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On the economics of the longevity risk transfer market

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Abstract

We present a model of a longevity risk transfer market with different market players (primary insurers, reinsurers, and capital market investors) and investigate how market dynamics and the market players' roles evolve with progressing market saturation. We find that reinsurers' appetite for longevity risk is the key driver in the early stage of market development. Since diversification benefits with other businesses decrease with every transaction, the reinsurance market is intrinsically antimonopolistic. With the increasing saturation of the reinsurance sector as a whole, its competitiveness shrinks leading to rising expected risk-adjusted returns for capital market investors. We show that in a saturated market, reinsurers should assume the entire longevity risk from primary insurers, diversify it within their business mix, and subsequently pass on only specific (nondiversifiable) components of the longevity risk to the capital markets. Our findings provide valuable suggestions on how to make the best use of the market's limited risk absorption capacity.

KEYWORDS

longevity risk hedging, longevity risk transfer market

JEL CLASSIFICATION

D40, D52, G22, G23

[Correction added on 12 June 2023, after first online publication: Author name is corrected to Jochen Ruß.]

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1 | INTRODUCTION

Since its beginning in the UK in 2006, the longevity risk transfer market has been dominated by global (re)insurers that have some appetite for longevity risk due to potential diversification benefits with their existing business mix. While the risk absorption capacity in the (re) insurance sector has been sufficient to meet the demand for longevity derisking so far, several practitioners and academics question its capability to cope with future increasing demand, compare Blake et al. (2019) and Kessler (2021). Michaelson and Mulholland (2014) estimate that the global public and private retirement obligations lie between \$60 and \$80 trillion. They warn that the risk of a potential increase in these liabilities due to an unexpected change in the trend of mortality improvements amounts to \$5–\$8 trillion, which “is far in excess of the amount of risk capital the global insurance industry could realistically bring to bear against this risk.”¹

This raises several questions. First, it calls for a better understanding of how the current market dynamics are driven by the available diversification capacity within the (re)insurance sector and how these dynamics change with increasing market saturation. In particular, a profound understanding is required if and how shrinking diversification opportunities impact (re)insurance prices and reduce the economic attractiveness of longevity derisking for hedgers. A shortage in supply of cost-efficient longevity derisking solutions may hamper the market’s potential for further risk transfers. This might lead to an economically suboptimal allocation of longevity risk among the market participants with potentially adverse impacts on the long-term financial stability of annuity providers. Furthermore, a deeper analysis is warranted of how risk capital provided by external investors can be optimally deployed to expand the market’s risk absorption capacity. However, this requires properly engineered instruments that reconcile longevity hedgers’ general demand for an effective and cost-efficient risk transfer with the investors’ request for attractive risk-adjusted returns, standardization, and manageable contract duration. A deeper understanding of these issues can explain current and predict future dynamics of the longevity risk transfer market, stimulate the innovation of adequate hedging solutions, and help expanding the market to a wider and more diversified set of risk takers.

The concept of a deep and liquid longevity risk transfer market that explicitly involves, besides annuity providers as primary hedgers and (re)insurers as first risk takers, also the capital markets as risk absorber goes back to Blake et al. (2006) and Cairns et al. (2008). They describe a hypothetical longevity risk transfer chain consisting of two separate links: First, longevity risk is transferred from annuity providers to the (re)insurance sector, typically by entering into a series of customized longevity swaps. Subsequently, in the second link of this chain, selected longevity risk components are further passed on to the capital markets through an index-based hedge. Since then, this concept has been taken up in several studies, including Blake et al. (2019), Cairns and El Boukfaoui (2021), and the references cited therein. However, previous studies are either limited to qualitative descriptions and discussions of a functioning longevity risk transfer market or solely focus on one link of this risk transfer chain, that is, on transactions between a longevity hedger and a hedge provider who is typically left unspecified (see, e.g., Börger, Freimann, et al., 2021; Cairns et al., 2014; Cairns & El Boukfaoui, 2021; Meyricke & Sherris, 2014; Ngai & Sherris, 2011). To the best of

¹Michaelson and Mulholland (2014), p. 19.

our knowledge, a comprehensive quantitative analysis of the dynamics of the longevity risk transfer market that simultaneously encompasses both links of this risk transfer chain has not been conducted in the literature yet.

Also, in spite of the paramount role that the reinsurance sector plays in this market, previous studies do not sufficiently consider its objectives and characteristics. Börger (2010) argues that (re)insurers typically evaluate and price longevity transactions with regard to the tied-up economic capital. This so-called cost-of-capital pricing approach is implemented and further refined by Freimann (2021), Levantesi and Menzietti (2017), and Zeddouk and Devolder (2019). However, all of these works have in common that they do not explicitly account for potential diversification effects within the reinsurer's business mix even though these may constitute a decisive competitive advantage. Finally, we are also not aware of any work that explicitly analyzes how the dynamics of the longevity risk transfer market evolve with increasing market saturation.

In this paper, we fill this gap in the literature by proposing a comprehensive stochastic model of a longevity risk transfer market that consists of the following components:

- *Market participants*: We consider the following three different types of market participants with own characteristics and (partly opposing) objectives:
 - *Primary longevity hedgers*: On the demand side, we consider primary writers of longevity risk, such as pension funds or life insurers, that evaluate a (partial) transfer of their longevity risk exposure. In the spirit of Börger, Freimann et al. (2021), we assume that they simultaneously seek a high *hedge effectiveness* (measuring the achieved level of risk reduction) and a high *capital efficiency* (measuring the reduction in the cost of capital after deduction of the hedging costs). Hypothetically speaking, primary hedgers would aim for a complete risk transfer, which reduces their capital charges to zero, if such a perfect hedge was available on a cost-free basis. However, due to the risk premium demanded by the counterparty, such a perfect hedge generally comes at a cost. These hedging costs reduce the capital efficiency with adverse effects on the economic attractiveness of hedging. If the hedging costs exceed the potential relief in the cost of capital, keeping the risk constitutes an economically viable alternative to hedging the risk.
 - *Reinsurers*: At the core of our framework is a reinsurer whose appetite for longevity risk is driven by potential diversification benefits with the existing business mix, which is categorized into three different lines of business (LOBs): longevity business, mortality business, and other assumably uncorrelated nonbiometric risks. The reinsurer relies on an internal economic capital model that explicitly takes into account diversification effects between these LOBs and evaluates and prices longevity transactions based on a so-called return on risk-adjusted capital (RORAC) approach with regard to an anticipated target return on equity. Depending on the specific business mix and the prevailing market environment, reinsurers might be on the supply or demand side, that is, either act as a taker or a hedger of longevity risk. While it might be economically reasonable to take on longevity risk from primary hedgers at one point, changing market conditions might also create incentives for passing on certain longevity risk components to the capital markets with the objective of freeing up economic capital.
 - *Capital market investors*: On the supply side, we consider capital market investors who might potentially be willing to take on some longevity risk if returns and the structuring of the transaction in terms of duration, standardization, and transparency seem attractive (in relation to the risks taken).

- *Hedging instruments:* For transferring longevity risk from one party to another, we consider a wide selection of cash flow and value hedges in terms of payout structure, maturity, and underlying index population (IP). In particular the latter allows for customized as well as index-based versions of the same base instrument that involve varying levels of population basis risk for the hedger.
- *Stochastic mortality model:* For the stochastic modeling of future mortality, we rely on the multipopulation actual mortality trend (AMT)/estimated mortality trend (EMT) modeling framework of Börger, Freimann et al. (2021) and Börger, Schupp, et al. (2021), which offers several appealing features for our purposes. First, it simultaneously models all components of longevity risk that are relevant for a proper assessment of population basis risk in index-based hedging, namely, long-term mortality trend risk of the overall population, potentially differing mortality characteristics in specific portfolio populations, and idiosyncratic fluctuations in portfolios of limited size. Second, it allows for pathwise derivations of economic capital and Solvency Capital Requirements (SCRs) for longevity risk in line with Solvency II guidelines over the entire term of a hedge contract.

Referring to our research questions outlined above, our primary concern is to analyze how the dynamics of this market evolve with increasing market saturation. For our numerical analyses, we therefore consider three exemplary “market stages” of rising saturation with respect to longevity risk-taking. These stages are characterized by the amount of longevity risk that is already being borne by the reinsurer: an *early stage* at which the reinsurer has not written any longevity business yet, an *advanced stage* with some longevity exposure, and a *saturated stage* at which longevity risk accounts for a substantial part of the reinsurer’s overall risk profile. On the basis of these market stages, our numerical analyses are carried out in two steps.

In the first step, we focus on longevity transfers from primary hedgers to the reinsurance sector and discuss the suitability of different hedge designs for this purpose. Our analyses demonstrate that prices a reinsurer can offer heavily depend on the reinsurer’s available diversification opportunities within its business mix. At the early market stage when longevity risk only makes up a marginal portion of the reinsurer’s overall risk profile, it is economically attractive for both parties to transfer longevity risk completely to the reinsurer taking full advantage of the available diversification capabilities. As more and more longevity risk is transferred to the reinsurance sector, reinsurance prices gradually increase due to shrinking marginal diversification benefits.

From these findings, we draw several far-reaching conclusions regarding the competitive dynamics of this market that complement the existing literature on longevity transfers in two main respects. First, we argue that this distinct price sensitivity with respect to the available diversification capacity constitutes a major “antimonopolistic” feature of this market: Since the competitiveness of established risk takers declines with increasing market share, this market will expand to a wider set of competing market players rather than towards a “winner-takes-it-all” market. Second, we clearly identify a level of market saturation beyond which longevity derisking becomes economically unattractive to primary hedgers. This provides clear evidence for the existence of a capacity constraint in the reinsurance sector, which was postulated by Blake et al. (2019) and Kessler (2021).

In the second step, we address the potential role of the capital market. We consider two potential market entry points for investors: transactions with reinsurers or with primary hedgers. We argue that the maximum risk premium that a longevity hedger would be willing to pay for a capital market instrument depends on the state of the competing reinsurance sector. Specifically, we show that the market risk premiums rise with increasing reinsurance-market

saturation and derive the maximum Sharpe ratios that a competitive investor can earn with different index-based instruments in the three considered market stages.

From this, we derive findings that again provide several novel insights into the dynamics of this market. First, our results suggest that the declining competitiveness of the reinsurance sector with increasing market saturation leaves more room for capital market investors. Second, we show that it can be of benefit to all market participants when investors engage in transactions with a reinsurer rather than directly with primary hedgers since risk components that are well diversifiable within the reinsurer's business mix should remain with the reinsurer and other components should be further passed on to the capital market to ensure an efficient usage of economic capital. Finally, we analyze the suitability of different hedge designs for reconciling reinsurers' and capital market investors' interests. In particular, we find that properly engineered index-based value hedges are most suitable for this second link of the risk transfer chain.

The remainder of the paper is organized as follows. In Section 2, we give a detailed descriptive presentation of our model setup. Supplementary technical details are provided in Appendix A. In Section 3, we provide an overview of our model parametrization. In particular, we distinguish three "market stages" characterized by the amount of longevity risk which the reinsurer has already assumed. We begin our numerical analyses in Section 4 with a brief discussion of the individual perspectives of the primary hedger and the reinsurer and analyze how reinsurance prices differ between the market stages. Subsequently in Section 5, we analyze longevity transfers from primary hedgers to reinsurers and discuss the suitability of different hedge designs for this purpose. In particular, we analyze how the market dynamics evolve along the three market stages. Section 6 then addresses the involvement of capital market investors. In particular, we derive the maximum Sharpe ratios that an investor can earn with various index-based instruments in the different market stages. This allows us to assess the suitability of different hedge designs for this link of the longevity risk transfer chain and to draw conclusions on the roles of the different market players. Our numerical analyses are complemented by some sensitivity analyses with respect to selected modeling assumptions in Section 7. Finally, Section 8 concludes.

2 | MODEL SETUP

As outlined in Section 1, we now establish our model for the longevity risk transfer market. To keep the model complexity at a reasonable level and the focus on longevity risk, we make several simplifying assumptions. First, we assume that all market participants are fully hedged against changes in interest rates and simply rely on a constant risk-free interest rate r for discounting purposes. Second, we do not explicitly deal with operational risk, operational costs, and counterparty credit risk.² Third, we assume that all market participants rely on the same mortality model and have access to the same underlying mortality data. In particular we do not consider model risk or potential asymmetries in mortality information.³

²Biffis et al. (2016) find that the costs of posting collateral for the purpose of mitigating counterparty credit risk in longevity hedging are generally immaterial.

