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Increasing Returns to Scale and the Long-Run Phillips Curve

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Increasing Returns to Scale and the Long-Run Phillips Curve

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Abstract

A growing body of empirical evidence shows that there exists a long-run positive tradeoff between inflation and real macroeconomic activity. Within a New Keynesian framework, we examine how increasing returns generate a positive long-run relation between inflation and output.

Keywords: Phillips curve, Inflation, Increasing returns, nominal inertia, monetary policy.

JEL classification code: E3, E20, E40, E50.

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1 Introduction

There is a growing body of empirical evidence that there exists a positive long-run relationship between inflation and real macroeconomic activity. According to the point estimates of Bernanke and Mihov (1998), a 1% deviation of non-borrowed reserves causes a 0.15% increase in output in the long run. Mankiw (2001), commenting on this result, writes that “...if one does not approach the data with a prior favouring long-run neutrality, one would not leave the data with that posterior. The data’s best guess is that monetary shocks leave permanent scars on the economy”. The analysis of Akerlof, Dickens and Perry (1996) shows that in the long run the unemployment rate can increase from 5.9% to 7.6% when reducing the inflation rate from 3% to nil. According to Akerlof, Dickens and Perry (2000), a rate of inflation of 2.6% implies a long-run equilibrium rate of unemployment that is 1.7 percentage points lower than either at no inflation or at a rate above 6 percent. Ball (1997) provides evidence indicating that countries which had comparatively large and long declines in inflation also tended to have comparatively large increases in their NAIRUs. Dolado, López-Salido and Vega (2000) consider Spanish data and find that, depending on the model specification, one percentage point of permanent disinflation implies a permanent output loss between 0.5 and 0.25 percentage points. King and Watson (1994) find that, over five years, the cost of a 1% permanent reduction in inflation amounts to a cumulative increase in unemployment ranging between 3.7% and 1.5%, depending on the identifying assumptions adopted. Karanasou, Sala and Snower (2003, 2005) find such a trade-off for the U.S. and the E.U. as well: a 10% increase in money growth leads to a fall of 3.14 percentage points in unemployment.

Studies analysing the steady state properties of the New Keynesian models, as King and Wolman (1996), Ascari (1998, 2000), Devereux and Yetman (2002) and Graham and Snower (2003), support long-run money non-superneutrality. Recent explanations of this pattern focused on three factors:

1. When prices are sticky due to Taylor or Calvo price staggering, inflation causes relative prices to vary over the contract period. The relative price variations lead consumers to substitute between goods - a phenomenon we may call "product cycling". If these goods are imperfect substitutes, then product cycling is inefficient, so that a rise in inflation leads to a fall in aggregate product demand.

2. When the output is produced by labor (or other productive factors), then product cycling gives rise to "labor cycling", i.e. substitutions among factors producing the products. In the presence of diminishing returns to labor, such labor cycling is inefficient, so that a rise in inflation leads to a further fall in output.

3. Under price staggering, the price of a product depends on the present and future price level. The greater the rate of time discount, the more closely the product price depends on the current (rather than the future)
price level - for the simple reason that the future is valued less. Thus, the
greater the rate of money growth and inflation, the lower will prices be
set relative to the money supply. Consequently real money balances rise,
leading to a rise in output.

The first two effects imply a negative relation between inflation and output,
whereas the third implies a positive effect. It can be shown that, except at
very low inflation rates, the first two effects dominate the third (Graham and
Snower, 2003).

The contribution of this paper is to examine the implications of increasing
returns on this analysis\footnote{The fact that increasing returns might have an impact on economic fluctuations was also pointed out by Leijonhufvud (1986).}. The empirical evidence indicates that increasing returns are observable with different strength in various sectors of the economy, including firms and plants both in the manufacturing and in the retail sector (see, for example, Betancourt and Malanoski, 1999, Ramey, 1991, and Roberts and Supina, 1997).

We show that in the presence of increasing returns, labor cycling leads to
efficiency gains. The greater the inflation rate, the greater the degree of la-
bor cycling and the greater these efficiency gains. Consequently, labor cycling
gives rise to a positive relation between inflation and output. For reasonable
calibrated values, we show that this effect is sufficiently strong as to generate
a positive inflation-output trade-off, even in the presence of product cycling.
An increase in money growth (and thus inflation) leads to a sufficiently large
increase in output to be roughly consonant with the empirical evidence above.
The upshot of our analysis is that returns to scale matter for the shape of the
long-run Phillips curve.

