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RESEARCH ARTICLE

First home or first away? Optimal ordering in two-legged ties

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Abstract

We model a two-legged tie between two teams as a two-stage contest where each team chooses its attacking and defending effort for each leg. Assuming that each team has a home advantage for one of the matches, we analyze teams' preferences of order of play under different tiebreaking rules. We show that the answer depends on the size of the home advantage, the relative strengths of both teams, and the tiebreaking rule. In particular, we identify under which conditions a team prefers to play first home or first away and under which conditions they are indifferent.

JEL CLASSIFICATION

D02, D72, Z2

1 | INTRODUCTION

In the UEFA Champions League semi-finals 2013, Germany's top soccer team FC Bayern München was paired with FC Barcelona, the Spanish top team, with the first leg in Germany and the return in Spain. As a response to this tie, Matthias Sammer, director of sports of Bayern München, said “I am absolutely optimistic. The fact that we first play home is not a disadvantage. We should try not to get a goal but to use every chance for scoring.”¹ The first leg in Munich ended 4:0 for Bayern, and the second leg was a 3:0 loss for Barcelona in their stadium Camp Nou. Three years later, in the UEFA Champions League round of 16, FC Barcelona was paired with the French top team Paris Saint-Germain. Again, the first leg took place away in Paris and the second leg at the Camp Nou in Barcelona. After a 4:0 loss away in Paris, Barcelona won 6–1 in the second leg and reached the next round.

So is it better to first play home or away in a two-legged tie? If we would take these two pairings as evidence for answering this question, we would have no clear proof. For Barcelona, the answer would be first away against Paris Saint-Germain but better first home against Bayern Munich. So what are relevant factors for responding to this question?

The present paper provides a game-theoretical framework for balancing the advantages of playing first at home with the advantages

of playing first away in such best-of-two contests. They are commonly used in association football, for example, in the Union of European Football Associations (UEFA) Champions League round of 16 or in the UEFA Cup Winner's Cup where all ties are two-legged. We model such a best-of-two-contest as a dynamic contest between two teams with each team as the home team in one leg. Each team, with a given strength of its defenders and forwards, chooses its attacking and defending effort for each match. A team's strength and effort choices are substitutes in that a stronger team is more successful in scoring or avoiding a goal than a weaker team when effort choices are the same. Two elements are central for our analysis, the home advantage and the tiebreaking procedure:

- The home advantage is well documented in professional soccer and refers to the phenomenon that in a soccer match, the team playing at its home field has an advantage over the visiting team. In our analysis, we model the home advantage by assuming that the home advantage stimulates the home team in their efforts to win the match and discourages the away team.
- The tiebreaking procedure comes into play if the aggregate score is tied after the two legs. To determine the winner of the contest, we consider four different methods to break the tie: under a coin toss, the tie is broken at random; under extra time, the tiebreaker consists of an additional period of play; under a penalty shootout,

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teams shoot alternately from the penalty mark; and under the away goals rule, the team that scored more goals away is considered the winner of the tie.² In general, these tiebreaking procedures are used sequentially, which is necessary, if the away goals rule and/or the extra time is used as a first tiebreaker. If, for example, the away goals are equal and/or the tie remains after the extra time, an additional second tiebreaking procedure is needed.³

In this context, our interests center around the discouragement effect usually observed in dynamic contests; see Malueg and Yates (2010). This effect describes the phenomenon that a player who lost in the first match might be discouraged in the second match so that competition and thus effort suffers.⁴ We ask three questions: First, what is the influence of varying home advantages on the discouragement effect when they reduce, respectively, increase, players' marginal cost of effort? Is it even possible that a player gives up the second match if the home advantage of its opponent in the second match is sufficiently high? Second, how do the different tiebreaking rules affect the discouragement effect? Which of these rules is best suited to mitigate the discouragement effect? Finally, what is the optimal order of play in a best-of two contest? When do teams prefer to play first home or away, that is, have its home advantage in the first or in the second match?

We show the following results: First, suppose that the tiebreaking rule specifies a coin toss, extra time, or a penalty shootout. In the absence of a home advantage, the effort choices of both teams in the second match are independent on which court the match takes place since their marginal effort costs are identical. But then the effort choices in the first match are independent of which team plays first at home. Hence, both teams are indifferent whether to play first home or away. This result changes if we introduce a home advantage for one team, for example, for Team 1. Then, Team 1 has a second match home advantage because its home advantage carries over to the first match. In the opposite scenario in which Team 1 has the first match on its home field, this spillover effect is not present. In sum, teams weakly prefer to play the second match at home.

Second, suppose that the tie is decided first by the away goals rule and then by a coin toss or by a penalty shootout. In the absence of a home advantage, the stronger team then prefers to play first at home, since a win in the first match is likely even with moderate attacking efforts. In case it does not win the first match, it can still profit from the away goals rule by shooting a goal in the second match.

Third, if the away goals rule does not lead to a winner during normal time and the overall contest is decided first by extra time, then by the away goals rule, and finally by a coin toss or a penalty shootout, then, in the absence of a home advantage, the away team in the second match is better off than the home team even if the home team is stronger than the visiting team. Hence, the away goals rule introduces a counterbalance for the visiting team to the home advantage of the home team.

Our results also shed new light on the introductory quote by Matthias Sammer and the widespread belief in professional soccer

that the team that plays the second leg at home has an overall advantage. The reasoning behind this second leg home advantage is that a team playing the first leg away should play defensive to reach a tie or a slight defeat and then use its home advantage in the second and decisive leg to win at home. However, our analysis shows that this reasoning depends crucially on the relative strength of both teams, the size of each home advantage, and the tiebreaking rules. Our results might explain why empirical studies show mixed results for a second leg home advantage. The present paper fills this gap since, from a theoretical perspective, no contribution explains these differing results in empirical studies.

The paper is organized as follows: After a literature review in Section 2, we introduce the basic model that describes a best-of-two contest in a game-theoretic framework. Section 4 characterizes equilibrium play, first for the second match, then for the first match. Whereas in the basic model we assume that a tie is decided by a coin toss with equal chances, Section 5 introduces the other three tiebreaking rules and analyzes their impact on behavior. In Section 6, we consider the question whether it is better to play first home or away. The paper concludes with some final remarks. Proofs are collected in Appendix A.

2 | LITERATURE REVIEW

The results of our model contribute to the literature on sport economics in three dimensions: First, we show that the effects of a home advantage depend on the rules of the game. Second, we analyze different tiebreaking rules in two-legged ties. And third, we identify the forces behind a first or second leg home advantage.

Our analysis rests on the well-known tendency in professional soccer and other team sports that home teams win more often than visiting teams; see, for example, Courneya and Carron (1992), Nevill (1999), or Pollard and Pollard (2005) for surveys in sports in general and Clarke and Norman (1995), Pollard (1986, 2006, 2008), Carmichael and Thomas (2005), Clarke (2005), Pollard and Gómez (2014a, 2014b), Ponzio and Scoppa (2018), and Van Damme and Baert (2018) for evidence in professional soccer. Whereas the existence of this home advantage is well established, the causes that attribute to the home advantage have been difficult to identify. Three main factors have been identified and empirically analyzed in the literature: familiarity of home grounds, travel effects, or crowd effects; see, for example, Courneya and Carron (1991), Pace and Carron (1992), Agnew and Carron (1994), Nevill et al. (1996), and Nevill (1999). The present study contributes to this literature by showing that the effects of a home advantage also depend on the rules of the game. In particular, in the context of two-legged ties, the tiebreaking procedures in place introduce an asymmetry between the home and the visiting team: A tie that is broken by extra time or a penalty shootout favors the home team and enlarges the effect the home advantage has on the win of the home team. If, however, the away goals rule applies, the visiting team is strengthened, and the implications of the home advantage are reduced.

In this sense, our results also contribute to the small theoretical literature that analyzes the effect of tiebreaking rules on teams' performances. Carrillo (2007), for example, suggests that penalty shootouts should take place before the start of extra time because this would avoid the common spectacle of two overtired teams during extra time. See also Lenten et al. (2013). Some other papers consider the design of penalty shootouts under fairness considerations, for example, Anbarci et al. (2015) or Vandebroek et al. (2018). Recently, in Jost (2021b), the away goals rule is considered as a tiebreaking rule during extra time. Whereas the analysis of that paper focuses on the competitive balance between teams, the present paper considers the effect of the away goals rule on teams' preferences of order of play in a best-of-two contest. This also implies that the home advantage is alternating between the two matches and not fixed as it is during extra time.

As regards the second leg home advantage in two-legged ties, there exist several empirical studies, which, however, show mixed results: Support for the a second leg home advantage is found by Page and Page (2007), Lidor et al. (2011), Flores et al. (2015), and Pic and Castellano (2017). However, there are also studies that do not find empirical evidence for the second leg home advantage hypothesis; see Eugster et al. (2011), Mueller-Langer and Andreoli-Versbach (2016), and Amez et al. (2020). And finally, Varela-Quintana et al. (2018) empirically show that teams that played the second leg as visitors had an increased probability of winning in normal time. Unfortunately, from a theoretical perspective, there is no contribution so far that might explain these differing results in the empirical studies. In this respect, our study contributes to this literature by showing how the home advantages of both teams together with their relative strengths and the tiebreaking rules determine a first or second leg home advantage. In this way, our results support a recent study by Waquil et al. (2020). They empirically study the influence of teams' abilities for reaching the next round in the Brazil Cup, a two-legged contest with three tiebreaking procedures: goal difference, away goals rule, and penalty shootout in the respectively order. They show with data from 1994 to 2017 that the ability spread between teams is the main factor that explains the winner. In particular, when considering the championship in general, there is evidence that the second leg home team wins the match in approximately 63% of the matches. However, this winning percentage is only 43% in matches that were decided by the away goals rule or the penalty shootout.

Our modeling of a single match in best-of-two contest as a multi-tasking contest also relates the present paper to the literature on sabotage in contests. In this literature, a player can choose productive effort to increase the probability of winning the contest as well as sabotage effort to reduce the winning probability of his opponent. In this stream of literature, Deutscher et al. (2013) and Doğan et al. (2019) also consider a soccer tournament between two teams. Whereas, in both papers, each team can choose productive and sabotage effort, the present paper assumes that each team consists of two member which are specialized in their activities.

In addition, our analysis also contributes to the growing literature on dynamic team contests; see Fu et al. (2015) and Häfner (2017).