³In practice, information asymmetries might play a role on both sides of the market. While primary holders of longevity risk might have a better understanding of their book characteristics, risk takers (such as specialized reinsurers or buyout firms) might have developed superior longevity risk valuation techniques. For a deeper discussion on these issues and the potential implications on the longevity risk transfer market, we refer to Biffis and Blake (2010, 2013) and Chen et al. (2023).

In Section 2.1, we present our underlying stochastic mortality modeling framework. Afterwards, we introduce the market participants and define their hedging or investment objectives in Section 2.2. Finally, Section 2.3 introduces the considered hedging instruments. We give a rather descriptive presentation of our setup in this section and refer the reader to Appendix A for technical details.

2.1 | Stochastic mortality modeling framework

Following Börger, Freimann et al. (2021) and Börger, Schupp et al. (2021), our model consists of two components: the so-called *AMT simulation model* and the so-called *EMT valuation model*.⁴

2.1.1 | AMT simulation model

The *AMT simulation model* is calibrated to historical mortality data and is used for projecting mortality in a stochastic simulation. To allow for portfolio-specific mortality characteristics and to analyze the associated population basis risk when using index-based hedges, we specify the model in a multipopulation setting. Building on a multipopulation extension of the model of Cairns et al. (2006), it jointly models the following components of longevity risk:

- First, it captures the risk arising from the uncertainty associated with the overall trend in future mortality improvements, which constitutes the principal longevity risk driver. This long-term mortality trend risk is modeled for a large *reference population*, for which the respective national population is a natural choice. Following Börger and Schupp (2018), we model the dynamics of both period effects of the mortality model as random fluctuations around some underlying continuous and piecewise linear trend. This trend is interpreted as the prevailing (but unobservable) AMT and can experience changes of random extent in both directions at any future point in time.
- Second, the model considers that mortality in specific portfolio populations might systematically differ from the reference population. Following Haberman et al. (2014) and Villegas et al. (2017), we rely on a so-called “characterization approach” based on a division of the reference population into N_{Sub} suitable subpopulations that differ in terms of their socioeconomic characteristics. The mortality characteristics of specific portfolio populations can then be derived from mortality in these subpopulations based on a suitable weighting. Within a common relative modeling approach, socioeconomic mortality differences are modeled via a multivariate random walk with drift (RWD).⁵
- Finally, there is the risk of idiosyncratic mortality fluctuations in portfolios of limited size, which is diversifiable in large populations. If relevant, this risk driver is captured by drawing realizations for survivors over time from a suitable binomial distribution.

⁴We refer the interested reader to Börger, Schupp et al. (2021) for a comprehensive discussion on which of the two model components is relevant for which kind of practical application.

⁵Also Haberman et al. (2014) rely on an RWD for modeling socioeconomic mortality differences in a similar setup. Alternatively, mean-reverting models of the class of vector autoregressive processes are often used in the field of multipopulation mortality modeling, see, for instance, Villegas et al. (2017) for an overview.

A clear distinction between these three risk drivers in the simulation framework is important for a proper assessment of population basis risk in index-based hedging (see Section 2.3).

2.1.2 | EMT valuation model

The valuation of hedges and associated risk capital reductions requires the computation of future SCRs in line with the Solvency II guidelines. To this end, best estimate liabilities need to be derived at future points in time for each simulation path. As argued by Börger, Schupp et al. (2021), an observer at some future point in time generally cannot determine the “true” underlying mortality dynamics. In particular, the prevailing mortality trend is unobservable since mortality data is sparse and blurred by random effects. Hence, the consistent derivation of best estimate mortality at future valuation dates based on observed mortality can only be based on an estimate for the prevailing AMT and therefore requires a second model component.

The so-called *EMT valuation model* consists of estimators for the dynamics of both systematic longevity risk drivers based only on mortality observed in previous years:

- Following Börger, Schupp et al. (2021), the EMT at any future point in time T is derived in an intuitive way by applying a weighted linear regression on the most recent observed period effects. It represents a reasonable time- T estimate for the unobservable AMT.
- Inspired by Cairns and El Boukfaoui (2021), we also derive so-called *experience ratios* that estimate the prevailing level and trend in socioeconomic mortality differences relative to the overall population. For consistency, we also rely on a suitable weighted linear regression on the most recent data points.

2.2 | Market participants and their hedging (or investment) objectives

In this section, we briefly discuss the hedging (or investment) objectives of the three market participants. The technical details (in particular on their computation of SCRs and economic capital) are given in Appendix A.2.

2.2.1 | Primary hedger

As primary hedger, we consider an annuity provider that seeks to (partly) hedge its exposure to longevity risk. For the sake of simplicity, we assume that the portfolio to be hedged is closed to new business and consists of immediate life annuities for a single cohort of size N_{Book} and age x_R at time zero that pay one unit of currency at the beginning of each until the beneficiary dies. The population of policyholders is interpreted as a subgroup of the reference population with specific socioeconomic characteristics with a portion of η_p belonging to subpopulation p and $\eta_1 + \dots + \eta_{N_{Sub}} = 1$. Assuming the portfolio size N_{Book} to be moderate, the hedger also faces idiosyncratic small sample risk.

SCRs for longevity risk

In line with Solvency II requirements, the primary hedger holds sufficient equity in every year to withstand all losses that may occur due to a change in (expected) mortality over a horizon of 1 year with a probability of at least 99.5%. We assume that SCRs are computed by means of a partial internal model for longevity risk⁶: First, the AMT simulation model simulates the three components of longevity risk over a horizon of 1 year, then the liabilities are reevaluated with the EMT valuation model, and finally the SCR is derived as the 99.5th percentile of the resulting change in the best estimate liabilities. This approach can also be applied to derive future SCRs: On the basis of a given outer simulation path in our combined AMT/EMT setup, at any time T , a nested 1-year simulation (analogously to the one at time zero) can be started to compute the SCR at time T .

Taking the associated cost of capital (and its uncertainty arising from stochastic future mortality) properly into account, the primary hedger's time-zero random present value of all future cash flows that are exposed to longevity risk is given by (cf. Börger, Freimann, et al., 2021)

$$\Pi := L(0) + CoC, \quad (1)$$

where $L(0)$ denotes the random present value of all future liabilities, that is, benefit payments to surviving annuitants, and CoC represents the random present value of all cost of capital for supporting the annuity portfolio over its lifetime based on a cost-of-capital rate of r_{CoC} . Note that the definition of Π coincides in terms of its mean with the Technical Provisions under Solvency II, which are defined as the sum of the best estimate liabilities and a risk margin computed with a cost-of-capital approach (cf. Börger, 2010).

Hedging objectives

With a longevity hedge H in place, the time-zero random present value of all future hedged liabilities is

$$\Pi_H := L(0) - H(0) + CoC_H, \quad (2)$$

where $H(0)$ is the time-zero random present value of all future cash flows from the hedging instrument (defined in Section 2.3), and CoC_H denotes the cost of capital for the hedged liabilities. Compared with the case without hedging, the inclusion of the hedging instrument has several implications (cf. Börger, Freimann, et al., 2021):

- The offsetting hedge payouts typically reduce the variability in future liabilities.
- The hedging costs, that is, the hedge provider's risk premium on top of the objective best estimate value, which is implicitly included in $H(0)$, typically increases the hedger's liabilities.
- The risk-mitigating effect reduces the SCRs over the term of the hedge, which reduces the cost of capital and therefore also Π_H .

⁶In particular, due to some structural shortcomings (cf. Börger, 2010), we do not deal with effects that arise from the specific structure of the Solvency II standard formula, that builds on a simplified one-off shock approach for longevity risk. The interested reader is referred to Börger, Freimann et al. (2021) for a recent study on the economic impact of longevity hedging under a risk-based internal model compared with that under the Solvency II standard formula.

Following Börger, Freimann et al. (2021), we assume that primary hedgers simultaneously have two hedging objectives:

- On the one hand, the hedge should effectively reduce the variability in future liabilities caused by uncertain future mortality. Hence, the *hedge effectiveness* of an instrument H is assessed by means of a risk measure ρ in terms of the achieved relative level of risk reduction, that is,

$$HE_{\rho}(H) := 1 - \frac{\rho(\Pi_H - \mathbb{E}(\Pi_H))}{\rho(\Pi - \mathbb{E}(\Pi))}. \quad (3)$$

The maximum hedge effectiveness of one means that longevity risk is completely eliminated.

- On the other hand, the hedge should provide a cost-efficient reduction in the cost of capital. Hence, the *capital efficiency* of a hedge H is measured as the expected proportionate cost of capital saving net of hedging costs relative to the unhedged case, that is,

$$CE(H) := 1 - \frac{\mathbb{E}(CoC_H) - \mathbb{E}(H(0))}{\mathbb{E}(CoC)}, \quad (4)$$

where the expected present value of all hedging instrument cash flows $\mathbb{E}(H(0))$ is typically negative representing the hedging costs (cf. Section 2.3). If a hedge is capital efficient, that is, if $CE(H) > 0$, the hedger has an economic incentive for derisking. However, if the costs are higher than the expected reduction in capital charges, that is, if $CE(H) < 0$, this economic incentive is lacking, and the hedge is not economically viable.

As shown by Börger, Freimann et al. (2021), in general the most effective hedge does not simultaneously represent the most capital efficient solution. In particular, hedgers might be willing to retain certain layers of their longevity risk exposure if the associated costs of hedging exceed the generated reduction in the cost of capital. Therefore, cost-efficient partial or index-based solutions might constitute economically viable options.

2.2.2 | Reinsurer

We consider a reinsurer whose appetite for taking on (or laying off) longevity risk is primarily driven by diversification effects with the existing business mix. His portfolio consists of three distinct LOBs that represent longevity business (\mathcal{L}), mortality business (\mathcal{M}), and other nonlife business (\mathcal{O}) (such as property/casualty). Relying on an internal economic capital model, the reinsurer evaluates and prices longevity transactions with regard to an anticipated target return on equity rate.

Parametrization of the LOBs

The LOBs are roughly specified as follows (a detailed description is given in Appendix A.2):

- *Longevity (\mathcal{L}):* A portfolio of immediate life annuities that consists of various cohorts, with cumulative face values of annual annuities $F^{\mathcal{L}}$.

- *Mortality* (\mathcal{M}): A portfolio of term life insurance policies with starting age x_M that mature at the retirement age x_R . The cumulative face value of death benefits is F^M .
- *Other* (\mathcal{O}): This LOB comprises all remaining nonlife risks, which are assumed to be uncorrelated with future mortality. To avoid over-complexity, we assume these risks to be lognormally distributed with mean F^O and Coefficient of Variation (CoV) CoV^O in each year $t \geq 0$.

The face values F^L , F^M , and F^O allow to scale the different LOBs and thereby to vary the reinsurer's risk profile. To obtain a "stable" business mix,⁷ the portfolio is projected as follows: the cohorts

age (or die) according to realized mortality, and a new cohort with starting age of x_R years (for the longevity book) or of x_M years (for the mortality book), respectively, enters the portfolio at the beginning of each year (the corresponding face values are specified in Appendix A.2).

Economic capital model

For the computation of economic capital, we assume that the reinsurer also relies on the 99.5% Value-at-Risk (VaR) risk measure as utilized under Solvency II in conjunction with a company-specific cost of economic capital rate (to be specified below).⁸ Accordingly, the economic capital in year t is derived as the 99.5% VaR of the portfolio-wide loss that may arise over the year, that is,

$$EC(t) := VaR_{99.5\%}(L^L(t) + L^M(t) + L^O(t)), \quad t \geq 0, \quad (5)$$

where $L(t)$ denotes the loss in year t for the respective LOB. Analogously to the primary hedger's SCRs, also $EC(t)$ is derived empirically in a nested Monte Carlo simulation: For each point in time along an outer simulation path, a nested 1-year simulation of all relevant stochastic risk factors is carried out to derive the VaR. Since a reinsurance portfolio is typically rather large, we assume that idiosyncratic small sample risk is negligible. Portfolio-wide economic capital provides a reasonable indicator for the overall risk profile as it implicitly accounts for interdependencies between the different LOBs. However, it does not provide insights into the individual risk contributions. A reasonable capital allocation principle is required to decompose the portfolio-wide economic capital into a sum of LOB-specific marginal risk contributions (cf. Rosen & Saunders, 2010; Tasche, 2008). The widely used *Euler allocation principle* yields the following additive decomposition:

$$EC(t) = EC^L(t) + EC^M(t) + EC^O(t), \quad t \geq 0, \quad (6)$$

where $EC^{LOB}(t)$ is interpreted as the marginal economic capital that is required to support each LOB in year t . Setting the expected present value of economic capital that is allocated to a specific LOB over all consecutive 1-year horizons in relation to the corresponding portfolio-wide capital, the *proportional risk contribution* of each LOB is measured as

⁷As an alternative to this going concern assumption, we also considered a "run-off" approach by assuming that the reinsurer is closed to new business in future years and found that our main findings and conclusions remain unaffected. For the sake of brevity, we omit details.