The paper is organized as follows. Section 1 presents our dynamic general
equilibrium model, which is quite standard, except for the inclusion of increasing
returns to scale. We derive the corresponding long-run Phillips curve. Section
2 clarifies the underlying intuition for our results. Section 3 concludes.

2 The Model

The economy has three markets: a perfectly competitive labour market, a mo-
nopolistically competitive intermediate goods market with staggered prices, and
a perfectly competitive final goods market. The money supply grows at rate
\((\mu - 1)\). All nominal values are detrended in terms of the money supply.

Consumers maximize their utility over consumption \((c_t)\), real money holdings
\((\frac{m_t}{p_t})\) and working time \((n_t)\) subject to the budget and resource constraints:
\[
\max_{(c_t,m_t,n_t)} \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t + V \left( \frac{m_t}{p_t} \right) - \kappa \frac{p_t^{1+\phi}}{1+\phi} \right]
\]

subject to \( p_t y_t = p_t c_t + m_t - \frac{m_{t-1}}{\mu} \)

\( p_t y_t = w t n_t + p_t \pi_t \)

where \( p_t \) is the aggregate price level, \( y_t \) is the level of output, \( \pi_t \) are profits, \( \beta \) is the time discount factor, and \( \kappa \) and \( \phi \) are positive constants. First-order conditions for consumption, labour and money holdings are

\[
\frac{1}{c_t} = \lambda_t \quad (1)
\]

\[-\kappa n_t^\phi + \lambda_t \frac{w_t}{p_t} = 0 \quad (2)
\]

\[
V_m \left( \frac{m_t}{p_t} \right) \frac{1}{p_t} - \frac{\lambda_t}{p_t} + \frac{\lambda_{t+1}}{p_{t+1}} \frac{1}{\mu} = 0 \quad (3)
\]

In the intermediate product market, each firm is an imperfectly competitive price setter, under Taylor price staggering. Specifically, the \( i \)’th firm sets the price \( p_{it} \) of the \( i \)’th good in period \( t \) for a contract period that lasts until period \( t+N \). This price is set so as to maximize its profit subject to its product demand (derived from (7), below) and its production function:

\[
\max_{(p_{i,t})} \mathbb{E}_t \sum_{i=0}^{N-1} \beta^{t+i} \left[ \frac{p_{i,t}}{p_{t+i}} y_{i,t+i} - \frac{w_{t+i}}{p_{t+i}} n_{t+i} \right] \quad (4)
\]

subject to \( y_{i,t+i} = \left( \frac{p_{i,t}}{p_{t+i}} \right)^{-\theta_p} y_{t+i} \)

\( y_{i,t+i} = n_{i,t+i}^\nu \quad (5) \)

where \( y_{i,t+i} \) is the \( i \)’th output at time \( t+i \), \( w_{t+i} \) is the nominal wage, \( n_{t+i} \) is employment, and \( y_{t+i} \) is aggregate output. The elasticity of substitution among intermediate goods, \( \theta_p \), is a positive constant. Since there are increasing returns to scale, \( \nu > 1 \) in the production function.

The first-order condition implies the following price setting equation:

\[
p_{i,t} = \frac{\theta_p}{v (\theta_p - 1)} \frac{\sum_{i=0}^{N-1} \beta^{t+i} y_{i,t+i}^\frac{1}{\nu} \frac{w_{t+i}}{p_{t+i}}}{\sum_{i=0}^{N-1} \beta^{t+i} y_{i,t+i}^\frac{1}{\nu} \frac{w_{t+i}}{p_{t+i}}} \quad (6)
\]

The second-order condition implies that \( v < \frac{\theta_p}{\theta_p - 1} \).

