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RESEARCH ARTICLE

Portfolio structures: Can they contribute to solving the low-risk puzzle?

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Abstract

This article expands on previous studies of the so-called low-risk puzzle with concepts from cooperative game theory. To allocate portfolio risk to single assets, previous studies used concepts such as the Shapley value. In these concepts, the marginal contributions of assets to risks of subsets of the portfolio are used to allocate portfolio risk to assets. In this article, beyond the marginal contributions, a structure on a set of assets is considered in the allocation of portfolio risk. This structure can model the branch, firm size or the region of the assets. Specifically, the Myerson value and the Spectrum value of cooperative game theory are applied. We show the application by means of a simulation study. In this context, considering an additional structure could enhance the analysis of the so-called low-risk puzzle.

JEL CLASSIFICATION

C71, G10, G11

1 | INTRODUCTION

Allocating portfolio risk to individual assets is an important step in (empirical) capital market research to perform risk–return analyses. In addition to classic measurement of the risk of an asset in the stand-alone view (for example asset variance), there are a number of concepts in capital market theory for distributing portfolio risk to individual assets. These market theories include the activity-based method (Hamlen et al., 1977), the Beta method (e.g., Homburg & Scherpereel, 2008), and the incremental approach (see, for instance, Jorion, 1985). In addition to these concepts, a growing body of literature has emerged that uses cooperative game theory, especially the Shapley value (Shapley, 1953), the Nucleolus (Schmeidler, 1969), and the τ value—also known as cost gap method (Tijs, 1987; Tijs & Driessen, 1986), for risk allocation (Auer & Hiller, 2019, 2021; Balog et al., 2017; Mussard & Terraza, 2008; Ortmann, 2016, 2018; Shalit, 2020).

The question of how the risk of a portfolio is distributed among the individual assets is a natural application of cooperative game

theory, since this theory addresses exactly such distribution problems. A very well-known example is the situation of voting committees. Value-like concepts (Hart, 1997) of cooperative game theory also offer the advantage of making unambiguous risk allocations and thus enable risk–return considerations. The Shapley value is the best-known value-like solution concept. Its advantage is that it takes into account all contributions that an asset makes to the risks of all other groups of assets and is thus more suitable than, for example, variance as a risk measure, which only takes into account the risk of the asset alone.¹

In addition to the Shapley concept, other value-like concepts exist such as the Nucleolus for the allocation of portfolio risk, which are considered in comparison in Auer and Hiller (2021). In addition to the consideration of all marginal contributions, cooperative game theory offers numerous possibilities to take into account further structures

¹One drawback of the Shapley value is that the weights of marginal contributions are predefined. Concepts considering variable weights are referred in the literature to as semi values (Monderer & Samet, 2002).

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between assets. These could be active groups (portfolios as part of the total stock market) in the sense of Aumann and Drèze (1974), opposing asset classes (in the sense of Owen, (1977)) or hierarchies between assets or asset classes (Gilles et al., 1992; van den Brink, 2008; van den Brink & Gilles, 1996). In this paper, graphs are used to incorporate similarities between assets into the analysis. Table 1 summarizes how different games of cooperative game theory have been applied in the field of portfolio theory so far and what gap our approach has left.

However, without applying cooperative game theory, a phenomenon in empirical capital market theory shows that the measurement of an asset's risk has so far been problematic. According to current theory, assets that generate a higher return also have to bear a higher risk (Lintner, 1965; Markowitz, 1952; Mossin, 1966; Rubinstein, 2002; Sharpe, 1964). Empirically, this cannot be confirmed and the opposite finding occurs—the so-called low-risk puzzle. Also, when using different classical measures for risk, the phenomenon still exists (Auer & Schuhmacher, 2021; Baker et al., 2011; Blitz et al., 2014; Dutt & Humphery-Jenner, 2013; Frazzini & Pedersen, 2014). The empirical evidence that assets with low risk (volatility) earn higher returns is stable for global large-cap assets and for the US, European, and Japanese markets. Also, the low-risk puzzle cannot be explained by other effects such as book-to-market ratio and free float market value. The puzzle is also robust to different measurement periods of volatility (Blitz & van Vliet, 2007). One reason for this empirical phenomenon may be the difficulty in allocating portfolio risk to individual assets (Baker et al., 2011). Auer and Hiller (2019, 2021) showed in simulation studies that using the Shapley value instead of classic measures of risk has the potential to solve the low-risk puzzle.