⁸In practice, the risk measure for the internal economic capital model follows internal risk management guidelines and might therefore differ from regulatory capital computations.

$$PRC(\mathcal{LOB}) := \frac{\sum_{t \geq 0} (1+r)^{-(t+1)} \mathbb{E}(EC^{\mathcal{LOB}}(t))}{\sum_{t \geq 0} (1+r)^{-(t+1)} \mathbb{E}(EC(t))}, \quad \mathcal{LOB} \in \{\mathcal{L}, \mathcal{M}, \mathcal{O}\}, \quad (7)$$

where $PRC(\mathcal{L}) + PRC(\mathcal{M}) + PRC(\mathcal{O}) = 1$. This measure characterizes the reinsurer's risk profile by indicating which proportion of the overall risk exposure originates from which LOB. It will be used in our numerical analyses to characterize different stages of the longevity risk transfer market.

Pricing longevity transactions

When entering into a hedge contract H to take on (or to lay off) longevity risk, the reinsurer's economic capital in year t is given by

$$EC(t) := \text{VaR}_{99.5\%}(L^{\mathcal{L}}(t) + L^{\mathcal{M}}(t) + L^{\mathcal{O}}(t) + L_H(t)), \quad t \geq 0, \quad (8)$$

where $L_H(t)$ denotes the additional potential loss that may arise from the instrument H in year t . The sensitivity of the reinsurer's economic capital with respect to the inclusion of the instrument H highly depends on the correlation with the existing business mix. Taking the arising diversification effects properly into account, the economic implications can be evaluated by considering the marginal economic capital $EC_H(t)$ that is allocated to the contract H by Euler's principle:

$$EC(t) = EC^{\mathcal{L}}(t) + EC^{\mathcal{M}}(t) + EC^{\mathcal{O}}(t) + EC_H(t), \quad t \geq 0. \quad (9)$$

- If the marginal risk contribution is positive, that is, if $EC_H(t) > 0$, the contract requires the provision of adequate economic capital.
- Otherwise, that is, if $EC_H(t) < 0$, the transaction decreases the overall risk exposure due to strong diversification effects with the existing portfolio, which frees up economic capital.

From an economic cost-of-capital perspective, the transaction should offer an attractive expected profit (or should be offered at attractive conditions, respectively) in relation to the tied-up (or freed-up, respectively) economic capital over its term. These considerations motivate the definition of the $RORAC^9$ over the multiyear term of the hedge contract as

$$RORAC(H) := \frac{\mathbb{E}(-H(0))}{\mathbb{E}\left(\sum_{t \geq 0} (1+r)^{-(t+1)} EC_H(t)\right)}, \quad (10)$$

where the numerator $\mathbb{E}(-H(0))$, which will be specified in Section 2.3, represents the expected profit if positive (or loss if negative, respectively) from the transaction for the reinsurer, and the denominator represents the present value of marginal economic capital, which is required to support the contract over its lifetime. Given an anticipated target return on equity rate roe , that explicitly includes a business margin on top of the regulatory

⁹Pricing and performance measurement by RORAC is widely used by both practitioners and academics and is generally applied in a one-period setup, see, for instance, Braun et al. (2018), Tasche (2008), and the references cited therein. In contrast, we extend the RORAC concept to a multiperiod setting by considering the expected profit over the entire term of the hedge contract in relation to the expected present value of the tied-up economic capital.

cost-of-capital rate, the reinsurer is invariant with respect to all longevity transactions H that satisfy $RORAC(H) = roe$:

- When offering longevity protection to primary hedgers, the hedging instrument's forward rates (specified in Section 2.3) are priced such that the contract generates in expectation the anticipated target return on equity.
- When laying off some longevity risk to capital market investors to free up economic capital, the reinsurer is willing to pay a risk premium to investors that does not exceed the generated cost of economic capital relief.

Hence, as long as longevity transactions meet this RORAC criterion, both taking on or laying off longevity risk constitutes an economically viable option. The price at which transactions meet this criterion depends on the state of the reinsurer. Transactions that fail this criterion do not provide the anticipated return on equity (or sufficient capital relief in relation to the costs of hedging, respectively) and are therefore not economically viable for the reinsurer. In our numerical analyses, we will rely on this RORAC criterion to analyze the economic sustainability of different longevity transactions in different market stages.

2.2.3 | Capital market investor

Following the considerations of Blake et al. (2019) (and works cited therein) on the development of a deep and liquid capital market for longevity risk, we assume that investors (such as private equity investors or investment funds) might be willing to enter the longevity risk transfer market under the following conditions:

- Transactions should offer an attractive return in relation to the risks taken. In light of the low correlation of longevity risk with traditional types of assets, taking on some longevity risk for an adequate risk-adjusted return might constitute a valuable investment strategy to improve the overall portfolio performance.
- The instrument should be structured in a standardized way based on transparent, reliable, sufficiently large, and publicly available mortality indices. This ensures market liquidity and leaves aside any opaque form of longevity risk that might be inherent in the hedger's book population.
- For the sake of a manageable contract duration and to maintain market liquidity, the term of the contract should be at most 20 years, preferably (much) shorter.

Investment objective

Following Loeys et al. (2007) and Barrieu and Veraart (2016), we measure the risk-adjusted return for a hedging instrument H with time to maturity of τ years in terms of the annualized Sharpe ratio of the instrument's cash flow profile, that is,

$$S(H) := \frac{1}{\sqrt{\tau}} \frac{\mathbb{E}(-H(0))}{\sqrt{\text{Var}(-H(0))}}, \quad (11)$$

where the numerator is generally positive and represents the expected profit for the investor, and the denominator measures its variability in terms of its standard deviation. This rather

simple performance measure offers several advantages: First, it allows for an intuitive interpretation and is widely known. Second, it is scale-invariant with respect to the size of the contract and comparable across different instruments with varying times to maturity. Third, it does not require any assumptions on the investor's existing portfolio, which would generally be difficult to make. Nevertheless, the individual threshold value above which a Sharpe ratio is attractive to investors will of course depend on their individual risk appetite, portfolio characteristics, and diversification opportunities.

Market entry points

From the investor's perspective, there are two potential market entry points:

- An investor could offer instruments to reinsurers that provide an economically attractive level of risk reduction in terms of their RORAC criterion.
- Alternatively, he could enter directly into a deal with primary hedgers. However, in the presence of the competing reinsurance sector, capital market instruments need to provide a higher capital efficiency at a comparable level of hedge effectiveness (or the other way around) to be attractive to primary hedgers.

In either case, the price that a longevity hedger would be willing to pay for a capital market instrument (and thereby also the maximum Sharpe ratio that investors can demand) depends, either directly or indirectly, on the appetite for longevity risk in the reinsurance sector. In our numerical analyses in Section 6, we analyze these market dynamics and address the question of the optimal market entry point for investors.

2.3 | Hedging instruments

In this section, we introduce longevity swaps, annuity forwards, and q -forwards as hedging instruments for the primary hedger. Following Börger, Freimann et al. (2021), we rely on different IPs for constructing customized as well as index-based versions of the same base instrument that involve varying levels of population basis risk for the hedger.

2.3.1 | Index populations

A hedging instrument can be linked to one of the following IPs:

- $IP = \mathcal{R}$: The hedge payout is linked to the mortality rates of the reference population and therefore solely covers the uncertainty associated with the overall long-term mortality evolution. The variability arising from socioeconomic mortality differences and small sample risk, however, remains with the hedger.
- $IP = \mathcal{S}$: In this case, the payouts are linked to the subpopulation-specific mortality rates and based on the prevailing experience ratios so that the hedge also provides protection against systematic changes in socioeconomic mortality differences.
- $IP = \mathcal{B}$: Finally, population basis risk can be eliminated by linking the payouts directly to the mortality in the hedger's portfolio population.

2.3.2 | Hedge payout structures

The time- t random present value of all future cash flows from the hedge contract reads as

$$H^{[IP]}(t) := \sum_{s>t}^{\tau} (1+r)^{-(s-t)} CF_H^{[IP]}(s), \quad IP \in \{\mathcal{B}, \mathcal{S}, \mathcal{R}\}, \quad (12)$$

where

- $CF_H^{[IP]}(s)$ denotes the hedging instrument's cash flow at time s (to be specified below), where a positive value represents a payment from the hedge provider to the hedger and vice versa,
- τ denotes the term of the contract in years, and
- IP represents the index population to which the hedge is linked.

All considered instruments are structured as forward contracts such that no payment is due at inception. The hedge provider's risk premium (on top of the objective best estimate rates) is implicitly included in the forward rates so that the time-zero expected present value of all hedging instrument cash flows $\mathbb{E}(H^{[IP]}(0))$ is typically negative representing the expected loss for the hedger, which corresponds in absolute terms to the time-zero value of all risk premiums. The value can be interpreted as the expected costs of hedging for the hedger or—with opposite sign—as the expected profit for the hedge provider, respectively. In what follows, we give an overview of the considered hedge payout structures. For the construction of the index-based instruments, we refer to Appendix A.3.

Longevity swaps

In a longevity swap with term τ , the hedger receives a sequence of payments of the form

$$CF_H^{[IP]}(t) := S_{x_{R+t},t}^{[IP]} - \widehat{S}_{x_{R+t},t}^{[IP]}, \quad 0 < t \leq \tau, \quad IP \in \{\mathcal{B}, \mathcal{S}, \mathcal{R}\}, \quad (13)$$

where the floating payments $S_{x_{R+t},t}^{[IP]}$ are either derived from the actual number of survivors in the hedger's book population (in case of $IP = \mathcal{B}$) or from a suitable survivor index (in case of $IP = \mathcal{S}, \mathcal{R}$). The corresponding forward rates $\widehat{S}_{x_{R+t},t}^{[IP]}$ are specified at inception by the hedge provider and comprise best estimate annuity payments plus a risk premium. By construction, a fully customized ($IP = \mathcal{B}$) unlimited ($\tau = \infty$) longevity swap provides a perfect hedge. For limited terms, long-term annuity obligations for oldest ages that are due after maturity τ remain uncovered.

Annuity forwards

In an annuity forward with maturity τ , the hedger receives the time- τ best estimate value of all future annuity payments in exchange for a forward liability resulting in a payment of

$$CF_H^{[IP]}(\tau) := LF^{[IP]}(\tau) - \widehat{LF}^{[IP]}(\tau), \quad IP \in \{\mathcal{B}, \mathcal{S}, \mathcal{R}\}. \quad (14)$$

Specifically, $LF^{[\mathcal{B}]}(\tau)$ equals the time- τ best estimate value of the hedger's future liabilities according to the then prevailing best estimate mortality assumptions. The forward liability $\widehat{LF}^{[IP]}(\tau)$ is the time-zero expectation of all liabilities after time τ plus a risk premium and again specified at inception by the hedge provider. For the index-based designs ($IP = \mathcal{S}, \mathcal{R}$), the payout is derived analogously based on a suitable liability index. This value hedge agreement protects the hedger against any changes in longevity level and trend assumptions that materialize in the underlying IP over the course of the hedge contract. In particular, the customized contract design based on $IP = \mathcal{B}$ factors in all observable mortality information from the hedger's portfolio population between time zero and maturity τ .

q-Forwards

In a q -forward with term τ , the hedger swaps realized mortality rates against a fixed forward rate, yielding a single hedge payout at maturity of the form

$$CF_H^{[IP]}(\tau) := \widehat{Q}_{x_R+\tau, \tau}^{[IP]} - Q_{x_R+\tau, \tau}^{[IP]}, \quad IP \in \{\mathcal{S}, \mathcal{R}\}. \quad (15)$$

Here, the floating lag $Q_{x_R+\tau, \tau}^{[IP]}$ is derived from the realized mortality rates in the underlying IP. In our numerical analyses, we focus on a single q -forward with given reference age $x_R + \tau$ and reference year τ and consider varying choices for τ .¹⁰ The calibration of the optimal number of q -forward contracts for the primary hedger¹¹ is described in Appendix A.3. The optimal number of q -forwards is calibrated at time zero and remains fixed up to maturity of the contract.