Whereas these two studies assume that players' ordering is fixed, we focus on the ordering of teams in the dynamics. In this sense, our study is closely related to the article by Fu and Lu (2020). Different to their model, however, we study teams' performance in a multi-tasking environment in which teams can choose both their defending and attacking efforts.

3 | THE BASIC MODEL

We consider a best-of-two contest between two soccer teams $i \in \{1,2\}$. We assume without loss of generality that Team 1 has a home advantage in the first match and Team 2 in the second match. Similar to Jost (2021a, 2021b), we concentrate on the two central team activities, defending and attacking, and assume that for each match $m \in \{1,2\}$, both teams choose their attacking and defending efforts (a_{im}, d_{im}) .⁵ The probability that team i scores a goal in match m depends on four factors; its attacking effort a_{im} , the ability α_i of its own forwards, the defending effort d_{jm} of the opponent team j , and the ability δ_j of the opposing defenders, $j \neq i$. The higher the attacking effort a_{im} or the ability α_i of team i 's forwards, the higher the probability to score a goal with decreasing marginal probability. On the other hand, the higher the defending effort d_{jm} or the ability δ_j of the opposing team's defenders, the lower the probability that team i can score a goal with increasing marginal probability. In particular, we apply the following success function, see Tullock (1980), and assume that

$$p_{im}(a_{1m}, d_{1m}, a_{2m}, d_{2m}) = \frac{\alpha_i a_{im}}{\alpha_i a_{im} + \delta_j d_{jm}}$$

is the probability that team i scores a goal in match m . Defining the relative strength of team i 's offensive as the ratio of its forwards quality α_i to the quality δ_j of the opposing defenders, $r_i = \alpha_i / \delta_j$, we denote team i as being offensive relative to its opponent j if $r_i > 1$ and as being defensive if $r_i < 1$. Two cases are of particular interest: Both teams are homogenous with identical strength, $r_1 = r_2$, or Team 1 is uniformly stronger than its opponent, $r_1 = 1/r_2 \geq 1$, the case of heterogeneous teams.

Marginal effort cost for both teams and both activities depend on whether a team is playing at its home field or not. If team i is the home team, it benefits from its home advantage and has marginal effort cost of $1 - b_i$, $b_i \in [0,1)$. At the same time, the other team j that is the away team suffers for team i 's home advantage and has marginal effort cost of $1 + b_j$. We call b_i the home advantage of team i .

Let g_{im} be the number of goals scored by team $i \in \{1,2\}$ in match $m \in \{1,2\}$. To focus on the main drivers of the discouragement effect and to keep the analysis tractable, note that we implicitly assumed that only one goal may be scored per team in one match, $g_{im} \in \{0,1\}$.⁶

Hence, each match has four possible outcomes $(g_{1m} - g_{2m}) \in \{(0-0), (1-1), (1-0), (0-1)\}$. The winner w_m of match m then is the team that scored more goals than the other team, that is,

$$w_m = \begin{cases} 1 & \text{if } g_{1m} > g_{2m} \\ 2 & \text{if } g_{2m} > g_{1m} \\ 0 & \text{if } g_{1m} = g_{2m} \end{cases}$$

The case of a tie, $w_m = 0$, where the number of goals in match m is equal, covers two outcomes, $(0-0)$ and $(1-1)$. For the overall contest with the two matches, we assume that the winning team $w \in \{1,2\}$ is determined by the aggregate number of goals in both matches, that is,

$$w = \begin{cases} 1 & \text{if } g_{11} + g_{12} > g_{21} + g_{22} \\ 2 & \text{if } g_{11} + g_{12} < g_{21} + g_{22} \end{cases}$$

In case, the aggregate score is tied after the two matches, $g_{11} + g_{12} = g_{21} + g_{22}$, we assume in the basic model that the tie is decided by drawing lots with equal chances.

For both matches in the basic model, both teams decide on their effort choices simultaneously, taking into account the number of goals scored by each team up to that point: For the first match, each team i chooses its efforts (a_{i1}, d_{i1}) independent of its opponents' choices (a_{j1}, d_{j1}) ; for the second match, team i chooses its efforts (a_{i2}, d_{i2}) also independent of its opponents tactic (a_{j2}, d_{j2}) but taking into account the outcome of the first match $(g_{11} - g_{21})$.

The losing team of the best-of-two contest values this outcome with 0. The winning team, however, values the contest prize with $V > 0$. With loss of generality, we normalize V to one, $V = 1$.⁷ Both teams are risk-neutral. Their objective is to maximize the expected prize net of effort costs.

4 | EQUILIBRIUM PLAY

We analyze the optimal strategies of both teams in the best-of-two contest using the subgame perfect equilibrium concept. Depending on the score of the first match, we first discuss equilibrium play in each of the subgames in the second match. We then turn to the play in the first match.

4.1 | Play in the second match

Let $(a_{12}, d_{12}, a_{22}, d_{22})$ be a tactic of both teams in the second match. Then the probability that team $i \in \{1,2\}$ wins the second match is given by $\text{Prob}(w_2 = i) = \text{Prob}(g_{i2} = 1) \cdot \text{Prob}(g_{j2} = 0)$, and the probability that the second match ends with a tie, $w_2 = 0$, with no or two goals in total is given by the corresponding counter probability.

Of course, when team $i \in \{1,2\}$ chooses its attacking and defending effort in the second match, it takes the outcome of the first match into account. We therefore consider its tactic $a_{i2}(w_1)$ and $d_{i2}(w_1)$ in each of the following three possible scenarios: the first match ends with a tie, $w_1 = 0$, the first match has a clear winner, either Team 1 for $(g_{11} - g_{21}) = (1-0)$ or Team 2 for $(g_{11} - g_{21}) = (0-1)$.

4.1.1 | Tie in the first match, $w_1 = 0$

Given a tie in the first match, the probability that team $i \in \{1,2\}$ wins the contest is given by

$$\text{Prob}(w = i | w_1 = 0) = \text{Prob}(w_2 = i) + \frac{1}{2} \cdot \text{Prob}(w_2 = 0).$$

Team i wins the best-of-two contest if it wins the second match or, in case of a tie, is drawn as the winner with a chance of 50%. Given these probabilities, the expected payoffs of both teams follow directly.

Proposition 1. Suppose that the first match ends with a tie and the second match is at Team 2's court. Then equilibrium play in the second match results in the following expected payoffs,

$$\begin{aligned} \pi_{12}^*(0) &= \frac{(1-b_2)^2}{2} \left(\frac{((1+b_2) + r_1(1-b_2))^2 + r_1^2((1-b_2) + r_2(1+b_2))^2}{((1+b_2) + r_1(1-b_2))^2((1-b_2) + r_2(1+b_2))^2} \right), \\ \pi_{22}^*(0) &= \frac{(1+b_2)^2}{2} \left(\frac{((1-b_2) + r_2(1+b_2))^2 + r_2^2((1+b_2) + r_1(1-b_2))^2}{((1+b_2) + r_1(1-b_2))^2((1-b_2) + r_2(1+b_2))^2} \right). \end{aligned}$$

Several remarks are worth noting: First, in the absence of a home advantage, $b_2 = 0$, homogeneous teams with $r_1 = r_2 = r$ would have identical expected payoffs equal to $(1+r^2)/2(1+r)^2$. This is because in case of a tie in the first match, no team has a disadvantage or advantage in the second match. Since both teams are identical in strength and no home advantage exists, payoffs have to be equal. Second, the presence of Team 2's home advantage in the second match encourages its activities but discouraged the ones of its opponent. Since the marginal benefits of an increase in attacking by one team are identical to the marginal benefits of an increase in defending by its opponent, the home advantage b_2 implies that the optimal attacking and defending efforts of Team 2 are higher than the defending and attacking efforts of Team 1 due to lower, respectively, higher, marginal effort costs. Proposition 1 shows that the factor is equal to the relative marginal effort costs $(1+b_2)/(1-b_2) > 1$. Whether this results in a higher scoring probability for Team 2 then depends on the relative strength of both teams. Of course, if Team 1 is weaker than Team 2, its scoring probability is lower due to the home advantage of Team 2. However, the scoring probability of Team 1 in equilibrium is higher than the scoring probability of Team 2, $\text{Prob}(g_{12} = 1) > \text{Prob}(g_{22} = 1)$, whenever Team 1 is sufficiently stronger than Team 2, $r_1 > r_2(1+b_2)^2/(1-b_2)^2$. In fact, Team 1's strength relative to the one of Team 2 has to outbalance the disadvantage due to its higher relative marginal effort costs. Given this is the case and Team 1 is an offensive team, its defending activities are stronger than its attacking activities, simply because the quality of its offense has to be supported with sufficient defending effort to make the team the overall winner. In fact, simple calculation shows that $d_{12}^*(0) > a_{12}^*(0)$. As a response, Team 2 will exert more attacking than defending effort,

$a_{22}^*(0) > d_{22}^*(0)$. Concerning the winning probabilities for the second match in equilibrium, the higher the home advantage, the more likely is a win of Team 2 independent of the relative strength of its offense.⁸ Moreover, the equilibrium probability that the match ends with a tie is decreasing in Team 2's home advantage if its offense is sufficiently high. Of course, the winning probability of Team 1 and its expected payoffs are decreasing in Team 2's home advantage, whereas the winning probability and the expected payoff of the home Team 2 are increasing in b_2 .

4.1.2 | Team 1 won the first match, $w_1 = 1$

Given a win in the first match by Team 1, the only possibility for Team 2 to win the overall contest is to score a goal in the second match. In all other cases, Team 1 is the overall winner. For given tactics $(a_{12}, d_{12}, a_{22}, d_{22})$, the probability that team $i \in \{1, 2\}$ then wins the contest is given by

$$\begin{aligned} \text{Prob}(w = 1 | w_1 = 1) &= \text{Prob}(w_2 = 1) \\ &\quad + \text{Prob}(w_2 = 0) + \frac{1}{2} \cdot \text{Prob}(w_2 = 2), \\ \text{Prob}(w = 2 | w_1 = 1) &= \frac{1}{2} \cdot \text{Prob}(w_2 = 2). \end{aligned}$$

The expected payoffs of both teams then follow directly.⁹

Proposition 2. Suppose Team 1 won the first match, $(1 - 0)$, and the second match is at Team 2's court.