This forms the starting point for our article. Whereas the Shapley value only considers marginal contributions of assets to risks of

subsets of the portfolio, we consider an additional structure at the set of assets. Specifically, this structure is a ranking of the assets. Assets that are close to each other in this ranking are more similar in terms of the structure. In practice, this structure can be for example: branches, company size, or regional origin of an asset or a company. We can illustrate our idea with the following example: Assume there are three assets: an American car manufacturer, an American technology company, and a European technology company. The first and the last assets have no similarity, while the second asset is situated between the two remaining assets and can “dock” to both. This situation is very similar to the technical modeling we consider within cooperative games. We assume a graph L that connects all players without making unnecessary cycles. Solution concepts that determine payoffs for the players in games with graphs exist in numerous forms. The best-known solution concept is the Myerson value (Myerson, 1977). The Spectrum value is a quite new value (Alvarez-Mozos et al., 2013). This concept was introduced for the analysis of political parties and takes into account the graph on the player set in a significantly different way compared to the Myerson value. Hence, both values capture a wide range in the field of games with graphs. In our study, we want to analyze whether the technical inclusion of another structure at a set of assets — a graph — can increase the number of corrections of the rankings between the assets in comparison to non-inclusion of this structure (application of the Shapley value). To the best of our knowledge, this article is the first to allocate portfolio risk to single assets considering an additional structure at a set of assets.

For our analysis, we perform a simulation study based on the ideas of Auer and Hiller (2019, 2021). A simulation was chosen as analytical framework for the following reasons. Firstly, we would like to further develop the analysis done with simulations by Auer and Hiller (2019, 2021). They chose this analytical framework

TABLE 1 Application of different games / values to portfolio theory.

games	values	application in portfolio theory	interpretation
no structure	Shapley (1953), Banzhaf (1965), Schmeidler (1969), Tijs and Driessen (1986), Tijs (1987)	Mussard and Terraza (2008), Balog et al. (2017), Ortmann ((2016), (2018)), Auer and Hiller ((2019), (2021)), Shalit (2020)	only one portfolio exists in the total stock market; no structures on the set of assets
(active) coalition structures	Aumann and Drèze (1974), Wiese (2007), Casajus (2009), Alonso-Meijide et al. (2015)	Hiller (2022)	portfolios as part of the total stock market
(bargaining) coalition structures	Owen (1977)	-	
cooperation structure/networks	Myerson (1977), Borm et al. (1992), Herings et al. (2008), Alvarez-Mozos et al. (2013), Navarro (2020)	this article	a structure on the set of assets like branch, firm size, or region is considered

because the low-risk puzzle is an empirical problem and the simulations can be used to represent all conceivable constellations on the portfolio market. Secondly, we think that strong, clear theoretical statements cannot be expected, given the large number of parameters (variances, means, covariances, graphs, and asset shares in the portfolio).

We look at an investor who is interested in combining individual assets in a portfolio and evaluate the riskiness of each asset based on payoffs according to Myerson or Spectrum. We design our setup as such that the low-risk puzzle exists in the individual asset variances. We then repeatedly randomly generate the covariance matrix of the assets. We consider two weighting schemes for asset shares in a market. Based on this data, we calculate Myerson payoffs and Spectrum payoffs and the percentage of cases in which they lead to asset rankings different from the individual asset variances. Three different graphs on the set of assets are considered in each of the calculations. In addition, we calculate the Shapley payoffs as benchmark and conclude whether there are more corrections using graph solution concepts.

