E[A_1] \cdot E[S_1]}{d_A d_S} = \frac{E[A_1] \cdot F_1}{d_A x}$$

Solving for x :

$$x = d_S \frac{F_1}{E[S_1]} = \frac{E[S_1]}{S_0} \frac{F_1}{E[S_1]} = \frac{F_1}{S_0} = \frac{1 + i_{HC}}{1 + i_{FC}}$$

Using this solution for x leads to Eq. (13).

Equation (19):

With $F_1 = (E[S_1] - RD_S)$ and

$$q_{FC} FCF_{Uu,FC,1} + (1 - q_{FC}) FCF_{Ud,FC,1} = E[FCF_{U,FC,1}] - RD_{FCF_{U,FC}}$$

we get from

$$V_{U,HC,0} = F_1 [q_{FC} FCF_{Uu,FC,1} + (1 - q_{FC}) FCF_{Ud,FC,1}] (1 + i_{HC})^{-1}$$

to

$$V_{U,HC,0} = (E[S_1] - RD_S) \left(E[FCF_{U,FC,1}] - RD_{FCF_{U,FC}} \right) (1 + i_{HC})^{-1}$$

Equation (25):

With r_Δ for the risk-equivalent rate to discount the difference between S_u and S_d , we can write:

$$V_{S,0} = \frac{1}{1 + E[r_{V_S}]} [pS_{u,1} + (1 - p)S_{d,1}] = p\Delta_S(1 + r_\Delta)^{-1} + S_{d,1}(1 + i_{HC})^{-1}$$

Since V_S and the present value of S_d are the same for both the RADR-valuation and the risk-neutral valuation, the present value of the difference must be the same for both approaches:

$$\begin{aligned} p\Delta_S(1 + r_\Delta)^{-1} &= q\Delta_S(1 + i_{HC})^{-1} \\ p(1 + r_\Delta)^{-1} &= q(1 + i_{HC})^{-1} \\ p\Delta_S &= q\Delta_S \frac{1 + r_\Delta}{1 + i_{HC}} \end{aligned}$$

Using these results and Eq. (23), we get to the definition of the risk discount RD_S :

$$\begin{aligned} RD_S = E[S_1] - F_1 &= q\Delta_S \frac{1 + r_\Delta}{1 + i_{HC}} - q\Delta_S = q\Delta_S \left(\frac{1 + r_\Delta}{1 + i_{HC}} - 1 \right) = q \frac{F_1 - S_{d,1}}{q} \frac{r_\Delta - i_{HC}}{1 + i_{HC}} \\ &= (F_1 - S_{d,1}) \frac{r_\Delta - i_{HC}}{1 + i_{HC}} = (F_1 - S_{d,1}) \frac{z_\Delta}{1 + i_{HC}} = (F_1 - S_{d,1}) \frac{\beta_\Delta E[MRP_{HC}]}{1 + i_{HC}} = (F_1 - S_{d,1}) \frac{E[MRP_{HC}]}{i_{HC} - r_{M,d}} \end{aligned}$$

Since the forward exchange rate is the certainty equivalent, adding this risk discount to the forward exchange rate leads to the expected spot exchange rate, as shown in Eq. (25).

Equation (26):

According to Eqs. (4) and (5), the present value of one unit of FC is:

$$V_{S,0} = S_0(1 + i_{FC})^{-1} = E[S_1] \left(1 + E[r_{V_S}] \right)^{-1}$$

Solving for the expected spot exchange rate, leads to:

$$E[S_1] = S_0 \frac{1 + E[r_{V_S}]}{1 + i_{FC}}$$

Substituting the spot exchange rate with the forward exchange rate defined in Eq. (1), and subtracting the forward exchange rate leads to the risk discount:

$$\begin{aligned}
 RD_S &= E[S_1] - F_1 = F_1 \frac{1 + E[r_{V_S}]}{1 + i_{HC}} - F_1 = F_1 \left[\frac{1 + E[r_{V_S}]}{1 + i_{HC}} - 1 \right] = F_1 \frac{E[r_{V_S}] - i_{HC}}{1 + i_{HC}} \\
 &= F_1 \frac{\beta_{V_S} E[MRP_{HC}]}{1 + i_{HC}} = S_0 \frac{\beta_{V_S} E[MRP_{HC}]}{1 + i_{FC}}
 \end{aligned}$$

Adding this risk discount to the certainty equivalent (F) leads to Eq. (26).