In the final product market, perfectly competitive firms buy an horizontally differentiated input, \( y_{i,t} \), to produce an homogeneous output, \( y_t \). They set output so as to maximize their profit subject to their production function:
\[
\max_{y_{i,t}} p_t y_t - \sum_{i=0}^{N-1} \frac{p_{i,t}}{\mu^i} y_{i,t} \\
\text{s.t.} y_t = \left( \sum_{i=0}^{N-1} y_{i,t}^{\theta_{p-1}} \right)^{\frac{\theta_{p}}{\theta_{p}-1}}
\]

Solving (7), we obtain the demand function for the intermediate good \( y_{i,t} \):

\[
y_{i,t} = \left( \frac{p_{i,t}}{p_t \mu^i} \right)^{-\theta_{p}} y_t
\]

The free entry condition gives the aggregate price index:

\[
p_t = \left[ \sum_{i=0}^{N-1} \left( \frac{p_{i,t}}{\mu^i} \right)^{1-\theta_{p}} \right]^{\frac{1}{1-\theta_{p}}}
\]

The general equilibrium is the solution of the equation system comprising the consumption condition (1), the leisure condition (2), the money balance condition (3), the production function (5), the price setting equation (6), the intermediate good demand (8), the price index (9), as well as the market clearing condition:

\[
y_t = c_t
\]

The steady state solution is outlined in the appendix.

We calibrate the system for standard parameter values showed in Table 1. The resulting long-run relation between money growth (equal to inflation) and output (for two different values of the elasticity of substitution \( \theta_{p} \)) is pictured in Fig. 1. Observe that a permanent increase in money growth has a sizable effect on the level of output. The classical dichotomy breaks down: money growth has a positive impact on the level of output in the long-run\(^2\). Furthermore, note that a rise in the elasticity of substitution \( \theta_{p} \) implies an increase in the output effect of monetary policy, given that substitution inefficiencies decrease. Fig. 2 shows that increasing the value of \( v \) makes the long-run Phillips curve steeper for \( v < 1 \) and flatter for \( v > 1 \). An explanation of this pattern is offered in the next section.

Our analysis does not however imply that the Phillips curve necessarily remains upward-sloping over the entire range of relevant money growth rates. The reason is that production functions often display increasing returns only as long as factor utilization is not too high. Once output exceeds some critical level, diminishing returns often set in and then the Phillips curve becomes downward-sloping. A straightforward way to model this concept is to replace (5) with a Skiba (1978) production function whereby \( v \) is greater than one when output is

---

\(^2\)As in King and Wolman (1996) under different assumptions.
smaller than a certain threshold level \( y^* \) and smaller for greater levels of output. Figure 3 shows how welfare evaluated as in Woodford (1998) would change with such a production function, setting \( y^* = y(\mu|\mu = 1.5) \) for illustrative purposes. Welfare increases in the increasing returns portion of the production function and decreases thereafter. This result is not sensitive to different values of \( \phi \).

Finally, we turn to the intuition underlying our results.

3 Intuition

As noted in the introductory section, aggregate price level inflation under staggered price setting leads an instability of relative prices that generates "product cycling" (households' substitutions among different products) and "labor cycling" (firms' substitutions among different labor types). Product cycling is inefficient when the products are imperfect substitutes; labor cycling is inefficient under diminishing returns, but efficient under increasing returns.

The nature of the latter inefficiency or efficiency is illustrated in Fig. 4, which pictures a total cost function. Under increasing returns, the marginal cost function is declining, and thus when production fluctuates between \( A + \epsilon \) and \( A - \epsilon \), there is an increase in efficiency due to the concavity of the cost function, as the average total cost is equal to \( C_2 \) and not to \( C_1 \). Conversely, under diminishing returns, the marginal cost function is increasing, so that when production fluctuations between \( B + \epsilon \) and \( B - \epsilon \) take place, there is a drop in efficiency, since the average total cost is equal to \( C_3 \) and not to \( C_4 \).

The greater is the elasticity of substitution among products, the greater are both product cycling and labour cycling. In presence of increasing returns, this generates greater economies leading to greater output gains after an increase in inflation\(^3\). To understand how the Phillips curve changes for different values of \( v \), it is necessary to keep in mind that firms choose their output and their price by equating their marginal cost to their marginal revenue. Over the contract period, the marginal cost schedule and the marginal revenue one shift in order to keep the price fixed and to let firms' output move. As showed by Figure 5, the absolute value of the elasticity of the marginal cost function is greater the more \( v \) is far from one. The greater is the elasticity of the marginal cost function, the smaller are the changes in output necessary to move the marginal cost schedule over the contract period. Small changes in output implies small (dis-)economies from cycling for \((v < 1)\ v > 1\) and so a flatter long run Phillips Curve\(^4\).