The remainder of our article is organized as follows. In Section 2, we present basic notations of cooperative game theory. Section 3 presents our simulation study. Finally, Section 4 summarizes and outlines directions for future research.

2 | COOPERATIVE GAME THEORY

A transferable utility (TU) game is a pair (N, v) . $N = \{1, 2, \dots, n\}$ is the set of players (assets). The coalition function v specifies for every subset $S \subseteq N$ a certain worth $v(S)$ reflecting the risk of a portfolio S , that is, $v: 2^N \rightarrow \mathbb{R}$ with $v(\emptyset) = 0$.

A value is an operator ϕ that assigns (unique) payoff vectors to all games (N, v) (i.e., uniquely determines a payoff for every player in every TU game). We interpret the payoff of player i as i 's contribution to portfolio risk $v(N)$. The most important value is the Shapley value. For calculating a player's payoff, rank orders δ on N are used. They are written as $(\delta_1, \dots, \delta_n)$ where δ_1 is the first player in the order and so on. The set of these orders is denoted by $R(N)$; $n!$ rank orders exist. The set of players before i in rank order δ including i is called $S_i(\delta)$. For player i , the Shapley payoff is (Shapley, 1953)

$$Sh_i(N, v) = \frac{1}{|R(N)|} \sum_{\delta \in R(N)} v(S_i(\delta)) - v(S_i(\delta) \setminus \{i\}). \tag{1}$$

In order to model a network structure on the set of players, the Myerson value (Myerson, 1977) has been introduced and axiomatized. The network can model communication relationships between players, distribution structures between firms, or similarities between players, to name just a few examples. Some further preliminaries are necessary to explain the Myerson value. First, a graph L on the set of players is considered. The set of possible pairwise links between players is called $L^N = \{\{i, j\} : i, j \in N, i \neq j\}$, whereat $\{i, j\}$ and $\{j, i\}$, respectively, (or ij and ji) is the direct link between players i and j . A cooperation structure CO on N is a graph (N, L) with $L \subseteq L^N$. A CO game is characterized by (N, v, L) . The graph L partitions N into connected components C_1, \dots, C_k . This partition is denoted by $N \setminus L$. Each player is in one component; $C_i \cap C_j = \emptyset, i \neq j, N = \bigcup C_j$. $N \setminus L(i)$ denotes the component of i . Two players i and j with $N \setminus L(i) = N \setminus L(j)$ are connected. The restricted coalitional function $v|_L$ is given by

$$v|_L(S) := \sum_{C \in S \setminus L(S)} v(C) \forall S \subseteq N. \tag{2}$$

The worth of a coalition S corresponds to the sum of the worths of its components. In the case $|S \setminus L(S)| = 1$, we have $v(S) = v|_L(S)$. A CO value is an operator ψ that assigns (unique) payoff vectors to all CO games (N, v, L) . The most popular value for CO games is the Myerson value (Myerson, 1977). According to this value, player i 's payoff is calculated by

$$My_i(N, v, L) = Sh_i(N, v|_L). \tag{3}$$

To apply the Myerson value there is no further requirement for L . To model an ordering of players in a game (N, v) , Alvarez-Mozos et al. (2013) introduced the so-called Spectrum value. This value assumes a graph L where all players in N are connected via one path. A path connecting all players in N is a sequence of players (i_1, i_2, \dots, i_n) with $\{i_k, i_{k+1}\} \in L$ for all $k = 1, \dots, n - 1$. Three examples of paths are shown in Figure 2. We denote graphs with only one path on N with \bar{L} . A rank order δ is an admissible permutation with respect to this path, if each player that is added in rank order is immediately to the left or immediately to the right of the set of predecessors. Hence, $S_i(\delta)$ is connected