Equation (28):

We can write analogously to Eq. (19), but with CAPM based certainty equivalents:

$$\begin{aligned}
 V_{U,HC,0} &= (E[S_1] - RD_S) (E[FCF_{U,FC,1}] - RD_{FCF_{U,FC}}) (1 + i_{HC})^{-1} \\
 &= [E[S_1] - \lambda_{HC} \text{cov}(S_1, r_{M,HC})] [E[FCF_{U,FC,1}] - \lambda_{FC} \text{cov}(FCF_{U,FC,1}, r_{M,FC})] (1 + i_{HC})^{-1}
 \end{aligned}$$

After multiplying, we get:

$$\begin{aligned}
 V_{U,HC,0} &= \left[\underbrace{E[FCF_{U,FC,1}] E[S_1]}_{(a)} - \underbrace{\lambda_{HC} \text{cov}(S_1, r_{M,HC}) E[FCF_{U,FC,1}]}_{(b)} \right. \\
 &\quad \left. - \underbrace{\lambda_{FC} \text{cov}(FCF_{U,FC,1}, r_{M,FC}) E[S_1]}_{(c)} + \underbrace{\lambda_{HC} \text{cov}(S_1, r_{M,HC}) \lambda_{FC} \text{cov}(FCF_{U,FC,1}, r_{M,FC})}_{(d)} \right] (1 + i_{HC})^{-1}
 \end{aligned}$$

Table 2 Risk parameters for the FC and the HC approach

| | |
|----------------------------------|---|
| Market price of risk $E[r_M]$ | <p>With HC-parameters to FC-market return</p> $E[r_{M,FC}] = \left(1 + E[r_{M,HC}]\right) E\left[\frac{S_0}{S_1}\right] + \text{cov}\left(r_{M,HC}, \frac{S_0}{S_1}\right) - 1$ <p>With FC-parameters to HC-market return</p> $E[r_{M,HC}] = \left(1 + E[r_{M,FC}]\right) (1 + E[\delta_S]) + \text{cov}\left(r_{M,FC}, \delta_S\right) - 1$ |
| σ_M^2 | <p>With HC-parameters to FC-variance</p> $\sigma_{M,FC}^2 = E[r_{M,FC}^2] - E[r_{M,FC}]^2$ <p>Using the link between the state-contingent market returns for the first term on the RHS, and solving for the expected value in the second term on the RHS</p> $\sigma_{M,FC}^2 = E\left[\left(\left(1 + r_{M,HC}\right) \frac{S_0}{S_1} - 1\right)^2\right] - \left[\left(1 + E[r_{M,HC}]\right) E\left[\frac{S_0}{S_1}\right] - 1 + \text{cov}\left(r_{M,HC}, \frac{S_0}{S_1}\right)\right]^2$ <p>With FC-parameters to HC-variance</p> $\sigma_{M,HC}^2 = E[r_{M,HC}^2] - E[r_{M,HC}]^2$ $= E\left[\left(\left(1 + r_{M,FC}\right) (1 + \delta_S) - 1\right)^2\right] - \left[\left(1 + E[r_{M,FC}]\right) (1 + E[\delta_S]) - 1 + \text{cov}\left(r_{M,FC}, \delta_S\right)\right]^2$ |

Table 2 (continued)

$$\text{cov}(FCF_{U, r_M}) = \text{cov}(r_U, r_M) V_U$$

With HC-parameters to FC-covariance (FCF_U)

$$\begin{aligned} \text{cov}(FCF_{U,FC}, r_{M,FC}) &= E[FCF_{U,FC} r_{M,FC}] - E[FCF_{U,FC}] E[r_{M,FC}] \\ &= E\left[FCF_{U,HC} S_1^{-1} \left((1 + r_{M,HC}) \frac{S_0}{S_1} - 1 \right) \right] \\ &\quad - \left[E[FCF_{U,HC}] E[S_1^{-1}] + \text{cov}(FCF_{U,HC}, S_1^{-1}) \right] \left[(1 + E[r_{M,HC}]) E\left[\frac{S_0}{S_1}\right] - 1 + \text{cov}\left(r_{M,HC}, \frac{S_0}{S_1}\right) \right] \end{aligned}$$