3 | PARAMETRIZATION OF MODELING FRAMEWORK

In this section, we give an overview of our model parameters, establish three different market stages, and provide the parametrization of the considered hedging instruments.

3.1 | Mortality model

For the simulation of future mortality, we follow Freimann (2021) and rely on his calibration of our stochastic mortality model to data of English and Welsh males. The corresponding national population represents the natural choice for the reference population, and we use data of English males sorted by quintiles of the Index of Multiple Deprivation (IMD) for the calibration of $N_{Sub} = 5$ suitable subpopulations of different socioeconomic status.¹² We refer the reader to

¹⁰The setup can be extended straightforwardly to a hedge portfolio of multiple q -forwards of different times to maturity.

¹¹This optimal number of q -forwards is required to properly assess the effectiveness of a q -forward portfolio for the primary hedger. For the reinsurer, however, this specific number of contracts does not play a role when deriving marginal economic capital based on Euler's infinitesimal allocation principle, see Appendix A.2.

¹²We are grateful to the Human Mortality Database and to the Office for National Statistics, respectively, for the provision of these data sets. Mortality data sorted by the IMD is frequently used to investigate and model socioeconomic mortality differences in England, compare, for example, Lyu et al. (2023) or Wen et al. (2023).

Freimann (2021) for all technical details, including the resulting model parameters. Throughout this work, we rely on 2500 outer simulation paths along with additional 2500 inner 1-year simulations for the computations of SCRs and economic capital.

3.2 | Liabilities and capital models

Our model parametrization is summarized in Table 1. We rely on the same set of socioeconomic weights for all longevity and mortality portfolios, which assigns an increasing weight to subpopulations of higher socioeconomic status to account for selection effects.¹³ For the assessment of hedge effectiveness, we follow Börger, Freimann et al. (2021) and rely on the 90% Tail-Value-at-Risk (TVaR).¹⁴ The reinsurer's anticipated target return on equity rate is set to 9%, which explicitly includes an anticipated business margin on top of the regulatory cost-of-capital rate of 6%.¹⁵ Finally, the face values F^{LOB} for the reinsurer's LOBs and the CoV for other nonlife risks are chosen such that reasonable market stages with respect to the LOB's risk contributions arise (see below).

3.3 | Market stages

We consider three different market stages with respect to the initial amount of longevity risk in the reinsurer's book by increasing the scaling parameter $F^{\mathcal{L}}$ while keeping the other two LOBs fixed. Table 2 shows the resulting proportional risk contributions, and we interpret the market stages as follows:

- *Early stage*: The reinsurer has not written any longevity business yet.
- *Advanced stage*: The reinsurer has accumulated a longevity exposure of size $F^{\mathcal{L}} = 4 \cdot 10^5$ in the past, which makes up approximately 28.5% of the overall risk profile.
- *Saturated stage*: The reinsurer has accumulated twice as much longevity business as in the previous stage. The resulting proportional risk contribution of approximately 60.8% indicates that longevity risk now accounts for a substantial part of the reinsurer's overall risk profile.

3.4 | Hedging instruments

Finally, Table 3 gives an overview of the IPs and maturities which we consider for the hedging instruments. For longevity swaps, we consider terms between 1 and 65 years, that is, until the underlying cohort reaches the age 130, which corresponds to the immanent limiting age in the AMT simulation model. Since annuity forwards and q -forwards are structurally intended for shorter hedge horizons, we consider times to maturity of up to 25 years (cf. Börger, Freimann, et al., 2021).

¹³We also considered different weights for the reinsurer's mortality book as well as for the primary hedger's annuity portfolio, but found that this does not have a significant impact on our main findings.

¹⁴We also considered alternative risk measures, such as variance and VaR, and found that the choice of risk measure does not have a substantial impact on our main results.

¹⁵We investigate the impact of this model parameter in Section 7.

TABLE 1 Liability and capital model parameters.

Category	Description	Parameter	Value
General	Retirement age	x_R	65
	Socioeconomic book composition	η	(10%, 15%, 20%, 25%, 30%)
	Risk-free interest rate	r	2%
Primary hedger	Cost-of-capital rate	r_{CoC}	6%
	Risk measure for hedge effectiveness	ρ	$TVaR_{90\%}$
	Initial book size	N_{Book}	10,000
Reinsurer	Face value of longevity exposure	$F^{\mathcal{L}}$	See Table 2
	Face value of mortality exposure	$F^{\mathcal{M}}$	$1 \cdot 10^8$
	Face value of other business	$F^{\mathcal{O}}$	$3 \cdot 10^5$
	Starting age of mortality book	x_M	50
	CoV of other business	$CoV^{\mathcal{O}}$	0.20
	Target return on equity rate	roe	9%

TABLE 2 Proportional risk contributions $PRC(\cdot)$ of each LOB in the three market stages.

Market stage	$PRC(\mathcal{L})(\%)$	$PRC(\mathcal{M})(\%)$	$PRC(\mathcal{O})(\%)$
Early ($F^{\mathcal{L}} = 0$)	0	38.5	61.5
Advanced ($F^{\mathcal{L}} = 4 \cdot 10^5$)	28.5	29.0	42.5
Saturated ($F^{\mathcal{L}} = 8 \cdot 10^5$)	60.8	16.9	22.3

Abbreviations: LOB, lines of business; PRC, proportional risk contribution.

TABLE 3 Index populations and maturities of hedging instruments.

Instrument	Description	Parameter	Values
Longevity swaps	Index population	IP	$\mathcal{B}, \mathcal{S}, \mathcal{R}$
	Maturity	τ	1, ..., 65
Annuity forwards	Index population	IP	$\mathcal{B}, \mathcal{S}, \mathcal{R}$
	Maturity	τ	1, ..., 25
q -Forwards	Index population	IP	\mathcal{S}, \mathcal{R}
	Maturity	τ	1, ..., 25

4 | INDIVIDUAL PERSPECTIVES OF PRIMARY HEDGERS AND REINSURERS

We start our numerical analyses with a brief discussion of the hedging instruments' attractiveness to the primary hedger (in Section 4.1) and the reinsurer (in Section 4.2) from their individual perspectives. The perspective of the capital market investor will be discussed in Section 6.

4.1 | Primary hedger: The benefits of hedging

As we will demonstrate in Section 4.2, prices for longevity protection (and hence the costs of hedging for the primary hedger) depend on the state of the reinsurer. To simplify the discussion in this section, we leave the reinsurer aside and disregard the hedging costs by simply assuming that all forward rates are derived at a best estimate basis without any risk loading. We will include the reinsurer's pricing approach (and the resulting costs for the hedger) in Section 5. For the sake of comprehensibility, this section recaps some findings of Börger, Freimann et al. (2021), who conduct a comprehensive analysis of these instruments from the perspective of a longevity hedger.

Figure 1 shows the hedge effectiveness (in the left panel) and the capital efficiency (in the right panel) of the considered instruments depending on their time to maturity. Throughout this work, we rely on the following visualization technique of Börger, Freimann et al. (2021): The dot size increases in the term of the contract, and different symbols are used to identify the three IPs: circles for fully customized hedges ($IP = \mathcal{B}$), squares for subpopulation-linked instruments ($IP = \mathcal{S}$), and crosses for instruments that are based on the reference population ($IP = \mathcal{R}$).

Comparing the instruments in terms of hedge effectiveness (in the left panel), we observe that the unlimited fully customized longevity swap offers the maximum hedge effectiveness of 100% since it provides a perfect cash flow hedge for the primary hedger's liabilities. When considering longevity swaps with shorter terms, however, the hedge effectiveness declines since annuity payments for oldest ages remain uncovered. Since long-term annuity obligations for oldest ages contain a substantial amount of longevity risk, rather long contract terms beyond 28 years are required to obtain a hedge effectiveness of at least 50% with longevity swaps. In contrast, value hedge agreements like annuity forwards or single q -forwards with fixed maturity offer a hedge effectiveness of up to 72% (or 60%, respectively) over rather short contract durations between 15 and 20 years. This is caused by the single lump sum payment at maturity that partly offsets

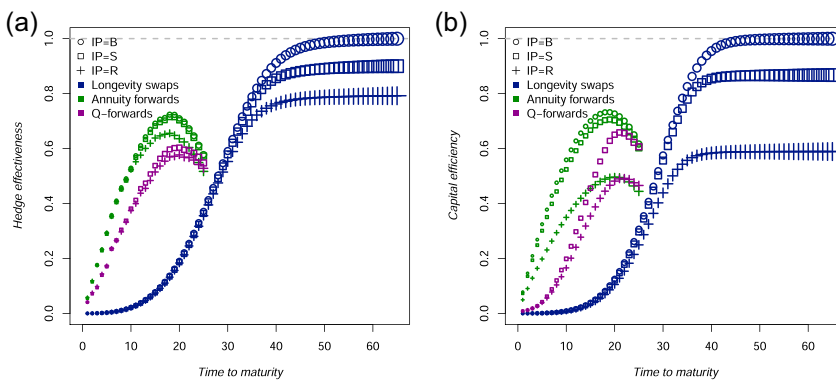


FIGURE 1 Hedge effectiveness (left panel) and capital efficiency (right panel) of the considered hedging instruments for the primary hedger under the assumption that all forward rates are derived at a best estimate basis. The symbol size increases with contract duration. (a) Hedge effectiveness and (b) capital efficiency. IP, index population.

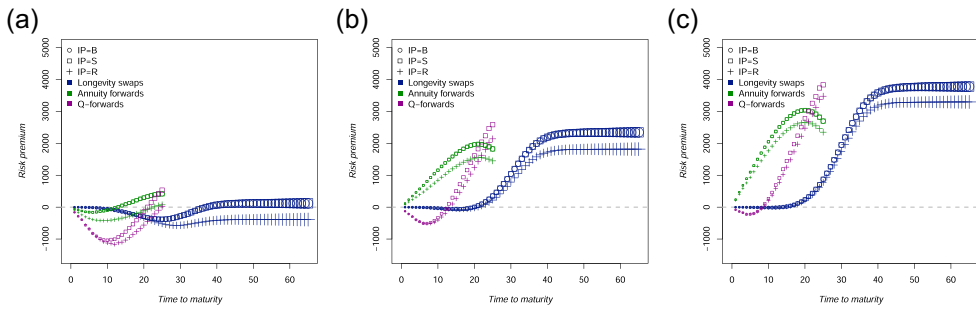


FIGURE 2 Risk premiums of various hedging instruments of different terms at the considered market stages: early stage (left panel), advanced stage (middle panel), and saturated stage (right panel). The symbol size increases with contract duration. (a) Early stage, (b) advanced stage, and (c) saturated stage. IP, index population.

a potential increase in liabilities arising from changing mortality over the term of the contract.

When comparing the results for any hedge payout structure among the IPs, we observe that compared with $IP = \mathcal{R}$, which only captures long-term mortality trend risk, hedge effectiveness gradually increases when additionally socioeconomic mortality differences ($IP = \mathcal{S}$) and small sample risk ($IP = \mathcal{B}$) are covered. Hence, we conclude that the latter two longevity risk drivers play a significant role for the primary hedger in our model parametrization. The lower levels of hedge effectiveness for the index-based instruments ($IP = \mathcal{S}, \mathcal{R}$) reflect the amount of population basis risk that arises from the imperfect correlation between the hedge indices and the mortality evolution of the primary hedger's book population.

When comparing the instruments in terms of capital efficiency in the right panel, we find structurally similar patterns: Again, the unlimited fully customized longevity swap provides the maximum risk capital reduction of 100%, value hedge agreements appear more suitable for shorter hedge horizons, and capital efficiency clearly increases when more longevity risk components are covered by the IP. The primary hedger has to accept significant “haircuts” for population basis risk when using index-based instruments, in particular for $IP = \mathcal{R}$, due to the significant variability in socioeconomic mortality differences.¹⁶

To summarize, when disregarding the costs of hedging, the unlimited fully customized longevity swap presents the most attractive instrument to the primary hedger since it simultaneously outperforms all other instruments in terms of both hedge effectiveness and capital efficiency. However, it requires a rather long contract duration to be effective. Value hedge agreements like annuity forwards or q -forwards are, by construction, much more suitable for shorter hedge horizons. Without taking any potential cost advantages of standardized index-based contracts into account for now, index-based hedging appears less attractive than customized hedging due to the involved population basis risk.