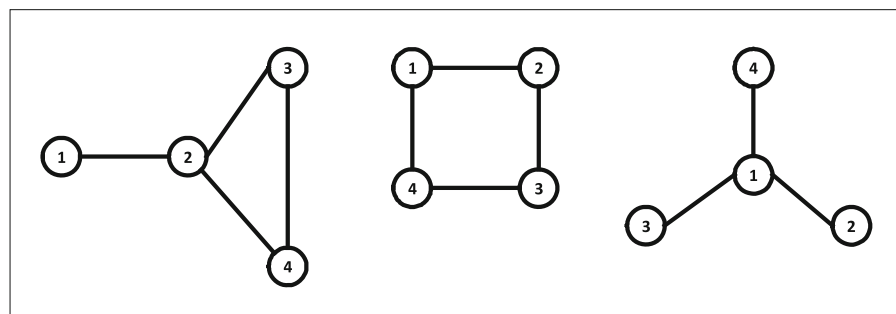


FIGURE 1 More possible graphs.

for all $i \in N$. The set of all admissible rank orders is denoted by $\overline{R(N)}$. A CO game with one path on N is characterized by (N, v, \overline{L}) . The Spectrum payoff of player i for (N, v, \overline{L}) is

$$Sp_i(N, v, \overline{L}) = \frac{1}{|\overline{R(N)}|} \sum_{\delta \in \overline{R(N)}} v(S_i(\delta)) - v(S_i(\delta) \setminus \{i\}). \tag{4}$$

The set of all CO games with one path is a subset of all CO games. Hence, the Myerson value is also defined for these games. The Myerson value and the Spectrum value differ in the way that the graph is considered for calculating players' payoffs. While the Myerson value restricts the coalition function, the Spectrum value changes the number of possible player rank orders.²

In addition to paths, a large number of graphs are conceivable that depict similarities or relationships between assets. Several examples are outlined in Figure 1. In this paper, we have focused on simple paths to test the suitability of the structure to solve the low-risk puzzle. The Spectrum value, which was developed with the aim of modeling the similarity of parties, can only be applied to paths.

To illustrate the potential of considering cooperation structures and CO values on the order of assets, we state one theorem. For this theorem, some additional preliminaries on variables that determine the portfolio risk are necessary. These variables are asset weights of the portfolio w_i with $\sum_{i \in N} w_i = 1$, standard deviation for an asset i noted by σ_i , variance for an asset i noted by σ_i^2 and correlations ρ_{ij} between two assets i and j . With these variables, portfolio risk for any $K \subseteq N, v(K)$, is determined by

$$v(K) = \sum_{i \in K} w_i^2 \cdot \sigma_i^2 + 2 \cdot \sum_{i \in K} \sum_{j \in K, j > i} w_i \cdot w_j \cdot \sigma_i \cdot \sigma_j \cdot \rho_{ij}. \tag{5}$$

Theorem 1. *In a portfolio with*

- n assets, $n > 3, w_i = \frac{1}{n}$ for all $i \in N$;
- $\sigma_i = \overline{\sigma}, 1 > \overline{\sigma} > -1, \overline{\sigma} \neq 0$, for all $i \in N$;
- $\rho_{ij} = \overline{\rho}, 1 > \overline{\rho} > -1, \overline{\rho} \neq 0$, for all $i, j \in N$; and
- a path with $(1, 2, \dots, n)$ with $\{i, i + 1\} \in L$ for all $i = 1, \dots, n - 1$,

a rank correction occurs if the Myerson value or the Spectrum value is used to allocate portfolio risk $v(N)$.

Pointers to this Theorem are in the Appendix A.³ In this theorem, assumptions regarding assets are very restrictive. For a realistic assessment of how many rank corrections occur, we use a simulation in the following section. Since this simulation is similar to those of

Auer and Hiller (2019, 2021) using the Shapley value without further cooperations structures, we can quantify the added value of considering CO games.