With FC-parameters to HC-covariance (FCF_U)

$$\begin{aligned} \text{cov}(FCF_{U,HC}, r_{M,HC}) &= E[FCF_{U,HC} r_{M,HC}] - E[FCF_{U,HC}] E[r_{M,HC}] \\ &= E\left[FCF_{U,FC} S_1 \left((1 + r_{M,FC}) (1 + \delta_S) - 1 \right) \right] \\ &\quad - \left[E[FCF_{U,FC}] E[S_1] + \text{cov}(FCF_{U,FC}, S_1) \right] \left[(1 + E[r_{M,FC}]) (1 + E[\delta_S]) - 1 + \text{cov}(r_{M,FC}, \delta_S) \right] \end{aligned}$$

Table 3 Risk parameters for the FC and the HC approach

| | | | |
|-------------------------|----------------------------|---|---|
| Market price of risk | $E[r_M]$ | With HC-parameters to FC-market return | $E[r_{M,FC}] = \left(1 + E\left[r_{M,HC}\right]\right) \frac{1+i_{FC}}{1+i_{HC}} - 1$ |
| | | With FC-parameters to HC-market return | $E[r_{M,HC}] = \left(1 + E\left[r_{M,FC}\right]\right) \frac{1+i_{HC}}{1+i_{FC}} - 1$ |
| | σ_M^2 | With HC-parameters to FC-variance | $\begin{aligned} \sigma_{M,FC}^2 &= E\left[r_{M,FC}^2\right] - E\left[r_{M,FC}\right]^2 \\ &= E\left[\left(\left(1 + r_{M,HC}\right) \frac{1+i_{FC}}{1+i_{HC}} - 1\right)^2\right] - \left[\left(1 + E\left[r_{M,HC}\right]\right) \frac{1+i_{FC}}{1+i_{HC}} - 1\right]^2 = \left(\frac{1+i_{FC}}{1+i_{HC}}\right)^2 \sigma_{M,HC}^2 \end{aligned}$ |
| | | With FC-parameters to HC-variance | $\sigma_{M,HC}^2 = \sigma_{M,FC}^2 \left(\frac{1+i_{HC}}{1+i_{FC}}\right)^2$ |
| Amount of relevant risk | $\text{cov}(r_U, r_M) V_U$ | With HC-parameters to FC-covariance (FCF_U) | $\text{cov}(r_{U,FC}, r_{M,FC}) = \text{cov}\left(\left(1 + r_{U,HC}\right) \frac{1+i_{FC}}{1+i_{HC}} - 1, \left(1 + r_{M,HC}\right) \frac{1+i_{FC}}{1+i_{HC}} - 1\right)$ |
| | | With FC-parameters to HC-covariance (FCF_U) | $= \left(\frac{1+i_{FC}}{1+i_{HC}}\right)^2 \text{cov}(r_{U,HC}, r_{M,HC})$ |
| | | With FC-parameters to HC-covariance (FCF_U) | $\text{cov}(r_{U,HC}, r_{M,HC}) = \text{cov}(r_{U,FC}, r_{M,FC}) \left(\frac{1+i_{HC}}{1+i_{FC}}\right)^2$ |

From FC to HC approach and back: risk parameters

If the uncovered interest parity is assumed to hold, forward exchange rates serve as expected spot exchange rates, leading to a simple transition:

Therefore:

$$\beta_{U,HC} = \frac{\text{cov}(r_{U,HC}, r_{M,HC})}{\sigma_{M,HC}^2} = \frac{\text{cov}(r_{U,FC}, r_{M,FC}) \left(\frac{1+i_{HC}}{1+i_{FC}} \right)^2}{\sigma_{M,FC}^2 \left(\frac{1+i_{HC}}{1+i_{FC}} \right)^2} = \frac{\text{cov}(r_{U,FC}, r_{M,FC})}{\sigma_{M,FC}^2} = \beta_{U,FC} \quad (56)$$

$$E[\text{MRP}_{HC}] = E[r_{M,HC}] - i_{HC} = \left(1 + E[r_{M,FC}] \right) \frac{1+i_{HC}}{1+i_{FC}} - 1 - \left[\left(1 + i_{FC} \right) \frac{1+i_{HC}}{1+i_{FC}} - 1 \right] = \text{MRP}_{FC} \frac{1+i_{HC}}{1+i_{FC}} \quad (57)$$