1. If Team 2 is sufficiently offensive, $r_2 \geq r_1(1 - b_2)^2 / (1 + b_2)^2$, equilibrium play in the second match results in the following expected payoffs,

$$\begin{aligned} \pi_{12}^*(1) &= 1 - \frac{r_2(1 + b_2)^2 (3r_1(1 - b_2)^2 + r_2(1 + b_2)^2 + 2(1 - b_2)(1 + b_2)(1 + r_1 r_2))}{2((1 + b_2) + r_1(1 - b_2))^2 ((1 - b_2) + r_2(1 + b_2))^2}, \\ \pi_{22}^*(1) &= \frac{r_2(1 + b_2)^2 (r_2(1 + b_2)^2 - r_1(1 - b_2)^2)}{2((1 + b_2) + r_1(1 - b_2))^2 ((1 - b_2) + r_2(1 + b_2))^2}. \end{aligned}$$

2. If Team 2 is not sufficiently offensive, $r_2 \leq r_1(1 - b_2)^2 / (1 + b_2)^2$, Team 2 gives up with positive probability, resulting in equilibrium payoffs,

$$\pi_{12}^*(1) = 1 - \frac{\sqrt{r_2 r_1} (1 - b_2) + r_2 (1 + b_2)}{2(\sqrt{r_2 r_1} + 1)^2 (1 - b_2)}, \pi_{22}^*(1) = 0.$$

The proposition distinguishes between two cases. Relative to the winning Team 1 of the first match, Team 2 with its home advantage is either sufficiently offensive to actively win the contest, or not sufficiently offensive to be discouraged and to give up the match.

Consider first the case in which Team 2 is sufficiently offensive and does not give up. Consulting Team 2's payoff, this requires that its payoff is positive as long as either its strength or its home advantage is sufficiently high compared to the strength of Team

1, $r_2(1 + b_2)^2 / (1 - b_2)^2 \geq r_1$. If this condition is satisfied, Team 2 exerts more effort in its offense, $d_{22}^*(1) > a_{22}^*(1)$ whenever Team 1's offense is weaker than Team 2's defense, that is, $r_1 < 1$. On the other hand, if Team 1's attacking abilities are higher than Team 2's defending abilities, $r_1 > 1$, Team 1 exerts more attacking than defending effort, $a_{12}^*(1) > d_{12}^*(1)$. In both cases, Team 1 then is more likely to win the second match whenever

$$r_1 > \frac{(1 + b_2) r_2 (1 + b_2) - (1 - b_2)}{(1 - b_2) r_2 (1 + b_2) + (1 - b_2)}.$$

In the second case, when Team 2 is not sufficiently offensive, $r_2 < r_1(1 - b_2)^2 / (1 + b_2)^2$, its expected payoff has to be zero. Giving up with certainty cannot be optimal. This would imply that Team 1's optimal response would be to play with some small attacking effort to win the second match, but then it is not optimal for Team 2 to give up. Hence, Team 2 either gives up with some probability or chooses positive attacking and defending efforts $(a_{22}^*(1), d_{22}^*(1))$. The proposition then shows that the probability of giving up is zero for $r_2(1 + b_2)^2 = r_1(1 - b_2)^2$ and increasing if the home advantage gets smaller or Team 2 gets weaker. Moreover, if Team 2 does not give up, Team 2 chooses higher effort levels than Team 1, that is, $a_{22}^*(1) > a_{12}^*(1)$ and $d_{22}^*(1) > d_{12}^*(1)$, since $\sqrt{r_2/r_1} < 1$. The probability that Team 2 then wins the second match is decreasing in the home advantage b_2 or the strength r_2 of Team 2.

4.1.3 | Team 2 won the first match, $w_1 = 2$

This scenario is different to the last one in that the role of each team is switched: The only possibility for Team 1 to win the contest is to win the second match, whereas in all other cases, Team 2 is the overall winner:

$$\begin{aligned} \text{Prob}(w = 1 | w_1 = 2) &= \frac{1}{2} \cdot \text{Prob}(w_2 = 1), \\ \text{Prob}(w = 2 | w_1 = 2) &= \text{Prob}(w_2 = 2) + \text{Prob}(w_2 = 0) + \frac{1}{2} \cdot \text{Prob}(w_2 = 1). \end{aligned}$$

This scenario, however, is similar to the last one in that Team 1 as the away team has higher effort costs, whereas the home Team 2 has lower effort costs due to its home advantage. Maximizing expected payoffs then gives the following¹⁰:

Proposition 3. Suppose Team 2 won the first match, $(0 - 1)$ and the second match is at Team 2's court.

1. If Team 1 is sufficiently offensive, $r_1 \geq r_2(1 + b_2)^2 / (1 - b_2)^2$, equilibrium play in the second match results in the following expected payoffs,

$$\begin{aligned} \pi_{12}^*(2) &= \frac{r_1(1 - b_2)^2 (r_1(1 - b_2)^2 - r_2(1 + b_2)^2)}{2((1 + b_2) + r_1(1 - b_2))^2 ((1 - b_2) + r_2(1 + b_2))^2}, \\ \pi_{22}^*(2) &= 1 - \frac{r_1(1 - b_2)^2 (3r_2(1 + b_2)^2 + r_1(1 - b_2)^2 + 2(1 - b_2)(1 + b_2)(1 + r_1 r_2))}{2((1 + b_2) + r_1(1 - b_2))^2 ((1 - b_2) + r_2(1 + b_2))^2}. \end{aligned}$$

2. If Team 1 is not sufficiently offensive, $r_1 \leq r_2(1+b_2)^2/(1-b_2)^2$, Team 1 gives up with positive probability, resulting in equilibrium payoffs,

$$\pi_{12}^*(2) = 0, \pi_{22}^*(2) = 1 - \frac{\sqrt{r_2 r_1}(1+b_2) + r_1(1-b_2)}{2(\sqrt{r_2 r_1} + 1)^2(1+b_2)}.$$

Compared to the case in which Team 1 won the first match, Team 2 now plays less offensive if Team 1 is not sufficiently offensive, $r_1 \leq r_2(1+b_2)^2/(1-b_2)^2$, that is, $a_{22}^*(1) > a_{22}^*(2)$. In fact, Team 2 then has a good chance to win the contest even if it lost the first match. This, however, requires a strong attacking effort. In case Team 2 won the first match, it is Team 1 that has to win the second match. Since Team 1 gives up with positive probability, this requires less attacking effort by Team 2.

If, in addition, the defense of Team 2 is stronger than the offense of Team 1, $r_1 \leq 1$, Team 2 exerts more effort in its defense if it lost instead of won the first match, that is, $d_{22}^*(1) > d_{22}^*(2)$. If it lost the first match, a goal by Team 1 would make Team 2 the overall loser. Hence, avoiding a goal by Team 1 is necessary. In the second case, defending is not that essential because Team 2 already won the first match and the offense of Team 1 is not as good as its defense, $r_1 \leq 1$.

On the other hand, if Team 1 is sufficiently offensive, $r_1 \geq r_2(1+b_2)^2/(1-b_2)^2$ and its offense stronger than the defense of Team 2, $r_1 \geq 1$, then $a_{12}^*(1) > a_{12}^*(2)$. That is, Team 1 invests more in the second match in attacking effort when it won the first match than when this match was lost. In the latter case, Team 1 is strong enough not to give up but will lose the overall match if the other team scores. Attacking only therefore is not beneficial. This is different in the first case, where Team 2 wins the overall contest if it scores. Since its offense is better than Team 2's defense, playing more offensive is beneficial.

If, however, in the same scenario Team 1's offense is weaker than the defense of Team 2, $r_1 \leq 1$, Team 1 invests more in the second match in defending effort, that is, $d_{12}^*(1) > d_{12}^*(2)$. The intuition is similar as before: when it won the first match, Team 1 is the overall winner when Team 2 does not score in the second match. Since the defense of Team 1 is better than the offense of Team 2, $1 \geq r_1 \geq r_2$, it is beneficial for Team 1 to avoid a goal. Again, this is different in the latter case where Team 1 lost the first match. Defending in this case does not help to become the overall winner.

4.2 | Play in the first match

Given the equilibrium play in the second match we will now consider teams' play in the first match where Team 1 is the home team. Let (a_{i1}, d_{i1}) be the tactic of team $i \in \{1, 2\}$ for the first match. Then the probability that team i wins the first match is given by $\text{Prob}(w_1 = i) = \text{Prob}(g_{i1} = 1) \cdot \text{Prob}(g_{j1} = 0)$ and the probability that the first match ends as a tie, $w_1 = 0$, is given by the corresponding counter

probability. Using the equilibrium payoffs in the second match, the expected payoffs of both teams for the first match then are given by

$$\begin{aligned} \pi_1 &= \sum_{i=0,1,2} \text{Prob}(w_1 = i) \cdot \pi_{12}^*(i) - a_{11}(1-b_1) - d_{11}(1-b_1) \\ &= \pi_{12}^*(0) + \sum_{i=1,2} \text{Prob}(w_1 = i) \cdot \Delta\pi_{12}^*(i, 0) - (a_{11} + d_{11})(1-b_1), \\ \pi_2 &= \sum_{i=0,1,2} \text{Prob}(w_1 = i) \cdot \pi_{22}^*(i) - a_{21}(1+b_1) - d_{21}(1+b_1) \\ &= \pi_{22}^*(0) + \sum_{i=1,2} \text{Prob}(w_1 = i) \cdot \Delta\pi_{22}^*(i, 0) - (a_{21} + d_{21})(1+b_1), \end{aligned}$$

with $\Delta\pi_{12}^*(i, 0) = \pi_{12}^*(i) - \pi_{12}^*(0)$ and $\Delta\pi_{22}^*(i, 0) = \pi_{22}^*(i) - \pi_{22}^*(0)$.