3 | SIMULATION

3.1 | General setting

Since our study is similar to articles by Auer and Hiller (2019, 2021), we adopt some of their simulation settings. We analyze a four-asset scenario, $N = \{1, 2, 3, 4\}$.⁴ We assume mean returns for each asset $\mu_1 > \mu_2 > \mu_3 > \mu_4$ and variances $\sigma_1^2 < \sigma_2^2 < \sigma_3^2 < \sigma_4^2$. Thus, the low-risk puzzle occurs. Further, we have a 4×4 covariance matrix \mathbb{V} of asset returns. For weights w_i , we have $\sum_{i \in N} w_i = 1$. We analyze two weighting schemes. The first is a naive scheme where all assets have the same weights; $w_i = \frac{1}{4} \forall i \in N$. The second weighting scheme has random weights assuming a uniform distribution of asset weights between 0 and 1. In addition, we adopt some further technical settings of the simulation by Auer and Hiller (2019). The covariance matrix \mathbb{V} and the asset weights in the second weight scheme are simulated 100,000 times, standard deviations of assets are between 0 and 10, and asset correlations range from -1 to 1. In contrast to Auer and Hiller (2019, 2021), we do not assume the invertibility of \mathbb{V} , because we do not consider minimum-variance weights. Therefore, our results may differ from Auer and Hiller (2019, 2021) with respect to the Shapley concept.

In addition, we assume three graphs on the set of assets (see Figure 2). One aspect to consider when choosing paths are “mirrored” paths—for example, $(1, 2, 3, 4)$ and $(4, 3, 2, 1)$ result in the same asset payoffs. Our choice of graphs allows one to analyze, for example, how the number of ranking corrections between Assets 1 and 2 change with increasing distance between both assets in the graph. The distance between 1 and 2 increases step by step from first path $(1, 2, 3, 4)$ to our last path $(1, 4, 3, 2)$. We predict some simulation results (see also

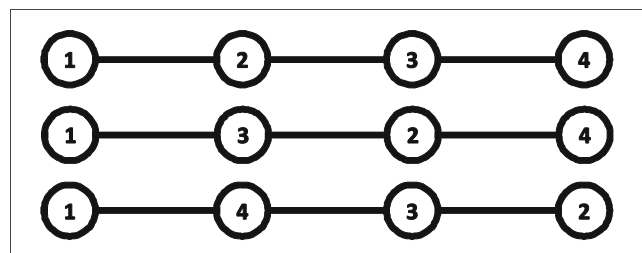


FIGURE 2 Graphs on the set of assets.

²Further values for CO games are the position value (Borm et al., 1992), the average tree solution (Herings et al., 2008), and the center value (Navarro, 2020). For a literature survey on CO games, see Slikker and van den Nouweland (2001) and Gilles (2010).

³A rank correction occurs if for at least two assets $i, j \in N$, we have $\sigma_i = \sigma_j$ and $\psi_i > \psi_j$.

⁴As such, four assets were used to make the results comparable with those of Auer and Hiller ((2019), (2021)). Additionally, using four assets and the chosen graphs, the effects of increasing the distance between two assets (in our case, Players 1 and 2) can be sufficiently modeled. In addition from perspective of portfolio theory, private investors typically construct small portfolios (Schuhmacher et al., 2021). Furthermore, they often follow passive strategies consisting of three asset classes (stocks, bonds, and commodities) combined with a risk-free asset (money) or, when focusing on stocks, limit their attention to four broad markets (North America, Europe, Pacific, and emerging markets).

TABLE 2 Asset 2, mean values.

	$Sh_2(N,v)$	$My_2(N,v,L)$	$Sp_2(N,v,L)$
Equal weights	-3.9392	-5.9722	-1.8726
Rand. weights	-2.6063	-5.2074	-0.0240

Theorem 1): One theoretical prediction is that when comparing path (1,2,3,4) and path (1,3,2,4), there will be more rank corrections between Assets 1 and 2 in (1,3,2,4) than in (1,2,3,4) since the similarity between the two assets decreases (the distance in the path increases), but Asset 2 retains its central role in the path.