References

- Baetge J, Kümmel J, Schulz R, Wiese J (2019) Darstellung der Discounted Cashflow-Verfahren (DCF-Verfahren) mit Beispiel. In: Peemöller VH (ed) Praxishandbuch der Unternehmensbewertung, 7th edn. Nwb, Herne, pp 409–569
- Bekaert G, Hodrick R (2018) International financial management, 3rd edn. Cambridge University Press, Cambridge
- Berk J, DeMarzo P (2020) Corporate finance, 5th edn. Pearson Education Limited, Harlow
- Brealey RA, Myers SC, Allen F (2019) Principles of corporate finance, 13th edn. McGraw-Hill, New York
- Breuer W (2001) Investition II: Entscheidungen bei Risiko. Gabler, Wiesbaden
- Breuer W (2015) Unternehmerisches Währungsmanagement, 3rd edn. Gabler, Wiesbaden
- Butler KC, O'Brien TJ, Utete G (2013) A fresh look at cross-border valuation and FX hedging decisions. *J Appl Finance* 23:61–83
- Chowdhry B, Coval JD (1998) Internal financing of multinational subsidiaries: debt vs. equity. *J Corp Finance* 4:87–106. [https://doi.org/10.1016/S0929-1199\(97\)00011-4](https://doi.org/10.1016/S0929-1199(97)00011-4)
- Click RW, Coval JD (2002) The theory and practice of international financial management. Upper Saddle River, New Jersey
- Cooper IA, Nyborg KG (2008) Tax-adjusted discount rates with investor taxes and risky debt. *Finance Manag* 37:365–379
- Cox JC, Ross SA, Rubinstein M (1979) Option pricing: a simplified approach. *J Finance Econ* 7:229–263. [https://doi.org/10.1016/0304-405X\(79\)90015-1](https://doi.org/10.1016/0304-405X(79)90015-1)
- Drukarczyk J, Schüler A (2016) Unternehmensbewertung, 7th edn. Vahlen, München
- Ejara D, Krapl A, O'Brien TJ, Ruiz de Vargas S (2019) Local, global, and international CAPM: for which countries does choice matter? *J Invest Manag*. <https://doi.org/10.2139/ssrn.3023501>
- Eiteman DK, Stonehill AI, Mofett MH (2016) Multinational business finance, 14th edn. Pearson, Harlow
- Fama EF (1977) Risk-adjusted discount rates and capital budgeting under uncertainty. *J Finance Econ* 5:3–24. [https://doi.org/10.1016/0304-405X\(77\)90027-7](https://doi.org/10.1016/0304-405X(77)90027-7)
- Friedrich T (2016) Corporate valuation and risk of insolvency. *DBW* 76:521–543
- Goodman LA (1960) On the exact variance of products. *J Am Stat Assoc* 55:708–713
- Haley CW, Schall LD (1979) The theory of financial decisions, 2nd edn. McGraw-Hill, New York
- Harris RS, Pringle JJ (1985) Risk-adjusted discount rates—extensions from the average-risk case. *J Finance Res* 8:237–244
- Hodder J, Senbet LW (1990) International capital structure equilibrium. *J Finance* 45:1495–1516. <https://doi.org/10.1111/j.1540-6261.1990.tb03725.x>
- Holthausen RW, Zmijewski ME (2020) Corporate valuation: theory, evidence & practice, 2nd edn. Cambridge Business Publishers, Cambridge
- Hull JC (2018) Options, futures, and other derivatives, 10th edn. Pearson, New York