Proposition 4. Given the equilibrium play in the second match and no team gives up in the first match, the equilibrium payoffs of team i , $i = 1, 2$, are¹¹

$$\begin{aligned} \pi_i^* &= \pi_{i2}^*(0) + \frac{r_i x_{i1}}{1 + r_i x_{i1}} \left(1 - \frac{r_j x_{j1}}{1 + r_j x_{j1}} \right) \Delta\pi_{i2}^*(i, 0) \\ &\quad + \frac{r_j x_{j1}}{1 + r_j x_{j1}} \left(1 - \frac{r_i x_{i1}}{1 + r_i x_{i1}} \right) \Delta\pi_{i2}^*(j, 0) - (a_{i1}^* + d_{i1}^*)(1 + \tau_i), \end{aligned}$$

with $\tau_i = (-1)^i b_1$ and

$$x_{i1} = - \frac{(1 - \tau_i) \Delta\pi_{i2}^*(i, 0) - r_j x_{j1} \Delta\pi_{i2}^*(j, 0)}{(1 + \tau_i) \Delta\pi_{j2}^*(i, 0) - r_j x_{j1} \Delta\pi_{j2}^*(j, 0)}.$$

The optimal attacking and defending efforts of both teams are given by outbalancing marginal benefits with marginal costs. Consider, for example, Team 1. Its marginal costs for both effort choices are given by $(1-b_1)$, which reflects its home advantage. Its marginal benefits depend on the particular activity it chooses:

- In case of attacking, the marginal benefits of an increase in effort a_{11} results from an increase of its marginal scoring probability $r_1 x_{11} / a_{11}(1+r_1 x_{11})^2$, and the resulting benefits from the second match,

$$\frac{r_2 x_{21}}{1 + r_2 x_{21}} (\pi_{12}^*(0) - \pi_{12}^*(2)) + \left(1 - \frac{r_2 x_{21}}{1 + r_2 x_{21}} \right) (\pi_{12}^*(1) - \pi_{12}^*(0)).$$

These benefits in the second match depend on whether Team 2 scores a goal or not, which happens with probability $r_2 x_{21} / (1 + r_2 x_{21})$. In the first case, the benefits of scoring a goal for Team 1 are to avoid a loss and to achieve a tie, $\pi_{12}^*(0) - \pi_{12}^*(2)$. In the second case, the benefits of scoring a goal for Team 1 are to win instead of playing tie, $\pi_{12}^*(1) - \pi_{12}^*(0)$.

- In case of defending, the marginal benefits of an increase in effort d_{11} result from a decrease of its opponents marginal scoring probability $r_2 / d_{11}(1+r_2 x_{21})^2$, and the resulting benefits of Team 2 from the second match,

$$\frac{r_1 x_{11}}{1+r_1 x_{11}}(\pi_{22}^*(0) - \pi_{22}^*(1)) + \left(1 - \frac{r_1 x_{11}}{1+r_1 x_{11}}\right)(\pi_{22}^*(2) - \pi_{22}^*(0)).$$

These benefits in the second match depend on whether Team 1 scores a goal or not which now happens with probability $r_1 x_{11}/(1+r_1 x_{11})$. In the first case, the benefits for Team 2 are to play a tie instead of loosing, $\pi_{22}^*(0) - \pi_{22}^*(1)$. In the latter case, the benefits for Team 2 are to win instead of reaching a tie, $\pi_{22}^*(2) - \pi_{22}^*(0)$.

5 | OTHER TIEBREAKING RULES

In the basic model, we assumed that if the aggregate score is tied after two matches, $g_{11} + g_{12} = g_{21} + g_{22}$, the tie is decided by a coin toss with equal chances. We now extend this basic model and consider three other rules for breaking the tie, the away goals rule, the extra time rule, and a penalty shoot-out. Since in many sports, in particular in soccer tournaments, these tiebreaking rules are used in sequence, we also analyze these rules in sequence.

5.1 | The away goals rule

The first additional tiebreaking rule we consider is the away goals rule. In case the aggregate score is tied after the two matches, $g_{11} + g_{12} = g_{21} + g_{22}$, the away goals rule determines that the team that scored more away goals is the winning team: If $g_{12} > g_{21}$, Team 1 is the winner. Otherwise, if $g_{21} > g_{12}$, the winner is Team 2. If away goals are equal, $g_{21} = g_{12}$, we still assume that the tie is decided by equal chances.

In our model, the away goals rule only applies if the first match ended with a tie. Different to our analysis of the basic model, we therefore have to differentiate between a tie in the first match without any goal, $(g_{11} - g_{21}) = (0 - 0)$, and a tie with two goals, $(g_{11} - g_{21}) = (1 - 1)$. The away goals rule then has the following implications on the second match.

5.1.1 | Tie in the first match with no goal, $(g_{11} - g_{21}) = (0 - 0)$

In this scenario, Team 2 wins the best-of-two contest if it wins the second match, whereas Team 1 is the overall winner as long as it scores a goal, independent of Team 2's scoring, that is,

$$\begin{aligned} \text{Prob}(w = 1 | (g_{11} - g_{21}) = (0 - 0)) &= \text{Prob}((g_{12} - g_{22}) = (1 - 0)) \\ &\quad + \text{Prob}((g_{12} - g_{22}) = (1 - 1)) \\ &\quad + \frac{1}{2} \cdot \text{Prob}((g_{12} : g_{22}) = (0 - 0)), \\ \text{Prob}(w = 2 | (g_{11} - g_{21}) = (0 - 0)) &= \text{Prob}((g_{12} : g_{22}) = (0 - 1)) \\ &\quad + \frac{1}{2} \cdot \text{Prob}((g_{12} - g_{22}) = (0 - 0)). \end{aligned}$$

Given these probabilities and the corresponding expected payoffs of both teams, the effect of the away goals rule on equilibrium play in the second match is as follows. If we compare these efforts with the ones in Proposition 1, then $a_{12}^*(0 - 0) > a_{12}^*(0)$ and $d_{12}^*(0 - 0) < d_{12}^*(0)$, that is, Team 1 chooses a higher offensive and a lower defensive effort in the presence of the away goals rule. The reason is straightforward: Different to the basic model, scoring a goal in the second match now makes Team 1 the overall winner for sure. In particular, Team 1 would be the winner of the contest after a $(1 - 1)$ in the second match. Hence, an increase in offensive activities implies higher marginal benefits: It not only increases the marginal probability of winning in case, the other team does not score a goal, but also in case the opponent scores in the second match. The latter effect is positive as a $(1 - 1)$ in the second match implies the overall win of the contest. Similarly, the marginal benefits of an increase of its defensive activities are reduced compared to scenario without the away goals rule. Of course, Team 2 responds in equilibrium with higher defensive and lower offensive efforts, $d_{22}^*(0 - 0) > d_{22}^*(0)$ and $a_{22}^*(0 - 0) < a_{22}^*(0)$.

To isolate the effect of the away goals rule from the home advantage for the equilibrium payoffs, suppose that $b_2 = 0$. Comparing the equilibrium payoffs $\pi_{12}^*(0 - 0)$ and $\pi_{22}^*(0 - 0)$ then shows that Team 1 is better off than Team 2, whenever $r_1 > r_2/(1+r_2)$. This inequality is true whenever Team 1 is strong in its offense, $r_1 \geq 1$. In particular, if both teams are equally strong, $r_1 = r_2$, Team 1's payoff is higher than the one of Team 2 by a factor of $(2r^2 + 1)$.

5.1.2 | Tie in the first match with two goals, $(g_{11} - g_{21}) = (1 - 1)$

The difference between this scenario and the last one concerns the winning probabilities of both teams. Team 2 now wins the best-of-two contest as long as Team 1 does not score, whereas Team 2 now is the overall winner only if it wins the second match:

$$\begin{aligned} \text{Prob}(w = 1 | (g_{11} - g_{21}) = (1 - 1)) &= \text{Prob}((g_{12} - g_{22}) = (1 - 0)) \\ &\quad + \frac{1}{2} \cdot \text{Prob}((g_{12} - g_{22}) = (1 - 1)), \\ \text{Prob}(w = 2 | (g_{11} - g_{21}) = (1 - 1)) &= \text{Prob}((g_{12} - g_{22}) = (0 - 1)) \\ &\quad + \text{Prob}((g_{12} - g_{22}) = (0 - 0)) \\ &\quad + \frac{1}{2} \cdot \text{Prob}((g_{12} - g_{22}) = (1 - 1)). \end{aligned}$$

Given these probabilities and the corresponding expected payoff of both teams, teams' equilibrium play in the presence of the away goals rule follows. If we compare these equilibrium efforts with the ones in Proposition 1, the effect of the away goals rule on equilibrium play is similar to the scenario of a $(0 - 0)$, that is, Team 1 plays more offensive and less defensive in the presence of the away goals rule, $a_{12}^*(1 - 1) > a_{12}^*(0)$ and $d_{12}^*(1 - 1) < d_{12}^*(0)$. The reason, however, is different. In case of $(1 - 1)$, Team 2 is more likely to win after a $(0 - 0)$, and, vice versa, Team 1 is more likely to loose. In particular, Team 1 would be the loser of the best-of-two contest after a $(0 - 0)$ in the second match. Hence, Team 1's benefits are increasing in its

offensive activities under the away goals rule whereas its benefits are decreasing in its defensive activities: An increase in offensive effort not only increases the marginal probability of winning but also reduces the marginal probability of a tie with no goals in the second match. The latter effect is positive as a (0–0) in the second match implies the best-of-two contest is lost.

The optimal response of Team 2 then implies $d_{22}^*(1-1) > d_{22}^*(0)$ and $a_{22}^*(1-1) < a_{22}^*(0)$, that is, Team 2 chooses higher defensive and lower offensive efforts in equilibrium. Of course, if Team 2 is stronger than Team 1, its defending effort is higher than its attacking effort, because preventing a goal makes the team the overall winner. That is, $d_{22}^*(1-1) > a_{22}^*(1-1)$. As a response, Team 1 will invest more in attacking than defending effort, $a_{12}^*(1-1) > d_{12}^*(1-1)$, since this is the only chance to be the overall winner.

Concerning equilibrium payoffs, the effect of the away goals rule is best illustrated without a home advantage. For $b_2 = 0$, Team 2 then is better off than Team 1, that is, $\pi_{22}^*(1-1) > \pi_{12}^*(1-1)$, whenever $1+r_2 > r_1$, which is true whenever Team 1 is weak in its offense, $r_1 \leq 1$. If both teams are equally strong, $r_1 = r_2$, Team 2's payoff is higher than the one of Team 1 by a factor of $(1+2/r^2)$. Of course, $\pi_{12}^*(1-1) < \pi_{12}^*(0-0)$ and $\pi_{22}^*(1-1) > \pi_{22}^*(0-0)$.

Finally, if we compare the equilibrium efforts for the two different ties, we find that $a_{12}^*(0-0) > a_{12}^*(1-1)$ whenever $r_2 > (1-b_2)/(1+b_2)$. Since a (0–0) in the first match implies that a goal of Team 1 in the second match makes the team the overall winner, it puts more effort in its offensive activities than for a (1–1) when Team 2 is sufficiently strong to score itself. Then, $d_{22}^*(1-1) < d_{22}^*(0-0)$ and Team 2 exerts more effort in its offense after a (1–1), that is, $a_{22}^*(1-1) > a_{22}^*(0-0)$, whenever $r_1 > (1+b_2)/(1-b_2)$. If Team 1 is sufficiently strong, a (1–1) in the first match would make Team 2 the overall loser, provided that it does not score in the second match. As a consequence, $d_{12}^*(1-1) < d_{12}^*(0-0)$.