Another theoretical consideration concerns the payoff from each asset when comparing the three solution concepts. To this end, we will first consider the effect of a path for the two CO values. If the Myerson value is considered, central assets in the path benefit because they connect the remaining assets. In our model, connecting assets leads to a lower portfolio risk $v(N)$ compared to the sum of individual risks. The central assets benefit is that their risk allocation decreases. Specifically, for Asset 2, paths (1,2,3,4) and (1,3,2,4) are expected to lead to a lower risk attribution when applying the Myerson value than when applying the Shapley value. Considering the Spectrum value, the opposite occurs. The central assets in the path are in front positions in many admissible rank orders $\bar{R}(N)$. Thus, they cannot achieve marginal contributions by gaining access to pre-existing assets; and thus, their risk allocation is higher compared to the Shapley value.⁵

3.2 | Results

In Table 2, we present results from our last theoretical consideration. For Asset 2, the mean values of the three solution concepts are noted. For the CO values, path (1,2,3,4) is considered. The mean values change as predicted. Compared to the Shapley concept, Asset 2 benefits (loses) from a central position with respect to the Myerson (Spectrum) concept.

In Tables 3–5, we present the results of our calculations concerning the risk rank corrections. The tables compile our results on the proportions of cases, in which a solution concept occurs for

- a rank correction between Assets 1 and 2,
- any partial correction (two assets change ranks),
- full corrections, and
- no corrections.

In our analysis, the results of the Shapley value are used as a benchmark if the inclusion of a structure increases the number of rank

corrections when using the Myerson value or the Spectrum value. This means that the low-risk puzzle has been solved to a greater extent. Any difference between rank corrections is significant since we use a simulation study and thus know the basic population.

In the case of equal asset weights, the Myerson value and the Spectrum value increase the number of rank corrections between Assets 1 and 2 for the paths (1,2,3,4) and (1,3,2,4) compared to the Shapley value. Path (1,3,2,4) achieves a higher number of corrections than (1,2,3,4). Comparing the Myerson value and the Spectrum value, the Myerson value leads to more ranking corrections between 1 and 2 for paths (1,2,3,4) and (1,3,2,4). If any directional correction is considered, both values for (N,v,\bar{L}) perform better than Shapley for all graphs with one path. In the analysis of any directional corrections, the Myerson value achieves a higher number than the Spectrum value. If we analyze full corrections of rank orders of assets, the results are not quite clear. With respect to our first theoretical prediction, for the Myerson value and the Spectrum value, we find more rank corrections between Assets 1 and 2 in path (1,3,2,4) than in path (1,2,3,4).

In the second case, we have random weights for assets in portfolio N . The Myerson value corrects more rankings between 1 and 2 for paths (1,2,3,4) and (1,3,2,4), whereas the Spectrum value outperforms the Shapley value for all paths. We have the same results as in the case of equal weights, where path (1,3,2,4) has a higher number of corrections than (1,2,3,4). In addition comparing both graph values, the Myerson value leads to more ranking corrections between 1 and 2 for paths (1,2,3,4) and (1,3,2,4) than the Spectrum value; this result is the same as in the case of equal asset weights. Analyzing any directional correction, the Myerson value corrects better than Shapley for all paths. Again, as in the case of equal asset weights, the Myerson value achieves a higher number of corrections than the Spectrum value. Considering full corrections, again, there are only small deviations between Shapley, Myerson, and Spectrum. Also, in this case, we find our first theoretical prediction confirmed; there are more rank corrections between 1 and 2 in path (1,3,2,4) than in path (1,2,3,4).