¹⁶This is mainly due to the rather affluent socioeconomic book structure and due to the RWD-assumption for modeling socioeconomic mortality differences in the AMT simulation model. We refer to Börger, Freimann et al. (2021) for a sensitivity analysis with regard to the stochastic model for projecting socioeconomic mortality deviations.

4.2 | Reinsurer: Pricing longevity transactions

Next, we consider the reinsurer as a supplier of these instruments. Figure 2 shows the time-zero value of the reinsurer's risk premiums (on top of the objective best estimate value) for the considered hedging instruments in the three market stages according to the applied RORAC pricing approach, that is, the expected profit $E(-H(0))$ given $RORAC(H) = roe$.

Before comparing the results among the three market stages, we note that the risk premiums for fully customized contracts ($IP = \mathcal{B}$) are always the same as for subpopulation-linked contracts ($IP = \mathcal{S}$) since diversifiable small sample risk is by assumption not priced by the reinsurer. For instruments that are linked to $IP = \mathcal{R}$, however, the reinsurer demands a slightly lower risk premium since socioeconomic mortality differences represent a systematic risk driver.

In the absence of any initial exposure to longevity risk (early market stage), the reinsurer would be willing to offer all considered instruments at a rather low risk premium. For instance, the risk premium for the unlimited fully customized longevity swap only amounts to 160, which corresponds to 0.1% of the expected liabilities to be hedged. Apparently, the reinsurer can benefit from substantial diversification effects with its other LOBs when adding some longevity exposure to its existing business mix. Interestingly, we find that some contracts can even be offered at a negative risk premium, that is, below their objective best estimate value. This is particularly the case for short-term contracts and can be traced back to significant diversification effects with the reinsurer's mortality business.¹⁷ We will further address the significant impact of the mortality business in Section 7.1.

For any given hedging instrument, the risk premium increases with increasing market saturation (from the left to the right panel) since reinsurance prices for longevity protection increase with shrinking diversification capacity in the reinsurance sector. For instance, the relative risk loading in relation to the expected liabilities to be hedged for the unlimited fully customized longevity swap increases from 0.10% at the early stage to 1.96% at the advanced stage and further to 3.15% at the saturated stage of the market. We conclude that reinsurance prices for longevity protection are significantly driven by the amount of longevity risk that is already being borne by the reinsurer.

This has several far-reaching implications on the competitive dynamics of this market. At an early development stage, those reinsurers with the most extensive diversification opportunities, in particular with mortality risk, have a considerable competitive advantage. However, since reinsurers' diversification opportunities are limited, their prices will gradually rise with each additional longevity acquisition. The resulting decline in competitiveness of risk takers with increasing market share constitutes a remarkable antimonopolistic feature of this market that impedes monopolization. As the longevity risk transfer market evolves, other reinsurers that have not written any longevity business yet might be able to enter the market at highly competitive prices and gain market shares. In consequence, our findings suggest that the longevity risk transfer market will expand to

¹⁷Of course, particularly in early stages of the market, actual prices could deviate from our model prices, for example, due to the hedger's willingness to pay or the reinsurer's costs for setting up the new business model.

a wider set of competing risk takers rather than develop towards a “winner-takes-it-all” market.

This raises the question of how these changing market dynamics affect the economic attractiveness of longevity derisking for the primary hedger. Our findings so far also call for a deeper analysis which instruments provide mutually optimal risk transfers in different market stages and which longevity transactions are economically suboptimal. We address these questions in Section 5.

5 | LONGEVITY TRANSFERS FROM PRIMARY HEDGERS TO REINSURERS

As a next step, we analyze longevity transactions between the primary hedger and the reinsurer, including the derived risk premiums, which represent the hedging costs for the primary hedger. Figure 3 shows the hedge effectiveness against the capital efficiency of the hedging instruments from the primary hedger’s point of view in the three considered market stages. All transactions are priced under the RORAC approach using the anticipated target return on equity. Hence, the reinsurer is invariant with respect to all instruments under consideration. The primary hedger, however, simultaneously evaluates the offered hedging instruments in terms of hedge effectiveness and capital efficiency and might favor some instruments over others.

We first note that for any hedge payout structure and in all market stages, the customized design ($IP = \mathcal{B}$) dominates its subpopulation-linked counterpart ($IP = \mathcal{S}$) which in turn dominates its reference-population-linked counterpart ($IP = \mathcal{R}$) in the sense that it simultaneously provides a higher hedge effectiveness and a higher capital efficiency. While socioeconomic mortality differences and small sample risk pose a significant risk to primary hedgers, they are of much less relevance to larger and more diversified reinsurers. Therefore, our model suggests that it is mutually economically beneficial to transfer these longevity risk components to the reinsurer regardless of the market stage.

We now compare the three market stages: At the early stage (left panel), we observe that the unlimited fully customized longevity swap simultaneously dominates all other instruments in terms of hedge effectiveness and capital efficiency. Hence, the primary hedger has no incentive to consider alternative instruments. From an economic point of view, both parties can mutually

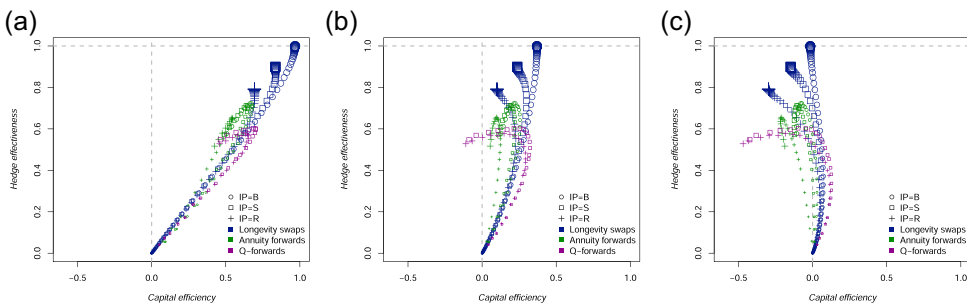


FIGURE 3 Hedge effectiveness (y -axis) versus capital efficiency (x -axis) for the primary hedger depending on the stage of the market. The symbol size increases with contract duration. (a) Early stage, (b) advanced stage, and (c) saturated stage. IP, index population.

benefit from completely transferring longevity risk to the reinsurer as they would fully exploit the available diversification opportunities. The fact that a perfect hedge with an effectiveness of 100% simultaneously offers a rather high capital efficiency of 96.8% (the remaining 3.2% correspond to the price of the hedge) indicates that the early market stage provides attractive conditions for annuity providers to completely swap out their longevity risk exposure in a cost-efficient way.

With increasing market saturation (from the left to the middle panel), however, the capital efficiency of all instruments declines due to the increasing risk premium resulting from the reinsurer's shrinking marginal diversification benefits as shown in Section 4.2. For instance, the capital efficiency of the unlimited customized longevity swap falls to 37%. Nevertheless, completely removing longevity risk by means of an unlimited fully customized longevity swap still presents the superior hedging solution to the primary hedger. This finding confirms the great practical success and the still prevailing market-dominating position of customized longevity swaps in the global longevity risk transfer market, compare Blake et al. (2019).

As the market further evolves (from the middle to the right panel), the capital efficiency of the unlimited customized longevity swap further declines and falls below zero. This means that the costs of hedging eventually exceed the hedger's expected reduction in the cost of capital. Hence, the major economic incentive of cost-efficient capital relief is no longer given for a complete longevity risk transfer. At the same time, partial hedging solutions such as shorter-term longevity swaps and q -forwards still provide a positive capital efficiency of up to 13% and therefore represent economically viable alternatives. These instruments transfer that part of the longevity risk exposure that is associated with younger-age mortality to the reinsurer, whereas longevity risk that is associated with mortality at older ages remains with the hedger. Therefore, they offer significantly lower levels of hedge effectiveness around 30%–40%. The fact that these instruments now dominate in terms of capital efficiency indicates that the reinsurer has a stronger appetite for short-term longevity risk associated with younger-age mortality at this market stage. While this risk is strongly negatively correlated with the reinsurer's mortality business and therefore still well diversifiable, the reinsurer's diversification opportunities for the remaining risks are exhausted at this level of market saturation. We will get back to this issue in Section 7.1.

To conclude, we find that increasing market saturation will lead to a capacity constraint in the reinsurance sector. Rising prices for reinsurance-based transactions will adversely affect the attractiveness of longevity derisking for primary hedgers. These findings confirm rough assessments of Michaelson and Mulholland (2014), who conclude that the "longevity risk inherent in the world's aggregate retirement obligations is far in excess of the amount of risk capital the global insurance industry could realistically bring to bear against this risk."¹⁸ Interestingly, our results suggests that the market will first reach a limit for longevity risk associated with old-age mortality. Since long-term obligations for older ages contain a substantial amount of longevity risk that would remain with the primary hedger, these supply constraints may lead to an economically suboptimal allocation of longevity risk among market participants. This may hamper the market's capability for further longevity risk transfers and may eventually even bring the market to a standstill. Therefore, a deeper analysis is warranted of how external risk capital can be optimally deployed to expand the market's risk absorption capacity. We will further address this issue in Section 6.

¹⁸Michaelson and Mulholland (2014), p. 19.

6 | INVOLVEMENT OF CAPITAL MARKET INVESTORS

As argued in Section 2.2.3, there are two potential market entry points for capital market investors: entering directly into a deal with the primary hedger or alternatively indirectly via reinsurers. Either way, the maximum risk premium (on top of the objective best estimate value) that the counterparty would be willing to pay for a capital market longevity hedge is bounded from above:

- From the primary hedger’s perspective, the risk premium needs to be low enough so that the instrument provides a higher level of capital efficiency at a comparable level of hedge effectiveness (or vice versa) compared with the reinsurance-based contracts that are available at the current market stage. Otherwise, the capital market investor would not be competitive as a longevity risk taker against the reinsurance sector.
- From the reinsurer’s perspective, the risk premium should be small enough in relation to the freed up economic capital to meet the RORAC criterion. Otherwise, there would be no sufficiently strong incentive for laying off some longevity risk.

Hence, in either case, the maximum risk premium that an external investor can demand for taking on longevity risk depends (either directly or indirectly) on the stage of the reinsurance market. Deducing the maximum risk premiums as outlined above, Figure 4 shows the maximum annualized Sharpe ratios as defined in Equation (11) that can be earned in the three considered market stages when offering various hedging instruments of different terms to the primary hedger (rhombus) or to the reinsurer (triangle), respectively. In accordance with the investor’s general request for a transparent and standardized instrument of manageable contract duration (cf. Section 2.2.3), we focus on index-based designs that are linked to the reference population ($IP = \mathcal{R}$) with a time to maturity of at most 20 years.

At the early market stage (left panel), we observe that none of the considered instruments provides a positive Sharpe ratio. Clearly, the market is dominated by the pronounced appetite for longevity risk in the reinsurance sector. Reinsurers do obviously not have any economic incentive for laying off longevity risk to external investors when they do not bear any substantial exposure to this risk, and the highly competitive prices for customized reinsurance-based deals at the early market stage make index-based capital market hedges generally unattractive to primary hedgers.

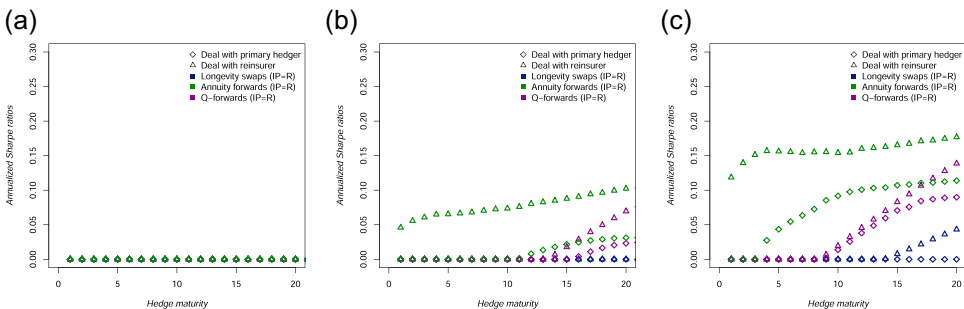


FIGURE 4 Maximum Sharpe ratios that can be earned with different index-based ($IP = \mathcal{R}$) instruments (y-axis) of different terms when entering into a deal with the primary hedger (rhombus) or with the reinsurer (triangle). Negative values are maximized with zero. (a) Early stage, (b) advanced stage, and (c) saturated stage. IP, index population.