Concerning payoffs, we have the following effect of the away goals rule for teams' success in the second match:

Proposition 5. The away goals rule has the following implications on teams' equilibrium payoffs in the second match on Team 2's home field:

$$\pi_{12}^*(0-0) > \pi_{12}^*(0) > \pi_{12}^*(1-1) \text{ and } \pi_{22}^*(1-1) > \pi_{22}^*(0) > \pi_{22}^*(0-0).$$

5.1.3 | Play in the first match

Of course, different equilibrium payoffs in the second match in case of a tie imply that teams' effort choices in the first match will also change under the away goals rule. For tactics (a_{i1}, d_{i1}) of team i in the first match, the probability that the match ends as a tie, $w_1 = 0$, with no or two goals, is given by $\text{Prob}((g-g)) = \text{Prob}(g_{i1} = g) \cdot \text{Prob}(g_{j1} = g)$ for $g \in \{0,1\}$. Using the equilibrium payoffs in the second match, the expected payoff of Team 1 for the entire match is now given by

$$\begin{aligned} \pi_1 = & \text{Prob}(w_1 = 1) \cdot \pi_{12}^*(1) + \text{Prob}(w_1 = 2) \cdot \pi_{12}^*(2) \\ & + \text{Prob}((0-0)) \cdot \pi_{12}^*(0-0) + \text{Prob}((1-1)) \cdot \pi_{12}^*(1-1) - a_{11}(1-b_1) \\ & - d_{11}(1-b_1), \end{aligned}$$

and for Team 2, expected payoffs are

$$\begin{aligned} \pi_2 = & \text{Prob}(w_1 = 1) \cdot \pi_{22}^*(1) + \text{Prob}(w_1 = 2) \cdot \pi_{22}^*(2) \\ & + \text{Prob}(0-0) \cdot \pi_{22}^*(0-0) + \text{Prob}((1-1)) \cdot \pi_{22}^*(1-1) - a_{21}(1+b_1) \\ & - d_{21}(1+b_1). \end{aligned}$$

As in the basic model, marginal benefits from attacking and defending have to be identical to its marginal costs in the optimum. Marginal benefits result from the marginal increase in shooting or preventing a goal, and the corresponding payoff spread in the second match. In case of attacking, this payoff spread depends on whether the other team scores or not. In case of defending, it depends on whether the own team scores or not. Different to the basic model, however, this payoff spread now relates to whether a tie is without or with two goals.

Proposition 6. In the presence of the away goals rule, if no team gives up in the first match, the tactics of each team i in the first match are as follows:

$$\begin{aligned} a_{i1}^* = & \frac{r_i x_{i1} \left(\pi_{i2}^*(\Delta g_{11} - 0) - \frac{r_j x_{j1}}{1+r_j x_{j1}} (\pi_{i2}^*(\Delta g_{11} - 1) - \pi_{i2}^*(\Delta g_{11} - 0)) \right)}{(1+\tau_i)(1+r_i x_{i1})^2}, \\ d_{i1}^* = & \frac{r_j \left(\pi_{i2}^*(0 - \Delta g_{21}) - \frac{r_i x_{i1}}{1+r_i x_{i1}} (\pi_{i2}^*(1 - \Delta g_{21}) - \pi_{i2}^*(0 - \Delta g_{21})) \right)}{(1-\tau_i)(1+r_j x_{j1})^2}, \end{aligned}$$

with $\tau_i = (-1)^i b_1$, $\pi_{i2}^*(\Delta g_{11} - g_{21}) = \pi_{i2}^*(0 - g_{21}) - \pi_{i2}^*(1 - g_{21})$, $\pi_{i2}^*(g_{11} - \Delta g_{21}) = \pi_{i2}^*(g_{11} - 1) - \pi_{i2}^*(g_{11} - 0)$, and

$$\begin{aligned} x_{11} = & \frac{(1+b_1)\pi_{12}^*(\Delta g_{11} - 0) + r_2 x_{21} \pi_{12}^*(\Delta g_{11} - 1)}{(1-b_1)\pi_{22}^*(\Delta g_{11} - 0) + r_2 x_{21} \pi_{22}^*(\Delta g_{11} - 1)}, \\ x_{21} = & \frac{(1-b_1)\pi_{22}^*(0 - \Delta g_{21}) + r_1 x_{11} \pi_{22}^*(1 - \Delta g_{21})}{(1+b_1)\pi_{12}^*(0 - \Delta g_{21}) + r_1 x_{11} \pi_{12}^*(1 - \Delta g_{21})}. \end{aligned}$$

The resulting expected payoffs then are

$$\begin{aligned} \pi_i^* = & \pi_{i2}^*(0) + \frac{r_i x_{i1}}{1+r_i x_{i1}} \left(1 - \frac{r_j x_{j1}}{1+r_j x_{j1}} \right) \Delta \pi_{i2}^*(i, 0) \\ & + \frac{r_j x_{j1}}{1+r_j x_{j1}} \left(1 - \frac{r_i x_{i1}}{1+r_i x_{i1}} \right) \Delta \pi_{i2}^*(j, 0) - (a_{i1}^* + d_{i1}^*)(1+\tau_i). \end{aligned}$$

5.2 | Extra time

As a third tiebreaking rule, we consider extra time. This method defines an additional period of play between teams that are tied at the

end of a match. In rule, extra time is used if the away goals rule does not lead to a clear winner. Teams then play two 15-min periods of extra time, and the team that leads at the end of 30 min wins the tie.

Different to away goals rule, the extra time does not change teams' playing in the best-of-two contest in our model. In fact, similar to our modeling of the best-of-two contest, we can simply model the extra time as a match with two half periods, where each team can choose its defending and attacking activities for each half as in our basic model. There is, however, a crucial difference to our basic model, namely, the alternating home advantage. Whereas in the basic model Team 1 had a home advantage in the first match and Team 2 in the second match, Team 2 now has a home advantage in both halves of the extra time. Hence, play in the second half does not change compared to Propositions 1 to 3, and play in the first half of extra time changes by replacing $\tau_i = (-1)^i b_1$ by $\tilde{\tau}_i = (-1)^{i+1} b_2$.

Of course, having this disadvantage for Team 1 also in the first half of extra time has a tremendous influence on its playing. Figure 1 compares the equilibrium attacking and defending effort choices of Team 1 in the first match with its choice in the first half of extra time for $r_1 = r_2 = 1$ and $b_1 = b_2 = b$.

The red and green lines represent Team 1's attacking and defending efforts, respectively, if it profits from its own home advantage b , whereas the blue and black lines are its attacking and defending efforts, respectively, if it plays on Team 2's court in the first half of extra time. Note, that when Team 1 plays in the best-of-two contest first at home, both efforts are increasing in its home advantage. The opposite is true when Team 1 plays the first half of extra time at Team 2's court. Hence, the higher b , the greater the difference in its attacking and defending efforts in the two scenarios. Of course, for $b = 0$, there is no difference in playing.

5.3 | Penalty shootout

As a fourth tiebreaking rule consider the penalty shootout. In the context of our model, we assume that a tie is decided by the following

penalty shootout procedure: Each team starts with one shot from the penalty mark. As in the second match, we assume that in the penalty shootout, the shooters of Team 2 benefit from their home advantage in their kicking activities as well as its goal keeper when Team 1's shooters are kicking. The higher the home advantage b_2 , the higher the probability that Team 2's kicking and catching is successful. In particular, we assume that the home team scores with a probability of $\frac{1}{2}(1 + b_2)$, whereas the away team only has a probability of $\frac{1}{2}(1 - b_2)$ for scoring a goal.¹² If after this round of kicks the teams have still an equal number of goals, additional rounds of one shot each are used until one team scores and the other misses. The winning team then is the team that scores the most goals at the end of the shootout.

Given the scoring probabilities of both teams, the probability that away Team 1 wins the match in the first round of the penalty shootout is $(1 - b_2)(1 - \frac{1}{2}(1 + b_2))/2$, and the corresponding probability for the home Team 2 is $(1 + b_2)(1 - \frac{1}{2}(1 - b_2))/2$. The shootout then continues to an additional round with probability $(1 - b_2^2)/2$. Successive application of these probabilities then leads to the following overall winning probabilities in the penalty shootout:

$$\text{Prob}(w = 2) = \sum_{p=1}^{\infty} \left(\left(\frac{1}{2}(1 + b_2) \left(1 - \frac{1}{2}(1 - b_2) \right) \right) \left(\frac{1}{2}(1 - b_2^2) \right)^{p-1} \right),$$

$$\text{Prob}(w = 1) = \sum_{p=1}^{\infty} \left(\left(\frac{1}{2}(1 - b_2) \left(1 - \frac{1}{2}(1 + b_2) \right) \right) \left(\frac{1}{2}(1 - b_2^2) \right)^{p-1} \right).$$

Proposition 7. In the penalty shootout, the expected winning probabilities are

$$\text{Prob}(w = 2|\text{PS}) = \frac{1(1 + b_2)^2}{2(1 + b_2^2)},$$

$$\text{Prob}(w = 1|\text{PS}) = \frac{1(1 - b_2)^2}{2(1 + b_2^2)}.$$

Note that for $b_2 = 0$, the winning probabilities of both teams are identical and equal to $\frac{1}{2}$. This implies that there is no difference

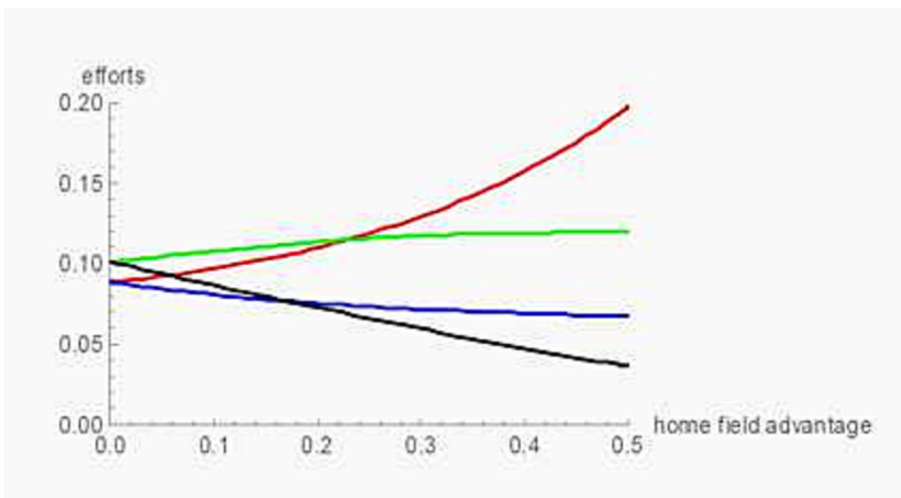


FIGURE 1 Attacking and defending effort choices of Team 1 in the first match and in the first half of extra time.

between a penalty shooting and a coin toss as tiebreaking rules in the absence of a home advantage.