In the next step, we take a closer look at the partial rank corrections. Specifically, we compute the risk–return relationship in cases with partial corrections. We use the least-squares method to a linear equation linking the mean return vector (with arbitrary values of [4 3 2 1] for the assets) to the risk allocation vector (computed as described above). In a very similar way, Auer and Hiller (2021) use this method to analyze which is the best remedy based on values of cooperative game theory to solve the low-risk anomaly. In case of a positive slope of the regression line, there is a solution of the low-risk-puzzle for average. In Table 6, the proportion of all simulations with a positive increase in the risk–return relationship is noted. Now, the Myerson value outperforms the Shapley value for (1,4,3,2) and with a low extent for (1,2,3,4). The Spectrum value has more positive sloped regression lines for (1,2,3,4) and (1,3,2,4) with respect to the Shapley value. Comparing both graph values, the Spectrum concept has more positive sloped regression lines than the Myerson concept for graphs (1,2,3,4) and (1,3,2,4) in both weight schemes.

⁵With respect to path (1,2,3,4), rank orders (1,2,3,4), (2,1,3,4), (2,3,1,4), (2,3,4,1), (3,2,1,4), (3,2,4,1), (3,4,2,1), and (4,3,2,1) are possible.

TABLE 3 Results for Shapley value.

	Corrections 1 and 2	Directional correction	Full correction	No correction
Equal weights	0.1825	0.4123	0.0001	0.5877
Rand. weights	0.2681	0.5133	0.0001	0.4868

TABLE 4 Results for Myerson value.

		Corrections 1 and 2	Directional correction	Full correction	No correction
Equal weights	1, 2, 3, 4	0.7581	0.9933	0.0000	0.0067
	1, 3, 2, 4	0.9928	0.9929	0.0000	0.0071
	1, 4, 3, 2	0.0309	0.7685	0.0001	0.2315
Rand. weights	1, 2, 3, 4	0.6799	0.9357	0.0000	0.0643
	1, 3, 2, 4	0.9409	0.9417	0.0000	0.0583
	1, 4, 3, 2	0.1380	0.8087	0.0001	0.1913

TABLE 5 Results for Spectrum value.

		Corrections 1 and 2	Directional correction	Full correction	No correction
Equal weights	1, 2, 3, 4	0.4171	0.8483	0.0013	0.1517
	1, 3, 2, 4	0.5168	0.8528	0.0001	0.1472
	1, 4, 3, 2	0.1626	0.4744	0.0000	0.5256
Rand. weights	1, 2, 3, 4	0.3102	0.8014	0.0005	0.1986
	1, 3, 2, 4	0.3761	0.7878	0.0000	0.2123
	1, 4, 3, 2	0.2838	0.5295	0.0001	0.4705

TABLE 6 Share positive risk–return slopes.

	Shapley		Myerson	Spectrum
Equal weights	0.0037	1, 2, 3, 4	0.0051	0.0459
		1, 3, 2, 4	0.0001	0.0133
		1, 4, 3, 2	0.0923	0.0018
Rand. weights	0.0031	1, 2, 3, 4	0.0037	0.0419
		1, 3, 2, 4	0.0003	0.0204
		1, 4, 3, 2	0.0680	0.0014

4 | RESEARCH OUTLOOK

In our article, we expand the application of cooperative game theory to the problem of allocation of portfolio risk to assets in portfolios. Our results show that considering an additional structure on a set of assets can improve the number of rank corrections and thus the number of cases in which the low-risk puzzle is solved.