With increasing market saturation, however, we observe that the Sharpe ratios rise (in the middle and right panel) for several instruments. Apparently, the declining competitiveness of the reinsurance sector leads to rising longevity risk premiums in the market. Interestingly, we find that the reinsurer (triangle) is willing to pay a higher or equal risk premium for any instrument compared with the primary hedger (rhombus) regardless of the market stage. In other words, our model clearly suggests that it is beneficial to all market participants when investors engage in transactions with a reinsurer rather than directly with primary hedgers. This key result can be traced back to some disadvantages of index-based hedging, which are of higher relevance to primary hedgers than to reinsurers. First, the accompanying population basis risk is naturally more relevant for primary writers of longevity risk than for larger and more diversified reinsurers. Second, as shown in Figure 1 in Section 4.1, hedges with maturities of up to 20 years typically provide medium levels of hedge effectiveness below 70%. For this level of risk reduction, the primary hedger can usually find more cost-efficient reinsurance-based offers in the market, in particular midterm customized longevity swaps with maturities between 30 and 40 years.

The evolution of the market dynamics with all three market participants can hence be summarized as follows: At the early stage of the market, reinsurers enter into a series of unlimited customized longevity swaps with a diverse set of annuity providers to completely absorb their longevity risk. As shown in Section 5, this constitutes a mutually economically optimal risk transfer as long as sufficient diversification capacity is available within the reinsurance sector. As soon as a reinsurer reaches a certain level of saturation with respect to longevity risk, it may pass on selected components of the acquired longevity risk to the capital markets. Well suited instruments for this second link of this risk transfer chain can be inferred from the derived Sharpe ratios in Figure 4. The fact that the Sharpe ratios significantly differ between the instruments indicates that the market demand for longevity protection is higher for certain longevity risk components (and for those instruments that are best suited for transferring these components, respectively) than for others.

We particularly observe that longevity swaps provide by far the lowest Sharpe ratios in all market stages. Even in the saturated stage, the corresponding Sharpe ratios do not exceed 0.05 and remain nonpositive for terms below 15 years. Value hedges however offer positive Sharpe ratios already for much shorter terms. Interestingly, the highest Sharpe ratios in all market stages can be obtained with annuity forwards. In fact, a hedge horizon of around 5 years is sufficient to provide Sharpe ratios of around 0.07 at the advanced market stage or of around 0.16 at the saturated market stage, respectively. Remarkably, an index-based annuity forward with a contract duration of only 1 year offers a potential Sharpe ratio of up to 0.12 at the saturated stage of the market, which by far exceeds all Sharpe ratios that can be earned with any of the considered longevity swaps.

On the basis of these findings, we infer that value hedges are much better suited than cash flow hedges for transferring longevity risk from the (re)insurance sector to the capital markets. One main reason is the efficiency of short-term value hedges in mitigating the adverse impact of a change in best estimate mortality assumptions on the hedger's liabilities. Since this is the principal driver of SCRs and economic capital for longevity risk, an appropriate value hedge agreement can lead to a significant capital relief for the hedger. Hence, this type of instrument offers an efficient risk transfer compressed to a manageable contract duration and is therefore suitable for reconciling reinsurers' and investors' interests.

Another major advantage of value hedges over cash flow hedges is that they can be optimally tailored to the reinsurer's diversification opportunities while keeping the contract duration short. In our mortality model, longevity risk that is associated with younger ages is significantly negatively correlated with the reinsurer's mortality exposure, whereas long-term

annuity obligations for older ages are harder to diversify. With regard to this, the reinsurer can retain the easily diversifiable risk components (i.e., small sample risk, socioeconomic mortality differences, and longevity risk associated with younger ages) and lay off the less diversifiable risks, for example, the short-term or midterm occurrence of a change in the long-term longevity trend in the overall population.

To conclude, we find that capital market investors would enter the market via reinsurers once the market has reached a certain degree of saturation. In this context, short- to midterm value hedges are structurally well suited for efficiently deploying external capital to expand the market’s risk absorption capacity. In return, investors can expect the highest risk-adjusted returns when taking on those components of longevity risk that are most difficult to diversify for reinsurers and for which the market demand is consequently highest. A properly engineered annuity forward can reconcile the reinsurers’ need for such a risk transfer with the capital market investors’ request for attractive return, standardization, transparency, and manageable contract duration.

7 | SENSITIVITY ANALYSIS

In this section, we analyze the sensitivity of our results and conclusions with respect to our modeling assumptions by altering selected parameters. Since the resulting effects can essentially be observed in any of the three market stages, we focus on the saturated stage to keep this section concise.

7.1 | Reinsurer’s diversification opportunities with mortality business

To further illustrate the impact of the reinsurer’s diversification opportunities with mortality business, we set the volume of the reinsurer’s existing and future mortality business to zero, that is, $F^M := 0$. Figure 5 shows the resulting hedge effectiveness against the capital efficiency of the considered hedging instruments from the perspective of the primary hedger in the

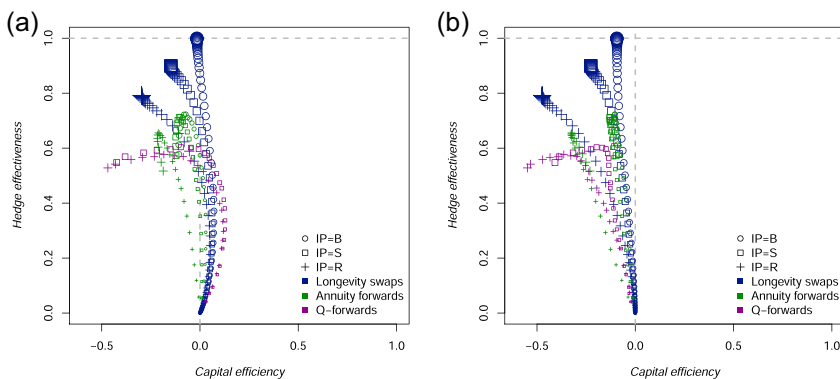


FIGURE 5 Hedge effectiveness (y-axis) versus capital efficiency (x-axis) for the primary hedger in the saturated stage of the market when the reinsurer benefits from diversification opportunities with mortality risk (left panel) or not (right panel). The symbol size increases with contract duration. (a) With mortality business (base case) and (b) without mortality business. IP, index population.

saturated stage of the market (right panel) compared with the case with mortality business (left panel).

First, we observe that the exclusion of the mortality business leads to a general decline in capital efficiency and consequently a shift to the left for all instruments in Figure 5. The decrease in diversification capacity leads to significantly higher prices for reinsurance-based transactions which in turn adversely affects the capital efficiency and the economic attractiveness of longevity hedging for primary hedgers. Hence, diversification opportunities with mortality business significantly contribute to the risk absorption capacity of the reinsurance sector. Without this source of diversification, the capacity limit in the reinsurance sector for longevity risk-taking would be even more constraining.

This decline in capital efficiency is more pronounced for instruments that are linked to younger-age mortality than for those that also transfer longevity risk associated with older-age mortality. For instance, the capital efficiency of an unlimited longevity swap declines by 7.8 percentage points, whereas it declines by 11 percentage points for its counterpart with only 30 years of coverage. Overall, we find that the left-bending curvature in the frontier of “efficient” instruments basically disappears when excluding the reinsurer’s mortality exposure. While capital efficient shorter-term hedges were still available in the base case, this does not hold anymore in the case without mortality diversification opportunities. Hence, our conclusion in Section 5 that the market will first become short of risk absorption capacity for older ages can be traced back to the assumption that the reinsurance sector can exploit significant diversification effects with mortality risk. Without this LOB, the supply shortage would be independent of the covered age range.

Regarding the second link of the longevity risk transfer chain, Figure 6 shows the maximum Sharpe ratios that an investor can earn with the considered index-based instruments in the base case (left panel) compared with the case without mortality business (right panel). The absence of diversification opportunities with mortality risk diminishes the competitiveness of the reinsurance sector and therefore leads to rising Sharpe ratios for longevity risk-taking. Again, the difference to the base case is most pronounced for those instruments that solely transfer longevity risk that is associated with younger ages, that is, short-term longevity swaps and q -forwards, due to the strong interdependencies with the reinsurer’s mortality business.

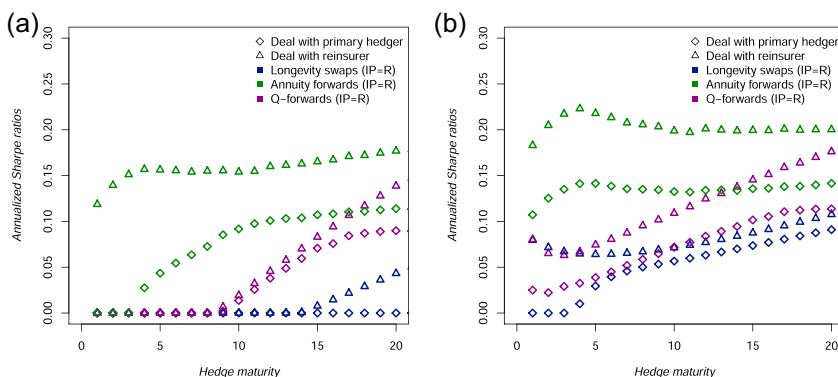


FIGURE 6 Maximal Sharpe ratios that can be earned with different index-based ($IP = \mathcal{R}$) instruments (y-axis) depending on their time to maturity (x-axis), with (left panel) and without (right panel) the reinsurer’s diversification opportunities with mortality business. (a) With mortality business (base case) and (b) without mortality business. IP, index population.

Nevertheless, our main conclusions from Section 6 regarding the involvement of capital market investors still hold true when leaving aside the reinsurer's mortality business: It is still more beneficial for investors to engage in transactions with reinsurers rather than directly with primary hedgers since this gives the potential for higher Sharpe ratios. For the purpose of passing on longevity risk to the capital markets, short-term index-based annuity forwards still appear to be structurally most suitable.

7.2 | Reinsurer's target return on equity rate

Now, we analyze the impact of the reinsurer's anticipated target return on equity rate *roe*. Figure 7 shows the hedge effectiveness against the capital efficiency of the considered instruments from the primary hedger's perspective when setting the reinsurer's target return on equity rate to 6% (left panel) or to 12% (right panel) compared with the base case of 9% (middle panel).

Naturally, a higher anticipated target return on equity causes a rise in reinsurance prices and consequently adversely affects the capital efficiency of longevity hedging for primary hedgers, which in particular means that the market reaches the stage at which reinsurance transactions are no longer attractive (and capital market investors should enter the market) sooner than in the base case. Again, this effect is most pronounced for those instruments that transfer long-term longevity risk associated with old-age mortality for reasons we discussed in Section 7.1. Hence, we conclude that a higher target return on equity further limits the overall risk absorption capacity in the reinsurance sector and therefore accelerates the market development process which we outlined towards the end of Section 5.

This is further confirmed by Figure 8 that shows the maximum Sharpe ratios that the investor can earn with different index-based instruments for the varying return on equity assumptions. An increase in the reinsurer's anticipated target return on equity implies that capital market investors can earn higher Sharpe ratios. The general patterns are very similar though in the three figures which means that our conclusions in Section 6 do not depend on the reinsurer's target return on equity rate.

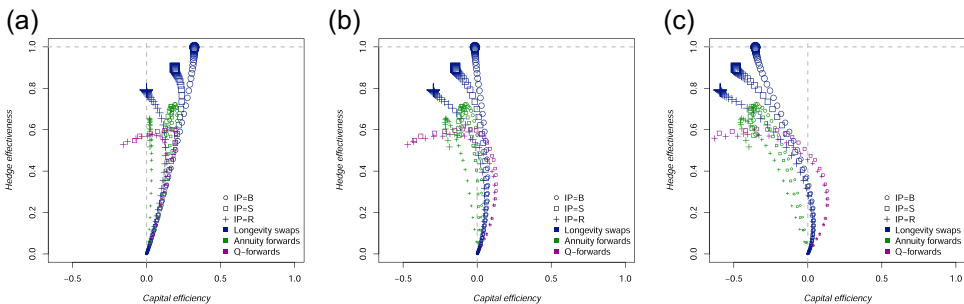


FIGURE 7 Hedge effectiveness (y-axis) versus capital efficiency (x-axis) for the primary hedger in the saturated stage of the market for different choices for the target return on equity rate. The symbol size increases with contract duration. (a) *roe* = 6%, (b) *roe* = 9% (base case), and (c) *roe* = 12%. IP, index population.