6 | FIRST HOME OR FIRST AWAY?

In principle, the question whether a team in a best-of-two contest is better off to play the first match at home or away is easy to answer: If we denote the overall equilibrium payoff π_i^* of team i as $\pi_i^*((r_1, b_1); (r_2, b_2))$ in case the first match is on Team 1's home field, we simply have to switch roles and take Team 2 as the team playing first at home. We then calculate in this scenario team i 's equilibrium payoff by $\pi_j^*((r_2, b_2); (r_1, b_1))$ and compare it with the one in the original scenario with Team 1 as the home team in the first match. Then team i prefers to play first at home instead of away, whenever

$$\pi_i^*((r_i, b_i); (r_j, b_j)) \geq \pi_j^*((r_j, b_j); (r_i, b_i)).$$

In the following, we argue that three factors determine the relation between these two payoffs: the size of the home advantage, the relative strengths of the teams, and the tiebreaking rule.

We start our discussion with the tiebreaking rule of the basic model. Suppose that in case of a tie after the second match, the winner is determined by lot. Then we can establish the following result:

Proposition 8. Suppose that a tie after two matches is decided by drawing lots with equal chances. In the absence of a home advantage, $b_1 = b_2 = 0$, teams are indifferent whether to play first home or away,

$$\pi_i^*((r_i, 0); (r_j, 0)) = \pi_j^*((r_j, 0); (r_i, 0)).$$

The reason for this result is straightforward: Without any home advantage, equilibrium efforts of both teams in the second match are independent of whether the match is at Team 1's or Team 2's home field since marginal effort costs of both teams in both scenarios are identical. Hence, the equilibrium payoffs in the second match are also independent of the order of play. But then the equilibrium payoffs in the first match are independent which team plays first at home, that is, the order of play has no influence on teams' overall equilibrium payoffs.

This result changes dramatically when we assume that one team has a positive home advantage. Since it is too complicated to calculate the optimal efforts and equilibrium payoffs in a closed form solution for the general case, we concentrate our discussion in the following on the two cases of homogeneous teams with $r_i = r_j$ and heterogeneous teams with $r_i = 1/r_j$.

Proposition 9. Suppose that a tie after two matches is decided by drawing lots with equal chances. In the presence of a home advantage, $b_1 \geq 0, b_2 \geq 0$, homogeneous

and heterogeneous teams prefer to play first away and then home,

$$\pi_i^*((r_i, b_i); (r_j, b_j)) > \pi_j^*((r_j, b_j); (r_i, b_i)),$$

for $r_i = r_j$ and $r_i = 1/r_j \geq 1$.

To see this result most clearly, consider two teams with identical strength, $r_1 = r_2 = r$ and assume that only Team 1 has a home advantage, $b_1 > 0, b_2 = 0$. Suppose that the first match is on Team 1's home field. From our discussion of the second match, we know that the team that lost the first match has zero payoff in the second match. In case of a tie in the match, no team has an advantage in the second match. But these payoff structures imply that in the first match, both teams have identical marginal benefits and only their marginal effort costs differ. In particular, Team 1's effort costs in the first match are linearly decreasing in its home advantage. Note that this implies that Team 1's efforts in the first match are higher than the ones of Team 2, and, therefore, also its winning probability.

If we compare this play to the one in a scenario in which the first match is on Team 2's home field, Team 1 is better off for two reasons: First, in this scenario, marginal effort costs are identical in the first match, but teams' marginal benefits differ. This is because in the second match, Team 1 profits from its home advantage, independent of the outcome in the first match. More important, however, is the fact that its payoffs in the second match are not only increasing linearly in its home advantage, but in a steeper way: In case of tie in the first match by $(b_1 + 1)^2/4$, in case of a loss by $b_1(b_1 + 1)^2/8$, and in case of a win in the first match, Team 1 receives its highest payoff anyway because Team 2 gives up with increasing probability in b_1 . In all three cases, Team 1 benefits from its home advantage and chooses higher efforts than its opponent and wins with a higher probability. Second, the higher expected payoff of Team 1 from the second match also implies that its effort choices in the first match are higher than the ones of Team 2. Team 1 therefore also has a higher probability to win the first match which makes Team 2's giving up even more likely. In sum, Team 1 benefits more from its home advantage when the first match is away and not on its home field, as Figure 2 shows for $r = 1$.

The blue line represents Team 1's payoff $\pi_2^*((1,0); (1, b_1))$ in case the first match is away, the red line is its payoff $\pi_1^*((1, b_1); (1,0))$ in the opposite case. Figure 2 illustrates that the payoff difference for Team 1 between playing first away than first home is higher the higher its home advantage. Of course, both payoffs are equal for $b_1 = 0$, as stated in the proposition above and equal to the ones for Team 2 since both teams are identical. The figure also shows that Team 2's payoffs in the two scenarios decline with Team 1's home advantage. The green line is $\pi_2^*((1, b_1); (1,0))$ and the black line $\pi_1^*((1,0); (1, b_1))$. Three points are worth noting: First, as Team 1, Team 2 prefers to play first away instead of first home. This follows directly from the fact that teams are playing a sequential rent seeking game where the gain of one party is the loss of the other party. Second, if the home advantage of Team 1 gets sufficiently high,

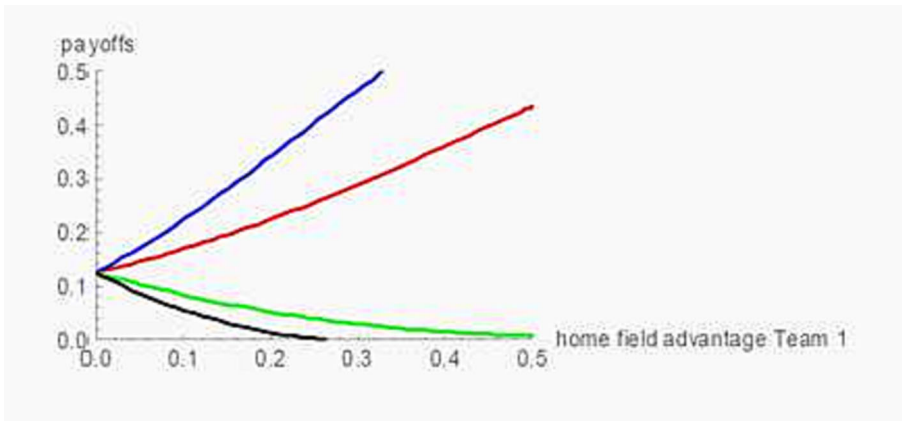


FIGURE 2 Homogeneous teams' payoffs in case first home and first away.

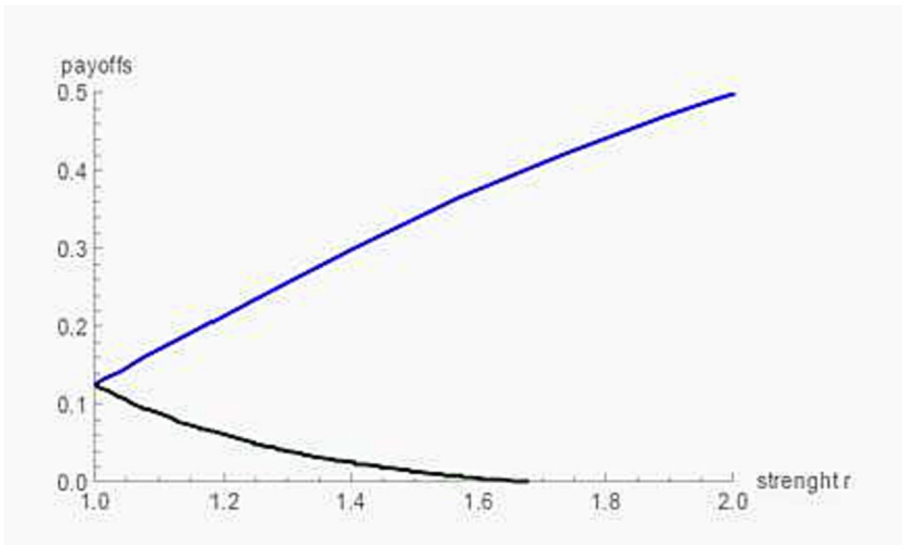


FIGURE 3 Heterogeneous team's payoffs in the absence of a home advantage.

$b_1 \geq \bar{b}_1$, Team 2 gives up the entire contest because its overall payoff is negative. Of course, the critical value \bar{b}_1 is higher, if the first match is on Team 1's home field than if Team 2 plays first home. And third, if Team 2 also benefits from a positive home field bias, $b_2 > 0$, the critical value \bar{b}_1 increases. Similar to the argumentation above, the preference of Team 2 to play first away than home does not change.

The case of heterogeneous teams with $r_i = 1/r_j = r \geq 1$ is similar to the case of homogeneous teams with the following modifications: Even if there exists no home advantage, the weaker Team 2 gives up the overall contest, if it is too weak, as Figure 3 shows.

In Figure 3, the black line represents Team 2's payoff and the blue line the one of Team 1. It shows that if Team 1 is too strong, $r \geq \bar{r}$, Team 2's payoff gets negative. Of course, if we introduce a home advantage for one team, this critical value \bar{r} would decrease in b_1 and increase in b_2 .

Our result that, in the presence of a home advantage, teams prefer to play first away and then at home is strengthened if we introduce an extra time or a penalty shootout as tiebreaking rules. In both cases, the home team of the second match either benefits from its own home advantage and/or avoids that its opponent benefits from its home advantage. In both cases, the home team of the second

match increases its probability of winning the best-of-two contest and therefore its overall payoff.

Things are different, however, if we consider the away goals rule as tiebreaker. In fact, the away goals rule introduces an away advantage into the contest. To see this most clearly, suppose that there exists no home advantage, $b_1 = b_2 = 0$. Then Proposition 8, stating that teams are indifferent whether to play first away or home, no longer holds true.