In sum, the long-run Phillips curve depends on the technologies available to the firms: increasing returns imply a positive relation between macroeconomic

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\(^3\)Conversely, it is possible to show that, in presence of decreasing returns to scale, the greater is \( \phi_p \) and the greater are the output losses deriving from higher inflation rates.

\(^4\)In a Taylor model it is possible to keep track of the output of the various cohorts of firms. It is worth considering that \( v = 1.02 \) implies a standard deviation of the levels of output of the cohorts of firms equal to 0.5% of that implied by \( v = 1.01 \). Instead, \( v = 0.67 \) implies a standard deviation equal to 14% of that produced by \( v = 0.95 \). This further supports the intuition offered for the effect of \( v \) on the slope of the long-run Phillips curve.
nominal activity and money growth; and - abstracting from the time discounting effect that is dominant at very low inflation rates - diminishing returns imply a negative relation.

4 Appendix: Solving the General Equilibrium System

For sake of simplicity we normalize the real wage to 1. Then given that \( p_{i,t} \) is constant in steady state at the value \( p_0 \), we used (9) to obtain:

\[
\frac{p_0}{p} = \left[ \frac{1 - \mu^N \theta_{\mu-1}}{1 - \mu(\theta_{\mu-1})} \right]^{\frac{1}{\theta_{\mu-1}}}
\]

From (6), we find the level of output for cohort zero:

\[
y_0 = \left\{ \frac{\theta_{p}}{v(\theta_{p} - 1)} \frac{p}{p_0} \sum_{i=0}^{N-1} \beta^i \mu^i \theta_{\mu-1} \right\}^{\frac{\theta_{\mu-1}}{\theta_{p}}} = \left\{ \frac{\theta_{p}}{v(\theta_{p} - 1)} \frac{p}{p_0} \frac{1 - \beta^N \mu^N \theta_{\mu-1}}{1 - \beta \mu (\theta_{\mu-1})} \right\}^{\frac{\theta_{\mu-1}}{\theta_{p}}}
\]

Then it is possible to use

\[
y_0 = \left( \frac{p_0}{p} \right)^{-\theta_{p}} y
\]

(13)
to derive the steady state level of aggregate output, \( y \).

The steady state values of \( y_{i,t+i} \) for \( i = 1, ..., N \) can be computed by taking the ratio of (8) for different cohorts. For instance for cohort zero and cohort one:

\[
\frac{y_0}{y_1} = \left( \frac{p_0}{p} \right)^{-\theta_{p}} \frac{y}{y} = \mu^{-\theta_{p}}
\]

(14)

where variables without subscripts are at their steady state values. From \( y_i \), one can derive \( n_i \) by using (5):

\[
n_i = y_i^{\frac{1}{\psi}}
\]

\( n_i \) is the demand for labour of cohort \( i \), therefore summing \( n_i \) over all the cohorts of the firms will gives \( n \), the aggregate quantity of labour. Having this in mind, it is possible to set \( \kappa \) endogenously by using (2)

\[
\kappa n_i^\phi = \lambda_i \frac{w_i}{p_i}
\]

the consumption first order condition.
\[ \frac{1}{c_t} = \lambda_t \]

and the aggregate equilibrium condition

\[ y_t = c_t \]

so that

\[ \kappa = n^{-\phi} \frac{1 - \phi}{y \ p} \]

where variables without time subscripts are variables at their steady state values.

Recalling that \( \frac{1}{c} = \lambda \) and \( y = c \) (3) yields the steady state value of \( \frac{m_t}{p_t} \):

\[ \frac{m}{p} = V^{-1} \left[ \lambda \left( 1 - \frac{1}{\mu} \right) \right] \]

References


Table 1 – Baseline Calibrated Parameter Values

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\theta_p$</th>
<th>$N$</th>
<th>$\beta$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>4</td>
<td>$0.98 ^{1/N}$</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Figure 1 – The Long-Run Output-Inflation Relationship for Different Values of $\theta_p$ and $\nu=1.01$

Figure 2 – The Long-Run Output-Inflation Relationship for Different Values of $\nu$ and $\theta_p=10$
Figure 3 – Welfare for different values of $\phi$ and percentage money growth rates with a Skiba (1978) production function

Note: $\nu$ is equal to 1.01 up to $y^* = y(\mu|\mu=1.5)$ and to 0.67 thereafter.

Figure 4 – The cost function with varying marginal cost.
Figure 5 – The elasticity of the marginal cost function for different values of $\nu$