We would like to leave the question of which CO value best solves the low-risk puzzle open at this point. The low-risk puzzle is an empirical problem and only empirical studies on the capital market with the help of CO values can ultimately provide information on which CO value makes the greatest contribution to solving this problem. Hence, our article could be a starting point for new empirical research. One

aspect in this research is which similarity (industry, region, firm size, etc.) should be modeled using graphs. Paths like those in our simulation can be used, as can more complex structures (see Figure 1). As an example, a recent generalization of the model in Alvarez-Mozos et al. (2013) could be used (Hellman & Peretz, 2018), for this purpose. In addition, it is conceivable that individual links have different weights (Calvo et al., 1999). If several structures are to be modeled at the same time, several graph games can be combined (van den Nouweland et al., 1992). Furthermore, analysis is possible to determine whether different structures are relevant for rank corrections in different asset classes. Subsequently, analysis is necessary to determine which CO value achieves the best results with the empirical data. In addition to the CO values used and mentioned, there are other CO values whose contribution to solving the low-risk puzzle can be empirically investigated (Gilles, 2010; Slikker & van den Nouweland, 2001).

As mentioned in Section 1, the Shapley value and the CO values take into account fixed weights for the marginal contributions of assets to the risks of subsets of the portfolio to allocate portfolio risk to single assets. However, the weights of marginal contributions could vary. Concepts of this type are referred to in the literature as semi values (Monderer & Samet, 2002). The idea behind varying marginal contributions is that investors do not have the complex formulas of, for example, the Myerson value or the Spectrum value in mind when evaluating marginal contributions in terms of determining the risk of

an asset. Hence, this approach could improve the application of cooperative game theory.

At best, implications for the theoretical development of capital market models can be drawn from these empirical studies.

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CONFLICT OF INTEREST STATEMENT

The authors have no relevant financial or non-financial interests to disclose.

APPENDIX A: Appendix

A.1 | Theorem 1

A.1.1 | Myerson value

With the assumptions on variables, $v(N)$ is determined by

$$\begin{aligned} v(N) &= n \cdot \left(\frac{1}{n}\right)^2 \cdot \bar{\sigma}^2 + 2 \cdot \frac{n \cdot (n-1)}{2} \cdot \left(\frac{1}{n}\right)^2 \cdot \bar{\sigma}^2 \cdot \bar{\rho} \\ &= \frac{1}{n} \cdot \bar{\sigma}^2 + \frac{(n-1)}{n} \cdot \bar{\sigma}^2 \cdot \bar{\rho}. \end{aligned} \quad (\text{A1})$$

Without cooperation structure, the Shapley payoffs of assets are $Sh_i(N, v) = \frac{1}{n} \cdot v(N)$. Hence, there are no rank corrections between assets. The coalition function v is subadditive; hence, $v_{\bar{L}}$ is subadditive.

In $v_{\bar{L}}$, any unconnected coalition has a Harsanyi dividend (Harsanyi, 1959) zero. A coalition K is connected, if there is a sequence of assets (i_1, i_2, \dots, i_j) with $\{i_s, i_{s+1}\} \in \bar{L}$ for all $s = 1, \dots, j-1$. The Harsanyi dividend of K in $v_{\bar{L}}$ is given by (van den Brink et al., 2007)

$$v(K) - v(K \setminus \{i_1\}) - v(K \setminus \{i_j\}) + v(K \setminus \{i_1, i_j\}). \quad (\text{A2})$$

This term is negative. The assets in the middle of path \bar{L} are member in more connected coalitions in \bar{L} than assets at the end of the path. Hence, Myerson payoffs of assets are not equal and rank corrections occur. For example, if the number of assets is odd, the asset in the middle of the path will have the lowest Myerson payoff, while the assets at the end of the path will have the highest (equal) payoffs.

A.1.2 | Spectrum value

In Alvarez-Mozos et al. (2013), symmetric weighted voting games are analyzed. These games are superadditive. The results from Claim 3 in

Alvarez-Mozos et al. (2013) can be transferred to our symmetric subadditive game. When n is large enough, assets located virtually half-between the center and one of the most extreme assets get the lowest payoffs. Hence, Spectrum payoffs of assets are not equal and rank corrections occur.

DATA AVAILABILITY STATEMENT

The datasets generated and analyzed during the current study are available from the corresponding author upon request.

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