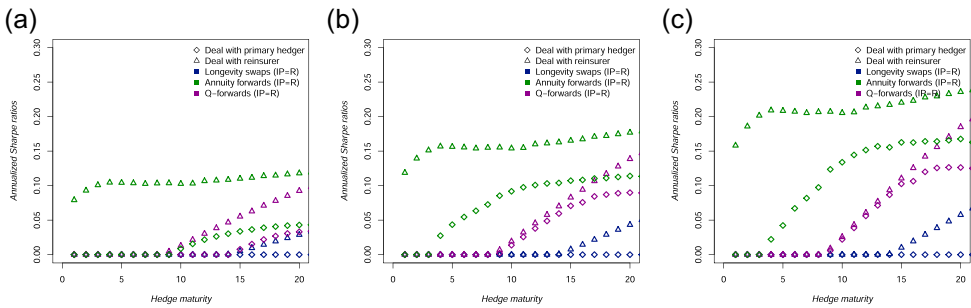


FIGURE 8 Maximal Sharpe ratios that can be earned with different index-based ($IP = \mathcal{R}$) instruments (y-axis) of different terms (x-axis) in the saturated stage of the market for different choices for the reinsurer's target return on equity. (a) $roe = 6\%$, (b) $roe = 9\%$ (base case), and (c) $roe = 12\%$. IP, index population.

8 | CONCLUSION

In this paper, we develop and implement a model of a longevity risk transfer market that consists of different market participants with individual characteristics and (partly opposing) objectives. Unlike previous studies that are mostly limited to transactions between two parties and typically leave the hedge provider unspecified, we particularly focus on the reinsurance sector with its diversification capacities. Depending on the reinsurers' risk profile and the prevailing market environment, they can act as either takers or hedgers of longevity risk.

Our numerical case study provides several novel and valuable insights into the dynamics of the longevity risk transfer market. At an early stage of market development, we demonstrate that comprehensive diversification opportunities with other LOBs, in particular with mortality risk, give reinsurers with little or no longevity risk exposure a decisive competitive edge. This does not leave room for capital market investors to earn a positive risk premium. These findings are in line with the market-dominating position of reinsurers that has been observed in the global longevity risk transfer market since its inception, compare Blake et al. (2019). With increasing market saturation, however, this competitive advantage lessens due to shrinking marginal diversification benefits. This constitutes a remarkable antimonopolistic feature of this market. In particular, it suggests that the market will tend to expand to a wider set of competing risk takers rather than develop towards a "winner-takes-it-all" market. We demonstrate that further longevity risk transfers into the reinsurance sector will ultimately lead to a critical level of saturation beyond which the prices for reinsurance-based longevity hedges become economically unattractive to primary hedgers. This provides clear evidence for a capacity constraint for longevity risk-taking in the reinsurance sector, which has been postulated by several authors, including Blake et al. (2019) and Kessler (2021).

The competitiveness of established reinsurers declines with increasing market saturation. This leads to rising market risk premiums for longevity risk-taking, which might attract capital market investors. We show that their optimal entry point is to assume systematic longevity risk from the reinsurance sector rather than taking on longevity risk directly from annuity providers. This is universally beneficial for all market participants. Our findings particularly support the concept of a longevity risk transfer chain as developed and further refined by Blake et al. (2006, 2019), Cairns et al. (2008), and Cairns and El Boukfaoui (2021): A reinsurer acts as intermediary between first writers of longevity risk and the capital markets and first enters into

a series of unlimited customized longevity swaps with a diverse set of primary hedgers. Subsequently, the most capital-intensive components of the acquired risks are further passed on to the capital markets by means of a suitable index-based hedge, whereas the more easily diversifiable risk components are retained.

Finally, we analyze the suitability of various hedge designs for each link of this risk transfer chain. While customized cash flow hedges turn out to be superior for transferring longevity risk from primary hedgers to the reinsurance sector, we find that value hedges are better suited for linking the reinsurance sector with the capital market. In particular, we demonstrate that index-based annuity forwards, even with rather short terms below 10 years, offer distinct structural advantages over cash flow hedges. These advantages become increasingly pronounced with progressing market saturation. With respect to the shown capacity constraint in the reinsurance sector, these findings are of particular practical relevance for efficiently expanding the market's risk absorption capacity. We show that index-based annuity forwards are particularly suited to reconcile longevity hedgers' demand for a cost-efficient and effective risk transfer with the capital market investors' request for attractive returns, standardization, and manageable contract durations. Such contract designs and any advanced forms or derivatives thereof clearly deserve greater attention in future works.

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APPENDIX A: MODEL SPECIFICATION

A.1 | Specification of the stochastic mortality model

Since our mortality model is presented in detail in Freimann (2021), we limit our descriptions in this section to those model features and technical details that are required for specifying the remaining components of our modeling framework. For further details, in particular on the model calibration and the stochastic projection of the period effects, we refer the reader to the aforementioned paper and the references cited therein.

A.1.1 | Multipopulation model structure

In our multipopulation extension of the Cairns–Blake–Dowd model, the subpopulations (denoted by $^{[p]}$) and the reference population (denoted by $^{[R]}$) are linked through the following relation:

$$q_{x,t}^{[p]} = \phi\left(q_{x,t}^{[R]}, \xi_{x,t}^{[p]}\right) := \text{logit}^{-1}\left(\text{logit}\left(q_{x,t}^{[R]}\right) + \xi_{x,t}^{[p]}\right), \quad p \in \{1, \dots, N_{Sub}\}. \tag{A1}$$

Here, $\xi_{x,t}^{[p]} := \kappa_t^{(1)[p]} + (x - \bar{x})\kappa_t^{(2)[p]}$ captures deviations between subpopulation-specific mortality and general population mortality. Throughout the paper, we rely on a straightforward logit-extrapolation for the oldest ages up to a limiting age of $\omega := 130$ years. To avoid unrealistic crossovers of subpopulation-specific and general population mortality at the oldest ages, we set the mortality rates of the subpopulations equal to those of the reference population as soon as such a crossover would occur, see Freimann (2021) for details.

A.1.2 | Pathwise derivation of best estimate mortality in the EMT valuation model

Given the observed mortality evolution up to time T , best estimate future mortality is derived at time T by following a central deterministic projection based on a reasonable time- T estimate for the prevailing trend in mortality improvements, the so-called EMT. Specifically for the reference population, this results in the following best estimate mortality rates:

$$\begin{aligned} \tilde{q}_{x,t}^{[R]}(T) &:= \text{logit}^{-1}\left(\tilde{\kappa}_T^{(1)[R]} + (t - T)EMT_T^{(1)[R]} + (x - \bar{x})\left(\tilde{\kappa}_T^{(2)[R]} + (t - T)EMT_T^{(2)[R]}\right)\right), \\ &t > T, \end{aligned} \tag{A2}$$

where the prevailing level $\tilde{\kappa}_T^{(i)[R]}$, $i = 1, 2$ and trend $EMT_T^{(i)[R]}$, $i = 1, 2$ in mortality are derived at time T by applying a suitable weighted linear regression on the most recent data points.

Analogously, the time- T best estimate mortality in the subpopulations is derived as

$$\tilde{q}_{x,t}^{[p]}(T) := \phi\left(\tilde{q}_{x,t}^{[R]}(T), \tilde{\xi}_{x,t}^{[p]}(T)\right), \quad t > T \tag{A3}$$

by using a time- T best estimate for the experience ratios of

$$\tilde{\xi}_{x,t}^{[p]}(T) := \tilde{\kappa}_T^{(1)[p]} + (t - T)\tilde{\nu}_T^{(1)[p]} + (x - \bar{x})\left(\tilde{\kappa}_T^{(2)[p]} + (t - T)\tilde{\nu}_T^{(2)[p]}\right), \quad t > T. \tag{A4}$$

The prevailing level $\tilde{\kappa}_T^{(i)[p]}$, $i = 1, 2$ and trend $\tilde{\nu}_T^{(i)[p]}$, $i = 1, 2$ in subpopulation-specific mortality differences are also derived at time T by means of a weighted linear regression on the most recent data points. Following Freimann (2021), all these weighted linear regressions are based on exponentially decaying weights going backwards in time.

A.2 | Parametrization of market participants

This appendix specifies the primary hedger's liabilities and the reinsurer's initial business mix and provides the technical details on the computation of SCRs and economic capital.

A.2.1 | Primary hedger

Liabilities and the effect of hedging

The time- t random present value of the primary hedger's unhedged liabilities is given by

$$L(t) := \sum_{p=1}^{N_{Sub}} \sum_{s>t} (1+r)^{-(s-t)} B_{x_R+s,s}^{[p]}, \quad t \geq 0, \quad (A5)$$

where $B_{x_R+s,s}^{[p]}$ denotes the number of surviving annuitants aged $x_R + s$ at time s that are assigned to subpopulation $p \in \{1, \dots, N_{Sub}\}$. The initial numbers of annuitants at the start of the simulation are given by $B_{x_R,0}^{[p]} := \eta_p N_{Book} \in \mathbb{N}$ and projected by iteratively drawing numbers of deaths from suitable binomial distributions (conditional on realized survivors and mortality).

The time- t best estimate of all future liabilities is derived with the EMT valuation model based on the actual number of survivors at time t and the then prevailing best estimate survival rates, that is,

$$BEL(t) := \sum_{p=1}^{N_{Sub}} B_{x_R+t,t}^{[p]} \sum_{s>t} (1+r)^{-(s-t)} \prod_{u=t}^{s-1} (1 - \tilde{q}_{x_R+u,u+1}^{[p]}(t)), \quad t \geq 0. \quad (A6)$$

By additionally considering the hedge cash flows, the time- t random present value of all future hedged liabilities and its time- t best estimate, respectively, are analogously given by

$$L(t) - H(t), \quad BEL(t) - BEH(t), \quad t \geq 0, \quad (A7)$$

where the hedge contract typically provides offsetting payments that reduce the variability. The time- t best estimate value of the hedge $BEH(t)$ is also derived based on a deterministic best estimate mortality projection under the EMT valuation model.¹⁹

Computation of longevity SCRs

Following Börger (2010), Börger, Freimann et al. (2021), and Börger, Schupp et al. (2021), the SCR for longevity risk in year t is derived as the 99.5% VaR of the change in best estimate liabilities between t and $t + 1$, that is,

¹⁹This provides a reasonable approach for the considered hedging instruments due to their symmetric forward-type payout structure.

$$SCR(t) := VaR_{99.5\%} \left(\frac{BEL(t + 1) + CF(t + 1)}{1 + r} \right) - BEL(t), \tag{A8}$$

where $CF(t + 1)$ denotes the annuity payments to surviving beneficiaries that are due at time $t + 1$.

With a hedge in place, the SCR is denoted as $SCR_H(t)$ and calculated based on the hedged liabilities, that is, as the 99.5% VaR of

$$\frac{BEL(t + 1) + CF(t + 1) - (BEH(t + 1) + CF_H(t + 1))}{1 + r} - (BEL(t) - BEH(t)), \tag{A9}$$

where $CF_H(t + 1)$ contains the offsetting hedge payouts that occur between t and $t + 1$. On the basis of the regulatory cost-of-capital rate r_{CoC} , the associated cost of capital for the liabilities (and its hedged counterpart, respectively) is defined as

$$CoC := \sum_{t \geq 0} \frac{r_{CoC} SCR(t)}{(1 + r)^{t+1}}, \quad CoC_H := \sum_{t \geq 0} \frac{r_{CoC} SCR_H(t)}{(1 + r)^{t+1}}, \tag{A10}$$

which also represent random variables due to stochastic future SCRs.