- Inselbag I, Kaufold H (1997) Two DCF approaches for valuing companies under alternative financing strategies (and how to choose between them). *J Appl Corp Finance* 10:114–122. <https://doi.org/10.1111/j.1745-6622.1997.tb00132.x>
- Koller T, Goedhart M, Wessels D (2015) *Valuation: measuring and managing the value of companies*, 6th edn. Wiley, New Jersey
- Kraus MV, Lahmann A (2016) Reconsidering the appropriate discount rate for tax shield valuation. *J Bus Econ* 86:477–512
- Kruschwitz L, Lodowicks A, Löffler A (2005) Zur Bewertung insolvenzbedrohter Unternehmen. *Die Betriebswirtschaft* 65:221–236. <https://doi.org/10.17169/refubium-24567>
- Krapl A, O'Brien TJ (2016) Estimating cost of equity: do you need to adjust for foreign exchange risk? *J Int Finance Manag Acc* 27:5–25. <https://doi.org/10.1111/jifm.12031>
- Lessard DR (1979) Evaluating international projects: an adjusted present value approach. *International financial management: theory and application*, 2nd edn. MIT, Boston, pp 570–584
- Lübbehüsen T (2000) *Steuern im Shareholder-Value-Ansatz: Der Einfluss von Steuern auf Unternehmensführung und -bewertung*. Erich Schmidt Verlag, Bielefeld
- Mehra R (1978) On the financing and investment decisions of multinational firms in the presence of exchange risk. *J Finance Quant Anal* 13:227–244. <https://doi.org/10.2307/2330384>
- Miles JA, Ezzell JR (1980) The weighted average cost of capital, perfect capital markets, and project life: a clarification. *J Finance Quant Anal* 15:719–730. <https://doi.org/10.2307/2330405>
- Modigliani F, Miller MH (1958) The cost of capital, corporation finance and the theory of investment. *Am Econ Rev* 48:261–297
- Modigliani F, Miller MH (1963) Corporate income taxes and the cost of capital—a correction. *Am Econ Rev* 53:433–443
- O'Brien T (2017) *Applied international finance II*, 2nd edn. BEP, New York
- Rubinstein ME (1973) A mean-variance synthesis of corporate financial theory. *J Finance* 28:167–181. <https://doi.org/10.1111/j.1540-6261.1973.tb01356.x>
- Ruiz de Vargas S (2019) Unternehmensbewertung im internationalen Kontext. In: Peemöller VH (ed) *Praxishandbuch Unternehmensbewertung*, 7th edn. NWB Verlag, Herne, pp 1639–1678
- Ruiz de Vargas S, Breuer W (2018) Corporate valuation in an international context with the global CAPM from a German perspective. <https://ssrn.com/abstract=3119049>. Accessed 19 Dec 2019
- Ruiz de Vargas S, Breuer W (2019) Globale vs. lokale Betafaktoren bei international zusammengesetzten Peergroups. In: Tönnies WA (ed) *Unternehmensbewertung: Festschrift für Bernhard Großfeld zum, vol 85. Geburtstag*, C.F. Müller, Heidelberg, pp 355–376
- Schall LD (1972) Asset valuation, firm investment, and firm diversification. *J Bus* 45:11–28
- Schüler A (2018a) A tool kit for discounted cash flow valuation: consistent and inconsistent ways to value risky cash flows. *J Bus Val Econ Loss Anal* 13:1–16. <https://doi.org/10.1515/jbvela-2016-0012>
- Schüler A (2018b) Aktienbasierte erfolgsabhängige Entlohnung & Unternehmensbewertung. *Schmalenbachs Z betriebswirtsch Forsch* 70:125–151. <https://doi.org/10.1007/s41471-017-0048-7>
- Senbet LW (1979) International capital market equilibrium and the multinational firm financing and investment policies. *J Finance Quant Anal* 14:455–480. <https://doi.org/10.2307/2330181>
- Sercu P (1980) A generalization of the international asset pricing model. *Revue de l'Association Française de Finance* 1:91–135
- Sercu P (2009) *International finance: theory into practice*. Princeton University Press, Princeton
- Siegel JJ (1972) Risk, interest rates and the forward exchange. *Q J Econ* 86:303–309
- Sick GA (1990) Tax-adjusted discount rates. *Manag Sci* 36:1432–1450
- Solnik BH (1974) An equilibrium model of the international capital market. *J Econ Theory* 8:500–524. [https://doi.org/10.1016/0022-0531\(74\)90024-6](https://doi.org/10.1016/0022-0531(74)90024-6)
- Solnik BH (1993) Currency hedging and Siegel's paradox: on Black's universal hedging rule. *Rev Int Econ* 1:180–187. <https://doi.org/10.1111/j.1467-9396.1993.tb00014.x>
- Stiglitz JE (1969) A re-examination of the Modigliani–Miller theorem. *Am Econ Rev* 59:784–793
- Stulz RM (1995) The cost of capital in internationally integrated markets: the case of Nestlé. *Eur Finance Manag* 1:11–22. <https://doi.org/10.1111/j.1468-036X.1995.tb00003.x>
- Zwirner T (1989) *Devisenkursrisiko, Unternehmen und Kapitalmarkt*. DUV, Wiesbaden

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