Proposition 10. Suppose that a tie after two matches is decided first by the away goals rule and then by drawing lots with equal chances. In the absence of a home advantage, $b_1 = b_2 = 0$, the stronger team prefers to play first at home and the weaker team first away,

$$\pi_1^*((1,0);(r,0)) - \pi_2^*((r,0);(1,0)) \begin{cases} > 0 & \text{if } r < 1 \\ < 0 & \text{if } r > 1 \end{cases}$$

Figure 4 illustrates this scenario.

The blue line represents the difference in payoffs $\pi_1^*((1,0);(r,0)) - \pi_2^*((r,0);(1,0))$ for Team 1 and the brown line the

FIGURE 4 Team's payoffs difference for first home versus first away.

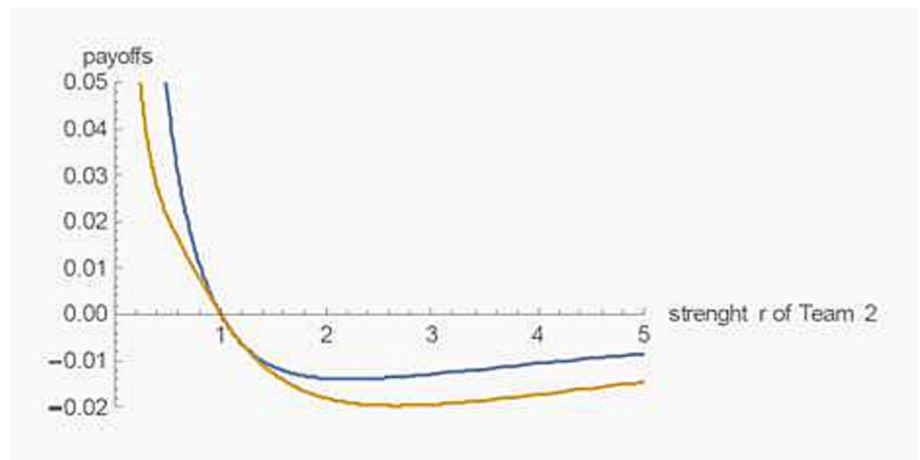


FIGURE 5 Team 1's effort choices first home and first away.

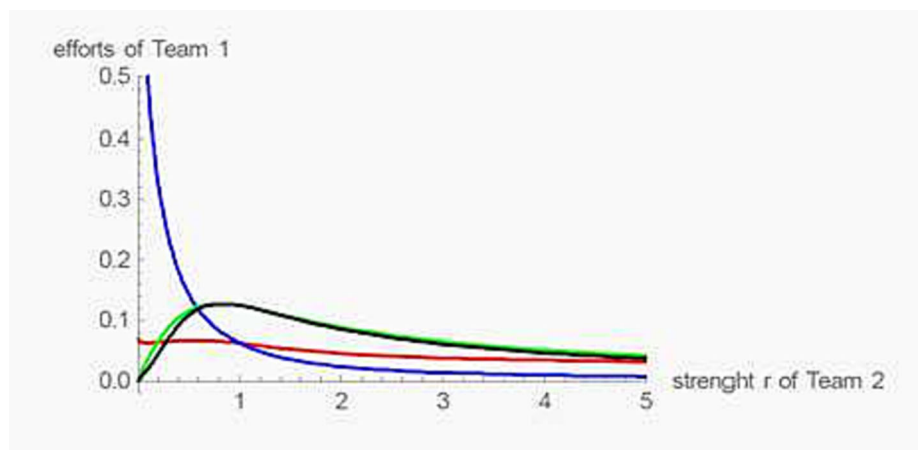
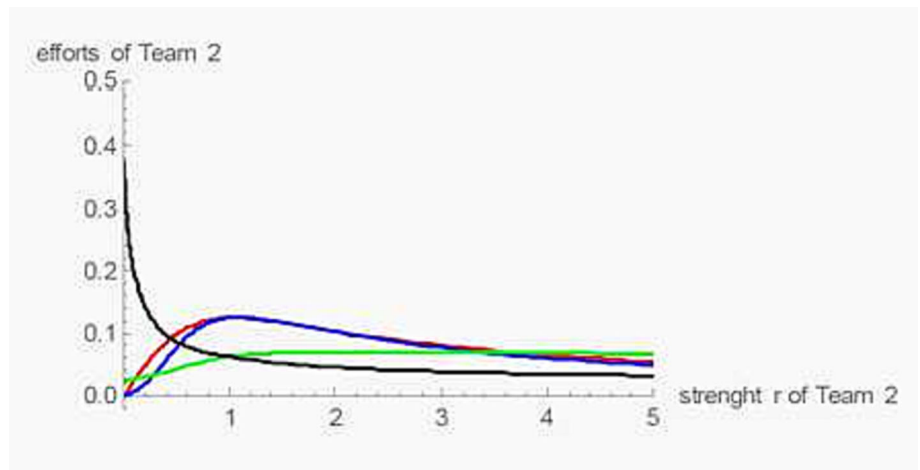


FIGURE 6 Team 2's effort choices first home and first away.



difference in payoffs for Team 2, $\pi_2^*((1,0);(r,0)) - \pi_1^*((r,0);(1,0))$.¹³ The reason for this result is as follows. Consider first Team 1. As long as Team 2 is weaker, $r < 1$, Team 1 prefers to play first at home since a win in the first match is likely with moderate attacking efforts due to its strength and the fact that even if this match ends without shooting a goal, Team 1 can still profit from the away goals rule by shooting a goal in the second match. This, however, is not the case if it plays first

on Team 2's court: To profit from the away goals rule, Team 1 now has to invest directly in its attacking efforts, whereas it could condition its attacking effort on the outcome of the first match in the other scenario. Figure 5 illustrates this argumentation by depicting Team 1's optimal effort choices in the two scenarios.

For $r < 1$, Team 1's attacking efforts if it play first at home, the red line, are lower than if it would play first away, the blue line, whereas

its defending efforts, the green and black lines, are almost identical, independent of the order of play. The opposite is true for its attacking efforts for $r > 1$ where defending efforts are again similar in the two scenarios. The argumentation for Team 2 is similar since its effort choices are complementary to the ones of Team 1; see Figure 6.

As long as Team 2 is weaker than Team 1, $r < 1$, its defending effort in case it plays first at home, the black line, are higher than in case it plays first away, the green line, as a response of Team 1's attacking efforts. Of course, its attacking efforts, the red and blue lines, are almost identical, independent of the order of play, similar to the defending efforts of Team 1.

7 | CONCLUSION

The present paper investigated, in a game theoretic model, the effect of a home advantage and different tiebreaking rules in a best-of-two contest on contestants' behavior. We showed that the question whether it is beneficial to have the home advantage in the first or second match depends crucially on the size of home advantages, on the relative strengths of the teams, and on the tiebreaking rules used to produce a winner in case of a tie. In particular, our analysis provides a consistent and compelling argumentation why playing the second match at home might be beneficial and how the away goals rule outbalance this advantage.

The main logic of our analysis remains robust for different variations of the basic model. We assumed, for example, that a home advantage identically decreases the costs for both activities, offense and defense. Alternatively, one could also argue that fans prefer a more offensive play of their team so that the home advantage only reduces the effort costs of attacking activities. Moreover, we assumed that each of the two matches only lasts one stage so that each team could score maximally one goal per match. On average, this number is lower than the average number of 2.8 to 3 goals per match.¹⁴ Assuming that each match lasts two halftimes as in Jost (2021a, 2021b) would be much more realistic but would make the analysis substantially more complex without changing the main intuition of our reasoning.

To evaluate our theoretical findings, an empirical analysis would be the next step of future research. As noted in the introduction, previous empirical studies yield ambiguous evidence. From a theoretical perspective, the present paper offers an explanation for these differing results in the empirical studies, namely, the heterogeneity in teams' abilities and the underlying tiebreaking rules in place.

Our model also points to a crucial difference between rule matches in a national league system such as the Bundesliga with a double round-robin tournament and in a supranational sports competitions such as the Champions League with a knock-out phase, in particular, in times of Corona with its ghost games. When the Bundesliga returned in May 2020 without fans, home teams were actually winning in 56 ghost games only 21% of the time, whereas home teams won 43% of the 223 games played before the Bundesliga closed down.¹⁵ This phenomenon is in line with our result that in the absence

of a home advantage the relative strengths of two teams crucially determine their winning probabilities and playing away or at home is irrelevant for the outcome of the match. However, this would not be true in the Champions League with its traditional two legged format. Our analysis shows that without any home advantage in both matches, the stronger team prefers to play first away whereas the weaker team is better off playing first at home, which crucially rests on the away goals rule.

An interesting extension of our modeling would be the inclusion of a contest designer as an additional player. So far, we considered the optimal order of play only from the perspective of the teams involved in the competition. Introducing such a contest designer of course implies that we had to qualify her objective function. Suppose that part of her objectives are to secure competitive balance between the teams, that is, to increase the uncertainty of the outcome of the best-of-two contest. Then our analysis would suggest that a weaker team should play the first match away and the second match at home. This, however, is not in line with the rule in many competitions where best-of-two contests involve seeded and unseeded teams, for example, in the UEFA Champions League round of 16. After the group stage, the group winners are seeded and play the second match at home while the group runners-up are unseeded and play the first match at home. And, of course, the abolition of the away goals rule in all UEFA club competitions from 2021/22 on completely runs counter to securing competitive balance. Our analysis then suggest that for the UEFA as organizer of the Champions League other objectives than competitive balance must be more relevant when deciding for this regulation.

Although we used the case of soccer to analyze varying home advantages and tiebreaking rules in best-of-two team contests, our analysis is of relevance also to other team contests outside of sports. Consider, for example, competition for research grants.¹⁶ Research teams apply with research proposals across several rounds of contests for funds. Each research team consists of several experts in their individual fields and each expert's special expertise its a source of home advantage. They are evaluated in their performance relative to the other experts in the same field. In case of a tie, a hearing procedure where each research team has to comment on general questions regarding its proposal, can be interpreted as an extra round with a penalty shootout. Another application of our modeling is internal labor markets.¹⁷ Consider, for example, a promotion tournament with two rounds in which each agent carries out a project with several tasks in each period. Depending on the type of project, the agents have different effort costs for each task. The agent who performs best in the majority of tasks in both periods is promoted to a higher position. Suppose, that in case of a tie, an agent who is better than his opponent on a task despite higher effort costs is evaluated higher than in case his opponent has higher effort costs. Then this tiebreaking procedure can be interpreted as an application of the away goals rule.