A.2.2 | Reinsurer

Specification and parametrization of the three LOBs

The reinsurer’s three different LOBs are parametrized as follows:

- *Longevity* (\mathcal{L}): The reinsurer is liable for the longevity risk that arises from a portfolio of immediate life annuities that consists of various cohorts. Similarly to the primary hedger, the loss arising from the longevity business in year t is defined as

$$L^\mathcal{L}(t) := \frac{BEL^\mathcal{L}(t + 1) + CF^\mathcal{L}(t + 1)}{1 + r} - BEL^\mathcal{L}(t), \tag{A11}$$

where the time- t best estimate value of all future liabilities is given by

$$BEL^\mathcal{L}(t) := \sum_{p=1}^{N_{Sub}} \sum_{x=x_R}^{\omega} B_{x,t}^{\mathcal{L}[p]} \sum_{s>t} (1 + r)^{-(s-t)} \prod_{u=t}^{s-1} \left(1 - \tilde{q}_{x+u-t, u+1}^{[p]}(t) \right), \tag{A12}$$

which includes all reinsured cohorts of age $x \in [x_R, \omega]$. At the start of the simulation, the initial number of x -year old annuitants from subpopulation $p \in \{1, \dots, N_{Sub}\}$ is given by $B_{x,0}^{\mathcal{L}[p]} := \alpha_x^\mathcal{L} \eta_p F^\mathcal{L} \in \mathbb{N}, x \geq x_R$. The age profile $\alpha_x^\mathcal{L}$ satisfies $\sum_{x \geq x_R} \alpha_x^\mathcal{L} = 1$ and is assumed to be proportional to the best estimate $(x - x_R)$ -year survival rates at time zero. At the beginning of each future year $t > 0$, $B_{x_R,t}^{\mathcal{L}[p]} := \alpha_{x_R}^\mathcal{L} \eta_p F^\mathcal{L}$ new annuitants of age x_R from subpopulation $p \in \{1, \dots, N_{Sub}\}$ enter into the portfolio.

- *Mortality* (\mathcal{M}): The reinsurer’s exposure to mortality risk arises from a portfolio of term life insurance policies with starting age x_M that mature at the retirement age x_R . Following the same principle as for the annuities, the loss in year t from the portfolio of reinsured term life insurance policies reads as

$$L^{\mathcal{M}}(t) := \frac{BEL^{\mathcal{M}}(t+1) + CF^{\mathcal{M}}(t+1)}{1+r} - BEL^{\mathcal{M}}(t) \quad (\text{A13})$$

based on the time- t best estimate liabilities

$$BEL^{\mathcal{M}}(t) := \sum_{p=1}^{N_{Sub}} \sum_{x=x_M}^{x_R-1} B_{x,t}^{\mathcal{M}[p]} \sum_{s=t+1}^{t+x_R-x} (1+r)^{-(s-t)} \prod_{u=t}^{s-1} \left(1 - \tilde{q}_{x+u-t, u+1}^{[p]}(t)\right) \tilde{q}_{x+s-t, s+1}^{[p]}(t). \quad (\text{A14})$$

Here, $B_{x,t}^{\mathcal{M}[p]}$ denotes the number of policyholders aged x at time t . The initial numbers of policyholders at time zero are given by $B_{x,0}^{\mathcal{M}[p]} := \alpha_x^{\mathcal{M}} \eta_p F^{\mathcal{M}}$, $x_M \leq x \leq x_R - 1$. The age profile again satisfies $\sum_x \alpha_x^{\mathcal{M}} = 1$ and is also assumed to be proportional to the $(x - x_R)$ -year best estimate survival rates. At the beginning of each future year $t > 0$, $B_{x_M,t}^{\mathcal{M}[p]} := \alpha_{x_M}^{\mathcal{M}} \eta_p F^{\mathcal{M}}$ new x_M -year old policyholders from subpopulation $p \in \{1, \dots, N_{Sub}\}$ enter into the portfolio.

- *Other* (\mathcal{O}): Following Asimit et al. (2016), we assume other nonbiometric risks $CF^{\mathcal{O}}(t)$ to be lognormally distributed with mean F_0 and given $CoV^{\mathcal{O}}$. Note that, when written in terms of the common parametrization of the lognormal distribution, this corresponds to a $\mathcal{LN}(\log(F^{\mathcal{O}}) - 0.5\sigma^2, \sigma^2)$ -distributed random variable, where $\sigma := \sqrt{\log(1 + CoV^{\mathcal{O}2})}$. The corresponding loss in year t is then given by $L^{\mathcal{O}}(t) := CF^{\mathcal{O}}(t) - \mathbb{E}(CF^{\mathcal{O}}(t))$.

Euler capital allocation

Under some technical differentiability assumptions on a positive homogeneous risk measure²⁰ ρ , Euler's theorem yields the following additive decomposition of the portfolio-wide economic capital in year t (cf. Rosen & Saunders, 2010):

$$\begin{aligned} EC(t) &:= \rho(L^{\mathcal{L}}(t) + L^{\mathcal{M}}(t) + L^{\mathcal{O}}(t)) \\ &= \sum_{\mathcal{LOB} \in \{\mathcal{L}, \mathcal{M}, \mathcal{O}\}} a_{\mathcal{LOB}} \frac{\partial \rho(a_{\mathcal{L}}L^{\mathcal{L}}(t) + a_{\mathcal{M}}L^{\mathcal{M}}(t) + a_{\mathcal{O}}L^{\mathcal{O}}(t))}{\partial a_{\mathcal{LOB}}} \Bigg|_{a_{\mathcal{L}}=a_{\mathcal{M}}=a_{\mathcal{O}}=1} \\ &:= \sum_{\mathcal{LOB} \in \{\mathcal{L}, \mathcal{M}, \mathcal{O}\}} EC^{\mathcal{LOB}}(t). \end{aligned} \quad (\text{A15})$$

We consider $\rho := VaR_{99.5\%}$ and in numerical computations, we approximate the marginal risk contributions $EC^{\mathcal{LOB}}(t)$, that represent the economic capital allocated to $\mathcal{LOB} \in \{\mathcal{L}, \mathcal{M}, \mathcal{O}\}$ in year t , via a common finite difference approach (cf. Asmussen & Glynn, 2007), that is,

$$EC^{\mathcal{LOB}}(t) \approx \frac{\rho(L^{\mathcal{L}}(t) + L^{\mathcal{M}}(t) + L^{\mathcal{O}}(t) + \varepsilon L^{\mathcal{LOB}}(t)) - \rho(L^{\mathcal{L}}(t) + L^{\mathcal{M}}(t) + L^{\mathcal{O}}(t) - \varepsilon L^{\mathcal{LOB}}(t))}{2\varepsilon} \quad (\text{A16})$$

²⁰A risk measure ρ is called positive homogeneous if $\rho(\lambda L) = \lambda \rho(L)$ holds for all $\lambda \geq 0$.

with sufficiently small $\epsilon := 0.01$.

For pricing under the RORAC criterion, the marginal economic capital that is allocated to the longevity transaction H is derived analogously as²¹

$$\begin{aligned}
 EC_H(t) &:= \frac{\partial \rho(a_C L^L(t) + a_M L^M(t) + a_O L^O(t) + a_H L_H(t))}{\partial a_H} \Bigg|_{a_C=a_M=a_O=a_H=1} \\
 &\quad \rho(L^L(t) + L^M(t) + L^O(t) + (1 + \epsilon)L_H(t)) \\
 &\approx \frac{-\rho(L^L(t) + L^M(t) + L^O(t) + (1 - \epsilon)L_H(t))}{2\epsilon},
 \end{aligned} \tag{A17}$$

where

$$L_H(t) := \frac{BEH(t + 1) + CF_H(t + 1)}{1 + r} - BEH(t) \tag{A18}$$

denotes the loss that may arise from the hedge contract (with the primary hedger in the one direction or with the capital market investor, respectively, in the other direction) in year t .

A.3 | Hedging instruments

In this section, we provide the technical details on the hedging instruments. Following Börger, Freimann et al. (2021), we construct the index-based instruments by replacing customized quantities by suitable indices that exclusively use the mortality information from the relevant IP. Hence, any change in (estimated) mortality that occurs during the term of the contract but does not originate from the underlying IP is no longer captured.

A.3.1 | Mortality indices and q -forwards

For each subpopulation $p \in \{1, \dots, N_{Sub}\}$, we define the following mortality indices:

$$q_{x,t}^{[IP(p)]} := \begin{cases} q_{x,t}^{[p]}, & IP = \mathcal{S}, \\ \phi\left(q_{x,t}^{[R]}, \xi_{x,t}^{[p]}(0)\right), & IP = \mathcal{R}. \end{cases} \tag{A19}$$

For $IP = \mathcal{R}$, the mortality rates are adjusted by the initial experience ratios to account for anticipated mortality differences between the hedger's portfolio population and the reference population. Based upon this, the floating lag of the q -forward contract is defined as

$$Q_{x_R+\tau,\tau}^{[IP]} := \sum_{p=1}^{N_{Sub}} n_{\tau}^{[p]} q_{x_R+\tau,\tau}^{[IP(p)]}, \quad IP \in \{\mathcal{S}, \mathcal{R}\}, \tag{A20}$$

²¹Alternatively, one could consider the *incremental* risk contribution of the hedge contract $\rho(L^L(t) + L^M(t) + L^O(t) + L_H(t)) - \rho(L^L(t) + L^M(t) + L^O(t))$. However, as pointed out by Rosen and Saunders (2010), this can be considered as a common finite difference approximation of the marginal contribution if the additional loss from the hedge contract is of infinitesimal size compared with the other terms, which is typically the case in our application.

where $n_\tau^{[p]}$ represents the number of q -forwards for each subpopulation $p \in \{1, \dots, N_{Sub}\}$. For a given term τ , the optimal number of q -forwards for the primary hedger is derived numerically by solving the following objective function:

$$\operatorname{argmin}_{n_\tau^{[p]}} \rho \left(L_H^{[p]}(0) - \mathbb{E} \left(L_H^{[p]}(0) \right) \right), \quad p \in \{1, \dots, N_{Sub}\}, \quad (\text{A21})$$

where $L_H^{[p]}(0) := L^{[p]}(0) - (1+r)^{-\tau} n_\tau^{[p]} \left(\mathbb{E} \left(q_{x_R+\tau, \tau}^{[IP(p)]} \right) - q_{x_R+\tau, \tau}^{[IP(p)]} \right)$. For comparability, we use the same number of q -forwards for $IP = \mathcal{S}$ and \mathcal{R} , compare Börger, Freimann, et al. (2021).

A.3.2 | Survivor indices and longevity swaps

The fully customized longevity swaps are based on the actual number of living policyholders

$$S_{x_R+t, t}^{[B]} := \sum_{p=1}^{N_{Sub}} B_{x_R+t, t}^{[p]} \quad (\text{A22})$$

at time t . The corresponding survivor indices for $IP = \mathcal{S}, \mathcal{R}$ are constructed through the best estimate survival rates that are implied by the previously introduced mortality indices, that is,

$$S_{x_R+t, t}^{[IP]} := \sum_{p=1}^{N_{Sub}} B_{x_R, 0}^{[p]} \prod_{s=0}^{t-1} \left(1 - q_{x_R+s, s+1}^{[IP(p)]} \right), \quad IP = \mathcal{S}, \mathcal{R}. \quad (\text{A23})$$

A.3.3 | Liability indices and annuity forwards

The floating lag of the fully customized annuity forward corresponds to the actual time- τ best estimate of the primary hedger's liabilities, that is,

$$LF^{[B]}(\tau) := BEL(\tau) = \sum_{p=1}^{N_{Sub}} B_{x_R+\tau, \tau}^{[p]} \sum_{s>\tau} (1+r)^{-(s-\tau)} \prod_{u=\tau}^{s-1} \left(1 - \tilde{q}_{x_R+u, u+1}^{[p]}(\tau) \right), \quad (\text{A24})$$

which is derived based on the actual number of living policyholders at time τ and the then prevailing best estimate mortality rates for the subpopulations. For the index-based designs, the actual number of survivors is replaced by the previously introduced survivor indices and the best estimate mortality assumptions are restricted to the underlying IP, that is,

$$LF^{[IP]}(\tau) := \sum_{p=1}^{N_{Sub}} S_{x_R+\tau, \tau}^{[IP(p)]} \sum_{s>\tau} (1+r)^{-(s-\tau)} \prod_{u=\tau}^{s-1} \left(1 - \tilde{q}_{x_R+u, u+1}^{[IP(p)]}(\tau) \right), \quad IP \in \{\mathcal{S}, \mathcal{R}\}. \quad (\text{A25})$$