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DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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ENDNOTES

- ¹ The original quote is "... ich bin total optimistisch. Dass wir zuerst zu Hause spielen, sehe ich nicht als Nachteil. Wir sollten versuchen, die Null zu halten, und alles nach vorne nehmen wir mit," see <https://www.dfb.de/news/detail/loew-wie-grosse-entscheidungsspiele-bei-wm-42139/>.
- ² The coin toss was used in soccer in the early 1970s and replaced by the penalty shootout. The away goals rule was introduced by UEFA in the 1965–1966 European Cup Winners' Cup and removed on June 24, 2021, in all UEFA club competitions from the 2021–2022 season onwards. Extra time and penalty shootouts are currently approved by the International Football Association Board in the Laws of the Game.
- ³ For example, in the UEFA Champions League round of 16 till 2021, all three tiebreaking procedures are used in sequence: First, the away goals rule applies, and if away goals are equal, the winner is decided first by extra time and, if the tie remains after the extra time, by a penalty shootout. But there are also examples where not all of three tiebreaking procedures are used. In the promotion playoffs in Italy's Serie B, for example, the away goals rule is not used, and the winner is decided directly by extra time. And in the annual matches of the Community Shield in England between the champions of the previous Premier League season and the holders of the FA Cup, a penalty shootout is used without extra time.
- ⁴ Note that we analyze the discouragement effect only in the context of a best-of-two contest. However, the UEFA Champions League and the UEFA Cup are multi-stage elimination tournaments where all winning teams in one round progress to the next round. As shown by Brown and Minor (2014) and Hill (2018), a discouragement effect then does not only come from current but also from potentially future competition. They show that the weaker the expected future competitor, the higher the contestants' effort in the current round. See also Footnote 7 how to integrate this feature into our model.
- ⁵ This, of course, is a simplifying assumption. In soccer, it is usually the coach that decides about the team's playing strategy and the players then choose the efforts in the game.
- ⁶ In a more elaborated model with more goals per match, these drivers would be intensified; see Section 6.
- ⁷ One possibility to integrate the discouragement effect of future competition into our single-stage elimination tournament, see Footnote 4, is to assume that each team values the next round as a function of its own strength r_i and the expected strength r_f of the future competitor. Let $V_i = V_i(r_i, r_f)$ the value of team i for winning the current best-of-two contest and progressing to the next round. Assuming that V_i is the higher, the weaker the expected future competitor, our analysis then would show that the contestants' efforts increase in the current round which would increase the probability that the stronger Team 1 wins the current round, as in Brown and Minor (2014).
- ⁸ Note that $\text{Prob}(g_{12} = 1)$ is decreasing in b_2 , whereas $\text{Prob}(g_{22} = 1)$ is increasing in b_2 .
- ⁹ This result is similar to Proposition 3 in Jost (2021b) where play is analyzed in the second half of extra time in case, team 1 is leading at half time, the away goals rule applies, and extra time is played at Team 1's home field. Note that in our model, the second match is at Team 2's court which implies that $b_2 = -b_1$.

- ¹⁰ This result is similar to Proposition 1 in Jost (2021b) where play is analyzed in the second half of extra time in case (1) the score at half time is $(1 - 1)$, (2) the away goals rule applies, and (3) extra time is played at Team 1's home field. Conditions (1) and (2) are identical to our scenario in Proposition 3 since the away team can only win the match if it wins the second half. And condition (3) implies that roles are revised, that is, $b_1 = -b_2$.
- ¹¹ The following characterization constitutes an interior solution to the maximization problem of both teams where both receive a positive payoff. Of course, there also exists corner solution where one team gives up the first leg with some positive probability. This happens, for example, if a team is sufficiently weak, or the home field advantage of the other team is sufficiently high. See also our discussion in Section 6.
- ¹² These probabilities can be endogenously derived by assuming that teams' abilities for kicking and catching are identical, but both activities involve effort costs.
- ¹³ Of course, there exists a critical value $\bar{r} < 1$ such that Team 2's payoff is negative for $r \leq \bar{r}$.
- ¹⁴ For the World Cup 2006, for example, there were approximately 2.6 goals per match, see Shepotylo (2010).
- ¹⁵ See <https://uk.reuters.com/article/uk-soccer-england-bundesliga/premier-league-hosts-beware-german-data-shows-end-of-home-advantage-idUKKBN23M2DS>.
- ¹⁶ See, for example, Gallini and Scotchmer (2002) as well as the discussion in Clark and Nilssen (2020).
- ¹⁷ See, for example, Lazear and Rosen (1981) and Rosen (1986) who started the theoretical literature on contests in the labor market context.

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APPENDIX

Proof of Propositions 1–6. All these proofs follow the same solution procedure: First, calculate teams' payoff functions; second, take the first order conditions to characterize the optimal efforts; third, solve these conditions simultaneously. As example, take Proposition 1: Teams' payoffs here read as

$$\begin{aligned}\pi_{12}(0) &= \frac{1}{2} \left(\frac{r_1 a_{12}}{r_1 a_{12} + d_{22}} + \left(1 - \frac{r_2 a_{22}}{r_2 a_{22} + d_{12}} \right) \right) - (a_{12} + d_{12})(1 + b_2), \\ \pi_{22}(0) &= \frac{1}{2} \left(\left(1 - \frac{r_1 a_{12}}{r_1 a_{12} + d_{22}} \right) + \frac{r_2 a_{22}}{r_2 a_{22} + d_{12}} \right) - (a_{22} + d_{22})(1 - b_2),\end{aligned}$$

so that the first order conditions are

$$\begin{aligned}\frac{\partial}{\partial a_{12}} \pi_{12} &= \frac{r_1}{2d_{22}(1 + r_1 x_{12})^2} = \frac{\partial}{\partial d_{12}} \pi_{12} = \frac{r_2 x_{22}}{2d_{12}(1 + r_2 x_{22})^2} = (1 + b_2), \\ \frac{\partial}{\partial a_{22}} \pi_{22} &= \frac{r_2}{2d_{12}(1 + r_2 x_{22})^2} = \frac{\partial}{\partial d_{22}} \pi_{22} = \frac{r_1 x_{12}}{2d_{22}(1 + r_1 x_{12})^2} = (1 - b_2),\end{aligned}$$

with $x_{12} = \frac{a_{12}}{d_{22}}$, $x_{22} = \frac{a_{22}}{d_{12}}$, and equilibrium efforts result by solving these four condition as

$$\begin{aligned}a_{12}^*(0) &= \frac{r_1(1 - b_2)}{2((1 + b_2) + r_1(1 - b_2))^2}, \quad d_{12}^*(0) = \frac{r_2(1 - b_2)}{2((1 - b_2) + r_2(1 + b_2))^2}, \\ a_{22}^*(0) &= \frac{r_2(1 + b_2)}{2((1 - b_2) + r_2(1 + b_2))^2}, \quad d_{22}^*(0) = \frac{r_1(1 + b_2)}{2((1 + b_2) + r_1(1 - b_2))^2}.\end{aligned}$$

■

Proof of Propositions 8–10. All these proofs follow from a comparison of the equilibrium payoffs given in Proposition 3, respectively, Proposition 6, with subscript ij when the first match is at i 's home court and the second match at j 's. As an example, take Proposition 8: For $b_1 = b_2 = 0$, Propositions 1–3 imply

$$a_{12}^{12}(0) = a_{12}^{21}(0) = d_{22}^{12}(0) = d_{22}^{21}(0) = \frac{r_1}{2(1+r_1)^2},$$

$$a_{22}^{12}(0) = a_{22}^{21}(0) = d_{12}^{12}(0) = d_{12}^{21}(0) = \frac{r_2}{2(1+r_2)^2},$$

and

$$a_{12}^{12}(1) = a_{12}^{21}(1) = d_{22}^{12}(1) = d_{22}^{21}(1) = \frac{r_1 r_2}{2(1+r_1)^2(1+r_2)},$$

$$a_{22}^{12}(1) = a_{22}^{21}(1) = d_{12}^{12}(1) = d_{12}^{21}(1) = \frac{r_2}{2(1+r_1)(1+r_2)^2},$$

$$a_{12}^{12}(2) = a_{12}^{21}(2) = \frac{1}{\sqrt{r_1 r_2}} d_{12}^{12}(2) = \frac{1}{\sqrt{r_1 r_2}} d_{12}^{21}(2) = \frac{\sqrt{r_1 r_2}}{2(1+\sqrt{r_1 r_2})^3} \text{ with } p^{12} = p^{21} = \sqrt{\frac{r_1}{r_2}},$$

$$a_{22}^{12}(2) = a_{22}^{21}(2) = \sqrt{r_1 r_2} d_{22}^{12}(2) = \sqrt{r_1 r_2} d_{22}^{21}(2) = \frac{r_1 \sqrt{r_1 r_2}}{2(1+\sqrt{r_1 r_2})^3},$$

for $r_1 \geq r_2$, otherwise,

$$a_{12}^{12}(1) = a_{12}^{21}(1) = \sqrt{r_1 r_2} d_{12}^{12}(1) = \sqrt{r_1 r_2} d_{12}^{21}(1) = \frac{r_2 \sqrt{r_1 r_2}}{2(1+\sqrt{r_1 r_2})^3},$$

$$a_{22}^{12}(1) = a_{22}^{21}(1) = \frac{1}{\sqrt{r_1 r_2}} d_{22}^{12}(1) = \frac{1}{\sqrt{r_1 r_2}} d_{22}^{21}(1) = \frac{\sqrt{r_1 r_2}}{2(1+\sqrt{r_1 r_2})^3} \text{ with } p^{12} = p^{21} = \sqrt{\frac{r_2}{r_1}},$$

$$a_{12}^{12}(2) = a_{12}^{21}(2) = d_{22}^{12}(2) = d_{22}^{21}(2) = \frac{r_1}{2(1+r_1)^2(1+r_2)},$$

$$a_{22}^{12}(2) = a_{22}^{21}(2) = d_{12}^{12}(2) = d_{12}^{21}(2) = \frac{r_1 r_2}{2(1+r_1)(1+r_2)^2}.$$

Since equilibrium efforts are independent of the sequence of matches, payoffs in the second match are also independent of which team plays first at home; hence, marginal incentives in the first match are identical, and since marginal costs are also identical, each team plays identical independent of the sequence of matches, leading to identical overall payoffs. ■

All proofs are available upon request.